```
> restart:with(linalg):with(difforms);with(liesymm):
> setup(x,y,z,t):defform(x=0,y=0,z=0,t=0,UU=0,VV=0,f=0,a=const,b=const,c=const,k=const,
mu=const,m=const,alpha=const,beta=const,n=const,omega=const,kappa=const,epsilon=const,
pi=const,p=const,e=const,N=const,H=const,Az=0,phi=0,Ax=0,Ay=0,Gamma=const,Omega=const,
gamma=const,q=const,g=const,sigma=const);
```

$$[\&\hat{}, d, defform, formpart, parity, scalarpart, simpform, wdegree] \quad (1)$$

Correlations, and closed Constitutive Currents

Maple Name: Constitutive Currents.mws

R. M. Kiehn

USES HOLDER NORMS, CONSTITUTIVE CURRENTS from D =epsilon E and B = Mu H in 4D

R. M. Kiehn

from maxwell/mws and maxwellplasma.mws and MapleEM.mws
Updated 12/12/97, 11/5/98, 10/24/2002 Correcting sign of T4 and d(F), 11/09/2003, Nov 22 2008

Last update: Dec 16, 2011,

Overview

On a variety of independent variables (x,y,z,t), consider a 4 component 1-form, Ao, with coefficients Ak = Ax,Ay,Az,phi.
Define the HolderNormN, of signature (a,b,c,e) degree p and homogeneity index N, as the function:

$$HolderNormN := \left(a\,Ax(x,y,z,t)^p + b\,Ay(x,y,z,t)^p + c\,Az(x,y,z,t)^p + e\,\phi(x,y,z,t)^p\right)^{\frac{N}{p}}$$

Define a scaled 1-form AN by dividing the 1-form Ao by the HolderNorm with index N.
AN = Ao/HolderNormN
also define AH=Ao/HolderNormH
Define the Jacobian matrix [**J**(AN)] of the Coefficents of of the scaled Action 1-form AN
Define the Adjoint matrix [**ADJ**(AN)] as the matrix of cofactors transposed of the Jacobian matrix computed from [**J**(AN)]
Define the Adjoint Current as the vector equal |**C**> to the product of the Adoint matrix times the vector AH (not necessarily AN).
|**C**> = [**ADJ**(AN)]|AH>

If 4 - 3N - H = 0, then |C> => |J> AND divergence |J> =0. (A conserved adjoint current !!!)

IF N = 1, H=1, and a=b=c=e=1, p=2, then the JAcobian matrix has zero determinant , and the similarity invariants of the JAcobian matrix can be related to the curvatures of the associated implicit surface.

The Adjoint current method will be demonstrate in another pdf file.

The 1-form of Action potentials

$$A = \Sigma_{k=1}^{3} A_k(x,y,z,t)dx^k - \phi(x,y,z,t)dt = \mathbf{A}\circ d\mathbf{r} - \phi dt.$$

The Engineering vector format of the field intensities

$$\mathbf{E} = -\partial\mathbf{A}/\partial t - grad\phi, \qquad \mathbf{B} = curl\ \mathbf{A} \equiv \partial A_k/\partial x^j - \partial A_j/\partial x^k.$$

The 2-form of Field intensities

$$F = dA = \{\partial A_k/\partial x^j - \partial A_j/\partial x^k\}dx^j\hat{}dx^k = F_{jk}dx^j\hat{}dx^k$$
$$= \mathbf{B}_z dx\hat{}dy + \mathbf{B}_x dy\hat{}dz + \mathbf{B}_y dz\hat{}dx + \mathbf{E}_x dx\hat{}dt + \mathbf{E}_y dy\hat{}dt + \mathbf{E}_z dz\hat{}dt$$

The Topological Torsion vector, T4,
The 3-form of Topological Torsion (note the minus sign)
and the 4-form of Topological Parity.

$$\begin{aligned}
\mathbf{T}_4 &= -[\mathbf{E}\times\mathbf{A} + \mathbf{B}\phi,\ \mathbf{A}\circ\mathbf{B}],\\
A\hat{}dA &= i(\mathbf{T}_4)\Omega_4,\\
&= T_4^x dy\hat{}dz\hat{}dt - T_4^y dx\hat{}dz\hat{}dt\\
&\quad + T_4^z dx\hat{}dy\hat{}dt - T_4^t dx\hat{}dy\hat{}dz,\\
dA\hat{}dA &= 2(\mathbf{E}\circ\mathbf{B})\ \Omega_4 = K\Omega_4,\\
&= \{\partial T_4^x/\partial x + \partial T_4^y/\partial y + \partial T_4^z/\partial z + \partial T_4^t/\partial t\}\ \Omega_4.
\end{aligned}$$

Some additional formulas (note the signs)

$$\textbf{The Work 1-form:}\qquad W = i(\rho\mathbf{V}_4)dA = i(J)F,$$
$$\Rightarrow\quad -\{\rho\mathbf{E} + \mathbf{J}\times\mathbf{B}\}\circ d\mathbf{r} + \{\mathbf{J}\circ\mathbf{E}\}dt.$$
$$\textbf{The Lorentz force}\quad :\quad -\{\mathbf{f}_{Lorentz}\}\circ d\mathbf{r}\ \textbf{component.}$$
$$\textbf{The dissipative power}\quad :\quad +\{\mathbf{J}\circ\mathbf{E}\}dt\ \textbf{component.}$$

Properties of the Topological Torsion vector T_4

$$\begin{aligned}
i(\mathbf{T}_4)\Omega_4 &= A\hat{}dA,\\
W &= i(\mathbf{T}_4)dA = \sigma\ A,\\
U &= i(\mathbf{T}_4)A = 0,\\
L_{(\mathbf{T}_4)}A &= \sigma\ A,\\
Q\hat{}dQ &= L_{(\mathbf{T}_4)}A\hat{}L_{(\mathbf{T}_4)}dA = \sigma^2 A\hat{}dA \neq 0,\\
dA\hat{}dA &= (2!)\ \sigma\ \Omega_4.
\end{aligned}$$

NOTES:

The fundamental references are my monographs Vol1 and Vol4, which can be found at http://www.lulu.com/kiehn
*
This Maple program computes Maxwell-Faraday formulas from the postulate of potentials F-dA=0.
Given a 1-form of Action on 4D space time, the E and B fields follow by exterior differentiation.
The 2-form F is the set of limit points for the 1-form, A.
The Maxwell Ampere equations are computed from the postulate of charge currents, J-dG=0.
The 2-form density, G, with components D and H, is constructed in several ways

1. The most simple assumption selects the Lorentz-Minkowski vacuum constitutive equations, D = epsilon E B = mu H.
2. A more complicated procedure selects the complex 6x6 constitutive matrix formulated by Post (see Vol 4)
3. Another procedure selects a chiral formulation for a the constitutive matrix. (see Vol4).
   The chiral additions to the Lorentz involve adding terms alpha*(g+I*gamma)*sqrt(mu/epsion) The factor alpha is more than likely equal to 1/(2*fine structure constant), which makes the effective chiral impedance the Hall Impedance ~ 137/2 Free Space Impedance.

*
The 1-form of Action not only encodes the electromagnetic potentials,
but also topologically encodes a thermodynamic system. (see Vol1).
The Potentials, not the charge current densities, are used as the computational starting point,
with functions defined on a basis variety if 4 dimensions (x,y,z,t).
This topological approach is more useful for the constuction of field, not necessarily particle, properties
of Plasma systems, where charge currents can be associated with collective states, not individual particles
*
The program also permits the study of homogeneous systems of various degrees, through the use of
Holder Norm divisors.
*
The procedure starts with the functional input the 4 potential, and computes
E,B, then D,H using a constitutive map, ,Jamperian,Jdisplacement,Jtotal, and the Charge density, rho,
as well as
the Torsion vector = -[ExA+Bphi, AdotB]
the Spin Vector = A x H + Dphi, AdotD,
the First Poincare invariant = F^G -A^J = (BdotH-DdotE) -(AdotJ-rho.phi),
the second Poincare Invariant = F^F = +2EdotB,
the Lorentz Force = - rho E +J x B,
the dissipation = JdotE,
the Poynting vector ExH,
the Topological Torsion, A^F
the Topological Spin A^G,
the 4D interaction energy density, AdotJ - rho phi,
the Work 1-form W = i(J,rho) F,
the internal energy U = AdotJ - rho phi
the Heat 1-form Q = W + dU
The program checks to see if Q^dQ is zero (hence {J,rho} is a reversible process) or not zero (hence {J,rho} is an irreversible process.
and the similarity invariants of the correlation Jacobian matrix computed from the (possibly scaled and homogeneous) Action 1-form of Potentials.
If A^F = 0, then the thermodynamic system is of PTD 2 or less
If A^F is not zero, but F^F is zero, then the thermodynamic system is a Closed non-equilibrium system that can exchange mass/energy or radiation, but not particles with its environment.
If F^F is not zero, then the thermodynamic system is an Open non-equilibrium system that can exchange particles as well as mass/energy or radiation with its environment
(see vol4)
It is remarkable that a choice of vector and scalar potential functions can lead to charge current 3-forms whose coefficients are proportional to the the vector and scalar potentials. J = chi A which is the form of a London current.
The are also cases where the Topological Spin current has coefficients that are proportional to the coefficients of the Lorentz force.

BE AWARE The algebra can be overpowering. Thank you Maple.

The main procedure

```
JCM:=proc(A1,A2,A3,phi,a,b,c,e,p,N,H,CH,sigma)
     local BFC,TFC,EF1,EF2,EF3,JAC,JDC,SFC,ExBC,S2:
global A1form,HEL,ExB,NAME,lambdaN,lambdaH,ACTIONN,ACTIONH,Action,Actionem,JACOB,ADJACOB,
ADJOINT,ADJOINTCURR,Xm,Yg,Za,Tk,EdotB,E,EXA,A,AA,BB,Ea,Eg,B,F,Fem,Torsion3_form,
Torsion3_formem,Parity4_form,Q,dQ,QdQ,dQdQ,QdQ4,DETJACOB,T3,T4,U,Uch,Uadj,JXB,JXBch,JXBadj,
AdotT4,DIVT4,DIVADJOINTCURR,ParityFFFF,ParityDIV_T4,Parity2EdotB,AdotJ,AdotJch,AdotJadj,
CHECK,CHECKch,CHECKadj,A4,DFcha,HFcha,JdotE,JdotEch,JdotEadj,J,Jch,Jadj,rho,rho_ch,rho_adj,
rho_E,rho_Ech,rho_Eadj,dWork,dWorkch,dWorkadj,Work_1form,Work_1formch,Work_1formadj,Work,
Workch,Workadj,DF,DFch,DFadj,HF,HFch,HFadj,EXH,EXHch,EXHadj,CD,CDch,CDadj,JA,JAch,JAadj,JTOT,
JTOTch,JTOTadj,DIVJTOT,DIVJTOTch,DIFJTOTadj,JD,JDch,JDadj,SP3,SP3ch,SP3adj,SP4,SP4ch,SP4adj,
dSP4,dSP4ch,dSP4adj,OPT,OPTch,OPTadj,AXH,AXHch,AXHadj,LAGF,LAGFch,LADFadj,PI,PIch,PIadj,LF,
LFcn,LFadj,SPIN3_form,dSPIN3_form,BH,DE,AJ,rhophi,CCB,AJJJ,AAAA,LONFAC,PTD,AxJ,Zfs,Zfsm,Zfse,
LFSPIN,SF,TF:
```

Compute Holder Norms with N and H homogeneity index. Note N and H can both be zero, But if 4 - 3N - H = 0, then the adjoint current is closed, dJ = 0.

```
> lambdaN:=subs(a=a,b=b,c=b,e=e,p=p,(a*A1^p+b*A2^p+c*A3^p+e*(-phi)^p)^(N/p));
> lambdaH:=subs(a=a,b=b,c=c,e=e,p=p,(a*A1^p+b*A2^p+c*A3^p+e*(-phi)^p)^(H/p));
```

Scale Action 1-forms with Holder divisors, compute Jacobian Correlation matrix of scaled Action and its Adjoint
Compute Action 1-form of Potentials,
the 2-form of field intensities F = dA , giving the E and B fields as coefficients
the 3-form of Topological Torison, A^F,
and the 4-form of Topological Parity.
Compare differential form methods with vector methods for Maxwell-Faraday equations.

```
> ACTIONN:=[A1/lambdaN,A2/lambdaN,A3/lambdaN,-phi/lambdaN];
> ACTIONH:=[A1/lambdaH,A2/lambdaH,A3/lambdaH,-phi/lambdaH]:
> JACOB:=jacobian(ACTIONN,[x,y,z,t]):
> ADJOINT:=adjoint(JACOB):
> DETJACOB:=factor(det(JACOB));
> Action:=wcollect(innerprod(ACTIONN,[d(x),d(y),d(z),d(t)]));
> A:=([ACTIONN[1],ACTIONN[2],ACTIONN[3]]);
> A4:=phi/lambdaN;
> BB:=curl(A,[x,y,z]);B:=[factor(BB[1]),factor(BB[2]),factor(BB[3])];
> Ea:=(-diff(A,t));CCB:=simplify(curl(curl(B,[x,y,z]),[x,y,z]));
> Eg:=evalm(grad(-phi/lambdaN,[x,y,z]));
> E:=([factor(Ea[1]+Eg[1]),factor(Ea[2]+Eg[2]),factor(Ea[3]+Eg[3])]);
> EdotB:=factor(E[1]*B[1]+E[2]*B[2]+E[3]*B[3]);
> Actionem:=innerprod(A,[d(x),d(y),d(z)])-phi/lambdaN*d(t);
> F:=wcollect(d(Action)):
> Fem:=E[1]*d(x)&^d(t)+E[2]*d(y)&^d(t)+E[3]*d(z)&^d(t)+B[1]*d(y)&^d(z)+B[2]*d(z)&^d(x)+B
  [3]*d(x)&^d(y);
> Torsion3_form:=wcollect((Action&^F));
> Parity4_form:=factor(F&^F);
```

Compute the Topological Torsion vector T4 using T4 = - [ExA + B phi, AdotB]
Compare the Parity coefficients of the 4-form and vector methods.
Note that A^A^F = 0 so that the 4-potentials are orthogonal to T4.

```
> EXA:=crossprod(E,A);
> T3:=([EXA[1]+B[1]*phi/lambdaN,EXA[2]+B[2]*phi/lambdaN,EXA[3]+B[3]*phi/lambdaN]):
> HEL:=factor(innerprod(A,B));
> T4:=-([factor(T3[1]),factor(T3[2]),factor(T3[3]),HEL]);
> AdotT4:=factor(A[1]*T4[1]+A[2]*T4[2]+A[3]*T4[3]-phi/lambdaN*T4[4]);
> DIVT4:=factor(diverge(T4,[x,y,z,t])):
> ParityDIV_T4:=factor(DIVT4);
> Parity2EdotB:=factor(2*EdotB);
> ParityFFFF:=factor(getcoeff(F&^F));
> ParityDIV_T4:=factor(DIVT4);
> Parity2EdotB:=factor(2*EdotB);
> ParityFFFF:=factor(getcoeff(F&^F));
> Torsion3_formem:=T4[1]&^d(y)&^d(z)&^d(t)-T4[2]&^d(x)&^d(z)&^d(t)+T4[3]&^d(x)&^d(y)&^d
  (t)-T4[4]&^d(x)&^d(y)&^d(z);

> if E = [0,0,0] and B = [0,0,0] then PTD:= 1 else PTD:=2 end if;
> if T4[1]<>0 or T4[2]<>0 or T4[3]<>0 or T4[4]<>0  then PTD:=3 else PTD:= PTD end if;
> if ParityFFFF <> 0 then PTD:=4 else PTD:=PTD end if;
```

Similarity Invariants of the (scaled) Jacobian matrix

```
> Xm:=factor(trace(JACOB));
> S2:=factor(trace(innerprod(JACOB,JACOB))):
> Yg:=factor((-(1/2)*((-trace(JACOB)*trace(JACOB)+S2))));
> Za:=factor((trace(adjoint(JACOB))));
> Tk:=factor(det(JACOB));
```

Print routines for E and B

```
> print(NAME);print(`*********************************    Differential Form Format    ***
  ****************************** `);
> print(`Action 1-form`= Action);
> print(`Intensity 2-form F=dA  `= F);
> print(`Topological Torsion 3-form    A^F` = (Torsion3_form));
> print(`Topological Parity 4-form     F^F` =  (Parity4_form));
> print(`*********************************  Using EM format  ****************************
  *****`);
> print(`E field`= simplify(E));
> print(`B field`= simplify(B));

> print(`Topological TORSION 4 vector   T4 = -[ExA + Bphi,AdotB] `= T4);
> print(`Helicity AdotB`= HEL);
> print(`Poincare II =2(E.B)`= 2*EdotB);
> print(`coefficient of Topological Parity 4-form ` = factor(getcoeff(Fem&^Fem)));
> print(`  Pfaff Topological Dimension     PTD`=PTD);
```

```
> print(`******************** Correlation Similarity Invariants of Jacobian of
  (Ak/lambda_N) ***************************`);
> print(`                                Xm or linear (Mean) curvature ` = Xm);
> print(`                         Yg or quadratic (GAUSS) curvature ` = Yg);
> print(` Za or Cubic (Interaction internal energy) curvature  ` = Za);
> print(`                   Tk or quartic (4D expansion) curvature  ` = Tk);
```

CONSTITUTIVE CURRENTS FROM D=epsilon E and H = B/mu
LORENTZ CURRENTS   use free space impedance

Use the Lorentz constitutive format D = epsilon E,   H = B/mu to compute the field Excitations (D and H).
The use the Maxwell Ampere equations to compute a Closed Current
J = curl H - dD/dt. (partial derivatives).   rho = div D

```
> Zfsm:=(1/epsilon)^(1/2);Zfse:=CH*(mu/epsilon)^(1/2);
> DF:=[factor(epsilon*E[1]+Zfse*B[1]),factor(epsilon*E[2]+Zfse*B[2]),factor(epsilon*E[3]+
  Zfse*B[3])];
> HF:=[factor(B[1]/mu-Zfse*E[1]),factor(B[2]/mu-Zfse*E[2]),factor(B[3]/mu-Zfse*E[3])];
  BH:=factor(innerprod(B,HF));DE:=factor(innerprod(DF,E));
> EXH:=(crossprod(E,HF));
> CD:=factor(diverge(DF,[x,y,z]));
> JAC:=curl(HF,[x,y,z]);
> JA:=[factor(JAC[1]),factor(JAC[2]),factor(JAC[3])];
> JDC:=-[diff(DF[1],t),diff(DF[2],t),diff(DF[3],t)];
> JD:=[factor(JDC[1]),factor(JDC[2]),factor(JDC[3])];
> JTOT:=[factor(factor(JAC[1]+sigma*E[1])+factor(JDC[1])),factor(factor(JAC[2]+sigma*E[2]
  )+factor(JDC[2])),factor(factor(JAC[3]+sigma*E[3])+factor(JDC[3])),factor(CD)];
> AdotJ:=factor(innerprod(ACTIONN,JTOT));
> JdotE:=factor(JTOT[1]*sigma*E[1]+JTOT[2]*sigma*E[2]+JTOT[3]*sigma*E[3]);
> J:=evalm([JTOT[1],JTOT[2],JTOT[3]]);AJ:=factor(innerprod(A,J));rhophi:=factor(-JTOT[4]*
  ACTIONN[4]);
> rho:=JTOT[4];
> rho_E:=simplify(rho*E);
> JXB:=(crossprod(J,B));
> LAGF:=factor(factor(innerprod(B,HF))-factor(innerprod(DF,E)));
> AdotJ:=factor(innerprod(JTOT,ACTIONN));
> PI:=factor(LAGF-AdotJ);
> DIVJTOT:=factor(diverge(JTOT,[x,y,z,t]));
> AAAA:=innerprod(A,A);AJJJ:=innerprod(A,J);
> AxJ:=simplify(crossprod([JTOT[1],JTOT[2],JTOT[3]],[A[1],A[2],A[3]]));
> if AxJ[1]<>0 and AxJ[2]<>0 and AxJ[3]<>0 then LONFAC:= 0 end if;
> if (AxJ[1]=0 and AxJ[2]=0 and AxJ[3]<>0) and A[1]<>0 then LONFAC:=factor(JTOT[1]/A[1])
  end if;
> if AxJ[1]<>0 and AxJ[2]=0 and AxJ[3]=0 and A[2]<>0 then LONFAC:=factor(JTOT[2]/A[2])
  end if;
> if AxJ[1]=0 and AxJ[2]<>0 and AxJ[3]=0 and A[1]<>0 then LONFAC:=factor(JTOT[1]/A[1])
  end if;
> if AxJ[1]=0 and AxJ[2]=0 and AxJ[3]=0 and A[1]<>0 then LONFAC:=+factor(JTOT[1]/A[1])
```

```
end if;
```

compute Topological Spin

```
> AXH:=(crossprod(A,HF));
> SP3:=([AXH[1]+DF[1]*phi/lambdaN,AXH[2]+DF[2]*phi/lambdaN,AXH[3]+DF[3]*phi/lambdaN]):
> OPT:=factor(innerprod(evalm(A),DF));
> SP4:=([factor(SP3[1]),factor(SP3[2]),factor(SP3[3]),factor(OPT)]);
> dSP4:=factor(diverge(SP4,[x,y,z,t]));
> SPIN3_form:= SP4[1]*d(y)&^d(z)&^d(t)-SP4[2]*d(x)&^d(z)&^d(t)+SP4[3]*d(x)&^d(y)&^d(t)-
  SP4[4]*d(x)&^d(y)&^d(z);
> dSPIN3_form:=factor(wcollect(d(SPIN3_form)));

> print(`**********      Compute Current using from Maxell-Ampere equations for
  constitutive equations with chirality CH  ***********`);print(`Chirality factor CH`=
  Zfse);
> print(`D field`= DF);
> print(`H field`= HF);
> print(`Poynting vector ExH`= simplify(EXH));
> print(`Amperian Current 4Vector     curlH-dD/dt=J4  `= JTOT);
> print(`Amerian charge density       divD = rho`= CD);
> print(`divergence Lorentz Current 4Vector,    4div(J4) `= factor(DIVJTOT));
print(`Topological SPIN 4 vector S4`=SP4);
print(`Topological SPIN 3-form`=SPIN3_form);
print(`Spin density rho_spin`= factor(SP4[4]));

> print(`LaGrange field energy density (B.H-D.E)`= LAGF);
```

Compute the Work, Heat and Internal Energy for a thermodynamic systems defined by the 1-form of Action, and the Closed Current. Determine if the process J is reversible or irreversible in a thermodynamic sense.

```
> Work:=-([factor(factor(rho*E[1])+factor(crossprod(J,B)[1])),factor(factor(rho*E[2])+
  factor(crossprod(J,B)[2])),factor(factor(rho*E[3])+factor(crossprod(J,B)[3])),-factor
  (JdotE)]);
> CHECK:=innerprod((Work),JTOT);
> Work_1form:=wcollect(innerprod(Work,[d(x),d(y),d(z),d(t)]));
> dWork:=wcollect(simplify(d(Work_1form)));
> LF:=-([factor(factor(rho*E[1])+factor(crossprod(J,B)[1])),factor(factor(rho*E[2])+
  factor(crossprod(J,B)[2])),factor(factor(rho*E[3])+factor(crossprod(J,B)[3]))]);
> U:=AdotJ;LFSPIN:=simplify(crossprod(LF,[SP4[1],SP4[2],SP4[3]]));
> if LFSPIN[1]+LFSPIN[2]+LFSPIN[3]=0 and LF[1]<>0 then LFSPIN:=SP4[1]/LF[1] else LFSPIN:=
  0  end if;
```

```
> SF:=-([factor(factor(SP4[4]*E[1])+factor(crossprod(SP3,B)[1])),factor(factor(SP4[4]*E
  [2])+factor(crossprod(SP3,B)[2])),factor(factor(SP4[4]*E[3])+factor(crossprod(SP3,B)[3]
  ))]);

> TF:=-([factor(factor(HEL*E[1])+factor(crossprod(T3,B)[1])),factor(factor(HEL*E[2])+
  factor(crossprod(T3,B)[2])),factor(factor(HEL*E[3])+factor(crossprod(T3,B)[3]))]);

> print(`    B.H`=BH);print(`    D.E`=DE);
> print(`    A.J`=AJ);print(`    -rho.phi`=rhophi);

> print(` Poincare I       (B.H - D.E)-(A.J - rho.phi) `= PI );
> print(`   London Coefficient      LC`=LONFAC);
>  print(`PROCA coefficient curlcurlB`=[factor(CCB[1]),factor(CCB[2]),factor(CCB[3])]);
  print(`  `);
> print(`Amperian Current 4Vector     curlH-dD/dt=J4  `= JTOT);

> print(` Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x
  B)   `= LF);

> print(`Amperian Dissipation  Jampere dot E   `= +JdotE );
> print(` Lorentz Force Spin factor  LFSPIN`=LFSPIN); print(`  `);

>

> print(`Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] `= T4);

> print(` Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E +
  J_torsion x B)   `= TF);
> print(`Torsion Dissipation  Jtorsion dot E   `= factor(innerprod(T3,E)));

>  print(`  `);print(`Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] `=
  SP4);
> print(` Lorentz Force 3 vector due to Spin current    SF = --(rho_spin E + J_spin x B)
   `= SF);
> print(`Spin Dissipation  J_spin dot E   `= factor(innerprod(SP3,E)));
```

```
> print(`  Disspative Force 3 vector`=[factor(LF[1]+mu*SF[1]+TF[1]),factor(LF[2]+mu*SF[2]
  +TF[2]),factor(LF[3]+mu*SF[3]+TF[3])]);
> print(`  Dissipation  `=factor(JTOT[4]+mu*SP4[4]+T4[1]));
```

```
> print(`******************************    END PROCEDURE   ****************************`)
  ;
> end:
```

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`-  vol 1 p. 397  vol1 p.397- Hedgehog, accretion discs`;r1:=(x^2+y^2);
> G:=alpha*z/(x^2+y^2+e*z^2)^(1/2);A1:=-G*b*(y)/r1;A2:=G*b*(x)/r1;
> A3:=0; phi:=0/(4*pi*epsilon)*q/(x^2+y^2+z^2)^(1/2);
```

Then call the procedure JCM(A 1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

NAME := - vol 1 p. 397 vol1 p.397- Hedgehog, accretion discs

$$r1 := x^2 + y^2$$

$$G := \frac{\alpha z}{\sqrt{x^2 + y^2 + e z^2}}$$

$$A1 := -\frac{\alpha z b y}{\sqrt{x^2 + y^2 + e z^2}\,(x^2 + y^2)}$$

$$A2 := \frac{\alpha z b x}{\sqrt{x^2 + y^2 + e z^2}\,(x^2 + y^2)}$$

$$A3 := 0$$

$$\phi := 0$$

- vol 1 p. 397 vol1 p.397- Hedgehog, accretion discs

******************************** *Differential Form Format* ****************************
****

$$\text{Action 1-form} = -\frac{\alpha z b y\, d(x)}{\sqrt{x^2 + y^2 + e z^2}\,(x^2 + y^2)} + \frac{\alpha z b x\, d(y)}{\sqrt{x^2 + y^2 + e z^2}\,(x^2 + y^2)}$$

$$\text{Intensity 2-form } F{=}dA \;= \left(\frac{\alpha z b\,(-x^2 y^2 - 2 y^4 + x^4 + x^2 e z^2 - y^2 e z^2)}{(x^2 + y^2 + e z^2)^{3/2}\,(x^2 + y^2)^2} - \frac{\alpha z b\,(2 x^4 + x^2 y^2 + x^2 e z^2 - y^4 - y^2 e z^2)}{(x^2 + y^2 + e z^2)^{3/2}\,(x^2 + y^2)^2} \right) (d(x))\ \&\char`^\ (d(y)) + \frac{\alpha b y\,(d(x))\ \&\char`^\ (d(z))}{(x^2 + y^2 + e z^2)^{3/2}}$$

$$-\frac{\alpha\, b\, x\, (d(y))\ \&\text{^}\ (d(z))}{(x^2+y^2+e\,z^2)^{3/2}}$$

Topological Torsion 3-form $A\text{^}F = 0$

Topological Parity 4-form $F\text{^}F = 0$

*********************************** *Using EM format* ***********************************

E field $= [0, 0, 0]$

$$B\ field = \left[-\frac{\alpha\, b\, x}{(x^2+y^2+e\,z^2)^{3/2}}, -\frac{\alpha\, b\, y}{(x^2+y^2+e\,z^2)^{3/2}}, -\frac{\alpha\, z\, b}{(x^2+y^2+e\,z^2)^{3/2}}\right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension PTD $= 2$

******************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************
****

Xm or linear (Mean) curvature $= 0$

$$Yg\ or\ quadratic\ (GAUSS)\ curvature = -\frac{(2\,x^2+2\,y^2+e\,z^2)\,\alpha^2\, z^2\, b^2}{(x^2+y^2+e\,z^2)^2\,(x^2+y^2)^2}$$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $= 0$

D field $= [0, 0, 0]$

$$H\ field = \left[-\frac{\alpha\, b\, x}{(x^2+y^2+e\,z^2)^{3/2}\,\mu}, -\frac{\alpha\, b\, y}{(x^2+y^2+e\,z^2)^{3/2}\,\mu}, -\frac{\alpha\, z\, b}{(x^2+y^2+e\,z^2)^{3/2}\,\mu}\right]$$

Poynting vector ExH $= EXH$

$$Amperian\ Current\ 4Vector\quad curlH\text{-}dD/dt\text{=}J4 = \left[-\frac{3\,\alpha\, z\, b\, y\,(-1+e)}{(x^2+y^2+e\,z^2)^{5/2}\,\mu}, \frac{3\,\alpha\, z\, b\, x\,(-1+e)}{(x^2+y^2+e\,z^2)^{5/2}\,\mu}, 0, 0\right]$$

Amerian charge density divD = rho $= 0$

divergence Lorentz Current 4Vector, 4div(J4) $= 0$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[-\frac{\alpha^2\, z^2\, b^2\, x}{(x^2+y^2+e\,z^2)^2\,(x^2+y^2)\,\mu}, -\frac{\alpha^2\, z^2\, b^2\, y}{(x^2+y^2+e\,z^2)^2\,(x^2+y^2)\,\mu},\right.$$

$$\frac{b^2 \alpha^2 z}{(x^2+y^2+e z^2)^2 \mu}, 0\Bigg]$$

$$\textit{Topological SPIN 3-form} = -\frac{\alpha^2 z^2 b^2 x \,\&\text{^}(d(y), d(z), d(t))}{(x^2+y^2+e z^2)^2 (x^2+y^2) \mu} + \frac{\alpha^2 z^2 b^2 y \,\&\text{^}(d(x), d(z), d(t))}{(x^2+y^2+e z^2)^2 (x^2+y^2) \mu} + \frac{b^2 \alpha^2 z \,\&\text{^}(d(x), d(y), d(t))}{(x^2+y^2+e z^2)^2 \mu}$$

$$\textit{Spin density rho\_spin} = 0$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{\alpha^2 b^2 (x^2+y^2+z^2)}{(x^2+y^2+e z^2)^3 \mu}$$

$$B.H = \frac{\alpha^2 b^2 (x^2+y^2+z^2)}{(x^2+y^2+e z^2)^3 \mu}$$

$$D.E = 0$$

$$A.J = \frac{3 \alpha^2 z^2 b^2 (-1+e)}{(x^2+y^2+e z^2)^3 \mu}$$

$$\text{-}rho.phi = 0$$

$$\textit{Poincare I} \qquad \textit{(B.H - D.E)-(A.J - rho.phi)} = -\frac{\alpha^2 b^2 (-x^2-y^2-4 z^2+3 e z^2)}{(x^2+y^2+e z^2)^3 \mu}$$

$$\textit{London Coefficient} \qquad LC = \frac{3 (-1+e) (x^2+y^2)}{(x^2+y^2+e z^2)^2 \mu}$$

$$\textit{PROCA coefficient curlcurlB} = \left[\frac{3 \alpha b x (-1+e) (4 e z^2-x^2-y^2)}{(x^2+y^2+e z^2)^{7/2}}, \frac{3 \alpha b y (-1+e) (4 e z^2-x^2-y^2)}{(x^2+y^2+e z^2)^{7/2}}, \frac{3 \alpha z b (-1+e) (2 e z^2-3 y^2-3 x^2)}{(x^2+y^2+e z^2)^{7/2}}\right]$$

$$\textit{Amperian Current 4Vector} \qquad \textit{curlH-dD/dt=J4} \quad = \left[-\frac{3 \alpha z b y (-1+e)}{(x^2+y^2+e z^2)^{5/2} \mu}, \frac{3 \alpha z b x (-1+e)}{(x^2+y^2+e z^2)^{5/2} \mu}, 0, 0\right]$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B)

$$= \left[\frac{3 \alpha^2 z^2 b^2 x (-1+e)}{(x^2+y^2+e z^2)^4 \mu}, \frac{3 \alpha^2 z^2 b^2 y (-1+e)}{(x^2+y^2+e z^2)^4 \mu}, -\frac{3 \alpha^2 z b^2 (-1+e) (x^2+y^2)}{(x^2+y^2+e z^2)^4 \mu}\right]$$

$$\textit{Amperian Dissipation Jampere dot E} \quad = 0$$

Lorentz Force Spin factor LFSPIN $= -\frac{1}{3}\,\frac{(x^2+y^2+e\,z^2)^2}{(x^2+y^2)\,(-1+e)}$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \left[-\frac{\alpha^2 z^2 b^2 x}{(x^2+y^2+e\,z^2)^2\,(x^2+y^2)\,\mu}, -\frac{\alpha^2 z^2 b^2 y}{(x^2+y^2+e\,z^2)^2\,(x^2+y^2)\,\mu}, \frac{b^2 \alpha^2 z}{(x^2+y^2+e\,z^2)^2\,\mu}, 0 \right]$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= \left[-\frac{\alpha^3 z\, b^3 y\,(x^2+y^2+z^2)}{(x^2+y^2+e\,z^2)^{7/2}\,(x^2+y^2)\,\mu}, \frac{b^3 \alpha^3 z\, x\,(x^2+y^2+z^2)}{(x^2+y^2+e\,z^2)^{7/2}\,(x^2+y^2)\,\mu}, 0 \right]$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \left[\frac{1}{(x^2+y^2+e\,z^2)^{15/2}\,\mu\,(x^2+y^2)} \left(b^2 \alpha^2 z \left(-3\,z\,x^3\,(x^2+y^2+e\,z^2)^{7/2} \right.\right.\right.$

$- \alpha\, b\, y^{11}\,\mu - 3\,z\,x\,(x^2+y^2+e\,z^2)^{7/2}\,y^2 + 3\,z\,x^3\,(x^2+y^2+e\,z^2)^{7/2}\,e - 5\,\alpha\, b\, y^3\,\mu\, x^8 - 10\,\alpha\, b\, y^5\,\mu\, x^6$

$- 10\,\alpha\, b\, y^7\,\mu\, x^4 - 5\,\alpha\, b\, y^9\,\mu\, x^2 - \alpha\, b\, y^9\,\mu\, z^2 - \alpha\, b\, y\,\mu\, x^{10} + 3\,z\,x\,(x^2+y^2+e\,z^2)^{7/2}\,e\,y^2$

$- 24\,\alpha\, b\, y^5\,\mu\, x^4\,e\,z^2 - 18\,\alpha\, b\, y^3\,\mu\, x^4\,e^2 z^4 - 16\,\alpha\, b\, y^7\,\mu\, x^2\,e\,z^2 - 18\,\alpha\, b\, y^5\,\mu\, x^2\,e^2 z^4 - 8\,\alpha\, b\, y^3\,\mu\, x^2\,e^3 z^6$

$- 12\,\alpha\, b\, y^3\,\mu\, z^4 x^4\,e - 12\,\alpha\, b\, y^5\,\mu\, z^4 x^2\,e - 12\,\alpha\, b\, y^3\,\mu\, z^6 x^2\,e^2 - 4\,\alpha\, b\, y\,\mu\, x^8\,e\,z^2 - 6\,\alpha\, b\, y\,\mu\, x^6\,e^2 z^4$

$- 4\,\alpha\, b\, y\,\mu\, x^4\,e^3 z^6 - \alpha\, b\, y\,\mu\, x^2\,e^4 z^8 - 4\,\alpha\, b\, y\,\mu\, z^4 x^6\,e - 6\,\alpha\, b\, y\,\mu\, z^6 x^4\,e^2 - 4\,\alpha\, b\, y\,\mu\, z^8 x^2\,e^3$

$- 4\,\alpha\, b\, y^9\,\mu\, e\,z^2 - 6\,\alpha\, b\, y^7\,\mu\, e^2 z^4 - 4\,\alpha\, b\, y^5\,\mu\, e^3 z^6 - \alpha\, b\, y^3\,\mu\, e^4 z^8 - 4\,\alpha\, b\, y^3\,\mu\, z^2 x^6 - 6\,\alpha\, b\, y^5\,\mu\, z^2 x^4$

$$-4\,\alpha\,b\,y^7\,\mu\,z^2\,x^2-4\,\alpha\,b\,y^7\,\mu\,z^4\,e-6\,\alpha\,b\,y^5\,\mu\,z^6\,e^2-4\,\alpha\,b\,y^3\,\mu\,z^8\,e^3-\alpha\,b\,y\,\mu\,z^{10}\,e^4-\alpha\,b\,y\,\mu\,z^2\,x^8$$

$$-16\,\alpha\,b\,y^3\,\mu\,x^6\,e\,z^2\big)\big),\ \frac{1}{\left(x^2+y^2+e\,z^2\right)^{15/2}\mu\left(x^2+y^2\right)}\Big(b^2\,\alpha^2\,z\,\Big(-3\,z\,y^3\left(x^2+y^2+e\,z^2\right)^{7/2}$$

$$+5\,b\,\alpha\,x^9\,\mu\,y^2+10\,b\,\alpha\,x^7\,\mu\,y^4+10\,b\,\alpha\,x^5\,\mu\,y^6+5\,b\,\alpha\,x^3\,\mu\,y^8+b\,\alpha\,x^9\,\mu\,z^2+b\,\alpha\,x\,\mu\,y^{10}$$

$$+24\,b\,\alpha\,x^5\,\mu\,e\,z^2\,y^4+18\,b\,\alpha\,x^5\,\mu\,y^2\,e^2\,z^4+16\,b\,\alpha\,x^3\,\mu\,y^6\,e\,z^2+18\,b\,\alpha\,x^3\,\mu\,y^4\,e^2\,z^4+8\,b\,\alpha\,x^3\,\mu\,y^2\,e^3\,z^6$$

$$+12\,b\,\alpha\,x^5\,\mu\,z^4\,e\,y^2+12\,b\,\alpha\,x^3\,\mu\,z^4\,e\,y^4+12\,b\,\alpha\,x^3\,\mu\,z^6\,y^2\,e^2+4\,b\,\alpha\,x\,\mu\,y^8\,e\,z^2+6\,b\,\alpha\,x\,\mu\,y^6\,e^2\,z^4$$

$$+4\,b\,\alpha\,x\,\mu\,y^4\,e^3\,z^6+b\,\alpha\,x\,\mu\,y^2\,e^4\,z^8+4\,b\,\alpha\,x\,\mu\,z^4\,y^6\,e+6\,b\,\alpha\,x\,\mu\,z^6\,y^4\,e^2+4\,b\,\alpha\,x\,\mu\,z^8\,y^2\,e^3+b\,\alpha\,x^{11}\,\mu$$

$$-3\,z\,y\left(x^2+y^2+e\,z^2\right)^{7/2}x^2+3\,z\,y^3\left(x^2+y^2+e\,z^2\right)^{7/2}e+4\,b\,\alpha\,x^9\,\mu\,e\,z^2+6\,b\,\alpha\,x^7\,\mu\,e^2\,z^4$$

$$+4\,b\,\alpha\,x^5\,\mu\,e^3\,z^6+b\,\alpha\,x^3\,\mu\,e^4\,z^8+4\,b\,\alpha\,x^7\,\mu\,z^2\,y^2+6\,b\,\alpha\,x^5\,\mu\,z^2\,y^4+4\,b\,\alpha\,x^3\,\mu\,z^2\,y^6+4\,b\,\alpha\,x^7\,\mu\,z^4\,e$$

$$+6\,b\,\alpha\,x^5\,\mu\,z^6\,e^2+4\,b\,\alpha\,x^3\,\mu\,z^8\,e^3+b\,\alpha\,x\,\mu\,z^{10}\,e^4+b\,\alpha\,x\,\mu\,z^2\,y^8+16\,b\,\alpha\,x^7\,\mu\,e\,z^2\,y^2+3\,z\,y\,\big(x^2$$

$$+y^2+e\,z^2\big)^{7/2}e\,x^2\Big)\Big),\ -\frac{3\,\alpha^2\,z\,b^2\,(-1+e)\left(x^2+y^2\right)}{\left(x^2+y^2+e\,z^2\right)^4\mu}\Bigg]$$

Dissipation $=0$

************************************ *END PROCEDURE* ********************************** **(2)**

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`-  vol 1 p. 397  vol4 p.147- Abrikosov Falaco vortex singularities.`;f:=(x^2+
  y^2);
> r1:=(f)^(2/2);A1:=1*b*0*(y)/r1;A2:=1*b*(-x)*0/r1;
> A3:=0; phi:=1/(4*pi*epsilon)*q/(x^2+y^2+z^2)^(1/2);
```

Then call the procedure JCM(A 1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := - vol 1 p. 397 vol4 p.147- Abrikosov Falaco vortex singularities.

$$f := x^2 + y^2$$

$$r1 := x^2 + y^2$$

$$A1 := 0$$

$$A2 := 0$$

$$A3 := 0$$

$$\phi := \frac{1}{4}\frac{q}{\pi\,\varepsilon\,\sqrt{x^2+y^2+z^2}}$$

- vol 1 p. 397 vol4 p.147- Abrikosov Falaco vortex singularities.

******************************** *Differential Form Format* ******************************** ****

$$\textit{Action 1-form} = -\frac{1}{4}\frac{q\,d(t)}{\pi\,\varepsilon\,\sqrt{x^2+y^2+z^2}}$$

$$\textit{Intensity 2-form F=dA} = \frac{1}{4}\frac{q\,x\,(d(x))\ \&\hat{}\ (d(t))}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}} + \frac{1}{4}\frac{q\,y\,(d(y))\ \&\hat{}\ (d(t))}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}} + \frac{1}{4}\frac{q\,z\,(d(z))\ \&\hat{}\ (d(t))}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}$$

Topological Torsion 3-form $\quad A\hat{}F = 0$

Topological Parity 4-form $\quad F\hat{}F = 0$

******************************** *Using EM format* ********************************

$$\textit{E field} = \left[\frac{1}{4}\frac{q\,x}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4}\frac{q\,y}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4}\frac{q\,z}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}\right]$$

$$\textit{B field} = [0, 0, 0]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension *PTD* = 2

******************* Correlation Similarity Invariants of Jacobian of (Ak/lambda_N) ***************
****

$$Xm \text{ or linear (Mean) curvature } = 0$$

$$Yg \text{ or quadratic (GAUSS) curvature } = 0$$

$$Za \text{ or Cubic (Interaction internal energy) curvature } = 0$$

$$Tk \text{ or quartic (4D expansion) curvature } = 0$$

********** Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH ***********

$$\text{Chirality factor } CH = 0$$

$$D\,field = \left[\frac{1}{4}\frac{q\,x}{\pi\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4}\frac{q\,y}{\pi\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4}\frac{q\,z}{\pi\,(x^2+y^2+z^2)^{3/2}}\right]$$

$$H\,field = [0, 0, 0]$$

$$\text{Poynting vector } ExH = EXH$$

$$\text{Amperian Current 4Vector} \quad curlH\text{-}dD/dt\text{=}J4 = [0, 0, 0, 0]$$

$$\text{Amerian charge density} \quad divD = rho = 0$$

$$\text{divergence Lorentz Current 4Vector,} \quad 4div(J4) = 0$$

$$\text{Topological SPIN 4 vector } S4 = \left[\frac{1}{16}\frac{q^2\,x}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon}, \frac{1}{16}\frac{q^2\,y}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon}, \frac{1}{16}\frac{q^2\,z}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon}, 0\right]$$

$$\text{Topological SPIN 3-form} = \frac{1}{16}\frac{q^2\,x\,\&\hat{}(d(y), d(z), d(t))}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon} - \frac{1}{16}\frac{q^2\,y\,\&\hat{}(d(x), d(z), d(t))}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon} + \frac{1}{16}\frac{q^2\,z\,\&\hat{}(d(x), d(y), d(t))}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon}$$

$$\text{Spin density } rho\_spin = 0$$

$$\text{LaGrange field energy density (B.H-D.E)} = -\frac{1}{16}\frac{q^2}{\varepsilon\,(x^2+y^2+z^2)^2\,\pi^2}$$

$$B.H = 0$$

$$D.E = \frac{1}{16}\frac{q^2}{\varepsilon\,(x^2+y^2+z^2)^2\,\pi^2}$$

$$A.J = 0$$

$$-rho.phi = 0$$

$$\textit{Poincare I} \quad \textit{(B.H - D.E)-(A.J - rho.phi)} = -\frac{1}{16}\,\frac{q^2}{\varepsilon\,(x^2+y^2+z^2)^2\,\pi^2}$$

$$\textit{London Coefficient} \quad LC = \frac{3\,(-1+e)\,(x^2+y^2)}{(x^2+y^2+e\,z^2)^2\,\mu}$$

$$\textit{PROCA coefficient curlcurlB} = [0, 0, 0]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} = [0, 0, 0, 0]$$

$$\textit{Lorentz Force 3 vector due to Ampere current} \quad \textit{FL = -(rho\_ampere E + J\_ampere x B)} = [0, 0, 0]$$

$$\textit{Amperian Dissipation Jampere dot E} = 0$$

$$\textit{Lorentz Force Spin factor LFSPIN} = 0$$

$$\textit{Topological Torsion current 4 vector} \quad \textit{T4 = -[ExA + B.phi,AdotB]} = [0, 0, 0, 0]$$

$$\textit{Lorentz Force 3 vector due to Torsion current} \quad \textit{TF = -(rho\_torsion E + J\_torsion x B)} = [0, 0, 0]$$

$$\textit{Torsion Dissipation Jtorsion dot E} = 0$$

$$\textit{Topological Spin current 4 vector} \quad \textit{TS4 = -[A x H + D.phi,AdotD]} = \left[\frac{1}{16}\,\frac{q^2 x}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon},\ \frac{1}{16}\,\frac{q^2 y}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon},\ \frac{1}{16}\,\frac{q^2 z}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon},\ 0\right]$$

$$\textit{Lorentz Force 3 vector due to Spin current} \quad \textit{SF = --(rho\_spin E + J\_spin x B)} = [0, 0, 0]$$

$$\textit{Spin Dissipation J\_spin dot E} = \frac{1}{64}\,\frac{q^3}{(x^2+y^2+z^2)^{5/2}\,\pi^3\,\varepsilon^2}$$

$$\textit{Disspative Force 3 vector} = [0, 0, 0]$$

$$\textit{Dissipation} = 0$$

****************************** END PROCEDURE *************************** **(3)**

Example Potentials from the Hopf map

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

```
NAME:=`Potentials from the Hopf Map   Chirality 0*(g+I*gamma)`:
> A1:=y;A2:=-x;A3:=t;phi:=z;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*(g+I*gamma),0);
```

****

$$A1 := y$$

$$A2 := -x$$

$$A3 := t$$

$$\phi := z$$

Potentials from the Hopf Map Chirality 0(gC I*gamma)*

********************************* *Differential Form Format* *********************************

$$\textit{Action 1-form} = y\, d(x) - x\, d(y) + t\, d(z) - z\, d(t)$$

$$\textit{Intensity 2-form F=dA} \;\; = -2\, (d(x))\ \&\hat{}\ (d(y)) - 2\, (d(z))\ \&\hat{}\ (d(t))$$

$$\textit{Topological Torsion 3-form} \quad A\hat{}F = -2\, t\, \&\hat{}(d(x), d(y), d(z)) + 2\, z\, \&\hat{}(d(x), d(y), d(t)) - 2\, y\, \&\hat{}(d(x), d(z), d(t)) + 2\, x\, \&\hat{}(d(y), d(z), d(t))$$

$$\textit{Topological Parity 4-form} \quad F\hat{}F = 8\, \&\hat{}(d(x), d(y), d(z), d(t))$$

********************************* *Using EM format* *********************************

$$\textit{E field} = [0, 0, -2]$$

$$\textit{B field} = [0, 0, -2]$$

$$\textit{Topological TORSION 4 vector} \quad T4 = \text{-}[ExA + Bphi, AdotB] = [2\,x, 2\,y, 2\,z, 2\,t]$$

$$\textit{Helicity AdotB} = -2\, t$$

$$\textit{Poincare II} = 2(E.B) = 8$$

$$\textit{coefficient of Topological Parity 4-form} = 8$$

$$\textit{Pfaff Topological Dimension} \quad PTD = 4$$

******************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ****************************

$$\textit{Xm or linear (Mean) curvature} = 0$$

$$\textit{Yg or quadratic (GAUSS) curvature} = 2$$

$$\textit{Za or Cubic (Interaction internal energy) curvature} = 0$$

$$\textit{Tk or quartic (4D expansion) curvature} = 1$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\textit{Chirality factor CH} = 0$$

$$D\ field = [0, 0, -2\,\varepsilon]$$

$$H\ field = \left[0, 0, -\frac{2}{\mu}\right]$$

$$Poynting\ vector\ ExH = EXH$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt\text{=}J4 \quad = [0, 0, 0, 0]$$

$$Amerian\ charge\ density \quad divD = rho = 0$$

$$divergence\ Lorentz\ Current\ 4Vector, \quad 4div(J4) \ = 0$$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[\frac{2\,x}{\mu}, \frac{2\,y}{\mu}, -2\,\varepsilon\,z, -2\,t\,\varepsilon\right]$$

$$Topological\ SPIN\ 3\text{-}form = \frac{2\,x\ \&\text{^}(d(y), d(z), d(t))}{\mu} - \frac{2\,y\ \&\text{^}(d(x), d(z), d(t))}{\mu} - 2\,\varepsilon\,z\ \&\text{^}(d(x), d(y), d(t)) + 2\,t\,\varepsilon\ \&\text{^}(d(x), d(y), d(z))$$

$$Spin\ density\ rho\_spin = -2\,t\,\varepsilon$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{4\,(-1 + \varepsilon\,\mu)}{\mu}$$

$$B.H = \frac{4}{\mu}$$

$$D.E = 4\,\varepsilon$$

$$A.J = 0$$

$$-rho.phi = 0$$

$$Poincare\ I \quad (B.H - D.E)\text{-}(A.J - rho.phi) \ = -\frac{4\,(-1 + \varepsilon\,\mu)}{\mu}$$

$$London\ Coefficient \quad LC = 0$$

$$PROCA\ coefficient\ curlcurlB = [0, 0, 0]$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt\text{=}J4 \quad = [0, 0, 0, 0]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Ampere\ current \quad FL = \text{-}(rho\_ampere\ E + J\_ampere\ x\ B) \quad = [0, 0, 0]$$

$$Amperian\ Dissipation\ \ Jampere\ dot\ E \quad = 0$$

$$Lorentz\ Force\ Spin\ factor\ \ LFSPIN = 0$$

$$Topological\ Torsion\ current\ 4\ vector \quad T4 = \text{-}[ExA + B.phi,AdotB] \ = [2\,x, 2\,y, 2\,z, 2\,t]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Torsion\ current \quad TF = \text{-}(rho\_torsion\ E + J\_torsion\ x\ B) \quad = [-4\,y, 4\,x, -4\,t]$$

$$Torsion\ Dissipation\ \ Jtorsion\ dot\ E \quad = 4\,z$$

$$\text{Topological Spin current 4 vector} \quad TS4 = \text{-[A x H + D.phi,AdotD]} = \left[\frac{2\,x}{\mu}, \frac{2\,y}{\mu}, -2\,\varepsilon\,z, -2\,t\,\varepsilon\right]$$

$$\text{Lorentz Force 3 vector due to Spin current} \quad SF = \text{--(rho\_spin E + J\_spin x B)} = \left[\frac{4\,y}{\mu}, -\frac{4\,x}{\mu}, -4\,t\,\varepsilon\right]$$

$$\text{Spin Dissipation J\_spin dot E} = 4\,\varepsilon\,z$$

$$\text{Disspative Force 3 vector} = [0, 0, -4\,t\,(\varepsilon\,\mu + 1)]$$

$$\text{Dissipation} = -2\,\mu\,t\,\varepsilon + 2\,x$$

******************************* *END PROCEDURE* ***************************** **(4)**

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`-  vol 1 p. 397  vol4 p.147- Abrikosov Falaco vortex singularities.`;f:=(x^2+
  y^2);
> r2:=1;r1:=(((f+1*kappa*z+(a*(1))^(2/2))));
> A1:=1*b*m*(y)/r2/r1;A2:=1*b*(-x)*m/r2/r1;
> A3:=0; phi:=0/(4*pi*epsilon)*q/(x^2+y^2+z^2)^(1/2);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := - vol 1 p. 397 vol4 p.147- Abrikosov Falaco vortex singularities.

$$f := x^2 + y^2$$

$$r2 := 1$$

$$r1 := x^2 + y^2 + \kappa\,z + a$$

$$A1 := \frac{b\,m\,y}{x^2 + y^2 + \kappa\,z + a}$$

$$A2 := -\frac{b\,x\,m}{x^2 + y^2 + \kappa\,z + a}$$

$$A3 := 0$$

$$\phi := 0$$

- vol 1 p. 397 vol4 p.147- Abrikosov Falaco vortex singularities.

********************************* *Differential Form Format* *****************************
****

$$\text{Action 1-form} = \frac{b\,m\,y\,d(x)}{x^2 + y^2 + \kappa\,z + a} - \frac{b\,x\,m\,d(y)}{x^2 + y^2 + \kappa\,z + a}$$

$$\text{Intensity 2-form F=dA} = \left(-\frac{b\,m\,(x^2 - y^2 + \kappa\,z + a)}{(x^2 + y^2 + \kappa\,z + a)^2} - \frac{b\,m\,(-x^2 + y^2 + \kappa\,z + a)}{(x^2 + y^2 + \kappa\,z + a)^2}\right)(d(x))\,\&\hat{}\,(d(y))$$

$$+\frac{b\,m\,y\,\kappa\,(d(x))\ \&\text{^}\ (d(z))}{(x^2+y^2+\kappa z+a)^2}-\frac{b\,x\,m\,\kappa\,(d(y))\ \&\text{^}\ (d(z))}{(x^2+y^2+\kappa z+a)^2}$$

Topological Torsion 3-form $\quad A\text{^}F=0$

Topological Parity 4-form $\quad F\text{^}F=0$

******************************** *Using EM format* ********************************

$E\,field=[0,0,0]$

$$B\,field=\left[-\frac{b\,x\,m\,\kappa}{(x^2+y^2+\kappa z+a)^2},-\frac{b\,m\,y\,\kappa}{(x^2+y^2+\kappa z+a)^2},-\frac{2\,b\,m\,(\kappa z+a)}{(x^2+y^2+\kappa z+a)^2}\right]$$

Topological TORSION 4 vector $\quad$ *T4 = -[ExA + Bphi,AdotB]* $=[0,0,0,0]$

Helicity AdotB $=0$

Poincare II =2(E.B) $=0$

coefficient of Topological Parity 4-form $=0$

Pfaff Topological Dimension $\quad PTD=2$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************
****

Xm or linear (Mean) curvature $=0$

$$\textit{Yg or quadratic (GAUSS) curvature} =\frac{b^2\,m^2\,(a+\kappa z-x^2-y^2)}{(x^2+y^2+\kappa z+a)^3}$$

Za or Cubic (Interaction internal energy) curvature $=0$

Tk or quartic (4D expansion) curvature $=0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $=0$

$D\,field=[0,0,0]$

$$H\,field=\left[-\frac{b\,x\,m\,\kappa}{(x^2+y^2+\kappa z+a)^2\,\mu},-\frac{b\,m\,y\,\kappa}{(x^2+y^2+\kappa z+a)^2\,\mu},-\frac{2\,b\,m\,(\kappa z+a)}{(x^2+y^2+\kappa z+a)^2\,\mu}\right]$$

Poynting vector ExH $=EXH$

$$\textit{Amperian Current 4Vector} \quad curlH\text{-}dD/dt=J4 =\left[\frac{2\,b\,m\,y\,(4\,\kappa z+4\,a-\kappa^2)}{(x^2+y^2+\kappa z+a)^3\,\mu},\right.$$
$$\left.-\frac{2\,b\,x\,m\,(4\,\kappa z+4\,a-\kappa^2)}{(x^2+y^2+\kappa z+a)^3\,\mu},0,0\right]$$

Amerian charge density $\quad divD = rho=0$

divergence Lorentz Current 4Vector, $\quad$ *4div(J4)* $=0$

$$\textit{Topological SPIN 4 vector S4} = \left[\frac{2\, b^2\, x\, m^2\, (\kappa\, z + a)}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu}, \frac{2\, b^2\, m^2\, y\, (\kappa\, z + a)}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu}, -\frac{b^2\, m^2\, \kappa\, (x^2 + y^2)}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu}, 0 \right]$$

$$\textit{Topological SPIN 3-form} = \frac{2\, b^2\, x\, m^2\, (\kappa\, z + a)\ \&\text{^}(d(y), d(z), d(t))}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu} - \frac{2\, b^2\, m^2\, y\, (\kappa\, z + a)\ \&\text{^}(d(x), d(z), d(t))}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu} - \frac{b^2\, m^2\, \kappa\, (x^2 + y^2)\ \&\text{^}(d(x), d(y), d(t))}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu}$$

$$\textit{Spin density rho\_spin} = 0$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{b^2\, m^2\, (x^2\, \kappa^2 + y^2\, \kappa^2 + 4\, \kappa^2\, z^2 + 8\, \kappa\, z\, a + 4\, a^2)}{(x^2 + y^2 + \kappa\, z + a)^4\, \mu}$$

$$B.H = \frac{b^2\, m^2\, (x^2\, \kappa^2 + y^2\, \kappa^2 + 4\, \kappa^2\, z^2 + 8\, \kappa\, z\, a + 4\, a^2)}{(x^2 + y^2 + \kappa\, z + a)^4\, \mu}$$

$$D.E = 0$$

$$A.J = \frac{2\, b^2\, m^2\, (4\, \kappa\, z + 4\, a - \kappa^2)\, (x^2 + y^2)}{(x^2 + y^2 + \kappa\, z + a)^4\, \mu}$$

$$\textit{-rho.phi} = 0$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)*

$$= \frac{b^2\, m^2\, (3\, x^2\, \kappa^2 + 3\, y^2\, \kappa^2 + 4\, \kappa^2\, z^2 + 8\, \kappa\, z\, a + 4\, a^2 - 8\, x^2\, \kappa\, z - 8\, \kappa\, z\, y^2 - 8\, x^2\, a - 8\, a\, y^2)}{(x^2 + y^2 + \kappa\, z + a)^4\, \mu}$$

$$\textit{London Coefficient} \quad LC = \frac{2\, (4\, \kappa\, z + 4\, a - \kappa^2)}{(x^2 + y^2 + \kappa\, z + a)^2\, \mu}$$

$$\textit{PROCA coefficient curlcurlB} = \left[-\frac{2\, b\, x\, m\, \kappa\, (\%1)}{(x^2 + y^2 + \kappa z + a)^4}, -\frac{2\, b\, m\, y\, \kappa\, (\%1)}{(x^2 + y^2 + \kappa z + a)^4}, -\frac{4\, b\, m\, (4\, \kappa z + 4\, a - \kappa^2)\, (a - 2\, x^2 + \kappa z - 2\, y^2)}{(x^2 + y^2 + \kappa z + a)^4} \right]$$

$$\%1 = -3\, \kappa^2 - 4\, x^2 - 4\, y^2 + 8\, \kappa\, z + 8\, a$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} = \left[\frac{2\, b\, m\, y\, (4\, \kappa\, z + 4\, a - \kappa^2)}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu}, -\frac{2\, b\, x\, m\, (4\, \kappa\, z + 4\, a - \kappa^2)}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu}, 0, 0 \right]$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B) =

$$\left[-\frac{4 b^2 x m^2 (4 \kappa z + 4 a - \kappa^2)(\kappa z + a)}{(x^2 + y^2 + \kappa z + a)^5 \mu}, -\frac{4 b^2 m^2 y (4 \kappa z + 4 a - \kappa^2)(\kappa z + a)}{(x^2 + y^2 + \kappa z + a)^5 \mu}, \frac{2 b^2 m^2 (4 \kappa z + 4 a - \kappa^2) \kappa (x^2 + y^2)}{(x^2 + y^2 + \kappa z + a)^5 \mu}\right]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= -\frac{1}{2} \frac{(x^2 + y^2 + \kappa z + a)^2}{4 \kappa z + 4 a - \kappa^2}$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] $= \left[\frac{2 b^2 x m^2 (\kappa z + a)}{(x^2 + y^2 + \kappa z + a)^3 \mu}, \frac{2 b^2 m^2 y (\kappa z + a)}{(x^2 + y^2 + \kappa z + a)^3 \mu}, -\frac{b^2 m^2 \kappa (x^2 + y^2)}{(x^2 + y^2 + \kappa z + a)^3 \mu}, 0\right]$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $= \left[\frac{b^3 m^3 y (\%1)}{(x^2 + y^2 + \kappa z + a)^5 \mu}, -\frac{b^3 m^3 x (\%1)}{(x^2 + y^2 + \kappa z + a)^5 \mu}, 0\right]$

$$\%1 = x^2 \kappa^2 + y^2 \kappa^2 + 4 \kappa^2 z^2 + 8 \kappa z a + 4 a^2$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \Big[\frac{1}{(x^2 + y^2 + \kappa z + a)^5 \mu} (b^2 m^2 (-16 x \kappa^2 z^2 - 32 x \kappa z a - 16 x a^2 + 4 x \kappa^3 z$

$+ 4 x \kappa^2 a + b m y \mu x^2 \kappa^2 + b m y^3 \mu \kappa^2 + 4 b m y \mu \kappa^2 z^2 + 8 b m y \mu \kappa z a + 4 b m y \mu a^2)),$

$-\frac{1}{(x^2 + y^2 + \kappa z + a)^5 \mu} (b^2 m^2 (16 y \kappa^2 z^2 + 32 y \kappa z a + 16 y a^2 - 4 y \kappa^3 z - 4 y \kappa^2 a + b m x^3 \mu \kappa^2$

$+ b m x \mu y^2 \kappa^2 + 4 b m x \mu \kappa^2 z^2 + 8 b m x \mu \kappa z a + 4 b m x \mu a^2)),$

$$\left.\frac{2\,b^2\,m^2\,\left(4\,\kappa\,z+4\,a-\kappa^2\right)\,\kappa\,\left(x^2+y^2\right)}{\left(x^2+y^2+\kappa\,z+a\right)^5\,\mu}\right]$$

$$\mathit{Dissipation}\ =0$$

***************************** *END PROCEDURE* **************************** **(5)**

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`-  vol 1 p. 397  vol4 p.147- Abrikosov vortex singularities.`;f:=(x^2+y^2);
> r2:=1;r1:=(((f+0*kappa*z+(a*(0))^(2/2))));
> A1:=1*b*m*(y)/r2/r1;A2:=1*b*(-x)*m/r2/r1;
> A3:=0; phi:=1/(4*pi*epsilon)*q/(x^2+y^2+z^2)^(1/2);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := - vol 1 p. 397 vol4 p.147- Abrikosov vortex singularities.

$$f := x^2 + y^2$$

$$r2 := 1$$

$$r1 := x^2 + y^2$$

$$A1 := \frac{b\,m\,y}{x^2 + y^2}$$

$$A2 := -\frac{b\,x\,m}{x^2 + y^2}$$

$$A3 := 0$$

$$\phi := \frac{1}{4}\,\frac{q}{\pi\,\varepsilon\,\sqrt{x^2 + y^2 + z^2}}$$

- vol 1 p. 397 vol4 p.147- Abrikosov vortex singularities.

******************************** *Differential Form Format* ****************************
****

$$\text{Action 1-form} = \frac{1}{4}\,\frac{(-q\,x^2 - q\,y^2)\,d(t)}{(x^2 + y^2)\,\pi\,\varepsilon\,\sqrt{x^2 + y^2 + z^2}} + \frac{b\,m\,y\,d(x)}{x^2 + y^2} - \frac{b\,x\,m\,d(y)}{x^2 + y^2}$$

$$\text{Intensity 2-form } F=dA \;= \frac{1}{4}\,\frac{q\,x\,(d(x))\;\&\text{^}\;(d(t))}{\pi\,\varepsilon\,(x^2 + y^2 + z^2)^{3/2}} + \frac{1}{4}\,\frac{q\,y\,(d(y))\;\&\text{^}\;(d(t))}{\pi\,\varepsilon\,(x^2 + y^2 + z^2)^{3/2}} + \frac{1}{4}\,\frac{q\,z\,(d(z))\;\&\text{^}\;(d(t))}{\pi\,\varepsilon\,(x^2 + y^2 + z^2)^{3/2}}$$

$$\text{Topological Torsion 3-form}\quad A\text{^}F = \left(\frac{1}{4}\,\frac{b\,x^2\,m\,q}{(x^2 + y^2)\,\pi\,\varepsilon\,(x^2 + y^2 + z^2)^{3/2}} + \frac{1}{4}\,\frac{b\,m\,y^2\,q}{(x^2 + y^2)\,\pi\,\varepsilon\,(x^2 + y^2 + z^2)^{3/2}}\right)\&\text{^}(d(x), d(y), d(t)) + \frac{1}{4}\,\frac{b\,m\,y\,q\,z\,\&\text{^}(d(x), d(z), d(t))}{(x^2 + y^2)\,\pi\,\varepsilon\,(x^2 + y^2 + z^2)^{3/2}} - \frac{1}{4}\,\frac{b\,x\,m\,q\,z\,\&\text{^}(d(y), d(z), d(t))}{(x^2 + y^2)\,\pi\,\varepsilon\,(x^2 + y^2 + z^2)^{3/2}}$$

$$\textit{Topological Parity 4-form} \quad F\text{^}F = 0$$

******************************** *Using EM format* ********************************

$$E\,field = \left[\frac{1}{4} \frac{q\,x}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4} \frac{q\,y}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4} \frac{q\,z}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}} \right]$$

$$B\,field = [0, 0, 0]$$

$$\textit{Topological TORSION 4 vector} \quad T4 = -[ExA + Bphi, AdotB] = \left[-\frac{1}{4} \frac{b\,x\,m\,q\,z}{(x^2+y^2)\,\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, -\frac{1}{4} \frac{b\,m\,y\,q\,z}{(x^2+y^2)\,\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4} \frac{b\,m\,q}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, 0 \right]$$

$$\textit{Helicity AdotB} = 0$$

$$\textit{Poincare II} = 2(E.B) = 0$$

$$\textit{coefficient of Topological Parity 4-form} = 0$$

$$\textit{Pfaff Topological Dimension} \quad PTD = 3$$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************************

$$\textit{Xm or linear (Mean) curvature} = 0$$

$$\textit{Yg or quadratic (GAUSS) curvature} = -\frac{b^2\,m^2}{(x^2+y^2)^2}$$

$$\textit{Za or Cubic (Interaction internal energy) curvature} = 0$$

$$\textit{Tk or quartic (4D expansion) curvature} = 0$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\textit{Chirality factor } CH = 0$$

$$D\,field = \left[\frac{1}{4} \frac{q\,x}{\pi\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4} \frac{q\,y}{\pi\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4} \frac{q\,z}{\pi\,(x^2+y^2+z^2)^{3/2}} \right]$$

$$H\,field = [0, 0, 0]$$

$$\textit{Poynting vector } ExH = EXH$$

$$\textit{Amperian Current 4Vector} \quad curlH\text{-}dD/dt\text{=}J4 = [0, 0, 0, 0]$$

$$\textit{Amerian charge density} \quad divD = rho = 0$$

$$\textit{divergence Lorentz Current 4Vector,} \quad 4div(J4) = 0$$

$$\textit{Topological SPIN 4 vector } S4 = \left[\frac{1}{16} \frac{q^2\,x}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon}, \frac{1}{16} \frac{q^2\,y}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon}, \frac{1}{16} \frac{q^2\,z}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon}, 0 \right]$$

$$\textit{Topological SPIN 3-form} = \frac{1}{16}\,\frac{q^2\,x\,\&\wedge(d(y),d(z),d(t))}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon} - \frac{1}{16}\,\frac{q^2\,y\,\&\wedge(d(x),d(z),d(t))}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon}$$

$$+\frac{1}{16}\,\frac{q^2\,z\,\&\wedge(d(x),d(y),d(t))}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon}$$

$$\textit{Spin density rho\_spin} = 0$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{1}{16}\,\frac{q^2}{\varepsilon\,(x^2+y^2+z^2)^2\,\pi^2}$$

$$B.H = 0$$

$$D.E = \frac{1}{16}\,\frac{q^2}{\varepsilon\,(x^2+y^2+z^2)^2\,\pi^2}$$

$$A.J = 0$$

$$\textit{-rho.phi} = 0$$

$$\textit{Poincare I} \qquad \textit{(B.H - D.E)-(A.J - rho.phi)} = -\frac{1}{16}\,\frac{q^2}{\varepsilon\,(x^2+y^2+z^2)^2\,\pi^2}$$

$$\textit{London Coefficient} \qquad LC = 0$$

$$\textit{PROCA coefficient curlcurlB} = [0, 0, 0]$$

$$\textit{Amperian Current 4Vector} \qquad \textit{curlH-dD/dt=J4} = [0, 0, 0, 0]$$

$$\textit{Lorentz Force 3 vector due to Ampere current} \quad \textit{FL = -(rho\_ampere E + J\_ampere x B)} = [0, 0, 0]$$

$$\textit{Amperian Dissipation Jampere dot E} = 0$$

$$\textit{Lorentz Force Spin factor LFSPIN} = 0$$

$$\textit{Topological Torsion current 4 vector} \quad \textit{T4 = -[ExA + B.phi,AdotB]} = \Big[$$

$$-\frac{1}{4}\,\frac{b\,x\,m\,q\,z}{(x^2+y^2)\,\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}},\ -\frac{1}{4}\,\frac{b\,m\,y\,q\,z}{(x^2+y^2)\,\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}},$$

$$\frac{1}{4}\,\frac{b\,m\,q}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}},\ 0\Big]$$

$$\textit{Lorentz Force 3 vector due to Torsion current} \quad \textit{TF = -(rho\_torsion E + J\_torsion x B)} = [0, 0, 0]$$

$$\textit{Torsion Dissipation Jtorsion dot E} = 0$$

$$\textit{Topological Spin current 4 vector} \quad \textit{TS4 = -[A x H + D.phi,AdotD]} = \Big[\frac{1}{16}\,\frac{q^2\,x}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon},$$

$$\frac{1}{16}\frac{q^2 y}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, \frac{1}{16}\frac{q^2 z}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, 0\Bigg]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= [0, 0, 0]$

Spin Dissipation J_spin dot E $= \frac{1}{64}\frac{q^3}{(x^2+y^2+z^2)^{5/2}\pi^3\varepsilon^2}$

Disspative Force 3 vector $= [0, 0, 0]$

Dissipation $= -\frac{1}{4}\frac{b\, x\, m\, q\, z}{(x^2+y^2)\,\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}$

******************************** *END PROCEDURE* ****************************** **(6)**

Note that there is a London current factor (an Amperian current proportional to A) and a Spin current with the Lorentz Force proportional to the Spin current, LF is proportional to Spin

---

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`-- wave rotation vol4 p159.`:

> Ax:=(y)*cos(-k*z+omega*t)/(1+x^2+y^2+z^2);Ay:=-x*cos(-k*z+omega*t)/(1+x^2+y^2+z^2);
> Az:=k*cos(-k*z+omega*t); phi:=omega*cos(-k*z+omega*t);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

$$Ax := \frac{y\cos(k z-\omega t)}{1+x^2+y^2+z^2}$$

$$Ay := -\frac{x\cos(k z-\omega t)}{1+x^2+y^2+z^2}$$

$$Az := k\cos(k z-\omega t)$$

$$\phi := \omega\cos(k z-\omega t)$$

-- wave rotation vol4 p159.

*********************************** *Differential Form Format* *******************************

****

$$\text{Action 1-form} = \frac{\cos(kz-\omega t)\,(-\omega-\omega x^2-\omega y^2-\omega z^2)\,d(t)}{1+x^2+y^2+z^2} + \frac{y\cos(kz-\omega t)\,d(x)}{1+x^2+y^2+z^2} - \frac{x\cos(kz-\omega t)\,d(y)}{1+x^2+y^2+z^2} + \frac{\cos(kz-\omega t)\,(k+kx^2+ky^2+kz^2)\,d(z)}{1+x^2+y^2+z^2}$$

$$\text{Intensity 2-form } F=dA \;= -\frac{y\sin(kz-\omega t)\,\omega\,(d(x))\,\&\hat{}\,(d(t))}{1+x^2+y^2+z^2} + \left(-\frac{\cos(kz-\omega t)\,(1+x^2-y^2+z^2)}{(1+x^2+y^2+z^2)^2} + \frac{\cos(kz-\omega t)\,(-1+x^2-y^2-z^2)}{(1+x^2+y^2+z^2)^2}\right)(d(x))\,\&\hat{}\,($$

$$\%1 = \sin(kz-\omega t)\,k + \sin(kz-\omega t)\,kx^2 + \sin(kz-\omega t)\,ky^2 + \sin(k$$

$$\text{Topological Torsion 3-form} \quad A\hat{}F = \frac{2\cos(kz-\omega t)^2\,\omega\,(1+z^2)\,\&\hat{}(d(x),d(y),d(t))}{(1+x^2+y^2+z^2)^2} - \frac{2\cos(kz-\omega t)^2\,k\,(1+z^2)\,\&\hat{}(d(x),d(y),d(z))}{(1+x^2+y^2+z^2)^2} + \left(\frac{\cos(kz-\omega t)}{1+x}\right.$$

$$\%1 = \sin(kz-\omega t)\,k + \sin(kz-\omega t)\,kx^2 + \sin(k$$

$$\text{Topological Parity 4-form} \qquad F\hat{}F = 0$$

******************************** *Using EM format* ********************************

$$E\,field = \left[-\frac{y\sin(kz-\omega t)\,\omega}{1+x^2+y^2+z^2}, \frac{x\sin(kz-\omega t)\,\omega}{1+x^2+y^2+z^2}, 0\right]$$

$$B\,field = \left[-\frac{x(\%1)}{(1+x^2+y^2+z^2)^2}, -\frac{y(\%1)}{(1+x^2+y^2+z^2)^2}, -\frac{2\cos(kz-\omega t)\,(1+z^2)}{(1+x^2+y^2+z^2)^2}\right]$$

$$\%1 = \sin(kz-\omega t)\,k + \sin(kz-\omega t)\,kx^2 + \sin(kz-\omega t)\,ky^2 + \sin(kz-\omega t)\,kz^2 + 2\cos(kz-\omega t)\,z$$

$$\text{Topological TORSION 4 vector} \quad T4 = -[ExA + Bphi, AdotB] = \left[\frac{2\cos(kz-\omega t)^2\,\omega xz}{(1+x^2+y^2+z^2)^2}, \frac{2\cos(kz-\omega t)^2\,\omega yz}{(1+x^2+y^2+z^2)^2}, \frac{2\cos(kz-\omega t)^2\,\omega\,(1+z^2)}{(1+x^2+y^2+z^2)^2}, \frac{2\cos(kz-\omega t)^2\,k\,(1+z^2)}{(1+x^2+y^2+z^2)^2}\right]$$

$$\text{Helicity AdotB} = -\frac{2\cos(kz-\omega t)^2\,k\,(1+z^2)}{(1+x^2+y^2+z^2)^2}$$

$$\text{Poincare II} = 2(E.B) = 0$$

$$\text{coefficient of Topological Parity 4-form} = 0$$

$$\text{Pfaff Topological Dimension} \qquad PTD = 3$$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

$$\text{Xm or linear (Mean) curvature} = -\sin(kz-\omega t)\,(k^2+\omega^2)$$

$$\text{Yg or quadratic (GAUSS) curvature} = -\frac{\cos(kz-\omega t)^2\,(x^2+y^2-1-z^2)}{(1+x^2+y^2+z^2)^3}$$

Za or Cubic (Interaction internal energy) curvature

$$= \frac{\cos(kz-\omega t)^2 \sin(kz-\omega t)\,(x^2+y^2-1-z^2)\,(k^2+\omega^2)}{(1+x^2+y^2+z^2)^3}$$

Tk or quartic (4D expansion) curvature $=0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $=0$

$$D\ field = \left[-\frac{\varepsilon\, y \sin(kz-\omega t)\,\omega}{1+x^2+y^2+z^2}, \frac{\varepsilon\, x \sin(kz-\omega t)\,\omega}{1+x^2+y^2+z^2}, 0 \right]$$

$$H\ field = \left[-\frac{x\,(\%1)}{(1+x^2+y^2+z^2)^2\,\mu}, -\frac{y\,(\%1)}{(1+x^2+y^2+z^2)^2\,\mu}, -\frac{2\cos(kz-\omega t)\,(1+z^2)}{(1+x^2+y^2+z^2)^2\,\mu} \right]$$

$$\%1 = \sin(kz-\omega t)\,k + \sin(kz-\omega t)\,k x^2 + \sin(kz-\omega t)\,k y^2 + \sin(kz-\omega t)\,k z^2 + 2\cos(kz-\omega t)\,z$$

$$Poynting\ vector\ ExH = \left[-\frac{2x\sin(kz-\omega t)\,\omega\cos(kz-\omega t)\,(1+z^2)}{(1+x^2+y^2+z^2)^3\,\mu}, -\frac{2y\sin(kz-\omega t)\,\omega\cos(kz-\omega t)\,(1+z^2)}{(1+x^2+y^2+z^2)^3\,\mu}, \frac{1}{(1+x^2+y^2+z^2)^3\,\mu}\left(\sin(kz-\omega t)\,\omega\,(\sin(kz-\omega t)\,k + \sin(kz-\omega t)\,k x^2 + \sin(kz-\omega t)\,k y^2 + \sin(kz-\omega t)\,k z^2 + 2\cos(kz-\omega t)\,z)\,(x^2+y^2)\right) \right]$$

$$\%13 = -10\cos(kz-\omega t) - k^2\cos(kz-\omega t) - 2k^2\cos(kz-\omega t)\,x^2 - 2k^2\cos(kz-\omega t)\,y^2 + 4\sin(kz-\omega t)\,kz$$

Amerian charge density $\quad$ *divD = rho* $= 0$

divergence Lorentz Current 4Vector, $\quad$ *4div(J4)* $= 0$

$$\textit{Topological SPIN 4 vector S4} = \Bigg[\frac{1}{(1+x^2+y^2+z^2)^3\,\mu} \Big(\cos(kz-\omega t)\,\big(2x\cos(kz-\omega t)$$

$$+ 2k^2y^3\sin(kz-\omega t) + 2k^2y\sin(kz-\omega t)\,x^2 + 2ky\cos(kz-\omega t)\,z + 2k^2y\sin(kz-\omega t)\,z^2$$

$$+ k^2y\sin(kz-\omega t) + k^2y^5\sin(kz-\omega t) - 2\varepsilon y\sin(kz-\omega t)\,\omega^2\mu x^2z^2 + k^2y\sin(kz-\omega t)\,x^4$$

$$+ 2k^2y^3\sin(kz-\omega t)\,x^2 + 2k^2y^3\sin(kz-\omega t)\,z^2 + k^2y\sin(kz-\omega t)\,z^4 + 2ky^3\cos(kz-\omega t)\,z$$

$$+ 2ky\cos(kz-\omega t)\,z^3 + 2\cos(kz-\omega t)\,xz^2 + 2ky\cos(kz-\omega t)\,zx^2 - \varepsilon y^5\sin(kz-\omega t)\,\omega^2\mu$$

$$-\varepsilon y \sin(kz-\omega t)\,\omega^2\mu - 2\varepsilon y^3 \sin(kz-\omega t)\,\omega^2\mu + 2k^2 y \sin(kz-\omega t)\,x^2 z^2 - 2\varepsilon y \sin(kz$$

$$-\omega t)\,\omega^2\mu x^2 - 2\varepsilon y \sin(kz-\omega t)\,\omega^2\mu z^2 - \varepsilon y \sin(kz-\omega t)\,\omega^2\mu x^4 - 2\varepsilon y^3 \sin(kz-\omega t)\,\omega^2\mu x^2$$

$$-2\varepsilon y^3 \sin(kz-\omega t)\,\omega^2\mu z^2 - \varepsilon y \sin(kz-\omega t)\,\omega^2\mu z^4)\Big), -\frac{1}{(1+x^2+y^2+z^2)^3\mu}\Big(\cos(kz$$

$$-\omega t)\,\big(2k^2x^3\sin(kz-\omega t) + k^2x^5\sin(kz-\omega t) - 2y\cos(kz-\omega t) + 2kx\cos(kz-\omega t)\,z$$

$$+2k^2x\sin(kz-\omega t)\,z^2 + 2k^2x\sin(kz-\omega t)\,y^2 + k^2x\sin(kz-\omega t) - 2\varepsilon x\sin(kz$$

$$-\omega t)\,\omega^2\mu y^2z^2 - 2\cos(kz-\omega t)\,yz^2 + 2kx\cos(kz-\omega t)\,zy^2 + 2k^2x\sin(kz-\omega t)\,y^2z^2$$

$$-\varepsilon x^5\sin(kz-\omega t)\,\omega^2\mu - \varepsilon x\sin(kz-\omega t)\,\omega^2\mu - 2\varepsilon x^3\sin(kz-\omega t)\,\omega^2\mu - 2\varepsilon x\sin(kz$$

$$-\omega t)\,\omega^2\mu y^2 - 2\varepsilon x\sin(kz-\omega t)\,\omega^2\mu z^2 - 2\varepsilon x^3\sin(kz-\omega t)\,\omega^2\mu y^2 - 2\varepsilon x^3\sin(kz$$

$$-\omega t)\,\omega^2\mu z^2 - \varepsilon x\sin(kz-\omega t)\,\omega^2\mu y^4 - \varepsilon x\sin(kz-\omega t)\,\omega^2\mu z^4 + 2k^2x^3\sin(kz-\omega t)\,y^2$$

$$+2k^2x^3\sin(kz-\omega t)\,z^2 + k^2x\sin(kz-\omega t)\,y^4 + k^2x\sin(kz-\omega t)\,z^4 + 2kx^3\cos(kz-\omega t)\,z$$

$$+2kx\cos(kz-\omega t)\,z^3\big)\Big), -\frac{1}{(1+x^2+y^2+z^2)^3\mu}\Big(\cos(kz-\omega t)\,\big(\sin(kz-\omega t)\,k + \sin(kz$$

$$-\omega t)\,kx^2 + \sin(kz-\omega t)\,ky^2 + \sin(kz-\omega t)\,kz^2 + 2\cos(kz-\omega t)\,z\big)\,(x^2+y^2)\Big),$$

$$-\frac{\cos(kz-\omega t)\,\varepsilon\sin(kz-\omega t)\,\omega\,(x^2+y^2)}{(1+x^2+y^2+z^2)^2}\Bigg]$$

Topological SPIN 3-form $= \frac{1}{(1+x^2+y^2+z^2)^3\mu}\Big(\cos(kz-\omega t)\,\big(2x\cos(kz-\omega t) + 2k^2y^3\sin(kz$

$$-\omega t) + 2k^2y\sin(kz-\omega t)\,x^2 + 2ky\cos(kz-\omega t)\,z + 2k^2y\sin(kz-\omega t)\,z^2 + k^2y\sin(kz$$

$$-\omega t) + k^2y^5\sin(kz-\omega t) - 2\varepsilon y\sin(kz-\omega t)\,\omega^2\mu x^2z^2 + k^2y\sin(kz-\omega t)\,x^4 + 2k^2y^3\sin(kz$$

$$-\omega t)\, x^2+2\,k^2 y^3 \sin(k z-\omega t)\, z^2+k^2 y \sin(k z-\omega t)\, z^4+2\,k y^3 \cos(k z-\omega t)\, z+2\,k y \cos(k z$$
$$-\omega t)\, z^3+2\cos(k z-\omega t)\, x z^2+2\,k y \cos(k z-\omega t)\, z x^2-\varepsilon y^5 \sin(k z-\omega t)\, \omega^2 \mu-\varepsilon y \sin(k z$$
$$-\omega t)\, \omega^2 \mu-2\,\varepsilon y^3 \sin(k z-\omega t)\, \omega^2 \mu+2\,k^2 y \sin(k z-\omega t)\, x^2 z^2-2\,\varepsilon y \sin(k z-\omega t)\, \omega^2 \mu x^2$$
$$-2\,\varepsilon y \sin(k z-\omega t)\, \omega^2 \mu z^2-\varepsilon y \sin(k z-\omega t)\, \omega^2 \mu x^4-2\,\varepsilon y^3 \sin(k z-\omega t)\, \omega^2 \mu x^2$$
$$-2\,\varepsilon y^3 \sin(k z-\omega t)\, \omega^2 \mu z^2-\varepsilon y \sin(k z-\omega t)\, \omega^2 \mu z^4)\ \&\char`^(d(y), d(z), d(t)))$$
$$+\frac{1}{(1+x^2+y^2+z^2)^3 \mu}\Big(\cos(k z-\omega t)\, \big(2\,k^2 x^3 \sin(k z-\omega t)+k^2 x^5 \sin(k z-\omega t)-2\,y \cos(k z$$
$$-\omega t)+2\,k x \cos(k z-\omega t)\, z+2\,k^2 x \sin(k z-\omega t)\, z^2+2\,k^2 x \sin(k z-\omega t)\, y^2+k^2 x \sin(k z$$
$$-\omega t)-2\,\varepsilon x \sin(k z-\omega t)\, \omega^2 \mu y^2 z^2-2\cos(k z-\omega t)\, y z^2+2\,k x \cos(k z-\omega t)\, z y^2$$
$$+2\,k^2 x \sin(k z-\omega t)\, y^2 z^2-\varepsilon x^5 \sin(k z-\omega t)\, \omega^2 \mu-\varepsilon x \sin(k z-\omega t)\, \omega^2 \mu-2\,\varepsilon x^3 \sin(k z$$
$$-\omega t)\, \omega^2 \mu-2\,\varepsilon x \sin(k z-\omega t)\, \omega^2 \mu y^2-2\,\varepsilon x \sin(k z-\omega t)\, \omega^2 \mu z^2-2\,\varepsilon x^3 \sin(k z-\omega t)\, \omega^2 \mu y^2$$
$$-2\,\varepsilon x^3 \sin(k z-\omega t)\, \omega^2 \mu z^2-\varepsilon x \sin(k z-\omega t)\, \omega^2 \mu y^4-\varepsilon x \sin(k z-\omega t)\, \omega^2 \mu z^4+2\,k^2 x^3 \sin(k z$$
$$-\omega t)\, y^2+2\,k^2 x^3 \sin(k z-\omega t)\, z^2+k^2 x \sin(k z-\omega t)\, y^4+k^2 x \sin(k z-\omega t)\, z^4+2\,k x^3 \cos(k z$$
$$-\omega t)\, z+2\,k x \cos(k z-\omega t)\, z^3\big)\ \&\char`^(d(x), d(z), d(t))\Big)-\frac{1}{(1+x^2+y^2+z^2)^3 \mu}\Big(\cos(k z$$
$$-\omega t)\, \big(\sin(k z-\omega t)\, k+\sin(k z-\omega t)\, k x^2+\sin(k z-\omega t)\, k y^2+\sin(k z-\omega t)\, k z^2+2\cos(k z$$
$$-\omega t)\, z\big)\, (x^2+y^2)\ \&\char`^(d(x), d(y), d(t))\Big)$$
$$+\frac{\cos(k z-\omega t)\, \varepsilon \sin(k z-\omega t)\, \omega\, (x^2+y^2)\ \&\char`^(d(x), d(y), d(z))}{(1+x^2+y^2+z^2)^2}$$

$$\textit{Spin density rho\_spin} = -\frac{\cos(k z-\omega t)\, \varepsilon \sin(k z-\omega t)\, \omega\, (x^2+y^2)}{(1+x^2+y^2+z^2)^2}$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{1}{(1+x^2+y^2+z^2)^4 \mu}\Big(4\cos(k z-\omega t)^2+8\cos(k z-\omega t)^2 z^2$$
$$+8\,x^2 \sin(k z-\omega t)\, k y^2 \cos(k z-\omega t)\, z+4\sin(k z-\omega t)\, k x^2 \cos(k z-\omega t)\, z+4\sin(k z$$
$$-\omega t)\, k y^2 \cos(k z-\omega t)\, z+4\,x^4 \sin(k z-\omega t)\, k \cos(k z-\omega t)\, z+4\,x^2 \sin(k z-\omega t)^2 k^2 y^2 z^2$$
$$+4\,x^2 \sin(k z-\omega t)\, k z^3 \cos(k z-\omega t)+4\,y^4 \sin(k z-\omega t)\, k \cos(k z-\omega t)\, z+4\,y^2 \sin(k z$$
$$-\omega t)\, k z^3 \cos(k z-\omega t)+k^2 \sin(k z-\omega t)^2 y^2+k^2 \sin(k z-\omega t)^2 x^2+2\sin(k z-\omega t)^2 k^2 x^4$$
$$+2\sin(k z-\omega t)^2 k^2 y^4+4\,x^2 \cos(k z-\omega t)^2 z^2+x^6 \sin(k z-\omega t)^2 k^2+4\,y^2 \cos(k z-\omega t)^2 z^2$$
$$+y^6 \sin(k z-\omega t)^2 k^2+4\cos(k z-\omega t)^2 z^4+4\sin(k z-\omega t)^2 k^2 x^2 y^2+2\sin(k z-\omega t)^2 k^2 x^2 z^2$$

$$+2\sin(kz-\omega t)^2 k^2 y^2 z^2+3x^2\sin(kz-\omega t)^2 k^2 y^4+x^2\sin(kz-\omega t)^2 k^2 z^4+3x^4\sin(kz$$
$$-\omega t)^2 k^2 y^2+2x^4\sin(kz-\omega t)^2 k^2 z^2+y^2\sin(kz-\omega t)^2 k^2 z^4+2y^4\sin(kz-\omega t)^2 k^2 z^2$$
$$-2\varepsilon\sin(kz-\omega t)^2\omega^2\mu x^2 z^2-3\varepsilon\sin(kz-\omega t)^2\omega^2\mu x^2 y^4-4\varepsilon\sin(kz-\omega t)^2\omega^2\mu x^2 y^2$$
$$-3\varepsilon\sin(kz-\omega t)^2\omega^2\mu x^4 y^2-\varepsilon\sin(kz-\omega t)^2\omega^2\mu z^4 x^2-2\varepsilon\sin(kz-\omega t)^2\omega^2\mu z^2 y^4$$
$$-\varepsilon\sin(kz-\omega t)^2\omega^2\mu z^4 y^2-2\varepsilon\sin(kz-\omega t)^2\omega^2\mu y^2 z^2-2\varepsilon\sin(kz-\omega t)^2\omega^2\mu x^4 z^2$$
$$-\varepsilon\sin(kz-\omega t)^2\omega^2\mu x^6-2\varepsilon\sin(kz-\omega t)^2\omega^2\mu x^4-2\varepsilon\sin(kz-\omega t)^2\omega^2\mu y^4-\varepsilon\sin(kz$$
$$-\omega t)^2\omega^2\mu y^2-\varepsilon\sin(kz-\omega t)^2\omega^2\mu y^6-\varepsilon\sin(kz-\omega t)^2\omega^2\mu x^2-4\varepsilon\sin(kz$$
$$-\omega t)^2\omega^2\mu x^2 z^2 y^2)$$

$$B.H=\frac{1}{(1+x^2+y^2+z^2)^4\mu}\left(4\cos(kz-\omega t)^2+8\cos(kz-\omega t)^2 z^2+8x^2\sin(kz-\omega t)\,k y^2\cos(kz\right.$$
$$-\omega t)\,z+4\sin(kz-\omega t)\,k x^2\cos(kz-\omega t)\,z+4\sin(kz-\omega t)\,k y^2\cos(kz-\omega t)\,z+4x^4\sin(kz$$
$$-\omega t)\,k\cos(kz-\omega t)\,z+4x^2\sin(kz-\omega t)^2 k^2 y^2 z^2+4x^2\sin(kz-\omega t)\,k z^3\cos(kz-\omega t)$$
$$+4y^4\sin(kz-\omega t)\,k\cos(kz-\omega t)\,z+4y^2\sin(kz-\omega t)\,k z^3\cos(kz-\omega t)+k^2\sin(kz-\omega t)^2 y^2$$
$$+k^2\sin(kz-\omega t)^2 x^2+2\sin(kz-\omega t)^2 k^2 x^4+2\sin(kz-\omega t)^2 k^2 y^4+4x^2\cos(kz-\omega t)^2 z^2$$
$$+x^6\sin(kz-\omega t)^2 k^2+4y^2\cos(kz-\omega t)^2 z^2+y^6\sin(kz-\omega t)^2 k^2+4\cos(kz-\omega t)^2 z^4$$
$$+4\sin(kz-\omega t)^2 k^2 x^2 y^2+2\sin(kz-\omega t)^2 k^2 x^2 z^2+2\sin(kz-\omega t)^2 k^2 y^2 z^2+3x^2\sin(kz$$
$$-\omega t)^2 k^2 y^4+x^2\sin(kz-\omega t)^2 k^2 z^4+3x^4\sin(kz-\omega t)^2 k^2 y^2+2x^4\sin(kz-\omega t)^2 k^2 z^2$$
$$\left.+y^2\sin(kz-\omega t)^2 k^2 z^4+2y^4\sin(kz-\omega t)^2 k^2 z^2\right)$$

$$D.E=\frac{\varepsilon\sin(kz-\omega t)^2\omega^2(x^2+y^2)}{(1+x^2+y^2+z^2)^2}$$

$$A.J=-\frac{1}{(1+x^2+y^2+z^2)^4\mu}\left(\cos(kz-\omega t)\left(-10\cos(kz-\omega t)-k^2\cos(kz-\omega t)-2k^2\cos(kz\right.\right.$$
$$-\omega t)\,x^2-2k^2\cos(kz-\omega t)\,y^2+4\sin(kz-\omega t)\,kz-2k^2\cos(kz-\omega t)\,z^2-\cos(kz-\omega t)\,k^2x^4$$
$$-\cos(kz-\omega t)\,k^2y^4-\cos(kz-\omega t)\,k^2z^4+4\sin(kz-\omega t)\,kz^3-2y^2\cos(kz-\omega t)-2x^2\cos(kz$$
$$-\omega t)-2\cos(kz-\omega t)\,z^2+4x^2\sin(kz-\omega t)\,kz+4y^2\sin(kz-\omega t)\,kz-2\cos(kz$$
$$-\omega t)\,k^2x^2y^2-2\cos(kz-\omega t)\,k^2x^2z^2-2\cos(kz-\omega t)\,k^2y^2z^2+\varepsilon\omega^2\cos(kz-\omega t)\,\mu$$
$$+2\varepsilon\omega^2\cos(kz-\omega t)\,\mu x^2+2\varepsilon\omega^2\cos(kz-\omega t)\,\mu y^2+2\varepsilon\omega^2\cos(kz-\omega t)\,\mu z^2+\varepsilon\omega^2\cos(kz$$
$$-\omega t)\,\mu x^4+\varepsilon\omega^2\cos(kz-\omega t)\,\mu y^4+\varepsilon\omega^2\cos(kz-\omega t)\,\mu z^4+2\varepsilon\omega^2\cos(kz-\omega t)\,\mu x^2y^2$$

$$+2\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)\,\mu\,x^2 z^2+2\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)\,\mu\,y^2 z^2\big)\,(x^2+y^2)\big)\big)$$

$$\textit{-rho.phi}=0$$

Poincare I $\quad$ *(B.H - D.E)-(A.J - rho.phi)* $=\dfrac{1}{(1+x^2+y^2+z^2)^4\,\mu}\big(-10\cos(k\,z-\omega\,t)^2 y^2-10\cos(k\,z-\omega\,t)^2 x^2+4\cos(k\,z-\omega\,t)^2+8\cos(k\,z-\omega\,t)^2 z^2+16\,x^2\sin(k\,z-\omega\,t)\,k\,y^2\cos(k\,z-\omega\,t)\,z+8\sin(k\,z-\omega\,t)\,k\,x^2\cos(k\,z-\omega\,t)\,z+8\sin(k\,z-\omega\,t)\,k\,y^2\cos(k\,z-\omega\,t)\,z+8\,x^4\sin(k\,z-\omega\,t)\,k\cos(k\,z-\omega\,t)\,z+4\,x^2\sin(k\,z-\omega\,t)^2 k^2 y^2 z^2+8\,x^2\sin(k\,z-\omega\,t)\,k\,z^3\cos(k\,z-\omega\,t)+8\,y^4\sin(k\,z-\omega\,t)\,k\cos(k\,z-\omega\,t)\,z+8\,y^2\sin(k\,z-\omega\,t)\,k\,z^3\cos(k\,z-\omega\,t)+k^2\sin(k\,z-\omega\,t)^2 y^2+k^2\sin(k\,z-\omega\,t)^2 x^2+2\sin(k\,z-\omega\,t)^2 k^2 x^4+2\sin(k\,z-\omega\,t)^2 k^2 y^4+2\,x^2\cos(k\,z-\omega\,t)^2 z^2+x^6\sin(k\,z-\omega\,t)^2 k^2+2\,y^2\cos(k\,z-\omega\,t)^2 z^2+y^6\sin(k\,z-\omega\,t)^2 k^2+4\cos(k\,z-\omega\,t)^2 z^4+4\sin(k\,z-\omega\,t)^2 k^2 x^2 y^2+2\sin(k\,z-\omega\,t)^2 k^2 x^2 z^2+2\sin(k\,z-\omega\,t)^2 k^2 y^2 z^2+3\,x^2\sin(k\,z-\omega\,t)^2 k^2 y^4+x^2\sin(k\,z-\omega\,t)^2 k^2 z^4+3\,x^4\sin(k\,z-\omega\,t)^2 k^2 y^2+2\,x^4\sin(k\,z-\omega\,t)^2 k^2 z^2+y^2\sin(k\,z-\omega\,t)^2 k^2 z^4+2\,y^4\sin(k\,z-\omega\,t)^2 k^2 z^2-2\,\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,x^2 z^2-3\,\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,x^2 y^4-4\,\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,x^2 y^2-3\,\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,x^4 y^2-\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,z^4 x^2-2\,\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,z^2 y^4-\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,z^4 y^2-2\,\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,y^2 z^2-2\,\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,x^4 z^2-\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,x^6-2\,\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,x^4-2\,\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,y^4-\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,y^2-\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,y^6-\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,x^2-4\,\varepsilon\sin(k\,z-\omega\,t)^2\omega^2\mu\,x^2 z^2 y^2+4\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,x^2 z^2 y^2-4\cos(k\,z-\omega\,t)^2 k^2 x^2 y^2-2\cos(k\,z-\omega\,t)^2 k^2 x^2 z^2-2\cos(k\,z-\omega\,t)^2 k^2 y^2 z^2-3\cos(k\,z-\omega\,t)^2 k^2 x^4 y^2-2\cos(k\,z-\omega\,t)^2 k^2 x^4 z^2-3\cos(k\,z-\omega\,t)^2 k^2 x^2 y^4-\cos(k\,z-\omega\,t)^2 k^2 x^2 z^4-2\cos(k\,z-\omega\,t)^2 k^2 y^4 z^2-\cos(k\,z-\omega\,t)^2 k^2 y^2 z^4-4\cos(k\,z-\omega\,t)^2 k^2 x^2 y^2 z^2+\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,x^2+\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,y^2+2\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,x^4+2\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,y^4+\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,x^6+\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,y^6+4\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,x^2 y^2+2\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,x^2 z^2+2\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,y^2 z^2+3\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,x^4 y^2+3\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,x^2 y^4+2\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,x^4 z^2+2\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,y^4 z^2+\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,z^4 x^2+\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\mu\,z^4 y^2-2\cos(k\,z-\omega\,t)^2 y^4-2\cos(k\,z-\omega\,t)^2 x^4-k^2\cos(k\,z-\omega\,t)^2 x^2-k^2\cos(k\,z-\omega\,t)^2 y^2-2\cos(k\,z-\omega\,t)^2 k^2 x^4-2\cos(k\,z-\omega\,t)^2 k^2 y^4-\cos(k\,z-\omega\,t)^2 k^2 x^6-\cos(k\,z-\omega\,t)^2 k^2 y^6-4\,y^2\cos(k\,z-\omega\,t)^2 x^2\big)$

London Coefficient $LC = 0$

$$PROCA\ coefficient\ curlcurlB = \left[-\frac{x(\%21)}{(1+x^2+y^2+z^2)^4}, -\frac{y(\%21)}{(1+x^2+y^2+z^2)^4} \right.$$

$$\%21 = 14\sin(k\,z - \omega\,t)\,k + 20\sin(k\,z - \omega\,t)\,k\,x^2 + 20\sin(k\,z - \omega\,t)\,k\,y^2 + 4\sin(k\,z - \omega\,t)\,k\,z^2 + 56\cos(k\,z - \omega\,t$$

$$\%13 = -10\cos(k\,z - \omega\,t) - k^2\cos(k\,z - \omega\,t) - 2\,k^2\cos(k\,z - \omega\,t)\,x^2 - 2\,k^2\cos(k\,z - \omega\,t)\,y^2 + 4\sin(k\,z - \omega\,t)\,k\,z$$

$$\%13 = -10\cos(kz-\omega t) - k^2\cos(kz-\omega t) - 2k^2\cos(kz-\omega t)\,x^2 - 2k^2\cos(kz-\omega t)\,y^2 + 4\sin(kz-\omega t)\,kz$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] $= \left[\frac{2\cos(kz-\omega t)^2\,\omega\,x\,z}{(1+x^2+y^2+z^2)^2}, \frac{2\cos(kz-\omega t)^2\,\omega\,y\,z}{(1+x^2+y^2+z^2)^2}, \frac{2\cos(kz-\omega t)^2\,\omega\,(1+z^2)}{(1+x^2+y^2+z^2)^2}, \frac{2\cos(kz-\omega t)^2\,k\,(1+z^2)}{(1+x^2+y^2+z^2)^2}\right]$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] $= \left[\frac{1}{(1+x^2+y^2+z^2)^3\,\mu}\Big(\cos(kz\right.$

$$-\omega t)\,\big(2x\cos(kz-\omega t) + 2k^2y^3\sin(kz-\omega t) + 2k^2y\sin(kz-\omega t)\,x^2 + 2ky\cos(kz-\omega t)\,z$$

$$+2\,k^2 y \sin(k z-\omega t)\, z^2+k^2 y \sin(k z-\omega t)+k^2 y^5 \sin(k z-\omega t)-2\,\varepsilon y \sin(k z-\omega t)\,\omega^2 \mu x^2 z^2$$

$$+k^2 y \sin(k z-\omega t)\, x^4+2\,k^2 y^3 \sin(k z-\omega t)\, x^2+2\,k^2 y^3 \sin(k z-\omega t)\, z^2+k^2 y \sin(k z-\omega t)\, z^4$$

$$+2\,k y^3 \cos(k z-\omega t)\, z+2\,k y \cos(k z-\omega t)\, z^3+2 \cos(k z-\omega t)\, x z^2+2\,k y \cos(k z-\omega t)\, z x^2$$

$$-\varepsilon y^5 \sin(k z-\omega t)\,\omega^2 \mu-\varepsilon y \sin(k z-\omega t)\,\omega^2 \mu-2\,\varepsilon y^3 \sin(k z-\omega t)\,\omega^2 \mu+2\,k^2 y \sin(k z$$

$$-\omega t)\, x^2 z^2-2\,\varepsilon y \sin(k z-\omega t)\,\omega^2 \mu x^2-2\,\varepsilon y \sin(k z-\omega t)\,\omega^2 \mu z^2-\varepsilon y \sin(k z-\omega t)\,\omega^2 \mu x^4$$

$$-2\,\varepsilon y^3 \sin(k z-\omega t)\,\omega^2 \mu x^2-2\,\varepsilon y^3 \sin(k z-\omega t)\,\omega^2 \mu z^2-\varepsilon y \sin(k z-\omega t)\,\omega^2 \mu z^4)),$$

$$-\frac{1}{(1+x^2+y^2+z^2)^3 \mu}\left(\cos(k z-\omega t)\,(2\,k^2 x^3 \sin(k z-\omega t)+k^2 x^5 \sin(k z-\omega t)-2\,y \cos(k z\right.$$

$$-\omega t)+2\,k x \cos(k z-\omega t)\, z+2\,k^2 x \sin(k z-\omega t)\, z^2+2\,k^2 x \sin(k z-\omega t)\, y^2+k^2 x \sin(k z$$

$$-\omega t)-2\,\varepsilon x \sin(k z-\omega t)\,\omega^2 \mu y^2 z^2-2 \cos(k z-\omega t)\, y z^2+2\,k x \cos(k z-\omega t)\, z y^2$$

$$+2\,k^2 x \sin(k z-\omega t)\, y^2 z^2-\varepsilon x^5 \sin(k z-\omega t)\,\omega^2 \mu-\varepsilon x \sin(k z-\omega t)\,\omega^2 \mu-2\,\varepsilon x^3 \sin(k z$$

$$-\omega t)\,\omega^2 \mu-2\,\varepsilon x \sin(k z-\omega t)\,\omega^2 \mu y^2-2\,\varepsilon x \sin(k z-\omega t)\,\omega^2 \mu z^2-2\,\varepsilon x^3 \sin(k z-\omega t)\,\omega^2 \mu y^2$$

$$-2\,\varepsilon x^3 \sin(k z-\omega t)\,\omega^2 \mu z^2-\varepsilon x \sin(k z-\omega t)\,\omega^2 \mu y^4-\varepsilon x \sin(k z-\omega t)\,\omega^2 \mu z^4+2\,k^2 x^3 \sin(k z$$

$$-\omega t)\, y^2+2\,k^2 x^3 \sin(k z-\omega t)\, z^2+k^2 x \sin(k z-\omega t)\, y^4+k^2 x \sin(k z-\omega t)\, z^4+2\,k x^3 \cos(k z$$

$$-\omega t)\, z+2\,k x \cos(k z-\omega t)\, z^3)),\ -\frac{1}{(1+x^2+y^2+z^2)^3 \mu}\left(\cos(k z-\omega t)\,(\sin(k z-\omega t)\, k\right.$$

$$+\sin(k z-\omega t)\, k x^2+\sin(k z-\omega t)\, k y^2+\sin(k z-\omega t)\, k z^2+2 \cos(k z-\omega t)\, z)\,(x^2+y^2)),$$

$$- \frac{\cos(k z - \omega t)\ \varepsilon \sin(k z - \omega t)\ \omega\ (x^2 + y^2)}{(1 + x^2 + y^2 + z^2)^2} \Bigg]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B)

$$= \Bigg[\frac{1}{(1 + x^2 + y^2 + z^2)^5\ \mu} \big(\cos(k z - \omega t)\ \big(4 \cos(k z - \omega t)^2 z^2 y^3 + \sin(k z - \omega t)^2 k^2 y^7 - 2 \cos(k z$$

$$- \omega t)\ k^2 x^5 \sin(k z - \omega t) - 4 k x^3 \cos(k z - \omega t)^2 z^3 - 4 k x \cos(k z - \omega t)^2 z^5 - 2 \sin(k z$$

$$- \omega t) \cos(k z - \omega t)\ k^2 x - 4 \sin(k z - \omega t) \cos(k z - \omega t)\ k^2 x^3 - 4 \cos(k z - \omega t)^2 k x z$$

$$- 8 \cos(k z - \omega t)^2 k x z^3 + 2 \sin(k z - \omega t)^2 k^2 y^3 z^2 + 3 \sin(k z - \omega t)^2 k^2 x^4 y^3 + 3 \sin(k z$$

$$- \omega t)^2 k^2 x^2 y^5 + 2 \sin(k z - \omega t)^2 k^2 y^5 z^2 + y \sin(k z - \omega t)^2 k^2 x^2 + 2 y \sin(k z - \omega t)^2 k^2 x^4$$

$$+ 4 y \cos(k z - \omega t)^2 z^2 x^2 + y \sin(k z - \omega t)^2 k^2 x^6 + \sin(k z - \omega t)^2 k^2 z^4 y^3 + 2 \cos(k z$$

$$- \omega t)\ \varepsilon x \sin(k z - \omega t)\ \omega^2 \mu y^4 z^2 + 4 \cos(k z - \omega t)\ \varepsilon x \sin(k z - \omega t)\ \omega^2 \mu y^2 z^4 - 8 \cos(k z$$

$$- \omega t)\ k^2 x^3 \sin(k z - \omega t)\ z^2 + 8 \cos(k z - \omega t)\ \varepsilon x \sin(k z - \omega t)\ \omega^2 \mu y^2 z^2 - 4 \cos(k z$$

$$- \omega t)\ k^2 x^3 \sin(k z - \omega t)\ y^2 - 2 \varepsilon \sin(k z - \omega t)^2 \omega^2 y \mu x^2 z^2 - 2 \varepsilon \sin(k z - \omega t)^2 \omega^2 y \mu x^4 z^2$$

$$- 4 \varepsilon \sin(k z - \omega t)^2 \omega^2 y^3 \mu x^2 z^2 - \varepsilon \sin(k z - \omega t)^2 \omega^2 y \mu x^2 z^4 + 4 \cos(k z - \omega t)\ \varepsilon x^3 \sin(k z$$

$$- \omega t)\ \omega^2 \mu y^2 z^2 - 6 \sin(k z - \omega t) \cos(k z - \omega t)\ k^2 x z^2 - 6 \sin(k z - \omega t) \cos(k z - \omega t)\ k^2 x z^4$$

$$- 4 \sin(k z - \omega t) \cos(k z - \omega t)\ k^2 x y^2 - 2 \cos(k z - \omega t)\ k^2 x \sin(k z - \omega t)\ y^4 - 4 k x \cos(k z$$

$$- \omega t)^2 z y^2 - 4 k x \cos(k z - \omega t)^2 z^3 y^2 - 4 \cos(k z - \omega t)\ k^2 x^3 \sin(k z - \omega t)\ z^4 - 2 \cos(k z$$

$- \omega t)\ k^2 x \sin(k z - \omega t)\ z^6 - 2 \cos(k z - \omega t)\ k^2 x^5 \sin(k z - \omega t)\ z^2 + 4 \sin(k z - \omega t)\ k y^3 \cos(k z$

$- \omega t)\ z + 4 \sin(k z - \omega t)\ k y^5 \cos(k z - \omega t)\ z + 4 \sin(k z - \omega t)^2\ k^2 x^2 z^2 y^3 + 4 \sin(k z$

$- \omega t)\ k z^3 \cos(k z - \omega t)\ y^3 + 2 y \sin(k z - \omega t)^2\ k^2 x^2 z^2 + 2 y \sin(k z - \omega t)^2\ k^2 x^4 z^2 + y \sin(k z$

$- \omega t)^2\ k^2 z^4 x^2 - \varepsilon \sin(k z - \omega t)^2\ \omega^2 y \mu x^2 - 2 \varepsilon \sin(k z - \omega t)^2\ \omega^2 y \mu x^4 - 4 \varepsilon \sin(k z$

$- \omega t)^2\ \omega^2 y^3 \mu x^2 - \varepsilon \sin(k z - \omega t)^2\ \omega^2 y \mu x^6 - 3 \varepsilon \sin(k z - \omega t)^2\ \omega^2 y^3 \mu x^4 - 3 \varepsilon \sin(k z$

$- \omega t)^2\ \omega^2 y^5 \mu x^2 - 2 \varepsilon \sin(k z - \omega t)^2\ \omega^2 y^3 \mu z^2 - 2 \varepsilon \sin(k z - \omega t)^2\ \omega^2 y^5 \mu z^2 - \varepsilon \sin(k z$

$- \omega t)^2\ \omega^2 y^3 \mu z^4 + 2 \sin(k z - \omega t)^2\ k^2 y^5 + 4 y \cos(k z - \omega t)^2 + 8 \cos(k z - \omega t)^2\ y z^2$

$+ 4 \cos(k z - \omega t)^2\ y z^4 + \sin(k z - \omega t)^2\ k^2 y^3 + 4 \cos(k z - \omega t)\ \varepsilon x^3 \sin(k z - \omega t)\ \omega^2 \mu y^2$

$+ 8 \cos(k z - \omega t)\ \varepsilon x^3 \sin(k z - \omega t)\ \omega^2 \mu z^2 + 2 \cos(k z - \omega t)\ \varepsilon x \sin(k z - \omega t)\ \omega^2 \mu y^4 + 4 \cos(k z$

$- \omega t)\ \varepsilon x^3 \sin(k z - \omega t)\ \omega^2 \mu z^4 + 2 \cos(k z - \omega t)\ \varepsilon x \sin(k z - \omega t)\ \omega^2 \mu z^6 + 2 \cos(k z$

$- \omega t)\ \varepsilon x^5 \sin(k z - \omega t)\ \omega^2 \mu z^2 + 6 \sin(k z - \omega t)\ \cos(k z - \omega t)\ \mu x z^2 \varepsilon \omega^2 + 6 \sin(k z$

$- \omega t)\ \cos(k z - \omega t)\ \mu x z^4 \varepsilon \omega^2 + 4 \sin(k z - \omega t)\ \cos(k z - \omega t)\ \mu x y^2 \varepsilon \omega^2 + 4 \sin(k z$

$- \omega t)^2\ k^2 x^2 y^3 - 4 k x^3 \cos(k z - \omega t)^2\ z + 2 \sin(k z - \omega t)\ \cos(k z - \omega t)\ \mu x \varepsilon \omega^2 + 4 \sin(k z$

$- \omega t)\ \cos(k z - \omega t)\ \mu x^3 \varepsilon \omega^2 - 8 \cos(k z - \omega t)\ k^2 x \sin(k z - \omega t)\ y^2 z^2 + 2 \cos(k z$

$$-\omega t)\ \varepsilon x^5 \sin(k z-\omega t)\ \omega^2 \mu-4 \cos(k z-\omega t)\ k^2 x^3 \sin(k z-\omega t)\ y^2 z^2-2 \cos(k z$$

$$-\omega t)\ k^2 x \sin(k z-\omega t)\ y^4 z^2-4 \cos(k z-\omega t)\ k^2 x \sin(k z-\omega t)\ y^2 z^4+4 y \sin(k z$$

$$-\omega t)\ k x^2 \cos(k z-\omega t)\ z+4 y \sin(k z-\omega t)\ k x^4 \cos(k z-\omega t)\ z+4 y \sin(k z-\omega t)\ k z^3 \cos(k z$$

$$-\omega t)\ x^2+8 \sin(k z-\omega t)\ k x^2 \cos(k z-\omega t)\ z y^3-\varepsilon \sin(k z-\omega t)^2 \omega^2 y^3 \mu-2 \varepsilon \sin(k z$$

$$-\omega t)^2 \omega^2 y^5 \mu-\varepsilon \sin(k z-\omega t)^2 \omega^2 y^7 \mu)),\ -\frac{1}{(1+x^2+y^2+z^2)^5 \mu}(\cos(k z-\omega t)\ (4 \sin(k z$$

$$-\omega t)\ k x^3 \cos(k z-\omega t)\ z+4 \sin(k z-\omega t)\ k x^5 \cos(k z-\omega t)\ z+4 \sin(k z-\omega t)^2 k^2 x^3 z^2 y^2$$

$$+4 \sin(k z-\omega t)\ k z^3 \cos(k z-\omega t)\ x^3+2 x \sin(k z-\omega t)^2 k^2 y^2 z^2+2 x \sin(k z-\omega t)^2 k^2 y^4 z^2$$

$$+x \sin(k z-\omega t)^2 k^2 z^4 y^2+4 k y \cos(k z-\omega t)^2 z^3 x^2+4 \cos(k z-\omega t)\ k^2 y^3 \sin(k z-\omega t)\ x^2$$

$$+8 \cos(k z-\omega t)\ k^2 y^3 \sin(k z-\omega t)\ z^2+2 \cos(k z-\omega t)\ k^2 y \sin(k z-\omega t)\ x^4+4 k y \cos(k z$$

$$-\omega t)^2 z x^2+4 \cos(k z-\omega t)\ k^2 y^3 \sin(k z-\omega t)\ z^4+2 \cos(k z-\omega t)\ k^2 y \sin(k z-\omega t)\ z^6$$

$$+2 \cos(k z-\omega t)\ k^2 y^5 \sin(k z-\omega t)\ z^2+6 \sin(k z-\omega t)\ \cos(k z-\omega t)\ k^2 y z^2+6 \sin(k z$$

$$-\omega t)\ \cos(k z-\omega t)\ k^2 y z^4+4 \sin(k z-\omega t)\ \cos(k z-\omega t)\ k^2 x^2 y-2 \varepsilon \sin(k z-\omega t)^2 \omega^2 x^5 \mu$$

$$-\varepsilon \sin(k z-\omega t)^2 \omega^2 x^7 \mu-\varepsilon \sin(k z-\omega t)^2 \omega^2 x^3 \mu-4 \varepsilon \sin(k z-\omega t)^2 \omega^2 x^3 \mu y^2 z^2$$

$$-2 \varepsilon \sin(k z-\omega t)^2 \omega^2 x \mu y^2 z^2-2 \varepsilon \sin(k z-\omega t)^2 \omega^2 x \mu y^4 z^2-\varepsilon \sin(k z-\omega t)^2 \omega^2 x \mu y^2 z^4$$

$$+\sin(kz-\omega t)^2 k^2 x^7 - 4\cos(kz-\omega t)\,\varepsilon y^3 \sin(kz-\omega t)\,\omega^2 \mu x^2 z^2 + \sin(kz-\omega t)^2 k^2 x^3$$

$$+2\sin(kz-\omega t)^2 k^2 x^5 + 4\cos(kz-\omega t)^2 z^2 x^3 + 4\cos(kz-\omega t)^2 x z^4 + 8\cos(kz-\omega t)^2 x z^2$$

$$-6\sin(kz-\omega t)\cos(kz-\omega t)\,\mu y z^4 \varepsilon \omega^2 - 4\sin(kz-\omega t)\cos(kz-\omega t)\,\mu x^2 y \varepsilon \omega^2 - 6\sin(kz$$

$$-\omega t)\cos(kz-\omega t)\,\mu y z^2 \varepsilon \omega^2 - 2\sin(kz-\omega t)\cos(kz-\omega t)\,\mu y \varepsilon \omega^2 - 4\sin(kz-\omega t)\cos(kz$$

$$-\omega t)\,\mu y^3 \varepsilon \omega^2 + 4x\sin(kz-\omega t)\,k y^2 \cos(kz-\omega t)\,z + 4x\sin(kz-\omega t)\,k y^4 \cos(kz-\omega t)\,z$$

$$+4x\sin(kz-\omega t)\,k z^3 \cos(kz-\omega t)\,y^2 + 8\sin(kz-\omega t)\,k x^3 \cos(kz-\omega t)\,z y^2 + 4\cos(kz$$

$$-\omega t)\,k^2 y^3 \sin(kz-\omega t)\,x^2 z^2 + 2\cos(kz-\omega t)\,k^2 y \sin(kz-\omega t)\,x^4 z^2 + 4\cos(kz$$

$$-\omega t)\,k^2 y \sin(kz-\omega t)\,x^2 z^4 - 2\cos(kz-\omega t)\,\varepsilon y^5 \sin(kz-\omega t)\,\omega^2 \mu + 8\cos(kz$$

$$-\omega t)\,k^2 y \sin(kz-\omega t)\,x^2 z^2 + 4\cos(kz-\omega t)^2 x - 8\cos(kz-\omega t)\,\varepsilon y \sin(kz-\omega t)\,\omega^2 \mu x^2 z^2$$

$$-2\cos(kz-\omega t)\,\varepsilon y \sin(kz-\omega t)\,\omega^2 \mu x^4 z^2 + 4\sin(kz-\omega t)^2 k^2 x^3 y^2 + 2\sin(kz-\omega t)\cos(kz$$

$$-\omega t)\,k^2 y + 4\sin(kz-\omega t)\cos(kz-\omega t)\,k^2 y^3 + 4\cos(kz-\omega t)^2 k y z + 8\cos(kz-\omega t)^2 k y z^3$$

$$+4ky^3\cos(kz-\omega t)^2 z + 2\cos(kz-\omega t)\,k^2 y^5 \sin(kz-\omega t) + 4ky^3\cos(kz-\omega t)^2 z^3$$

$$+4ky\cos(kz-\omega t)^2 z^5 + 2\sin(kz-\omega t)^2 k^2 x^3 z^2 + 3\sin(kz-\omega t)^2 k^2 x^5 y^2 + 3\sin(kz$$

$$-\omega t)^2 k^2 x^3 y^4 + 2\sin(kz-\omega t)^2 k^2 x^5 z^2 + x\sin(kz-\omega t)^2 k^2 y^2 + 2x\sin(kz-\omega t)^2 k^2 y^4$$

$$+4\,x\cos(k\,z-\omega\,t)^2\,z^2\,y^2+x\sin(k\,z-\omega\,t)^2\,k^2\,y^6+\sin(k\,z-\omega\,t)^2\,k^2\,z^4\,x^3-4\,\varepsilon\sin(k\,z$$

$$-\omega\,t)^2\,\omega^2\,x^3\,\mu\,y^2-2\,\varepsilon\sin(k\,z-\omega\,t)^2\,\omega^2\,x^3\,\mu\,z^2-3\,\varepsilon\sin(k\,z-\omega\,t)^2\,\omega^2\,x^5\,\mu\,y^2-2\,\varepsilon\sin(k\,z$$

$$-\omega\,t)^2\,\omega^2\,x^5\,\mu\,z^2-3\,\varepsilon\sin(k\,z-\omega\,t)^2\,\omega^2\,x^3\,\mu\,y^4-\varepsilon\sin(k\,z-\omega\,t)^2\,\omega^2\,x^3\,\mu\,z^4-\varepsilon\sin(k\,z$$

$$-\omega\,t)^2\,\omega^2\,x\,\mu\,y^2-2\,\varepsilon\sin(k\,z-\omega\,t)^2\,\omega^2\,x\,\mu\,y^4-\varepsilon\sin(k\,z-\omega\,t)^2\,\omega^2\,x\,\mu\,y^6-2\cos(k\,z$$

$$-\omega\,t)\,\varepsilon\,y\sin(k\,z-\omega\,t)\,\omega^2\,\mu\,x^4-4\cos(k\,z-\omega\,t)\,\varepsilon\,y^3\sin(k\,z-\omega\,t)\,\omega^2\,\mu\,x^2-8\cos(k\,z$$

$$-\omega\,t)\,\varepsilon\,y^3\sin(k\,z-\omega\,t)\,\omega^2\,\mu\,z^2-2\cos(k\,z-\omega\,t)\,\varepsilon\,y^5\sin(k\,z-\omega\,t)\,\omega^2\,\mu\,z^2-4\cos(k\,z$$

$$-\omega\,t)\,\varepsilon\,y^3\sin(k\,z-\omega\,t)\,\omega^2\,\mu\,z^4-2\cos(k\,z-\omega\,t)\,\varepsilon\,y\sin(k\,z-\omega\,t)\,\omega^2\,\mu\,z^6-4\cos(k\,z$$

$$-\omega\,t)\,\varepsilon\,y\sin(k\,z-\omega\,t)\,\omega^2\,\mu\,x^2\,z^4)\Big),\ \frac{1}{\mu\,(1+x^2+y^2+z^2)^4}\Big((x^2+y^2)\,\big(-\varepsilon\sin(k\,z$$
$$-\omega\,t)\,\omega^2\,\mu\,x^2+k^2\sin(k\,z-\omega\,t)\,x^2+k^2\sin(k\,z-\omega\,t)\,z^2+k^2\sin(k\,z-\omega\,t)+k^2\sin(k\,z-\omega\,t)\,y^2$$
$$-\varepsilon\sin(k\,z-\omega\,t)\,\omega^2\,\mu\,y^2-\varepsilon\sin(k\,z-\omega\,t)\,\omega^2\,\mu\,z^2-\varepsilon\sin(k\,z-\omega\,t)\,\omega^2\,\mu+2\,k\cos(k\,z-\omega\,t)\,z\big)$$
$$\cos(k\,z-\omega\,t)\,\big(\sin(k\,z-\omega\,t)\,k+\sin(k\,z-\omega\,t)\,k\,x^2+\sin(k\,z-\omega\,t)\,k\,y^2+\sin(k\,z-\omega\,t)\,k\,z^2$$
$$+2\cos(k\,z-\omega\,t)\,z\big)\Big)\Big]$$

Spin Dissipation J_spin dot E $=-\dfrac{1}{(1+x^2+y^2+z^2)^3\,\mu}\Big((x^2+y^2)\,\big(-\varepsilon\sin(k\,z-\omega\,t)\,\omega^2\,\mu\,x^2$

$$+k^2\sin(k\,z-\omega\,t)\,x^2+k^2\sin(k\,z-\omega\,t)\,z^2+k^2\sin(k\,z-\omega\,t)+k^2\sin(k\,z-\omega\,t)\,y^2-\varepsilon\sin(k\,z$$
$$-\omega\,t)\,\omega^2\,\mu\,y^2-\varepsilon\sin(k\,z-\omega\,t)\,\omega^2\,\mu\,z^2-\varepsilon\sin(k\,z-\omega\,t)\,\omega^2\,\mu+2\,k\cos(k\,z-\omega\,t)\,z\big)\cos(k\,z$$
$$-\omega\,t)\sin(k\,z-\omega\,t)\,\omega\Big)$$

Disspative Force 3 vector $=\Big[\dfrac{1}{(1+x^2+y^2+z^2)^5\,\mu}\Big(\cos(k\,z-\omega\,t)\,\big(4\cos(k\,z-\omega\,t)\,\varepsilon\,x\sin(k\,z$

$$-\omega\,t)\,\omega^2\,\mu^2\,y^2\,z^4+8\cos(k\,z-\omega\,t)\,\varepsilon\,x\sin(k\,z-\omega\,t)\,\omega^2\,\mu^2\,y^2\,z^2-4\,x^3\cos(k\,z-\omega\,t)\,z^2-4\,\mu\cos(k\,z$$

$$- \omega t)\ k^2 x \sin(k z - \omega t)\ y^2 z^4 - 2\ k^2 \cos(k z - \omega t)\ x + 2 \cos(k z - \omega t)\ \varepsilon x \sin(k z - \omega t)\ \omega^2 \mu^2 y^4 z^2$$

$$+ 4\ \mu y \sin(k z - \omega t)\ k x^2 \cos(k z - \omega t)\ z - 20\ x \cos(k z - \omega t)\ - 4 \cos(k z - \omega t)\ x z^4 - 2 \cos(k z$$

$$- \omega t)\ k^2 x^5 - 4\ x y^2 \cos(k z - \omega t)\ + 4\ \mu y \cos(k z - \omega t)^2 + 4\ \mu y \sin(k z - \omega t)\ k x^4 \cos(k z - \omega t)\ z$$

$$+ 4\ \mu y \sin(k z - \omega t)\ k z^3 \cos(k z - \omega t)\ x^2 + 8\ \mu \sin(k z - \omega t)\ k x^2 \cos(k z - \omega t)\ z y^3 + 6 \sin(k z$$

$$- \omega t)\ \cos(k z - \omega t)\ \mu^2 x z^2 \varepsilon \omega^2 + 6 \sin(k z - \omega t)\ \cos(k z - \omega t)\ \mu^2 x z^4 \varepsilon \omega^2 + 4 \sin(k z$$

$$- \omega t)\ \cos(k z - \omega t)\ \mu^2 x y^2 \varepsilon \omega^2 + 8\ x \sin(k z - \omega t)\ k z - 4 \cos(k z - \omega t)\ k^2 x y^2 - 6 \cos(k z$$

$$- \omega t)\ k^2 x z^2 - 4 \cos(k z - \omega t)\ k^2 x^3 + 4\ \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x y^2 + 6\ \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x z^2$$

$$+ 2\ \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x + 4\ \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^3 - 24 \cos(k z - \omega t)\ x z^2 - 8\ x \cos(k z$$

$$- \omega t)\ k^2 y^2 z^2 + 8\ x y^2 \sin(k z - \omega t)\ k z + 8\ x y^2 \sin(k z - \omega t)\ k z^3 - 2\ x \cos(k z - \omega t)\ k^2 y^4 z^2$$

$$- 4\ x \cos(k z - \omega t)\ k^2 y^2 z^4 - 4 \cos(k z - \omega t)\ k^2 x^3 y^2 z^2 + 2\ \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^5 + 4\ \mu \sin(k z$$

$$- \omega t)^2 k^2 x^2 y^3 - \varepsilon \sin(k z - \omega t)^2 \omega^2 y^3 \mu^2 - 4\ \mu k x^3 \cos(k z - \omega t)^2 z - 2\ \mu \cos(k z$$

$$- \omega t)\ k^2 x^5 \sin(k z - \omega t)\ - 4\ \mu k x^3 \cos(k z - \omega t)^2 z^3 - 4\ \mu k x \cos(k z - \omega t)^2 z^5 - 2\ \mu \sin(k z$$

$$- \omega t)\ \cos(k z - \omega t)\ k^2 x - 4\ \mu \sin(k z - \omega t)\ \cos(k z - \omega t)\ k^2 x^3 - 4\ \mu \cos(k z - \omega t)^2 k x z$$

$$- 8\ \mu \cos(k z - \omega t)^2 k x z^3 + 2\ \mu \sin(k z - \omega t)^2 k^2 y^3 z^2 + 3\ \mu \sin(k z - \omega t)^2 k^2 x^4 y^3 + 3\ \mu \sin(k z$$

$- \omega\, t)^2\, k^2\, x^2\, y^5 + 2\,\mu \sin(k\, z - \omega\, t)^2\, k^2\, y^5\, z^2 + \mu\, y \sin(k\, z - \omega\, t)^2\, k^2\, x^2 + 2\,\mu\, y \sin(k\, z - \omega\, t)^2\, k^2\, x^4$

$+ 4\,\mu\, y \cos(k\, z - \omega\, t)^2\, z^2\, x^2 + \mu\, y \sin(k\, z - \omega\, t)^2\, k^2\, x^6 + \mu \sin(k\, z - \omega\, t)^2\, k^2\, z^4\, y^3 - 2\,\varepsilon \sin(k\, z$

$- \omega\, t)^2\, \omega^2\, y^5\, \mu^2 - \varepsilon \sin(k\, z - \omega\, t)^2\, \omega^2\, y^7\, \mu^2 + 4\,\mu \cos(k\, z - \omega\, t)^2\, y\, z^4 + \mu \sin(k\, z - \omega\, t)^2\, k^2\, y^3$

$+ 2\,\mu \sin(k\, z - \omega\, t)^2\, k^2\, y^5 + 4\,\mu \cos(k\, z - \omega\, t)^2\, z^2\, y^3 + \mu \sin(k\, z - \omega\, t)^2\, k^2\, y^7 - 6 \cos(k\, z$

$- \omega\, t)\, k^2\, x\, z^4 - 8 \cos(k\, z - \omega\, t)\, k^2\, x^3\, z^2 + 8\, x^3 \sin(k\, z - \omega\, t)\, k\, z - 4 \cos(k\, z - \omega\, t)\, k^2\, x^3\, y^2$

$- 2\, x \cos(k\, z - \omega\, t)\, k^2\, z^6 + 8\, x \sin(k\, z - \omega\, t)\, k\, z^5 - 4\, x\, y^2 \cos(k\, z - \omega\, t)\, z^2 - 2\, x \cos(k\, z - \omega\, t)\, k^2\, y^4$

$+ 16\, x \sin(k\, z - \omega\, t)\, k\, z^3 + 8\, x^3 \sin(k\, z - \omega\, t)\, k\, z^3 - 2 \cos(k\, z - \omega\, t)\, k^2\, x^5\, z^2 - 4 \cos(k\, z$

$- \omega\, t)\, k^2\, x^3\, z^4 + 8\,\mu \cos(k\, z - \omega\, t)^2\, y\, z^2 + 4 \cos(k\, z - \omega\, t)\, \varepsilon\, x^3 \sin(k\, z - \omega\, t)\, \omega^2\, \mu^2\, y^2\, z^2$

$+ 2\, x\, \varepsilon\, \omega^2 \cos(k\, z - \omega\, t)\, \mu\, y^4 + 6\, x\, \varepsilon\, \omega^2 \cos(k\, z - \omega\, t)\, \mu\, z^4 - 6\,\mu \sin(k\, z - \omega\, t) \cos(k\, z - \omega\, t)\, k^2\, x\, z^2$

$- 6\,\mu \sin(k\, z - \omega\, t) \cos(k\, z - \omega\, t)\, k^2\, x\, z^4 - 4\,\mu \sin(k\, z - \omega\, t) \cos(k\, z - \omega\, t)\, k^2\, x\, y^2 + 2 \sin(k\, z$

$- \omega\, t) \cos(k\, z - \omega\, t)\, \mu^2\, x\, \varepsilon\, \omega^2 + 4 \sin(k\, z - \omega\, t) \cos(k\, z - \omega\, t)\, \mu^2\, x^3\, \varepsilon\, \omega^2 - 4\,\mu \cos(k\, z$

$- \omega\, t)\, k^2\, x^3 \sin(k\, z - \omega\, t)\, y^2 - 8\,\mu \cos(k\, z - \omega\, t)\, k^2\, x^3 \sin(k\, z - \omega\, t)\, z^2 - 2\,\mu \cos(k\, z$

$- \omega\, t)\, k^2\, x \sin(k\, z - \omega\, t)\, y^4 - 4\,\mu\, k\, x \cos(k\, z - \omega\, t)^2\, z\, y^2 - 4\,\mu\, k\, x \cos(k\, z - \omega\, t)^2\, z^3\, y^2 - 4\,\mu \cos(k\, z$

$- \omega\, t)\, k^2\, x^3 \sin(k\, z - \omega\, t)\, z^4 - 2\,\mu \cos(k\, z - \omega\, t)\, k^2\, x \sin(k\, z - \omega\, t)\, z^6 - 2\,\mu \cos(k\, z$

$$- \omega t)\ k^2 x^5 \sin(k z - \omega t)\ z^2 + 4 \mu \sin(k z - \omega t)\ k y^3 \cos(k z - \omega t)\ z + 4 \mu \sin(k z - \omega t)\ k y^5 \cos(k z$$

$$- \omega t)\ z + 4 \mu \sin(k z - \omega t)^2 k^2 x^2 z^2 y^3 + 4 \mu \sin(k z - \omega t)\ k z^3 \cos(k z - \omega t)\ y^3 + 2 \mu y \sin(k z$$

$$- \omega t)^2 k^2 x^2 z^2 + 2 \mu y \sin(k z - \omega t)^2 k^2 x^4 z^2 + \mu y \sin(k z - \omega t)^2 k^2 z^4 x^2 - \varepsilon \sin(k z$$

$$- \omega t)^2 \omega^2 y \mu^2 x^2 - 2 \varepsilon \sin(k z - \omega t)^2 \omega^2 y \mu^2 x^4 - 4 \varepsilon \sin(k z - \omega t)^2 \omega^2 y^3 \mu^2 x^2 - \varepsilon \sin(k z$$

$$- \omega t)^2 \omega^2 y \mu^2 x^6 - 3 \varepsilon \sin(k z - \omega t)^2 \omega^2 y^3 \mu^2 x^4 - 3 \varepsilon \sin(k z - \omega t)^2 \omega^2 y^5 \mu^2 x^2 - 2 \varepsilon \sin(k z$$

$$- \omega t)^2 \omega^2 y^3 \mu^2 z^2 - 2 \varepsilon \sin(k z - \omega t)^2 \omega^2 y^5 \mu^2 z^2 - \varepsilon \sin(k z - \omega t)^2 \omega^2 y^3 \mu^2 z^4 + 2 \cos(k z$$

$$- \omega t)\ \varepsilon x^5 \sin(k z - \omega t)\ \omega^2 \mu^2 + 4 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^3 z^4 + 2 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^5 z^2$$

$$+ 4 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^3 y^2 + 8 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^3 z^2 + 2 x \varepsilon \omega^2 \cos(k z - \omega t)\ \mu z^6$$

$$- 4 x^3 \cos(k z - \omega t)\ + 4 x \varepsilon \omega^2 \cos(k z - \omega t)\ \mu y^2 z^4 + 2 x \varepsilon \omega^2 \cos(k z - \omega t)\ \mu y^4 z^2$$

$$+ 8 x \varepsilon \omega^2 \cos(k z - \omega t)\ \mu y^2 z^2 + 4 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^3 y^2 z^2 + 4 \cos(k z - \omega t)\ \varepsilon x^3 \sin(k z$$

$$- \omega t)\ \omega^2 \mu^2 y^2 + 8 \cos(k z - \omega t)\ \varepsilon x^3 \sin(k z - \omega t)\ \omega^2 \mu^2 z^2 + 2 \cos(k z - \omega t)\ \varepsilon x \sin(k z$$

$$- \omega t)\ \omega^2 \mu^2 y^4 + 4 \cos(k z - \omega t)\ \varepsilon x^3 \sin(k z - \omega t)\ \omega^2 \mu^2 z^4 + 2 \cos(k z - \omega t)\ \varepsilon x \sin(k z$$

$$- \omega t)\ \omega^2 \mu^2 z^6 + 2 \cos(k z - \omega t)\ \varepsilon x^5 \sin(k z - \omega t)\ \omega^2 \mu^2 z^2 - 2 \varepsilon \sin(k z - \omega t)^2 \omega^2 y \mu^2 x^2 z^2$$

$$- 2 \varepsilon \sin(k z - \omega t)^2 \omega^2 y \mu^2 x^4 z^2 - 4 \varepsilon \sin(k z - \omega t)^2 \omega^2 y^3 \mu^2 x^2 z^2 - \varepsilon \sin(k z - \omega t)^2 \omega^2 y \mu^2 x^2 z^4$$

$$-8\,\mu\cos(k\,z-\omega\,t)\;k^2\,x\sin(k\,z-\omega\,t)\;y^2\,z^2-4\,\mu\cos(k\,z-\omega\,t)\;k^2\,x^3\sin(k\,z-\omega\,t)\;y^2\,z^2$$

$$-2\,\mu\cos(k\,z-\omega\,t)\;k^2\,x\sin(k\,z-\omega\,t)\;y^4\,z^2)\big)\big),\;-\frac{1}{(1+x^2+y^2+z^2)^5\,\mu}\big(\cos(k\,z$$

$$-\omega\,t)\;\big(4\,y^3\cos(k\,z-\omega\,t)\;z^2+2\cos(k\,z-\omega\,t)\;k^2\,y^5+4\cos(k\,z-\omega\,t)^2\,\mu\,x+4\cos(k\,z-\omega\,t)\;y\,z^4$$

$$-4\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)\;\mu\,y^3\,z^4-2\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)\;\mu\,y^5\,z^2-4\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)\;\mu\,x^2\,y^3$$

$$-8\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)\;\mu\,y^3\,z^2-2\,y\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)\;\mu\,z^6-2\,y\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)\;\mu\,x^4$$

$$-6\,y\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)\;\mu\,z^4+4\,\mu\sin(k\,z-\omega\,t)\;k\,x^3\cos(k\,z-\omega\,t)\;z+4\,\mu\sin(k\,z$$

$$-\omega\,t)\;k\,x^5\cos(k\,z-\omega\,t)\;z+4\,\mu\sin(k\,z-\omega\,t)^2\,k^2\,x^3\,z^2\,y^2+4\,\mu\sin(k\,z-\omega\,t)\;k\,z^3\cos(k\,z-\omega\,t)\;x^3$$

$$+2\,\mu\,x\sin(k\,z-\omega\,t)^2\,k^2\,y^2\,z^2+2\,\mu\,x\sin(k\,z-\omega\,t)^2\,k^2\,y^4\,z^2+\mu\,x\sin(k\,z-\omega\,t)^2\,k^2\,z^4\,y^2$$

$$+4\,\mu\,k\,y\cos(k\,z-\omega\,t)^2\,z^3\,x^2+4\,\mu\cos(k\,z-\omega\,t)\;k^2\,y^3\sin(k\,z-\omega\,t)\;x^2+8\,\mu\cos(k\,z$$

$$-\omega\,t)\;k^2\,y^3\sin(k\,z-\omega\,t)\;z^2+2\,\mu\cos(k\,z-\omega\,t)\;k^2\,y\sin(k\,z-\omega\,t)\;x^4+4\,\mu\,k\,y\cos(k\,z-\omega\,t)^2\,z\,x^2$$

$$+4\,\mu\cos(k\,z-\omega\,t)\;k^2\,y^3\sin(k\,z-\omega\,t)\;z^4+2\,\mu\cos(k\,z-\omega\,t)\;k^2\,y\sin(k\,z-\omega\,t)\;z^6+2\,\mu\cos(k\,z$$

$$-\omega\,t)\;k^2\,y^5\sin(k\,z-\omega\,t)\;z^2+6\,\mu\sin(k\,z-\omega\,t)\;\cos(k\,z-\omega\,t)\;k^2\,y\,z^2+6\,\mu\sin(k\,z-\omega\,t)\;\cos(k\,z$$

$$-\omega\,t)\;k^2\,y\,z^4+4\,\mu\sin(k\,z-\omega\,t)\;\cos(k\,z-\omega\,t)\;k^2\,x^2\,y-2\sin(k\,z-\omega\,t)\;\cos(k\,z-\omega\,t)\;\mu^2\,y\,\varepsilon\,\omega^2$$

$$-4\sin(k\,z-\omega\,t)\;\cos(k\,z-\omega\,t)\;\mu^2\,y^3\,\varepsilon\,\omega^2-2\cos(k\,z-\omega\,t)\;\varepsilon\,y^5\sin(k\,z-\omega\,t)\;\omega^2\,\mu^2$$

$$-4\,\varepsilon\sin(k\,z-\omega\,t)^2\,\omega^2\,x^3\,\mu^2\,y^2-2\,\varepsilon\sin(k\,z-\omega\,t)^2\,\omega^2\,x^3\,\mu^2\,z^2-3\,\varepsilon\sin(k\,z-\omega\,t)^2\,\omega^2\,x^5\,\mu^2\,y^2$$

$$-4\cos(k\,z-\omega\,t)\,\varepsilon\,y^3\sin(k\,z-\omega\,t)\,\omega^2\,\mu^2\,x^2\,z^2-4\cos(k\,z-\omega\,t)\,\varepsilon\,y\sin(k\,z-\omega\,t)\,\omega^2\,\mu^2\,x^2\,z^4$$

$$+4\,y^3\cos(k\,z-\omega\,t)+4\,y\,x^2\cos(k\,z-\omega\,t)+8\cos(k\,z-\omega\,t)\,k^2\,y^3\,z^2-8\,y^3\sin(k\,z-\omega\,t)\,k\,z$$

$$+4\cos(k\,z-\omega\,t)\,k^2\,x^2\,y^3+2\,y\cos(k\,z-\omega\,t)\,k^2\,z^6-8\,y\sin(k\,z-\omega\,t)\,k\,z^5+4\,y\,x^2\cos(k\,z-\omega\,t)\,z^2$$

$$+2\,y\cos(k\,z-\omega\,t)\,k^2\,x^4-16\,y\sin(k\,z-\omega\,t)\,k\,z^3-8\,y^3\sin(k\,z-\omega\,t)\,k\,z^3+2\cos(k\,z-\omega\,t)\,k^2\,y^5\,z^2$$

$$+4\cos(k\,z-\omega\,t)\,k^2\,y^3\,z^4+\mu\sin(k\,z-\omega\,t)^2\,k^2\,x^3+2\,\mu\sin(k\,z-\omega\,t)^2\,k^2\,x^5+4\,\mu\cos(k\,z$$

$$-\omega\,t)^2\,z^2\,x^3+\mu\sin(k\,z-\omega\,t)^2\,k^2\,x^7+4\,\mu\cos(k\,z-\omega\,t)^2\,x\,z^4+8\,\mu\cos(k\,z-\omega\,t)^2\,x\,z^2+6\cos(k\,z$$

$$-\omega\,t)\,k^2\,y\,z^4-2\,\varepsilon\sin(k\,z-\omega\,t)^2\,\omega^2\,x^5\,\mu^2\,z^2-3\,\varepsilon\sin(k\,z-\omega\,t)^2\,\omega^2\,x^3\,\mu^2\,y^4-\varepsilon\sin(k\,z$$

$$-\omega\,t)^2\,\omega^2\,x^3\,\mu^2\,z^4-\varepsilon\sin(k\,z-\omega\,t)^2\,\omega^2\,x\,\mu^2\,y^2-2\,\varepsilon\sin(k\,z-\omega\,t)^2\,\omega^2\,x\,\mu^2\,y^4-\varepsilon\sin(k\,z$$

$$-\omega\,t)^2\,\omega^2\,x\,\mu^2\,y^6+2\,k^2\cos(k\,z-\omega\,t)\,y+20\,y\cos(k\,z-\omega\,t)-8\cos(k\,z-\omega\,t)\,\varepsilon\,y\sin(k\,z$$

$$-\omega\,t)\,\omega^2\,\mu^2\,x^2\,z^2-2\cos(k\,z-\omega\,t)\,\varepsilon\,y\sin(k\,z-\omega\,t)\,\omega^2\,\mu^2\,x^4\,z^2-8\,y\sin(k\,z-\omega\,t)\,k\,z+4\cos(k\,z$$

$$-\omega\,t)\,k^2\,x^2\,y+6\cos(k\,z-\omega\,t)\,k^2\,y\,z^2+4\cos(k\,z-\omega\,t)\,k^2\,y^3-4\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)\,\mu\,x^2\,y$$

$$-6\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)\,\mu\,y\,z^2-2\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)\,\mu\,y-4\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)\,\mu\,y^3+24\cos(k\,z$$

$$-\omega\,t)\,y\,z^2-6\sin(k\,z-\omega\,t)\cos(k\,z-\omega\,t)\,\mu^2\,y\,z^2\,\varepsilon\,\omega^2-6\sin(k\,z-\omega\,t)\cos(k\,z-\omega\,t)\,\mu^2\,y\,z^4\,\varepsilon\,\omega^2$$

$$-4\sin(kz-\omega t)\cos(kz-\omega t)\mu^2 x^2 y\varepsilon\omega^2+4\mu\cos(kz-\omega t)k^2 y\sin(kz-\omega t)x^2 z^4$$

$$+8\mu\cos(kz-\omega t)k^2 y\sin(kz-\omega t)x^2 z^2-4y\varepsilon\omega^2\cos(kz-\omega t)\mu x^2 z^4-2y\varepsilon\omega^2\cos(kz$$

$$-\omega t)\mu x^4 z^2-8y\varepsilon\omega^2\cos(kz-\omega t)\mu x^2 z^2-4\varepsilon\omega^2\cos(kz-\omega t)\mu x^2 y^3 z^2-2\cos(kz$$

$$-\omega t)\varepsilon y\sin(kz-\omega t)\omega^2\mu^2 x^4-4\cos(kz-\omega t)\varepsilon y^3\sin(kz-\omega t)\omega^2\mu^2 x^2-8\cos(kz$$

$$-\omega t)\varepsilon y^3\sin(kz-\omega t)\omega^2\mu^2 z^2-2\cos(kz-\omega t)\varepsilon y^5\sin(kz-\omega t)\omega^2\mu^2 z^2-4\cos(kz$$

$$-\omega t)\varepsilon y^3\sin(kz-\omega t)\omega^2\mu^2 z^4-2\cos(kz-\omega t)\varepsilon y\sin(kz-\omega t)\omega^2\mu^2 z^6-4\varepsilon\sin(kz$$

$$-\omega t)^2\omega^2 x^3\mu^2 y^2 z^2-2\varepsilon\sin(kz-\omega t)^2\omega^2 x\mu^2 y^2 z^2-2\varepsilon\sin(kz-\omega t)^2\omega^2 x\mu^2 y^4 z^2$$

$$-\varepsilon\sin(kz-\omega t)^2\omega^2 x\mu^2 y^2 z^4+4\mu x\sin(kz-\omega t)ky^2\cos(kz-\omega t)z+4\mu x\sin(kz$$

$$-\omega t)ky^4\cos(kz-\omega t)z+4\mu x\sin(kz-\omega t)kz^3\cos(kz-\omega t)y^2+8\mu\sin(kz$$

$$-\omega t)kx^3\cos(kz-\omega t)zy^2+4\mu\cos(kz-\omega t)k^2 y^3\sin(kz-\omega t)x^2 z^2+2\mu\cos(kz$$

$$-\omega t)k^2 y\sin(kz-\omega t)x^4 z^2+8y\cos(kz-\omega t)k^2 x^2 z^2-8yx^2\sin(kz-\omega t)kz-8yx^2\sin(kz$$

$$-\omega t)kz^3+2y\cos(kz-\omega t)k^2 x^4 z^2+4y\cos(kz-\omega t)k^2 x^2 z^4+4\cos(kz-\omega t)k^2 x^2 y^3 z^2$$

$$-2\varepsilon\omega^2\cos(kz-\omega t)\mu y^5-2\varepsilon\sin(kz-\omega t)^2\omega^2 x^5\mu^2-\varepsilon\sin(kz-\omega t)^2\omega^2 x^7\mu^2+4\mu\sin(kz$$

$$-\omega t)^2 k^2 x^3 y^2+2\mu\sin(kz-\omega t)\cos(kz-\omega t)k^2 y+4\mu\sin(kz-\omega t)\cos(kz-\omega t)k^2 y^3$$

$$+4\,\mu\cos(k z-\omega t)^2 k y z+8\,\mu\cos(k z-\omega t)^2 k y z^3-\varepsilon\sin(k z-\omega t)^2\,\omega^2 x^3\mu^2$$

$$+4\,\mu k y^3\cos(k z-\omega t)^2 z+2\,\mu\cos(k z-\omega t)\,k^2 y^5\sin(k z-\omega t)\,+4\,\mu k y^3\cos(k z-\omega t)^2 z^3$$

$$+4\,\mu k y\cos(k z-\omega t)^2 z^5+2\,\mu\sin(k z-\omega t)^2 k^2 x^3 z^2+3\,\mu\sin(k z-\omega t)^2 k^2 x^5 y^2+3\,\mu\sin(k z$$

$$-\omega t)^2 k^2 x^3 y^4+2\,\mu\sin(k z-\omega t)^2 k^2 x^5 z^2+\mu x\sin(k z-\omega t)^2 k^2 y^2+2\,\mu x\sin(k z-\omega t)^2 k^2 y^4$$

$$+4\,\mu x\cos(k z-\omega t)^2 z^2 y^2+\mu x\sin(k z-\omega t)^2 k^2 y^6+\mu\sin(k z-\omega t)^2 k^2 z^4 x^3)\,),$$

$$\frac{1}{(1+x^2+y^2+z^2)^5\,\mu}\,(\,(\sin(k z-\omega t)\,k+\sin(k z-\omega t)\,k x^2+\sin(k z-\omega t)\,k y^2+\sin(k z$$

$$-\omega t)\,k z^2+2\cos(k z-\omega t)\,z)\,(x^2+y^2)\,\big(2\,\mu\cos(k z-\omega t)^2 k x^2 z+2\,\mu\cos(k z-\omega t)^2 k y^2 z$$

$$-\mu^2\cos(k z-\omega t)\,\varepsilon\sin(k z-\omega t)\,\omega^2+2\,\mu\cos(k z-\omega t)\,k^2 x^2\sin(k z-\omega t)\,+2\,\mu\cos(k z$$

$$-\omega t)\,k^2 y^2\sin(k z-\omega t)\,+\mu\cos(k z-\omega t)\,k^2 x^4\sin(k z-\omega t)\,+2\,\mu\cos(k z-\omega t)\,k^2\sin(k z$$

$$-\omega t)\,z^2+\mu\cos(k z-\omega t)\,k^2\sin(k z-\omega t)\,z^4+\mu\cos(k z-\omega t)\,k^2 y^4\sin(k z-\omega t)\,+10\cos(k z$$

$$-\omega t)\,+k^2\cos(k z-\omega t)\,+2\,k^2\cos(k z-\omega t)\,x^2+2\,k^2\cos(k z-\omega t)\,y^2-4\sin(k z-\omega t)\,k z$$

$$+2\,k^2\cos(k z-\omega t)\,z^2+2\,\mu\cos(k z-\omega t)\,k^2 x^2\sin(k z-\omega t)\,z^2+2\,\mu\cos(k z-\omega t)\,k^2 x^2\sin(k z$$

$$-\omega t)\,y^2+2\,\mu\cos(k z-\omega t)\,k^2 y^2\sin(k z-\omega t)\,z^2-2\,\mu^2\cos(k z-\omega t)\,\varepsilon y^2\sin(k z-\omega t)\,\omega^2$$

$$-\mu^2\cos(k z-\omega t)\,\varepsilon y^4\sin(k z-\omega t)\,\omega^2-2\,\mu^2\cos(k z-\omega t)\,\varepsilon\sin(k z-\omega t)\,\omega^2 z^2-\mu^2\cos(k z$$

$$-\omega t)\,\varepsilon\sin(k z-\omega t)\,\omega^2 z^4-2\,\mu^2\cos(k z-\omega t)\,\varepsilon x^2\sin(k z-\omega t)\,\omega^2-\mu^2\cos(k z$$

$$-\omega t)\,\varepsilon x^4\sin(k z-\omega t)\,\omega^2+\cos(k z-\omega t)\,k^2 x^4+\cos(k z-\omega t)\,k^2 y^4+\cos(k z-\omega t)\,k^2 z^4$$

$$-4\sin(k z-\omega t)\,k z^3+2\,y^2\cos(k z-\omega t)\,+2\,x^2\cos(k z-\omega t)\,+2\cos(k z-\omega t)\,z^2-4\,x^2\sin(k z$$

$$-\omega t)\,k z-4\,y^2\sin(k z-\omega t)\,k z+2\cos(k z-\omega t)\,k^2 x^2 y^2+2\cos(k z-\omega t)\,k^2 x^2 z^2+2\cos(k z$$

$$-\omega t)\,k^2 y^2 z^2-\varepsilon\,\omega^2\cos(k z-\omega t)\,\mu-2\,\varepsilon\,\omega^2\cos(k z-\omega t)\,\mu x^2-2\,\varepsilon\,\omega^2\cos(k z-\omega t)\,\mu y^2$$

$$-2\,\varepsilon\,\omega^2\cos(k z-\omega t)\,\mu z^2-\varepsilon\,\omega^2\cos(k z-\omega t)\,\mu x^4-\varepsilon\,\omega^2\cos(k z-\omega t)\,\mu y^4-\varepsilon\,\omega^2\cos(k z$$

$$-\omega t)\,\mu z^4-2\,\varepsilon\,\omega^2\cos(k z-\omega t)\,\mu x^2 y^2-2\,\varepsilon\,\omega^2\cos(k z-\omega t)\,\mu x^2 z^2-2\,\varepsilon\,\omega^2\cos(k z-\omega t)\,\mu y^2 z^2$$

$$+2\,\mu\cos(k z-\omega t)^2 k z^3+\mu\cos(k z-\omega t)\,k^2\sin(k z-\omega t)\,+2\,\mu\cos(k z-\omega t)^2 k z-2\,\mu^2\cos(k z$$

$- \omega\, t)\ \varepsilon\, x^2 \sin(k\, z - \omega\, t)\ \omega^2\, y^2 - 2\, \mu^2 \cos(k\, z - \omega\, t)\ \varepsilon\, x^2 \sin(k\, z - \omega\, t)\ \omega^2\, z^2 - 2\, \mu^2 \cos(k\, z$

$- \omega\, t)\ \varepsilon\, y^2 \sin(k\, z - \omega\, t)\ \omega^2\, z^2))\,]$

$$Dissipation\ = - \frac{\cos(k\, z - \omega\, t)\ \omega\ (\mu\, \varepsilon \sin(k\, z - \omega\, t)\ x^2 + \mu\, \varepsilon \sin(k\, z - \omega\, t)\ y^2 - 2 \cos(k\, z - \omega\, t)\ z\, x)}{(1 + x^2 + y^2 + z^2)^2}$$

****************************** *END PROCEDURE* ****************************** **(7)**

Enter the name of the problem, and the components of the 4 potential
p-2, n=4

```
> NAME:=`Example 7a = Index 1 Irreversible solution EdotB < 0 (kinematic out) Type 1`;
```

```
> Ax:=y*z;Ay:=-x*z;Az:=C*t*1;phi:=+C*z*z*1;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

$$NAME := \textit{Example 7a = Index 1 Irreversible solution EdotB } < 0 \textit{ (kinematic out) Type 1}$$

$$Ax := y\,z$$

$$Ay := -x\,z$$

$$Az := C\,t$$

$$\phi := C\,z^2$$

Example 7a = Index 1 Irreversible solution EdotB ! 0 (kinematic out) Type 1

******************************** *Differential Form Format* ********************************

$$\textit{Action 1-form} = y\,z\,d(x) - x\,z\,d(y) + C\,t\,d(z) - C\,z^2\,d(t)$$

$$\textit{Intensity 2-form F=dA} = -2\,z\,(d(x))\,\&\hat{}\,(d(y)) - y\,(d(x))\,\&\hat{}\,(d(z)) + x\,(d(y))\,\&\hat{}\,(d(z)) + (-C - 2\,C\,z)\,(d(z))\,\&\hat{}\,(d(t))$$

$$\textit{Topological Torsion 3-form} \quad A\hat{}F = 2\,z^3\,C\,\&\hat{}(d(x), d(y), d(t)) - 2\,C\,t\,z\,\&\hat{}(d(x), d(y), d(z)) + \left(-y\,z\,C\,(1+2\,z) + C\,z^2\,y\right)\&\hat{}(d(x), d(z), d(t)) + \left(x\,z\,C\,(1+2\,z) - C\,z^2\,x\right)\&\hat{}(d(y), d(z), d(t))$$

$$\textit{Topological Parity 4-form} \quad F\hat{}F = 4\,C\,(1+2\,z)\,z\,\&\hat{}(d(x), d(y), d(z), d(t))$$

******************************** *Using EM format* ********************************

$$E\,\textit{field} = [0, 0, -C\,(1+2\,z)]$$

$$B\,\textit{field} = [x, y, -2\,z]$$

$$\textit{Topological TORSION 4 vector} \quad T4 = \textit{-[ExA + Bphi,AdotB]} = [z\,x\,C\,(1+z), z\,y\,C\,(1+z), 2\,z^3\,C, 2\,C\,t\,z]$$

$$\textit{Helicity AdotB} = -2\,C\,t\,z$$

$$\textit{Poincare II =2(E.B)} = 4\,C\,(1+2\,z)\,z$$

$$\textit{coefficient of Topological Parity 4-form} = 4\,C\,(1+2\,z)\,z$$

$$\textit{Pfaff Topological Dimension} \quad PTD = 4$$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

$$\textit{Xm or linear (Mean) curvature} = 0$$

$$\textit{Yg or quadratic (GAUSS) curvature} = z\left(z + 2\,C^2\right)$$

$$\textit{Za or Cubic (Interaction internal energy) curvature} = 0$$

Tk or quartic (4D expansion) curvature $= 2\, z^3\, C^2$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor $CH = 0$

$$D\ field = \left[0,\ 0,\ -\varepsilon\, C\, (1 + 2\, z)\right]$$

$$H\ field = \left[\frac{x}{\mu},\ \frac{y}{\mu},\ -\frac{2\, z}{\mu}\right]$$

Poynting vector $ExH = EXH$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \left[0,\ 0,\ 0,\ -2\, \varepsilon\, C\right]$

Amerian charge density *divD = rho* $= -2\, \varepsilon\, C$

divergence Lorentz Current 4Vector, *4div(J4)* $= 0$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[-\frac{-2\, x\, z^2 + C\, t\, y}{\mu},\ \frac{C\, t\, x + 2\, y\, z^2}{\mu},\ -\frac{z\left(-y^2 - x^2 + \varepsilon\, C^2\, \mu\, z + 2\, \varepsilon\, C^2\, \mu\, z^2\right)}{\mu},\ -C^2\, t\, \varepsilon\, (1 + 2\, z)\right]$$

$$Topological\ SPIN\ 3\text{-}form = -\frac{\left(-2\, x\, z^2 + C\, t\, y\right)\ \&\text{^}(d(y), d(z), d(t))}{\mu} - \frac{\left(C\, t\, x + 2\, y\, z^2\right)\ \&\text{^}(d(x), d(z), d(t))}{\mu} - \frac{z\left(-y^2 - x^2 + \varepsilon\, C^2\, \mu\, z + 2\, \varepsilon\, C^2\, \mu\, z^2\right)\ \&\text{^}(d(x), d(y), d(t))}{\mu} + C^2\, t\, \varepsilon\, (1 + 2\, z)\ \&\text{^}(d(x), d(y), d(z))$$

Spin density rho_spin $= -C^2\, t\, \varepsilon\, (1 + 2\, z)$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{-x^2 - y^2 - 4\, z^2 + \varepsilon\, C^2\, \mu + 4\, \varepsilon\, C^2\, \mu\, z + 4\, \varepsilon\, C^2\, \mu\, z^2}{\mu}$$

$$B.H = \frac{x^2 + y^2 + 4\, z^2}{\mu}$$

$$D.E = \varepsilon\, C^2\, (1 + 2\, z)^2$$

$$A.J = 0$$

$$\text{-}rho.phi = -2\, C^2\, z^2\, \varepsilon$$

$$Poincare\ I\quad (B.H - D.E)\text{-}(A.J - rho.phi) = -\frac{-x^2 - y^2 - 4\, z^2 + \varepsilon\, C^2\, \mu + 4\, \varepsilon\, C^2\, \mu\, z + 6\, \varepsilon\, C^2\, \mu\, z^2}{\mu}$$

London Coefficient $LC = 0$

$$PROCA\ coefficient\ curlcurlB = [0, 0, 0]$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt{=}J4 \quad = [0, 0, 0, -2\,\varepsilon\,C]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Ampere\ current \quad FL = \text{-}(rho\_ampere\ E + J\_ampere\ x\ B) \quad = [0, 0, -2\,C^2\,\varepsilon\,(1+2\,z)]$$

$$Amperian\ Dissipation\ \ Jampere\ dot\ E \quad = 0$$

$$Lorentz\ Force\ Spin\ factor\ \ LFSPIN = 0$$

$$Topological\ Torsion\ current\ 4\ vector \quad T4 = \text{-}[ExA + B.phi, AdotB] = [z\,x\,C\,(1+z), z\,y\,C\,(1+z), 2\,z^3\,C, 2\,C\,t\,z]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Torsion\ current \quad TF = \text{-}(rho\_torsion\ E + J\_torsion\ x\ B) \quad = [-2\,C\,z^2\,y\,(1+2\,z), 2\,C\,z^2\,x\,(1+2\,z), -2\,C^2\,t\,z\,(1+2\,z)]$$

$$Torsion\ Dissipation\ \ Jtorsion\ dot\ E \quad = 2\,z^3\,C^2\,(1+2\,z)$$

$$Topological\ Spin\ current\ 4\ vector \quad TS4 = \text{-}[A\ x\ H + D.phi, AdotD] = \left[-\frac{-2\,x\,z^2 + C\,t\,y}{\mu}, \frac{C\,t\,x + 2\,y\,z^2}{\mu}, -\frac{z\,(-y^2 - x^2 + \varepsilon\,C^2\,\mu\,z + 2\,\varepsilon\,C^2\,\mu\,z^2)}{\mu}, -C^2\,t\,\varepsilon\,(1+2\,z)\right]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Spin\ current \quad SF = \text{--}(rho\_spin\ E + J\_spin\ x\ B) \quad = \left[-\frac{z\,(-2\,C\,t\,x - 4\,y\,z^2 - y^3 - y\,x^2 + y\,\varepsilon\,C^2\,\mu\,z + 2\,y\,\varepsilon\,C^2\,\mu\,z^2)}{\mu}, \frac{z\,(-x\,y^2 - x^3 + x\,\varepsilon\,C^2\,\mu\,z + 2\,x\,\varepsilon\,C^2\,\mu\,z^2 - 4\,x\,z^2 + 2\,C\,t\,y)}{\mu}, -\frac{C\,t\,(\varepsilon\,C^2\,\mu + 4\,\varepsilon\,C^2\,\mu\,z + 4\,\varepsilon\,C^2\,\mu\,z^2 - x^2 - y^2)}{\mu}\right]$$

$$Spin\ Dissipation\ \ J\_spin\ dot\ E \quad = \frac{z\,(-y^2 - x^2 + \varepsilon\,C^2\,\mu\,z + 2\,\varepsilon\,C^2\,\mu\,z^2)\,C\,(1+2\,z)}{\mu}$$

$$Disspative\ Force\ 3\ vector = [-z\,(-4\,y\,z^2 + 2\,y\,\varepsilon\,C^2\,\mu\,z^2 + 4\,C\,z^2\,y + y\,\varepsilon\,C^2\,\mu\,z + 2\,z\,y\,C - 2\,C\,t\,x - y^3 - y\,x^2), z\,(2\,x\,\varepsilon\,C^2\,\mu\,z^2 - 4\,x\,z^2 + 4\,C\,z^2\,x + x\,\varepsilon\,C^2\,\mu\,z + 2\,z\,x\,C - x\,y^2 - x^3 + 2\,C\,t\,y), -C\,(2\,\varepsilon\,C + 4\,C\,\varepsilon\,z + C^2\,t\,\varepsilon\,\mu + 4\,C^2\,t\,\varepsilon\,\mu\,z + 4\,C^2\,t\,\varepsilon\,\mu\,z^2 - t\,x^2 - t\,y^2 + 2\,C\,t\,z + 4\,C\,t\,z^2)]$$

$$Dissipation \quad = -C\,(2\,\varepsilon + C\,t\,\varepsilon\,\mu + 2\,C\,t\,\varepsilon\,\mu\,z - x\,z - x\,z^2) \tag{8}$$

******************************* *END PROCEDURE* ***************************** **(8)**

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

p.155 vol4

```
> NAME:=`Example EVANS B# p155 vol4`;

> Ax:=y*cos(-kappa*z+omega*t)/(0+x^2+y^2+1*z^2);Ay:=-x*cos(-kappa*z+omega*t)/(0+x^2+
  y^2+1*z^2);Az:=0*kappa*cos(-kappa*z+omega*t);phi:=0*omega*cos(-kappa*z+omega*t);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0);
```

****

$$NAME := Example\ EVANS\ B\#\ p155\ vol4$$

$$Ax := \frac{y\cos(-\kappa z+\omega t)}{x^2+y^2+z^2}$$

$$Ay := -\frac{x\cos(-\kappa z+\omega t)}{x^2+y^2+z^2}$$

$$Az := 0$$

$$\phi := 0$$

Example EVANS B# p155 vol4

********************************* *Differential Form Format* *****************************
****

$$Action\ 1\text{-}form = \frac{y\cos(-\kappa z+\omega t)\ d(x)}{x^2+y^2+z^2} - \frac{x\cos(-\kappa z+\omega t)\ d(y)}{x^2+y^2+z^2}$$

$$Intensity\ 2\text{-}form\ F{=}dA = \frac{y\sin(-\kappa z+\omega t)\ \omega\ (d(x))\ \&\hat{}\ (d(t))}{x^2+y^2+z^2} + \left(-\frac{\cos(-\kappa z+\omega t)\ (x^2-y^2+z^2)}{(x^2+y^2+z^2)^2} + \frac{\cos(-\kappa z+\omega t)\ (x^2-y^2-z^2)}{(x^2+y^2+z^2)^2}\right)(d(x))\ \&\hat{}\ (d(y))$$

$$\%1 = \sin(-\kappa z+\omega t)\ \kappa x^2 + \sin(-\kappa z+\omega t)\ \kappa y^2 + \sin(-\kappa z+\omega t)$$

$$Topological\ Torsion\ 3\text{-}form \quad A\hat{}F = 0$$

$$Topological\ Parity\ 4\text{-}form \quad F\hat{}F = 0$$

******************************** *Using EM format* ********************************

$$E\ field = \left[\frac{y\sin(-\kappa z+\omega t)\ \omega}{x^2+y^2+z^2}, -\frac{x\sin(-\kappa z+\omega t)\ \omega}{x^2+y^2+z^2}, 0\right]$$

$$B\ field = \left[\frac{x(\%1)}{(x^2+y^2+z^2)^2}, \frac{y(\%1)}{(x^2+y^2+z^2)^2}, -\frac{2\cos(-\kappa z+\omega t)\ z^2}{(x^2+y^2+z^2)^2}\right]$$

$$\%1 = \sin(-\kappa z+\omega t)\ \kappa x^2 + \sin(-\kappa z+\omega t)\ \kappa y^2 + \sin(-\kappa z+\omega t)\ \kappa z^2 - 2\cos(-\kappa z+\omega t)\ z$$

$$Topological\ TORSION\ 4\ vector \quad T4 = -[ExA + Bphi, AdotB] = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ II = 2(E.B) = 0$$

$$coefficient\ of\ Topological\ Parity\ 4\text{-}form\ = 0$$

$$Pfaff\ Topological\ Dimension\quad PTD = 2$$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************************

$$Xm\ or\ linear\ (Mean)\ curvature\ = 0$$

$$Yg\ or\ quadratic\ (GAUSS)\ curvature\ = -\frac{\cos(-\kappa z+\omega t)^2\,(x^2+y^2-z^2)}{(x^2+y^2+z^2)^3}$$

$$Za\ or\ Cubic\ (Interaction\ internal\ energy)\ curvature\ = 0$$

$$Tk\ or\ quartic\ (4D\ expansion)\ curvature\ = 0$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$Chirality\ factor\ CH = 0$$

$$D\,field = \left[\frac{\varepsilon\, y \sin(-\kappa z+\omega t)\,\omega}{x^2+y^2+z^2}, -\frac{\varepsilon\, x \sin(-\kappa z+\omega t)\,\omega}{x^2+y^2+z^2}, 0\right]$$

$$H\,field = \left[\frac{x(\%1)}{(x^2+y^2+z^2)^2\mu}, \frac{y(\%1)}{(x^2+y^2+z^2)^2\mu}, -\frac{2\cos(-\kappa z+\omega t)\,z^2}{(x^2+y^2+z^2)^2\mu}\right]$$

$$\%1 = \sin(-\kappa z+\omega t)\,\kappa x^2+\sin(-\kappa z+\omega t)\,\kappa y^2+\sin(-\kappa z+\omega t)\,\kappa z^2-2\cos(-\kappa z+\omega t)\,z$$

$$Poynting\ vector\ ExH = \left[\frac{2x\sin(-\kappa z+\omega t)\,\omega\cos(-\kappa z+\omega t)\,z^2}{(x^2+y^2+z^2)^3\mu},\right.$$

$$\frac{2y\sin(-\kappa z+\omega t)\,\omega\cos(-\kappa z+\omega t)\,z^2}{(x^2+y^2+z^2)^3\mu}, \frac{1}{\mu(x^2+y^2+z^2)^3}(\sin(-\kappa z+\omega t)\,\omega\,(\sin(-\kappa z$$

$$\left.+\omega t)\,\kappa x^2+\sin(-\kappa z+\omega t)\,\kappa y^2+\sin(-\kappa z+\omega t)\,\kappa z^2-2\cos(-\kappa z+\omega t)\,z)\,(x^2+y^2))\right]$$

$$Amperian\ Current\ 4Vector\quad curlH\text{-}dD/dt\text{=}J4\ = \left[\frac{y(\%4)}{(x^2+y^2+z^2)^2\mu}, -\frac{x(\%4)}{(x^2+y^2+z^2)^2\mu}, 0, 0\right]$$

$$\%1 = \varepsilon\cos(-\kappa z+\omega t)\,\omega^2\mu x^2$$

$$\%2 = \varepsilon\cos(-\kappa z+\omega t)\,\omega^2\mu y^2$$

$$\%3 = \varepsilon\cos(-\kappa z+\omega t)\,\omega^2\mu z^2$$

$$\%4 = \cos(-\kappa z+\omega t)\,\kappa^2x^2+4\sin(-\kappa z+\omega t)\,\kappa z+\cos(-\kappa z+\omega t)\,\kappa^2z^2+\cos(-\kappa z+\omega t)\,\kappa^2y^2+2\cos(-\kappa z+$$

$$Amerian\ charge\ density\quad divD\ =\ rho = 0$$

$$divergence\ Lorentz\ Current\ 4Vector,\quad 4div(J4)\ = 0$$

$$\textit{Topological SPIN 4 vector S4} = \left[\frac{2\,x\cos(-\kappa z+\omega t)^2 z^2}{(x^2+y^2+z^2)^3\,\mu}, \frac{2\,y\cos(-\kappa z+\omega t)^2 z^2}{(x^2+y^2+z^2)^3\,\mu}, \right.$$

$$\frac{1}{\mu\,(x^2+y^2+z^2)^3}\left(\cos(-\kappa z+\omega t)\,\left(\sin(-\kappa z+\omega t)\,\kappa x^2+\sin(-\kappa z+\omega t)\,\kappa y^2+\sin(-\kappa z\right.\right.$$

$$\left.\left.\left.+\omega t)\,\kappa z^2-2\cos(-\kappa z+\omega t)\,z\right)\,(x^2+y^2)\right), \frac{\cos(-\kappa z+\omega t)\,\varepsilon\sin(-\kappa z+\omega t)\,\omega\,(x^2+y^2)}{(x^2+y^2+z^2)^2}\right]$$

$$\textit{Topological SPIN 3-form} = \frac{2\,x\cos(-\kappa z+\omega t)^2 z^2\,\&\text{^}(d(y),d(z),d(t))}{(x^2+y^2+z^2)^3\,\mu}$$

$$-\frac{2\,y\cos(-\kappa z+\omega t)^2 z^2\,\&\text{^}(d(x),d(z),d(t))}{(x^2+y^2+z^2)^3\,\mu}+\frac{1}{\mu\,(x^2+y^2+z^2)^3}\left(\cos(-\kappa z+\omega t)\,\left(\sin(-\kappa z\right.\right.$$

$$\left.+\omega t)\,\kappa x^2+\sin(-\kappa z+\omega t)\,\kappa y^2+\sin(-\kappa z+\omega t)\,\kappa z^2-2\cos(-\kappa z+\omega t)\,z\right)\,(x^2+y^2)\,\&\text{^}(d(x),$$

$$\left.d(y),d(t))\right)-\frac{\cos(-\kappa z+\omega t)\,\varepsilon\sin(-\kappa z+\omega t)\,\omega\,(x^2+y^2)\,\&\text{^}(d(x),d(y),d(z))}{(x^2+y^2+z^2)^2}$$

$$\textit{Spin density rho\_spin} = \frac{\cos(-\kappa z+\omega t)\,\varepsilon\sin(-\kappa z+\omega t)\,\omega\,(x^2+y^2)}{(x^2+y^2+z^2)^2}$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{1}{\mu\,(x^2+y^2+z^2)^3}\left(-\sin(-\kappa z+\omega t)^2\kappa^2 x^4-2\sin(-\kappa z\right.$$

$$+\omega t)^2\kappa^2 x^2 y^2-\sin(-\kappa z+\omega t)^2\kappa^2 x^2 z^2+4\sin(-\kappa z+\omega t)\,\kappa x^2\cos(-\kappa z+\omega t)\,z-\sin(-\kappa z$$

$$+\omega t)^2\kappa^2 y^2 z^2-\sin(-\kappa z+\omega t)^2\kappa^2 y^4-4\cos(-\kappa z+\omega t)^2 z^2+4\sin(-\kappa z+\omega t)\,\kappa y^2\cos(-\kappa z$$

$$+\omega t)\,z+\varepsilon\sin(-\kappa z+\omega t)^2\omega^2\mu x^4+2\,\varepsilon\sin(-\kappa z+\omega t)^2\omega^2\mu x^2 y^2+\varepsilon\sin(-\kappa z+\omega t)^2\omega^2\mu x^2 z^2$$

$$\left.+\varepsilon\sin(-\kappa z+\omega t)^2\omega^2\mu y^4+\varepsilon\sin(-\kappa z+\omega t)^2\omega^2\mu y^2 z^2\right)$$

$$B.H = \frac{1}{\mu\,(x^2+y^2+z^2)^3}\left(\sin(-\kappa z+\omega t)^2\kappa^2 x^4+2\sin(-\kappa z+\omega t)^2\kappa^2 x^2 y^2+\sin(-\kappa z+\omega t)^2\kappa^2 x^2 z^2\right.$$

$$-4\sin(-\kappa z+\omega t)\,\kappa x^2\cos(-\kappa z+\omega t)\,z+\sin(-\kappa z+\omega t)^2\kappa^2 y^2 z^2+\sin(-\kappa z+\omega t)^2\kappa^2 y^4$$

$$\left.+4\cos(-\kappa z+\omega t)^2 z^2-4\sin(-\kappa z+\omega t)\,\kappa y^2\cos(-\kappa z+\omega t)\,z\right)$$

$$D.E = \frac{\varepsilon\sin(-\kappa z+\omega t)^2\omega^2\,(x^2+y^2)}{(x^2+y^2+z^2)^2}$$

$$A.J = \frac{1}{\mu\,(x^2+y^2+z^2)^3}\left(\cos(-\kappa z+\omega t)\,\left(\cos(-\kappa z+\omega t)\,\kappa^2 x^2+4\sin(-\kappa z+\omega t)\,\kappa z+\cos(-\kappa z\right.\right.$$

$$+\omega t)\,\kappa^2 z^2+\cos(-\kappa z+\omega t)\,\kappa^2 y^2+2\cos(-\kappa z+\omega t)\,-\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\mu x^2-\varepsilon\cos(-\kappa z$$

$+\omega t)\ \omega^2 \mu y^2 - \varepsilon \cos(-\kappa z+\omega t)\ \omega^2 \mu z^2)\ (x^2+y^2))$

$$\text{-}rho.phi = 0$$

Poincare I $\quad$ *(B.H - D.E)-(A.J - rho.phi)* $= -\dfrac{1}{\mu\,(x^2+y^2+z^2)^3}\big(-\sin(-\kappa z+\omega t)^2 \kappa^2 x^4 - 2\,\varepsilon \cos(-\kappa z$

$+\omega t)^2 \omega^2 \mu x^2 y^2 - \varepsilon \cos(-\kappa z+\omega t)^2 \omega^2 \mu z^2 x^2 - \varepsilon \cos(-\kappa z+\omega t)^2 \omega^2 \mu z^2 y^2 - \sin(-\kappa z$

$+\omega t)^2 \kappa^2 y^4 - 2 \sin(-\kappa z+\omega t)^2 \kappa^2 x^2 y^2 - \sin(-\kappa z+\omega t)^2 \kappa^2 x^2 z^2 - \sin(-\kappa z+\omega t)^2 \kappa^2 y^2 z^2$

$+8 \sin(-\kappa z+\omega t)\ \kappa x^2 \cos(-\kappa z+\omega t)\ z + 8 \sin(-\kappa z+\omega t)\ \kappa y^2 \cos(-\kappa z+\omega t)\ z + 2\,\varepsilon \sin($

$-\kappa z+\omega t)^2 \omega^2 \mu x^2 y^2 + \varepsilon \sin(-\kappa z+\omega t)^2 \omega^2 \mu x^2 z^2 + \varepsilon \sin(-\kappa z+\omega t)^2 \omega^2 \mu y^2 z^2 + \cos(-\kappa z$

$+\omega t)^2 x^4 \kappa^2 + \cos(-\kappa z+\omega t)^2 y^4 \kappa^2 + 2 x^2 \cos(-\kappa z+\omega t)^2 + 2 y^2 \cos(-\kappa z+\omega t)^2 + \varepsilon \sin($

$-\kappa z+\omega t)^2 \omega^2 \mu x^4 + \varepsilon \sin(-\kappa z+\omega t)^2 \omega^2 \mu y^4 - \varepsilon \cos(-\kappa z+\omega t)^2 \omega^2 \mu x^4 - \varepsilon \cos(-\kappa z$

$+\omega t)^2 \omega^2 \mu y^4 + 2 \cos(-\kappa z+\omega t)^2 x^2 \kappa^2 y^2 + x^2 \cos(-\kappa z+\omega t)^2 \kappa^2 z^2 + \cos(-\kappa z+\omega t)^2 \kappa^2 y^2 z^2$

$-4 \cos(-\kappa z+\omega t)^2 z^2\big)$

London Coefficient $\quad LC = \dfrac{1}{(x^2+y^2+z^2)\ \mu \cos(-\kappa z+\omega t)}\big(\cos(-\kappa z+\omega t)\ \kappa^2 x^2 + 4 \sin(-\kappa z$

$+\omega t)\ \kappa z + \cos(-\kappa z+\omega t)\ \kappa^2 z^2 + \cos(-\kappa z+\omega t)\ \kappa^2 y^2 + 2 \cos(-\kappa z+\omega t) - \varepsilon \cos(-\kappa z$

$+\omega t)\ \omega^2 \mu x^2 - \varepsilon \cos(-\kappa z+\omega t)\ \omega^2 \mu y^2 - \varepsilon \cos(-\kappa z+\omega t)\ \omega^2 \mu z^2\big)$

PROCA coefficient curlcurlB $= \left[\dfrac{x\,(\%6)}{(x^2+y^2+z^2)^3}, \dfrac{y\,(\%6)}{(x^2+y^2+z^2)^3}, \dfrac{2\,(2 x^2 \cos(-\kappa z+\omega t) + 4 x^2 \sin(-\kappa z+\omega t)\ \kappa z - x^2 \cos(-\kappa z+\omega t)\ \kappa^2 z^2 -}{}\right.$

$\%1 = 2 x^2 \sin(-$

$\%2 = 6 x^2 \cos(-$

$\%3 = 2 x^2 \sin(-$

$\%4 = 6 \cos(-\kappa z$

$\%5 = 2 \sin(-\kappa z$

$\%6 = \sin(-\kappa z+\omega t)\ \kappa^3 x^4 + \%1 - \%2 + \%3 + 6 \sin(-\kappa z+\omega t)\ \kappa x^2 - 8 \cos(-\kappa z+\omega t)\ z + 6 \sin(-\kappa z+\omega t)\ \kappa$

Amperian Current 4Vector curlH-dD/dt=J4 $= \left[\frac{y(\%4)}{(x^2+y^2+z^2)^2 \mu}, -\frac{x(\%4)}{(x^2+y^2+z^2)^2 \mu}, 0, 0 \right]$

$$\%1 = \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu x^2$$

$$\%2 = \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu y^2$$

$$\%3 = \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu z^2$$

$$\%4 = \cos(-\kappa z + \omega t)\ \kappa^2 x^2 + 4 \sin(-\kappa z + \omega t)\ \kappa z + \cos(-\kappa z + \omega t)\ \kappa^2 z^2 + \cos(-\kappa z + \omega t)\ \kappa^2 y^2 + 2\cos(-\kappa z +$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= \left[-\frac{2x(\%4)\cos(-\kappa z+\omega t)\, z^2}{(x^2+y^2+z^2)^4 \mu}, -\frac{2y(\%4)\cos(-\kappa z+\omega t)\, z^2}{(x^2+y^2+z^2)^4 \mu}, \right.$

$$\%1 = \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu x^2$$

$$\%2 = \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu y^2$$

$$\%3 = \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu z^2$$

$$\%4 = \cos(-\kappa z + \omega t)\ \kappa^2 x^2 + 4 \sin(-\kappa z + \omega t)\ \kappa z + \cos(-\kappa z + \omega t)\ \kappa^2 z^2 + \cos(-\kappa z$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= -\left(\cos(-\kappa z + \omega t)\ (x^2+y^2+z^2)\right) \Big/ \left(\cos(-\kappa z + \omega t)\ \kappa^2 x^2 + 4 \sin(-\kappa z + \omega t)\ \kappa z + \cos(-\kappa z + \omega t)\ \kappa^2 z^2 + \cos(-\kappa z + \omega t)\ \kappa^2 y^2 + 2\cos(-\kappa z + \omega t) - \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu x^2 - \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu y^2 - \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu z^2\right)$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \left[\frac{2x\cos(-\kappa z+\omega t)^2 z^2}{(x^2+y^2+z^2)^3 \mu}, \frac{2y\cos(-\kappa z+\omega t)^2 z^2}{(x^2+y^2+z^2)^3 \mu}, \frac{1}{\mu (x^2+y^2+z^2)^3} \left(\cos(-\kappa z+\omega t)\ \left(\sin(-\kappa z+\omega t)\ \kappa x^2 + \sin(-\kappa z+\omega t)\ \kappa y^2 + \sin(-\kappa z+\omega t)\ \kappa z^2 - 2\cos(-\kappa z+\omega t)\ z\right)\ (x^2+y^2)\right), \frac{\cos(-\kappa z+\omega t)\ \varepsilon \sin(-\kappa z+\omega t)\ \omega\ (x^2+y^2)}{(x^2+y^2+z^2)^2} \right]$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $= \left[-\frac{\cos(-\kappa z+\omega t)\, y\, (\%11)}{(x^2+y^2+z^2)^4 \mu}, \frac{x \cos(-\kappa z+\omega t)}{(x^2+y^2+z^2}\right.$

$$\%1 = 2 \sin(-\kappa z+\omega t)^2 \kappa^2 x^2 y^2$$

$$\%2 = \sin(-\kappa z+\omega t)^2 \kappa^2 x^2 z^2$$

$$\%3 = 4 \sin(-\kappa z+\omega t)\, \kappa x^2 \cos(-\kappa z+\omega t)\, z$$

$$\%4 = \sin(-\kappa z+\omega t)^2 \kappa^2 y^2 z^2$$

$$\%5 = 4 \sin(-\kappa z+\omega t)\, \kappa y^2 \cos(-\kappa z+\omega t)\, z$$

$$\%6 = \varepsilon \sin(-\kappa z+\omega t)^2 \omega^2 \mu x^4$$

$$\%7 = 2 \varepsilon \sin(-\kappa z+\omega t)^2 \omega^2 \mu x^2 y^2$$

$$\%8 = \varepsilon \sin(-\kappa z+\omega t)^2 \omega^2 \mu x^2 z^2$$

$$\%9 = \varepsilon \sin(-\kappa z+\omega t)^2 \omega^2 \mu y^4$$

$$\%10 = \varepsilon \sin(-\kappa z+\omega t)^2 \omega^2 \mu y^2 z^2$$

$$\%11 = -\sin(-\kappa z+\omega t)^2 \kappa^2 x^4 - \%1 - \%2 + \%3 - \%4 - \sin(-\kappa z+\omega t)^2 \kappa^2 y^4 - 4 \cos(-\kappa z+\omega t)^2 z^2 + \%5 + \%6$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \left[-\frac{1}{(x^2+y^2+z^2)^4 \mu} \left(\cos(-\kappa z+\omega t)\, \left(2 x^3 z^2 \cos(-\kappa z+\omega t)\, \kappa^2\right.\right.\right.$

$+ 8 x z^3 \kappa \sin(-\kappa z+\omega t) + 2 x z^4 \cos(-\kappa z+\omega t)\, \kappa^2 + 2 x z^2 \cos(-\kappa z+\omega t)\, \kappa^2 y^2 + 4 \cos(-\kappa z$

$+ \omega t)\, z^2 x - 2 x^3 z^2 \varepsilon \cos(-\kappa z+\omega t)\, \omega^2 \mu - 2 x z^2 \varepsilon \cos(-\kappa z+\omega t)\, \omega^2 \mu y^2 - 2 x z^4 \varepsilon \cos(-\kappa z$

$+ \omega t)\, \omega^2 \mu - \mu y \sin(-\kappa z+\omega t)^2 \kappa^2 x^4 - 2 \mu y^3 \sin(-\kappa z+\omega t)^2 \kappa^2 x^2 - \mu y \sin(-\kappa z+\omega t)^2 \kappa^2 x^2 z^2$

$+ 4 \mu y \sin(-\kappa z+\omega t)\, \kappa x^2 \cos(-\kappa z+\omega t)\, z - \mu y^3 \sin(-\kappa z+\omega t)^2 \kappa^2 z^2 - \mu y^5 \sin(-\kappa z+\omega t)^2 \kappa^2$

$- 4 \mu y \cos(-\kappa z+\omega t)^2 z^2 + 4 \mu y^3 \sin(-\kappa z+\omega t)\, \kappa \cos(-\kappa z+\omega t)\, z + \mu^2 y \varepsilon \sin(-\kappa z$

$+ \omega t)^2 \omega^2 x^4 + 2 \mu^2 y^3 \varepsilon \sin(-\kappa z+\omega t)^2 \omega^2 x^2 + \mu^2 y \varepsilon \sin(-\kappa z+\omega t)^2 \omega^2 x^2 z^2 + \mu^2 y^5 \varepsilon \sin($

$$-\kappa z+\omega t)^2\,\omega^2+\mu^2 y^3\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2 z^2)),\ \frac{1}{(x^2+y^2+z^2)^4\,\mu}\big(\cos(-\kappa z+\omega t)\,($$

$$-2\,y z^2\cos(-\kappa z+\omega t)\,\kappa^2 x^2-8\,y z^3\,\kappa\sin(-\kappa z+\omega t)-2\,y z^4\cos(-\kappa z+\omega t)\,\kappa^2-2\,y^3 z^2\cos(-\kappa z$$

$$+\omega t)\,\kappa^2-4\cos(-\kappa z+\omega t)\,z^2 y+2\,y z^2\,\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\mu x^2+2\,y^3 z^2\,\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\mu$$

$$+2\,y z^4\,\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\mu-\mu x^5\sin(-\kappa z+\omega t)^2\,\kappa^2-2\,\mu x^3\sin(-\kappa z+\omega t)^2\,\kappa^2 y^2$$

$$-\mu x^3\sin(-\kappa z+\omega t)^2\,\kappa^2 z^2+4\,\mu x^3\sin(-\kappa z+\omega t)\,\kappa\cos(-\kappa z+\omega t)\,z-\mu x\sin(-\kappa z$$

$$+\omega t)^2\,\kappa^2 y^2 z^2-\mu x\sin(-\kappa z+\omega t)^2\,\kappa^2 y^4-4\,\mu x\cos(-\kappa z+\omega t)^2 z^2+4\,\mu x\sin(-\kappa z$$

$$+\omega t)\,\kappa y^2\cos(-\kappa z+\omega t)\,z+\mu^2 x^5\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2+2\,\mu^2 x^3\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2 y^2$$

$$+\mu^2 x^3\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2 z^2+\mu^2 x\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2 y^4+\mu^2 x\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2 y^2 z^2)$$

$$\big),\ -\frac{1}{(x^2+y^2+z^2)^4\,\mu}\big(\big(\cos(-\kappa z+\omega t)\,\kappa^2 x^2+4\sin(-\kappa z+\omega t)\,\kappa z+\cos(-\kappa z+\omega t)\,\kappa^2 z^2$$

$$+\cos(-\kappa z+\omega t)\,\kappa^2 y^2+2\cos(-\kappa z+\omega t)-\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\mu x^2-\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\mu y^2$$

$$-\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\mu z^2\big)\,\big(\sin(-\kappa z+\omega t)\,\kappa x^2+\sin(-\kappa z+\omega t)\,\kappa y^2+\sin(-\kappa z+\omega t)\,\kappa z^2$$

$$-2\cos(-\kappa z+\omega t)\,z\big)\,(x^2+y^2)\big)\big]$$

$$Dissipation=\frac{\mu\cos(-\kappa z+\omega t)\,\varepsilon\sin(-\kappa z+\omega t)\,\omega\,(x^2+y^2)}{(x^2+y^2+z^2)^2}$$

*****************************   *END PROCEDURE*   ***************************** **(9)**

```
> EH:=crossprod(E,HF):EH[1];EH[2];factor(EH[3]);
```

$$\frac{2\,x\sin(-\kappa z+\omega t)\,\omega\cos(-\kappa z+\omega t)\,z^2}{(x^2+y^2+z^2)^3\,\mu}$$

$$\frac{2\,y\sin(-\kappa z+\omega t)\,\omega\cos(-\kappa z+\omega t)\,z^2}{(x^2+y^2+z^2)^3\,\mu}$$

(10)

$$\frac{1}{\mu\,(x^2+y^2+z^2)^3}\left(\sin(-\kappa z+\omega t)\,\omega\,(\sin(-\kappa z+\omega t)\,\kappa x^2+\sin(-\kappa z+\omega t)\,\kappa y^2+\sin(-\kappa z+\omega t)\,\kappa z^2 - 2\cos(-\kappa z+\omega t)\,z)\,(x^2+y^2)\right) \quad \textbf{(10)}$$

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.
p=2, n=2, euclidean signature index 0.   p160,vol4

```
> NAME:=`Example A-- Hopf signature index 0 or 1. The 1-form is divided by the Holder
  norm any p, n=2  EdotB =0 `;
```

```
> Ax:=y*1*Omega;Ay:=-x*Omega;Az:=C*t;phi:=z*C;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,-1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME :=

Example A-- Hopf signature index 0 or 1. The 1-form is divided by the Holder norm any p, n=2 EdotB = 0

$$Ax := y\,\Omega$$

$$Ay := -x\,\Omega$$

$$Az := C\,t$$

$$\phi := C\,z$$

Example A-- Hopf signature index 0 or 1. The 1-form is divided by the Holder norm any p, n=2 EdotB =0

********************************* *Differential Form Format* *********************************

$$\textit{Action 1-form} = y\,\Omega\,d(x) - x\,\Omega\,d(y) + C\,t\,d(z) - C\,z\,d(t)$$

$$\textit{Intensity 2-form F=dA} \; = -2\,\Omega\,(d(x))\ \&\hat{}\ (d(y)) - 2\,C\,(d(z))\ \&\hat{}\ (d(t))$$

$$\textit{Topological Torsion 3-form} \quad A\hat{}F = -2\,C\,t\,\Omega\,\&\hat{}(d(x),d(y),d(z)) + 2\,C\,z\,\Omega\,\&\hat{}(d(x),d(y),d(t)) - 2\,y\,\Omega\,C\,\&\hat{}(d(x),d(z),d(t)) + 2\,x\,\Omega\,C\,\&\hat{}(d(y),d(z),d(t))$$

$$\textit{Topological Parity 4-form} \quad F\hat{}F = 8\,\Omega\,C\,\&\hat{}(d(x),d(y),d(z),d(t))$$

********************************* *Using EM format* *********************************

$$\textit{E field} = [0, 0, -2\,C]$$

$$\textit{B field} = [0, 0, -2\,\Omega]$$

$$\textit{Topological TORSION 4 vector} \quad T4 = \text{-[ExA + Bphi,AdotB]} = [2\,x\,\Omega\,C, 2\,y\,\Omega\,C, 2\,C\,z\,\Omega, 2\,C\,t\,\Omega]$$

$$\textit{Helicity AdotB} = -2\,C\,t\,\Omega$$

Poincare II =2(E.B) $= 8\,\Omega\,C$

coefficient of Topological Parity 4-form $= 8\,\Omega\,C$

Pfaff Topological Dimension PTD $= 4$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************************

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= \Omega^2 + C^2$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= \Omega^2\,C^2$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $= 0$

D field $= [0, 0, -2\,\varepsilon\,C]$

$$H\ field = \left[0, 0, -\frac{2\,\Omega}{\mu}\right]$$

Poynting vector ExH $=$ *EXH*

Amperian Current 4Vector curlH-dD/dt=J4 $= [0, 0, 0, 0]$

Amerian charge density divD = rho $= 0$

divergence Lorentz Current 4Vector, 4div(J4) $= 0$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[\frac{2\,x\,\Omega^2}{\mu}, \frac{2\,y\,\Omega^2}{\mu}, -2\,\varepsilon\,C^2\,z, -2\,C^2\,t\,\varepsilon\right]$$

$$Topological\ SPIN\ 3\text{-}form = \frac{2\,x\,\Omega^2\,\&\text{^}(d(y), d(z), d(t))}{\mu} - \frac{2\,y\,\Omega^2\,\&\text{^}(d(x), d(z), d(t))}{\mu}$$
$$-2\,\varepsilon\,C^2\,z\,\&\text{^}(d(x), d(y), d(t)) + 2\,C^2\,t\,\varepsilon\,\&\text{^}(d(x), d(y), d(z))$$

Spin density rho_spin $= -2\,C^2\,t\,\varepsilon$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{4\,(-\Omega^2 + \varepsilon\,C^2\,\mu)}{\mu}$$

$$B.H = \frac{4\,\Omega^2}{\mu}$$

D.E $= 4\,\varepsilon\,C^2$

A.J $= 0$

-rho.phi $= 0$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* $= -\frac{4\,(-\Omega^2 + \varepsilon\, C^2\,\mu)}{\mu}$

London Coefficient $LC = 0$

PROCA coefficient curlcurlB $= [0, 0, 0]$

Amperian Current 4Vector *curlH-dD/dt=J4* $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Ampere current *FL = -(rho_ampere E + J_ampere x B)* $= [0, 0, 0]$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector *T4 = -[ExA + B.phi,AdotB]* $= [2\,x\,\Omega\, C, 2\,y\,\Omega\, C, 2\, C\, z\,\Omega, 2\, C\, t\,\Omega]$

Lorentz Force 3 vector due to Torsion current *TF = -(rho_torsion E + J_torsion x B)* $= [-4\,y\,\Omega^2\, C,$
$4\,x\,\Omega^2\, C, -4\, C^2\, t\,\Omega]$

Torsion Dissipation Jtorsion dot E $= 4\, C^2\, z\,\Omega$

Topological Spin current 4 vector *TS4 = -[A x H + D.phi,AdotD]* $= \left[\frac{2\,x\,\Omega^2}{\mu}, \frac{2\,y\,\Omega^2}{\mu}, -2\,\varepsilon\, C^2\, z,\right.$
$\left. -2\, C^2\, t\,\varepsilon\right]$

Lorentz Force 3 vector due to Spin current *SF = --(rho_spin E + J_spin x B)* $= \left[\frac{4\,y\,\Omega^3}{\mu}, -\frac{4\,x\,\Omega^3}{\mu},\right.$
$\left. -4\, C^3\, t\,\varepsilon\right]$

Spin Dissipation J_spin dot E $= 4\,\varepsilon\, C^3\, z$

Disspative Force 3 vector $= [-4\,y\,\Omega^2\,(-\Omega + C), 4\,x\,\Omega^2\,(-\Omega + C), -4\, C^2\, t\,(C\,\varepsilon\,\mu + \Omega)]$

Dissipation $= -2\, C\,(C\, t\,\varepsilon\,\mu - x\,\Omega)$

****************************** *END PROCEDURE* **************************** **(11)**

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.
p=2, n=4, euclidean signature index 0 p 160 vol4

```
> NAME:=`Example A-- Hopf signature index 0. The 1-form is divided by the Holder norm any
  p=2, n=4 `;
```

```
> Ax:=y*Omega;Ay:=-x*Omega;Az:=C*t*1;phi:=z*C*1;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,4,0*alpha*(g+I*gamma),0):
```

****

NAME := Example A-- Hopf signature index 0. The 1-form is divided by the Holder norm any p=2, n=4

$$Ax := y\,\Omega$$

$$Ay := -x\,\Omega$$

$$Az := C\,t$$

$$\phi := C\,z$$

Example A-- Hopf signature index 0. The 1-form is divided by the Holder norm any p=2, n=4

********************************* *Differential Form Format* *********************************

$$\textit{Action 1-form} = y\,\Omega\,d(x) - x\,\Omega\,d(y) + C\,t\,d(z) - C\,z\,d(t)$$

$$\textit{Intensity 2-form F=dA} \;= -2\,\Omega\,(d(x))\ \&\wedge\ (d(y)) - 2\,C\,(d(z))\ \&\wedge\ (d(t))$$

$$\textit{Topological Torsion 3-form}\quad A\wedge F = -2\,C\,t\,\Omega\,\&\wedge(d(x), d(y), d(z)) + 2\,C\,z\,\Omega\,\&\wedge(d(x), d(y), d(t)) - 2\,y\,\Omega\,C\,\&\wedge(d(x), d(z), d(t)) + 2\,x\,\Omega\,C\,\&\wedge(d(y), d(z), d(t))$$

$$\textit{Topological Parity 4-form}\quad F\wedge F = 8\,\Omega\,C\,\&\wedge(d(x), d(y), d(z), d(t))$$

********************************* *Using EM format* *********************************

$$\textit{E field} = [0, 0, -2\,C]$$

$$\textit{B field} = [0, 0, -2\,\Omega]$$

$$\textit{Topological TORSION 4 vector}\quad T4 = \text{-[ExA + Bphi,AdotB]} = [2\,x\,\Omega\,C, 2\,y\,\Omega\,C, 2\,C\,z\,\Omega, 2\,C\,t\,\Omega]$$

$$\textit{Helicity AdotB} = -2\,C\,t\,\Omega$$

$$\textit{Poincare II =2(E.B)} = 8\,\Omega\,C$$

$$\textit{coefficient of Topological Parity 4-form} = 8\,\Omega\,C$$

$$\textit{Pfaff Topological Dimension}\quad PTD = 4$$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************************

$$\textit{Xm or linear (Mean) curvature} = 0$$

$$\textit{Yg or quadratic (GAUSS) curvature} = \Omega^2 + C^2$$

$$\textit{Za or Cubic (Interaction internal energy) curvature} = 0$$

$$\textit{Tk or quartic (4D expansion) curvature} = \Omega^2\,C^2$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\textit{Chirality factor CH} = 0$$

$$D\,field = [0, 0, -2\,\varepsilon\,C]$$

$$H\,field = \left[0, 0, -\frac{2\,\Omega}{\mu}\right]$$

$$Poynting\ vector\ ExH = EXH$$

$$Amperian\ Current\ 4Vector\quad curlH\text{-}dD/dt{=}J4\ = [0, 0, 0, 0]$$

$$Amerian\ charge\ density\quad divD\ =\ rho = 0$$

$$divergence\ Lorentz\ Current\ 4Vector,\quad 4div(J4)\ = 0$$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[\frac{2\,x\,\Omega^2}{\mu}, \frac{2\,y\,\Omega^2}{\mu}, -2\,\varepsilon\,C^2\,z, -2\,C^2\,t\,\varepsilon\right]$$

$$Topological\ SPIN\ 3\text{-}form = \frac{2\,x\,\Omega^2\ \&\text{^}(d(y), d(z), d(t))}{\mu} - \frac{2\,y\,\Omega^2\ \&\text{^}(d(x), d(z), d(t))}{\mu}$$
$$-2\,\varepsilon\,C^2\,z\ \&\text{^}(d(x), d(y), d(t)) + 2\,C^2\,t\,\varepsilon\ \&\text{^}(d(x), d(y), d(z))$$

$$Spin\ density\ rho\_spin = -2\,C^2\,t\,\varepsilon$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{4\,(-\Omega^2 + \varepsilon\,C^2\,\mu)}{\mu}$$

$$B.H = \frac{4\,\Omega^2}{\mu}$$

$$D.E = 4\,\varepsilon\,C^2$$

$$A.J = 0$$

$$\text{-}rho.phi = 0$$

$$Poincare\ I\qquad (B.H - D.E)\text{-}(A.J - rho.phi)\ = -\frac{4\,(-\Omega^2 + \varepsilon\,C^2\,\mu)}{\mu}$$

$$London\ Coefficient\qquad LC = 0$$

$$PROCA\ coefficient\ curlcurlB = [0, 0, 0]$$

$$Amperian\ Current\ 4Vector\quad curlH\text{-}dD/dt{=}J4\ = [0, 0, 0, 0]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Ampere\ current\quad FL = \text{-}(rho\_ampere\ E + J\_ampere\ x\ B)\ = [0, 0, 0]$$

$$Amperian\ Dissipation\ \ Jampere\ dot\ E\ = 0$$

$$Lorentz\ Force\ Spin\ factor\ \ LFSPIN = 0$$

$$Topological\ Torsion\ current\ 4\ vector\quad T4 = \text{-}[ExA + B.phi, AdotB]\ = [2\,x\,\Omega\,C, 2\,y\,\Omega\,C, 2\,C\,z\,\Omega, 2\,C\,t\,\Omega]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Torsion\ current\quad TF = \text{-}(rho\_torsion\ E + J\_torsion\ x\ B)\ = [-4\,y\,\Omega^2\,C,$$
$$4\,x\,\Omega^2\,C, -4\,C^2\,t\,\Omega]$$

$$\textit{Torsion Dissipation Jtorsion dot E} \quad = 4\, C^2 z\, \Omega$$

$$\textit{Topological Spin current 4 vector} \quad \textit{TS4 = -[A x H + D.phi,AdotD]} \; = \left[\frac{2\, x\, \Omega^2}{\mu}, \frac{2\, y\, \Omega^2}{\mu}, -2\, \varepsilon\, C^2 z, -2\, C^2 t\, \varepsilon \right]$$

$$\textit{Lorentz Force 3 vector due to Spin current} \quad \textit{SF = --(rho\_spin E + J\_spin x B)} \; = \left[\frac{4\, y\, \Omega^3}{\mu}, -\frac{4\, x\, \Omega^3}{\mu}, -4\, C^3 t\, \varepsilon \right]$$

$$\textit{Spin Dissipation J\_spin dot E} \quad = 4\, \varepsilon\, C^3 z$$

$$\textit{Disspative Force 3 vector} = \left[-4\, y\, \Omega^2 \left(-\Omega + C \right), 4\, x\, \Omega^2 \left(-\Omega + C \right), -4\, C^2 t \left(C\, \varepsilon\, \mu + \Omega \right) \right]$$

$$\textit{Dissipation} \quad = -2\, C \left(C\, t\, \varepsilon\, \mu - x\, \Omega \right)$$

******************************* *END PROCEDURE* ******************************* **(12)**

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

p=2, n=4, Minkowski signature index 1 negative ambiguity p 161 vol4

```
> NAME:=`Example B-- Hopf signature index 1. The 1-form is divided by the Holder norm p=
  2, n=4 `;

> Ax:=-y;Ay:=+x;Az:=-C*t;phi:=-z*C;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,-1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example B-- Hopf signature index 1. The 1-form is divided by the Holder norm p=2, n=4

$$Ax := -y$$

$$Ay := x$$

$$Az := -C\, t$$

$$\phi := -C\, z$$

Example B-- Hopf signature index 1. The 1-form is divided by the Holder norm p=2, n=4

*********************************** *Differential Form Format* **********************************

$$\textit{Action 1-form} = -y\, d(x) + x\, d(y) - C\, t\, d(z) + C\, z\, d(t)$$

$$\textit{Intensity 2-form F=dA} \quad = 2\, (d(x))\, \&\hat{}\, (d(y)) + 2\, C\, (d(z))\, \&\hat{}\, (d(t))$$

$$\textit{Topological Torsion 3-form} \quad \textit{A\^{}F} = -2\, C\, t\, \&\hat{}(d(x), d(y), d(z)) + 2\, C\, z\, \&\hat{}(d(x), d(y), d(t)) - 2\, y\, C\, \&\hat{}(d(x), d(z), d(t)) + 2\, x\, C\, \&\hat{}(d(y), d(z), d(t))$$

$$\textit{Topological Parity 4-form} \quad \textit{F\^{}F} = 8\, C\, \&\hat{}(d(x), d(y), d(z), d(t))$$

*********************************** Using EM format ***********************************

$E\,field = [0, 0, 2\,C]$

$B\,field = [0, 0, 2]$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [2\,x\,C, 2\,y\,C, 2\,C\,z, 2\,C\,t]$

Helicity AdotB $= -2\,C\,t$

Poincare II =2(E.B) $= 8\,C$

coefficient of Topological Parity 4-form $= 8\,C$

Pfaff Topological Dimension PTD $= 4$

******************** Correlation Similarity Invariants of Jacobian of (Ak/lambda_N) ***************
****

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= C^2 + 1$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= C^2$

********** Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH ***********

Chirality factor $CH = 0$

$D\,field = [0, 0, 2\,\varepsilon\,C]$

$$H\,field = \left[0, 0, \frac{2}{\mu}\right]$$

Poynting vector ExH = EXH

Amperian Current 4Vector curlH-dD/dt=J4 $= [0, 0, 0, 0]$

Amerian charge density divD = rho $= 0$

divergence Lorentz Current 4Vector, 4div(J4) $= 0$

$$\text{Topological SPIN 4 vector } S4 = \left[\frac{2\,x}{\mu}, \frac{2\,y}{\mu}, -2\,\varepsilon\,C^2\,z, -2\,C^2\,t\,\varepsilon\right]$$

$$\text{Topological SPIN 3-form} = \frac{2\,x\,\&\text{^}(d(y), d(z), d(t))}{\mu} - \frac{2\,y\,\&\text{^}(d(x), d(z), d(t))}{\mu} - 2\,\varepsilon\,C^2\,z\,\&\text{^}(d(x), d(y), d(t)) + 2\,C^2\,t\,\varepsilon\,\&\text{^}(d(x), d(y), d(z))$$

Spin density rho_spin $= -2\,C^2\,t\,\varepsilon$

$$\text{LaGrange field energy density (B.H-D.E)} = -\frac{4\,(-1 + \varepsilon\,C^2\,\mu)}{\mu}$$

$$B.H = \frac{4}{\mu}$$

$$D.E = 4\,\varepsilon\,C^2$$

$$A.J = 0$$

$$-rho.phi = 0$$

$$Poincare\ I \qquad (B.H - D.E)\text{-}(A.J - rho.phi) = -\frac{4\left(-1+\varepsilon C^2 \mu\right)}{\mu}$$

$$London\ Coefficient \qquad LC = 0$$

$$PROCA\ coefficient\ curlcurlB = [0, 0, 0]$$

$$Amperian\ Current\ 4Vector \qquad curlH\text{-}dD/dt\text{=}J4 \quad = [0, 0, 0, 0]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Ampere\ current \quad FL = \text{-}(rho\_ampere\ E + J\_ampere\ x\ B) \quad = [0, 0, 0]$$

$$Amperian\ Dissipation\ \ Jampere\ dot\ E \quad = 0$$

$$Lorentz\ Force\ Spin\ factor\ \ LFSPIN = 0$$

$$Topological\ Torsion\ current\ 4\ vector \quad T4 = \text{-}[ExA + B.phi,AdotB] = [2\,x\,C, 2\,y\,C, 2\,C\,z, 2\,C\,t]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Torsion\ current \quad TF = \text{-}(rho\_torsion\ E + J\_torsion\ x\ B) \quad = \left[4\,y\,C, -4\,x\,C, 4\,C^2\,t\right]$$

$$Torsion\ Dissipation\ \ Jtorsion\ dot\ E \quad = -4\,C^2\,z$$

$$Topological\ Spin\ current\ 4\ vector \quad TS4 = \text{-}[A\ x\ H + D.phi,AdotD] = \left[\frac{2\,x}{\mu}, \frac{2\,y}{\mu}, -2\,\varepsilon\,C^2\,z, -2\,C^2\,t\,\varepsilon\right]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Spin\ current \quad SF = \text{--}(rho\_spin\ E + J\_spin\ x\ B) \quad = \left[-\frac{4\,y}{\mu}, \frac{4\,x}{\mu}, 4\,C^3\,t\,\varepsilon\right]$$

$$Spin\ Dissipation\ \ J\_spin\ dot\ E \quad = -4\,\varepsilon\,C^3\,z$$

$$Disspative\ Force\ 3\ vector = \left[4\,y\,(C-1), -4\,x\,(C-1), 4\,C^2\,t\,\left(C\,\varepsilon\,\mu+1\right)\right]$$

$$Dissipation \quad = -2\,C\,\left(C\,t\,\varepsilon\,\mu - x\right)$$

****************************** *END PROCEDURE* ****************************** **(13)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 1a-- Real Linear Polarization A^G<>0, A^F = 0  INBOUND `;
> theta:=(k*z+omega*t);
> Ax:=cos(theta);Ay:=cos(theta);Az:=0;phi:=0;
```

****

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0);
```

NAME := Example 1a-- Real Linear Polarization A^G <>0, A^F = 0 INBOUND

$$\theta := k z + \omega t$$

$$Ax := \cos(k z + \omega t)$$

$$Ay := \cos(k z + \omega t)$$

$$Az := 0$$

$$\phi := 0$$

Example 1a-- Real Linear Polarization A^G! O 0, A^F = 0 INBOUND

*********************************** Differential Form Format **

$$\textit{Action 1-form} = \cos(k z + \omega t)\ d(x) + \cos(k z + \omega t)\ d(y)$$

$$\textit{Intensity 2-form F=dA} = \sin(k z + \omega t)\ \omega\ (d(x))\ \&\text{^}\ (d(t)) + \sin(k z + \omega t)\ k\ (d(x))\ \&\text{^}\ (d(z)) + \sin(k z + \omega t)\ \omega\ (d(y))\ \&\text{^}\ (d(t)) + \sin(k z + \omega t)\ k\ (d(y))\ \&\text{^}\ (d(z))$$

$$\textit{Topological Torsion 3-form} \quad A\text{^}F = 0$$

$$\textit{Topological Parity 4-form} \quad F\text{^}F = 0$$

*********************************** Using EM format ************************************

$$\textit{E field} = [\sin(k z + \omega t)\ \omega, \sin(k z + \omega t)\ \omega, 0]$$

$$\textit{B field} = [\sin(k z + \omega t)\ k, -\sin(k z + \omega t)\ k, 0]$$

$$\textit{Topological TORSION 4 vector} \quad T4 = -[ExA + Bphi, AdotB] = [0, 0, 0, 0]$$

$$\textit{Helicity AdotB} = 0$$

$$\textit{Poincare II} = 2(E.B) = 0$$

$$\textit{coefficient of Topological Parity 4-form} = 0$$

$$\textit{Pfaff Topological Dimension} \quad PTD = 2$$

********************* Correlation Similarity Invariants of Jacobian of (Ak/lambda_N) *****************************

$$\textit{Xm or linear (Mean) curvature} = 0$$

$$\textit{Yg or quadratic (GAUSS) curvature} = 0$$

$$\textit{Za or Cubic (Interaction internal energy) curvature} = 0$$

$$\textit{Tk or quartic (4D expansion) curvature} = 0$$

*********** Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH ************

$$\textit{Chirality factor CH} = 0$$

$$\textit{D field} = [\varepsilon \sin(k z + \omega t)\ \omega, \varepsilon \sin(k z + \omega t)\ \omega, 0]$$

$$H\,field = \left[\frac{\sin(kz+\omega t)\,k}{\mu}, -\frac{\sin(kz+\omega t)\,k}{\mu}, 0\right]$$

$$Poynting\ vector\ ExH = \left[\begin{array}{ccc} 0 & 0 & \frac{2\,\omega\,k\,(-1+\cos(kz+\omega t)^2)}{\mu} \end{array}\right]$$

Amperian Current 4Vector   curlH-dD/dt=J4 $= [\%1, \%1, 0, 0]$

$$\%1 = \frac{\cos(kz+\omega t)\,(k^2-\varepsilon\,\omega^2\,\mu)}{\mu}$$

Amerian charge density   divD = rho $= 0$

divergence Lorentz Current 4Vector,   4div(J4) $= 0$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[0, 0, -\frac{2\cos(kz+\omega t)\sin(kz+\omega t)\,k}{\mu}, 2\cos(kz+\omega t)\,\varepsilon\sin(kz+\omega t)\,\omega\right]$$

$$Topological\ SPIN\ 3\text{-}form = -\frac{2\cos(kz+\omega t)\sin(kz+\omega t)\,k\,\&\wedge(d(x), d(y), d(t))}{\mu} - 2\cos(kz+\omega t)\,\varepsilon\sin(kz+\omega t)\,\omega\,\&\wedge(d(x), d(y), d(z))$$

$$Spin\ density\ rho\_spin = 2\cos(kz+\omega t)\,\varepsilon\sin(kz+\omega t)\,\omega$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = \frac{2\sin(kz+\omega t)^2\,(k^2-\varepsilon\,\omega^2\,\mu)}{\mu}$$

$$B.H = \frac{2\sin(kz+\omega t)^2\,k^2}{\mu}$$

$$D.E = 2\,\varepsilon\sin(kz+\omega t)^2\,\omega^2$$

$$A.J = \frac{2\cos(kz+\omega t)^2\,(k^2-\varepsilon\,\omega^2\,\mu)}{\mu}$$

$$-rho.phi = 0$$

Poincare I   (B.H - D.E)-(A.J - rho.phi) $=$

$$-\frac{2\,(k^2-\varepsilon\,\omega^2\,\mu)\,(\cos(kz+\omega t)-\sin(kz+\omega t))\,(\cos(kz+\omega t)+\sin(kz+\omega t))}{\mu}$$

$$London\ Coefficient\quad LC = \frac{k^2-\varepsilon\,\omega^2\,\mu}{\mu}$$

$$PROCA\ coefficient\ curlcurlB = [\sin(kz+\omega t)\,k^3, -\sin(kz+\omega t)\,k^3, 0]$$

Amperian Current 4Vector curlH-dD/dt=J4 $= [\%1, \%1, 0, 0]$

$$\%1 = \frac{\cos(k z + \omega t)\,(k^2 - \varepsilon\,\omega^2\,\mu)}{\mu}$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= \left[0, 0, \frac{2\cos(k z + \omega t)\,(k^2 - \varepsilon\,\omega^2\,\mu)\,\sin(k z + \omega t)\,k}{\mu}\right]$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \left[0, 0, -\frac{2\cos(k z + \omega t)\,\sin(k z + \omega t)\,k}{\mu}, 2\cos(k z + \omega t)\,\varepsilon\,\sin(k z + \omega t)\,\omega\right]$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= [\%1, \%1, 0]$

$$\%1 = \frac{2\cos(k z + \omega t)\,\sin(k z + \omega t)^2\,(k^2 - \varepsilon\,\omega^2\,\mu)}{\mu}$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \left[\%1, \%1, \frac{2\cos(k z + \omega t)\,(k^2 - \varepsilon\,\omega^2\,\mu)\,\sin(k z + \omega t)\,k}{\mu}\right]$

$$\%1 = 2\cos(k z + \omega t)\,\sin(k z + \omega t)^2\,(k^2 - \varepsilon\,\omega^2\,\mu)$$

Dissipation $= 2\,\mu\cos(k z + \omega t)\,\varepsilon\,\sin(k z + \omega t)\,\omega$

****************************** *END PROCEDURE* **************************** **(14)**

```
> factor(B[3]);
```

0 **(15)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 1b-- Real Linear Polarization A^G<>0, A^F = 0  OUTBOUND `;
```

```
> theta:=(-k*z+omega*t);
> Ax:=cos(theta);Ay:=cos(theta);Az:=0;phi:=0;
```
Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)
```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,g+I*gamma,0);
```
****

NAME := *Example 1b-- Real Linear Polarization A^G<>0, A^F = 0 OUTBOUND*

$$\theta := -k z + \omega t$$

$$Ax := \cos(k z - \omega t)$$

$$Ay := \cos(k z - \omega t)$$

$$Az := 0$$

$$\phi := 0$$

Example 1b-- Real Linear Polarization A^G! O 0, A^F = 0 OUTBOUND

******************************** *Differential Form Format* ******************************

Action 1-form $= d(x)\cos(k z - \omega t) + d(y)\cos(k z - \omega t)$

Intensity 2-form F=dA $= -\sin(k z - \omega t)\,\omega\,(d(x))\ \&\wedge\ (d(t)) + \sin(k z - \omega t)\,k\,(d(x))\ \&\wedge\ (d(z)) - \sin(k z - \omega t)\,\omega\,(d(y))\ \&\wedge\ (d(t)) + \sin(k z - \omega t)\,k\,(d(y))\ \&\wedge\ (d(z))$

Topological Torsion 3-form *A^F* $= 0$

Topological Parity 4-form *F^F* $= 0$

******************************** *Using EM format* ********************************

E field $= [-\sin(k z - \omega t)\,\omega, -\sin(k z - \omega t)\,\omega, 0]$

B field $= [\sin(k z - \omega t)\,k, -\sin(k z - \omega t)\,k, 0]$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension PTD $= 2$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= 0$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\textit{Chirality factor CH} = (g + \mathrm{I}\,\gamma)\sqrt{\frac{\mu}{\varepsilon}}$$

$$\textit{D field} = \left[-\mathrm{I}\left(-\mathrm{I}\,\varepsilon\,\omega + \mathrm{I}\sqrt{\frac{\mu}{\varepsilon}}\,k\,g - \sqrt{\frac{\mu}{\varepsilon}}\,k\,\gamma \right)\sin(k\,z - \omega\,t), \mathrm{I}\left(\mathrm{I}\,\varepsilon\,\omega + \mathrm{I}\sqrt{\frac{\mu}{\varepsilon}}\,k\,g - \sqrt{\frac{\mu}{\varepsilon}}\,k\,\gamma \right)\sin(k\,z - \omega\,t), 0 \right]$$

$$\textit{H field} = \left[-\frac{\mathrm{I}\left(\mathrm{I}\,k + \mathrm{I}\sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\mu\,g - \sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\mu\,\gamma \right)\sin(k\,z - \omega\,t)}{\mu}, -\frac{\mathrm{I}\left(-\mathrm{I}\,k + \mathrm{I}\sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\mu\,g - \sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\mu\,\gamma \right)\sin(k\,z - \omega\,t)}{\mu}, 0 \right]$$

$$\textit{Poynting vector ExH} = \left[\begin{matrix} 0 & 0 & -\frac{2\,\omega\,k\,(-1 + \cos(k\,z - \omega\,t)^2)}{\mu} \end{matrix} \right]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} = [\%1, \%1, 0, 0]$$

$$\%1 = \frac{\cos(k\,z - \omega\,t)\,(k^2 - \varepsilon\,\omega^2\,\mu)}{\mu}$$

$$\textit{Amerian charge density} \qquad \textit{divD} = \textit{rho} = 0$$

$$\textit{divergence Lorentz Current 4Vector,} \quad \textit{4div(J4)} = 0$$

$$\textit{Topological SPIN 4 vector S4} = \left[0, 0, -\frac{2\cos(k\,z - \omega\,t)\sin(k\,z - \omega\,t)\,k}{\mu}, -2\cos(k\,z - \omega\,t)\sin(k\,z - \omega\,t)\,\varepsilon\,\omega \right]$$

$$\textit{Topological SPIN 3-form} = -\frac{2\cos(k\,z - \omega\,t)\sin(k\,z - \omega\,t)\,k\,\&\hat{}(d(x), d(y), d(t))}{\mu} + 2\cos(k\,z - \omega\,t)\sin(k\,z - \omega\,t)\,\varepsilon\,\omega\,\&\hat{}(d(x), d(y), d(z))$$

$$\textit{Spin density rho\_spin} = -2\cos(k\,z - \omega\,t)\sin(k\,z - \omega\,t)\,\varepsilon\,\omega$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{2\sin(k\,z - \omega\,t)^2\,(k^2 - \varepsilon\,\omega^2\,\mu)}{\mu}$$

$$\textit{B.H} = \frac{2\sin(k\,z - \omega\,t)^2\,k^2}{\mu}$$

$$\textit{D.E} = 2\sin(k\,z - \omega\,t)^2\,\omega^2\,\varepsilon$$

$$A.J = \frac{2\cos(kz-\omega t)^2 (k^2-\varepsilon\omega^2\mu)}{\mu}$$

$$\textit{-rho.phi} = 0$$

Poincare I (B.H - D.E)-(A.J - rho.phi) =

$$-\frac{2(k^2-\varepsilon\omega^2\mu)(\cos(kz-\omega t)-\sin(kz-\omega t))(\cos(kz-\omega t)+\sin(kz-\omega t))}{\mu}$$

London Coefficient $LC = \frac{k^2-\varepsilon\omega^2\mu}{\mu}$

PROCA coefficient curlcurlB $= [k^3\sin(kz-\omega t), -k^3\sin(kz-\omega t), 0]$

Amperian Current 4Vector curlH-dD/dt=J4 $= [\%1, \%1, 0, 0]$

$$\%1 = \frac{\cos(kz-\omega t)(k^2-\varepsilon\omega^2\mu)}{\mu}$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= \left[0, 0, \frac{2\cos(kz-\omega t)(k^2-\varepsilon\omega^2\mu)\sin(kz-\omega t)\,k}{\mu}\right]$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \left[0, 0, -\frac{2\cos(kz-\omega t)\sin(kz-\omega t)\,k}{\mu}, -2\cos(kz-\omega t)\sin(kz-\omega t)\,\varepsilon\omega\right]$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= [\%1, \%1, 0]$

$$\%1 = \frac{2\cos(kz-\omega t)\sin(kz-\omega t)^2(k^2-\varepsilon\omega^2\mu)}{\mu}$$

Spin Dissipation J_spin dot E $= 0$

$$Disspative\ Force\ 3\ vector = \left[\%1, \%1, \frac{2\cos(kz - \omega t)\left(k^2 - \varepsilon\omega^2\mu\right)\sin(kz - \omega t)\, k}{\mu} \right]$$

$$\%1 = 2\cos(kz - \omega t)\sin(kz - \omega t)^2\left(k^2 - \varepsilon\omega^2\mu\right)$$

$$Dissipation = -2\mu\cos(kz - \omega t)\sin(kz - \omega t)\,\varepsilon\omega$$

****************************** *END PROCEDURE* **************************** **(16)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 2a-- Real Circular Polarization A^G=0, A^F <> 0 INBOUND`;
> theta:=(k*z+omega*t);
> Ax:=cos(theta);Ay:=sin(theta);Az:=0;phi:=0;
```

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,g+I*gamma,0):
```

****

NAME := *Example 2a-- Real Circular Polarization A^G=0, A^F <> 0 INBOUND*

$$\theta := k\,z + \omega\,t$$

$$Ax := \cos(k\,z + \omega\,t)$$

$$Ay := \sin(k\,z + \omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

Example 2a-- Real Circular Polarization A^G=0, A^F ! O 0 INBOUND

******************************** *Differential Form Format* ********************************

$$\textit{Action 1-form} = \cos(k\,z + \omega\,t)\ d(x) + \sin(k\,z + \omega\,t)\ d(y)$$

$$\textit{Intensity 2-form F=dA} = \sin(k\,z + \omega\,t)\ \omega\ (d(x))\ \&\text{^}\ (d(t)) + \sin(k\,z + \omega\,t)\ k\ (d(x))\ \&\text{^}\ (d(z)) - \cos(k\,z + \omega\,t)\ \omega\ (d(y))\ \&\text{^}\ (d(t)) - \cos(k\,z + \omega\,t)\ k\ (d(y))\ \&\text{^}\ (d(z))$$

$$\textit{Topological Torsion 3-form}\quad A\text{^}F = \left(-\sin(k\,z + \omega\,t)^2\,\omega - \cos(k\,z + \omega\,t)^2\,\omega\right)\ \&\text{^}(d(x), d(y), d(t)) + \left(-\sin(k\,z + \omega\,t)^2\,k - \cos(k\,z + \omega\,t)^2\,k\right)\ \&\text{^}(d(x), d(y), d(z))$$

$$\textit{Topological Parity 4-form}\quad F\text{^}F = 0$$

******************************** *Using EM format* ********************************

$$\textit{E field} = [\sin(k\,z + \omega\,t)\ \omega,\ -\cos(k\,z + \omega\,t)\ \omega,\ 0]$$

$$\textit{B field} = [-\cos(k\,z + \omega\,t)\ k,\ -\sin(k\,z + \omega\,t)\ k,\ 0]$$

$$\textit{Topological TORSION 4 vector}\quad T4 = -[ExA + Bphi, AdotB] = [0, 0, -\omega\,(\%1), k\,(\%1)]$$

$$\%1 = \cos(k\,z + \omega\,t)^2 + \sin(k\,z + \omega\,t)^2$$

$$\textit{Helicity AdotB} = -k\left(\cos(k\,z + \omega\,t)^2 + \sin(k\,z + \omega\,t)^2\right)$$

$$\textit{Poincare II} = 2(E.B) = 0$$

$$\textit{coefficient of Topological Parity 4-form} = 0$$

$$\textit{Pfaff Topological Dimension}\quad PTD = 3$$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************

****

Xm or linear (Mean) curvature $=0$

Yg or quadratic (GAUSS) curvature $=0$

Za or Cubic (Interaction internal energy) curvature $=0$

Tk or quartic (4D expansion) curvature $=0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\textit{Chirality factor CH} = (g + \mathrm{I}\,\gamma)\sqrt{\frac{\mu}{\varepsilon}}$$

$$\textit{D field} = \left[\mathrm{I}\left(-\mathrm{I}\,\varepsilon \sin(k z + \omega t)\,\omega + \mathrm{I}\sqrt{\frac{\mu}{\varepsilon}}\cos(k z + \omega t)\,k\,g - \sqrt{\frac{\mu}{\varepsilon}}\cos(k z + \omega t)\,k\,\gamma\right), -\varepsilon\cos(k z + \omega t)\,\omega - \sqrt{\frac{\mu}{\varepsilon}}\sin(k z + \omega t)\,k\,g - \mathrm{I}\sqrt{\frac{\mu}{\varepsilon}}\sin(k z + \omega t)\,k\,\gamma, 0\right]$$

$$\textit{H field} = \left[-\frac{\cos(k z + \omega t)\,k + \sqrt{\frac{\mu}{\varepsilon}}\sin(k z + \omega t)\,\omega\,\mu\,g + \mathrm{I}\sqrt{\frac{\mu}{\varepsilon}}\sin(k z + \omega t)\,\omega\,\mu\,\gamma}{\mu}, -\frac{\mathrm{I}\left(-\mathrm{I}\sin(k z + \omega t)\,k + \mathrm{I}\sqrt{\frac{\mu}{\varepsilon}}\cos(k z + \omega t)\,\omega\,\mu\,g - \sqrt{\frac{\mu}{\varepsilon}}\cos(k z + \omega t)\,\omega\,\mu\,\gamma\right)}{\mu}, 0\right]$$

$$\textit{Poynting vector ExH} = \left[\begin{array}{ccc} 0 & 0 & -\frac{\omega k}{\mu} \end{array}\right]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} = \left[\frac{\cos(k z + \omega t)\,(k^2 - \varepsilon\,\omega^2\,\mu)}{\mu}, \frac{\sin(k z + \omega t)\,(k^2 - \varepsilon\,\omega^2\,\mu)}{\mu}, 0, 0\right]$$

Amerian charge density *divD = rho* $=0$

divergence Lorentz Current 4Vector, 4div(J4) $=0$

$$\textit{Topological SPIN 4 vector S4} = \left[0, 0, \mathrm{I}\,(\%1)\,(\%2)\,(\mathrm{I}\,g - \gamma)\sqrt{\frac{\mu}{\varepsilon}}\,\omega, -\mathrm{I}\,(\%1)\,(\%2)\,(\mathrm{I}\,g - \gamma)\sqrt{\frac{\mu}{\varepsilon}}\,k\right]$$

$$\%1 = \cos(k z + \omega t) + \mathrm{I}\sin(k z + \omega t)$$

$$\%2 = -\cos(k z + \omega t) + \mathrm{I}\sin(k z + \omega t)$$

$$\textit{Topological SPIN 3-form} = \mathrm{I}(\%1)(\%2)(\mathrm{I}g-\gamma)\sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\&\hat{}(d(x),d(y),d(t)) + \mathrm{I}(\%1)(\%2)(\mathrm{I}g-\gamma)\sqrt{\frac{\mu}{\varepsilon}}\,k\,\&\hat{}(d(x),d(y),d(z))$$

$$\%1 = \cos(kz+\omega t) + \mathrm{I}\sin(kz+\omega t)$$

$$\%2 = -\cos(kz+\omega t) + \mathrm{I}\sin(kz+\omega t)$$

$$\textit{Spin density rho\_spin} = -\mathrm{I}\,(\cos(kz+\omega t) + \mathrm{I}\sin(kz+\omega t))\,(-\cos(kz+\omega t) + \mathrm{I}\sin(kz+\omega t))\,(\mathrm{I}g-\gamma)\sqrt{\frac{\mu}{\varepsilon}}\,k$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{(k^2-\varepsilon\omega^2\mu)\,(\cos(kz+\omega t)^2+\sin(kz+\omega t)^2)}{\mu}$$

$$B.H = \frac{k^2\,(\cos(kz+\omega t)^2+\sin(kz+\omega t)^2)}{\mu}$$

$$D.E = \varepsilon\omega^2\,(\cos(kz+\omega t)^2+\sin(kz+\omega t)^2)$$

$$A.J = \frac{(k^2-\varepsilon\omega^2\mu)\,(\cos(kz+\omega t)^2+\sin(kz+\omega t)^2)}{\mu}$$

$$\textit{-rho.phi} = 0$$

$$\textit{Poincare I} \quad \textit{(B.H - D.E)-(A.J - rho.phi)} = 0$$

$$\textit{London Coefficient} \quad LC = \frac{k^2-\varepsilon\omega^2\mu}{\mu}$$

$$\textit{PROCA coefficient curlcurlB} = [-\cos(kz+\omega t)\,k^3,\ -\sin(kz+\omega t)\,k^3,\ 0]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} = \left[\frac{\cos(kz+\omega t)\,(k^2-\varepsilon\omega^2\mu)}{\mu},\ \frac{\sin(kz+\omega t)\,(k^2-\varepsilon\omega^2\mu)}{\mu},\ 0,\ 0\right]$$

$$\textit{Lorentz Force 3 vector due to Ampere current} \quad \textit{FL = -(rho\_ampere E + J\_ampere x B)} = [0, 0, 0]$$

$$\textit{Amperian Dissipation Jampere dot E} = 0$$

$$\textit{Lorentz Force Spin factor LFSPIN} = 0$$

$$\textit{Topological Torsion current 4 vector} \quad \textit{T4 = -[ExA + B.phi,AdotB]} = [0, 0, -\omega(\%1), k(\%1)]$$

$$\%1 = \cos(kz+\omega t)^2 + \sin(kz+\omega t)^2$$

$$\textit{Lorentz Force 3 vector due to Torsion current} \quad \textit{TF = -(rho\_torsion E + J\_torsion x B)} = [0, 0, 0]$$

$$\textit{Torsion Dissipation Jtorsion dot E} = 0$$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \left[0, 0, \mathrm{I}(\%1)(\%2)(\mathrm{I}g-\gamma)\sqrt{\frac{\mu}{\varepsilon}}\,\omega, -\mathrm{I}(\%1)(\%2)(\mathrm{I}g-\gamma)\sqrt{\frac{\mu}{\varepsilon}}\,k\right]$

$$\%1 = \cos(kz+\omega t) + \mathrm{I}\sin(kz+\omega t)$$

$$\%2 = -\cos(kz+\omega t) + \mathrm{I}\sin(kz+\omega t)$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= [0, 0, 0]$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= [0, 0, 0]$

Dissipation $= -\mathrm{I}\mu\left(\cos(kz+\omega t) + \mathrm{I}\sin(kz+\omega t)\right)\left(-\cos(kz+\omega t) + \mathrm{I}\sin(kz+\omega t)\right)(\mathrm{I}g - \gamma)\sqrt{\frac{\mu}{\varepsilon}}\,k$

******************************* *END PROCEDURE* ******************************* **(17)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 2b-- Real Circular Polarization A^G<>0, A^F = 0  OUTBOUND `;
> theta:=(-k*z+omega*t);
> Ax:=cos(theta);Ay:=-sin(theta);Az:=0;phi:=0;
```

****

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,alpha,0):
```

NAME := Example 2b-- Real Circular Polarization A^G<>0, A^F = 0 OUTBOUND

$$\theta := -kz + \omega t$$

$$Ax := \cos(kz - \omega t)$$

$$Ay := \sin(kz - \omega t)$$

$$Az := 0$$

$$\phi := 0$$

Example 2b-- Real Circular Polarization A^G! O 0, A^F = 0 OUTBOUND

******************************** *Differential Form Format* ********************************

Action 1-form $= d(x)\cos(kz-\omega t) + \sin(kz-\omega t)\, d(y)$

Intensity 2-form F=dA $= -\sin(kz-\omega t)\,\omega\,(d(x))\ \&\hat{}\ (d(t)) + \sin(kz-\omega t)\,k\,(d(x))\ \&\hat{}\ (d(z)) + \cos(kz-\omega t)\,\omega\,(d(y))\ \&\hat{}\ (d(t)) - \cos(kz-\omega t)\,k\,(d(y))\ \&\hat{}\ (d(z))$

Topological Torsion 3-form $A\hat{}F = \left(\sin(kz-\omega t)^2\omega + \cos(kz-\omega t)^2\omega\right)\ \&\hat{}(d(x), d(y), d(t)) + \left(-\sin(kz-\omega t)^2 k - \cos(kz-\omega t)^2 k\right)\ \&\hat{}(d(x), d(y), d(z))$

Topological Parity 4-form $F\hat{}F = 0$

********************************* *Using EM format* *********************************

$$E\ field = [-\sin(k z - \omega t)\ \omega, \cos(k z - \omega t)\ \omega, 0]$$

$$B\ field = [-\cos(k z - \omega t)\ k, -\sin(k z - \omega t)\ k, 0]$$

Topological TORSION 4 vector   T4 = -[ExA + Bphi,AdotB] $= [0, 0, \omega(\%1), k(\%1)]$

$$\%1 = \cos(k z - \omega t)^2 + \sin(k z - \omega t)^2$$

$$Helicity\ AdotB = -k\left(\cos(k z - \omega t)^2 + \sin(k z - \omega t)^2\right)$$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension   *PTD* $= 3$

******************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************
****

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= 0$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

**********   *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$Chirality\ factor\ CH = \alpha\sqrt{\frac{\mu}{\varepsilon}}$$

$$D\ field = \left[-\varepsilon\sin(k z - \omega t)\ \omega - \alpha\sqrt{\frac{\mu}{\varepsilon}}\cos(k z - \omega t)\ k, \varepsilon\cos(k z - \omega t)\ \omega - \alpha\sqrt{\frac{\mu}{\varepsilon}}\sin(k z - \omega t)\ k, 0\right]$$

$$H\ field = \left[-\frac{\cos(k z - \omega t)\ k - \alpha\sqrt{\frac{\mu}{\varepsilon}}\sin(k z - \omega t)\ \omega\mu}{\mu}, -\frac{\sin(k z - \omega t)\ k + \alpha\sqrt{\frac{\mu}{\varepsilon}}\cos(k z - \omega t)\ \omega\mu}{\mu}, 0\right]$$

$$Poynting\ vector\ ExH = \left[\begin{matrix} 0 & 0 & \frac{\omega k}{\mu}\end{matrix}\right]$$

Amperian Current 4Vector   *curlH-dD/dt=J4* $= \left[\frac{\cos(k z - \omega t)\ (k^2 - \varepsilon\,\omega^2\mu)}{\mu},\right.$

$$\frac{\sin(k z - \omega t)\,(k^2 - \varepsilon\,\omega^2\,\mu)}{\mu}, 0, 0\Bigg]$$

Amerian charge density *divD = rho* $= 0$

divergence Lorentz Current 4Vector, *4div(J4)* $= 0$

$$\textit{Topological SPIN 4 vector S4} = \left[0, 0, -\alpha \sqrt{\frac{\mu}{\varepsilon}}\,\omega\,(\%1), -\alpha \sqrt{\frac{\mu}{\varepsilon}}\,k\,(\%1)\right]$$

$$\%1 = \cos(k z - \omega t)^2 + \sin(k z - \omega t)^2$$

$$\textit{Topological SPIN 3-form} = -\alpha \sqrt{\frac{\mu}{\varepsilon}}\,\omega\,(\%1)\,\&\hat{}(d(x), d(y), d(t)) + \alpha \sqrt{\frac{\mu}{\varepsilon}}\,k\,(\%1)\,\&\hat{}(d(x), d(y), d(z))$$

$$\%1 = \cos(k z - \omega t)^2 + \sin(k z - \omega t)^2$$

$$\textit{Spin density rho\_spin} = -\alpha \sqrt{\frac{\mu}{\varepsilon}}\,k\,\left(\cos(k z - \omega t)^2 + \sin(k z - \omega t)^2\right)$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{(k^2 - \varepsilon\,\omega^2\,\mu)\,\left(\cos(k z - \omega t)^2 + \sin(k z - \omega t)^2\right)}{\mu}$$

$$B.H = \frac{k^2\,\left(\cos(k z - \omega t)^2 + \sin(k z - \omega t)^2\right)}{\mu}$$

$$D.E = \varepsilon\,\omega^2\,\left(\cos(k z - \omega t)^2 + \sin(k z - \omega t)^2\right)$$

$$A.J = \frac{(k^2 - \varepsilon\,\omega^2\,\mu)\,\left(\cos(k z - \omega t)^2 + \sin(k z - \omega t)^2\right)}{\mu}$$

-rho.phi $= 0$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* $= 0$

$$\textit{London Coefficient} \quad LC = \frac{k^2 - \varepsilon\,\omega^2\,\mu}{\mu}$$

$$\textit{PROCA coefficient curlcurlB} = \left[-k^3 \cos(k z - \omega t), -k^3 \sin(k z - \omega t), 0\right]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} = \left[\frac{\cos(k z - \omega t)\,(k^2 - \varepsilon\,\omega^2\,\mu)}{\mu},\right.$$

$$\left.\frac{\sin(k z - \omega t)\,(k^2 - \varepsilon\,\omega^2\,\mu)}{\mu}, 0, 0\right]$$

Lorentz Force 3 vector due to Ampere current *FL = -(rho_ampere E + J_ampere x B)* $= [0, 0, 0]$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] $= [0, 0, \omega(\%1), k(\%1)]$

$$\%1 = \cos(k z - \omega t)^2 + \sin(k z - \omega t)^2$$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] $= \left[0, 0, -\alpha\sqrt{\frac{\mu}{\varepsilon}}\,\omega(\%1), -\alpha\sqrt{\frac{\mu}{\varepsilon}}\,k(\%1)\right]$

$$\%1 = \cos(k z - \omega t)^2 + \sin(k z - \omega t)^2$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $= [0, 0, 0]$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= [0, 0, 0]$

Dissipation $= -\mu\,\alpha\sqrt{\frac{\mu}{\varepsilon}}\,k\left(\cos(k z - \omega t)^2 + \sin(k z - \omega t)^2\right)$

****************************** *END PROCEDURE* ****************************** **(18)**

Enter the name of the procedure and then the components of the 4 potential

```
> NAME:=`Example 3a-- Complex Linear Polarization A^G<>0, A^F = 0  INBOUND `;
> theta:=(k*z+omega*t);
> Ax:=cos(theta);Ay:=I*cos(theta);Az:=0;phi:=0;
```

****

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,gamma,0):
```

NAME := Example 3a-- Complex Linear Polarization A^G<>0, A^F = 0 INBOUND

$$\theta := k z + \omega t$$

$$Ax := \cos(k z + \omega t)$$

$$Ay := I\cos(k z + \omega t)$$

$$Az := 0$$

$$\phi := 0$$

Example 3a-- Complex Linear Polarization A^G! O 0, A^F = 0 INBOUND

********************************* *Differential Form Format* *****************************
****

Action 1-form $= \cos(k z + \omega t)\ d(x) + I\cos(k z + \omega t)\ d(y)$

Intensity 2-form F=dA $= \sin(k z + \omega t)\ \omega\ (d(x))\ \&\hat{}\ (d(t)) + \sin(k z + \omega t)\ k\ (d(x))\ \&\hat{}\ (d(z))$
$+ I\sin(k z + \omega t)\ \omega\ (d(y))\ \&\hat{}\ (d(t)) + I\sin(k z + \omega t)\ k\ (d(y))\ \&\hat{}\ (d(z))$

Topological Torsion 3-form $A \wedge F = 0$

Topological Parity 4-form $F \wedge F = 0$

********************************* *Using EM format* *********************************

$$E\,field = [\sin(k z + \omega t)\,\omega, I \sin(k z + \omega t)\,\omega, 0]$$

$$B\,field = [I \sin(k z + \omega t)\,k, -\sin(k z + \omega t)\,k, 0]$$

Topological TORSION 4 vector $T4 = -[ExA + Bphi, AdotB] = [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension $PTD = 2$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ************** ************

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= 0$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$Chirality\ factor\ CH = \gamma \sqrt{\frac{\mu}{\varepsilon}}$$

$$D\,field = \left[-I \sin(k z + \omega t)\left(I \varepsilon \omega - \sqrt{\frac{\mu}{\varepsilon}}\,k \gamma\right), \sin(k z + \omega t)\left(I \varepsilon \omega - \sqrt{\frac{\mu}{\varepsilon}}\,k \gamma\right), 0\right]$$

$$H\,field = \left[\frac{\sin(k z + \omega t)\left(I k - \sqrt{\frac{\mu}{\varepsilon}}\,\omega \mu \gamma\right)}{\mu}, \frac{I \sin(k z + \omega t)\left(I k - \sqrt{\frac{\mu}{\varepsilon}}\,\omega \mu \gamma\right)}{\mu}, 0\right]$$

Poynting vector ExH $= EXH$

$$Amperian\ Current\ 4Vector\quad curlH\text{-}dD/dt{=}J4 = \left[\frac{\cos(k z + \omega t)\,(k^2 - \varepsilon \omega^2 \mu)}{\mu}, \frac{I \cos(k z + \omega t)\,(k^2 - \varepsilon \omega^2 \mu)}{\mu}, 0, 0\right]$$

Amerian charge density $divD = rho = 0$

divergence Lorentz Current 4Vector, $4div(J4) = 0$

Topological SPIN 4 vector $S4 = [0, 0, 0, 0]$

Topological SPIN 3-form $= 0$

$$Spin\ density\ rho\_spin = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = 0$$

$$B.H = 0$$

$$D.E = 0$$

$$A.J = 0$$

$$\text{-}rho.phi = 0$$

$$Poincare\ I \quad (B.H - D.E)\text{-}(A.J - rho.phi) = 0$$

$$London\ Coefficient \quad LC = \frac{k^2 - \varepsilon\,\omega^2\,\mu}{\mu}$$

$$PROCA\ coefficient\ curlcurlB = \left[\mathrm{I}\sin(k\,z + \omega\,t)\,k^3,\ -\sin(k\,z + \omega\,t)\,k^3,\ 0\right]$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt\text{=}J4 \ = \left[\frac{\cos(k\,z + \omega\,t)\,\left(k^2 - \varepsilon\,\omega^2\,\mu\right)}{\mu},\ \frac{\mathrm{I}\cos(k\,z + \omega\,t)\,\left(k^2 - \varepsilon\,\omega^2\,\mu\right)}{\mu},\ 0,\ 0\right]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Ampere\ current \quad FL = \text{-}(rho\_ampere\ E + J\_ampere\ x\ B) \ = [0, 0, 0]$$

$$Amperian\ Dissipation\ \ Jampere\ dot\ E \ = 0$$

$$Lorentz\ Force\ Spin\ factor\ \ LFSPIN = 0$$

$$Topological\ Torsion\ current\ 4\ vector \quad T4 = \text{-}[ExA + B.phi,AdotB] \ = [0, 0, 0, 0]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Torsion\ current \quad TF = \text{-}(rho\_torsion\ E + J\_torsion\ x\ B) \ = [0, 0, 0]$$

$$Torsion\ Dissipation\ \ Jtorsion\ dot\ E \ = 0$$

$$Topological\ Spin\ current\ 4\ vector \quad TS4 = \text{-}[A\ x\ H + D.phi,AdotD] \ = [0, 0, 0, 0]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Spin\ current \quad SF = \text{--}(rho\_spin\ E + J\_spin\ x\ B) \ = [0, 0, 0]$$

$$Spin\ Dissipation\ \ J\_spin\ dot\ E \ = 0$$

$$Disspative\ Force\ 3\ vector = [0, 0, 0]$$

$$Dissipation \ = 0$$

***************************** END PROCEDURE *************************** **(19)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 3b-- Complex Linear Polarization A^G = 0, A^F = 0   OUTBOUND`;
> theta:=(-k*z+omega*t);
> Ax:=cos(theta);Ay:=I*cos(theta);Az:=0;phi:=0;
```

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,gamma,0):
```

****

NAME := Example 3b-- Complex Linear Polarization A^G = 0, A^F = 0   OUTBOUND

$$\theta := -k\,z + \omega\,t$$

$$Ax := \cos(k\,z - \omega\,t)$$

$$Ay := I\cos(k\,z - \omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

Example 3b-- Complex Linear Polarization A^G = 0, A^F = 0   OUTBOUND

******************************** *Differential Form Format* ********************************

$$\text{Action 1-form} = d(x)\cos(k\,z-\omega\,t) + I\cos(k\,z-\omega\,t)\,d(y)$$

$$\text{Intensity 2-form F=dA} = -\sin(k\,z-\omega\,t)\,\omega\,(d(x))\ \&\hat{}\ (d(t)) + \sin(k\,z-\omega\,t)\,k\,(d(x))\ \&\hat{}\ (d(z)) - I\sin(k\,z-\omega\,t)\,\omega\,(d(y))\ \&\hat{}\ (d(t)) + I\sin(k\,z-\omega\,t)\,k\,(d(y))\ \&\hat{}\ (d(z))$$

$$\text{Topological Torsion 3-form}\quad A\hat{}F = 0$$

$$\text{Topological Parity 4-form}\quad F\hat{}F = 0$$

******************************** *Using EM format* ********************************

$$\text{E field} = [-\sin(k\,z-\omega\,t)\,\omega,\ -I\sin(k\,z-\omega\,t)\,\omega,\ 0]$$

$$\text{B field} = [I\sin(k\,z-\omega\,t)\,k,\ -\sin(k\,z-\omega\,t)\,k,\ 0]$$

$$\text{Topological TORSION 4 vector}\quad T4 = -[ExA + Bphi, AdotB] = [0, 0, 0, 0]$$

$$\text{Helicity AdotB} = 0$$

$$\text{Poincare II} = 2(E.B) = 0$$

$$\text{coefficient of Topological Parity 4-form} = 0$$

$$\text{Pfaff Topological Dimension}\quad PTD = 2$$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

$$\text{Xm or linear (Mean) curvature} = 0$$

$$\text{Yg or quadratic (GAUSS) curvature} = 0$$

$$\text{Za or Cubic (Interaction internal energy) curvature} = 0$$

$$\text{Tk or quartic (4D expansion) curvature} = 0$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\text{Chirality factor } CH = \gamma\sqrt{\frac{\mu}{\varepsilon}}$$

$$D\,field = \left[I\left(I\varepsilon\omega + \sqrt{\frac{\mu}{\varepsilon}}\,k\gamma\right)\sin(kz-\omega t),\ -\left(I\varepsilon\omega + \sqrt{\frac{\mu}{\varepsilon}}\,k\gamma\right)\sin(kz-\omega t),\ 0\right]$$

$$H\,field = \left[\frac{\sin(kz-\omega t)\left(Ik + \sqrt{\frac{\mu}{\varepsilon}}\,\omega\mu\gamma\right)}{\mu},\ \frac{I\sin(kz-\omega t)\left(Ik + \sqrt{\frac{\mu}{\varepsilon}}\,\omega\mu\gamma\right)}{\mu},\ 0\right]$$

Poynting vector ExH = EXH

Amperian Current 4Vector curlH-dD/dt=J4 $= \left[\frac{\cos(kz-\omega t)\,(k^2-\varepsilon\omega^2\mu)}{\mu},\ \frac{I\cos(kz-\omega t)\,(k^2-\varepsilon\omega^2\mu)}{\mu},\ 0,\ 0\right]$

Amerian charge density divD = rho $= 0$

divergence Lorentz Current 4Vector, 4div(J4) $= 0$

Topological SPIN 4 vector S4 $= [0, 0, 0, 0]$

Topological SPIN 3-form $= 0$

Spin density rho_spin $= 0$

LaGrange field energy density (B.H-D.E) $= 0$

B.H $= 0$

D.E $= 0$

A.J $= 0$

-rho.phi $= 0$

Poincare I (B.H - D.E)-(A.J - rho.phi) $= 0$

London Coefficient $LC = \frac{k^2-\varepsilon\omega^2\mu}{\mu}$

PROCA coefficient curlcurlB $= [I\sin(kz-\omega t)\,k^3,\ -k^3\sin(kz-\omega t),\ 0]$

Amperian Current 4Vector curlH-dD/dt=J4 $= \left[\frac{\cos(kz-\omega t)\,(k^2-\varepsilon\omega^2\mu)}{\mu},\ \frac{I\cos(kz-\omega t)\,(k^2-\varepsilon\omega^2\mu)}{\mu},\ 0,\ 0\right]$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= [0, 0, 0]$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B)   = [0, 0, 0]

Torsion Dissipation Jtorsion dot E   = 0

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] = [0, 0, 0, 0]

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B)   = [0, 0, 0]

Spin Dissipation J_spin dot E   = 0

Disspative Force 3 vector = [0, 0, 0]

Dissipation   = 0

****************************** *END PROCEDURE* ****************************   **(20)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 4a-- Complex Circular Polarization A^G<>0, A^F <> 0   INBOUND`;
> theta:=(k*z+omega*t);
> Ax:=cos(theta);Ay:=I*sin(theta);Az:=0;phi:=0;
```

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,gamma,0):
```

****

NAME := *Example 4a-- Complex Circular Polarization A^G <>0, A^F <> 0   INBOUND*

$$\theta := k\,z + \omega\,t$$

$$Ax := \cos(k\,z + \omega\,t)$$

$$Ay := \mathrm{I}\sin(k\,z + \omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

Example 4a-- Complex Circular Polarization A^G! O 0, A^F ! O 0   INBOUND

********************************* *Differential Form Format* ****************************
****

Action 1-form $= \cos(k\,z + \omega\,t)\ d(x) + \mathrm{I}\sin(k\,z + \omega\,t)\ d(y)$

Intensity 2-form F=dA $= \sin(k\,z + \omega\,t)\ \omega\ (d(x))\ \&\text{^}\ (d(t)) + \sin(k\,z + \omega\,t)\ k\ (d(x))\ \&\text{^}\ (d(z))$
$- \mathrm{I}\cos(k\,z + \omega\,t)\ \omega\ (d(y))\ \&\text{^}\ (d(t)) - \mathrm{I}\cos(k\,z + \omega\,t)\ k\ (d(y))\ \&\text{^}\ (d(z))$

Topological Torsion 3-form $A\text{^}F = \left(-\mathrm{I}\sin(k\,z + \omega\,t)^2\,\omega - \mathrm{I}\cos(k\,z + \omega\,t)^2\,\omega\right)\ \&\text{^}(d(x), d(y), d(t))$
$+ \left(-\mathrm{I}\sin(k\,z + \omega\,t)^2\,k - \mathrm{I}\cos(k\,z + \omega\,t)^2\,k\right)\ \&\text{^}(d(x), d(y), d(z))$

Topological Parity 4-form $F\text{^}F = 0$

********************************* *Using EM format* *********************************

E field $= [\sin(k\,z + \omega\,t)\ \omega,\ -\mathrm{I}\cos(k\,z + \omega\,t)\ \omega,\ 0]$

$$B\,field = [-I\cos(kz+\omega t)\,k,\ -\sin(kz+\omega t)\,k,\ 0]$$

$$Topological\ TORSION\ 4\ vector\quad T4 = -[ExA + Bphi, AdotB] = [0,\ 0,\ I(\%1)(\%2)\,\omega,\ -I(\%1)(\%2)\,k]$$

$$\%1 = \cos(kz+\omega t) + I\sin(kz+\omega t)$$

$$\%2 = -\cos(kz+\omega t) + I\sin(kz+\omega t)$$

$$Helicity\ AdotB = I\,(\cos(kz+\omega t) + I\sin(kz+\omega t))\,(-\cos(kz+\omega t) + I\sin(kz+\omega t))\,k$$

$$Poincare\ II = 2(E.B) = 0$$

$$coefficient\ of\ Topological\ Parity\ 4\text{-}form = 0$$

$$Pfaff\ Topological\ Dimension\quad PTD = 3$$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

$$Xm\ or\ linear\ (Mean)\ curvature = 0$$

$$Yg\ or\ quadratic\ (GAUSS)\ curvature = 0$$

$$Za\ or\ Cubic\ (Interaction\ internal\ energy)\ curvature = 0$$

$$Tk\ or\ quartic\ (4D\ expansion)\ curvature = 0$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$Chirality\ factor\ CH = \gamma\sqrt{\frac{\mu}{\varepsilon}}$$

$$D\,field = \left[-I\left(I\varepsilon\sin(kz+\omega t)\,\omega + \sqrt{\frac{\mu}{\varepsilon}}\cos(kz+\omega t)\,k\gamma\right),\ I\left(-\varepsilon\cos(kz+\omega t)\,\omega + I\sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)\,k\gamma\right),\ 0\right]$$

$$H\,field = \left[\frac{I\left(-\cos(kz+\omega t)\,k + I\sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)\,\omega\mu\gamma\right)}{\mu},\ \frac{I\left(I\sin(kz+\omega t)\,k + \sqrt{\frac{\mu}{\varepsilon}}\cos(kz+\omega t)\,\omega\mu\gamma\right)}{\mu},\ 0\right]$$

$$Poynting\ vector\ ExH = \left[\begin{array}{ccc} 0 & 0 & \frac{\omega k\,(-1 + 2\cos(kz+\omega t)^2)}{\mu}\end{array}\right]$$

$$Amperian\ Current\ 4Vector\quad curlH\text{-}dD/dt = J4 = \left[\frac{\cos(kz+\omega t)\,(k^2 - \varepsilon\omega^2\mu)}{\mu},\right.$$

$$\frac{\mathrm{I}\sin(kz+\omega t)\,(k^2-\varepsilon\,\omega^2\,\mu)}{\mu},0,0\Bigg]$$

Amerian charge density *divD = rho* $=0$

divergence Lorentz Current 4Vector, *4div(J4)* $=0$

Topological SPIN 4 vector S4 $= \Bigg[0, 0,$

$$\frac{\mathrm{I}\left(2\,\mathrm{I}\sin(kz+\omega t)\cos(kz+\omega t)\,k+\cos(kz+\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\mu\,\gamma+\sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)^2\,\omega\,\mu\,\gamma\right)}{\mu},$$

$$-\mathrm{I}\left(2\,\mathrm{I}\sin(kz+\omega t)\,\varepsilon\cos(kz+\omega t)\,\omega+\cos(kz+\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,k\,\gamma+\sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)^2\,k\,\gamma\right)\Bigg]$$

Topological SPIN 3-form $=\frac{1}{\mu}\Bigg(\mathrm{I}\left(2\,\mathrm{I}\sin(kz+\omega t)\cos(kz+\omega t)\,k+\cos(kz+\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\mu\,\gamma\right.$

$$\left.+\sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)^2\,\omega\,\mu\,\gamma\right)\&\hat{}(d(x),d(y),d(t))\Bigg)+\mathrm{I}\Big(2\,\mathrm{I}\sin(kz+\omega t)\,\varepsilon\cos(kz+\omega t)\,\omega$$

$$+\cos(kz+\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,k\,\gamma+\sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)^2\,k\,\gamma\Big)\&\hat{}(d(x),d(y),d(z))$$

Spin density rho_spin $=-\mathrm{I}\Big(2\,\mathrm{I}\sin(kz+\omega t)\,\varepsilon\cos(kz+\omega t)\,\omega+\cos(kz+\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,k\,\gamma$

$$+\sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)^2\,k\,\gamma\Big)$$

LaGrange field energy density (B.H-D.E) $=$

$$-\frac{(k^2-\varepsilon\,\omega^2\,\mu)\,(\cos(kz+\omega t)-\sin(kz+\omega t))\,(\cos(kz+\omega t)+\sin(kz+\omega t))}{\mu}$$

$$B.H=-\frac{k^2\,(\cos(kz+\omega t)-\sin(kz+\omega t))\,(\cos(kz+\omega t)+\sin(kz+\omega t))}{\mu}$$

$$D.E=-\varepsilon\,\omega^2\,(\cos(kz+\omega t)-\sin(kz+\omega t))\,(\cos(kz+\omega t)+\sin(kz+\omega t))$$

$$A.J=\frac{(k^2-\varepsilon\,\omega^2\,\mu)\,(\cos(kz+\omega t)-\sin(kz+\omega t))\,(\cos(kz+\omega t)+\sin(kz+\omega t))}{\mu}$$

-rho.phi $=0$

Poincare I   *(B.H - D.E)-(A.J - rho.phi)* =

$$-\frac{2\left(k^2-\varepsilon\,\omega^2\mu\right)\left(\cos(k z+\omega t)-\sin(k z+\omega t)\right)\left(\cos(k z+\omega t)+\sin(k z+\omega t)\right)}{\mu}$$

London Coefficient $LC = \dfrac{k^2-\varepsilon\,\omega^2\mu}{\mu}$

PROCA coefficient curlcurlB $= \left[-\mathrm{I}\cos(k z+\omega t)\,k^3,\ -\sin(k z+\omega t)\,k^3,\ 0\right]$

Amperian Current 4Vector   *curlH-dD/dt=J4* $= \left[\dfrac{\cos(k z+\omega t)\left(k^2-\varepsilon\,\omega^2\mu\right)}{\mu},\right.$

$$\left.\frac{\mathrm{I}\sin(k z+\omega t)\left(k^2-\varepsilon\,\omega^2\mu\right)}{\mu},\ 0,\ 0\right]$$

Lorentz Force 3 vector due to Ampere current   *FL = -(rho_ampere E + J_ampere x B)* $= \left[0,\ 0,\right.$

$$\left.\frac{2\cos(k z+\omega t)\left(k^2-\varepsilon\,\omega^2\mu\right)\sin(k z+\omega t)\,k}{\mu}\right]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector   *T4 = -[ExA + B.phi,AdotB]* $= \left[0,\ 0,\ \mathrm{I}\,(\%1)\,(\%2)\,\omega,\ -\mathrm{I}\,(\%1)\,(\%2)\,k\right]$

$\%1 = \cos(k z+\omega t) + \mathrm{I}\sin(k z+\omega t)$

$\%2 = -\cos(k z+\omega t) + \mathrm{I}\sin(k z+\omega t)$

Lorentz Force 3 vector due to Torsion current   *TF = -(rho_torsion E + J_torsion x B)* $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   *TS4 = -[A x H + D.phi,AdotD]* $= \left[0,\ 0,\right.$

$$\frac{\mathrm{I}\left(2\,\mathrm{I}\sin(k z+\omega t)\cos(k z+\omega t)\,k+\cos(k z+\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\mu\,\gamma+\sqrt{\frac{\mu}{\varepsilon}}\,\sin(k z+\omega t)^2\,\omega\,\mu\,\gamma\right)}{\mu},$$

$$\left.-\mathrm{I}\left(2\,\mathrm{I}\sin(k z+\omega t)\,\varepsilon\cos(k z+\omega t)\,\omega+\cos(k z+\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,k\,\gamma+\sqrt{\frac{\mu}{\varepsilon}}\,\sin(k z+\omega t)^2\,k\,\gamma\right)\right]$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B)

$$= \left[\frac{2\cos(kz+\omega t)\,\sin(kz+\omega t)^2\,(k^2-\varepsilon\omega^2\mu)}{\mu}, -\frac{2\,\mathrm{I}\cos(kz+\omega t)^2\sin(kz+\omega t)\,(k^2-\varepsilon\omega^2\mu)}{\mu}, 0 \right]$$

Spin Dissipation J_spin dot E $=0$

Disspative Force 3 vector $= \left[2\cos(kz+\omega t)\,\sin(kz+\omega t)^2\,(k^2-\varepsilon\omega^2\mu), -2\,\mathrm{I}\cos(kz+\omega t)^2\sin(kz+\omega t)\,(k^2-\varepsilon\omega^2\mu), \frac{2\cos(kz+\omega t)\,(k^2-\varepsilon\omega^2\mu)\sin(kz+\omega t)\,k}{\mu} \right]$

Dissipation $= -\mathrm{I}\,\mu \left(2\,\mathrm{I}\sin(kz+\omega t)\,\varepsilon\cos(kz+\omega t)\,\omega + \cos(kz+\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,k\gamma + \sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)^2\,k\gamma \right)$

****************************** *END PROCEDURE* ***************************** **(21)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 4b-- Complex Circular Polarization A^G<>0, A^F <> 0   OUTBOUND`;
> theta:=(-k*z+omega*t);
> Ax:=cos(theta);Ay:=-I*sin(theta);Az:=0;phi:=0;
```

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,gamma,0):
```

****

NAME := Example 4b-- Complex Circular Polarization A^G <>0, A^F <> 0   OUTBOUND

$$\theta := -kz+\omega t$$

$$Ax := \cos(kz-\omega t)$$

$$Ay := \mathrm{I}\sin(kz-\omega t)$$

$$Az := 0$$

$$\phi := 0$$

Example 4b-- Complex Circular Polarization A^G! O 0, A^F ! O 0   OUTBOUND

********************************* *Differential Form Format* *********************************

Action 1-form $= d(x)\cos(kz-\omega t) + \mathrm{I}\sin(kz-\omega t)\,d(y)$

Intensity 2-form F=dA $= -\sin(kz-\omega t)\,\omega\,(d(x))\ \&\wedge\ (d(t)) + \sin(kz-\omega t)\,k\,(d(x))\ \&\wedge\ (d(z))$

$+ \mathrm{I}\cos(k z - \omega t)\ \omega\ (d(y))\ \&\hat{}\ (d(t)) - \mathrm{I}\cos(k z - \omega t)\ k\ (d(y))\ \&\hat{}\ (d(z))$

Topological Torsion 3-form $A\hat{}F = \left(\mathrm{I}\sin(k z - \omega t)^2\ \omega + \mathrm{I}\cos(k z - \omega t)^2\ \omega\right)\ \&\hat{}(d(x), d(y), d(t)) + \left(-\mathrm{I}\sin(k z - \omega t)^2\ k - \mathrm{I}\cos(k z - \omega t)^2\ k\right)\ \&\hat{}(d(x), d(y), d(z))$

Topological Parity 4-form $F\hat{}F = 0$

******************************** *Using EM format* ********************************

$$E\ field = [-\sin(k z - \omega t)\ \omega, \mathrm{I}\cos(k z - \omega t)\ \omega, 0]$$

$$B\ field = [-\mathrm{I}\cos(k z - \omega t)\ k, -\sin(k z - \omega t)\ k, 0]$$

Topological TORSION 4 vector $T4 = -[ExA + Bphi, AdotB] = [0, 0, -\mathrm{I}(\%1)(\%2)\ \omega, -\mathrm{I}(\%1)(\%2)\ k]$

$$\%1 = -\cos(k z - \omega t) + \mathrm{I}\sin(k z - \omega t)$$

$$\%2 = \cos(k z - \omega t) + \mathrm{I}\sin(k z - \omega t)$$

$$Helicity\ AdotB = \mathrm{I}\ (-\cos(k z - \omega t) + \mathrm{I}\sin(k z - \omega t))\ (\cos(k z - \omega t) + \mathrm{I}\sin(k z - \omega t))\ k$$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension $PTD = 3$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= 0$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********* *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$Chirality\ factor\ CH = \gamma\sqrt{\frac{\mu}{\varepsilon}}$$

$$D\ field = \left[\mathrm{I}\left(\mathrm{I}\,\varepsilon\sin(k z - \omega t)\ \omega - \gamma\sqrt{\frac{\mu}{\varepsilon}}\ \cos(k z - \omega t)\ k\right), \mathrm{I}\left(\varepsilon\cos(k z - \omega t)\ \omega + \mathrm{I}\sqrt{\frac{\mu}{\varepsilon}}\ \sin(k z - \omega t)\ k\,\gamma\right), 0\right]$$

$$H\ field = \left[-\frac{\mathrm{I}\left(\cos(k z - \omega t)\ k + \mathrm{I}\,\gamma\sqrt{\frac{\mu}{\varepsilon}}\ \sin(k z - \omega t)\ \omega\,\mu\right)}{\mu},\right.$$

$$\frac{I\left(I\sin(kz-\omega t)\,k-\gamma\sqrt{\frac{\mu}{\varepsilon}}\cos(kz-\omega t)\,\omega\,\mu\right)}{\mu}, 0\Bigg]$$

$$\textit{Poynting vector ExH} = \left[\begin{array}{ccc} 0 & 0 & -\frac{\omega k\left(-1+2\cos(kz-\omega t)^2\right)}{\mu}\end{array}\right]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} \quad = \Bigg[\frac{\cos(kz-\omega t)\left(k^2-\varepsilon\,\omega^2\mu\right)}{\mu},$$

$$\frac{I\sin(kz-\omega t)\left(k^2-\varepsilon\,\omega^2\mu\right)}{\mu}, 0, 0\Bigg]$$

$$\textit{Amerian charge density} \quad \textit{divD = rho} = 0$$

$$\textit{divergence Lorentz Current 4Vector,} \quad \textit{4div(J4)} = 0$$

$$\textit{Topological SPIN 4 vector S4} = \Bigg[0, 0,$$

$$\frac{I\left(2I\sin(kz-\omega t)\cos(kz-\omega t)\,k-\cos(kz-\omega t)^2\gamma\sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\mu-\gamma\sqrt{\frac{\mu}{\varepsilon}}\sin(kz-\omega t)^2\omega\,\mu\right)}{\mu},$$

$$I\left(2I\cos(kz-\omega t)\sin(kz-\omega t)\,\varepsilon\,\omega-\cos(kz-\omega t)^2\gamma\sqrt{\frac{\mu}{\varepsilon}}\,k-\sqrt{\frac{\mu}{\varepsilon}}\sin(kz-\omega t)^2k\,\gamma\right)\Bigg]$$

$$\textit{Topological SPIN 3-form} = \frac{1}{\mu}\Bigg(I\Bigg(2I\sin(kz-\omega t)\cos(kz-\omega t)\,k-\cos(kz-\omega t)^2\gamma\sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\mu$$

$$-\gamma\sqrt{\frac{\mu}{\varepsilon}}\sin(kz-\omega t)^2\omega\,\mu\Bigg)\,\&\hat{}(d(x), d(y), d(t))\Bigg)-I\Bigg(2I\cos(kz-\omega t)\sin(kz-\omega t)\,\varepsilon\,\omega$$

$$-\cos(kz-\omega t)^2\gamma\sqrt{\frac{\mu}{\varepsilon}}\,k-\sqrt{\frac{\mu}{\varepsilon}}\sin(kz-\omega t)^2k\,\gamma\Bigg)\,\&\hat{}(d(x), d(y), d(z))$$

$$\textit{Spin density rho\_spin} = I\Bigg(2I\cos(kz-\omega t)\sin(kz-\omega t)\,\varepsilon\,\omega-\cos(kz-\omega t)^2\gamma\sqrt{\frac{\mu}{\varepsilon}}\,k$$

$$-\sqrt{\frac{\mu}{\varepsilon}}\sin(kz-\omega t)^2k\,\gamma\Bigg)$$

LaGrange field energy density (B.H-D.E) =

$$-\frac{(k^2-\varepsilon\,\omega^2\,\mu)\,(\cos(k\,z-\omega\,t)-\sin(k\,z-\omega\,t))\,(\cos(k\,z-\omega\,t)+\sin(k\,z-\omega\,t))}{\mu}$$

$$B.H = -\frac{k^2\,(\cos(k\,z-\omega\,t)-\sin(k\,z-\omega\,t))\,(\cos(k\,z-\omega\,t)+\sin(k\,z-\omega\,t))}{\mu}$$

$$D.E = -\varepsilon\,\omega^2\,(\cos(k\,z-\omega\,t)-\sin(k\,z-\omega\,t))\,(\cos(k\,z-\omega\,t)+\sin(k\,z-\omega\,t))$$

$$A.J = \frac{(k^2-\varepsilon\,\omega^2\,\mu)\,(\cos(k\,z-\omega\,t)-\sin(k\,z-\omega\,t))\,(\cos(k\,z-\omega\,t)+\sin(k\,z-\omega\,t))}{\mu}$$

$$-rho.phi = 0$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* =

$$-\frac{2\,(k^2-\varepsilon\,\omega^2\,\mu)\,(\cos(k\,z-\omega\,t)-\sin(k\,z-\omega\,t))\,(\cos(k\,z-\omega\,t)+\sin(k\,z-\omega\,t))}{\mu}$$

London Coefficient $LC = \frac{k^2-\varepsilon\,\omega^2\,\mu}{\mu}$

PROCA coefficient curlcurlB $= [-I\cos(k\,z-\omega\,t)\,k^3,\ -k^3\sin(k\,z-\omega\,t),\ 0]$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \left[\frac{\cos(k\,z-\omega\,t)\,(k^2-\varepsilon\,\omega^2\,\mu)}{\mu},\ \frac{I\sin(k\,z-\omega\,t)\,(k^2-\varepsilon\,\omega^2\,\mu)}{\mu},\ 0,\ 0\right]$

Lorentz Force 3 vector due to Ampere current *FL = -(rho_ampere E + J_ampere x B)* $= \left[0,\ 0,\ \frac{2\cos(k\,z-\omega\,t)\,(k^2-\varepsilon\,\omega^2\,\mu)\sin(k\,z-\omega\,t)\,k}{\mu}\right]$

Amperian Dissipation *Jampere dot E* $= 0$

Lorentz Force Spin factor *LFSPIN* $= 0$

Topological Torsion current 4 vector *T4 = -[ExA + B.phi,AdotB]* $= [0,\ 0,\ -I(\%1)(\%2)\,\omega,\ -I(\%1)(\%2)\,k]$

$$\%1 = -\cos(k\,z-\omega\,t)+I\sin(k\,z-\omega\,t)$$

$$\%2 = \cos(k\,z-\omega\,t)+I\sin(k\,z-\omega\,t)$$

Lorentz Force 3 vector due to Torsion current *TF = -(rho_torsion E + J_torsion x B)* $= [0, 0, 0]$

Torsion Dissipation *Jtorsion dot E* $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \left[0, 0, \frac{I\left(2 I \sin(kz-\omega t)\cos(kz-\omega t) k - \cos(kz-\omega t)^2 \gamma \sqrt{\frac{\mu}{\varepsilon}}\,\omega\mu - \gamma\sqrt{\frac{\mu}{\varepsilon}}\sin(kz-\omega t)^2\omega\mu\right)}{\mu},\right.$

$\left. I\left(2 I \cos(kz-\omega t)\sin(kz-\omega t)\,\varepsilon\omega - \cos(kz-\omega t)^2\gamma\sqrt{\frac{\mu}{\varepsilon}}\,k - \sqrt{\frac{\mu}{\varepsilon}}\sin(kz-\omega t)^2 k\gamma\right)\right]$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B)

$$= \left[\frac{2\sin(kz-\omega t)^2\cos(kz-\omega t)\left(k^2-\varepsilon\omega^2\mu\right)}{\mu}, -\frac{2 I\cos(kz-\omega t)^2\sin(kz-\omega t)\left(k^2-\varepsilon\omega^2\mu\right)}{\mu}, 0\right]$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \left[2\sin(kz-\omega t)^2\cos(kz-\omega t)\left(k^2-\varepsilon\omega^2\mu\right), -2 I\cos(kz-\omega t)^2\sin(kz-\omega t)\left(k^2-\varepsilon\omega^2\mu\right), \frac{2\cos(kz-\omega t)\left(k^2-\varepsilon\omega^2\mu\right)\sin(kz-\omega t)\,k}{\mu}\right]$

Dissipation $= I\mu\left(2 I\cos(kz-\omega t)\sin(kz-\omega t)\,\varepsilon\omega - \cos(kz-\omega t)^2\gamma\sqrt{\frac{\mu}{\varepsilon}}\,k - \sqrt{\frac{\mu}{\varepsilon}}\sin(kz-\omega t)^2 k\gamma\right)$

****************************** *END PROCEDURE* **************************** **(22)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 5a-- waveguide TM mode  (group kinematic in, wave in)`;
> theta:=(k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=-vg*f(x,y)*cos(theta);
```

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,gamma,0):
```

****

NAME := Example 5a-- waveguide TM mode (group kinematic in, wave in)

$$\theta := kz + \omega t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z + \omega t)$$

$$\phi := -vg \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z + \omega t)$$

Example 5a-- waveguide TM mode (group kinematic in, wave in)

******************************** *Differential Form Format* ******************************
****

Action 1-form $= \left(vg \cos(k z + \omega t)\, x(x,y)^2 + vg \cos(k z + \omega t)\, y(x,y)^2\right) d(t) + \left(\cos(k z + \omega t)\, x(x,y)^2 + \cos(k z + \omega t)\, y(x,y)^2\right) d(z)$

Intensity 2-form F=dA $= \left(2\, vg \cos(k z + \omega t)\, y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 2\, vg \cos(k z + \omega t)\, x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right)\right) (d(x)) \,\&\wedge\, (d(t)) + \left(2 \cos(k z + \omega t)\, y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 2 \cos(k z + \omega t)\, x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right)\right) (d(x)) \,\&\wedge\, (d(z)) + \left(2\, vg \cos(k z + \omega t)\, y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) + 2\, vg \cos(k z + \omega t)\, x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right)\right) (d(y)) \,\&\wedge\, (d(t)) + \left(2 \cos(k z + \omega t)\, y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) + 2 \cos(k z + \omega t)\, x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right)\right) (d(y)) \,\&\wedge\, (d(z)) + \left(\sin(k z + \omega t)\, \omega\, x(x,y)^2 + \sin(k z + \omega t)\, \omega\, y(x,y)^2 - vg \sin(k z + \omega t)\, k\, x(x,y)^2 - vg \sin(k z + \omega t)\, k\, y(x,y)^2\right) (d(z)) \,\&\wedge\, (d(t))$

Topological Torsion 3-form *A^F* $= 0$

Topological Parity 4-form *F^F* $= 0$

******************************** *Using EM format* ********************************

E field $= \left[2\, vg \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right)\right) \cos(k z + \omega t),\ 2\, vg \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)\right) \cos(k z + \omega t),\ -\sin(k z + \omega t) \left(x(x,y)^2 + y(x,y)^2\right) (-\omega + vg\, k)\right]$

B field $= \left[2 \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)\right) \cos(k z + \omega t),\ -2 \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right)\right) \cos(k z + \omega t),\ 0\right]$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension $PTD = 2$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

$$Xm \text{ or linear (Mean) curvature } = -\sin(k z + \omega t)\,(x(x,y)^2 + y(x,y)^2)\,(k + \omega\, vg)$$

$$Yg \text{ or quadratic (GAUSS) curvature } = 0$$

$$Za \text{ or Cubic (Interaction internal energy) curvature } = 0$$

$$Tk \text{ or quartic (4D expansion) curvature } = 0$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\text{Chirality factor } CH = \gamma \sqrt{\frac{\mu}{\varepsilon}}$$

$$D\,field = \left[2\cos(k z + \omega t)\left(\varepsilon\, vg\, x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + \varepsilon\, vg\, y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right) + \gamma\sqrt{\frac{\mu}{\varepsilon}}\, x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + \left(\frac{\partial}{\partial y} y(x,y)\right) y(x,y)\sqrt{\frac{\mu}{\varepsilon}}\,\gamma\right), -2\cos(k z + \omega t)\left(-\varepsilon\, vg\, x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) - \left(\frac{\partial}{\partial y} y(x,y)\right) y(x,y)\, vg\, \varepsilon + \gamma\sqrt{\frac{\mu}{\varepsilon}}\, x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + \gamma\sqrt{\frac{\mu}{\varepsilon}}\, y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right), -\varepsilon\sin(k z + \omega t)\,(x(x,y)^2 + y(x,y)^2)\,(-\omega + vg\, k)\right]$$

$$H\,field = \left[-\frac{1}{\mu}\left(2\cos(k z + \omega t)\left(-x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) - y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right) + \gamma\sqrt{\frac{\mu}{\varepsilon}}\, vg\,\mu\, x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + \gamma\sqrt{\frac{\mu}{\varepsilon}}\, vg\,\mu\, y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\right), -\frac{1}{\mu}\left(2\cos(k z + \omega t)\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right) + \gamma\sqrt{\frac{\mu}{\varepsilon}}\, vg\,\mu\, x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + \gamma\sqrt{\frac{\mu}{\varepsilon}}\, vg\,\mu\, y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right)\right), \gamma\sqrt{\frac{\mu}{\varepsilon}}\sin(k z + \omega t)\,(x(x,y)^2 + y(x,y)^2)\,(-\omega + vg\, k)\right]$$

$$\text{Poynting vector } ExH = \left[-\frac{1}{\mu}\left(2\cos(k z + \omega t)\sin(k z + \omega t)\,(x(x,y)^2 + y(x,y)^2)\,(-\omega + vg\, k)\,\left(x(x,\right.\right.\right.$$

$$y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)\Big)\Big), -\frac{1}{\mu}\Big(2\cos(kz+\omega t)\sin(kz+\omega t)\,(x(x,y)^2$$

$$+y(x,y)^2)\,(-\omega+vg\,k)\,\Big(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Big)\Big), -\frac{1}{\mu}\Big(4\cos(kz$$

$$+\omega t)^2\,vg\,\Big(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,$$

$$y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)$$

$$+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Big)\Big)\Big]$$

Amperian Current 4Vector *curlH-dD/dt=J4* $=\Big[$

$$-\frac{2\sin(kz+\omega t)\,(k-\omega\,\mu\,\varepsilon\,vg)\,\Big(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)}{\mu},$$

$$-\frac{2\sin(kz+\omega t)\,(k-\omega\,\mu\,\varepsilon\,vg)\,\Big(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Big)}{\mu},$$

$$-\frac{1}{\mu}\Big(\cos(kz+\omega t)\,\Big(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\,\omega^2\,\mu\,x(x,y)^2-\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2+\varepsilon\,\omega^2\,\mu\,y(x,y)^2-\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\Big)\Big),$$

$$\varepsilon\cos(kz+\omega t)\,\Big(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial x}y(x,y)\right)^2$$

$$+2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial^2}{\partial y^2}y(x,$$

$$y)\right)y(x,y)\,vg+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2vg+k\,\omega\,x(x,y)^2-k^2\,vg\,x(x,y)^2+y(x,y)^2\,\omega\,k-k^2\,vg\,y(x,y)^2\Big)\Big]$$

Amerian charge density *divD = rho* $=\varepsilon\cos(kz+\omega t)\,\Big(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,\right.$

$$y)\Big) + 2\, vg \left(\frac{\partial}{\partial x}\, y(x,y)\right)^2 + 2\, vg\, y(x,y) \left(\frac{\partial^2}{\partial x^2}\, y(x,y)\right) + 2\, vg \left(\frac{\partial}{\partial y}\, x(x,y)\right)^2 + 2\, vg\, x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2}\, x(x,y)\right) + 2 \left(\frac{\partial^2}{\partial y^2}\, y(x,y)\right) y(x,y)\, vg + 2 \left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 vg + k\, \omega\, x(x,y)^2 - k^2\, vg\, x(x,y)^2$$

$$+ y(x,y)^2\, \omega\, k - k^2\, vg\, y(x,y)^2\Big)$$

divergence Lorentz Current 4Vector, *4div(J4)* $= 0$

Topological SPIN 4 vector S4 $= \Big[$

$$-\frac{1}{\mu}\Big(2\, \left(x(x,y)^2 + y(x,y)^2\right) \cos(k\, z + \omega\, t)^2 \left(-1 + vg^2\, \varepsilon\, \mu\right) \Big(x(x,y) \left(\frac{\partial}{\partial x}\, x(x,y)\right) + y(x,$$

$$y) \left(\frac{\partial}{\partial x}\, y(x,y)\right)\Big)\Big),$$

$$-\frac{1}{\mu}\Big(2\, \left(x(x,y)^2 + y(x,y)^2\right) \cos(k\, z + \omega\, t)^2 \left(-1 + vg^2\, \varepsilon\, \mu\right) \Big(x(x,y) \left(\frac{\partial}{\partial y}\, x(x,y)\right) + y(x,$$

$$y) \left(\frac{\partial}{\partial y}\, y(x,y)\right)\Big)\Big), \varepsilon \sin(k\, z + \omega\, t) \left(x(x,y)^2 + y(x,y)^2\right)^2 (-\omega + vg\, k)\, vg \cos(k\, z + \omega\, t),$$

$$-\left(x(x,y)^2 + y(x,y)^2\right)^2 \cos(k\, z + \omega\, t)\, \varepsilon \sin(k\, z + \omega\, t)\, (-\omega + vg\, k)\Big]$$

Topological SPIN 3-form $= -\frac{1}{\mu}\Big(2\, \left(x(x,y)^2 + y(x,y)^2\right) \cos(k\, z + \omega\, t)^2 \left(-1 + vg^2\, \varepsilon\, \mu\right) \Big(x(x,$

$$y) \left(\frac{\partial}{\partial x}\, x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial x}\, y(x,y)\right)\Big) \,\&\text{^}(d(y), d(z), d(t))\Big) + \frac{1}{\mu}\Big(2\, \Big(x(x,y)^2 + y(x,$$

$$y)^2\Big) \cos(k\, z + \omega\, t)^2 \left(-1 + vg^2\, \varepsilon\, \mu\right) \Big(x(x,y) \left(\frac{\partial}{\partial y}\, x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial y}\, y(x,y)\right)\Big) \,\&\text{^}(d(x), d(z),$$

$$d(t))\Big) + \varepsilon \sin(k\, z + \omega\, t) \left(x(x,y)^2 + y(x,y)^2\right)^2 (-\omega + vg\, k)\, vg \cos(k\, z + \omega\, t) \,\&\text{^}(d(x), d(y), d(t))$$

$$+ \left(x(x,y)^2 + y(x,y)^2\right)^2 \cos(k\, z + \omega\, t)\, \varepsilon \sin(k\, z + \omega\, t)\, (-\omega + vg\, k) \,\&\text{^}(d(x), d(y), d(z))$$

Spin density rho_spin $= -\left(x(x,y)^2 + y(x,y)^2\right)^2 \cos(k\, z + \omega\, t)\, \varepsilon \sin(k\, z + \omega\, t)\, (-\omega + vg\, k)$

LaGrange field energy density (B.H-D.E) $= -\frac{1}{\mu}\Big(-4 \left(\frac{\partial}{\partial y}\, x(x,y)\right)^2 x(x,y)^2 \cos(k\, z + \omega\, t)^2 - 8 \Big(\frac{\partial}{\partial y}\, y(x,$

$$y)\Big) y(x,y) \left(\frac{\partial}{\partial y}\, x(x,y)\right) x(x,y) \cos(k\, z + \omega\, t)^2 - 4\, y(x,y)^2 \left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 \cos(k\, z + \omega\, t)^2$$

$$-4\left(\frac{\partial}{\partial x}x(x,y)\right)^2 x(x,y)^2\cos(kz+\omega t)^2-8\left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)x(x,$$

$$y)\cos(kz+\omega t)^2-4\left(\frac{\partial}{\partial x}y(x,y)\right)^2 y(x,y)^2\cos(kz+\omega t)^2+4\,\varepsilon\,\mu\cos(kz+\omega t)^2 vg^2 x(x,$$

$$y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+8\,\varepsilon\,\mu\cos(kz+\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)$$

$$+4\,\varepsilon\,\mu\cos(kz+\omega t)^2 vg^2 y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+4\,\varepsilon\,\mu\cos(kz+\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$+8\,\varepsilon\,\mu\cos(kz+\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+4\,\varepsilon\,\mu\cos(kz$$

$$+\omega t)^2 vg^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 y(x,y)^2+\varepsilon\,\mu\sin(kz+\omega t)^2 x(x,y)^4\omega^2-2\,\varepsilon\,\mu\sin(kz$$

$$+\omega t)^2\omega\,vg\,k\,x(x,y)^4+\varepsilon\,\mu\sin(kz+\omega t)^2 x(x,y)^4 vg^2 k^2+2\,\varepsilon\,\mu\sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2\omega^2$$

$$-4\,\varepsilon\,\mu\sin(kz+\omega t)^2\omega\,vg\,k\,x(x,y)^2 y(x,y)^2+2\,\varepsilon\,\mu\sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2$$

$$+\varepsilon\,\mu\sin(kz+\omega t)^2 y(x,y)^4\omega^2-2\,\varepsilon\,\mu\sin(kz+\omega t)^2\omega\,vg\,k\,y(x,y)^4+\varepsilon\,\mu\sin(kz+\omega t)^2 y(x,$$

$$y)^4 vg^2 k^2\Big)$$

$$B.H=\frac{1}{\mu}\Bigg(4\cos(kz+\omega t)^2\Bigg(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)$$

$$+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)$$

$$+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Bigg)\Bigg)$$

$$D.E=\varepsilon\Bigg(4\cos(kz+\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+8\cos(kz+\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial x}x(x,$$

$$y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+4\cos(kz+\omega t)^2 vg^2 y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+4\cos(kz$$

$$+\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+8\cos(kz+\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,$$

$$y)\right)+4\cos(kz+\omega t)^2 vg^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 y(x,y)^2+\sin(kz+\omega t)^2 x(x,y)^4\omega^2-2\sin(kz$$

$$+\omega t)^2\omega\,vg\,k\,x(x,y)^4+\sin(kz+\omega t)^2 x(x,y)^4 vg^2 k^2+2\sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2\omega^2$$

$$-4\sin(kz+\omega t)^2\omega\,vg\,k\,x(x,y)^2 y(x,y)^2+2\sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2+\sin(kz$$

$$+\omega t)^2 y(x,y)^4 \omega^2 - 2\sin(kz+\omega t)^2 \omega\, vg\, k\, y(x,y)^4 + \sin(kz+\omega t)^2 y(x,y)^4 vg^2 k^2)$$

$$A.J = -\frac{1}{\mu}\Bigg(\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\Bigg(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$$
$$+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 + 2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right) + 2\left(\frac{\partial}{\partial y}x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$
$$+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 + 2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right) + \varepsilon\,\omega^2\mu\,x(x,y)^2 - \varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2 + \varepsilon\,\omega^2\mu\,y(x,y)^2$$
$$-\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\Bigg)\Bigg)$$

$$-rho.phi = -vg\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\,\varepsilon\Bigg(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2 + 2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,$$
$$y)\right) + 2\,vg\left(\frac{\partial}{\partial x}y(x,y)\right)^2 + 2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right) + 2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2 + 2\,vg\,x(x,$$
$$y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right) + 2\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)y(x,y)\,vg + 2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 vg + k\,\omega\,x(x,y)^2 - k^2\,vg\,x(x,y)^2$$
$$+y(x,y)^2\,\omega\,k - k^2\,vg\,y(x,y)^2\Bigg)$$

Poincare I (B.H - D.E)-(A.J - rho.phi) $= -\frac{1}{\mu}\Bigg(2\cos(kz+\omega t)^2 y(x,y)^3 vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)$

$$+2\cos(kz+\omega t)^2 x(x,y)^3 vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial y^2}x(x,y)\right) + 2\cos(kz+\omega t)^2 x(x,y)^3 vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$$
$$+2\cos(kz+\omega t)^2 x(x,y)^2 vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}y(x,y)\right)^2 + 2\cos(kz+\omega t)^2 x(x,y)^2 vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}y(x,y)\right)^2$$
$$-\cos(kz+\omega t)^2 x(x,y)^4 vg^2\,\varepsilon\,\mu\,k^2 + 2\cos(kz+\omega t)^2 y(x,y)^2 vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}x(x,y)\right)^2 + 2\cos(kz$$
$$+\omega t)^2 y(x,y)^2 vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}x(x,y)\right)^2 - \cos(kz+\omega t)^2 y(x,y)^4 vg^2\,\varepsilon\,\mu\,k^2 - 2\cos(kz$$
$$+\omega t)^2 x(x,y)^2\,\varepsilon\,\omega^2\mu\,y(x,y)^2 - 8\left(\frac{\partial}{\partial y}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)x(x,y)\cos(kz+\omega t)^2$$
$$-8\left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)x(x,y)\cos(kz+\omega t)^2 + \varepsilon\,\mu\sin(kz+\omega t)^2 x(x,y)^4\omega^2$$
$$+\varepsilon\,\mu\sin(kz+\omega t)^2 y(x,y)^4\omega^2 + 4\cos(kz+\omega t)^2 x(x,y)^2\,\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2 - 2\cos(kz$$

$$+ \omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - \cos(k z + \omega t)^2 x(x,y)^4 \varepsilon \omega^2 \mu - \cos(k z + \omega t)^2 y(x,y)^4 \varepsilon \omega^2 \mu$$

$$- 2 \cos(k z + \omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - 2 \cos(k z + \omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2$$

$$- 2 \cos(k z + \omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 2 \cos(k z + \omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2$$

$$- 6 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 x(x,y)^2 \cos(k z + \omega t)^2 - 6 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 \cos(k z + \omega t)^2$$

$$- 6 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 x(x,y)^2 \cos(k z + \omega t)^2 - 6 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 y(x,y)^2 \cos(k z + \omega t)^2$$

$$- 2 \cos(k z + \omega t)^2 x(x,y)^2 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y) - 2 \cos(k z + \omega t)^2 x(x,y)^2 y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right)$$

$$- 2 \cos(k z + \omega t)^2 y(x,y)^2 x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - 2 \cos(k z + \omega t)^2 y(x,y)^2 x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$- 2 \cos(k z + \omega t)^2 x(x,y)^3 \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 8 \varepsilon \mu \cos(k z + \omega t)^2 vg^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 8 \varepsilon \mu \cos(k z + \omega t)^2 vg^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)$$

$$- 4 \varepsilon \mu \sin(k z + \omega t)^2 \omega\, vg\, k\, x(x,y)^2 y(x,y)^2 - 2 \varepsilon \mu \sin(k z + \omega t)^2 \omega\, vg\, k\, y(x,y)^4$$

$$+ 2 \varepsilon \mu \sin(k z + \omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 - 2 \varepsilon \mu \sin(k z + \omega t)^2 \omega\, vg\, k\, x(x,y)^4$$

$$+ 6 \varepsilon \mu \cos(k z + \omega t)^2 vg^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 6 \varepsilon \mu \cos(k z + \omega t)^2 vg^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2$$

$$+ 6 \varepsilon \mu \cos(k z + \omega t)^2 vg^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 6 \varepsilon \mu \cos(k z + \omega t)^2 vg^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 y(x,y)^2$$

$$+ \varepsilon \mu \sin(k z + \omega t)^2 x(x,y)^4 vg^2 k^2 + 2 \varepsilon \mu \sin(k z + \omega t)^2 x(x,y)^2 y(x,y)^2 \omega^2 + \varepsilon \mu \sin(k z$$

$$+ \omega t)^2 y(x,y)^4 vg^2 k^2 - 2 \cos(k z + \omega t)^2 x(x,y)^3 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) - 2 \cos(k z + \omega t)^2 y(x,$$

$$y)^3 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + 2 \cos(k z + \omega t)^2 y(x,y)^3 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + 2 \cos(k z + \omega t)^2 x(x,$$

$$y)^2 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) y(x,y) + 2 \cos(k z + \omega t)^2 x(x,y)^2 vg^2 \varepsilon \mu\, y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right)$$

$$+2\cos(kz+\omega t)^2 y(x,y)^2 vg^2 \varepsilon \mu x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2\cos(kz+\omega t)^2 y(x,y)^2 vg^2 \varepsilon \mu x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\cos(kz+\omega t)^2 x(x,y)^4 \varepsilon \omega \mu vg k + 2\cos(kz+\omega t)^2 y(x,y)^4 \varepsilon \omega \mu vg k - 2\cos(kz+\omega t)^2 x(x,y)^2 vg^2 \varepsilon \mu k^2 y(x,y)^2\Big)$$

London Coefficient $\quad LC = \dfrac{k^2 - \varepsilon \omega^2 \mu}{\mu}$

PROCA coefficient curlcurlB $= \Big[-2\cos(kz+\omega t) \Big(2 \left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y \partial x} x(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial y \partial x^2} x(x,y)\right) + 2\left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial y \partial x} y(x,y)\right) + \left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y \partial x^2} y(x,y)\right) + 3\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial y^3} x(x,y)\right) + 3\left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y^3} y(x,y)\right) - k^2 x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) - k^2 y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\Big), 2\cos(kz+\omega t)\Big(-k^2 x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) - k^2 y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right) + 3\left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial x^3} x(x,y)\right) + 3\left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial x^3} y(x,y)\right) + 2\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y \partial x} x(x,y)\right) + \left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial y^2 \partial x} x(x,y)\right) + 2\left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial y \partial x} y(x,y)\right) + \left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y^2 \partial x} y(x,y)\right)\Big), 0\Big]$

Amperian Current 4Vector $\quad$ *curlH-dD/dt=J4* $= \Big[$

$$-\frac{2\sin(kz+\omega t)\,(k-\omega\mu\varepsilon vg)\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)}{\mu},$$

$$-\frac{2\sin(kz+\omega t)\,(k-\omega\mu\varepsilon vg)\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right)}{\mu},$$

$$-\frac{1}{\mu}\left(\cos(kz+\omega t)\left(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\,\omega^2\mu\,x(x,y)^2-\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2+\varepsilon\,\omega^2\mu\,y(x,y)^2-\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\right)\right),$$

$$\varepsilon\cos(kz+\omega t)\left(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)y(x,y)\,vg+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2vg+k\,\omega\,x(x,y)^2-k^2\,vg\,x(x,y)^2+y(x,y)^2\,\omega\,k-k^2\,vg\,y(x,y)^2\right)\Bigg]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= \left[-\frac{2\cos(kz+\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)}{\mu}\right.$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $=[0,0,0,0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $=[0,0,0]$

Torsion Dissipation Jtorsion dot E $=0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \Bigg[$

$$-\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\left(-1+vg^2\varepsilon\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\right),$$

$$-\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\left(-1+vg^2\varepsilon\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\right),\varepsilon\sin(kz+\omega t)\left(x(x,y)^2+y(x,y)^2\right)^2(-\omega+vg\,k)\,vg\cos(kz+\omega t),$$

$$-\left(x(x,y)^2+y(x,y)^2\right)^2\cos(kz+\omega t)\,\varepsilon\sin(kz+\omega t)\,(-\omega+vg\,k)\Big]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $=\Big[0,0,-\frac{1}{\mu}\Big(\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)\Big(-4\left(\frac{\partial}{\partial y}x(x,y)\right)^2x(x,y)^2\cos(kz+\omega t)^2-8\left(\frac{\partial}{\partial y}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)x(x,y)\cos(kz+\omega t)^2-4\,y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2\cos(kz+\omega t)^2$

$-4\left(\frac{\partial}{\partial x}x(x,y)\right)^2x(x,y)^2\cos(kz+\omega t)^2-8\left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)x(x,y)\cos(kz+\omega t)^2-4\left(\frac{\partial}{\partial x}y(x,y)\right)^2y(x,y)^2\cos(kz+\omega t)^2+4\,\varepsilon\,\mu\cos(kz+\omega t)^2vg^2x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+8\,\varepsilon\,\mu\cos(kz+\omega t)^2vg^2x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)$

$+4\,\varepsilon\,\mu\cos(kz+\omega t)^2vg^2y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+4\,\varepsilon\,\mu\cos(kz+\omega t)^2vg^2x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$

$+8\,\varepsilon\,\mu\cos(kz+\omega t)^2vg^2x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+4\,\varepsilon\,\mu\cos(kz+\omega t)^2vg^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2y(x,y)^2+\varepsilon\,\mu\sin(kz+\omega t)^2x(x,y)^4\omega^2-2\,\varepsilon\,\mu\sin(kz+\omega t)^2\omega\,vg\,k\,x(x,y)^4+\varepsilon\,\mu\sin(kz+\omega t)^2x(x,y)^4vg^2k^2+2\,\varepsilon\,\mu\sin(kz+\omega t)^2x(x,y)^2y(x,y)^2\omega^2$

$-4\,\varepsilon\,\mu\sin(kz+\omega t)^2\omega\,vg\,k\,x(x,y)^2y(x,y)^2+2\,\varepsilon\,\mu\sin(kz+\omega t)^2x(x,y)^2y(x,y)^2vg^2k^2$

$+\varepsilon\,\mu\sin(kz+\omega t)^2y(x,y)^4\omega^2-2\,\varepsilon\,\mu\sin(kz+\omega t)^2\omega\,vg\,k\,y(x,y)^4+\varepsilon\,\mu\sin(kz+\omega t)^2y(x,y)^4vg^2k^2\Big)\Big)\Big]$

Spin Dissipation J_spin dot E $= -\frac{1}{\mu}\Big(\big(x(x,y)^2 + y(x,y)^2\big)\cos(kz+\omega t)\, vg\Big(-4\left(\frac{\partial}{\partial y}x(x,y)\right)^2 x(x,$

$$y)^2\cos(kz+\omega t)^2 - 8\left(\frac{\partial}{\partial y}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)x(x,y)\cos(kz+\omega t)^2 - 4\,y(x,$$

$$y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2\cos(kz+\omega t)^2 - 4\left(\frac{\partial}{\partial x}x(x,y)\right)^2 x(x,y)^2\cos(kz+\omega t)^2 - 8\left(\frac{\partial}{\partial x}y(x,y)\right)y(x,$$

$$y)\left(\frac{\partial}{\partial x}x(x,y)\right)x(x,y)\cos(kz+\omega t)^2 - 4\left(\frac{\partial}{\partial x}y(x,y)\right)^2 y(x,y)^2\cos(kz+\omega t)^2$$

$$+ 4\,\varepsilon\,\mu\cos(kz+\omega t)^2\, vg^2\, x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2 + 8\,\varepsilon\,\mu\cos(kz+\omega t)^2\, vg^2\, x(x,y)\left(\frac{\partial}{\partial x}x(x,$$

$$y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) + 4\,\varepsilon\,\mu\cos(kz+\omega t)^2\, vg^2\, y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 + 4\,\varepsilon\,\mu\cos(kz$$

$$+\omega t)^2\, vg^2\, x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2 + 8\,\varepsilon\,\mu\cos(kz+\omega t)^2\, vg^2\, x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,$$

$$y)\left(\frac{\partial}{\partial y}y(x,y)\right) + 4\,\varepsilon\,\mu\cos(kz+\omega t)^2\, vg^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 y(x,y)^2 + \varepsilon\,\mu\sin(kz+\omega t)^2\, x(x,y)^4\,\omega^2$$

$$-2\,\varepsilon\,\mu\sin(kz+\omega t)^2\,\omega\, vg\, k\, x(x,y)^4 + \varepsilon\,\mu\sin(kz+\omega t)^2\, x(x,y)^4\, vg^2\, k^2 + 2\,\varepsilon\,\mu\sin(kz$$

$$+\omega t)^2\, x(x,y)^2\, y(x,y)^2\,\omega^2 - 4\,\varepsilon\,\mu\sin(kz+\omega t)^2\,\omega\, vg\, k\, x(x,y)^2\, y(x,y)^2 + 2\,\varepsilon\,\mu\sin(kz$$

$$+\omega t)^2\, x(x,y)^2\, y(x,y)^2\, vg^2\, k^2 + \varepsilon\,\mu\sin(kz+\omega t)^2\, y(x,y)^4\,\omega^2 - 2\,\varepsilon\,\mu\sin(kz+\omega t)^2\,\omega\, vg\, k\, y(x,y)^4$$

$$+\varepsilon\,\mu\sin(kz+\omega t)^2\, y(x,y)^4\, vg^2\, k^2\Big)\Big)$$

Disspative Force 3 vector $= \left[\frac{2\cos(kz+\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)(\%5)}{\mu}, \frac{2\cos(kz+\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}\, y\right.\right.}{\mu}\right.$

Dissipation $= -\varepsilon\cos(kz+\omega t)\left(-2\, vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2 - 2\, vg\, x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right) - 2\, vg\left(\frac{\partial}{\partial x}y(x,\right.\right.$

$y)\Big)^2 - 2\, vg\, y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) - 2\, vg \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$

$-2 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) y(x,y)\, vg - 2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 vg - k\, \omega\, x(x,y)^2 + k^2\, vg\, x(x,y)^2 - y(x,y)^2\, \omega\, k$

$+ k^2\, vg\, y(x,y)^2 - \mu \sin(k z + \omega t)\, x(x,y)^4\, \omega + \mu \sin(k z + \omega t)\, x(x,y)^4\, vg\, k - 2\, \mu \sin(k z$

$+ \omega t)\, x(x,y)^2\, y(x,y)^2\, \omega + 2\, \mu \sin(k z + \omega t)\, x(x,y)^2\, y(x,y)^2\, vg\, k - \omega\, y(x,y)^4 \sin(k z + \omega t)\, \mu$

$+ \mu \sin(k z + \omega t)\, y(x,y)^4\, vg\, k)$

***************************** *END PROCEDURE* *************************** **(23)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 5b-- waveguide TM mode  (group kinematic in, wave out)`;
> theta:=(-k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=-vg*f(x,y)*cos(theta);
```

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,gamma,0):
```

****

NAME := Example 5b-- waveguide TM mode (group kinematic in, wave out)

$$\theta := -k z + \omega t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z - \omega t)$$

$$\phi := -vg \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z - \omega t)$$

Example 5b-- waveguide TM mode (group kinematic in, wave out)

******************************** *Differential Form Format* ****************************
****

Action 1-form $= \left(vg \cos(k z - \omega t)\, x(x,y)^2 + vg \cos(k z - \omega t)\, y(x,y)^2\right) d(t) + \left(\cos(k z - \omega t)\, x(x,y)^2 + \cos(k z - \omega t)\, y(x,y)^2\right) d(z)$

Intensity 2-form F=dA $= \Big(2\, vg \cos(k z - \omega t)\, y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 2\, vg \cos(k z - \omega t)\, x(x,$

$y) \left(\frac{\partial}{\partial x} x(x,y)\right)\Big)\, (d(x))\ \&\hat{}\ (d(t)) + \Big(2 \cos(k z - \omega t)\, y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 2 \cos(k z$

$- \omega t)\, x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right)\Big)\, (d(x))\ \&\hat{}\ (d(z)) + \Big(2\, vg \cos(k z - \omega t)\, y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)$

$+ 2\, vg \cos(k z - \omega t)\, x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right)\Big)\, (d(y))\ \&\hat{}\ (d(t)) + \Big(2 \cos(k z - \omega t)\, y(x,$

$y) \left(\frac{\partial}{\partial y} y(x,y)\right) + 2\cos(kz - \omega t)\, x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right)\Big) (d(y))$ &^ $(d(z)) + \Big(-\sin(kz - \omega t)\, \omega\, x(x,y)^2 - \sin(kz - \omega t)\, \omega\, y(x,y)^2 - vg \sin(kz - \omega t)\, k\, x(x,y)^2 - vg \sin(kz - \omega t)\, k\, y(x,y)^2\Big) (d(z))$ &^ $(d(t))$

Topological Torsion 3-form A^$F = 0$

Topological Parity 4-form F^$F = 0$

********************************* *Using EM format* *********************************

$$E\,field = \left[2\, vg \left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\cos(kz - \omega t),\ 2\, vg \left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right)\cos(kz - \omega t),\ -\sin(kz - \omega t)\left(x(x,y)^2 + y(x,y)^2\right)(\omega + vg\, k)\right]$$

$$B\,field = \left[2\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right)\cos(kz - \omega t),\ -2\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\cos(kz - \omega t),\ 0\right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension PTD $= 2$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************
****

Xm or linear (Mean) curvature $= -\sin(kz - \omega t)\left(x(x,y)^2 + y(x,y)^2\right)(k - \omega\, vg)$

Yg or quadratic (GAUSS) curvature $= 0$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$Chirality\ factor\ CH = \gamma\sqrt{\frac{\mu}{\varepsilon}}$$

$$D\,field = \left[2\cos(kz - \omega t)\left(\varepsilon\, vg\, x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + \varepsilon\, vg\, y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right) + \gamma\sqrt{\frac{\mu}{\varepsilon}}\, x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + \left(\frac{\partial}{\partial y} y(x,y)\right) y(x,y)\sqrt{\frac{\mu}{\varepsilon}}\,\gamma\right),\ 2\cos(kz - \omega t)\left(\varepsilon\, vg\, x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right)\right.\right.$$

$$+\left(\frac{\partial}{\partial y}\, y(x,y)\right) y(x,y)\; vg\,\varepsilon - \gamma\sqrt{\frac{\mu}{\varepsilon}}\; x(x,y)\left(\frac{\partial}{\partial x}\, x(x,y)\right) - \gamma\sqrt{\frac{\mu}{\varepsilon}}\; y(x,y)\left(\frac{\partial}{\partial x}\, y(x,y)\right)\Bigg),$$

$$-\varepsilon \sin(k\,z-\omega\,t)\,\left(x(x,y)^2+y(x,y)^2\right)\,(\omega+vg\,k)\Bigg]$$

$$H\,field = \Bigg[\frac{1}{\mu}\Bigg(2\cos(k\,z-\omega\,t)\,\Bigg(x(x,y)\left(\frac{\partial}{\partial y}\, x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}\, y(x,y)\right) - \gamma\sqrt{\frac{\mu}{\varepsilon}}\; vg\,\mu\, x(x,y)\left(\frac{\partial}{\partial x}\, x(x,y)\right) - \gamma\sqrt{\frac{\mu}{\varepsilon}}\; vg\,\mu\, y(x,y)\left(\frac{\partial}{\partial x}\, y(x,y)\right)\Bigg)\Bigg), -\frac{1}{\mu}\Bigg(2\cos(k\,z-\omega\,t)\,\Bigg(x(x,y)\left(\frac{\partial}{\partial x}\, x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}\, y(x,y)\right) + \gamma\sqrt{\frac{\mu}{\varepsilon}}\; vg\,\mu\, x(x,y)\left(\frac{\partial}{\partial y}\, x(x,y)\right) + \gamma\sqrt{\frac{\mu}{\varepsilon}}\; vg\,\mu\, y(x,y)\left(\frac{\partial}{\partial y}\, y(x,y)\right)\Bigg)\Bigg), \gamma\sqrt{\frac{\mu}{\varepsilon}}\;\sin(k\,z-\omega\,t)\,\left(x(x,y)^2+y(x,y)^2\right)\,(\omega+vg\,k)\Bigg]$$

$$Poynting\ vector\ ExH = \Bigg[-\frac{1}{\mu}\Bigg(2\cos(k\,z-\omega\,t)\,\sin(k\,z-\omega\,t)\,\left(x(x,y)^2+y(x,y)^2\right)\,(\omega+vg\,k)\,\Bigg(x(x,y)\left(\frac{\partial}{\partial x}\, x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}\, y(x,y)\right)\Bigg)\Bigg), -\frac{1}{\mu}\Bigg(2\cos(k\,z-\omega\,t)\,\sin(k\,z-\omega\,t)\,\left(x(x,y)^2+y(x,y)^2\right)\,(\omega+vg\,k)\,\Bigg(x(x,y)\left(\frac{\partial}{\partial y}\, x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}\, y(x,y)\right)\Bigg)\Bigg), -\frac{1}{\mu}\Bigg(4\cos(k\,z-\omega\,t)^2\, vg\,\Bigg(x(x,y)^2\left(\frac{\partial}{\partial y}\, x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial}{\partial y}\, x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial y}\, y(x,y)\right) + y(x,y)^2\left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 + x(x,y)^2\left(\frac{\partial}{\partial x}\, x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial}{\partial x}\, x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial x}\, y(x,y)\right) + y(x,y)^2\left(\frac{\partial}{\partial x}\, y(x,y)\right)^2\Bigg)\Bigg)\Bigg]$$

$$Amperian\ Current\ 4Vector\quad curlH\text{-}dD/dt=J4\ = \Bigg[$$

$$-\frac{2\sin(k\,z-\omega\,t)\,(k+\omega\,\mu\,\varepsilon\, vg)\,\Bigg(x(x,y)\left(\frac{\partial}{\partial x}\, x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}\, y(x,y)\right)\Bigg)}{\mu},$$

$$-\frac{2\sin(kz-\omega t)\,(k+\omega\mu\varepsilon vg)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)}{\mu},$$

$$-\frac{1}{\mu}\Bigg(\cos(kz-\omega t)\Bigg(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\,\omega^2\mu\,x(x,y)^2+\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2+\varepsilon\,\omega^2\mu\,y(x,y)^2+\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\Bigg)\Bigg),$$

$$-\cos(kz-\omega t)\,\varepsilon\Bigg(-2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2-2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-2\,vg\left(\frac{\partial}{\partial x}y(x,y)\right)^2-2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)-2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2-2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-2\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)y(x,y)\,vg-2\left(\frac{\partial}{\partial y}y(x,y)\right)^2vg+k\,\omega\,x(x,y)^2+k^2\,vg\,x(x,y)^2+y(x,y)^2\,\omega\,k+k^2\,vg\,y(x,y)^2\Bigg)\Bigg]$$

Amerian charge density $\quad divD = rho = -\cos(kz-\omega t)\,\varepsilon\Bigg(-2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2-2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-2\,vg\left(\frac{\partial}{\partial x}y(x,y)\right)^2-2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)-2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2-2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-2\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)y(x,y)\,vg-2\left(\frac{\partial}{\partial y}y(x,y)\right)^2vg+k\,\omega\,x(x,y)^2+k^2\,vg\,x(x,y)^2+y(x,y)^2\,\omega\,k+k^2\,vg\,y(x,y)^2\Bigg)$

divergence Lorentz Current 4Vector, $\quad$ *4div(J4)* $=0$

Topological SPIN 4 vector S4 $=\Bigg[$

$$-\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(-1+vg^2\,\varepsilon\,\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\right),$$

$$-\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(-1+vg^2\,\varepsilon\,\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,\right.\right.$$

$y) \left(\frac{\partial}{\partial y} y(x,y)\right)\Big)\Big)$, $\varepsilon \sin(k z-\omega t) \left(x(x,y)^2+y(x,y)^2\right)^2 (\omega+vg\,k)\, vg \cos(k z-\omega t)$,

$-\left(x(x,y)^2+y(x,y)^2\right)^2 \cos(k z-\omega t)\, \varepsilon \sin(k z-\omega t)\, (\omega+vg\,k)\Big]$

$$\textit{Topological SPIN 3-form} = -\frac{1}{\mu}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k z-\omega t)^2\left(-1+vg^2\varepsilon\mu\right)\Big(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)\,\&\hat{}(d(y),d(z),d(t))\Big)+\frac{1}{\mu}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k z-\omega t)^2\left(-1+vg^2\varepsilon\mu\right)\Big(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Big)\,\&\hat{}(d(x),d(z),d(t))\Big)+\varepsilon\sin(k z-\omega t)\left(x(x,y)^2+y(x,y)^2\right)^2(\omega+vg\,k)\,vg\cos(k z-\omega t)\,\&\hat{}(d(x),d(y),d(t))+\left(x(x,y)^2+y(x,y)^2\right)^2\cos(k z-\omega t)\,\varepsilon\sin(k z-\omega t)\,(\omega+vg\,k)\,\&\hat{}(d(x),d(y),d(z))$$

$$\textit{Spin density rho\_spin} = -\left(x(x,y)^2+y(x,y)^2\right)^2\cos(k z-\omega t)\,\varepsilon\sin(k z-\omega t)\,(\omega+vg\,k)$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{1}{\mu}\Big(-4\left(\frac{\partial}{\partial x}x(x,y)\right)^2 x(x,y)^2\cos(k z-\omega t)^2-8\left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)x(x,y)\cos(k z-\omega t)^2-4\left(\frac{\partial}{\partial x}y(x,y)\right)^2 y(x,y)^2\cos(k z-\omega t)^2-8\left(\frac{\partial}{\partial y}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)x(x,y)\cos(k z-\omega t)^2-4\,y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2\cos(k z-\omega t)^2-4\left(\frac{\partial}{\partial y}x(x,y)\right)^2 x(x,y)^2\cos(k z-\omega t)^2+4\,\varepsilon\mu\cos(k z-\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+8\,\varepsilon\mu\cos(k z-\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+4\,\varepsilon\mu\cos(k z-\omega t)^2 vg^2 y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+4\,\varepsilon\mu\cos(k z-\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+8\,\varepsilon\mu\cos(k z-\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+4\,\varepsilon\mu\cos(k z-\omega t)^2 vg^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 y(x,y)^2+\varepsilon\mu\sin(k z-\omega t)^2 x(x,y)^4\omega^2+2\,\varepsilon\mu\sin(k z-\omega t)^2\omega\,vg\,k\,x(x,y)^4+\varepsilon\mu\sin(k z-\omega t)^2 x(x,y)^4 vg^2 k^2+2\,\varepsilon\mu\sin(k z-\omega t)^2 x(x,y)^2 y(x,y)^2\omega^2+4\,\varepsilon\mu\sin(k z-\omega t)^2\omega\,vg\,k\,x(x,y)^2 y(x,y)^2+2\,\varepsilon\mu\sin(k z-\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2+\varepsilon\mu\sin(k z-\omega t)^2 y(x,y)^4\omega^2+2\,\varepsilon\mu\sin(k z-\omega t)^2\omega\,vg\,k\,y(x,y)^4+\varepsilon\mu\sin(k z-\omega t)^2 y(x,$$

$y)^4\, vg^2\, k^2\big)$

$$B.H = \frac{1}{\mu}\left(4\cos(kz-\omega t)^2\left(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\right)\right)$$

$$D.E = \varepsilon\left(4\cos(kz-\omega t)^2\, vg^2\, x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+8\cos(kz-\omega t)^2\, vg^2\, x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+4\cos(kz-\omega t)^2\, vg^2\, y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+4\cos(kz-\omega t)^2\, vg^2\, x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+8\cos(kz-\omega t)^2\, vg^2\, x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+4\cos(kz-\omega t)^2\, vg^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 y(x,y)^2+\sin(kz-\omega t)^2\, x(x,y)^4\,\omega^2+2\sin(kz-\omega t)^2\,\omega\, vg\, k\, x(x,y)^4+\sin(kz-\omega t)^2\, x(x,y)^4\, vg^2\, k^2+2\sin(kz-\omega t)^2\, x(x,y)^2\, y(x,y)^2\,\omega^2+4\sin(kz-\omega t)^2\,\omega\, vg\, k\, x(x,y)^2\, y(x,y)^2+2\sin(kz-\omega t)^2\, x(x,y)^2\, y(x,y)^2\, vg^2\, k^2+\sin(kz-\omega t)^2\, y(x,y)^4\,\omega^2+2\sin(kz-\omega t)^2\,\omega\, vg\, k\, y(x,y)^4+\sin(kz-\omega t)^2\, y(x,y)^4\, vg^2\, k^2\right)$$

$$A.J = -\frac{1}{\mu}\left(\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\,\omega^2\,\mu\, x(x,y)^2+\varepsilon\,\omega\,\mu\, vg\, k\, x(x,y)^2+\varepsilon\,\omega^2\,\mu\, y(x,y)^2+\varepsilon\,\omega\,\mu\, vg\, k\, y(x,y)^2\right)\right)$$

$$-rho.phi = vg\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\,\varepsilon\left(-2\, vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2-2\, vg\, x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-2\, vg\left(\frac{\partial}{\partial x}y(x,y)\right)^2-2\, vg\, y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)-2\, vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2-2\, vg\, x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-2\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)y(x,y)\, vg-2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 vg+k\,\omega\, x(x,y)^2+k^2\, vg\, x(x,y)^2\right.$$

$$+ y(x,y)^2 \omega k + k^2 vg\, y(x,y)^2 \Big)$$

Poincare I (B.H - D.E)-(A.J - rho.phi) $= \frac{1}{\mu} \Big(2 \cos(k z - \omega t)^2 \varepsilon \omega \mu\, vg\, k\, x(x,y)^4 + 2 \cos(k z - \omega t)^2 \varepsilon \omega \mu\, vg\, k\, y(x,y)^4 + 2 \cos(k z - \omega t)^2 vg^2 \varepsilon \mu k^2 x(x,y)^2 y(x,y)^2 + 2 \cos(k z - \omega t)^2 y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y) \right) x(x,y)^2 + 2 \cos(k z - \omega t)^2 x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y) \right) y(x,y)^2 + 2 \cos(k z - \omega t)^2 y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y) \right) x(x,y)^2 + 2 \cos(k z - \omega t)^2 x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y) \right) y(x,y)^2 + \cos(k z - \omega t)^2 vg^2 \varepsilon \mu k^2 y(x,y)^4 + \cos(k z - \omega t)^2 vg^2 \varepsilon \mu k^2 x(x,y)^4 - 2 \cos(k z - \omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial x} y(x,y) \right)^2 x(x,y)^2 - 2 \cos(k z - \omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial x} x(x,y) \right)^2 y(x,y)^2 + 2 \cos(k z - \omega t)^2 \varepsilon \omega^2 \mu\, x(x,y)^2 y(x,y)^2 - 2 \cos(k z - \omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial y} x(x,y) \right)^2 y(x,y)^2 - 2 \cos(k z - \omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial y} y(x,y) \right)^2 x(x,y)^2 + \cos(k z - \omega t)^2 \varepsilon \omega^2 \mu\, y(x,y)^4 - 6 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 - 6 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 - 6 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 - 6 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 y(x,y)^2 - \varepsilon \mu \sin(k z - \omega t)^2 x(x,y)^4 vg^2 k^2 - 8 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) - 8 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) + 2 \cos(k z - \omega t)^2 x(x,y)^3 \left(\frac{\partial^2}{\partial y^2} x(x,y) \right) + 2 \cos(k z - \omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial x^2} y(x,y) \right) + 2 \cos(k z - \omega t)^2 x(x,y)^3 \left(\frac{\partial^2}{\partial x^2} x(x,y) \right) - 2 \varepsilon \mu \sin(k z - \omega t)^2 \omega\, vg\, k\, x(x,y)^4 - 2 \varepsilon \mu \sin(k z - \omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 - 2 \varepsilon \mu \sin(k z - \omega t)^2 \omega\, vg\, k\, y(x,y)^4 + 2 \cos(k z - \omega t)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 x(x,y)^2 + 2 \cos(k z - \omega t)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 y(x,y)^2 + 2 \cos(k z$

$$-\omega t)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 x(x,y)^2 + 2\cos(kz-\omega t)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 y(x,y)^2 + 2\cos(kz$$

$$-\omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + \cos(kz-\omega t)^2 \varepsilon \omega^2 \mu x(x,y)^4 + 4\cos(kz$$

$$-\omega t)^2 \varepsilon \omega \mu vg\, k\, x(x,y)^2 y(x,y)^2 + 8\left(\frac{\partial}{\partial x} y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) x(x,y) \cos(kz-\omega t)^2$$

$$+8\left(\frac{\partial}{\partial y} y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) x(x,y) \cos(kz-\omega t)^2 - \varepsilon \mu \sin(kz-\omega t)^2 x(x,y)^4 \omega^2$$

$$+6\left(\frac{\partial}{\partial x} x(x,y)\right)^2 x(x,y)^2 \cos(kz-\omega t)^2 + 6\left(\frac{\partial}{\partial x} y(x,y)\right)^2 y(x,y)^2 \cos(kz-\omega t)^2 + 6\, y(x,$$

$$y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 \cos(kz-\omega t)^2 + 6\left(\frac{\partial}{\partial y} x(x,y)\right)^2 x(x,y)^2 \cos(kz-\omega t)^2 - 2\varepsilon\mu \sin(kz$$

$$-\omega t)^2 x(x,y)^2 y(x,y)^2 \omega^2 - \varepsilon\mu \sin(kz-\omega t)^2 y(x,y)^4 vg^2 k^2 - 4\varepsilon\mu \sin(kz-\omega t)^2 \omega\, vg\, k\, x(x,$$

$$y)^2 y(x,y)^2 - 2\cos(kz-\omega t)^2 vg^2 \varepsilon\mu x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) y(x,y)^2 - 2\cos(kz-\omega t)^2 vg^2 \varepsilon\mu y(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) x(x,y)^2 - 2\cos(kz-\omega t)^2 vg^2 \varepsilon\mu \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) y(x,y)\, x(x,y)^2 - 2\cos(kz$$

$$-\omega t)^2 vg^2 \varepsilon\mu x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) y(x,y)^2 - 2\cos(kz-\omega t)^2 vg^2 \varepsilon\mu y(x,y)^3 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right)$$

$$-2\cos(kz-\omega t)^2 vg^2 \varepsilon\mu x(x,y)^3 \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - 2\cos(kz-\omega t)^2 vg^2 \varepsilon\mu \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) y(x,y)^3$$

$$-2\cos(kz-\omega t)^2 vg^2 \varepsilon\mu x(x,y)^3 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) - \varepsilon\mu \sin(kz-\omega t)^2 y(x,y)^4 \omega^2 \Big)$$

$$\textit{London Coefficient} \qquad LC = \frac{k^2 - \varepsilon\omega^2\mu}{\mu}$$

$$\textit{PROCA coefficient curlcurlB} = \Big[-2\cos(kz-\omega t) \left(2\left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,\right.\right.$$

$$y)\Big) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2} x(x,y)\right) + 2\left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} y(x,y)\right) + \Big(\frac{\partial}{\partial y} y(x,$$

$$y)\Big) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2} y(x,y)\right) + 3\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + x(x,$$

$$y) \left(\frac{\partial^3}{\partial y^3} x(x,y)\right) + 3\left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y^3} y(x,y)\right) - k^2 x(x,y) \Big(\frac{\partial}{\partial y} x(x,$$

$$y)\Big) - k^2 y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\Big), 2\cos(kz-\omega t)\Big(-k^2 x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) - k^2 y(x,y)\Big(\frac{\partial}{\partial x} y(x,$$

$$y)\Big) + 3\left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial x^3} x(x,y)\right) + 3\left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right)$$

$$+ y(x,y)\left(\frac{\partial^3}{\partial x^3} y(x,y)\right) + 2\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right) + \left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + x(x,$$

$$y)\left(\frac{\partial^3}{\partial y^2\,\partial x} x(x,y)\right) + 2\left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} y(x,y)\right) + \left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + y(x,$$

$$y)\left(\frac{\partial^3}{\partial y^2\,\partial x} y(x,y)\right)\Big)\Big), 0\Big]$$

Amperian Current 4Vector   curlH-dD/dt=J4 $=\Big[$

$$-\frac{2\sin(kz-\omega t)(k+\omega\mu\varepsilon vg)\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)}{\mu},$$

$$-\frac{2\sin(kz-\omega t)(k+\omega\mu\varepsilon vg)\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right)}{\mu},$$

$$-\frac{1}{\mu}\Big(\cos(kz-\omega t)\Big(2\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2\left(\frac{\partial}{\partial x} y(x,y)\right)^2 + 2y(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + 2\left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 2y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + \varepsilon\omega^2\mu x(x,y)^2 + \varepsilon\omega\mu vg\,k\,x(x,y)^2 + \varepsilon\omega^2\mu y(x,y)^2 + \varepsilon\omega\mu vg\,k\,y(x,y)^2\Big)\Big),$$

$$-\cos(kz-\omega t)\,\varepsilon\Big(-2vg\left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 2vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - 2vg\left(\frac{\partial}{\partial x} y(x,y)\right)^2$$

$$-2vg\,y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) - 2vg\left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 2vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) - 2\Big(\frac{\partial^2}{\partial y^2} y(x,$$

$$y)\Big)y(x,y)\,vg - 2\left(\frac{\partial}{\partial y} y(x,y)\right)^2 vg + k\omega x(x,y)^2 + k^2 vg\,x(x,y)^2 + y(x,y)^2\omega k + k^2 vg\,y(x,y)^2\Big)\Big]$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B) $= \left[\dfrac{2\cos(kz-\omega t)^2 \left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\left(\right.}{\mu} \right.$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] $=[0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $=[0, 0, 0]$

Torsion Dissipation Jtorsion dot E $=0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] $=\Bigg[$

$$-\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(-1+vg^2\varepsilon\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\right),$$

$$-\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(-1+vg^2\varepsilon\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\right), \varepsilon\sin(kz-\omega t)\left(x(x,y)^2+y(x,y)^2\right)^2(\omega+vg\,k)\,vg\cos(kz-\omega t),$$

$$-\left(x(x,y)^2+y(x,y)^2\right)^2\cos(kz-\omega t)\,\varepsilon\sin(kz-\omega t)\,(\omega+vg\,k)\Bigg]$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $=\left[0, 0, -\frac{1}{\mu}\left(\left(x(x,y)^2\right.\right.\right.$

$$+y(x,y)^2)\,\cos(kz-\omega t)\,\Big(-4\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2 x(x,y)^2\cos(kz-\omega t)^2-8\Big(\frac{\partial}{\partial x}y(x,y)\Big)\,y(x,$$

$$y)\,\Big(\frac{\partial}{\partial x}x(x,y)\Big)\,x(x,y)\,\cos(kz-\omega t)^2-4\Big(\frac{\partial}{\partial x}y(x,y)\Big)^2 y(x,y)^2\cos(kz-\omega t)^2-8\Big(\frac{\partial}{\partial y}y(x,$$

$$y)\Big)\,y(x,y)\,\Big(\frac{\partial}{\partial y}x(x,y)\Big)\,x(x,y)\,\cos(kz-\omega t)^2-4\,y(x,y)^2\Big(\frac{\partial}{\partial y}y(x,y)\Big)^2\cos(kz-\omega t)^2$$

$$-4\Big(\frac{\partial}{\partial y}x(x,y)\Big)^2 x(x,y)^2\cos(kz-\omega t)^2+4\,\varepsilon\,\mu\cos(kz-\omega t)^2\,vg^2\,x(x,y)^2\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2$$

$$+8\,\varepsilon\,\mu\cos(kz-\omega t)^2\,vg^2\,x(x,y)\,\Big(\frac{\partial}{\partial x}x(x,y)\Big)\,y(x,y)\,\Big(\frac{\partial}{\partial x}y(x,y)\Big)+4\,\varepsilon\,\mu\cos(kz$$

$$-\omega t)^2\,vg^2\,y(x,y)^2\Big(\frac{\partial}{\partial x}y(x,y)\Big)^2+4\,\varepsilon\,\mu\cos(kz-\omega t)^2\,vg^2\,x(x,y)^2\Big(\frac{\partial}{\partial y}x(x,y)\Big)^2$$

$$+8\,\varepsilon\,\mu\cos(kz-\omega t)^2\,vg^2\,x(x,y)\,\Big(\frac{\partial}{\partial y}x(x,y)\Big)\,y(x,y)\,\Big(\frac{\partial}{\partial y}y(x,y)\Big)+4\,\varepsilon\,\mu\cos(kz$$

$$-\omega t)^2\,vg^2\Big(\frac{\partial}{\partial y}y(x,y)\Big)^2 y(x,y)^2+\varepsilon\,\mu\sin(kz-\omega t)^2\,x(x,y)^4\,\omega^2+2\,\varepsilon\,\mu\sin(kz$$

$$-\omega t)^2\,\omega\,vg\,k\,x(x,y)^4+\varepsilon\,\mu\sin(kz-\omega t)^2\,x(x,y)^4\,vg^2\,k^2+2\,\varepsilon\,\mu\sin(kz-\omega t)^2\,x(x,y)^2\,y(x,y)^2\,\omega^2$$

$$+4\,\varepsilon\,\mu\sin(kz-\omega t)^2\,\omega\,vg\,k\,x(x,y)^2\,y(x,y)^2+2\,\varepsilon\,\mu\sin(kz-\omega t)^2\,x(x,y)^2\,y(x,y)^2\,vg^2\,k^2$$

$$+\varepsilon\,\mu\sin(kz-\omega t)^2\,y(x,y)^4\,\omega^2+2\,\varepsilon\,\mu\sin(kz-\omega t)^2\,\omega\,vg\,k\,y(x,y)^4+\varepsilon\,\mu\sin(kz-\omega t)^2\,y(x,$$

$$y)^4\,vg^2\,k^2))\,]$$

Spin Dissipation J_spin dot E $=-\frac{1}{\mu}\Big(\big(x(x,y)^2+y(x,y)^2\big)\cos(kz-\omega t)\,vg\,\Big(-4\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2 x(x,$

$$y)^2\cos(kz-\omega t)^2-8\Big(\frac{\partial}{\partial x}y(x,y)\Big)\,y(x,y)\,\Big(\frac{\partial}{\partial x}x(x,y)\Big)\,x(x,y)\,\cos(kz-\omega t)^2-4\Big(\frac{\partial}{\partial x}y(x,$$

$$y)\Big)^2 y(x,y)^2\cos(kz-\omega t)^2-8\Big(\frac{\partial}{\partial y}y(x,y)\Big)\,y(x,y)\,\Big(\frac{\partial}{\partial y}x(x,y)\Big)\,x(x,y)\,\cos(kz-\omega t)^2$$

$$-4\,y(x,y)^2\Big(\frac{\partial}{\partial y}y(x,y)\Big)^2\cos(kz-\omega t)^2-4\Big(\frac{\partial}{\partial y}x(x,y)\Big)^2 x(x,y)^2\cos(kz-\omega t)^2$$

$$+4\,\varepsilon\,\mu\cos(kz-\omega t)^2\,vg^2\,x(x,y)^2\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2+8\,\varepsilon\,\mu\cos(kz-\omega t)^2\,vg^2\,x(x,y)\,\Big(\frac{\partial}{\partial x}x(x,$$

$$y)\Big)\,y(x,y)\,\Big(\frac{\partial}{\partial x}y(x,y)\Big)+4\,\varepsilon\,\mu\cos(kz-\omega t)^2\,vg^2\,y(x,y)^2\Big(\frac{\partial}{\partial x}y(x,y)\Big)^2+4\,\varepsilon\,\mu\cos(kz$$

$$-\omega t)^2\,vg^2\,x(x,y)^2\Big(\frac{\partial}{\partial y}x(x,y)\Big)^2+8\,\varepsilon\,\mu\cos(kz-\omega t)^2\,vg^2\,x(x,y)\,\Big(\frac{\partial}{\partial y}x(x,y)\Big)\,y(x,$$

$y)\left(\frac{\partial}{\partial y}y(x,y)\right)+4\,\varepsilon\,\mu\cos(k\,z-\omega\,t)^2\,vg^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2y(x,y)^2+\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,x(x,y)^4\,\omega^2$
$+2\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,\omega\,vg\,k\,x(x,y)^4+\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,x(x,y)^4\,vg^2\,k^2+2\,\varepsilon\,\mu\sin(k\,z$
$-\omega\,t)^2\,x(x,y)^2\,y(x,y)^2\,\omega^2+4\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,\omega\,vg\,k\,x(x,y)^2\,y(x,y)^2+2\,\varepsilon\,\mu\sin(k\,z$
$-\omega\,t)^2\,x(x,y)^2\,y(x,y)^2\,vg^2\,k^2+\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,y(x,y)^4\,\omega^2+2\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,\omega\,vg\,k\,y(x,y)^4$
$+\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,y(x,y)^4\,vg^2\,k^2))$

$$Disspative\ Force\ 3\ vector=\left[-\frac{2\cos(kz-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)(\%5)}{\mu},-\frac{2\cos(kz-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial\right.}\right.}{\mu}\right.$$

$Dissipation\ =\cos(k\,z-\omega\,t)\,\varepsilon\left(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial x}y(x,y)\right)^2\right.$
$+2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial^2}{\partial y^2}y(x,\right.$
$\left.y)\right)y(x,y)\,vg+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2vg-k\,\omega\,x(x,y)^2-k^2\,vg\,x(x,y)^2-y(x,y)^2\,\omega\,k-k^2\,vg\,y(x,y)^2$
$-\mu\sin(k\,z-\omega\,t)\,x(x,y)^4\,\omega-\mu\sin(k\,z-\omega\,t)\,x(x,y)^4\,vg\,k-2\,\mu\sin(k\,z-\omega\,t)\,x(x,y)^2\,y(x,y)^2\,\omega$
$-2\,\mu\sin(k\,z-\omega\,t)\,x(x,y)^2\,y(x,y)^2\,vg\,k-\omega\,y(x,y)^4\sin(k\,z-\omega\,t)\,\mu-\mu\sin(k\,z-\omega\,t)\,y(x,$
$\left.y)^4\,vg\,k\right)$

********************************** *END PROCEDURE* ******************************* **(24)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 5c-- waveguide TM mode phi (group kinematic out, wave in)`;
> theta:=(k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=+vg*f(x,y)*cos(theta);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 5c-- waveguide TM mode phi (group kinematic out, wave in)

$$\theta := k\,z + \omega\,t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z + \omega\,t)$$

$$\phi := vg\,\left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z + \omega\,t)$$

Example 5c-- waveguide TM mode phi (group kinematic out, wave in)

******************************** *Differential Form Format* ********************************

$$\textit{Action 1-form} = \left(-vg\cos(k\,z+\omega\,t)\,x(x,y)^2 - vg\cos(k\,z+\omega\,t)\,y(x,y)^2\right)d(t) + \left(\cos(k\,z+\omega\,t)\,x(x,y)^2 + \cos(k\,z+\omega\,t)\,y(x,y)^2\right)d(z)$$

$$\textit{Intensity 2-form F=dA} = \left(-2\,vg\cos(k\,z+\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) - 2\,vg\cos(k\,z+\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)\right)(d(x))\,\&\text{^}\,(d(t)) + \left(2\cos(k\,z+\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) + 2\cos(k\,z+\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)\right)(d(x))\,\&\text{^}\,(d(z)) + \left(-2\,vg\cos(k\,z+\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) - 2\,vg\cos(k\,z+\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)\right)(d(y))\,\&\text{^}\,(d(t)) + \left(2\cos(k\,z+\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) + 2\cos(k\,z+\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)\right)(d(y))\,\&\text{^}\,(d(z)) + \left(\sin(k\,z+\omega\,t)\,\omega\,x(x,y)^2 + \sin(k\,z+\omega\,t)\,\omega\,y(x,y)^2 + vg\sin(k\,z+\omega\,t)\,k\,x(x,y)^2 + vg\sin(k\,z+\omega\,t)\,k\,y(x,y)^2\right)(d(z))\,\&\text{^}\,(d(t))$$

Topological Torsion 3-form $A\text{^}F = 0$

Topological Parity 4-form $F\text{^}F = 0$

******************************** *Using EM format* ********************************

$$\textit{E field} = \left[-2\,vg\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z+\omega\,t),\ -2\,vg\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z+\omega\,t),\ \sin(k\,z+\omega\,t)\left(x(x,y)^2 + y(x,y)^2\right)(\omega + vg\,k)\right]$$

$$B\,field = \left[2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k z+\omega t),\ -2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k z+\omega t),\ 0\right]$$

Topological TORSION 4 vector   T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension   PTD = 2

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************************

$$Xm\ or\ linear\ (Mean)\ curvature = -\sin(k z+\omega t)\left(x(x,y)^2+y(x,y)^2\right)(k-\omega\, vg)$$

Yg or quadratic (GAUSS) curvature = 0

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

**********   *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

$$D\,field = \left[-2\,\varepsilon\, vg\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k z+\omega t),\ -2\,\varepsilon\, vg\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k z+\omega t),\ \varepsilon\sin(k z+\omega t)\left(x(x,y)^2+y(x,y)^2\right)(\omega+vg\,k)\right]$$

$$H\,field = \left[\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k z+\omega t)}{\mu},\ -\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k z+\omega t)}{\mu},\ 0\right]$$

$$Poynting\ vector\ ExH = \left[\frac{1}{\mu}\left(2\sin(k z+\omega t)\left(x(x,y)^2+y(x,y)^2\right)(\omega+vg\,k)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k z+\omega t)\right),\ \frac{1}{\mu}\left(2\sin(k z+\omega t)\left(x(x,y)^2+y(x,y)^2\right)(\omega\right.\right.$$

$$+ vg\, k) \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \cos(k z + \omega t) \Big), \frac{1}{\mu} \Big(4\, vg \cos(k z + \omega t)^2 \Big(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2\, x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2\, x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 \Big) \Big) \Big]$$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \Big[$

$$-\frac{2 \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \sin(k z + \omega t) \, (k + \omega \mu \varepsilon\, vg)}{\mu},$$

$$-\frac{2 \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \sin(k z + \omega t) \, (k + \omega \mu \varepsilon\, vg)}{\mu},$$

$$-\frac{1}{\mu} \Big(\cos(k z + \omega t) \Big(2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y) \right) + 2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 + 2\, y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y) \right) + 2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2\, x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y) \right) + 2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + 2\, y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y) \right) + \varepsilon\, \omega^2 \mu\, x(x,y)^2 + \varepsilon\, \omega\, \mu\, vg\, k\, x(x,y)^2 + \varepsilon\, \omega^2 \mu\, y(x,y)^2 + \varepsilon\, \omega\, \mu\, vg\, k\, y(x,y)^2 \Big) \Big),$$

$$-\varepsilon \cos(k z + \omega t) \Big(2\, vg \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y) \right) + 2\, vg \left(\frac{\partial}{\partial x} y(x,y) \right)^2 + 2\, vg\, y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y) \right) + 2\, vg \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y) \right) + 2 \left(\frac{\partial^2}{\partial y^2} y(x,y) \right) y(x,y)\, vg + 2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 vg - k\, \omega\, x(x,y)^2 - k^2\, vg\, x(x,y)^2 - y(x,y)^2\, \omega\, k - k^2\, vg\, y(x,y)^2 \Big) \Big]$$

Amerian charge density $divD = rho = -\varepsilon \cos(k z + \omega t) \Big(2\, vg \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y) \right) + 2\, vg \left(\frac{\partial}{\partial x} y(x,y) \right)^2 + 2\, vg\, y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y) \right) + 2\, vg \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y) \right) + 2 \left(\frac{\partial^2}{\partial y^2} y(x,y) \right) y(x,y)\, vg + 2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 vg - k\, \omega\, x(x,y)^2 - k^2\, vg\, x(x,y)^2$

$$-y(x,y)^2\,\omega\,k-k^2\,vg\,y(x,y)^2\Big)$$

divergence Lorentz Current 4Vector, *4div(J4)* $=0$

Topological SPIN 4 vector S4 $=\Big[$

$$-\frac{1}{\mu}\left(2\,\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\left(-1+vg^2\,\varepsilon\,\mu\right)\right),$$

$$-\frac{1}{\mu}\left(2\,\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\left(-1+vg^2\,\varepsilon\,\mu\right)\right),\varepsilon\sin(k\,z+\omega\,t)\,\left(x(x,y)^2+y(x,y)^2\right)^2(\omega+vg\,k)\,vg\cos(k\,z+\omega\,t),\left(x(x,y)^2+y(x,y)^2\right)^2\cos(k\,z+\omega\,t)\,\varepsilon\sin(k\,z+\omega\,t)\,(\omega+vg\,k)\Big]$$

Topological SPIN 3-form $=-\frac{1}{\mu}\left(2\,\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\left(-1+vg^2\,\varepsilon\,\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\&\hat{}(d(y),d(z),d(t))\right)+\frac{1}{\mu}\left(2\,\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\left(-1+vg^2\,\varepsilon\,\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\&\hat{}(d(x),d(z),d(t))\right)+\varepsilon\sin(k\,z+\omega\,t)\,\left(x(x,y)^2+y(x,y)^2\right)^2(\omega+vg\,k)\,vg\cos(k\,z+\omega\,t)\,\&\hat{}(d(x),d(y),d(t))-\left(x(x,y)^2+y(x,y)^2\right)^2\cos(k\,z+\omega\,t)\,\varepsilon\sin(k\,z+\omega\,t)\,(\omega+vg\,k)\,\&\hat{}(d(x),d(y),d(z))$

Spin density rho_spin $=\left(x(x,y)^2+y(x,y)^2\right)^2\cos(k\,z+\omega\,t)\,\varepsilon\sin(k\,z+\omega\,t)\,(\omega+vg\,k)$

LaGrange field energy density (B.H-D.E) $=-\frac{1}{\mu}\Big(-4\left(\frac{\partial}{\partial x}x(x,y)\right)^2x(x,y)^2\cos(k\,z+\omega\,t)^2-8\left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)x(x,y)\cos(k\,z+\omega\,t)^2-4\left(\frac{\partial}{\partial x}y(x,y)\right)^2y(x,y)^2\cos(k\,z+\omega\,t)^2-8\left(\frac{\partial}{\partial y}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)x(x,y)\cos(k\,z+\omega\,t)^2-4\,y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2\cos(k\,z+\omega\,t)^2-4\left(\frac{\partial}{\partial y}x(x,y)\right)^2x(x,y)^2\cos(k\,z+\omega\,t)^2+4\,\varepsilon\,\mu\cos(k\,z+\omega\,t)^2\,vg^2\,x(x,$

$$y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 8\,\varepsilon\,\mu\cos(k\,z+\omega\,t)^2\,vg^2\,x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)$$

$$+ 4\,\varepsilon\,\mu\cos(k\,z+\omega\,t)^2\,vg^2\,y(x,y)^2\left(\frac{\partial}{\partial x} y(x,y)\right)^2 + 4\,\varepsilon\,\mu\cos(k\,z+\omega\,t)^2\,vg^2\,x(x,y)^2\left(\frac{\partial}{\partial y} x(x,y)\right)^2$$

$$+ 8\,\varepsilon\,\mu\cos(k\,z+\omega\,t)^2\,vg^2\,x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right) + 4\,\varepsilon\,\mu\cos(k\,z$$

$$+\omega\,t)^2\,vg^2\left(\frac{\partial}{\partial y} y(x,y)\right)^2 y(x,y)^2 + \varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,x(x,y)^4\,\omega^2 + 2\,\varepsilon\,\mu\sin(k\,z$$

$$+\omega\,t)^2\,\omega\,vg\,k\,x(x,y)^4 + \varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,x(x,y)^4\,vg^2\,k^2 + 2\,\varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,x(x,y)^2\,y(x,y)^2\,\omega^2$$

$$+ 4\,\varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,\omega\,vg\,k\,x(x,y)^2\,y(x,y)^2 + 2\,\varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,x(x,y)^2\,y(x,y)^2\,vg^2\,k^2$$

$$+ \varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,y(x,y)^4\,\omega^2 + 2\,\varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,\omega\,vg\,k\,y(x,y)^4 + \varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,y(x,$$

$$y)^4\,vg^2\,k^2\Big)$$

$$B.H = \frac{1}{\mu}\Bigg(4\cos(k\,z+\omega\,t)^2\Bigg(x(x,y)^2\left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)$$

$$+ y(x,y)^2\left(\frac{\partial}{\partial y} y(x,y)\right)^2 + x(x,y)^2\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)$$

$$+ y(x,y)^2\left(\frac{\partial}{\partial x} y(x,y)\right)^2\Bigg)\Bigg)$$

$$D.E = \varepsilon\Bigg(4\,vg^2\cos(k\,z+\omega\,t)^2\left(\frac{\partial}{\partial x} x(x,y)\right)^2 x(x,y)^2 + 8\,vg^2\cos(k\,z+\omega\,t)^2\,x(x,y)\left(\frac{\partial}{\partial x} x(x,\right.$$

$$\left.y)\right) y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right) + 4\,vg^2\cos(k\,z+\omega\,t)^2\left(\frac{\partial}{\partial x} y(x,y)\right)^2 y(x,y)^2 + 4\,vg^2\cos(k\,z$$

$$+\omega\,t)^2\left(\frac{\partial}{\partial y} x(x,y)\right)^2 x(x,y)^2 + 8\,vg^2\cos(k\,z+\omega\,t)^2\,x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)$$

$$+ 4\,vg^2\cos(k\,z+\omega\,t)^2\left(\frac{\partial}{\partial y} y(x,y)\right)^2 y(x,y)^2 + \sin(k\,z+\omega\,t)^2\,x(x,y)^4\,\omega^2 + 2\sin(k\,z$$

$$+\omega\,t)^2\,\omega\,vg\,k\,x(x,y)^4 + \sin(k\,z+\omega\,t)^2\,x(x,y)^4\,vg^2\,k^2 + 2\sin(k\,z+\omega\,t)^2\,x(x,y)^2\,y(x,y)^2\,\omega^2$$

$$+ 4\sin(k\,z+\omega\,t)^2\,\omega\,vg\,k\,x(x,y)^2\,y(x,y)^2 + 2\sin(k\,z+\omega\,t)^2\,x(x,y)^2\,y(x,y)^2\,vg^2\,k^2 + \sin(k\,z$$

$$+\omega\,t)^2\,y(x,y)^4\,\omega^2 + 2\sin(k\,z+\omega\,t)^2\,\omega\,vg\,k\,y(x,y)^4 + \sin(k\,z+\omega\,t)^2\,y(x,y)^4\,vg^2\,k^2\Bigg)$$

$$A.J = -\frac{1}{\mu}\Bigg(\left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\Bigg(2\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right)$$

$$+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$

$$+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\,\omega^2\,\mu\,x(x,y)^2+\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2+\varepsilon\,\omega^2\,\mu\,y(x,y)^2$$

$$+\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\Bigg)\Bigg)$$

$$\textit{-rho.phi}=-vg\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\,\varepsilon\left(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,\right.\right.$$

$$\left.y)\right)+2\,vg\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)y(x,y)\,vg+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2vg-k\,\omega\,x(x,y)^2-k^2\,vg\,x(x,y)^2$$

$$\left.-y(x,y)^2\,\omega\,k-k^2\,vg\,y(x,y)^2\right)$$

Poincare I $\quad$ *(B.H - D.E)-(A.J - rho.phi)* $= -\frac{1}{\mu}\Bigg(-8\left(\frac{\partial}{\partial y}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)x(x,y)\cos(k\,z$

$$+\omega\,t)^2-8\left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)x(x,y)\cos(k\,z+\omega\,t)^2+\varepsilon\,\mu\sin(k\,z$$

$$+\omega\,t)^2\,x(x,y)^4\,\omega^2+\varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,y(x,y)^4\,\omega^2-2\cos(k\,z+\omega\,t)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2x(x,y)^2$$

$$-2\cos(k\,z+\omega\,t)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2y(x,y)^2-2\cos(k\,z+\omega\,t)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2x(x,y)^2$$

$$-2\cos(k\,z+\omega\,t)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2y(x,y)^2-2\cos(k\,z+\omega\,t)^2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)y(x,y)^2$$

$$-2\cos(k\,z+\omega\,t)^2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)x(x,y)^2-2\cos(k\,z+\omega\,t)^2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)y(x,y)^2$$

$$-2\cos(k\,z+\omega\,t)^2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)x(x,y)^2-4\cos(k\,z+\omega\,t)^2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2\,y(x,y)^2$$

$$-6\left(\frac{\partial}{\partial y}x(x,y)\right)^2x(x,y)^2\cos(k\,z+\omega\,t)^2-6\,y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2\cos(k\,z+\omega\,t)^2$$

$$-6\left(\frac{\partial}{\partial x}x(x,y)\right)^2x(x,y)^2\cos(k\,z+\omega\,t)^2-6\left(\frac{\partial}{\partial x}y(x,y)\right)^2y(x,y)^2\cos(k\,z+\omega\,t)^2$$

$$+8\,\varepsilon\,\mu\cos(k\,z+\omega\,t)^2\,vg^2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+8\,\varepsilon\,\mu\cos(k\,z$$

$$+\omega t)^2 vg^2 x(x, y) \left(\frac{\partial}{\partial y} x(x, y)\right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y)\right) + 4 \varepsilon \mu \sin(k z + \omega t)^2 \omega vg k x(x, y)^2 y(x, y)^2$$

$$+ 2 \varepsilon \mu \sin(k z + \omega t)^2 \omega vg k y(x, y)^4 + 2 \varepsilon \mu \sin(k z + \omega t)^2 x(x, y)^2 y(x, y)^2 vg^2 k^2 + 2 \varepsilon \mu \sin(k z$$

$$+\omega t)^2 \omega vg k x(x, y)^4 + 6 \varepsilon \mu \cos(k z + \omega t)^2 vg^2 x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y)\right)^2 + 6 \varepsilon \mu \cos(k z$$

$$+\omega t)^2 vg^2 y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y)\right)^2 + 6 \varepsilon \mu \cos(k z + \omega t)^2 vg^2 x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y)\right)^2$$

$$+ 6 \varepsilon \mu \cos(k z + \omega t)^2 vg^2 \left(\frac{\partial}{\partial y} y(x, y)\right)^2 y(x, y)^2 + \varepsilon \mu \sin(k z + \omega t)^2 x(x, y)^4 vg^2 k^2$$

$$+ 2 \varepsilon \mu \sin(k z + \omega t)^2 x(x, y)^2 y(x, y)^2 \omega^2 + \varepsilon \mu \sin(k z + \omega t)^2 y(x, y)^4 vg^2 k^2 - 2 \cos(k z$$

$$+\omega t)^2 \varepsilon \omega^2 \mu x(x, y)^2 y(x, y)^2 + 2 \cos(k z + \omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial y} x(x, y)\right)^2 y(x, y)^2 + 2 \cos(k z$$

$$+\omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial y} y(x, y)\right)^2 x(x, y)^2 - \cos(k z + \omega t)^2 vg^2 \varepsilon \mu k^2 y(x, y)^4 - 2 \cos(k z$$

$$+\omega t)^2 vg^2 \varepsilon \mu k^2 x(x, y)^2 y(x, y)^2 - 2 \cos(k z + \omega t)^2 \varepsilon \omega \mu vg k y(x, y)^4 + 2 \cos(k z$$

$$+\omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial y^2} y(x, y)\right) y(x, y)^3 + 2 \cos(k z + \omega t)^2 vg^2 \varepsilon \mu x(x, y)^3 \left(\frac{\partial^2}{\partial y^2} x(x, y)\right)$$

$$+ 2 \cos(k z + \omega t)^2 vg^2 \varepsilon \mu y(x, y)^3 \left(\frac{\partial^2}{\partial x^2} y(x, y)\right) + 2 \cos(k z + \omega t)^2 vg^2 \varepsilon \mu x(x, y)^3 \left(\frac{\partial^2}{\partial x^2} x(x, y)\right)$$

$$+ 2 \cos(k z + \omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial y^2} y(x, y)\right) y(x, y) x(x, y)^2 + 2 \cos(k z + \omega t)^2 vg^2 \varepsilon \mu x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x, y)\right) y(x, y)^2 + 2 \cos(k z + \omega t)^2 vg^2 \varepsilon \mu x(x, y) \left(\frac{\partial^2}{\partial x^2} x(x, y)\right) y(x, y)^2 + 2 \cos(k z$$

$$+\omega t)^2 vg^2 \varepsilon \mu y(x, y) \left(\frac{\partial^2}{\partial x^2} y(x, y)\right) x(x, y)^2 - 2 \cos(k z + \omega t)^2 \varepsilon \omega \mu vg k x(x, y)^4 - \cos(k z$$

$$+\omega t)^2 vg^2 \varepsilon \mu k^2 x(x, y)^4 + 2 \cos(k z + \omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial x} y(x, y)\right)^2 x(x, y)^2 + 2 \cos(k z$$

$$+\omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial x} x(x, y)\right)^2 y(x, y)^2 - \cos(k z + \omega t)^2 \varepsilon \omega^2 \mu y(x, y)^4 - \cos(k z$$

$$+\omega t)^2 \varepsilon \omega^2 \mu x(x, y)^4 - 2 \cos(k z + \omega t)^2 x(x, y)^3 \left(\frac{\partial^2}{\partial x^2} x(x, y)\right) - 2 \cos(k z + \omega t)^2 x(x,$$

$y)^3 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) - 2\cos(kz+\omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - 2\cos(kz+\omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial x^2} y(x,$
$y)\Big)\Big)$

London Coefficient $\quad LC = \dfrac{k^2 - \varepsilon\,\omega^2\,\mu}{\mu}$

PROCA coefficient curlcurlB $= \Big[-2\cos(kz+\omega t)\Big(2\left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,$
$y)\right)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2} x(x,y)\right) + 2\left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} y(x,y)\right) + \left(\frac{\partial}{\partial y} y(x,$
$y)\right)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2} y(x,y)\right) + 3\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + x(x,$
$y)\left(\frac{\partial^3}{\partial y^3} x(x,y)\right) + 3\left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y^3} y(x,y)\right) - k^2 x(x,y)\left(\frac{\partial}{\partial y} x(x,$
$y)\right) - k^2 y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\Big), 2\cos(kz+\omega t)\Big(-k^2 x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) - k^2 y(x,y)\left(\frac{\partial}{\partial x} y(x,$
$y)\right) + 3\left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial x^3} x(x,y)\right) + 3\left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right)$
$+ y(x,y)\left(\frac{\partial^3}{\partial x^3} y(x,y)\right) + 2\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right) + \left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + x(x,$
$y)\left(\frac{\partial^3}{\partial y^2\,\partial x} x(x,y)\right) + 2\left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} y(x,y)\right) + \left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + y(x,$
$y)\left(\frac{\partial^3}{\partial y^2\,\partial x} y(x,y)\right)\Big), 0\Big]$

Amperian Current 4Vector $\quad$ *curlH-dD/dt=J4* $= \Big[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\sin(kz+\omega t)\,(k+\omega\,\mu\,\varepsilon\,vg)}{\mu},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right)\sin(kz+\omega t)\,(k+\omega\,\mu\,\varepsilon\,vg)}{\mu},$$

$-\frac{1}{\mu}\Big(\cos(kz+\omega t)\Big(2\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2\left(\frac{\partial}{\partial x} y(x,y)\right)^2 + 2\,y(x,$

$$y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + 2\left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 2\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + \varepsilon\,\omega^2\,\mu\,x(x,y)^2 + \varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2 + \varepsilon\,\omega^2\,\mu\,y(x,y)^2 + \varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\Big)\Big),$$

$$-\varepsilon\cos(k\,z+\omega\,t)\left(2\,vg\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2\,vg\left(\frac{\partial}{\partial x} y(x,y)\right)^2\right.$$

$$+ 2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + 2\,vg\left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\left(\frac{\partial^2}{\partial y^2} y(x,\right.$$

$$\left.y)\right)y(x,y)\,vg + 2\left(\frac{\partial}{\partial y} y(x,y)\right)^2 vg - k\,\omega\,x(x,y)^2 - k^2\,vg\,x(x,y)^2 - y(x,y)^2\,\omega\,k - k^2\,vg\,y(x,y)^2\Big)\Big]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= \left[-\frac{2\cos(kz+\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)}{\mu}\right.$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \Big[$

$$-\frac{1}{\mu}\left(2\left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\right)($$

$$-1+vg^2\,\varepsilon\,\mu)\Big),$$

$$-\frac{1}{\mu}\Big(2\,\big(x(x,y)^2+y(x,y)^2\big)\cos(k\,z+\omega\,t)^2\Big(x(x,y)\Big(\frac{\partial}{\partial y}x(x,y)\Big)+y(x,y)\Big(\frac{\partial}{\partial y}y(x,y)\Big)\Big)\big($$

$$-1+vg^2\,\varepsilon\,\mu)\Big),\varepsilon\sin(k\,z+\omega\,t)\,\big(x(x,y)^2+y(x,y)^2\big)^2\,(\omega+vg\,k)\,vg\cos(k\,z+\omega\,t),\big(x(x,y)^2$$

$$+y(x,y)^2\big)^2\cos(k\,z+\omega\,t)\,\varepsilon\sin(k\,z+\omega\,t)\,(\omega+vg\,k)\Big]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $=\Big[0,0,-\frac{1}{\mu}\Big(\cos(k\,z$

$$+\omega\,t)\,\big(x(x,y)^2+y(x,y)^2\big)\Big(-4\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2x(x,y)^2\cos(k\,z+\omega\,t)^2-8\Big(\frac{\partial}{\partial x}y(x,y)\Big)y(x,$$

$$y)\Big(\frac{\partial}{\partial x}x(x,y)\Big)x(x,y)\cos(k\,z+\omega\,t)^2-4\Big(\frac{\partial}{\partial x}y(x,y)\Big)^2y(x,y)^2\cos(k\,z+\omega\,t)^2-8\Big(\frac{\partial}{\partial y}y(x,$$

$$y)\Big)y(x,y)\Big(\frac{\partial}{\partial y}x(x,y)\Big)x(x,y)\cos(k\,z+\omega\,t)^2-4\,y(x,y)^2\Big(\frac{\partial}{\partial y}y(x,y)\Big)^2\cos(k\,z+\omega\,t)^2$$

$$-4\Big(\frac{\partial}{\partial y}x(x,y)\Big)^2x(x,y)^2\cos(k\,z+\omega\,t)^2+4\,\varepsilon\,\mu\cos(k\,z+\omega\,t)^2\,vg^2\,x(x,y)^2\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2$$

$$+8\,\varepsilon\,\mu\cos(k\,z+\omega\,t)^2\,vg^2\,x(x,y)\Big(\frac{\partial}{\partial x}x(x,y)\Big)y(x,y)\Big(\frac{\partial}{\partial x}y(x,y)\Big)+4\,\varepsilon\,\mu\cos(k\,z$$

$$+\omega\,t)^2\,vg^2\,y(x,y)^2\Big(\frac{\partial}{\partial x}y(x,y)\Big)^2+4\,\varepsilon\,\mu\cos(k\,z+\omega\,t)^2\,vg^2\,x(x,y)^2\Big(\frac{\partial}{\partial y}x(x,y)\Big)^2$$

$$+8\,\varepsilon\,\mu\cos(k\,z+\omega\,t)^2\,vg^2\,x(x,y)\Big(\frac{\partial}{\partial y}x(x,y)\Big)y(x,y)\Big(\frac{\partial}{\partial y}y(x,y)\Big)+4\,\varepsilon\,\mu\cos(k\,z$$

$$+\omega\,t)^2\,vg^2\Big(\frac{\partial}{\partial y}y(x,y)\Big)^2y(x,y)^2+\varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,x(x,y)^4\,\omega^2+2\,\varepsilon\,\mu\sin(k\,z$$

$$+\omega\,t)^2\,\omega\,vg\,k\,x(x,y)^4+\varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,x(x,y)^4\,vg^2\,k^2+2\,\varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,x(x,y)^2\,y(x,y)^2\,\omega^2$$

$$+4\,\varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,\omega\,vg\,k\,x(x,y)^2\,y(x,y)^2+2\,\varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,x(x,y)^2\,y(x,y)^2\,vg^2\,k^2$$

$$+\varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,y(x,y)^4\,\omega^2+2\,\varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,\omega\,vg\,k\,y(x,y)^4+\varepsilon\,\mu\sin(k\,z+\omega\,t)^2\,y(x,$$

$$y)^4\,vg^2\,k^2\Big)\Big)\Big]$$

Spin Dissipation J_spin dot E $=\frac{1}{\mu}\Big(vg\cos(k\,z+\omega\,t)\,\big(x(x,y)^2+y(x,y)^2\big)\Big(-4\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2x(x,$

$$y)^2 \cos(kz+\omega t)^2 - 8 \left(\frac{\partial}{\partial x} y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) x(x,y) \cos(kz+\omega t)^2 - 4 \left(\frac{\partial}{\partial x} y(x,$$

$$y)\right)^2 y(x,y)^2 \cos(kz+\omega t)^2 - 8 \left(\frac{\partial}{\partial y} y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) x(x,y) \cos(kz+\omega t)^2$$

$$- 4^{y(x,y)^2} \left(\frac{\partial}{\partial y} y(x,y)\right)^2 \cos(kz+\omega t)^2 - 4 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 x(x,y)^2 \cos(kz+\omega t)^2$$

$$+ 4 \varepsilon \mu^{\cos(kz+\omega t)^2} vg^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 8 \varepsilon \mu \cos(kz+\omega t)^2 vg^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,$$

$$y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 4 \varepsilon \mu \cos(kz+\omega t)^2 vg^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + 4 \varepsilon \mu \cos(kz$$

$$+ \omega t)^2 vg^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 8 \varepsilon \mu \cos(kz+\omega t)^2 vg^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial y} y(x,y)\right) + 4 \varepsilon \mu \cos(kz+\omega t)^2 vg^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 y(x,y)^2 + \varepsilon \mu \sin(kz+\omega t)^2 x(x,y)^4 \omega^2$$

$$+ 2 \varepsilon \mu \sin(kz+\omega t)^2 \omega\, vg\, k\, x(x,y)^4 + \varepsilon \mu \sin(kz+\omega t)^2 x(x,y)^4 vg^2 k^2 + 2 \varepsilon \mu \sin(kz$$

$$+ \omega t)^2 x(x,y)^2 y(x,y)^2 \omega^2 + 4 \varepsilon \mu \sin(kz+\omega t)^2 \omega\, vg\, k\, x(x,y)^2 y(x,y)^2 + 2 \varepsilon \mu \sin(kz$$

$$+ \omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 + \varepsilon \mu \sin(kz+\omega t)^2 y(x,y)^4 \omega^2 + 2 \varepsilon \mu \sin(kz+\omega t)^2 \omega\, vg\, k\, y(x,y)^4$$

$$+ \varepsilon \mu \sin(kz+\omega t)^2 y(x,y)^4 vg^2 k^2))$$

$$\textit{Disspative Force 3 vector} = \left[-\frac{2\cos(kz+\omega t)^2 \left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right) (\%5)}{\mu}, -\frac{2\cos(kz+\omega t)^2 \left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial}\right.\right.}{\mu}\right.$$

$$\textit{Dissipation} = -\varepsilon \cos(kz+\omega t) \left(2\, vg \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2\, vg \left(\frac{\partial}{\partial x} y(x,\right.\right.$$

$$y)\Big)^2 + 2\, vg\, y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + 2\, vg \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$+ 2 \left(\frac{\partial^2}{\partial y^2} y(x, y)\right) y(x, y)\, vg + 2 \left(\frac{\partial}{\partial y} y(x, y)\right)^2 vg - k\, \omega\, x(x, y)^2 - k^2\, vg\, x(x, y)^2 - y(x, y)^2\, \omega\, k$$

$$- k^2\, vg\, y(x, y)^2 - \mu \sin(k\, z + \omega\, t)\, x(x, y)^4\, \omega - \mu \sin(k\, z + \omega\, t)\, x(x, y)^4\, vg\, k - 2\, \mu \sin(k\, z$$

$$+ \omega\, t)\, x(x, y)^2\, y(x, y)^2\, \omega - 2\, \mu \sin(k\, z + \omega\, t)\, x(x, y)^2\, y(x, y)^2\, vg\, k - \omega\, y(x, y)^4 \sin(k\, z + \omega\, t)\, \mu$$

$$- \mu \sin(k\, z + \omega\, t)\, y(x, y)^4\, vg\, k)$$

****************************** *END PROCEDURE* **************************** **(25)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 5d -- waveguide TM mode (group kinematic out, wave out)`;
> theta:=(k*z-omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=+vg*f(x,y)*cos(theta);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 5d -- waveguide TM mode (group kinematic out, wave out)

$$\theta := k\,z - \omega\,t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z - \omega\,t)$$

$$\phi := vg\,\left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z - \omega\,t)$$

Example 5d -- waveguide TM mode (group kinematic out, wave out)

******************************** *Differential Form Format* ****************************
****

$$Action\ 1\text{-}form = \left(-vg\cos(k\,z-\omega\,t)\,x(x,y)^2 - vg\cos(k\,z-\omega\,t)\,y(x,y)^2\right)d(t) + \left(\cos(k\,z-\omega\,t)\,x(x,y)^2 + \cos(k\,z-\omega\,t)\,y(x,y)^2\right)d(z)$$

$$Intensity\ 2\text{-}form\ F{=}dA = \left(-2\,vg\cos(k\,z-\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) - 2\,vg\cos(k\,z-\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)\right)(d(x))\,\&\hat{}\,(d(t)) + \left(2\cos(k\,z-\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) + 2\cos(k\,z-\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)\right)(d(x))\,\&\hat{}\,(d(z)) + \left(-2\,vg\cos(k\,z-\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) - 2\,vg\cos(k\,z-\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)\right)(d(y))\,\&\hat{}\,(d(t)) + \left(2\cos(k\,z-\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) + 2\cos(k\,z-\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)\right)(d(y))\,\&\hat{}\,(d(z)) + \left(-\sin(k\,z-\omega\,t)\,\omega\,x(x,y)^2 - \sin(k\,z-\omega\,t)\,\omega\,y(x,y)^2 + vg\sin(k\,z-\omega\,t)\,k\,x(x,y)^2 + vg\sin(k\,z-\omega\,t)\,k\,y(x,y)^2\right)(d(z))\,\&\hat{}\,(d(t))$$

Topological Torsion 3-form $A\hat{}F = 0$

Topological Parity 4-form $F\hat{}F = 0$

********************************* *Using EM format* *********************************

$$E\,field = \left[-2\,vg\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z-\omega\,t),\ -2\,vg\left(x(x,\right.\right.$$

$$y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Big)\cos(kz-\omega t),\sin(kz-\omega t)\left(x(x,y)^2+y(x,y)^2\right)(-\omega+vg\,k)\Big]$$

$$B\,field=\Big[2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz-\omega t),-2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz-\omega t),0\Big]$$

Topological TORSION 4 vector   T4 = -[ExA + Bphi,AdotB] $=[0,0,0,0]$

Helicity AdotB $=0$

Poincare II =2(E.B) $=0$

coefficient of Topological Parity 4-form $=0$

Pfaff Topological Dimension   PTD $=2$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************************

Xm or linear (Mean) curvature $=-\sin(kz-\omega t)\left(x(x,y)^2+y(x,y)^2\right)(k+\omega\,vg)$

Yg or quadratic (GAUSS) curvature $=0$

Za or Cubic (Interaction internal energy) curvature $=0$

Tk or quartic (4D expansion) curvature $=0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $=0$

$$D\,field=\Big[-2\,\varepsilon\,vg\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz-\omega t),-2\,\varepsilon\,vg\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz-\omega t),\varepsilon\sin(kz-\omega t)\left(x(x,y)^2+y(x,y)^2\right)(-\omega+vg\,k)\Big]$$

$$H\,field=\left[\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz-\omega t)}{\mu},-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz-\omega t)}{\mu},0\right]$$

$$Poynting\ vector\ ExH=\Big[\frac{1}{\mu}\Big(2\sin(kz-\omega t)\left(x(x,y)^2+y(x,y)^2\right)(-\omega+vg\,k)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)\right.$$

$$+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)\cos(kz-\omega t)\Big),\ \frac{1}{\mu}\Big(2\sin(kz-\omega t)\,(x(x,y)^2+y(x,y)^2)\,(-\omega$$

$$+vg\,k)\Big(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Big)\cos(kz-\omega t)\Big),\ \frac{1}{\mu}\Big(4\,vg\cos(kz$$

$$-\omega t)^2\Big(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,$$

$$y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)$$

$$+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Big)\Big)\Big]$$

Amperian Current 4Vector *curlH-dD/dt=J4* $=\Big[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz-\omega t)\,(k-\omega\,\mu\,\varepsilon\,vg)}{\mu},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\sin(kz-\omega t)\,(k-\omega\,\mu\,\varepsilon\,vg)}{\mu},$$

$$-\frac{1}{\mu}\Big(\cos(kz-\omega t)\Big(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\,\omega^2\mu\,x(x,y)^2+\varepsilon\,\omega^2\mu\,y(x,y)^2-\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2-\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\Big)\Big),$$

$$-\cos(kz-\omega t)\,\varepsilon\Big(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial x}y(x,y)\right)^2$$

$$+2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,$$

$$y)\right)^2vg+2\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)y(x,y)\,vg+k\,\omega\,x(x,y)^2+y(x,y)^2\,\omega\,k-k^2\,vg\,x(x,y)^2-k^2\,vg\,y(x,y)^2\Big)\Big]$$

Amerian charge density *divD = rho* $=-\cos(kz-\omega t)\,\varepsilon\Big(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,$

$y)\Big) + 2\,vg\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 + 2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right) + 2\,vg\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 + 2\,vg\,x(x,$

$y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right) + 2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 vg + 2\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right) y(x,y)\,vg + k\,\omega\,x(x,y)^2 + y(x,y)^2\,\omega\,k$

$-k^2\,vg\,x(x,y)^2 - k^2\,vg\,y(x,y)^2\Big)$

divergence Lorentz Current 4Vector, *4div(J4)* $= 0$

Topological SPIN 4 vector S4 $= \Big[$

$-\frac{1}{\mu}\Big(2\,\big(x(x,y)^2+y(x,y)^2\big)\cos(k\,z-\omega\,t)^2\,\big(-1+vg^2\,\varepsilon\,\mu\big)\Big(x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)+y(x,$

$y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)\Big)\Big),$

$-\frac{1}{\mu}\Big(2\,\big(x(x,y)^2+y(x,y)^2\big)\cos(k\,z-\omega\,t)^2\,\big(-1+vg^2\,\varepsilon\,\mu\big)\Big(x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)+y(x,$

$y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)\Big)\Big),\ \varepsilon\sin(k\,z-\omega\,t)\,\big(x(x,y)^2+y(x,y)^2\big)^2\,(-\omega+vg\,k)\,vg\cos(k\,z-\omega\,t),$

$\big(x(x,y)^2+y(x,y)^2\big)^2\cos(k\,z-\omega\,t)\,\varepsilon\sin(k\,z-\omega\,t)\,(-\omega+vg\,k)\Big]$

Topological SPIN 3-form $= -\frac{1}{\mu}\Big(2\,\big(x(x,y)^2+y(x,y)^2\big)\cos(k\,z-\omega\,t)^2\,\big(-1+vg^2\,\varepsilon\,\mu\big)\Big(x(x,$

$y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)\Big)\ \&\hat{}(d(y),d(z),d(t))\Big)+\frac{1}{\mu}\Big(2\,\big(x(x,y)^2+y(x,$

$y)^2\big)\cos(k\,z-\omega\,t)^2\,\big(-1+vg^2\,\varepsilon\,\mu\big)\Big(x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)\Big)\ \&\hat{}(d(x),d(z),$

$d(t))\Big)+\varepsilon\sin(k\,z-\omega\,t)\,\big(x(x,y)^2+y(x,y)^2\big)^2\,(-\omega+vg\,k)\,vg\cos(k\,z-\omega\,t)\ \&\hat{}(d(x),d(y),d(t))$

$-\big(x(x,y)^2+y(x,y)^2\big)^2\cos(k\,z-\omega\,t)\,\varepsilon\sin(k\,z-\omega\,t)\,(-\omega+vg\,k)\ \&\hat{}(d(x),d(y),d(z))$

Spin density rho_spin $= \big(x(x,y)^2+y(x,y)^2\big)^2\cos(k\,z-\omega\,t)\,\varepsilon\sin(k\,z-\omega\,t)\,(-\omega+vg\,k)$

LaGrange field energy density (B.H-D.E) $= -\frac{1}{\mu}\Big(-4\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 x(x,y)^2\cos(k\,z-\omega\,t)^2-8\,\Big(\frac{\partial}{\partial x}\,y(x,$

$y)\Big)\,y(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)x(x,y)\cos(k\,z-\omega\,t)^2-4\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 y(x,y)^2\cos(k\,z-\omega\,t)^2$

$$-8\left(\frac{\partial}{\partial y}\, y(x,y)\right) y(x,y)\left(\frac{\partial}{\partial y}\, x(x,y)\right) x(x,y)\cos(k z-\omega t)^2-4\, y(x,y)^2\left(\frac{\partial}{\partial y}\, y(x,y)\right)^2\cos(k z-\omega t)^2-4\left(\frac{\partial}{\partial y}\, x(x,y)\right)^2 x(x,y)^2\cos(k z-\omega t)^2+4\,\varepsilon\,\mu\cos(k z-\omega t)^2\, vg^2\, x(x,y)^2\left(\frac{\partial}{\partial x}\, x(x,y)\right)^2+8\,\varepsilon\,\mu\cos(k z-\omega t)^2\, vg^2\, x(x,y)\left(\frac{\partial}{\partial x}\, x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial x}\, y(x,y)\right)$$

$$+4\,\varepsilon\,\mu\cos(k z-\omega t)^2\, vg^2\, y(x,y)^2\left(\frac{\partial}{\partial x}\, y(x,y)\right)^2+4\,\varepsilon\,\mu\cos(k z-\omega t)^2\, vg^2\, x(x,y)^2\left(\frac{\partial}{\partial y}\, x(x,y)\right)^2+8\,\varepsilon\,\mu\cos(k z-\omega t)^2\, vg^2\, x(x,y)\left(\frac{\partial}{\partial y}\, x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial y}\, y(x,y)\right)+4\,\varepsilon\,\mu\cos(k z-\omega t)^2\, vg^2\left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 y(x,y)^2+\varepsilon\,\mu\sin(k z-\omega t)^2\, x(x,y)^4\,\omega^2-2\,\varepsilon\,\mu\sin(k z-\omega t)^2\,\omega\, vg\, k\, x(x,y)^4+\varepsilon\,\mu\sin(k z-\omega t)^2\, x(x,y)^4\, vg^2\, k^2+2\,\varepsilon\,\mu\sin(k z-\omega t)^2\, x(x,y)^2\, y(x,y)^2\,\omega^2$$

$$-4\,\varepsilon\,\mu\sin(k z-\omega t)^2\,\omega\, vg\, k\, x(x,y)^2\, y(x,y)^2+2\,\varepsilon\,\mu\sin(k z-\omega t)^2\, x(x,y)^2\, y(x,y)^2\, vg^2\, k^2+\varepsilon\,\mu\sin(k z-\omega t)^2\, y(x,y)^4\,\omega^2-2\,\varepsilon\,\mu\sin(k z-\omega t)^2\,\omega\, vg\, k\, y(x,y)^4+\varepsilon\,\mu\sin(k z-\omega t)^2\, y(x,y)^4\, vg^2\, k^2)$$

$$B.H=\frac{1}{\mu}\left(4\cos(k z-\omega t)^2\left(x(x,y)^2\left(\frac{\partial}{\partial y}\, x(x,y)\right)^2+2\, x(x,y)\left(\frac{\partial}{\partial y}\, x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial y}\, y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}\, y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}\, x(x,y)\right)^2+2\, x(x,y)\left(\frac{\partial}{\partial x}\, x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial x}\, y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}\, y(x,y)\right)^2\right)\right)$$

$$D.E=\varepsilon\left(4\cos(k z-\omega t)^2\, vg^2\, x(x,y)^2\left(\frac{\partial}{\partial x}\, x(x,y)\right)^2+8\cos(k z-\omega t)^2\, vg^2\, x(x,y)\left(\frac{\partial}{\partial x}\, x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial x}\, y(x,y)\right)+4\cos(k z-\omega t)^2\, vg^2\, y(x,y)^2\left(\frac{\partial}{\partial x}\, y(x,y)\right)^2+4\cos(k z-\omega t)^2\, vg^2\, x(x,y)^2\left(\frac{\partial}{\partial y}\, x(x,y)\right)^2+8\cos(k z-\omega t)^2\, vg^2\, x(x,y)\left(\frac{\partial}{\partial y}\, x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial y}\, y(x,y)\right)\right.$$

$$+4\cos(k z-\omega t)^2\, vg^2\left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 y(x,y)^2+\sin(k z-\omega t)^2\, x(x,y)^4\,\omega^2-2\sin(k z-\omega t)^2\,\omega\, vg\, k\, x(x,y)^4+\sin(k z-\omega t)^2\, x(x,y)^4\, vg^2\, k^2+2\sin(k z-\omega t)^2\, x(x,y)^2\, y(x,y)^2\,\omega^2-4\sin(k z-\omega t)^2\,\omega\, vg\, k\, x(x,y)^2\, y(x,y)^2+2\sin(k z-\omega t)^2\, x(x,y)^2\, y(x,y)^2\, vg^2\, k^2+\sin(k z$$

$$- \omega t)^2 y(x,y)^4 \omega^2 - 2 \sin(k z - \omega t)^2 \omega\, vg\, k\, y(x,y)^4 + \sin(k z - \omega t)^2 y(x,y)^4 vg^2 k^2)$$

$$A.J = -\frac{1}{\mu}\Bigg(\left(x(x,y)^2 + y(x,y)^2\right) \cos(k z - \omega t)^2 \Bigg(2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right)$$

$$+ 2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + 2\, y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + 2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\, x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$+ 2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 2\, y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + \varepsilon \omega^2 \mu\, x(x,y)^2 + \varepsilon \omega^2 \mu\, y(x,y)^2 - \varepsilon \omega \mu\, vg\, k\, x(x,y)^2$$

$$- \varepsilon \omega \mu\, vg\, k\, y(x,y)^2 \Bigg)\Bigg)$$

$$\text{-rho.phi} = -vg \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z - \omega t)^2 \varepsilon \Bigg(2\, vg \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,$$

$$y)\right) + 2\, vg \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + 2\, vg\, y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + 2\, vg \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\, vg\, x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 vg + 2 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) y(x,y)\, vg + k \omega\, x(x,y)^2 + y(x,y)^2 \omega k$$

$$- k^2 vg\, x(x,y)^2 - k^2 vg\, y(x,y)^2 \Bigg)$$

Poincare I $\quad$ *(B.H - D.E)-(A.J - rho.phi)* $$= -\frac{1}{\mu}\Big(2 \cos(k z - \omega t)^2 \varepsilon \omega \mu\, vg\, k\, x(x,y)^4 + 2 \cos(k z$$

$$- \omega t)^2 \varepsilon \omega \mu\, vg\, k\, y(x,y)^4 - 2 \cos(k z - \omega t)^2 vg^2 \varepsilon \mu k^2 x(x,y)^2 y(x,y)^2 - \cos(k z$$

$$- \omega t)^2 vg^2 \varepsilon \mu k^2 y(x,y)^4 - \cos(k z - \omega t)^2 vg^2 \varepsilon \mu k^2 x(x,y)^4 + 2 \cos(k z$$

$$- \omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial x} y(x,y)\right)^2 x(x,y)^2 + 2 \cos(k z - \omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial x} x(x,y)\right)^2 y(x,y)^2$$

$$- 2 \cos(k z - \omega t)^2 \varepsilon \omega^2 \mu\, x(x,y)^2 y(x,y)^2 + 2 \cos(k z - \omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial y} x(x,y)\right)^2 y(x,y)^2$$

$$+ 2 \cos(k z - \omega t)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial y} y(x,y)\right)^2 x(x,y)^2 - 2 \cos(k z - \omega t)^2 x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) y(x,y)^2$$

$$- 2 \cos(k z - \omega t)^2 y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) x(x,y)^2 - 2 \cos(k z - \omega t)^2 x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) y(x,y)^2$$

$$- 2 \cos(k z - \omega t)^2 y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) x(x,y)^2 + 2 \cos(k z - \omega t)^2 vg^2 \varepsilon \mu\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,$$

$$y)\Big)\,y(x,y)^2+2\cos(k\,z-\omega\,t)^2\,vg^2\,\varepsilon\,\mu\,y(x,y)\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)x(x,y)^2+2\cos(k\,z$$

$$-\omega\,t)^2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)y(x,y)\,x(x,y)^2+2\cos(k\,z-\omega\,t)^2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,\right.$$

$$y)\Big)\,y(x,y)^2-\cos(k\,z-\omega\,t)^2\,\varepsilon\,\omega^2\,\mu\,y(x,y)^4+6\,\varepsilon\,\mu\cos(k\,z-\omega\,t)^2\,vg^2\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2$$

$$+6\,\varepsilon\,\mu\cos(k\,z-\omega\,t)^2\,vg^2\,y(x,y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2+6\,\varepsilon\,\mu\cos(k\,z-\omega\,t)^2\,vg^2\,x(x,y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2$$

$$+6\,\varepsilon\,\mu\cos(k\,z-\omega\,t)^2\,vg^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2y(x,y)^2+\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,x(x,y)^4\,vg^2\,k^2$$

$$+8\,\varepsilon\,\mu\cos(k\,z-\omega\,t)^2\,vg^2\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)+8\,\varepsilon\,\mu\cos(k\,z$$

$$-\omega\,t)^2\,vg^2\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)-2\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,\omega\,vg\,k\,x(x,y)^4$$

$$+2\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,x(x,y)^2\,y(x,y)^2\,vg^2\,k^2-2\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,\omega\,vg\,k\,y(x,y)^4-2\cos(k\,z$$

$$-\omega\,t)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2x(x,y)^2-2\cos(k\,z-\omega\,t)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2y(x,y)^2-2\cos(k\,z$$

$$-\omega\,t)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2x(x,y)^2-2\cos(k\,z-\omega\,t)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2y(x,y)^2-\cos(k\,z$$

$$-\omega\,t)^2\,\varepsilon\,\omega^2\,\mu\,x(x,y)^4+4\cos(k\,z-\omega\,t)^2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2\,y(x,y)^2-8\left(\frac{\partial}{\partial x}\,y(x,y)\right)y(x,$$

$$y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)x(x,y)\cos(k\,z-\omega\,t)^2-8\left(\frac{\partial}{\partial y}\,y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)x(x,y)\cos(k\,z$$

$$-\omega\,t)^2+\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,x(x,y)^4\,\omega^2-6\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2x(x,y)^2\cos(k\,z-\omega\,t)^2$$

$$-6\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2y(x,y)^2\cos(k\,z-\omega\,t)^2-6\,y(x,y)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2\cos(k\,z-\omega\,t)^2$$

$$-6\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2x(x,y)^2\cos(k\,z-\omega\,t)^2+2\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,x(x,y)^2\,y(x,y)^2\,\omega^2$$

$$+\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,y(x,y)^4\,vg^2\,k^2-4\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,\omega\,vg\,k\,x(x,y)^2\,y(x,y)^2-2\cos(k\,z$$

$$-\omega\,t)^2\,x(x,y)^3\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)-2\cos(k\,z-\omega\,t)^2\,x(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)-2\cos(k\,z$$

$$-\omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - 2\cos(kz - \omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + \varepsilon\mu\sin(kz$$

$$-\omega t)^2 y(x,y)^4 \omega^2 + 2\cos(kz-\omega t)^2 vg^2 \varepsilon\mu\, y(x,y)^3 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + 2\cos(kz$$

$$-\omega t)^2 vg^2 \varepsilon\mu\, x(x,y)^3 \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2\cos(kz-\omega t)^2 vg^2 \varepsilon\mu \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) y(x,y)^3$$

$$+ 2\cos(kz-\omega t)^2 vg^2 \varepsilon\mu\, x(x,y)^3 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right)\Bigg)$$

London Coefficient $\quad LC = \dfrac{k^2 - \varepsilon\omega^2\mu}{\mu}$

PROCA coefficient curlcurlB $= \Bigg[-2\cos(kz-\omega t)\Bigg(2\left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,$

$$y)\right)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2} x(x,y)\right) + 2\left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} y(x,y)\right) + \left(\frac{\partial}{\partial y} y(x,$$

$$y)\right)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2} y(x,y)\right) + 3\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + x(x,$$

$$y)\left(\frac{\partial^3}{\partial y^3} x(x,y)\right) + 3\left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y^3} y(x,y)\right) - k^2 x(x,y)\left(\frac{\partial}{\partial y} x(x,$$

$$y)\right) - k^2 y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\Bigg), 2\cos(kz-\omega t)\Bigg(-k^2 x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) - k^2 y(x,y)\left(\frac{\partial}{\partial x} y(x,$$

$$y)\right) + 3\left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial x^3} x(x,y)\right) + 3\left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right)$$

$$+ y(x,y)\left(\frac{\partial^3}{\partial x^3} y(x,y)\right) + 2\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right) + \left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + x(x,$$

$$y)\left(\frac{\partial^3}{\partial y^2\,\partial x} x(x,y)\right) + 2\left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} y(x,y)\right) + \left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + y(x,$$

$$y)\left(\frac{\partial^3}{\partial y^2\,\partial x} y(x,y)\right)\Bigg), 0\Bigg]$$

Amperian Current 4Vector $\quad$ *curlH-dD/dt=J4* $\quad = \Bigg[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\sin(kz-\omega t)\,(k-\omega\mu\varepsilon\, vg)}{\mu},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\sin(kz-\omega t)\,(k-\omega\mu\varepsilon\,vg)}{\mu},$$

$$-\frac{1}{\mu}\Bigg(\cos(kz-\omega t)\Bigg(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\,\omega^2\mu\,x(x,y)^2+\varepsilon\,\omega^2\mu\,y(x,y)^2-\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2-\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\Bigg)\Bigg),$$

$$-\cos(kz-\omega t)\,\varepsilon\Bigg(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2vg+2\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)y(x,y)\,vg+k\,\omega\,x(x,y)^2+y(x,y)^2\,\omega\,k-k^2\,vg\,x(x,y)^2-k^2\,vg\,y(x,y)^2\Bigg)\Bigg]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $=\Bigg[\frac{2\cos(kz-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)(}{\mu}$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $=[0,0,0,0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $=[0,0,0]$

Torsion Dissipation Jtorsion dot E $=0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] =

$$\left[-\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(-1+vg^2\,\varepsilon\,\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\right),$$

$$-\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(-1+vg^2\,\varepsilon\,\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\right),\ \varepsilon\sin(kz-\omega t)\left(x(x,y)^2+y(x,y)^2\right)^2(-\omega+vg\,k)\,vg\cos(kz-\omega t),$$

$$\left(x(x,y)^2+y(x,y)^2\right)^2\cos(kz-\omega t)\,\varepsilon\sin(kz-\omega t)\,(-\omega+vg\,k)\Big]$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) =

$$\Big[0,\ 0,\ -\frac{1}{\mu}\Big(\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)\Big(-4\left(\frac{\partial}{\partial x}x(x,y)\right)^2x(x,y)^2\cos(kz-\omega t)^2-8\left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)x(x,y)\cos(kz-\omega t)^2-4\left(\frac{\partial}{\partial x}y(x,y)\right)^2y(x,y)^2\cos(kz-\omega t)^2-8\left(\frac{\partial}{\partial y}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)x(x,y)\cos(kz-\omega t)^2-4\,y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2\cos(kz-\omega t)^2$$

$$-4\left(\frac{\partial}{\partial y}x(x,y)\right)^2x(x,y)^2\cos(kz-\omega t)^2+4\,\varepsilon\,\mu\cos(kz-\omega t)^2vg^2x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2$$

$$+8\,\varepsilon\,\mu\cos(kz-\omega t)^2vg^2x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+4\,\varepsilon\,\mu\cos(kz-\omega t)^2vg^2y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+4\,\varepsilon\,\mu\cos(kz-\omega t)^2vg^2x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$+8\,\varepsilon\,\mu\cos(kz-\omega t)^2vg^2x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+4\,\varepsilon\,\mu\cos(kz-\omega t)^2vg^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2y(x,y)^2+\varepsilon\,\mu\sin(kz-\omega t)^2x(x,y)^4\omega^2-2\,\varepsilon\,\mu\sin(kz-\omega t)^2\omega\,vg\,k\,x(x,y)^4+\varepsilon\,\mu\sin(kz-\omega t)^2x(x,y)^4vg^2k^2+2\,\varepsilon\,\mu\sin(kz-\omega t)^2x(x,y)^2y(x,y)^2\omega^2$$

$$-4\,\varepsilon\,\mu\sin(kz-\omega t)^2\omega\,vg\,k\,x(x,y)^2y(x,y)^2+2\,\varepsilon\,\mu\sin(kz-\omega t)^2x(x,y)^2y(x,y)^2vg^2k^2$$

$+\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,y(x,y)^4\,\omega^2-2\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,\omega\,vg\,k\,y(x,y)^4+\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,y(x,y)^4\,vg^2\,k^2)\,)\,]$

Spin Dissipation J_spin dot E $=\frac{1}{\mu}\Bigg(\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)\;vg\,\Bigg(-4\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 x(x,y)^2\cos(k\,z-\omega\,t)^2-8\left(\frac{\partial}{\partial x}\,y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)x(x,y)\cos(k\,z-\omega\,t)^2-4\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 y(x,y)^2\cos(k\,z-\omega\,t)^2-8\left(\frac{\partial}{\partial y}\,y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)x(x,y)\cos(k\,z-\omega\,t)^2$
$-4\,y(x,y)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2\cos(k\,z-\omega\,t)^2-4\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 x(x,y)^2\cos(k\,z-\omega\,t)^2$
$+4\,\varepsilon\,\mu\cos(k\,z-\omega\,t)^2\,vg^2\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2+8\,\varepsilon\,\mu\cos(k\,z-\omega\,t)^2\,vg^2\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)+4\,\varepsilon\,\mu\cos(k\,z-\omega\,t)^2\,vg^2\,y(x,y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2+4\,\varepsilon\,\mu\cos(k\,z-\omega\,t)^2\,vg^2\,x(x,y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2+8\,\varepsilon\,\mu\cos(k\,z-\omega\,t)^2\,vg^2\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)+4\,\varepsilon\,\mu\cos(k\,z-\omega\,t)^2\,vg^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 y(x,y)^2+\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,x(x,y)^4\,\omega^2$
$-2\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,\omega\,vg\,k\,x(x,y)^4+\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,x(x,y)^4\,vg^2\,k^2+2\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,x(x,y)^2\,y(x,y)^2\,\omega^2-4\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,\omega\,vg\,k\,x(x,y)^2\,y(x,y)^2+2\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,x(x,y)^2\,y(x,y)^2\,vg^2\,k^2+\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,y(x,y)^4\,\omega^2-2\,\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,\omega\,vg\,k\,y(x,y)^4$
$+\varepsilon\,\mu\sin(k\,z-\omega\,t)^2\,y(x,y)^4\,vg^2\,k^2\Bigg)\Bigg)$

$$Disspative\ Force\ 3\ vector = \left[\frac{2\cos(kz-\omega t)^2 \left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)(\%5)}{\mu}, \frac{2\cos(kz-\omega t)^2 \left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y\right.\right.}{\mu}\right.$$

$$Dissipation = \cos(kz-\omega t)\,\varepsilon\left(-2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2 - 2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right) - 2\,vg\left(\frac{\partial}{\partial x}y(x,y)\right)^2 - 2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right) - 2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2 - 2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right) - 2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 vg - 2\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)y(x,y)\,vg - k\,\omega\,x(x,y)^2 - y(x,y)^2\,\omega\,k + k^2\,vg\,x(x,y)^2 + k^2\,vg\,y(x,y)^2 - \mu\sin(kz-\omega t)\,x(x,y)^4\,\omega + \mu\sin(kz-\omega t)\,x(x,y)^4\,vg\,k - 2\,\mu\sin(kz-\omega t)\,x(x,y)^2\,y(x,y)^2\,\omega + 2\,\mu\sin(kz-\omega t)\,x(x,y)^2\,y(x,y)^2\,vg\,k - \omega\,y(x,y)^4\sin(kz-\omega t)\,\mu + \mu\sin(kz-\omega t)\,y(x,y)^4\,vg\,k\right)$$

***************************** *END PROCEDURE* ***************************** **(26)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 6a-- Wave guide TTM (kinematic in, wave in)`;
> theta:=(k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=-(omega/k)*f(x,y)*cos(theta);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 6a-- Wave guide TTM (kinematic in, wave in)

$$\theta := k\,z + \omega\,t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right)\cos(kz + \omega t)$$

$$\phi := -\frac{\omega\left(x(x,y)^2 + y(x,y)^2\right)\cos(kz + \omega t)}{k}$$

Example 6a-- Wave guide TTM (kinematic in, wave in)

*********************************** Differential Form Format *************************************

$$\text{Action 1-form} = \left(x(x,y)^2 + y(x,y)^2\right)\cos(kz + \omega t)\, d(z) + \frac{\omega\left(x(x,y)^2 + y(x,y)^2\right)\cos(kz + \omega t)\, d(t)}{k}$$

$$\text{Intensity 2-form } F{=}dA \;\; = 2\,(\%1)\cos(kz + \omega t)\,(d(x))\,\&\hat{}\,(d(z)) + 2\,(\%2)\cos(kz + \omega t)\,(d(y))\,\&\hat{}\,(d(z)) + \frac{2\,\omega\,(\%1)\cos(kz + \omega t)\,(d(x))\,\&\hat{}\,(d(t))}{k} +$$

$$\%1 = x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)$$

$$\%2 = x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)$$

Topological Torsion 3-form $A\hat{}F = 0$

Topological Parity 4-form $F\hat{}F = 0$

*********************************** Using EM format ***********************************

$$E\,field = \left[\frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz + \omega t)}{k}, \frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz + \omega t)}{k}, 0\right]$$

$$B\,field = \left[2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz + \omega t), -2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz + \omega t), 0\right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension PTD $= 2$

********************* Correlation Similarity Invariants of Jacobian of (Ak/lambda_N) *****************************

$$\text{Xm or linear (Mean) curvature} = -\frac{\left(x(x,y)^2 + y(x,y)^2\right)\sin(kz + \omega t)\left(k^2 + \omega^2\right)}{k}$$

Yg or quadratic (GAUSS) curvature $= 0$

$$\textit{Za or Cubic (Interaction internal energy) curvature} \ = 0$$

$$\textit{Tk or quartic (4D expansion) curvature} \ = 0$$

*********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\textit{Chirality factor } CH = 0$$

$$D\,field = \left[\frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz+\omega t)}{k},\right.$$
$$\left.\frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz+\omega t)}{k}, 0\right]$$

$$H\,field = \left[\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz+\omega t)}{\mu},\right.$$
$$\left.-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz+\omega t)}{\mu}, 0\right]$$

$$\textit{Poynting vector ExH} = \left[0, 0, -\frac{1}{\mu k}\left(4\,\omega\cos(kz+\omega t)^2\left(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,\right.\right.\right.$$
$$y)\Big)\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,\right.$$
$$\left.\left.\left.y)\Big)\,y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\right)\right)\right]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} \ = \left[-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz+\omega t)\left(k^2-\varepsilon\,\omega^2\mu\right)}{\mu k}, -\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+\right.}{}\right.$$

$$\%1 = \left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)$$

$$\textit{Amerian charge density} \quad divD = rho = \frac{1}{k}\left(2\,\varepsilon\cos(kz+\omega t)\,\omega\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,\right.\right.\right.$$
$$y)\Big)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$
$$\left.\left.+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\right)\right)$$

$$\textit{divergence Lorentz Current 4Vector,} \quad \textit{4div(J4)} \ = 0$$

Topological SPIN 4 vector S4

$$= \left[\frac{1}{\mu k^2} \left(2 \left(x(x,y)^2 + y(x,y)^2 \right) \cos(k z + \omega t)^2 \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x, y) \right) \right) \left(k^2 - \varepsilon \omega^2 \mu \right) \right), \frac{1}{\mu k^2} \left(2 \left(x(x,y)^2 + y(x,y)^2 \right) \cos(k z + \omega t)^2 \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x, y) \right) \right) \left(k^2 - \varepsilon \omega^2 \mu \right) \right), 0, 0 \right]$$

$$\textit{Topological SPIN 3-form} = \frac{1}{\mu k^2} \left(2 \left(x(x,y)^2 + y(x,y)^2 \right) \cos(k z + \omega t)^2 \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x, y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \left(k^2 - \varepsilon \omega^2 \mu \right) \&\text{^}(d(y), d(z), d(t)) \right) - \frac{1}{\mu k^2} \left(2 \left(x(x,y)^2 + y(x, y)^2 \right) \cos(k z + \omega t)^2 \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \left(k^2 - \varepsilon \omega^2 \mu \right) \&\text{^}(d(x), d(z), d(t)) \right)$$

$$\textit{Spin density rho\_spin} = 0$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{1}{\mu k^2} \left(4 \cos(k z + \omega t)^2 \left(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2 x(x, y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2 x(x, y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 \right) \left(k^2 - \varepsilon \omega^2 \mu \right) \right)$$

$$B.H = \frac{1}{\mu} \left(4 \cos(k z + \omega t)^2 \left(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 \right) \right)$$

$$D.E = \frac{1}{k^2} \left(4 \varepsilon \omega^2 \cos(k z + \omega t)^2 \left(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} y(x, y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x, y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 \right) \right)$$

$$A.J = -\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)\right.\right.$$
$$+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$
$$\left.\left.+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\right)\right)$$

$$-rho.phi = -\frac{1}{k^2}\left(2\,\omega^2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\,\varepsilon\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)\right.\right.$$
$$+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$
$$\left.\left.+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\right)\right)$$

Poincare I $\quad$ *(B.H - D.E)-(A.J - rho.phi)* $= \frac{1}{\mu k^2}\left(2\cos(kz+\omega t)^2\left(k^2-\varepsilon\,\omega^2\mu\right)\left(3\,x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+4\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+3\,y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+3\,x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+4\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+3\,y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2\right.\right.$

$$+x(x,y)^3\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2x(x,y)^2+\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,x(x,y)^2+x(x,y)^3\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2x(x,y)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)x(x,y)^2+\left(\frac{\partial}{\partial x}x(x,y)\right)^2y(x,y)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)y(x,y)^2+\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)^3+\left(\frac{\partial}{\partial y}x(x,y)\right)^2y(x,y)^2$$
$$\left.\left.+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)y(x,y)^2+y(x,y)^3\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\right)\right)$$

London Coefficient $\quad LC = \frac{k^2-\varepsilon\,\omega^2\mu}{\mu}$

PROCA coefficient curlcurlB $= \left[-2\cos(kz+\omega t)\left(2\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}x(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+x(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2}x(x,y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}y(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+y(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2}y(x,y)\right)+3\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+x(x,\right.\right.$

$$y)\left(\frac{\partial^3}{\partial y^3}x(x,y)\right)+3\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+y(x,y)\left(\frac{\partial^3}{\partial y^3}y(x,y)\right)-k^2x(x,y)\left(\frac{\partial}{\partial y}x(x,$$

$$y)\right)-k^2y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Bigg), 2\cos(kz+\omega t)\left(-k^2x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)-k^2y(x,y)\left(\frac{\partial}{\partial x}y(x,\right.\right.$$

$$y)\right)+3\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+x(x,y)\left(\frac{\partial^3}{\partial x^3}x(x,y)\right)+3\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)$$

$$+y(x,y)\left(\frac{\partial^3}{\partial x^3}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}x(x,y)\right)+\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+x(x,$$

$$y)\left(\frac{\partial^3}{\partial y^2\,\partial x}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}y(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+y(x,$$

$$y)\left(\frac{\partial^3}{\partial y^2\,\partial x}y(x,y)\right)\Bigg), 0\Bigg]$$

Amperian Current 4Vector curlH-dD/dt=J4 $= \left[-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz+\omega t)\left(k^2-\varepsilon\omega^2\mu\right)}{\mu k}, -\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+\right.}{}\right.$

$$\%1=\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= \left[\frac{4\cos(kz+\omega t)^2(\%1)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,\right.\right.}{\mu k^2}\right.$

$$\%1=\left(\frac{\partial}{\partial x}x(x,y)\right)$$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $=[0,0,0,0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $=[0,0,0]$

Torsion Dissipation Jtorsion dot E $=0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD]

$$=\left[\frac{1}{\mu k^2}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,\right.\right.\right.\right.$$

$$y)\right)\Bigg)\left(k^2-\varepsilon\omega^2\mu\right)\Bigg),$$

$$\frac{1}{\mu k^2}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\left(k^2-\varepsilon\omega^2\mu\right)\right),0,0\Bigg]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B)

$$=\Bigg[0,0,\frac{1}{\mu k^2}\Bigg(4\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^3\left(k^2-\varepsilon\omega^2\mu\right)\Bigg(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Bigg)\Bigg)\Bigg]$$

Spin Dissipation J_spin dot E

$$=\frac{1}{\mu k^3}\Bigg(4\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^3\left(k^2-\varepsilon\omega^2\mu\right)\omega\Bigg(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Bigg)\Bigg)$$

Disspative Force 3 vector

$$=\left[\frac{4\cos(kz+\omega t)^2(\%1)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\left(k^2-\varepsilon\omega^2\mu\right)}{\mu k^2},\frac{4\cos(kz+\omega t)^2(\%1)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x\right.\right.}{\mu}\right.$$

$$\%1=\left(\frac{\partial}{\partial x}x(x,y)\right)^2$$

Dissipation

$$=\frac{1}{k}\Bigg(2\varepsilon\cos(kz+\omega t)\omega\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg)$$

***************************** *END PROCEDURE* *************************** **(27)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 6b-- Wave guide TTE (kinematic in, wave out)`;\
> theta:=(k*z-omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=-(omega/k)*f(x,y)*cos(theta);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 6b-- Wave guide TTE (kinematic in, wave out)

$$\theta := k\,z - \omega\,t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z-\omega\,t)$$

$$\phi := -\frac{\omega\left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z-\omega\,t)}{k}$$

Example 6b-- Wave guide TTE (kinematic in, wave out)

******************************** *Differential Form Format* ****************************
****

$$Action\ 1\text{-}form = \left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z-\omega\,t)\ d(z) + \frac{\omega\left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z-\omega\,t)\ d(t)}{k}$$

$$Intensity\ 2\text{-}form\ F{=}dA\ = -2\left(x(x,y)^2 + y(x,y)^2\right)\sin(kz-\omega t)\,\omega\,(d(z))\ \&\text{^}\ (d(t)) + 2\,(\%1)\cos(kz-\omega t)\,(d(x))\ \&\text{^}\ (d(z)) + 2\,(\%2)\cos(kz-\omega t)\,(d(y))$$

$$\%1 = x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x$$

$$\%2 = x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x$$

Topological Torsion 3-form $A\text{^}F = 0$

Topological Parity 4-form $F\text{^}F = 0$

******************************** *Using EM format* ********************************

$$E\ field = \left[\frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{k},\right.$$

$$\frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{k}, -2\left(x(x,y)^2 + y(x,\right.$$

$$\left.y)^2\right)\sin(k\,z-\omega\,t)\,\omega\Big]$$

$$B\ field = \left[2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z-\omega\,t), -2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)\right.\right.$$

$$+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)\cos(kz-\omega t),0\Big]$$

Topological TORSION 4 vector   T4 = -[ExA + Bphi,AdotB] $=[0,0,0,0]$

Helicity AdotB $=0$

Poincare II =2(E.B) $=0$

coefficient of Topological Parity 4-form $=0$

Pfaff Topological Dimension   PTD $=2$

******************** Correlation Similarity Invariants of Jacobian of (Ak/lambda_N) ****************************

Xm or linear (Mean) curvature =

$$-\frac{\left(x(x,y)^2+y(x,y)^2\right)\sin(kz-\omega t)\,(k-\omega)\,(k+\omega)}{k}$$

Yg or quadratic (GAUSS) curvature $=0$

Za or Cubic (Interaction internal energy) curvature $=0$

Tk or quartic (4D expansion) curvature $=0$

***********   Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH ************

Chirality factor CH $=0$

$$D\,field=\left[\frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz-\omega t)}{k},\right.$$

$$\frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz-\omega t)}{k},-2\,\varepsilon\left(x(x,y)^2+y(x,\right.$$

$$\left.y)^2\right)\sin(kz-\omega t)\,\omega\Big]$$

$$H\,field=\left[\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz-\omega t)}{\mu},\right.$$

$$\left.-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz-\omega t)}{\mu},0\right]$$

$$Poynting\ vector\ ExH=\Big[$$

$$-\frac{1}{\mu}\left(4\left(x(x,y)^2+y(x,y)^2\right)\sin(kz-\omega t)\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz-\omega t)\right),$$

$$-\frac{1}{\mu}\left(4\left(x(x,y)^2+y(x,y)^2\right)\sin(kz-\omega t)\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz-\omega t)\right), -\frac{1}{\mu k}\left(4\,\omega\cos(kz-\omega t)^2\left(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\right)\right)\Bigg]$$

Amperian Current 4Vector *curlH-dD/dt=J4* =

$$\Bigg[-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz-\omega t)\left(k^2+\varepsilon\,\omega^2\mu\right)}{\mu k},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\sin(kz-\omega t)\left(k^2+\varepsilon\,\omega^2\mu\right)}{\mu k},$$

$$-\frac{1}{\mu}\left(2\cos(kz-\omega t)\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\,\omega^2\mu\,x(x,y)^2+\varepsilon\,\omega^2\mu\,y(x,y)^2\right)\right), \frac{1}{k}\left(2\,\varepsilon\,\omega\cos(kz-\omega t)\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-k^2x(x,y)^2-k^2y(x,y)^2\right)\right)\Bigg]$$

Amerian charge density $\quad divD = rho = \frac{1}{k}\left(2\,\varepsilon\,\omega\cos(k z-\omega t)\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-k^2x(x,y)^2-k^2y(x,y)^2\right)\right)$

divergence Lorentz Current 4Vector, $\quad$ *4div(J4)* $=0$

Topological SPIN 4 vector S4

$$=\left[\frac{1}{\mu k^2}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k z-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\left(k^2-\varepsilon\,\omega^2\mu\right)\right),\right.$$

$$\frac{1}{\mu k^2}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k z-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\left(k^2-\varepsilon\,\omega^2\mu\right)\right),\frac{2\,\varepsilon\left(x(x,y)^2+y(x,y)^2\right)^2\sin(k z-\omega t)\,\omega^2\cos(k z-\omega t)}{k},\text{-}2\left(x(x,y)^2\right.$$

$$\left.\left.+y(x,y)^2\right)^2\cos(k z-\omega t)\,\varepsilon\sin(k z-\omega t)\,\omega\right]$$

Topological SPIN 3-form $=\frac{1}{\mu k^2}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k z-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\left(k^2-\varepsilon\,\omega^2\mu\right)\&\text{^}(d(y),d(z),d(t))\right)-\frac{1}{\mu k^2}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k z-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\left(k^2-\varepsilon\,\omega^2\mu\right)\&\text{^}(d(x),d(z),d(t))\right)+\frac{2\,\varepsilon\left(x(x,y)^2+y(x,y)^2\right)^2\sin(k z-\omega t)\,\omega^2\cos(k z-\omega t)\,\&\text{^}(d(x),d(y),d(t))}{k}+2\left(x(x,y)^2+y(x,y)^2\right)^2\cos(k z-\omega t)\,\varepsilon\sin(k z-\omega t)\,\omega\,\&\text{^}(d(x),d(y),d(z))$

Spin density rho_spin $=\text{-}2\left(x(x,y)^2+y(x,y)^2\right)^2\cos(k z-\omega t)\,\varepsilon\sin(k z-\omega t)\,\omega$

LaGrange field energy density (B.H-D.E) $=\frac{1}{\mu k^2}\left(4\left(\cos(k z-\omega t)^2k^2x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\cos(k z-\omega t)^2k^2x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+\cos(k z-\omega t)^2k^2y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2\right.\right.$

$$+\cos(kz-\omega t)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\cos(kz-\omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + \cos(kz-\omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - \varepsilon\,\omega^2 \mu \cos(kz-\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 2\,\varepsilon\,\omega^2 \mu \cos(kz-\omega t)^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) - \varepsilon\,\omega^2 \mu \cos(kz-\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - \varepsilon\,\omega^2 \mu \cos(kz-\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 2\,\varepsilon\,\omega^2 \mu \cos(kz-\omega t)^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) - \varepsilon\,\omega^2 \mu \cos(kz-\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - \varepsilon\,\omega^2 \mu \sin(kz-\omega t)^2 k^2 x(x,y)^4 - 2\,\varepsilon\,\omega^2 \mu \sin(kz-\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 - \varepsilon\,\omega^2 \mu \sin(kz-\omega t)^2 k^2 y(x,y)^4 \Big)\Big)$$

$$B.H = \frac{1}{\mu}\left(4\cos(kz-\omega t)^2 \left(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\,x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) + y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\,x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2\right)\right)$$

$$D.E = \frac{1}{k^2}\left(4\,\varepsilon\,\omega^2 \left(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 x(x,y)^2 \cos(kz-\omega t)^2 + 2\left(\frac{\partial}{\partial x} y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) x(x,y) \cos(kz-\omega t)^2 + \left(\frac{\partial}{\partial x} y(x,y)\right)^2 y(x,y)^2 \cos(kz-\omega t)^2 + \left(\frac{\partial}{\partial y} x(x,y)\right)^2 x(x,y)^2 \cos(kz-\omega t)^2 + 2\left(\frac{\partial}{\partial y} y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) x(x,y) \cos(kz-\omega t)^2 + y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 \cos(kz-\omega t)^2 + \sin(kz-\omega t)^2 k^2 x(x,y)^4 + 2\sin(kz-\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 + \sin(kz-\omega t)^2 k^2 y(x,y)^4\right)\right)$$

$$A.J = -\frac{1}{\mu}\left(2\left(x(x,y)^2 + y(x,y)^2\right)\cos(kz-\omega t)^2 \left(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + \varepsilon\,\omega^2 \mu\, x(x,y)^2 + \varepsilon\,\omega^2 \mu\, y(x,y)^2\right)\right)$$

$$-rho.phi = -\frac{1}{k^2}\left(2\,\omega^2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\,\varepsilon\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)\right.\right.$$

$$+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$

$$\left.\left.+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-k^2x(x,y)^2-k^2y(x,y)^2\right)\right)$$

Poincare I (B.H - D.E)-(A.J - rho.phi) $= \frac{1}{\mu k^2}\left(2\left(3\cos(kz-\omega t)^2k^2x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2\right.\right.$

$$+3\cos(kz-\omega t)^2k^2y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+3\cos(kz-\omega t)^2k^2x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2$$

$$+3\cos(kz-\omega t)^2k^2y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\cos(kz-\omega t)^2x(x,y)^4k^2\varepsilon\omega^2\mu-\cos(kz$$

$$-\omega t)^2x(x,y)^2\varepsilon\omega^2\mu\left(\frac{\partial}{\partial x}y(x,y)\right)^2-\cos(kz-\omega t)^2x(x,y)^2\varepsilon\omega^2\mu\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\cos(kz$$

$$-\omega t)^2y(x,y)^4k^2\varepsilon\omega^2\mu-\cos(kz-\omega t)^2y(x,y)^2\varepsilon\omega^2\mu\left(\frac{\partial}{\partial x}x(x,y)\right)^2-\cos(kz-\omega t)^2y(x,$$

$$y)^2\varepsilon\omega^2\mu\left(\frac{\partial}{\partial y}x(x,y)\right)^2+\cos(kz-\omega t)^2x(x,y)^2k^2y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\cos(kz$$

$$-\omega t)^2x(x,y)^2k^2y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\cos(kz-\omega t)^2y(x,y)^2k^2x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$$

$$+\cos(kz-\omega t)^2y(x,y)^2k^2x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+4\cos(kz-\omega t)^2k^2x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,$$

$$y)\left(\frac{\partial}{\partial x}y(x,y)\right)-3\varepsilon\omega^2\mu\cos(kz-\omega t)^2x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2-3\varepsilon\omega^2\mu\cos(kz$$

$$-\omega t)^2y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2-3\varepsilon\omega^2\mu\cos(kz-\omega t)^2x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$-3\varepsilon\omega^2\mu\cos(kz-\omega t)^2y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-2\varepsilon\omega^2\mu\sin(kz-\omega t)^2k^2x(x,y)^4$$

$$+4\cos(kz-\omega t)^2k^2x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)-\cos(kz-\omega t)^2x(x,$$

$$y)^3\varepsilon\omega^2\mu\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-\cos(kz-\omega t)^2y(x,y)^3\varepsilon\omega^2\mu\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-\cos(kz-\omega t)^2x(x,$$

$$y)^3\varepsilon\omega^2\mu\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-\cos(kz-\omega t)^2y(x,y)^3\varepsilon\omega^2\mu\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)-4\varepsilon\omega^2\mu\sin(kz$$

$-\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 - 2\varepsilon\omega^2\mu\sin(kz-\omega t)^2 k^2 y(x,y)^4 + 4\cos(kz-\omega t)^2 x(x,$

$y)^2 k^2 \varepsilon\omega^2\mu y(x,y)^2 - \cos(kz-\omega t)^2 y(x,y)^2 \varepsilon\omega^2\mu x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right) - \cos(kz$

$-\omega t)^2 x(x,y)^2\varepsilon\omega^2\mu y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right) - \cos(kz-\omega t)^2 y(x,y)^2\varepsilon\omega^2\mu x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$

$-\cos(kz-\omega t)^2 x(x,y)^2\varepsilon\omega^2\mu y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right) + \cos(kz-\omega t)^2 y(x,y)^3 k^2\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)$

$+\cos(kz-\omega t)^2 y(x,y)^3 k^2\left(\frac{\partial^2}{\partial y^2}y(x,y)\right) + \cos(kz-\omega t)^2 x(x,y)^3 k^2\left(\frac{\partial^2}{\partial x^2}x(x,y)\right) + \cos(kz$

$-\omega t)^2 x(x,y)^3 k^2\left(\frac{\partial^2}{\partial y^2}x(x,y)\right) + \cos(kz-\omega t)^2 y(x,y)^2 k^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2 + \cos(kz$

$-\omega t)^2 x(x,y)^2 k^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 + \cos(kz-\omega t)^2 x(x,y)^2 k^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 + \cos(kz$

$-\omega t)^2 y(x,y)^2 k^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2 - 4\varepsilon\omega^2\mu\cos(kz-\omega t)^2 x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,\right.$

$\left.\left.\left.y)\right) - 4\varepsilon\omega^2\mu\cos(kz-\omega t)^2 x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\right)$

$$\textit{London Coefficient} \quad LC = \frac{k^2-\varepsilon\omega^2\mu}{\mu}$$

$\textit{PROCA coefficient curlcurlB} = \left[-2\cos(kz-\omega t)\left(2\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}x(x,y)\right) + \left(\frac{\partial}{\partial y}x(x,\right.\right.\right.$

$\left.y)\right)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2}x(x,y)\right) + 2\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}y(x,y)\right) + \left(\frac{\partial}{\partial y}y(x,\right.$

$\left.y)\right)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2}y(x,y)\right) + 3\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right) + x(x,$

$y)\left(\frac{\partial^3}{\partial y^3}x(x,y)\right) + 3\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y^3}y(x,y)\right) - k^2 x(x,y)\left(\frac{\partial}{\partial y}x(x,\right.$

$\left.\left.y)\right) - k^2 y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right), 2\cos(kz-\omega t)\left(-k^2 x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) - k^2 y(x,y)\left(\frac{\partial}{\partial x}y(x,\right.\right.$

$\left.y)\right) + 3\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial x^3}x(x,y)\right) + 3\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)$

$+y(x,y)\left(\frac{\partial^3}{\partial x^3}y(x,y)\right) + 2\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}x(x,y)\right) + \left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right) + x(x,$

$y)\left(\frac{\partial^3}{\partial y^2\,\partial x}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}y(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+y(x,$
$y)\left(\frac{\partial^3}{\partial y^2\,\partial x}y(x,y)\right)\Big)\Big),0\Big]$

Amperian Current 4Vector curlH-dD/dt=J4 $=\Bigg[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz-\omega t)\left(k^2+\varepsilon\,\omega^2\mu\right)}{\mu k},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\sin(kz-\omega t)\left(k^2+\varepsilon\,\omega^2\mu\right)}{\mu k},$$

$-\frac{1}{\mu}\Bigg(2\cos(kz-\omega t)\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,$

$y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,\right.$
$\left.y)\right)+\varepsilon\,\omega^2\mu\,x(x,y)^2+\varepsilon\,\omega^2\mu\,y(x,y)^2\Bigg)\Bigg),\ \frac{1}{k}\Bigg(2\,\varepsilon\,\omega\cos(kz-\omega t)\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,$
$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,\right.$
$\left.y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-k^2x(x,y)^2-k^2y(x,y)^2\Bigg)\Bigg)\Bigg]$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= \left[\frac{4\cos(kz-\omega t)^2 \left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right) (}{\mu k^2} \right.$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $=0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{1}{\mu k^2} \left(2\left(x(x,y)^2 + y(x,y)^2\right)\cos(kz-\omega t)^2 \left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\left(k^2 - \varepsilon\,\omega^2\mu\right)\right), \right.$$

$$\frac{1}{\mu k^2} \left(2\left(x(x,y)^2 + y(x,y)^2\right)\cos(kz-\omega t)^2 \left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right)\left(k^2 - \varepsilon\,\omega^2\mu\right)\right), \frac{2\varepsilon\left(x(x,y)^2 + y(x,y)^2\right)^2 \sin(kz-\omega t)\,\omega^2 \cos(kz-\omega t)}{k}, -2\left(x(x,y)^2 + y(x,y)^2\right)^2 \cos(kz-\omega t)\,\varepsilon \sin(kz-\omega t)\,\omega \Bigg]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= \left[0, 0, \frac{1}{\mu k^2}\left(4\left(x(x,y)^2\right.\right.\right.$

$$+ y(x,y)^2\big) \cos(kz-\omega t) \Big(\cos(kz-\omega t)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\cos(kz-\omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) + \cos(kz-\omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + \cos(kz-\omega t)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\cos(kz-\omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + \cos(kz-\omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - \varepsilon\omega^2\mu\cos(kz-\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 2\varepsilon\omega^2\mu\cos(kz-\omega t)^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) - \varepsilon\omega^2\mu\cos(kz-\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - \varepsilon\omega^2\mu\cos(kz-\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 2\varepsilon\omega^2\mu\cos(kz-\omega t)^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) - \varepsilon\omega^2\mu\cos(kz-\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - \varepsilon\omega^2\mu\sin(kz-\omega t)^2 k^2 x(x,y)^4 - 2\varepsilon\omega^2\mu\sin(kz-\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 - \varepsilon\omega^2\mu\sin(kz-\omega t)^2 k^2 y(x,y)^4\Big)\Big)\Big]$$

Spin Dissipation J_spin dot E $= \frac{1}{\mu k^3}\Big(4\left(x(x,y)^2 + y(x,y)^2\right)\cos(kz-\omega t)\,\omega \Big(\cos(kz-\omega t)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\cos(kz-\omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) + \cos(kz-\omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + \cos(kz-\omega t)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\cos(kz-\omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + \cos(kz-\omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - \varepsilon\omega^2\mu\cos(kz-\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 2\varepsilon\omega^2\mu\cos(kz-\omega t)^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) - \varepsilon\omega^2\mu\cos(kz-\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - \varepsilon\omega^2\mu\cos(kz-\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 2\varepsilon\omega^2\mu\cos(kz-\omega t)^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) - \varepsilon\omega^2\mu\cos(kz-\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - \varepsilon\omega^2\mu\sin(kz-\omega t)^2 k^2 x(x,y)^4 - 2\varepsilon\omega^2\mu\sin(kz-\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 - \varepsilon\omega^2\mu\sin(kz-\omega t)^2 k^2 y(x,y)^4\Big)\Big)$

$$\textit{Disspative Force 3 vector} = \left[\frac{4\cos(kz-\omega t)^2 \left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)(\%5)}{\mu k^2}, \frac{4\cos(kz-\omega t)^2 \left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y\right.\right.}{\mu k^2}\right.$$

$$\begin{aligned}\textit{Dissipation} &= \frac{1}{k}\Bigg(2\,\varepsilon\,\omega\cos(kz-\omega t)\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right) + \left(\frac{\partial}{\partial x}y(x,y)\right)^2 \\ &+ y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right) + \left(\frac{\partial}{\partial y}x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right) + \left(\frac{\partial}{\partial y}y(x,y)\right)^2 + y(x, \\ &y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right) - k^2x(x,y)^2 - k^2y(x,y)^2 - \mu\sin(kz-\omega t)\,k\,x(x,y)^4 - 2\,\mu\sin(kz \\ &-\omega t)\,k\,x(x,y)^2y(x,y)^2 - \mu\sin(kz-\omega t)\,k\,y(x,y)^4\Bigg)\Bigg)\end{aligned}$$

****************************** *END PROCEDURE* ****************************** **(28)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 6c-- Wave guide TTM (kinematic out, wave in)`;
> theta:=(k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=+(omega/k)*f(x,y)*cos(theta);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 6c-- Wave guide TTM (kinematic out, wave in)

$$\theta := kz + \omega t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right)\cos(kz+\omega t)$$

$$\phi := \frac{\omega\,(x(x,y)^2 + y(x,y)^2)\cos(kz+\omega t)}{k}$$

Example 6c-- Wave guide TTM (kinematic out, wave in)

********************************* *Differential Form Format* ********************************

Action 1-form $= (x(x,y)^2 + y(x,y)^2)\cos(kz+\omega t)\, d(z) - \frac{\omega\,(x(x,y)^2 + y(x,y)^2)\cos(kz+\omega t)\, d(t)}{k}$

Intensity 2-form F=dA $= 2\,(x(x,y)^2 + y(x,y)^2)\sin(kz+\omega t)\,\omega\,(d(z))\ \&\hat{}\ (d(t)) + 2\,(\%1)\cos(kz+\omega t)\,(d(x))\ \&\hat{}\ (d(z)) + 2\,(\%2)\cos(kz+\omega t)\,(d(y))\ \&$

$\%1 = x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x\right.$

$\%2 = x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x\right.$

Topological Torsion 3-form $A\hat{}F = 0$

Topological Parity 4-form $F\hat{}F = 0$

********************************* *Using EM format* *********************************

$$E\,field = \left[-\frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz+\omega t)}{k},\ -\frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz+\omega t)}{k},\ 2\,(x(x,y)^2 + y(x,y)^2)\sin(kz+\omega t)\,\omega\right]$$

$$B\,field = \left[2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz+\omega t),\ -2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz+\omega t),\ 0\right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension PTD $= 2$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

Xm or linear (Mean) curvature =

$$-\frac{\left(x(x,y)^2+y(x,y)^2\right)\sin(kz+\omega t)\,(k-\omega)\,(k+\omega)}{k}$$

Yg or quadratic (GAUSS) curvature $=0$

Za or Cubic (Interaction internal energy) curvature $=0$

Tk or quartic (4D expansion) curvature $=0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor $CH=0$

$$D\,field=\left[-\frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz+\omega t)}{k},\right.$$
$$\left.-\frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz+\omega t)}{k},2\,\varepsilon\left(x(x,y)^2+y(x,y)^2\right)\sin(kz+\omega t)\,\omega\right]$$

$$H\,field=\left[\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz+\omega t)}{\mu},\right.$$
$$\left.-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz+\omega t)}{\mu},0\right]$$

Poynting vector ExH

$$=\left[\frac{1}{\mu}\left(4\left(x(x,y)^2+y(x,y)^2\right)\sin(kz+\omega t)\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz+\omega t)\right),\right.$$

$$\frac{1}{\mu}\left(4\left(x(x,y)^2+y(x,y)^2\right)\sin(kz+\omega t)\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz+\omega t)\right),\frac{1}{\mu k}\left(4\,\omega\cos(kz+\omega t)^2\left(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,\right.\right.$$

$$y)\Big)\,y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)+y(x,y)^{2}\left(\frac{\partial}{\partial x}\,y(x,y)\right)^{2}\Big)\Big)\Big]$$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \Big[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)\right)\sin(k\,z+\omega\,t)\,\left(k^{2}+\varepsilon\,\omega^{2}\,\mu\right)}{\mu\,k},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)\right)\sin(k\,z+\omega\,t)\,\left(k^{2}+\varepsilon\,\omega^{2}\,\mu\right)}{\mu\,k},$$

$$-\frac{1}{\mu}\Bigg(2\cos(k\,z+\omega\,t)\,\Bigg(\left(\frac{\partial}{\partial x}\,x(x,y)\right)^{2}+x(x,y)\left(\frac{\partial^{2}}{\partial x^{2}}\,x(x,y)\right)+\left(\frac{\partial}{\partial x}\,y(x,y)\right)^{2}+y(x,$$

$$y)\left(\frac{\partial^{2}}{\partial x^{2}}\,y(x,y)\right)+\left(\frac{\partial}{\partial y}\,x(x,y)\right)^{2}+x(x,y)\left(\frac{\partial^{2}}{\partial y^{2}}\,x(x,y)\right)+\left(\frac{\partial}{\partial y}\,y(x,y)\right)^{2}+y(x,y)\left(\frac{\partial^{2}}{\partial y^{2}}\,y(x,$$

$$y)\right)+\varepsilon\,\omega^{2}\,\mu\,x(x,y)^{2}+\varepsilon\,\omega^{2}\,\mu\,y(x,y)^{2}\Bigg)\Bigg),\ -\frac{1}{k}\Bigg(2\,\varepsilon\cos(k\,z+\omega\,t)\,\omega\,\Bigg(\left(\frac{\partial}{\partial x}\,x(x,y)\right)^{2}+x(x,$$

$$y)\left(\frac{\partial^{2}}{\partial x^{2}}\,x(x,y)\right)+\left(\frac{\partial}{\partial x}\,y(x,y)\right)^{2}+y(x,y)\left(\frac{\partial^{2}}{\partial x^{2}}\,y(x,y)\right)+\left(\frac{\partial}{\partial y}\,x(x,y)\right)^{2}+x(x,y)\left(\frac{\partial^{2}}{\partial y^{2}}\,x(x,$$

$$y)\right)+\left(\frac{\partial}{\partial y}\,y(x,y)\right)^{2}+y(x,y)\left(\frac{\partial^{2}}{\partial y^{2}}\,y(x,y)\right)-k^{2}\,x(x,y)^{2}-k^{2}\,y(x,y)^{2}\Bigg)\Bigg)\Big]$$

Amerian charge density *divD = rho* $= -\frac{1}{k}\Bigg(2\,\varepsilon\cos(k\,z+\omega\,t)\,\omega\,\Bigg(\left(\frac{\partial}{\partial x}\,x(x,y)\right)^{2}+x(x,y)\left(\frac{\partial^{2}}{\partial x^{2}}\,x(x,$

$$y)\right)+\left(\frac{\partial}{\partial x}\,y(x,y)\right)^{2}+y(x,y)\left(\frac{\partial^{2}}{\partial x^{2}}\,y(x,y)\right)+\left(\frac{\partial}{\partial y}\,x(x,y)\right)^{2}+x(x,y)\left(\frac{\partial^{2}}{\partial y^{2}}\,x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial y}\,y(x,y)\right)^{2}+y(x,y)\left(\frac{\partial^{2}}{\partial y^{2}}\,y(x,y)\right)-k^{2}\,x(x,y)^{2}-k^{2}\,y(x,y)^{2}\Bigg)\Bigg)$$

divergence Lorentz Current 4Vector, *4div(J4)* $= 0$

Topological SPIN 4 vector S4

$$=\Bigg[\frac{1}{\mu\,k^{2}}\Bigg(2\left(x(x,y)^{2}+y(x,y)^{2}\right)\cos(k\,z+\omega\,t)^{2}\,\Bigg(x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}\,y(x,$$

$$y)\Big)\Big)\left(k^2-\varepsilon\,\omega^2\,\mu\right)\Big),$$

$$\frac{1}{\mu\,k^2}\Big(2\,\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\,\Big(x(x,y)\,\left(\frac{\partial}{\partial y}\,x(x,y)\right)+y(x,y)\,\Big(\frac{\partial}{\partial y}\,y(x,$$

$$y)\Big)\Big)\left(k^2-\varepsilon\,\omega^2\,\mu\right)\Big),\ \frac{2\,\varepsilon\,\left(x(x,y)^2+y(x,y)^2\right)^2\sin(k\,z+\omega\,t)\,\omega^2\cos(k\,z+\omega\,t)}{k},\ 2\,\big(x(x,y)^2$$

$$+y(x,y)^2\big)^2\cos(k\,z+\omega\,t)\,\varepsilon\sin(k\,z+\omega\,t)\,\omega\Big]$$

Topological SPIN 3-form $= \frac{1}{\mu\,k^2}\Big(2\,\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\,\Big(x(x,y)\,\left(\frac{\partial}{\partial x}\,x(x,y)\right)+y(x,$

$$y)\,\left(\frac{\partial}{\partial x}\,y(x,y)\right)\Big)\left(k^2-\varepsilon\,\omega^2\,\mu\right)\ \&\hat{}\,(d(y),d(z),d(t))\Big)-\frac{1}{\mu\,k^2}\Big(2\,\big(x(x,y)^2+y(x,$$

$$y)^2\big)\cos(k\,z+\omega\,t)^2\,\Big(x(x,y)\,\left(\frac{\partial}{\partial y}\,x(x,y)\right)+y(x,y)\,\left(\frac{\partial}{\partial y}\,y(x,y)\right)\Big)\left(k^2-\varepsilon\,\omega^2\,\mu\right)\ \&\hat{}\,(d(x),d(z),$$

$$d(t))\Big)+\frac{2\,\varepsilon\,\left(x(x,y)^2+y(x,y)^2\right)^2\sin(k\,z+\omega\,t)\,\omega^2\cos(k\,z+\omega\,t)\ \&\hat{}\,(d(x),d(y),d(t))}{k}$$

$$-2\,\left(x(x,y)^2+y(x,y)^2\right)^2\cos(k\,z+\omega\,t)\,\varepsilon\sin(k\,z+\omega\,t)\,\omega\ \&\hat{}\,(d(x),d(y),d(z))$$

Spin density rho_spin $= 2\,\left(x(x,y)^2+y(x,y)^2\right)^2\cos(k\,z+\omega\,t)\,\varepsilon\sin(k\,z+\omega\,t)\,\omega$

LaGrange field energy density (B.H-D.E) $= \frac{1}{\mu\,k^2}\Big(4\,\Big(\cos(k\,z+\omega\,t)^2\,k^2\,x(x,y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2+2\cos(k\,z$

$$+\omega\,t)^2\,k^2\,x(x,y)\,\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,y)\,\left(\frac{\partial}{\partial y}\,y(x,y)\right)+\cos(k\,z+\omega\,t)^2\,k^2\,y(x,y)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2$$

$$+\cos(k\,z+\omega\,t)^2\,k^2\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2+2\cos(k\,z+\omega\,t)^2\,k^2\,x(x,y)\,\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,$$

$$y)\,\left(\frac{\partial}{\partial x}\,y(x,y)\right)+\cos(k\,z+\omega\,t)^2\,k^2\,y(x,y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2-\varepsilon\,\omega^2\,\mu\cos(k\,z+\omega\,t)^2\,x(x,$$

$$y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2-2\,\varepsilon\,\omega^2\,\mu\cos(k\,z+\omega\,t)^2\,x(x,y)\,\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,y)\,\left(\frac{\partial}{\partial x}\,y(x,y)\right)$$

$$-\varepsilon\,\omega^2\,\mu\cos(k\,z+\omega\,t)^2\,y(x,y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2-\varepsilon\,\omega^2\,\mu\cos(k\,z+\omega\,t)^2\,x(x,y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2$$

$$-2\,\varepsilon\,\omega^2\,\mu\cos(k\,z+\omega\,t)^2\,x(x,y)\,\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,y)\,\left(\frac{\partial}{\partial y}\,y(x,y)\right)-\varepsilon\,\omega^2\,\mu\cos(k\,z$$

$$+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - \varepsilon\,\omega^2 \mu \sin(kz+\omega t)^2 k^2 x(x,y)^4 - 2\,\varepsilon\,\omega^2 \mu \sin(kz$$

$$+\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 - \varepsilon\,\omega^2 \mu \sin(kz+\omega t)^2 k^2 y(x,y)^4\Big)\Big)$$

$$B.H = \frac{1}{\mu}\Bigg(4\cos(kz+\omega t)^2 \Bigg(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)$$

$$+ y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)$$

$$+ y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2\Bigg)\Bigg)$$

$$D.E = \frac{1}{k^2}\Bigg(4\,\varepsilon\,\omega^2 \Bigg(\cos(kz+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\cos(kz+\omega t)^2 x(x,y)\left(\frac{\partial}{\partial x} x(x,$$

$$y)\right) y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right) + \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + \cos(kz+\omega t)^2 x(x,$$

$$y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\cos(kz+\omega t)^2 x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right) + \cos(kz$$

$$+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + \sin(kz+\omega t)^2 k^2 x(x,y)^4 + 2\sin(kz+\omega t)^2 k^2 x(x,y)^2 y(x,y)^2$$

$$+ \sin(kz+\omega t)^2 k^2 y(x,y)^4\Bigg)\Bigg)$$

$$A.J = -\frac{1}{\mu}\Bigg(2\left(x(x,y)^2 + y(x,y)^2\right)\cos(kz+\omega t)^2 \Bigg(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right)$$

$$+ \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$+ \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + \varepsilon\,\omega^2 \mu\, x(x,y)^2 + \varepsilon\,\omega^2 \mu\, y(x,y)^2\Bigg)\Bigg)$$

$$-rho.phi = -\frac{1}{k^2}\Bigg(2\,\omega^2 \left(x(x,y)^2 + y(x,y)^2\right)\cos(kz+\omega t)^2 \varepsilon \Bigg(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right)$$

$$+ \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$+ \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - k^2 x(x,y)^2 - k^2 y(x,y)^2\Bigg)\Bigg)$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* $= \frac{1}{\mu k^2}\Bigg(2\Bigg(\cos(kz+\omega t)^2 x(x,y)^2 k^2 y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right)$

$$- \cos(kz+\omega t)^2 x(x,y)^2 \varepsilon \omega^2 \mu \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 2\cos(kz+\omega t)^2 y(x,y)^4 k^2 \varepsilon \omega^2 \mu - \cos(kz$$

$$+ \omega t)^2 y(x,y)^2 \varepsilon \omega^2 \mu \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - \cos(kz+\omega t)^2 y(x,y)^2 \varepsilon \omega^2 \mu \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\cos(kz$$

$$+ \omega t)^2 x(x,y)^4 k^2 \varepsilon \omega^2 \mu - \cos(kz+\omega t)^2 x(x,y)^2 \varepsilon \omega^2 \mu \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - \cos(kz+\omega t)^2 y(x,$$

$$y)^3 \varepsilon \omega^2 \mu \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + \cos(kz+\omega t)^2 x(x,y)^2 k^2 y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - \cos(kz$$

$$+ \omega t)^2 x(x,y)^2 \varepsilon \omega^2 \mu\, y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - \cos(kz+\omega t)^2 y(x,y)^2 \varepsilon \omega^2 \mu\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right)$$

$$- \cos(kz+\omega t)^2 y(x,y)^2 \varepsilon \omega^2 \mu\, x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) - \cos(kz+\omega t)^2 x(x,y)^2 \varepsilon \omega^2 \mu\, y(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + 4\cos(kz+\omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) + 4\cos(kz$$

$$+ \omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) - 3\varepsilon \omega^2 \mu \cos(kz+\omega t)^2 x(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 3\varepsilon \omega^2 \mu \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - 3\varepsilon \omega^2 \mu \cos(kz$$

$$+ \omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 3\varepsilon \omega^2 \mu \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2$$

$$- 2\varepsilon \omega^2 \mu \sin(kz+\omega t)^2 k^2 x(x,y)^4 - 2\varepsilon \omega^2 \mu \sin(kz+\omega t)^2 k^2 y(x,y)^4 + \cos(kz+\omega t)^2 y(x,$$

$$y)^2 k^2 x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + \cos(kz+\omega t)^2 y(x,y)^2 k^2 x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + \cos(kz$$

$$+ \omega t)^2 y(x,y)^3 k^2 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + \cos(kz+\omega t)^2 x(x,y)^3 k^2 \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + \cos(kz$$

$$+ \omega t)^2 x(x,y)^3 k^2 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + \cos(kz+\omega t)^2 y(x,y)^3 k^2 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + 4\cos(kz$$

$$+ \omega t)^2 x(x,y)^2 k^2 \varepsilon \omega^2 \mu\, y(x,y)^2 - \cos(kz+\omega t)^2 y(x,y)^3 \varepsilon \omega^2 \mu \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) - \cos(kz$$

$$+ \omega t)^2 x(x,y)^3 \varepsilon \omega^2 \mu \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - \cos(kz+\omega t)^2 x(x,y)^3 \varepsilon \omega^2 \mu \left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$- 4\varepsilon \omega^2 \mu \sin(kz+\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 + 3\cos(kz+\omega t)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2$$

$$+3\cos(kz+\omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 3\cos(kz+\omega t)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2$$

$$+3\cos(kz+\omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + \cos(kz+\omega t)^2 y(x,y)^2 k^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2$$

$$+\cos(kz+\omega t)^2 x(x,y)^2 k^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + \cos(kz+\omega t)^2 x(x,y)^2 k^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + \cos(kz$$

$$+\omega t)^2 y(x,y)^2 k^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 4\,\varepsilon\,\omega^2\,\mu\cos(kz+\omega t)^2 x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial x} y(x,$$

$$y)\right) - 4\,\varepsilon\,\omega^2\,\mu\cos(kz+\omega t)^2 x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\Big)\Big)$$

London Coefficient $\quad LC = \dfrac{k^2 - \varepsilon\,\omega^2\,\mu}{\mu}$

PROCA coefficient curlcurlB $= \Bigg[-2\cos(kz+\omega t)\Bigg(2\left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,$

$$y)\right)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2} x(x,y)\right) + 2\left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} y(x,y)\right) + \left(\frac{\partial}{\partial y} y(x,$$

$$y)\right)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2} y(x,y)\right) + 3\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + x(x,$$

$$y)\left(\frac{\partial^3}{\partial y^3} x(x,y)\right) + 3\left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y^3} y(x,y)\right) - k^2 x(x,y)\left(\frac{\partial}{\partial y} x(x,$$

$$y)\right) - k^2 y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\Bigg), -2\cos(kz+\omega t)\Bigg(k^2 x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + k^2 y(x,y)\left(\frac{\partial}{\partial x} y(x,$$

$$y)\right) - 3\left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - x(x,y)\left(\frac{\partial^3}{\partial x^3} x(x,y)\right) - 3\left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right)$$

$$-y(x,y)\left(\frac{\partial^3}{\partial x^3} y(x,y)\right) - 2\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right) - \left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) - x(x,$$

$$y)\left(\frac{\partial^3}{\partial y^2\,\partial x} x(x,y)\right) - 2\left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} y(x,y)\right) - \left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - y(x,$$

$$y)\left(\frac{\partial^3}{\partial y^2\,\partial x} y(x,y)\right)\Bigg), 0\Bigg]$$

Amperian Current 4Vector $\quad$ *curlH-dD/dt=J4* $\quad = \Bigg[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz+\omega t)\left(k^2+\varepsilon\omega^2\mu\right)}{\mu k},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\sin(kz+\omega t)\left(k^2+\varepsilon\omega^2\mu\right)}{\mu k},$$

$$-\frac{1}{\mu}\left(2\cos(kz+\omega t)\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\omega^2\mu x(x,y)^2+\varepsilon\omega^2\mu y(x,y)^2\right)\right), -\frac{1}{k}\left(2\varepsilon\cos(kz+\omega t)\,\omega\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-k^2x(x,y)^2-k^2y(x,y)^2\right)\right)\Bigg]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= \left[\frac{4\cos(kz+\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)(\ldots}{\mu k^2}\right.$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $=[0,0,0,0]$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{1}{\mu k^2} \left(2 \left(x(x,y)^2 + y(x,y)^2 \right) \cos(k z + \omega t)^2 \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \left(k^2 - \varepsilon \omega^2 \mu \right) \right), \right.$$

$$\frac{1}{\mu k^2} \left(2 \left(x(x,y)^2 + y(x,y)^2 \right) \cos(k z + \omega t)^2 \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \left(k^2 - \varepsilon \omega^2 \mu \right) \right), \frac{2 \varepsilon \left(x(x,y)^2 + y(x,y)^2 \right)^2 \sin(k z + \omega t) \, \omega^2 \cos(k z + \omega t)}{k}, 2 \left(x(x,y)^2 \right.$$

$$\left. \left. + y(x,y)^2 \right)^2 \cos(k z + \omega t) \, \varepsilon \sin(k z + \omega t) \, \omega \right]$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B)

$$= \left[0, 0, -\frac{1}{\mu k^2} \left(4 \left(x(x,y)^2 + y(x,y)^2 \right) \cos(k z + \omega t) \left(\varepsilon \omega^2 \mu \sin(k z + \omega t)^2 k^2 x(x,y)^4 + 2 \varepsilon \omega^2 \mu \sin(k z + \omega t)^2 k^2 x(x,y)^2 y(x,y)^2 + \varepsilon \omega^2 \mu \sin(k z + \omega t)^2 k^2 y(x,y)^4 - \cos(k z + \omega t)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 \right. \right. \right.$$

$$- 2 \cos(k z + \omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) - \cos(k z + \omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 - \cos(k z + \omega t)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 - 2 \cos(k z + \omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) - \cos(k z + \omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2$$

$$+ \varepsilon \omega^2 \mu \cos(k z + \omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2 \varepsilon \omega^2 \mu \cos(k z + \omega t)^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) + \varepsilon \omega^2 \mu \cos(k z + \omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 + \varepsilon \omega^2 \mu \cos(k z + \omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2 \varepsilon \omega^2 \mu \cos(k z + \omega t)^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right)$$

$$+ \varepsilon \omega^2 \mu \cos(k z + \omega t)^2 y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 \Big) \Big) \Big) \Big]$$

Spin Dissipation J_spin dot E $= \frac{1}{\mu k^3} \Big(4 \left(x(x, y)^2 + y(x, y)^2 \right) \cos(k z + \omega t) \, \omega \Big(\varepsilon \omega^2 \mu \sin(k z + \omega t)^2 k^2 x(x, y)^4 + 2 \varepsilon \omega^2 \mu \sin(k z + \omega t)^2 k^2 x(x, y)^2 y(x, y)^2 + \varepsilon \omega^2 \mu \sin(k z + \omega t)^2 k^2 y(x, y)^4 - \cos(k z + \omega t)^2 k^2 x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 - 2 \cos(k z + \omega t)^2 k^2 x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) - \cos(k z + \omega t)^2 k^2 y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 - \cos(k z + \omega t)^2 k^2 x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 - 2 \cos(k z + \omega t)^2 k^2 x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) - \cos(k z + \omega t)^2 k^2 y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 + \varepsilon \omega^2 \mu \cos(k z + \omega t)^2 x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 + 2 \varepsilon \omega^2 \mu \cos(k z + \omega t)^2 x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) + \varepsilon \omega^2 \mu \cos(k z + \omega t)^2 y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 + \varepsilon \omega^2 \mu \cos(k z + \omega t)^2 x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 + 2 \varepsilon \omega^2 \mu \cos(k z + \omega t)^2 x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) + \varepsilon \omega^2 \mu \cos(k z + \omega t)^2 y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 \Big) \Big)$

Disspative Force 3 vector $= \left[\frac{4 \cos(kz + \omega t)^2 \left(x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) + y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) \right) (\%5)}{\mu k^2}, \frac{4 \cos(kz + \omega t)^2 \left(x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) + y(x, y) \left(\frac{\partial}{\partial y} y \right.\right.}{\mu k^2} \right.$

$$Dissipation = \frac{1}{k}\left(2\,\varepsilon\cos(k\,z+\omega\,t)\,\omega\left(-\left(\frac{\partial}{\partial x}x(x,y)\right)^2 - x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right) - \left(\frac{\partial}{\partial x}y(x,y)\right)^2 - y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right) - \left(\frac{\partial}{\partial y}x(x,y)\right)^2 - x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right) - \left(\frac{\partial}{\partial y}y(x,y)\right)^2 - y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right) + k^2\,x(x,y)^2 + k^2\,y(x,y)^2 + \mu\sin(k\,z+\omega\,t)\,k\,x(x,y)^4 + 2\,\mu\sin(k\,z+\omega\,t)\,k\,x(x,y)^2\,y(x,y)^2 + \mu\sin(k\,z+\omega\,t)\,k\,y(x,y)^4\right)\right)$$

******************************** *END PROCEDURE* ****************************** **(29)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 6d-- Wave guide TTM (kinematic out, wave out)`;
> theta:=(k*z-omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=+(omega/k)*f(x,y)*cos(theta);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 6d-- Wave guide TTM (kinematic out, wave out)

$$\theta := k\,z-\omega\,t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)$$

$$\phi := \frac{\omega\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)}{k}$$

Example 6d-- Wave guide TTM (kinematic out, wave out)

*********************************** *Differential Form Format* **********************************

$$Action\ 1\text{-}form = \left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)\,d(z) - \frac{\omega\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)\,d(t)}{k}$$

$$Intensity\ 2\text{-}form\ F{=}dA = 2\,(\%1)\cos(k z-\omega t)\,(d(x))\ \&\hat{}\ (d(z)) + 2\,(\%2)\cos(k z-\omega t)\,(d(y))\ \&\hat{}\ (d(z)) - \frac{2\,\omega\,(\%1)\cos(k z-\omega t)\,(d(x))\ \&\hat{}\ (d(t))}{k} -$$

$$\%1 = x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)$$

$$\%2 = x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)$$

Topological Torsion 3-form $A\hat{}F = 0$

Topological Parity 4-form $F\hat{}F = 0$

******************************** *Using EM format* ********************************

$$E\,field = \left[-\frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{k},\right.$$
$$\left. -\frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{k}, 0\right]$$

$$B\,field = \left[2\left(x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)\right)\cos(k\,z-\omega\,t), -2\left(x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)\right.\right.$$
$$\left.\left.+y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)\right)\cos(k\,z-\omega\,t), 0\right]$$

Topological TORSION 4 vector $T4$ = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension $PTD = 2$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************
****

$$Xm \text{ or linear (Mean) curvature } = -\frac{\left(x(x,y)^2+y(x,y)^2\right)\sin(k\,z-\omega\,t)\left(k^2+\omega^2\right)}{k}$$

Yg or quadratic (GAUSS) curvature $= 0$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor $CH = 0$

$$D\,field = \left[-\frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{k},\right.$$
$$\left. -\frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{k}, 0\right]$$

$$H\,field = \left[\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{\mu},\right.$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz-\omega t)}{\mu}, 0\Bigg]$$

Poynting vector ExH $=\Bigg[0, 0, \frac{1}{\mu k}\Bigg(4\,\omega\cos(kz-\omega t)^2\Bigg(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x, y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x, y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Bigg)\Bigg)\Bigg]$

Amperian Current 4Vector curlH-dD/dt=J4 $=\Bigg[-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz-\omega t)\left(k^2-\varepsilon\omega^2\mu\right)}{\mu k}, -\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+\right.}{}$

$$\%1=\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right.$$

Amerian charge density divD = rho $=-\frac{1}{k}\Bigg(2\,\varepsilon\,\omega\cos(kz-\omega t)\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x, y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg)$

divergence Lorentz Current 4Vector, 4div(J4) $=0$

Topological SPIN 4 vector S4

$$=\Bigg[\frac{1}{\mu k^2}\Bigg(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x, y)\right)\right)\left(k^2-\varepsilon\,\omega^2\mu\right)\Bigg),$$

$$\frac{1}{\mu k^2}\Bigg(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x, y)\right)\right)\left(k^2-\varepsilon\,\omega^2\mu\right)\Bigg), 0, 0\Bigg]$$

Topological SPIN 3-form $=\frac{1}{\mu k^2}\Bigg(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x, y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\left(k^2-\varepsilon\,\omega^2\mu\right)\ \&\hat{}\,(d(y), d(z), d(t))\Bigg)-\frac{1}{\mu k^2}\Bigg(2\left(x(x,y)^2+y(x,\right.$

$$y)^2)\cos(kz-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\left(k^2-\varepsilon\,\omega^2\mu\right)\,\&\hat{}(d(x),d(z),d(t))\Big)$$

$$\textit{Spin density rho\_spin}=0$$

$$\textit{LaGrange field energy density (B.H-D.E)}=\frac{1}{\mu k^2}\left(4\cos(kz-\omega t)^2\left(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\right)\left(k^2-\varepsilon\,\omega^2\mu\right)\right)$$

$$B.H=\frac{1}{\mu}\left(4\cos(kz-\omega t)^2\left(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\right)\right)$$

$$D.E=\frac{1}{k^2}\left(4\,\varepsilon\,\omega^2\cos(kz-\omega t)^2\left(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\right)\right)$$

$$A.J=-\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\right)\right)$$

$$\textit{-rho.phi}=-\frac{1}{k^2}\left(2\,\omega^2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\,\varepsilon\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\right)\right)$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* $$= \frac{1}{\mu k^2}\left(2\cos(kz-\omega t)^2\left(k^2-\varepsilon\omega^2\mu\right)\left(3x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+4x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+3y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+3x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+4x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+3y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^3\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2x(x,y)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)x(x,y)^2+x(x,y)^3\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2x(x,y)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)x(x,y)^2+\left(\frac{\partial}{\partial x}x(x,y)\right)^2y(x,y)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)y(x,y)^2+y(x,y)^3\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2y(x,y)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)y(x,y)^2+y(x,y)^3\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\right)\right)$$

London Coefficient $$LC=\frac{k^2-\varepsilon\omega^2\mu}{\mu}$$

PROCA coefficient curlcurlB $$=\left[2\cos(kz-\omega t)\left(-2\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}x(x,y)\right)-\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-x(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2}x(x,y)\right)-2\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}y(x,y)\right)-\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)-y(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2}y(x,y)\right)-3\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-x(x,y)\left(\frac{\partial^3}{\partial y^3}x(x,y)\right)-3\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-y(x,y)\left(\frac{\partial^3}{\partial y^3}y(x,y)\right)+k^2x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+k^2y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right), -2\cos(kz-\omega t)\left(k^2x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+k^2y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)-3\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-x(x,y)\left(\frac{\partial^3}{\partial x^3}x(x,y)\right)-3\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)-y(x,y)\left(\frac{\partial^3}{\partial x^3}y(x,y)\right)-2\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}x(x,y)\right)-\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-x(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}x(x,y)\right)-2\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}y(x,y)\right)-\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-y(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}y(x,y)\right)\right), 0\right]$$

Amperian Current 4Vector curlH-dD/dt=J4 $= \left[-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz-\omega t)\left(k^2-\varepsilon\omega^2\mu\right)}{\mu k}, -\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+\right.}{}\right.$

$$\%1 = \left(\frac{\partial}{\partial x}x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right) + \left(\frac{\partial}{\partial x}y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right) + \left(\frac{\partial}{\partial y}x(x,y)\right.$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= \left[\frac{4\cos(kz-\omega t)^2(\%1)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,\right.\right.}{\mu k^2}\right.$

$$\%1 = \left(\frac{\partial}{\partial x}x(x,y)\right.$$

$$\textit{Amperian Dissipation Jampere dot E} \quad = 0$$

$$\textit{Lorentz Force Spin factor LFSPIN} = 0$$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{1}{\mu k^2}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\left(k^2-\varepsilon\omega^2\mu\right)\right),\right.$$

$$\left.\frac{1}{\mu k^2}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\left(k^2-\varepsilon\omega^2\mu\right)\right), 0, 0\right]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B)

$$= \left[0, 0, \frac{1}{\mu k^2}\left(4\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^3\left(k^2-\varepsilon\omega^2\mu\right)\left(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\right)\right)\right]$$

$$\textit{Spin Dissipation J\_spin dot E} \;= -\frac{1}{\mu k^3}\Bigg(4\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^3\left(k^2-\varepsilon\omega^2\mu\right)\omega\Bigg(x(x, y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 + x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x, y)\right)^2\Bigg)\Bigg)$$

$$\textit{Disspative Force 3 vector} = \left[\frac{4\cos(kz-\omega t)^2\,(\%1)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\left(k^2-\varepsilon\omega^2\mu\right)}{\mu k^2}, \frac{4\cos(kz-\omega t)^2\,(\%1)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x\right.\right.}{\mu}\right.$$

$$\%1 = \left(\frac{\partial}{\partial x}x(x,y)\right)^2$$

$$\textit{Dissipation} \;= -\frac{1}{k}\Bigg(2\,\varepsilon\,\omega\cos(kz-\omega t)\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x, y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg)$$

***************************** *END PROCEDURE* ***************************** **(30)**

****

Enter the name of the problem, and the components of the 4 potential
p-2, n=4

```
> NAME:=`Example 7a = Index 1 Irreversible solution EdotB < 0 (kinematic out) Type 1`;
> Holder:=(x^2+y^2+z^2-c^2*t^2)^(4/2);
> Ax:=y/Holder;Ay:=-x/Holder;Az:=c*t/Holder;phi:=+c*z/Holder;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

NAME := Example 7a = Index 1 Irreversible solution EdotB $<$ 0 (kinematic out) Type 1

$$Holder := \left(x^2+y^2+z^2-c^2t^2\right)^2$$

$$Ax := \frac{y}{\left(x^2+y^2+z^2-c^2t^2\right)^2}$$

$$Ay := -\frac{x}{\left(x^2+y^2+z^2-c^2t^2\right)^2}$$

$$Az := \frac{c\,t}{(x^2+y^2+z^2-c^2t^2)^2}$$

$$\phi := \frac{c\,z}{(x^2+y^2+z^2-c^2t^2)^2}$$

Example 7a = Index 1 Irreversible solution EdotB ! 0 (kinematic out) Type 1

******************************** *Differential Form Format* ****************************
****

$$Action\ 1\text{-}form = -\frac{c\,z\,d(t)}{(-x^2-y^2-z^2+c^2t^2)^2} + \frac{y\,d(x)}{(-x^2-y^2-z^2+c^2t^2)^2} - \frac{x\,d(y)}{(-x^2-y^2-z^2+c^2t^2)^2} + \frac{c\,t\,d(z)}{(-x^2-y^2-z^2+c^2t^2)^2}$$

$$Intensity\ 2\text{-}form\ F{=}dA = \left(-\frac{4\,c\,z\,x}{(-x^2-y^2-z^2+c^2t^2)^3} + \frac{4\,y\,c^2\,t}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(x))\ \&\text{^}\ (d(t))$$
$$+\left(-\frac{3\,x^2-y^2-z^2+c^2t^2}{(-x^2-y^2-z^2+c^2t^2)^3} - \frac{-x^2+3\,y^2-z^2+c^2t^2}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(x))\ \&\text{^}\ (d(y)) + \Bigg($$
$$-\frac{4\,y\,z}{(-x^2-y^2-z^2+c^2t^2)^3} + \frac{4\,c\,t\,x}{(-x^2-y^2-z^2+c^2t^2)^3}\Bigg)(d(x))\ \&\text{^}\ (d(z)) + \Bigg($$
$$-\frac{4\,c\,z\,y}{(-x^2-y^2-z^2+c^2t^2)^3} - \frac{4\,x\,c^2\,t}{(-x^2-y^2-z^2+c^2t^2)^3}\Bigg)(d(y))\ \&\text{^}\ (d(t))$$
$$+\left(\frac{4\,x\,z}{(-x^2-y^2-z^2+c^2t^2)^3} + \frac{4\,c\,t\,y}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(y))\ \&\text{^}\ (d(z)) + \Bigg($$
$$-\frac{c\,(-x^2-y^2+3\,z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3} + \frac{c\,(x^2+y^2+z^2+3\,c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3}\Bigg)(d(z))\ \&\text{^}\ (d(t))$$

$$Topological\ Torsion\ 3\text{-}form\quad A\text{^}F = \Bigg(\frac{2\,c\,z\,(x^2+y^2-z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5} + \frac{4\,x\,c\,(c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}$$
$$-\frac{4\,y\,c\,(c\,t\,x+y\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}\Bigg)\&\text{^}(d(x), d(y), d(t)) + \Bigg(-\frac{2\,c\,t\,(x^2+y^2-z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5}$$
$$+\frac{4\,x\,(-y\,z+c\,t\,x)}{(-x^2-y^2-z^2+c^2t^2)^5} + \frac{4\,y\,(c\,t\,y+x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}\Bigg)\&\text{^}(d(x), d(y), d(z))$$
$$+\left(\frac{2\,y\,c\,(x^2+y^2-z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5} - \frac{4\,c\,z\,(-y\,z+c\,t\,x)}{(-x^2-y^2-z^2+c^2t^2)^5} - \frac{4\,c^2\,t\,(c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}\right)\&\text{^}(d(x),$$

$$d(z), d(t)) + \left(\frac{4 c^2 t (c t x + y z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5} - \frac{2 x c (x^2 + y^2 - z^2 + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5} - \frac{4 c z (c t y + x z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5} \right) \&\text{^}(d(y), d(z), d(t))$$

$$\textit{Topological Parity 4-form} \quad F\text{^}F = -\frac{8 c \&\text{^}(d(x), d(y), d(z), d(t))}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}$$

******************************* *Using EM format* *******************************

$$E\,field = \left[\frac{4 c (c t y - x z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, -\frac{4 c (c t x + y z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, \frac{2 c (x^2 + y^2 - z^2 + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3} \right]$$

$$B\,field = \left[\frac{4 (c t y + x z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, -\frac{4 (-y z + c t x)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, -\frac{2 (x^2 + y^2 - z^2 + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3} \right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= \left[\frac{2 x c}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}, \frac{2 y c}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}, \frac{2 c z}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}, \frac{2 c t}{(-x^2 - y^2 - z^2 + c^2 t^2)^4} \right]$

$$\textit{Helicity AdotB} = -\frac{2 c t}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}$$

$$\textit{Poincare II =2(E.B)} = -\frac{8 c}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}$$

$$\textit{coefficient of Topological Parity 4-form} = -\frac{8 c}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}$$

$$\textit{Pfaff Topological Dimension} \quad PTD = 4$$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************
****

$$\textit{Xm or linear (Mean) curvature} = \frac{4 c t z (1 + c^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}$$

Yg or quadratic (GAUSS) curvature =

$$-\frac{3 c^4 t^2 - c^2 t^2 - 3 c^2 z^2 + x^2 c^2 + y^2 c^2 - 3 x^2 + z^2 - 3 y^2}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}$$

$$\textit{Za or Cubic (Interaction internal energy) curvature} = \frac{4 c t z (1 + c^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^7}$$

$$\textit{Tk or quartic (4D expansion) curvature} = -\frac{3 c^2}{(-x^2 - y^2 - z^2 + c^2 t^2)^8}$$

***********   Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH ************

Chirality factor CH $= 0$

$$D\,field = \left[\frac{4\,\varepsilon\,c\,(c\,t\,y - x\,z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, -\frac{4\,\varepsilon\,c\,(c\,t\,x + y\,z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, \frac{2\,\varepsilon\,c\,(x^2 + y^2 - z^2 + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3} \right]$$

$$H\,field = \left[\frac{4\,(c\,t\,y + x\,z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3\,\mu}, -\frac{4\,(-y\,z + c\,t\,x)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3\,\mu}, -\frac{2\,(x^2 + y^2 - z^2 + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3\,\mu} \right]$$

Poynting vector ExH

$$= \left[\frac{16\,c^2\,(x^2 + y^2 - z^2 + c^2 t^2)\,t\,x}{(-x^2 - y^2 - z^2 + c^2 t^2)^6\,\mu}\quad \frac{16\,c^2\,(x^2 + y^2 - z^2 + c^2 t^2)\,t\,y}{(-x^2 - y^2 - z^2 + c^2 t^2)^6\,\mu}\quad \frac{32\,c^2\,t\,z\,(x^2 + y^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^6\,\mu} \right]$$

Amperian Current 4Vector   curlH-dD/dt=J4

$$= \left[\frac{4\,(y\,x^2 + y^3 + y\,z^2 + 5\,y\,c^2 t^2 - 6\,c\,t\,x\,z)\,(\varepsilon\,c^2\,\mu - 1)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4\,\mu}, -\frac{4\,(x^3 + x\,y^2 + x\,z^2 + 5\,c^2 t^2\,x + 6\,c\,t\,y\,z)\,(\varepsilon\,c^2\,\mu - 1)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4\,\mu}, \frac{8\,c\,t\,(2\,x^2 + 2\,y^2 - z^2 + c^2 t^2)\,(\varepsilon\,c^2\,\mu - 1)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4\,\mu}, 0 \right]$$

Amerian charge density   divD = rho $= 0$

divergence Lorentz Current 4Vector,   4div(J4) $= 0$

Topological SPIN 4 vector S4

$$= \left[\frac{2\,(x^3 + x\,y^2 - x\,z^2 + 3\,c^2 t^2\,x - 2\,c\,t\,y\,z + 2\,\varepsilon\,c^3\,\mu\,t\,y\,z - 2\,\varepsilon\,c^2\,\mu\,x\,z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5\,\mu}, -\frac{2\,(-3\,y\,c^2 t^2 - 2\,c\,t\,x\,z - y\,x^2 - y^3 + y\,z^2 + 2\,\varepsilon\,c^3\,\mu\,t\,x\,z + 2\,\varepsilon\,c^2\,\mu\,y\,z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5\,\mu}, \frac{2\,z\,(2\,y^2 + 2\,x^2 + \varepsilon\,c^2\,\mu\,x^2 + \varepsilon\,c^2\,\mu\,y^2 - \varepsilon\,c^2\,z^2\,\mu + \varepsilon\,c^4\,\mu\,t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5\,\mu}, \frac{2\,\varepsilon\,c^2\,t\,(3\,y^2 + 3\,x^2 - z^2 + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5} \right]$$

Topological SPIN 3-form

$$= \frac{2\,(x^3 + x\,y^2 - x\,z^2 + 3\,c^2 t^2\,x - 2\,c\,t\,y\,z + 2\,\varepsilon\,c^3\,\mu\,t\,y\,z - 2\,\varepsilon\,c^2\,\mu\,x\,z^2)\ \&\hat{}(d(y), d(z), d(t))}{(-x^2 - y^2 - z^2 + c^2 t^2)^5\,\mu}$$

$$+ \frac{2\,(-3\,y\,c^2 t^2 - 2\,c\,t\,x\,z - y\,x^2 - y^3 + y\,z^2 + 2\,\varepsilon\,c^3\,\mu\,t\,x\,z + 2\,\varepsilon\,c^2\,\mu\,y\,z^2)\ \&\hat{}(d(x), d(z), d(t))}{(-x^2 - y^2 - z^2 + c^2 t^2)^5\,\mu}$$

$$+ \frac{2\,z\,(2\,y^2 + 2\,x^2 + \varepsilon\,c^2\,\mu\,x^2 + \varepsilon\,c^2\,\mu\,y^2 - \varepsilon\,c^2\,z^2\,\mu + \varepsilon\,c^4\,\mu\,t^2)\ \&\hat{}(d(x), d(y), d(t))}{(-x^2 - y^2 - z^2 + c^2 t^2)^5\,\mu}$$

$$-\frac{2\,\varepsilon\,c^2\,t\,(3\,y^2+3\,x^2-z^2+c^2\,t^2)\ \&\hat{}(d(x),d(y),d(z))}{(-x^2-y^2-z^2+c^2\,t^2)^5}$$

$$\textit{Spin density rho\_spin} = \frac{2\,\varepsilon\,c^2\,t\,(3\,y^2+3\,x^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5}$$

LaGrange field energy density (B.H-D.E) =

$$-\frac{4\,(6\,y^2\,c^2\,t^2+2\,x^2\,z^2+2\,y^2\,z^2+6\,x^2\,c^2\,t^2+x^4+2\,x^2\,y^2+y^4+z^4-2\,c^2\,t^2\,z^2+c^4\,t^4)\,(\varepsilon\,c^2\,\mu-1)}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}$$

$$B.H = \frac{4\,(6\,y^2\,c^2\,t^2+2\,x^2\,z^2+2\,y^2\,z^2+6\,x^2\,c^2\,t^2+x^4+2\,x^2\,y^2+y^4+z^4-2\,c^2\,t^2\,z^2+c^4\,t^4)}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}$$

$$D.E = \frac{4\,\varepsilon\,c^2\,(6\,y^2\,c^2\,t^2+2\,x^2\,z^2+2\,y^2\,z^2+6\,x^2\,c^2\,t^2+x^4+2\,x^2\,y^2+y^4+z^4-2\,c^2\,t^2\,z^2+c^4\,t^4)}{(-x^2-y^2-z^2+c^2\,t^2)^6}$$

$$A.J = \frac{4\,(\varepsilon\,c^2\,\mu-1)\,(2\,x^2\,y^2+y^4+y^2\,z^2+9\,y^2\,c^2\,t^2+x^4+x^2\,z^2+9\,x^2\,c^2\,t^2-2\,c^2\,t^2\,z^2+2\,c^4\,t^4)}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}$$

$$-rho.phi = 0$$

Poincare I (B.H - D.E)-(A.J - rho.phi) =

$$-\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}\Big(4\,(\varepsilon\,c^2\,\mu-1)\,(15\,y^2\,c^2\,t^2+3\,x^2\,z^2+3\,y^2\,z^2+15\,x^2\,c^2\,t^2+2\,x^4$$
$$+4\,x^2\,y^2+2\,y^4+z^4-4\,c^2\,t^2\,z^2+3\,c^4\,t^4)\Big)$$

$$\textit{London Coefficient}\quad LC = 0$$

PROCA coefficient curlcurlB =

$$\Bigg[-\frac{24\,(3\,c\,t\,y\,x^2+3\,c\,t\,y\,z^2+7\,x\,z\,c^2\,t^2+3\,c\,t\,y^3+5\,c^3\,t^3\,y+y^2\,z\,x+x^3\,z+x\,z^3)}{(-x^2-y^2-z^2+c^2\,t^2)^5},$$
$$\frac{24\,(-7\,y\,z\,c^2\,t^2+3\,z^2\,c\,t\,x+3\,c\,t\,x\,y^2-y\,z\,x^2+3\,c\,t\,x^3+5\,c^3\,t^3\,x-y^3\,z-y\,z^3)}{(-x^2-y^2-z^2+c^2\,t^2)^5},$$
$$\frac{8\,(4\,x^2\,y^2+2\,y^4-z^4+x^2\,z^2+y^2\,z^2+5\,c^4\,t^4-4\,c^2\,t^2\,z^2+17\,x^2\,c^2\,t^2+17\,y^2\,c^2\,t^2+2\,x^4)}{(-x^2-y^2-z^2+c^2\,t^2)^5}\Bigg]$$

Amperian Current 4Vector curlH-dD/dt=J4 $=$

$$\Bigg[\frac{4\,(y\,x^2+y^3+y\,z^2+5\,y\,c^2\,t^2-6\,c\,t\,x\,z)\,(\varepsilon\,c^2\,\mu-1)}{(-x^2-y^2-z^2+c^2\,t^2)^4\,\mu},$$
$$-\frac{4\,(x^3+x\,y^2+x\,z^2+5\,c^2\,t^2\,x+6\,c\,t\,y\,z)\,(\varepsilon\,c^2\,\mu-1)}{(-x^2-y^2-z^2+c^2\,t^2)^4\,\mu},$$

$$\frac{8\,c\,t\,(2\,x^2+2\,y^2-z^2+c^2\,t^2)\,(\varepsilon\,c^2\,\mu-1)}{(-x^2-y^2-z^2+c^2\,t^2)^4\,\mu},0\Bigg]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $=\Bigg[$

$$-\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^7\,\mu}\big(8\,(\varepsilon\,c^2\,\mu-1)\,(x^5+2\,x^3\,y^2+14\,x^3\,c^2\,t^2+x\,y^4+14\,x\,y^2\,c^2\,t^2-x\,z^4$$

$$-8\,x\,z^2\,c^2\,t^2+9\,c^4\,t^4\,x-2\,c\,t\,y\,z\,x^2-2\,c\,t\,y^3\,z-2\,c\,t\,y\,z^3+2\,c^3\,t^3\,y\,z)\big),$$

$$-\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^7\,\mu}\big(8\,(\varepsilon\,c^2\,\mu-1)\,(14\,c^2\,t^2\,y\,x^2+2\,c\,t\,x^3\,z+14\,c^2\,t^2\,y^3+2\,c\,t\,y^2\,z\,x$$

$$-8\,c^2\,t^2\,y\,z^2+2\,c\,t\,x\,z^3+9\,c^4\,t^4\,y-2\,c^3\,t^3\,x\,z+y\,x^4+2\,y^3\,x^2+y^5-y\,z^4)\big),$$

$$-\frac{16\,(\varepsilon\,c^2\,\mu-1)\,z\,(x^2+y^2)\,(x^2+y^2+11\,c^2\,t^2+z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^7\,\mu}\Bigg]$$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $=\Bigg[\frac{2\,x\,c}{(-x^2-y^2-z^2+c^2\,t^2)^4},$

$$\frac{2\,y\,c}{(-x^2-y^2-z^2+c^2\,t^2)^4},\frac{2\,c\,z}{(-x^2-y^2-z^2+c^2\,t^2)^4},\frac{2\,c\,t}{(-x^2-y^2-z^2+c^2\,t^2)^4}\Bigg]$$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B)

$$=\Bigg[\frac{4\,y\,c}{(-x^2-y^2-z^2+c^2\,t^2)^6},-\frac{4\,x\,c}{(-x^2-y^2-z^2+c^2\,t^2)^6},\frac{4\,c^2\,t}{(-x^2-y^2-z^2+c^2\,t^2)^6}\Bigg]$$

Torsion Dissipation Jtorsion dot E $=-\frac{4\,c^2\,z}{(-x^2-y^2-z^2+c^2\,t^2)^6}$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD]

$$=\Bigg[\frac{2\,(x^3+x\,y^2-x\,z^2+3\,c^2\,t^2\,x-2\,c\,t\,y\,z+2\,\varepsilon\,c^3\,\mu\,t\,y\,z-2\,\varepsilon\,c^2\,\mu\,x\,z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu},$$

$$-\frac{2\,(-3\,y\,c^2\,t^2-2\,c\,t\,x\,z-y\,x^2-y^3+y\,z^2+2\,\varepsilon\,c^3\,\mu\,t\,x\,z+2\,\varepsilon\,c^2\,\mu\,y\,z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu},$$

$$\frac{2\,z\,(2\,y^2+2\,x^2+\varepsilon\,c^2\,\mu\,x^2+\varepsilon\,c^2\,\mu\,y^2-\varepsilon\,c^2\,z^2\,\mu+\varepsilon\,c^4\,\mu\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu},\ \frac{2\,\varepsilon\,c^2\,t\,(3\,y^2+3\,x^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5}\Bigg]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= \Bigg[$

$$-\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^8\,\mu}(4\,(2\,c\,t\,y^2\,z\,x-4\,c^2\,t^2\,y\,x^2+2\,c\,t\,x^3\,z+4\,c^2\,t^2\,y\,z^2+2\,c\,t\,x\,z^3-2\,c^3\,t^3\,x\,z$$

$$-y^5+6\,\varepsilon\,c^4\,\mu\,t^2\,y^3+2\,\varepsilon\,c^6\,\mu\,t^4\,y-4\,c^2\,t^2\,y^3-3\,c^4\,t^4\,y-2\,y\,x^2\,z^2-2\,\varepsilon\,c^3\,\mu\,t\,x\,z\,y^2-y\,x^4-2\,y^3\,x^2$$

$$-2\,y^3\,z^2-y\,z^4+6\,\varepsilon\,c^4\,\mu\,t^2\,y\,x^2-2\,\varepsilon\,c^3\,\mu\,t\,x^3\,z-2\,\varepsilon\,c^3\,\mu\,t\,x\,z^3+2\,\varepsilon\,c^5\,\mu\,t^3\,x\,z-2\,\varepsilon\,c^4\,\mu\,y\,z^2\,t^2)),$$

$$\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^8\,\mu}(4\,(-x^5-2\,x^3\,y^2-2\,x^3\,z^2-x\,y^4-x\,z^4-2\,x\,y^2\,z^2-3\,c^4\,t^4\,x-4\,x\,y^2\,c^2\,t^2$$

$$+4\,x\,z^2\,c^2\,t^2-2\,c\,t\,y^3\,z-2\,c\,t\,y\,z^3+2\,c^3\,t^3\,y\,z+6\,\varepsilon\,c^4\,\mu\,t^2\,x\,y^2+2\,z\,\varepsilon\,c^3\,\mu\,x^2\,t\,y-4\,x^3\,c^2\,t^2$$

$$+6\,\varepsilon\,c^4\,\mu\,t^2\,x^3+2\,\varepsilon\,c^6\,\mu\,t^4\,x+2\,z\,\varepsilon\,c^3\,\mu\,y^3\,t+2\,\varepsilon\,c^3\,z^3\,\mu\,t\,y-2\,z\,\varepsilon\,c^5\,\mu\,t^3\,y-2\,\varepsilon\,c^4\,\mu\,t^2\,x\,z^2$$

$$-2\,c\,t\,y\,z\,x^2)),\ -\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^8\,\mu}(4\,c\,t\,(6\,\varepsilon\,c^2\,\mu\,x^2\,y^2+3\,\varepsilon\,c^2\,\mu\,y^4+4\,\varepsilon\,c^2\,\mu\,y^2\,z^2$$

$$+4\,\varepsilon\,c^4\,\mu\,y^2\,t^2+3\,\varepsilon\,c^2\,\mu\,x^4+4\,\varepsilon\,c^2\,\mu\,x^2\,z^2+4\,\varepsilon\,c^4\,\mu\,x^2\,t^2+\varepsilon\,c^2\,\mu\,z^4-2\,\varepsilon\,c^4\,\mu\,t^2\,z^2+\varepsilon\,c^6\,\mu\,t^4-4\,x^2\,y^2$$

$$-2\,x^2\,z^2-2\,y^2\,z^2-2\,x^4-6\,x^2\,c^2\,t^2-6\,y^2\,c^2\,t^2-2\,y^4))\Big]$$

Spin Dissipation J_spin dot E $= \frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^8\,\mu}(4\,c^3\,z\,(6\,\varepsilon\,c^2\,\mu\,t^2\,y^2+6\,\varepsilon\,c^2\,\mu\,t^2\,x^2+\varepsilon\,\mu\,x^4$

$$+\varepsilon\,\mu\,y^4-2\,\varepsilon\,c^2\,\mu\,t^2\,z^2+\varepsilon\,c^4\,\mu\,t^4+2\,\varepsilon\,\mu\,x^2\,z^2+2\,\varepsilon\,\mu\,y^2\,z^2-8\,y^2\,t^2-8\,x^2\,t^2+\varepsilon\,\mu\,z^4+2\,\varepsilon\,\mu\,x^2\,y^2))$$

Disspative Force 3 vector $= \Bigg[-\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^8\,\mu}(4\,(2\,\mu\,c\,t\,y^2\,z\,x+6\,\varepsilon\,c^4\,\mu^2\,t^2\,y\,x^2$

$$-2\,\varepsilon\,c^3\,\mu^2\,t\,x^3\,z-2\,\varepsilon\,c^3\,\mu^2\,t\,x\,z^3+2\,\varepsilon\,c^5\,\mu^2\,t^3\,x\,z-2\,\varepsilon\,c^4\,\mu^2\,y\,z^2\,t^2+2\,y\,c^3\,\mu\,x^2\,t^2+2\,x^7+4\,x^3\,y^2\,z^2$$

$$+2\,y^4\,x\,z^2-2\,y^2\,z^4\,x+26\,x^5\,c^2\,t^2-10\,x^3\,c^4\,t^4-8\,x^2\,y^3\,c\,t\,z+12\,y^2\,z^2\,c^2\,t^2\,x+8\,x^2\,c^3\,t^3\,y\,z-4\,x^4\,c\,t\,y\,z$$

$-2x^5\varepsilon c^2\mu z^2-8x^2z^3cty+2x^3z^4\varepsilon c^2\mu-6x^5\varepsilon c^2\mu y^2-26x^5\varepsilon c^4\mu t^2-6x^3y^4\varepsilon c^2\mu$

$+10x^3c^6t^4\varepsilon\mu-2xy^6\varepsilon c^2\mu+2xz^6\varepsilon c^2\mu+18c^8t^6x\varepsilon\mu-4\mu c^2t^2yx^2+2\mu ctx^3z+4\mu c^2t^2yz^2$

$+2\mu ctxz^3-2\mu c^3t^3xz+6\varepsilon c^4\mu^2t^2y^3+2\varepsilon c^6\mu^2t^4y-2yc\mu x^2z^2+2yc^3\mu t^2z^2+8x^2y^3\varepsilon c^3\mu tz$

$-12y^2c^4t^2\varepsilon\mu xz^2-8x^2c^5t^3\varepsilon\mu yz+4x^4\varepsilon c^3\mu tyz+8x^2z^3\varepsilon c^3\mu ty-\mu yz^4-\mu yx^4-18c^6t^6x$

$-2\mu y^3z^2-2\mu y^3x^2-y^5c\mu-14xz^4c^2t^2+34xz^2c^4t^4-4ctyz^5+8c^3t^3yz^3-4c^5t^5yz$

$-4\mu c^2t^2y^3-3\mu c^4t^4y-2\mu yx^2z^2-2y^3c\mu x^2-yc\mu x^4+2y^3c^3\mu t^2-2y^3c\mu z^2-yc\mu z^4$

$-yc^5\mu t^4-2xz^6-\mu y^5-4x^3y^2\varepsilon c^2\mu z^2+4y^5\varepsilon c^3\mu tz-2y^4\varepsilon c^2\mu xz^2+8y^3z^3\varepsilon c^3\mu t$

$+2y^2z^4\varepsilon c^2\mu x-8y^3c^5t^3\varepsilon\mu z-12x^3c^4t^2\varepsilon\mu z^2-52x^3y^2\varepsilon c^4\mu t^2-26xy^4\varepsilon c^4\mu t^2$

$+10xy^2c^6t^4\varepsilon\mu+14xz^4\varepsilon c^4\mu t^2-34xz^2c^6t^4\varepsilon\mu+4c^3tyz^5\varepsilon\mu-8c^5t^3yz^3\varepsilon\mu+4c^7t^5yz\varepsilon\mu$

$+2x^5z^2-2x^3z^4+6x^5y^2+6x^3y^4+2y^6x+52x^3y^2c^2t^2+26y^4c^2t^2x-4y^5ctz-8y^3z^3ct$

$-10y^2c^4t^4x+8y^3c^3t^3z+12x^3c^2t^2z^2-2x^7\varepsilon c^2\mu-2\varepsilon c^3\mu^2txzy^2)),$

$-\frac{1}{(-x^2-y^2-z^2+c^2t^2)^8\mu}(4(2y^7+26c^2t^2y^5-10c^4t^4y^3-2yz^4x^2+4x^2y^3z^2+2x^4yz^2-2yz^6$

$+\mu x^5+8ctx^3zy^2-26c^4t^2y^5\varepsilon\mu+10c^6t^4y^3\varepsilon\mu+4cty^4zx+12c^2t^2yz^2x^2+8ctxz^3y^2$

$+18c^8t^6y\varepsilon\mu-8c^3t^3xzy^2-2yx^6\varepsilon c^2\mu-6y^3x^4\varepsilon c^2\mu-6y^5x^2\varepsilon c^2\mu-2y^5\varepsilon c^2z^2\mu$

$$+ 2 y^3 z^4 \varepsilon c^2 \mu + 2 y z^6 \varepsilon c^2 \mu - 14 c^2 t^2 y z^4 + 34 c^4 t^4 y z^2 + 4 c t x z^5 - 8 c^3 t^3 x z^3 + 4 c^5 t^5 x z$$

$$+ 2 \mu x y^2 z^2 + 3 \mu c^4 t^4 x + 4 \mu x^3 c^2 t^2 + 2 x^3 c \mu y^2 - 2 x^3 c^3 \mu t^2 + 2 x^3 c \mu z^2 + x c \mu y^4 + x c \mu z^4$$

$$+ x c^5 \mu t^4 - 18 c^6 t^6 y + 2 \mu x^3 y^2 + 2 \mu x^3 z^2 + \mu x y^4 + \mu x z^4 + x^5 c \mu - 12 c^4 t^2 y x^2 \varepsilon z^2 \mu$$

$$- 8 c^3 t x^3 z \varepsilon \mu y^2 - 4 c^3 t y^4 z x \varepsilon \mu - 8 c^3 t y^2 z^3 x \varepsilon \mu + 8 c^5 t^3 y^2 z x \varepsilon \mu + 4 \mu x y^2 c^2 t^2 - 4 \mu x z^2 c^2 t^2$$

$$+ 2 \mu c t y^3 z + 2 \mu c t y z^3 - 2 \mu c^3 t^3 y z - 6 \varepsilon c^4 \mu^2 t^2 x^3 - 2 \varepsilon c^6 \mu^2 t^4 x + 2 x c \mu y^2 z^2 - 2 x c^3 \mu t^2 z^2$$

$$- 2 x c^3 \mu y^2 t^2 + 6 y^3 x^4 + 2 y x^6 + 6 y^5 x^2 - 2 y^3 z^4 + 2 y^5 z^2 + 52 c^2 t^2 y^3 x^2 + 26 c^2 t^2 y x^4 + 4 c t x^5 z$$

$$+ 12 c^2 t^2 y^3 z^2 + 8 c t x^3 z^3 - 10 c^4 t^4 y x^2 - 8 c^3 t^3 x^3 z - 2 y^7 \varepsilon c^2 \mu - 6 \varepsilon c^4 \mu^2 t^2 x y^2 - 2 z \varepsilon c^3 \mu^2 y^3 t$$

$$- 2 \varepsilon c^3 z^3 \mu^2 t y + 2 z \varepsilon c^5 \mu^2 t^3 y + 2 \varepsilon c^4 \mu^2 t^2 x z^2 + 2 \mu c t y z x^2 - 26 c^4 t^2 y x^4 \varepsilon \mu - 52 c^4 t^2 y^3 x^2 \varepsilon \mu$$

$$+ 10 c^6 t^4 y x^2 \varepsilon \mu - 4 c^3 t x^5 z \varepsilon \mu - 8 c^3 t x^3 z^3 \varepsilon \mu + 8 c^5 t^3 x^3 z \varepsilon \mu - 12 c^4 t^2 y^3 \varepsilon z^2 \mu + 14 c^4 t^2 y z^4 \varepsilon \mu$$

$$- 34 c^6 t^4 y z^2 \varepsilon \mu - 4 c^3 t x z^5 \varepsilon \mu + 8 c^5 t^3 x z^3 \varepsilon \mu - 4 c^7 t^5 x z \varepsilon \mu - 2 y x^4 \varepsilon c^2 z^2 \mu - 4 y^3 x^2 \varepsilon c^2 z^2 \mu$$

$$+ 2 y z^4 \varepsilon c^2 \mu x^2 - 2 z \varepsilon c^3 \mu^2 x^2 t y)), \; - \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^8 \mu} (4 (-16 \varepsilon c^2 \mu x^2 y^2 z^3$$
$$- 40 \varepsilon c^4 \mu x^2 t^2 z^3 - 40 \varepsilon c^4 \mu y^2 t^2 z^3 - 12 z \varepsilon c^2 \mu x^4 y^2 - 40 z \varepsilon c^4 \mu x^4 t^2 - 12 z \varepsilon c^2 \mu x^2 y^4$$
$$+ 44 z \varepsilon c^6 \mu x^2 t^4 - 40 z \varepsilon c^4 \mu y^4 t^2 + 44 z \varepsilon c^6 \mu y^2 t^4 + 6 c^3 t \mu^2 \varepsilon x^2 y^2 + 4 c^3 t \mu^2 \varepsilon y^2 z^2 + 4 c^3 t \mu^2 \varepsilon x^2 z^2$$
$$- 80 z \varepsilon c^4 \mu x^2 y^2 t^2 + 3 c^3 t \mu^2 \varepsilon y^4 - 2 c^2 t \mu x^2 y^2 + 16 x^2 y^2 z^3 + 12 z x^4 y^2 + 12 z x^2 y^4 - c^6 t^5 \mu + 4 z x^6$$
$$+ 8 y^4 z^3 + 4 y^2 z^5 + 8 x^4 z^3 + 4 x^2 z^5 + 40 z y^4 c^2 t^2 - 44 z y^2 c^4 t^4 + 40 z x^4 c^2 t^2 - 44 z x^2 c^4 t^4$$
$$+ 40 y^2 z^3 c^2 t^2 + 40 x^2 c^2 t^2 z^3 + c^7 t^5 \mu^2 \varepsilon - 2 c t \mu x^4 - 6 c^3 t^3 \mu x^2 - 6 c^3 t^3 \mu y^2 - 2 c t \mu y^4 + 2 c^4 t^3 \mu x^2$$
$$- c^2 t \mu x^4 + 2 c^4 t^3 \mu y^2 - c^2 t \mu y^4 + 2 c^4 t^3 \mu z^2 - c^2 t \mu z^4 + 4 z y^6 + 80 z x^2 y^2 c^2 t^2 - 4 z \varepsilon c^2 \mu x^6$$
$$- 4 z \varepsilon c^2 \mu y^6 - 8 \varepsilon c^2 \mu x^4 z^3 - 4 \varepsilon c^2 \mu x^2 z^5 - 8 \varepsilon c^2 \mu y^4 z^3 - 4 \varepsilon c^2 \mu y^2 z^5 + 4 c^5 t^3 \mu^2 \varepsilon y^2$$

$$+3c^3 t \mu^2 \varepsilon x^4 + 4c^5 t^3 \mu^2 \varepsilon x^2 + c^3 t \mu^2 \varepsilon z^4 - 2c^5 t^3 \mu^2 \varepsilon z^2 - 4ct\mu x^2 y^2 - 2ct\mu x^2 z^2 - 2ct\mu y^2 z^2$$

$$-2c^2 t\mu x^2 z^2 - 2c^2 t \mu y^2 z^2)))]$$

$$Dissipation = \frac{2c\,(3cty^2\varepsilon\mu + 3x^2 ct\varepsilon\mu - \mu\varepsilon ctz^2 + \mu\varepsilon c^3 t^3 - x^3 - xy^2 - xz^2 + c^2 t^2 x)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}$$

******************************* END PROCEDURE ******************************** **(31)**

```
> NAME:=`Example 7b = Index 1 Irreversible solution EdotB < 0 (kinematic out) Type 1`;
> Holder:=(x^2+y^2+z^2-c^2*t^2)^(4/2);

> Ax:=m*y/Holder;Ay:=-m*x/Holder;Az:=-c*t/Holder;phi:=z*c/Holder;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 7b = Index 1 Irreversible solution EdotB < 0 (kinematic out) Type 1

$$Holder := (x^2 + y^2 + z^2 - c^2 t^2)^2$$

$$Ax := \frac{my}{(x^2 + y^2 + z^2 - c^2 t^2)^2}$$

$$Ay := -\frac{mx}{(x^2 + y^2 + z^2 - c^2 t^2)^2}$$

$$Az := -\frac{ct}{(x^2 + y^2 + z^2 - c^2 t^2)^2}$$

$$\phi := \frac{cz}{(x^2 + y^2 + z^2 - c^2 t^2)^2}$$

Example 7b = Index 1 Irreversible solution EdotB ! 0 (kinematic out) Type 1

********************************* Differential Form Format *********************************

$$Action\ 1\text{-}form = -\frac{cz\,d(t)}{(-x^2 - y^2 - z^2 + c^2 t^2)^2} + \frac{my\,d(x)}{(-x^2 - y^2 - z^2 + c^2 t^2)^2} - \frac{mx\,d(y)}{(-x^2 - y^2 - z^2 + c^2 t^2)^2} - \frac{ct\,d(z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^2}$$

$$Intensity\ 2\text{-}form\ F{=}dA = \left(-\frac{4czx}{(-x^2 - y^2 - z^2 + c^2 t^2)^3} + \frac{4myc^2 t}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}\right)(d(x))\ \&\text{^}\ (d(t))$$

$$+\left(-\frac{m\left(3x^2-y^2-z^2+c^2t^2\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}-\frac{m\left(-x^2+3y^2-z^2+c^2t^2\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}\right)(d(x))\ \&\wedge\ (d(y))+\Bigg($$

$$-\frac{4myz}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}-\frac{4ctx}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}\Bigg)(d(x))\ \&\wedge\ (d(z))+\Bigg($$

$$-\frac{4czy}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}-\frac{4mxc^2t}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}\Bigg)(d(y))\ \&\wedge\ (d(t))$$

$$+\left(\frac{4mxz}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}-\frac{4cty}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}\right)(d(y))\ \&\wedge\ (d(z))+\Bigg($$

$$-\frac{c\left(-x^2-y^2+3z^2+c^2t^2\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}-\frac{c\left(x^2+y^2+z^2+3c^2t^2\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}\Bigg)(d(z))\ \&\wedge\ (d(t))$$

Topological Torsion 3-form $\quad A\wedge F=\Bigg(\frac{2czm\left(x^2+y^2-z^2+c^2t^2\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}+\frac{4mxc\,(mcty-xz)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}$

$$-\frac{4myc\,(mxct+yz)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}\Bigg)\&\wedge(d(x),d(y),d(t))+\Bigg(\frac{2ctm\left(x^2+y^2-z^2+c^2t^2\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}$$

$$-\frac{4mx\,(myz+ctx)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}-\frac{4my\,(cty-mxz)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}\Bigg)\&\wedge(d(x),d(y),d(z))+\Bigg($$

$$-\frac{4myc\left(c^2t^2+z^2\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}+\frac{4cz\,(myz+ctx)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}+\frac{4c^2t\,(mcty-xz)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}\Bigg)\&\wedge(d(x),d(z),$$

$$d(t))+\Bigg(-\frac{4c^2t\,(mxct+yz)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}+\frac{4mxc\left(c^2t^2+z^2\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}$$

$$+\frac{4cz\,(cty-mxz)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}\Bigg)\&\wedge(d(y),d(z),d(t))$$

Topological Parity 4-form $\quad F\wedge F=\frac{16mc\left(c^2t^2+z^2\right)\&\wedge(d(x),d(y),d(z),d(t))}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}$

******************************** *Using EM format* ********************************

$$E\,field=\left[\frac{4c\,(mcty-xz)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3},-\frac{4c\,(mxct+yz)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3},-\frac{4c\left(c^2t^2+z^2\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}\right]$$

$$B\,field=\left[-\frac{4\,(cty-mxz)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3},\frac{4\,(myz+ctx)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3},-\frac{2m\left(x^2+y^2-z^2+c^2t^2\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}\right]$$

Topological TORSION 4 vector $\quad T4 = -[ExA + Bphi, AdotB] = \Bigg[0, 0, \frac{2czm}{\left(-x^2-y^2-z^2+c^2t^2\right)^4},$

$$- \frac{2\, m\, c\, t}{(-x^2 - y^2 - z^2 + c^2 t^2)^4} \Bigg]$$

$$\textit{Helicity AdotB} = \frac{2\, m\, c\, t}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}$$

$$\textit{Poincare II =2(E.B)} = \frac{16\, m\, c\, (c^2 t^2 + z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}$$

$$\textit{coefficient of Topological Parity 4-form} \; = \frac{16\, m\, c\, (c^2 t^2 + z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}$$

Pfaff Topological Dimension $\quad PTD = 4$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************************

$$\textit{Xm or linear (Mean) curvature} \; = \frac{4\, c\, t\, z\, (c-1)\, (1+c)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}$$

Yg or quadratic (GAUSS) curvature

$$= \frac{3\, c^4 t^2 + m^2 c^2 t^2 - 3\, c^2 z^2 + x^2 c^2 + y^2 c^2 - m^2 z^2 + 3\, m^2 y^2 + 3\, m^2 x^2}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}$$

$$\textit{Za or Cubic (Interaction internal energy) curvature} \; = \frac{4\, m^2 c\, t\, z\, (c-1)\, (1+c)}{(-x^2 - y^2 - z^2 + c^2 t^2)^7}$$

$$\textit{Tk or quartic (4D expansion) curvature} \; = \frac{3\, m^2 c^2}{(-x^2 - y^2 - z^2 + c^2 t^2)^8}$$

********* *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor $CH = 0$

$$\textit{D field} = \left[\frac{4\, \varepsilon\, c\, (m\, c\, t\, y - x\, z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, - \frac{4\, \varepsilon\, c\, (m\, x\, c\, t + y\, z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, - \frac{4\, \varepsilon\, c\, (c^2 t^2 + z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3} \right]$$

$$\textit{H field} = \left[- \frac{4\, (c\, t\, y - m\, x\, z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3 \mu}, \frac{4\, (m\, y\, z + c\, t\, x)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3 \mu}, - \frac{2\, m\, (x^2 + y^2 - z^2 + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3 \mu} \right]$$

$$\textit{Poynting vector ExH} = \Bigg[\frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^6 \mu} \big(8\, c\, \big(m^2 x^3 c\, t + m^2 x\, c\, t\, y^2 - m^2 x\, c\, t\, z^2 + m^2 x\, c^3 t^3$$

$$+ m\, y\, z\, x^2 + m\, y^3 z + m\, y\, z^3 + 3\, m\, y\, z\, c^2 t^2 + 2\, c^3 t^3 x + 2\, z^2 c\, t\, x\big)\big),$$

$$\frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^6 \mu} \big(8\, c\, \big(2\, c^3 t^3 y - 3\, m\, x\, z\, c^2 t^2 + 2\, c\, t\, y\, z^2 - m\, x\, z^3 + m^2 c\, t\, y\, x^2 + m^2 c\, t\, y^3$$

$$- m^2 c t y z^2 + m^2 c^3 t^3 y - m x^3 z - m x z y^2)), \frac{16 c^2 t z (m^2 y^2 - x^2 + m^2 x^2 - y^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^6 \mu}\Bigg]$$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \Bigg[\frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^4 \mu} (4 (-m y x^2 - m y^3 - m y z^2$

$- 5 m y c^2 t^2 - 6 c t x z + \varepsilon c^2 \mu m y x^2 + \varepsilon c^2 \mu m y^3 + \varepsilon c^2 \mu m y z^2 + 5 \varepsilon c^4 \mu m y t^2 - 6 \varepsilon c^3 \mu t x z))$

$, - \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^4 \mu} (4 (-m x^3 - m x y^2 - m x z^2 - 5 m x c^2 t^2 + 6 c t y z + \varepsilon c^2 \mu m x^3$

$+ \varepsilon c^2 \mu m x y^2 + \varepsilon c^2 \mu m x z^2 + 5 \varepsilon c^4 \mu m x t^2 + 6 \varepsilon c^3 \mu t y z)),$

$- \frac{8 c t (-2 x^2 - 2 y^2 + z^2 - c^2 t^2 + \varepsilon c^2 \mu x^2 + \varepsilon c^2 \mu y^2 + 4 \varepsilon c^2 z^2 \mu + 2 \varepsilon c^4 \mu t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4 \mu},$

$- \frac{8 \varepsilon c z (x^2 + y^2 + z^2 + 5 c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4} \Bigg]$

Amerian charge density *divD = rho* $= - \frac{8 \varepsilon c z (x^2 + y^2 + z^2 + 5 c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}$

divergence Lorentz Current 4Vector, *4div(J4)* $= 0$

Topological SPIN 4 vector S4

$= \Bigg[\frac{2 (m^2 x^3 + m^2 x y^2 - m^2 x z^2 + m^2 x c^2 t^2 + 2 c t m y z + 2 c^2 t^2 x + 2 \varepsilon c^3 z \mu m t y - 2 \varepsilon c^2 \mu x z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5 \mu},$

$- \frac{2 (-2 y c^2 t^2 + 2 c t m x z - m^2 y x^2 - m^2 y^3 + m^2 y z^2 - m^2 y c^2 t^2 + 2 \varepsilon c^3 z \mu m x t + 2 \varepsilon c^2 \mu y z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5 \mu},$

$- \frac{4 z (-m^2 y^2 - m^2 x^2 + \varepsilon c^4 \mu t^2 + \varepsilon c^2 z^2 \mu)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5 \mu}, \frac{4 \varepsilon c^2 t (m^2 y^2 + m^2 x^2 + c^2 t^2 + z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5} \Bigg]$

Topological SPIN 3-form $= \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^5 \mu} (2 (m^2 x^3 + m^2 x y^2 - m^2 x z^2 + m^2 x c^2 t^2 + 2 c t m y z$

$+ 2 c^2 t^2 x + 2 \varepsilon c^3 z \mu m t y - 2 \varepsilon c^2 \mu x z^2)$ &^(d(y), d(z), d(t)))

$+ \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^5 \mu} (2 (-2 y c^2 t^2 + 2 c t m x z - m^2 y x^2 - m^2 y^3 + m^2 y z^2 - m^2 y c^2 t^2$

$+ 2 \varepsilon c^3 z \mu m x t + 2 \varepsilon c^2 \mu y z^2)$ &^(d(x), d(z), d(t)))

$$-\frac{4\,z\left(-m^2 y^2 - m^2 x^2 + \varepsilon\, c^4 \mu\, t^2 + \varepsilon\, c^2 z^2 \mu\right) \&\wedge(d(x), d(y), d(t))}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5 \mu}$$

$$-\frac{4\,\varepsilon\, c^2\, t\left(m^2 y^2 + m^2 x^2 + c^2 t^2 + z^2\right) \&\wedge(d(x), d(y), d(z))}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5}$$

$$\textit{Spin density rho\_spin} = \frac{4\,\varepsilon\, c^2\, t\left(m^2 y^2 + m^2 x^2 + c^2 t^2 + z^2\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5}$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^6 \mu}\Big(4\,\big(-4\,y^2 c^2 t^2 - 2\,m^2 x^2 z^2 - 2\,m^2 y^2 z^2 - 4\,x^2 c^2 t^2 - m^2 x^4 - 2\,m^2 y^2 x^2 - 2\,m^2 x^2 c^2 t^2 - m^2 y^4 - 2\,m^2 y^2 c^2 t^2 - m^2 z^4 + 2\,m^2 c^2 t^2 z^2 - m^2 c^4 t^4 + 4\,m^2 y^2 \varepsilon\, c^4 \mu\, t^2 + 4\,\varepsilon\, c^2 \mu\, x^2 z^2 + 4\,m^2 x^2 \varepsilon\, c^4 \mu\, t^2 + 4\,\varepsilon\, c^2 \mu\, y^2 z^2 + 4\,\varepsilon\, c^6 \mu\, t^4 + 8\,\varepsilon\, c^4 \mu\, t^2 z^2 + 4\,\varepsilon\, c^2 \mu\, z^4\big)\Big)$$

$$\textit{B.H} = \frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^6 \mu}\Big(4\,\big(4\,y^2 c^2 t^2 + 2\,m^2 x^2 z^2 + 2\,m^2 y^2 z^2 + 4\,x^2 c^2 t^2 + m^2 x^4 + 2\,m^2 y^2 x^2 + 2\,m^2 x^2 c^2 t^2 + m^2 y^4 + 2\,m^2 y^2 c^2 t^2 + m^2 z^4 - 2\,m^2 c^2 t^2 z^2 + m^2 c^4 t^4\big)\Big)$$

$$\textit{D.E} = \frac{16\,\varepsilon\, c^2\left(m^2 y^2 c^2 t^2 + x^2 z^2 + m^2 x^2 c^2 t^2 + y^2 z^2 + c^4 t^4 + 2\,c^2 t^2 z^2 + z^4\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^6}$$

$$\textit{A.J} = \frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^6 \mu}\Big(4\,\big(m^2 y^4 \varepsilon\, c^2 \mu + m^2 x^4 \varepsilon\, c^2 \mu - 2\,c^4 t^4 + 2\,\varepsilon\, c^4 \mu\, y^2 t^2 + 2\,\varepsilon\, c^4 \mu\, x^2 t^2 + 8\,\varepsilon\, c^4 \mu\, t^2 z^2 - m^2 x^2 z^2 - m^2 y^2 z^2 - m^2 x^4 - m^2 y^4 + m^2 y^2 \varepsilon\, c^2 \mu\, z^2 + 2\,m^2 y^2 \varepsilon\, c^2 \mu\, x^2 + 5\,m^2 y^2 \varepsilon\, c^4 \mu\, t^2 + m^2 x^2 \varepsilon\, c^2 \mu\, z^2 + 5\,m^2 x^2 \varepsilon\, c^4 \mu\, t^2 + 2\,c^2 t^2 z^2 - 4\,x^2 c^2 t^2 - 4\,y^2 c^2 t^2 - 2\,m^2 y^2 x^2 - 5\,m^2 x^2 c^2 t^2 - 5\,m^2 y^2 c^2 t^2 + 4\,\varepsilon\, c^6 \mu\, t^4\big)\Big)$$

$$\textit{-rho.phi} = -\frac{8\,c^2 z^2 \varepsilon\left(x^2 + y^2 + z^2 + 5\,c^2 t^2\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^6}$$

$$\textit{Poincare I} \quad \textit{(B.H - D.E)-(A.J - rho.phi)} = -\frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^6 \mu}\Big(4\,\big(m^2 y^4 \varepsilon\, c^2 \mu + m^2 x^4 \varepsilon\, c^2 \mu - 2\,c^4 t^4 + 2\,\varepsilon\, c^4 \mu\, y^2 t^2 + 6\,\varepsilon\, c^2 \mu\, x^2 z^2 + 6\,\varepsilon\, c^2 \mu\, y^2 z^2 + 2\,\varepsilon\, c^4 \mu\, x^2 t^2 + 26\,\varepsilon\, c^4 \mu\, t^2 z^2 - 3\,m^2 x^2 z^2 - 3\,m^2 y^2 z^2 - m^2 c^4 t^4 - 2\,m^2 x^4 - 2\,m^2 y^4 - m^2 z^4 + m^2 y^2 \varepsilon\, c^2 \mu\, z^2 + 2\,m^2 y^2 \varepsilon\, c^2 \mu\, x^2 + 9\,m^2 y^2 \varepsilon\, c^4 \mu\, t^2 + m^2 x^2 \varepsilon\, c^2 \mu\, z^2 + 9\,m^2 x^2 \varepsilon\, c^4 \mu\, t^2 + 2\,c^2 t^2 z^2 - 8\,x^2 c^2 t^2 - 8\,y^2 c^2 t^2 - 4\,m^2 y^2 x^2 - 7\,m^2 x^2 c^2 t^2 - 7\,m^2 y^2 c^2 t^2 + 2\,m^2 c^2 t^2 z^2 + 6\,\varepsilon\, c^2 \mu\, z^4 + 8\,\varepsilon\, c^6 \mu\, t^4\big)\Big)$$

$$\textit{London Coefficient} \quad LC = 0$$

PROCA coefficient curlcurlB

$$= \left[\frac{24\left(-m x^3 z - m x z^3 + 3 c t y x^2 + 3 c t y z^2 + 3 c t y^3 + 5 c^3 t^3 y - m x z y^2 - 7 m x z c^2 t^2\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5}, \right.$$

$$- \frac{24\left(m y^3 z + m y z^3 + 3 z^2 c t x + 3 c t x y^2 + 3 c t x^3 + 5 c^3 t^3 x + m y z x^2 + 7 m y z c^2 t^2\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5},$$

$$\left. \frac{8 m\left(4 x^2 y^2 + 2 y^4 - z^4 + x^2 z^2 + y^2 z^2 + 5 c^4 t^4 - 4 c^2 t^2 z^2 + 17 x^2 c^2 t^2 + 17 y^2 c^2 t^2 + 2 x^4\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5} \right]$$

Amperian Current 4Vector curlH-dD/dt=J4 $= \left[\frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^4 \mu} \left(4\left(-m y x^2 - m y^3 - m y z^2\right.\right.\right.$

$$\left.\left. - 5 m y c^2 t^2 - 6 c t x z + \varepsilon c^2 \mu m y x^2 + \varepsilon c^2 \mu m y^3 + \varepsilon c^2 \mu m y z^2 + 5 \varepsilon c^4 \mu m y t^2 - 6 \varepsilon c^3 \mu t x z\right)\right)$$

$$, - \frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^4 \mu} \left(4\left(-m x^3 - m x y^2 - m x z^2 - 5 m x c^2 t^2 + 6 c t y z + \varepsilon c^2 \mu m x^3\right.\right.$$

$$\left.\left. + \varepsilon c^2 \mu m x y^2 + \varepsilon c^2 \mu m x z^2 + 5 \varepsilon c^4 \mu m x t^2 + 6 \varepsilon c^3 \mu t y z\right)\right),$$

$$- \frac{8 c t\left(-2 x^2 - 2 y^2 + z^2 - c^2 t^2 + \varepsilon c^2 \mu x^2 + \varepsilon c^2 \mu y^2 + 4 \varepsilon c^2 z^2 \mu + 2 \varepsilon c^4 \mu t^2\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^4 \mu},$$

$$\left. - \frac{8 \varepsilon c z\left(x^2 + y^2 + z^2 + 5 c^2 t^2\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^4} \right]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= \Bigg[$

$$- \frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^7 \mu} \left(8\left(-m^2 x^5 - 4 c^4 t^4 x + 4 \varepsilon c^2 z^2 \mu y^2 x - 8 x y^2 c^2 t^2 + 4 x z^2 c^2 t^2\right.\right.$$

$$+ 4 \varepsilon c^2 z^2 \mu x^3 + 4 \varepsilon c^2 z^4 \mu x - 2 m c t y^3 z - 2 m c t y z^3 + 2 m c^3 t^3 y z - 6 m^2 x y^2 c^2 t^2 + 4 m^2 x z^2 c^2 t^2$$

$$+ \varepsilon c^2 \mu m^2 x^5 + 4 \varepsilon c^4 \mu t^2 x y^2 + 6 \varepsilon c^4 \mu m^2 x^3 t^2 + \varepsilon c^2 \mu m^2 x y^4 + 2 \varepsilon c^2 \mu m^2 x^3 y^2 - 2 m c t y z x^2$$

$$+ 5 \varepsilon c^6 \mu m^2 x t^4 + 6 m z \varepsilon c^3 \mu x^2 t y - \varepsilon c^2 \mu m^2 x z^4 - m^2 x y^4 + m^2 x z^4 - 6 m z \varepsilon c^5 \mu t^3 y$$

$$+ 6 m \varepsilon c^3 z^3 \mu t y + 6 m z \varepsilon c^3 \mu y^3 t + 6 \varepsilon c^4 \mu m^2 x y^2 t^2 - 4 \varepsilon c^4 \mu m^2 x z^2 t^2 - 2 m^2 x^3 y^2 - 6 m^2 x^3 c^2 t^2$$

$-5\,m^2\,x\,c^4\,t^4-8\,x^3\,c^2\,t^2+4\,\varepsilon\,c^4\,\mu\,t^2\,x^3+8\,\varepsilon\,c^6\,\mu\,t^4\,x+36\,\varepsilon\,c^4\,\mu\,t^2\,x\,z^2)\,)$,

$-\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^7\,\mu}\,(8\,(-8\,c^2\,t^2\,y\,x^2+4\,c^2\,t^2\,y\,z^2+4\,\varepsilon\,c^2\,z^2\,\mu\,y^3+4\,\varepsilon\,c^2\,z^4\,\mu\,y-2\,m^2\,y^3\,x^2$

$+m^2\,y\,z^4-m^2\,y\,x^4-6\,m^2\,y\,x^2\,c^2\,t^2+4\,m^2\,y\,z^2\,c^2\,t^2+\varepsilon\,c^2\,\mu\,m^2\,y^5+2\,c\,t\,m\,x^3\,z+2\,c\,t\,m\,x\,z^3$

$-2\,c^3\,t^3\,m\,x\,z+6\,\varepsilon\,c^4\,\mu\,m^2\,y\,x^2\,t^2-4\,\varepsilon\,c^4\,\mu\,m^2\,y\,z^2\,t^2-6\,c^3\,t\,\varepsilon\,\mu\,x^3\,m\,z-6\,c^3\,t\,\varepsilon\,z^3\,\mu\,m\,x$

$+6\,c^5\,t^3\,\varepsilon\,\mu\,m\,x\,z+4\,\varepsilon\,c^4\,\mu\,t^2\,y^3+8\,\varepsilon\,c^6\,\mu\,t^4\,y-8\,c^2\,t^2\,y^3-4\,c^4\,t^4\,y-6\,m^2\,y^3\,c^2\,t^2-5\,m^2\,y\,c^4\,t^4$

$-m^2\,y^5+4\,\varepsilon\,c^2\,z^2\,\mu\,y\,x^2-6\,c^3\,t\,\varepsilon\,\mu\,y^2\,m\,x\,z-\varepsilon\,c^2\,\mu\,m^2\,y\,z^4+5\,\varepsilon\,c^6\,\mu\,m^2\,y\,t^4+\varepsilon\,c^2\,\mu\,m^2\,y\,x^4$

$+2\,\varepsilon\,c^2\,\mu\,m^2\,y^3\,x^2+6\,\varepsilon\,c^4\,\mu\,m^2\,y^3\,t^2+2\,c\,t\,m\,x\,z\,y^2+4\,\varepsilon\,c^4\,\mu\,t^2\,y\,x^2+36\,\varepsilon\,c^4\,\mu\,y\,z^2\,t^2)\,)$,

$-\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^7\,\mu}\,(16\,z\,(m^2\,y^4\,\varepsilon\,c^2\,\mu+m^2\,x^4\,\varepsilon\,c^2\,\mu-4\,\varepsilon\,c^4\,\mu\,y^2\,t^2+2\,\varepsilon\,c^2\,\mu\,x^2\,z^2$
$+2\,\varepsilon\,c^2\,\mu\,y^2\,z^2-4\,\varepsilon\,c^4\,\mu\,x^2\,t^2+12\,\varepsilon\,c^4\,\mu\,t^2\,z^2-m^2\,x^2\,z^2-m^2\,y^2\,z^2-m^2\,x^4-m^2\,y^4+m^2\,y^2\,\varepsilon\,c^2\,\mu\,z^2$
$+2\,m^2\,y^2\,\varepsilon\,c^2\,\mu\,x^2+5\,m^2\,y^2\,\varepsilon\,c^4\,\mu\,t^2+m^2\,x^2\,\varepsilon\,c^2\,\mu\,z^2+5\,m^2\,x^2\,\varepsilon\,c^4\,\mu\,t^2-6\,x^2\,c^2\,t^2-6\,y^2\,c^2\,t^2$
$-2\,m^2\,y^2\,x^2-5\,m^2\,x^2\,c^2\,t^2-5\,m^2\,y^2\,c^2\,t^2+2\,\varepsilon\,c^2\,\mu\,z^4+10\,\varepsilon\,c^6\,\mu\,t^4)\,)\,]$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $=\left[0,\,0,\,\frac{2\,c\,z\,m}{(-x^2-y^2-z^2+c^2\,t^2)^4},\right.$
$\left.-\frac{2\,m\,c\,t}{(-x^2-y^2-z^2+c^2\,t^2)^4}\right]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $=\Bigg[$

$-\frac{8\,c\,m^2\,(c^2\,t^2+z^2)\,y}{(-x^2-y^2-z^2+c^2\,t^2)^7},\,\frac{8\,c\,m^2\,(c^2\,t^2+z^2)\,x}{(-x^2-y^2-z^2+c^2\,t^2)^7},\,\frac{8\,m\,c^2\,t\,(c^2\,t^2+z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^7}\Bigg]$

Torsion Dissipation Jtorsion dot E $=\frac{8\,c^2\,m\,z\,(c^2\,t^2+z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^7}$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{2\left(m^2 x^3 + m^2 x y^2 - m^2 x z^2 + m^2 x c^2 t^2 + 2 c t m y z + 2 c^2 t^2 x + 2 \varepsilon c^3 z \mu m t y - 2 \varepsilon c^2 \mu x z^2\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5 \mu}, \right.$$

$$-\frac{2\left(-2 y c^2 t^2 + 2 c t m x z - m^2 y x^2 - m^2 y^3 + m^2 y z^2 - m^2 y c^2 t^2 + 2 \varepsilon c^3 z \mu m x t + 2 \varepsilon c^2 \mu y z^2\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5 \mu},$$

$$\left. -\frac{4 z\left(-m^2 y^2 - m^2 x^2 + \varepsilon c^4 \mu t^2 + \varepsilon c^2 z^2 \mu\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5 \mu}, \frac{4 \varepsilon c^2 t\left(m^2 y^2 + m^2 x^2 + c^2 t^2 + z^2\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5} \right]$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $= \Big[$

$$-\frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^8 \mu}\left(4 m\left(-2 m^2 y^3 z^2 - 2 c^2 t^2 y x^2 + 2 c^2 t^2 y z^2 - 2 m^2 y x^2 z^2 + 2 \varepsilon c^2 z^2 \mu y^3\right.\right.$$

$$+ 2 \varepsilon c^2 z^4 \mu y - 2 m^2 y^3 x^2 - m^2 y z^4 - m^2 y x^4 - 2 m^2 y x^2 c^2 t^2 + 2 m^2 y z^2 c^2 t^2 - 2 c t m x^3 z$$

$$- 2 c t m x z^3 + 2 c^3 t^3 m x z + 4 \varepsilon c^4 \mu m^2 y x^2 t^2 - 2 c^3 t \varepsilon \mu x^3 m z - 2 c^3 t \varepsilon z^3 \mu m x + 2 c^5 t^3 \varepsilon \mu m x z$$

$$+ 4 \varepsilon c^6 \mu t^4 y - 2 c^2 t^2 y^3 - 2 c^4 t^4 y - 2 m^2 y^3 c^2 t^2 - m^2 y c^4 t^4 - m^2 y^5 + 2 \varepsilon c^2 z^2 \mu y x^2$$

$$\left.\left. - 2 c^3 t \varepsilon \mu y^2 m x z + 4 \varepsilon c^4 \mu m^2 y^3 t^2 - 2 c t m x z y^2 + 10 \varepsilon c^4 \mu y z^2 t^2\right)\right),$$

$$\frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^8 \mu}\left(4 m\left(-m^2 x^5 - 2 c^4 t^4 x + 2 \varepsilon c^2 z^2 \mu y^2 x - 2 x y^2 c^2 t^2 + 2 x z^2 c^2 t^2\right.\right.$$

$$+ 2 \varepsilon c^2 z^2 \mu x^3 + 2 \varepsilon c^2 z^4 \mu x + 2 m c t y^3 z + 2 m c t y z^3 - 2 m c^3 t^3 y z - 2 m^2 x y^2 c^2 t^2 + 2 m^2 x z^2 c^2 t^2$$

$$+ 4 \varepsilon c^4 \mu m^2 x^3 t^2 + 2 m c t y z x^2 + 2 m z \varepsilon c^3 \mu x^2 t y - 2 m^2 x^3 z^2 - m^2 x y^4 - m^2 x z^4 - 2 m z \varepsilon c^5 \mu t^3 y$$

$$+ 2 m \varepsilon c^3 z^3 \mu t y + 2 m z \varepsilon c^3 \mu y^3 t + 4 \varepsilon c^4 \mu m^2 x y^2 t^2 - 2 m^2 x^3 y^2 - 2 m^2 x^3 c^2 t^2 - m^2 x c^4 t^4$$

$$\left.\left. - 2 x^3 c^2 t^2 + 4 \varepsilon c^6 \mu t^4 x + 10 \varepsilon c^4 \mu t^2 x z^2 - 2 m^2 x y^2 z^2\right)\right),$$

$$\frac{1}{\left(-x^2-y^2-z^2+c^2t^2\right)^8\mu}\left(8ct\left(2m^2y^2\varepsilon c^4\mu t^2+2m^2x^2\varepsilon c^4\mu t^2+2\varepsilon c^6\mu t^4+4\varepsilon c^4\mu t^2z^2\right.\right.$$
$$+2\varepsilon c^2\mu z^4-2m^2y^2x^2-m^2x^2z^2-m^2y^2z^2+2\varepsilon c^2\mu x^2z^2+2\varepsilon c^2\mu y^2z^2-2x^2c^2t^2-2y^2c^2t^2$$
$$\left.\left.-m^2x^4-m^2x^2c^2t^2-m^2y^4-m^2y^2c^2t^2\right)\right)\Big]$$

Spin Dissipation J_spin dot E $= \frac{1}{\left(-x^2-y^2-z^2+c^2t^2\right)^8\mu}\left(8cz\left(2m^2y^2\varepsilon c^4\mu t^2+2m^2x^2\varepsilon c^4\mu t^2\right.\right.$
$$+2\varepsilon c^6\mu t^4+4\varepsilon c^4\mu t^2z^2+2\varepsilon c^2\mu z^4-2m^2y^2x^2-m^2x^2z^2-m^2y^2z^2+2\varepsilon c^2\mu x^2z^2+2\varepsilon c^2\mu y^2z^2$$
$$\left.\left.-2x^2c^2t^2-2y^2c^2t^2-m^2x^4-m^2x^2c^2t^2-m^2y^4-m^2y^2c^2t^2\right)\right)$$

Disspative Force 3 vector $= \Big[-\frac{1}{\left(-x^2-y^2-z^2+c^2t^2\right)^8\mu}\left(4\left(-6\varepsilon c^2\mu m^2x^5y^2-16\varepsilon c^4\mu t^2x^3y^2\right.\right.$
$$-8\varepsilon c^4\mu t^2xy^4-6\varepsilon c^2\mu m^2x^3y^4-2\varepsilon c^2\mu m^2xy^6+4mctyzx^4-2m\mu c^2t^2yx^2+2m\mu c^2t^2yz^2$$
$$+2m\mu^2\varepsilon c^2z^2y^3+2m\mu^2\varepsilon c^2z^4y-2m^3\mu yx^2c^2t^2+2m^3\mu yz^2c^2t^2-2m^2\mu ctx^3z-2m^2\mu ctxz^3$$
$$+2m^2\mu c^3t^3xz+4m\mu^2\varepsilon c^6t^4y+4m^3\mu^2\varepsilon c^4y^3t^2-2c^3m^2y\mu x^2t^2-2cm^2y\mu x^2z^2$$
$$+4m^2x^3z^2c^2t^2+10m^2y^4c^2t^2x+20x^3c^2t^2m^2y^2-2y^2c^4t^4m^2x-8y^3c^3t^3mz+16\varepsilon c^8\mu t^6x$$
$$-10m^2x^5\varepsilon c^4\mu t^2-8\varepsilon c^6\mu y^2t^4x-2\varepsilon c^2\mu x^5z^2m^2+2\varepsilon c^2\mu x^3z^4m^2-72\varepsilon c^4\mu x^3z^2t^2$$
$$+2\varepsilon c^6\mu x^3t^4m^2+56\varepsilon c^6\mu t^4z^2x+4m^2y^2z^2c^2t^2x-8x^2c^3t^3myz+2\varepsilon c^2\mu z^6m^2x$$
$$-64\varepsilon c^4\mu z^4t^2x+10\varepsilon c^8\mu t^6m^2x-8\varepsilon c^6\mu x^3t^4+2m^2x^5z^2+16x^5c^2t^2+2m^2xy^6$$
$$-16\varepsilon c^2z^2\mu y^2x^3-8\varepsilon c^2z^2\mu y^4x-16\varepsilon c^2z^4\mu y^2x+8mcty^3zx^2+8mctyz^3x^2+8xy^2c^2t^2z^2$$
$$-8\varepsilon c^2z^2\mu x^5-16\varepsilon c^2z^4\mu x^3-8\varepsilon c^2z^6\mu x+10m^2x^5c^2t^2-2x^3c^4t^4m^2-8y^2c^4t^4x+4mcty^5z$$

$$+ 8\, m\, c\, t\, y^3\, z^3 + 4\, m\, c\, t\, y\, z^5 - 8\, m\, c^3\, t^3\, y\, z^3 + 4\, m\, c^5\, t^5\, y\, z - 6\, m^2\, x\, z^4\, c^2\, t^2 + 18\, m^2\, x\, z^2\, c^4\, t^4$$

$$- 2\, \varepsilon\, c^2\, \mu\, m^2\, x^7 - 8\, \varepsilon\, c^4\, \mu\, t^2\, x^5 - 2\, m^3\, \mu\, y\, x^2\, z^2 - 2\, m\, \mu\, c^2\, t^2\, y^3 - 2\, m\, \mu\, c^4\, t^4\, y - 2\, m^3\, \mu\, y^3\, c^2\, t^2$$

$$- m^3\, \mu\, y\, c^4\, t^4 + 2\, c^5\, m^2\, y\, \mu\, t^4 - 2\, c\, m^2\, y^3\, \mu\, z^2 - 2\, c\, m^2\, y\, \mu\, z^4 + 16\, c^4\, t^4\, x\, z^2 + 32\, x^3\, y^2\, c^2\, t^2 + 16\, x\, y^4\, c^2\, t^2$$

$$+ 8\, x^3\, z^2\, c^2\, t^2 - 8\, x\, z^4\, c^2\, t^2 + 2\, m^2\, x\, y^4\, z^2 - 2\, m^2\, x\, z^4\, y^2 + 4\, m^2\, x^3\, y^2\, z^2 - 10\, m^2\, x\, c^6\, t^6 - 2\, m^3\, \mu\, y^3\, z^2$$

$$- 2\, m^3\, \mu\, y^3\, x^2 - m^3\, \mu\, y\, z^4 - m^3\, \mu\, y\, x^4 - 12\, \varepsilon\, c^3\, \mu\, z^5\, t\, m\, y - 12\, \varepsilon\, c^7\, \mu\, t^5\, m\, y\, z - 4\, \varepsilon\, c^4\, \mu\, y^2\, t^2\, m^2\, x\, z^2$$

$$- 24\, \varepsilon\, c^3\, \mu\, x^2\, z^3\, t\, m\, y + 24\, \varepsilon\, c^5\, \mu\, x^2\, t^3\, m\, y\, z - 2\, m^2\, \mu^2\, c^3\, t\, \varepsilon\, y^2\, x\, z - 2\, c^3\, m^2\, y^3\, \mu\, t^2 - 12\, m\, z\, \varepsilon\, c^3\, \mu\, x^4\, t\, y$$

$$- 24\, m\, z\, \varepsilon\, c^3\, \mu\, x^2\, t\, y^3 - 10\, m^2\, y^4\, \varepsilon\, c^4\, \mu\, t^2\, x - 20\, \varepsilon\, c^4\, \mu\, y^2\, t^2\, m^2\, x^3 + 2\, \varepsilon\, c^6\, \mu\, y^2\, t^4\, m^2\, x + 24\, \varepsilon\, c^5\, \mu\, y^3\, t^3\, m\, z$$

$$- 4\, \varepsilon\, c^2\, \mu\, x^3\, z^2\, m^2\, y^2 - 4\, \varepsilon\, c^4\, \mu\, x^3\, z^2\, m^2\, t^2 - 2\, \varepsilon\, c^2\, \mu\, y^4\, z^2\, m^2\, x + 2\, \varepsilon\, c^2\, \mu\, y^2\, z^4\, m^2\, x - 24\, \varepsilon\, c^3\, \mu\, y^3\, z^3\, t\, m$$

$$- 72\, \varepsilon\, c^4\, \mu\, y^2\, z^2\, t^2\, x + 6\, \varepsilon\, c^4\, \mu\, t^2\, z^4\, m^2\, x - 18\, \varepsilon\, c^6\, \mu\, t^4\, z^2\, m^2\, x + 24\, \varepsilon\, c^5\, \mu\, t^3\, z^3\, m\, y + 2\, m^2\, x^7 - 8\, x^3\, c^4\, t^4$$

$$- m^3\, \mu\, y^5 - 2\, m^2\, x^3\, z^4 - 8\, c^6\, t^6\, x + 6\, m^2\, x^3\, y^4 + 6\, m^2\, x^5\, y^2 - 2\, m^2\, x\, z^6 - 12\, m\, z\, \varepsilon\, c^3\, \mu\, y^5\, t$$

$$+ 4\, m^3\, \mu^2\, \varepsilon\, c^4\, y\, x^2\, t^2 - 2\, m^2\, \mu^2\, c^3\, t\, \varepsilon\, x^3\, z - 2\, m^2\, \mu^2\, c^3\, t\, \varepsilon\, z^3\, x + 2\, m^2\, \mu^2\, c^5\, t^3\, \varepsilon\, x\, z + 2\, m\, \mu^2\, \varepsilon\, c^2\, z^2\, y\, x^2$$

$$- 2\, m^2\, \mu\, c\, t\, x\, z\, y^2 + 10\, m\, \mu^2\, \varepsilon\, c^4\, y\, z^2\, t^2\big)\big),\ -\frac{1}{\left(-x^2 - y^2 - z^2 + c^2\, t^2\right)^8\, \mu}\big(4\,\big(16\, c^2\, t^2\, y\, x^4$$

$$+ 32\, c^2\, t^2\, y^3\, x^2 + 8\, c^2\, t^2\, y^3\, z^2 - 8\, c^2\, t^2\, y\, z^4 + 16\, c^4\, t^4\, y\, z^2 + 4\, m^2\, y^3\, x^2\, z^2 - 2\, m^2\, y\, z^4\, x^2 + 2\, m^2\, y\, x^4\, z^2$$

$$- 10\, m^2\, y\, c^6\, t^6 + 2\, m^3\, \mu\, x^3\, z^2 + m^3\, \mu\, x\, y^4 + m^3\, \mu\, x\, z^4 + 2\, m^3\, \mu\, x^3\, y^2 - 8\, c^6\, t^4\, y\, x^2\, \varepsilon\, \mu + 4\, c^2\, t^2\, y\, z^2\, m^2\, x^2$$

$$+56\,c^6\,t^4\,y\,z^2\,\varepsilon\,\mu-64\,c^4\,t^2\,y\,z^4\,\varepsilon\,\mu-2\,\varepsilon\,c^2\,z^2\,\mu\,y^5\,m^2+2\,\varepsilon\,c^2\,z^4\,\mu\,y^3\,m^2+2\,m^2\,y\,z^6\,\varepsilon\,c^2\,\mu$$

$$-10\,\varepsilon\,c^4\,\mu\,t^2\,y^5\,m^2+2\,\varepsilon\,c^6\,\mu\,t^4\,y^3\,m^2-72\,c^4\,t^2\,y^3\,\varepsilon\,z^2\,\mu+10\,m^2\,y\,c^8\,t^6\,\varepsilon\,\mu+8\,y^2\,c^3\,t^3\,m\,x\,z$$

$$-72\,c^4\,t^2\,y\,x^2\,\varepsilon\,z^2\,\mu-4\,\varepsilon\,c^2\,z^2\,\mu\,y^3\,m^2\,x^2+2\,\varepsilon\,c^2\,z^4\,\mu\,y\,m^2\,x^2-20\,m^2\,y^3\,x^2\,\varepsilon\,c^4\,\mu\,t^2+6\,m^2\,y\,z^4\,\varepsilon\,c^4\,\mu\,t^2$$

$$-10\,m^2\,y\,x^4\,\varepsilon\,c^4\,\mu\,t^2-2\,m^2\,y\,x^4\,\varepsilon\,c^2\,z^2\,\mu+2\,m^2\,y\,x^2\,c^6\,t^4\,\varepsilon\,\mu-18\,m^2\,y\,z^2\,c^6\,t^4\,\varepsilon\,\mu-24\,c^5\,t^3\,m\,x^3\,z\,\varepsilon\,\mu$$

$$+24\,c^3\,t\,m\,x^3\,z^3\,\varepsilon\,\mu-24\,c^5\,t^3\,m\,x\,z^3\,\varepsilon\,\mu+12\,c^3\,t\,m\,x\,z^5\,\varepsilon\,\mu+12\,c^7\,t^5\,m\,x\,z\,\varepsilon\,\mu-4\,m^2\,y^3\,c^4\,t^2\,\varepsilon\,z^2\,\mu$$

$$+20\,c^2\,t^2\,y^3\,x^2\,m^2+10\,c^2\,t^2\,y\,x^4\,m^2+4\,c^2\,t^2\,y^3\,z^2\,m^2-8\,c^6\,t^4\,y^3\,\varepsilon\,\mu-2\,c^4\,t^4\,y\,m^2\,x^2+16\,c^8\,t^6\,y\,\varepsilon\,\mu$$

$$+8\,x^3\,c^3\,t^3\,m\,z-4\,m^2\,y\,x^2\,c^4\,t^2\,\varepsilon\,z^2\,\mu+10\,c^2\,t^2\,y^5\,m^2-2\,c^4\,t^4\,y^3\,m^2-8\,x^2\,c^4\,t^4\,y-16\,\varepsilon\,c^2\,z^2\,\mu\,y^3\,x^2$$

$$-16\,\varepsilon\,c^2\,z^4\,\mu\,y\,x^2-6\,\varepsilon\,c^2\,\mu\,m^2\,y^5\,x^2-8\,c\,t\,m\,x^3\,z\,y^2-8\,c\,t\,m\,x\,z^3\,y^2-16\,\varepsilon\,c^4\,\mu\,t^2\,y^3\,x^2-8\,\varepsilon\,c^2\,z^2\,\mu\,y\,x^4$$

$$-2\,\varepsilon\,c^2\,\mu\,m^2\,y\,x^6-6\,\varepsilon\,c^2\,\mu\,m^2\,y^3\,x^4-4\,c\,t\,m\,x\,z\,y^4+8\,c^2\,t^2\,y\,x^2\,z^2-8\,\varepsilon\,c^2\,z^2\,\mu\,y^5-16\,\varepsilon\,c^2\,z^4\,\mu\,y^3$$

$$-4\,m^3\,\mu^2\,\varepsilon\,c^4\,x^3\,t^2-4\,m\,\mu^2\,\varepsilon\,c^6\,t^4\,x+2\,c^3\,m^2\,x\,\mu\,y^2\,t^2+2\,c\,m^2\,x\,\mu\,y^2\,z^2-2\,m^2\,\mu^2\,z\,\varepsilon\,c^3\,x^2\,t\,y$$

$$-8\,\varepsilon\,c^4\,\mu\,t^2\,y\,x^4+2\,m\,\mu\,x\,y^2\,c^2\,t^2-2\,m\,\mu\,x\,z^2\,c^2\,t^2-2\,m\,\mu^2\,\varepsilon\,c^2\,z^2\,x^3-2\,m\,\mu^2\,\varepsilon\,c^2\,z^4\,x-2\,m^2\,\mu\,c\,t\,y^3\,z$$

$$-2\,m^2\,\mu\,c\,t\,y\,z^3+2\,m^2\,\mu\,c^3\,t^3\,y\,z+2\,m^3\,\mu\,x\,y^2\,c^2\,t^2-2\,m^3\,\mu\,x\,z^2\,c^2\,t^2-24\,c^5\,t^3\,m\,x\,z\,y^2\,\varepsilon\,\mu$$

$$+24\,c^3\,t\,m\,x\,z^3\,y^2\,\varepsilon\,\mu+2\,m^2\,y^7-2\,m\,\mu^2\,\varepsilon\,c^2\,z^2\,y^2\,x-2\,m^2\,\mu\,c\,t\,y\,z\,x^2+2\,m^2\,\mu^2\,z\,\varepsilon\,c^5\,t^3\,y$$

$$-2\,m^2\,\mu^2\,\varepsilon\,c^3\,z^3\,t\,y-2\,m^2\,\mu^2\,z\,\varepsilon\,c^3\,y^3\,t-4\,m^3\,\mu^2\,\varepsilon\,c^4\,x\,y^2\,t^2-10\,m\,\mu^2\,\varepsilon\,c^4\,t^2\,x\,z^2-8\,y^3\,c^4\,t^4$$

$$- 8\,\varepsilon\, c^2 z^6 \mu\, y - 6\, m^2 y\, z^4 c^2 t^2 + 18\, m^2 y\, z^2 c^4 t^4 - 2\,\varepsilon\, c^2 \mu\, m^2 y^7 - 4\, c\, t\, m\, x^5 z - 8\, c\, t\, m\, x^3 z^3 - 4\, c\, t\, m\, x\, z^5$$

$$+ 8\, c^3 t^3 m\, x\, z^3 - 4\, c^5 t^5 m\, x\, z - 8\,\varepsilon\, c^4 \mu\, t^2 y^5 + 2\, m\,\mu\, c^4 t^4 x + 2\, m^3 \mu\, x^3 c^2 t^2 + m^3 \mu\, x\, c^4 t^4 + 2\, m\,\mu\, x^3 c^2 t^2$$

$$+ 2\, m^3 \mu\, x\, y^2 z^2 + 2\, c^3 m^2 x^3 \mu\, t^2 - 2\, c^5 m^2 x\,\mu\, t^4 + 2\, c\, m^2 x^3 \mu\, z^2 + 2\, c\, m^2 x\,\mu\, z^4 + 24\, c^3 t\,\varepsilon\,\mu\, x^3 m\, z\, y^2$$

$$+ 12\, c^3 t\,\varepsilon\,\mu\, y^4 m\, x\, z + 6\, m^2 y^3 x^4 + 6\, m^2 y^5 x^2 - 2\, m^2 y^3 z^4 - 2\, m^2 y\, z^6 + 2\, m^2 y\, x^6 + 16\, c^2 t^2 y^5 - 8\, c^6 t^6 y$$

$$+ 2\, m^2 y^5 z^2 + m^3 \mu\, x^5 + 12\, c^3 t\,\varepsilon\,\mu\, x^5 m\, z \big) \big),\ - \frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^8 \mu} \Big(8 \big(10\, z\, m^2 x^2 \varepsilon\, c^6 \mu\, t^4$$

$$+ 10\, z\, m^2 y^2 \varepsilon\, c^6 \mu\, t^4 - 8\, m^2 y^2 z^3 \varepsilon\, c^4 \mu\, t^2 - 8\, m^2 x^2 z^3 \varepsilon\, c^2 \mu\, y^2 - 8\, m^2 x^2 z^3 \varepsilon\, c^4 \mu\, t^2 + 8\, m^2 x^2 z^3 c^2 t^2$$

$$+ 8\, m^2 y^2 z^3 c^2 t^2 - 4\,\varepsilon\, c^2 \mu\, y^4 z^3 - 8\,\varepsilon\, c^2 \mu\, y^2 z^5 - 20\,\varepsilon\, c^4 \mu\, t^2 z^5 + 4\,\varepsilon\, c^6 \mu\, t^4 z^3 - 4\,\varepsilon\, c^2 \mu\, x^4 z^3$$

$$- 10\, z\, m^2 y^2 c^4 t^4 + 20\, z\,\varepsilon\, c^8 \mu\, t^6 + 8\, z\, m^2 x^4 c^2 t^2 + 8\, z\, m^2 y^4 c^2 t^2 + 24\, z\, x^2 c^2 t^2 y^2 - 10\, z\, m^2 x^2 c^4 t^4$$

$$- 8\,\varepsilon\, c^2 \mu\, x^2 z^5 - 4\, c^5 t^3 \mu^2 \varepsilon\, z^2 - 2\, c^3 t\,\mu^2 \varepsilon\, z^4 + c\, t\,\mu\, m^2 x^4 + c^3 t^3 \mu\, m^2 x^2 + c\, t\,\mu\, m^2 y^4 + c^3 t^3 \mu\, m^2 y^2$$

$$+ m\, c^4 t^3 \mu\, x^2 + m\, c^4 t^3 \mu\, y^2 + m\, c^2 t\,\mu\, z^4 - 8\, z\, m^2 x^4 \varepsilon\, c^4 \mu\, t^2 - 6\, z\, m^2 x^4 \varepsilon\, c^2 \mu\, y^2 + 16\, z\,\varepsilon\, c^4 \mu\, y^2 t^2 x^2$$

$$- 6\, z\, m^2 y^4 \varepsilon\, c^2 \mu\, x^2 - 8\, z\, m^2 y^4 \varepsilon\, c^4 \mu\, t^2 - 8\,\varepsilon\, c^2 \mu\, x^2 z^3 y^2 - 12\,\varepsilon\, c^4 \mu\, x^2 z^3 t^2 - 4\, m^2 x^4 \varepsilon\, c^2 \mu\, z^3$$

$$- 12\,\varepsilon\, c^4 \mu\, y^2 t^2 z^3 - 4\, m^2 y^4 \varepsilon\, c^2 \mu\, z^3 + 16\, z\, m^2 y^2 x^2 c^2 t^2 + 8\, z\,\varepsilon\, c^4 \mu\, x^4 t^2 - 28\, z\,\varepsilon\, c^6 \mu\, x^2 t^4$$

$$- 2\, z\, m^2 y^6 \varepsilon\, c^2 \mu - 2\, z\, m^2 x^6 \varepsilon\, c^2 \mu + 8\, z\,\varepsilon\, c^4 \mu\, y^4 t^2 - 28\, z\,\varepsilon\, c^6 \mu\, y^2 t^4 - 2\, m^2 y^2 z^5 \varepsilon\, c^2 \mu$$

$$- 2\, m^2 x^2 z^5 \varepsilon\, c^2 \mu - 2\, c^5 t^3 \mu^2 m^2 y^2 \varepsilon - 2\, c^5 t^3 \mu^2 m^2 x^2 \varepsilon + 2\, c\, t\,\mu\, m^2 y^2 x^2 + c\, t\,\mu\, m^2 x^2 z^2 + c\, t\,\mu\, m^2 y^2 z^2$$

$$- 2\, c^3 t\,\mu^2 \varepsilon\, x^2 z^2 - 2\, c^3 t\,\mu^2 \varepsilon\, y^2 z^2 + m\, c^2 t\,\mu\, x^2 z^2 + m\, c^2 t\,\mu\, y^2 z^2 - 4\,\varepsilon\, c^2 \mu\, z^7 + 8\, m^2 x^2 z^3 y^2$$

$$+ 12\, x^2 c^2 t^2 z^3 + 12\, y^2 c^2 t^2 z^3 + 6\, z\, m^2 x^4 y^2 + 6\, z\, m^2 y^4 x^2 + 12\, z\, x^4 c^2 t^2 - 12\, z\, x^2 c^4 t^4 + 12\, z\, y^4 c^2 t^2$$

$$- 12\, z\, y^2 c^4 t^4 - 2\, c^7 t^5 \mu^2 \varepsilon + 2\, c^3 t^3 \mu\, x^2 + 2\, c^3 t^3 \mu\, y^2 - m\, c^6 t^5 \mu + 2\, z\, m^2 y^6 + 4\, m^2 x^4 z^3 + 2\, m^2 x^2 z^5$$

$$+ 4\, m^2 y^4 z^3 + 2\, m^2 y^2 z^5 + 2\, z\, m^2 x^6 - 16\, z\, m^2 y^2 \varepsilon\, c^4 \mu\, x^2 t^2 \big) \Big) \Big]$$

$$\mathit{Dissipation} = - \frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5} \big(4\,\varepsilon\, c \big(-2\, x^4 z - 4\, y^2 x^2 z - 4\, x^2 z^3 - 8\, c^2 t^2 x^2 z - 2\, y^4 z - 4\, y^2 z^3$$

$$- 8\, y^2 c^2 t^2 z - 2\, z^5 - 8\, c^2 t^2 z^3 + 10\, z\, c^4 t^4 - c\, t\,\mu\, m^2 y^2 - c\, t\,\mu\, m^2 x^2 - c^3 t^3 \mu - c\, t\,\mu\, z^2 \big) \big)$$

****************************** *END PROCEDURE* **************************** **(32)**

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.
p=2, n=4

```
> NAME:=`Example 8a Index 1 Irreversible solution EdotB >0 (kinematic out) Type 2`;
```

```
> Holder:=(x^2+y^2+z^2-(c*t)^2)^(4/2);
> Ax:=c*t*1/Holder;Ay:=-z*1/Holder;Az:=y*1/Holder;phi:=+x*c*1/Holder;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 8a Index 1 Irreversible solution EdotB >0 (kinematic out) Type 2

$$Holder := \left(x^2+y^2+z^2-c^2\,t^2\right)^2$$

$$Ax := \frac{c\,t}{\left(x^2+y^2+z^2-c^2\,t^2\right)^2}$$

$$Ay := -\frac{z}{\left(x^2+y^2+z^2-c^2\,t^2\right)^2}$$

$$Az := \frac{y}{\left(x^2+y^2+z^2-c^2\,t^2\right)^2}$$

$$\phi := \frac{x\,c}{\left(x^2+y^2+z^2-c^2\,t^2\right)^2}$$

Example 8a Index 1 Irreversible solution EdotB ○ 0 (kinematic out) Type 2

*********************************** *Differential Form Format* ******************************
****

$$Action\ 1\text{-}form = -\frac{x\,c\,d(t)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^2}+\frac{c\,t\,d(x)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^2}-\frac{z\,d(y)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^2}+\frac{y\,d(z)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^2}$$

$$Intensity\ 2\text{-}form\ F=dA\ = \left(-\frac{c\left(3\,x^2-y^2-z^2+c^2\,t^2\right)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3}+\frac{c\left(x^2+y^2+z^2+3\,c^2\,t^2\right)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3}\right)(d(x))\ \&\hat{}\ (d(t))$$
$$+\left(-\frac{4\,x\,z}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3}-\frac{4\,c\,t\,y}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3}\right)(d(x))\ \&\hat{}\ (d(y))+\Bigg($$
$$-\frac{4\,c\,t\,z}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3}+\frac{4\,y\,x}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3}\Bigg)(d(x))\ \&\hat{}\ (d(z))+\Bigg($$
$$-\frac{4\,x\,c\,y}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3}-\frac{4\,z\,c^2\,t}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3}\Bigg)(d(y))\ \&\hat{}\ (d(t))$$
$$+\left(\frac{-x^2-y^2+3\,z^2+c^2\,t^2}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3}+\frac{-x^2+3\,y^2-z^2+c^2\,t^2}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3}\right)(d(y))\ \&\hat{}\ (d(z))+\Bigg($$

$$-\frac{4\,c\,z\,x}{(-x^2-y^2-z^2+c^2t^2)^3}+\frac{4\,y\,c^2\,t}{(-x^2-y^2-z^2+c^2t^2)^3}\Bigg)\,(d(z))\ \&\hat{}\ (d(t))$$

Topological Torsion 3-form $\quad A\hat{}F=\Bigg(\frac{4\,x\,c\,(c\,t\,y+x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}+\frac{2\,z\,c\,(-x^2+y^2+z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5}$

$$-\frac{4\,c^2\,t\,(c\,t\,z+y\,x)}{(-x^2-y^2-z^2+c^2t^2)^5}\Bigg)\,\&\hat{}(d(x),d(y),d(t))+\Bigg(-\frac{4\,y\,(c\,t\,y+x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}$$

$$-\frac{4\,z\,(c\,t\,z-y\,x)}{(-x^2-y^2-z^2+c^2t^2)^5}+\frac{2\,c\,t\,(-x^2+y^2+z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5}\Bigg)\,\&\hat{}(d(x),d(y),d(z))$$

$$+\Bigg(\frac{4\,c^2\,t\,(c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}+\frac{4\,x\,c\,(c\,t\,z-y\,x)}{(-x^2-y^2-z^2+c^2t^2)^5}$$

$$-\frac{2\,y\,c\,(-x^2+y^2+z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5}\Bigg)\,\&\hat{}(d(x),d(z),d(t))+\Bigg(\frac{4\,y\,c\,(c\,t\,z+y\,x)}{(-x^2-y^2-z^2+c^2t^2)^5}$$

$$-\frac{4\,z\,c\,(c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}-\frac{2\,x\,c\,(-x^2+y^2+z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5}\Bigg)\,\&\hat{}(d(y),d(z),d(t))$$

Topological Parity 4-form $\quad F\hat{}F=\frac{8\,c\,\&\hat{}(d(x),d(y),d(z),d(t))}{(-x^2-y^2-z^2+c^2t^2)^4}$

******************************** *Using EM format* ********************************

E field $=\left[\frac{2\,c\,(-x^2+y^2+z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3},-\frac{4\,c\,(c\,t\,z+y\,x)}{(-x^2-y^2-z^2+c^2t^2)^3},\frac{4\,c\,(c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2t^2)^3}\right]$

B field $=\left[\frac{2\,(-x^2+y^2+z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3},\frac{4\,(c\,t\,z-y\,x)}{(-x^2-y^2-z^2+c^2t^2)^3},-\frac{4\,(c\,t\,y+x\,z)}{(-x^2-y^2-z^2+c^2t^2)^3}\right]$

Topological TORSION 4 vector $\quad$ *T4 = -[ExA + Bphi,AdotB]* $=\Bigg[-\frac{2\,x\,c}{(-x^2-y^2-z^2+c^2t^2)^4},$

$$-\frac{2\,y\,c}{(-x^2-y^2-z^2+c^2t^2)^4},-\frac{2\,c\,z}{(-x^2-y^2-z^2+c^2t^2)^4},-\frac{2\,c\,t}{(-x^2-y^2-z^2+c^2t^2)^4}\Bigg]$$

Helicity AdotB $=\frac{2\,c\,t}{(-x^2-y^2-z^2+c^2t^2)^4}$

Poincare II =2(E.B) $=\frac{8\,c}{(-x^2-y^2-z^2+c^2t^2)^4}$

coefficient of Topological Parity 4-form $=\frac{8\,c}{(-x^2-y^2-z^2+c^2t^2)^4}$

Pfaff Topological Dimension $\quad PTD=4$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************

****

$$Xm \text{ or linear (Mean) curvature } = \frac{4\,c\,t\,x\,(1+c^2)}{(-x^2-y^2-z^2+c^2 t^2)^3}$$

$$Yg \text{ or quadratic (GAUSS) curvature } =$$

$$-\frac{3\,c^4 t^2 - c^2 t^2 - 3\,x^2 c^2 + y^2 c^2 + c^2 z^2 + x^2 - 3\,y^2 - 3\,z^2}{(-x^2-y^2-z^2+c^2 t^2)^5}$$

$$Za \text{ or Cubic (Interaction internal energy) curvature } = \frac{4\,c\,t\,x\,(1+c^2)}{(-x^2-y^2-z^2+c^2 t^2)^7}$$

$$Tk \text{ or quartic (4D expansion) curvature } = -\frac{3\,c^2}{(-x^2-y^2-z^2+c^2 t^2)^8}$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $= 0$

$$D\,field = \left[\frac{2\,\varepsilon\,c\,(-x^2+y^2+z^2+c^2 t^2)}{(-x^2-y^2-z^2+c^2 t^2)^3},\ -\frac{4\,\varepsilon\,c\,(c\,t\,z+y\,x)}{(-x^2-y^2-z^2+c^2 t^2)^3},\ \frac{4\,\varepsilon\,c\,(c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2 t^2)^3} \right]$$

$$H\,field = \left[\frac{2\,(-x^2+y^2+z^2+c^2 t^2)}{(-x^2-y^2-z^2+c^2 t^2)^3\,\mu},\ \frac{4\,(c\,t\,z-y\,x)}{(-x^2-y^2-z^2+c^2 t^2)^3\,\mu},\ -\frac{4\,(c\,t\,y+x\,z)}{(-x^2-y^2-z^2+c^2 t^2)^3\,\mu} \right]$$

Poynting vector ExH

$$= \left[\frac{32\,c^2\,t\,x\,(z^2+y^2)}{(-x^2-y^2-z^2+c^2 t^2)^6\,\mu}\quad \frac{16\,c^2\,(-x^2+y^2+z^2+c^2 t^2)\,t\,y}{(-x^2-y^2-z^2+c^2 t^2)^6\,\mu}\quad \frac{16\,c^2\,(-x^2+y^2+z^2+c^2 t^2)\,t\,z}{(-x^2-y^2-z^2+c^2 t^2)^6\,\mu} \right]$$

Amperian Current 4Vector *curlH-dD/dt=J4*

$$= \left[\frac{8\,c\,t\,(-x^2+2\,y^2+2\,z^2+c^2 t^2)\,(\varepsilon\,c^2\mu-1)}{(-x^2-y^2-z^2+c^2 t^2)^4\,\mu},\right.$$

$$-\frac{4\,(x^2 z+y^2 z+z^3+5\,z\,c^2 t^2+6\,x\,c\,t\,y)\,(\varepsilon\,c^2\mu-1)}{(-x^2-y^2-z^2+c^2 t^2)^4\,\mu},$$

$$\left.\frac{4\,(y\,x^2+y^3+y\,z^2+5\,y\,c^2 t^2-6\,c\,t\,x\,z)\,(\varepsilon\,c^2\mu-1)}{(-x^2-y^2-z^2+c^2 t^2)^4\,\mu},\ 0\right]$$

Amerian charge density *divD = rho* $= 0$

divergence Lorentz Current 4Vector, *4div(J4)* $= 0$

Topological SPIN 4 vector S4 =

$$\left[\frac{2\,x\,(2\,z^2+2\,y^2-\varepsilon\,c^2\mu\,x^2+\varepsilon\,c^2\mu\,y^2+\varepsilon\,c^2 z^2\mu+\varepsilon\,c^4\mu\,t^2)}{(-x^2-y^2-z^2+c^2 t^2)^5\,\mu},\right.$$

$$-\frac{2\,(y\,x^2-y^3-y\,z^2-3\,y\,c^2 t^2-2\,c\,t\,x\,z+2\,\varepsilon\,c^3\mu\,t\,x\,z+2\,\varepsilon\,c^2\mu\,y\,x^2)}{(-x^2-y^2-z^2+c^2 t^2)^5\,\mu},$$

$$\frac{2\left(3 z c^2 t^2-2 x c t y-x^2 z+y^2 z+z^3+2 \varepsilon c^3 \mu x t y-2 \varepsilon c^2 \mu x^2 z\right)}{\left(-x^2-y^2-z^2+c^2 t^2\right)^5 \mu},$$

$$\left.\frac{2 \varepsilon c^2 t\left(c^2 t^2-x^2+3 y^2+3 z^2\right)}{\left(-x^2-y^2-z^2+c^2 t^2\right)^5}\right]$$

Topological SPIN 3-form

$$= \frac{2 x\left(2 z^2+2 y^2-\varepsilon c^2 \mu x^2+\varepsilon c^2 \mu y^2+\varepsilon c^2 z^2 \mu+\varepsilon c^4 \mu t^2\right) \&\text{^}(d(y), d(z), d(t))}{\left(-x^2-y^2-z^2+c^2 t^2\right)^5 \mu}$$

$$+\frac{2\left(y x^2-y^3-y z^2-3 y c^2 t^2-2 c t x z+2 \varepsilon c^3 \mu t x z+2 \varepsilon c^2 \mu y x^2\right) \&\text{^}(d(x), d(z), d(t))}{\left(-x^2-y^2-z^2+c^2 t^2\right)^5 \mu}$$

$$+\frac{2\left(3 z c^2 t^2-2 x c t y-x^2 z+y^2 z+z^3+2 \varepsilon c^3 \mu x t y-2 \varepsilon c^2 \mu x^2 z\right) \&\text{^}(d(x), d(y), d(t))}{\left(-x^2-y^2-z^2+c^2 t^2\right)^5 \mu}$$

$$-\frac{2 \varepsilon c^2 t\left(c^2 t^2-x^2+3 y^2+3 z^2\right) \&\text{^}(d(x), d(y), d(z))}{\left(-x^2-y^2-z^2+c^2 t^2\right)^5}$$

$$\textit{Spin density rho\_spin} = \frac{2 \varepsilon c^2 t\left(c^2 t^2-x^2+3 y^2+3 z^2\right)}{\left(-x^2-y^2-z^2+c^2 t^2\right)^5}$$

LaGrange field energy density (B.H-D.E) =

$$-\frac{4\left(x^4+2 x^2 y^2+2 x^2 z^2-2 x^2 c^2 t^2+y^4+2 y^2 z^2+6 y^2 c^2 t^2+z^4+6 c^2 t^2 z^2+c^4 t^4\right)\left(\varepsilon c^2 \mu-1\right)}{\left(-x^2-y^2-z^2+c^2 t^2\right)^6 \mu}$$

$$B.H=\frac{4\left(x^4+2 x^2 y^2+2 x^2 z^2-2 x^2 c^2 t^2+y^4+2 y^2 z^2+6 y^2 c^2 t^2+z^4+6 c^2 t^2 z^2+c^4 t^4\right)}{\left(-x^2-y^2-z^2+c^2 t^2\right)^6 \mu}$$

$$D.E=\frac{4 \varepsilon c^2\left(x^4+2 x^2 y^2+2 x^2 z^2-2 x^2 c^2 t^2+y^4+2 y^2 z^2+6 y^2 c^2 t^2+z^4+6 c^2 t^2 z^2+c^4 t^4\right)}{\left(-x^2-y^2-z^2+c^2 t^2\right)^6}$$

$$A.J=\frac{4\left(\varepsilon c^2 \mu-1\right)\left(-2 x^2 c^2 t^2+9 y^2 c^2 t^2+9 c^2 t^2 z^2+2 c^4 t^4+x^2 z^2+2 y^2 z^2+z^4+x^2 y^2+y^4\right)}{\left(-x^2-y^2-z^2+c^2 t^2\right)^6 \mu}$$

$$-rho.phi=0$$

Poincare I *(B.H - D.E)-(A.J - rho.phi) =*

$$-\frac{1}{\left(-x^2-y^2-z^2+c^2 t^2\right)^6 \mu}\left(4\left(\varepsilon c^2 \mu-1\right)\left(x^4+3 x^2 y^2+3 x^2 z^2-4 x^2 c^2 t^2+2 y^4+4 y^2 z^2\right.\right.$$
$$\left.\left.+15 y^2 c^2 t^2+2 z^4+15 c^2 t^2 z^2+3 c^4 t^4\right)\right)$$

London Coefficient $LC=0$

PROCA coefficient curlcurlB $= \Bigg[$

$$-\frac{8\,(x^2 y^2 + 2\,y^4 + 2\,z^4 + x^2 z^2 + 4\,y^2 z^2 + 5\,c^4 t^4 + 17\,c^2 t^2 z^2 - 4\,x^2 c^2 t^2 + 17\,y^2 c^2 t^2 - x^4)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5},$$

$$-\frac{24\,(3\,y^2 z\,c\,t - 7\,y\,x\,c^2 t^2 + 3\,c\,t\,x^2 z - y\,x^3 - y^3 x + 3\,c\,t\,z^3 + 5\,c^3 t^3 z - y\,x\,z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5},$$

$$\frac{24\,(3\,c\,t\,y\,x^2 + 3\,c\,t\,y\,z^2 + 7\,x\,z\,c^2 t^2 + 3\,c\,t\,y^3 + 5\,c^3 t^3 y + y^2 z\,x + x^3 z + x\,z^3)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}\Bigg]$$

Amperian Current 4Vector curlH-dD/dt=J4 $= \left[\frac{8\,c\,t\,(-x^2 + 2\,y^2 + 2\,z^2 + c^2 t^2)\,(\varepsilon\,c^2 \mu - 1)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4 \mu},\right.$

$$-\frac{4\,(x^2 z + y^2 z + z^3 + 5\,z\,c^2 t^2 + 6\,x\,c\,t\,y)\,(\varepsilon\,c^2 \mu - 1)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4 \mu},$$

$$\left.\frac{4\,(y\,x^2 + y^3 + y\,z^2 + 5\,y\,c^2 t^2 - 6\,c\,t\,x\,z)\,(\varepsilon\,c^2 \mu - 1)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4 \mu}, 0\right]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= \Bigg[$

$$-\frac{16\,(\varepsilon\,c^2 \mu - 1)\,x\,(z^2 + y^2)\,(x^2 + y^2 + 11\,c^2 t^2 + z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^7 \mu}, -\frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^7 \mu}(8\,(\varepsilon\,c^2 \mu - 1)\,(-y\,x^4 - 8\,c^2 t^2 y\,x^2 + y^5 + 2\,y^3 z^2 + 14\,c^2 t^2 y^3 + y\,z^4 + 14\,c^2 t^2 y\,z^2 + 9\,c^4 t^4 y + 2\,c\,t\,x^3 z + 2\,c\,t\,y^2 z\,x + 2\,c\,t\,x\,z^3 - 2\,c^3 t^3 x\,z)),$$

$$-\frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^7 \mu}(8\,(\varepsilon\,c^2 \mu - 1)\,(-8\,c^2 t^2 x^2 z - 2\,c\,t\,y\,x^3 + 14\,y^2 c^2 t^2 z - 2\,c\,t\,y^3 x + 14\,c^2 t^2 z^3 - 2\,c\,t\,y\,x\,z^2 + 9\,z\,c^4 t^4 + 2\,c^3 t^3 y\,x - x^4 z + y^4 z + 2\,y^2 z^3 + z^5))\Bigg]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= \left[-\frac{2\,x\,c}{(-x^2 - y^2 - z^2 + c^2 t^2)^4},\right.$

$$\left.-\frac{2\,y\,c}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}, -\frac{2\,c\,z}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}, -\frac{2\,c\,t}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}\right]$$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $= \left[-\frac{4c^2 t}{(-x^2-y^2-z^2+c^2t^2)^6}, \frac{4zc}{(-x^2-y^2-z^2+c^2t^2)^6}, -\frac{4yc}{(-x^2-y^2-z^2+c^2t^2)^6} \right]$

Torsion Dissipation Jtorsion dot E $= \frac{4c^2 x}{(-x^2-y^2-z^2+c^2t^2)^6}$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{2x\,(2z^2+2y^2-\varepsilon c^2\mu x^2+\varepsilon c^2\mu y^2+\varepsilon c^2 z^2\mu+\varepsilon c^4\mu t^2)}{(-x^2-y^2-z^2+c^2t^2)^5\,\mu}, \right.$$
$$-\frac{2\,(y x^2-y^3-y z^2-3 y c^2 t^2-2 c t x z+2\varepsilon c^3\mu t x z+2\varepsilon c^2\mu y x^2)}{(-x^2-y^2-z^2+c^2t^2)^5\,\mu},$$
$$\frac{2\,(3 z c^2 t^2-2 x c t y-x^2 z+y^2 z+z^3+2\varepsilon c^3\mu x t y-2\varepsilon c^2\mu x^2 z)}{(-x^2-y^2-z^2+c^2t^2)^5\,\mu},$$
$$\left. \frac{2\varepsilon c^2 t\,(c^2t^2-x^2+3y^2+3z^2)}{(-x^2-y^2-z^2+c^2t^2)^5} \right]$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $= \Big[$

$$-\frac{1}{(-x^2-y^2-z^2+c^2t^2)^8\,\mu}\big(4ct\,(-2\varepsilon c^4\mu x^2t^2+4\varepsilon c^4\mu y^2t^2+4\varepsilon c^4\mu t^2z^2+\varepsilon c^6\mu t^4+\varepsilon c^2\mu x^4$$
$$+4\varepsilon c^2\mu x^2y^2+4\varepsilon c^2\mu x^2z^2+3\varepsilon c^2\mu y^4+6\varepsilon c^2\mu y^2z^2+3\varepsilon c^2\mu z^4-6y^2c^2t^2-2y^4-6c^2t^2z^2$$
$$-2z^4-2x^2y^2-4y^2z^2-2x^2z^2)\big), \frac{1}{(-x^2-y^2-z^2+c^2t^2)^8\,\mu}\big(4\,(-2cty x^3-2cty^3x$$
$$+2c^3t^3yx+2\varepsilon c^3\mu x^3ty+2\varepsilon c^3\mu xty^3-2\varepsilon c^5\mu xt^3y+6z^3\varepsilon c^4\mu t^2-y^4z-2y^2z^3-x^4z$$
$$-2x^2z^3-z^5+2z\varepsilon c^6\mu t^4+2\varepsilon c^3\mu xtyz^2-2\varepsilon c^4\mu t^2x^2z+6\varepsilon c^4\mu t^2y^2z-2ctyxz^2-2y^2x^2z$$
$$-4c^2t^2z^3-3zc^4t^4-4y^2c^2t^2z+4c^2t^2x^2z)\big), -\frac{1}{(-x^2-y^2-z^2+c^2t^2)^8\,\mu}\big(4\,(2cty^2zx$$
$$+4c^2t^2yx^2+2ctx^3z-4c^2t^2yz^2+2ctxz^3-2c^3t^3xz-y^5+6\varepsilon c^4\mu t^2y^3+2\varepsilon c^6\mu t^4y$$

$-4\,c^2\,t^2\,y^3 - 3\,c^4\,t^4\,y - 2\,y\,x^2\,z^2 - 2\,\varepsilon\,c^3\,\mu\,t\,x\,z\,y^2 - y\,x^4 - 2\,y^3\,x^2 - 2\,y^3\,z^2 - y\,z^4 - 2\,\varepsilon\,c^4\,\mu\,t^2\,y\,x^2$
$-2\,\varepsilon\,c^3\,\mu\,t\,x^3\,z - 2\,\varepsilon\,c^3\,\mu\,t\,x\,z^3 + 2\,\varepsilon\,c^5\,\mu\,t^3\,x\,z + 6\,\varepsilon\,c^4\,\mu\,y\,z^2\,t^2)\,)\,]$

Spin Dissipation J_spin dot E $= \dfrac{1}{(-x^2 - y^2 - z^2 + c^2\,t^2)^8\,\mu}\,(4\,c^3\,x\,(2\,\varepsilon\,\mu\,y^2\,z^2 - 8\,t^2\,y^2 - 8\,t^2\,z^2$
$+2\,\varepsilon\,\mu\,x^2\,z^2 + \varepsilon\,\mu\,z^4 + 2\,\varepsilon\,\mu\,y^2\,x^2 + \varepsilon\,\mu\,y^4 + 6\,\varepsilon\,c^2\,\mu\,t^2\,y^2 + \varepsilon\,\mu\,x^4 - 2\,\varepsilon\,c^2\,\mu\,t^2\,x^2 + \varepsilon\,c^4\,\mu\,t^4$
$+6\,\varepsilon\,c^2\,\mu\,t^2\,z^2)\,)$

Disspative Force 3 vector $= \Big[\,-\dfrac{1}{(-x^2 - y^2 - z^2 + c^2\,t^2)^8\,\mu}\,(4\,(-40\,\varepsilon\,c^4\,\mu\,t^2\,x^3\,y^2 - 40\,\varepsilon\,c^4\,\mu\,t^2\,x\,y^4 + 4\,z^2\,x^5$

$+8\,z^4\,x^3 + 4\,z^6\,x + 44\,\varepsilon\,c^6\,\mu\,y^2\,t^4\,x - 40\,\varepsilon\,c^4\,\mu\,x^3\,z^2\,t^2 + 44\,\varepsilon\,c^6\,\mu\,t^4\,z^2\,x - 40\,\varepsilon\,c^4\,\mu\,z^4\,t^2\,x - 2\,c\,t\,\mu\,x^2\,z^2$

$-4\,c\,t\,\mu\,y^2\,z^2 - 16\,\varepsilon\,c^2\,z^2\,\mu\,y^2\,x^3 - 12\,\varepsilon\,c^2\,z^2\,\mu\,y^4\,x - 12\,\varepsilon\,c^2\,z^4\,\mu\,y^2\,x + 4\,c^5\,t^3\,\mu^2\,\varepsilon\,z^2 + 3\,c^3\,t\,\mu^2\,\varepsilon\,z^4$

$+80\,x\,y^2\,c^2\,t^2\,z^2 - 4\,\varepsilon\,c^2\,z^2\,\mu\,x^5 - 8\,\varepsilon\,c^2\,z^4\,\mu\,x^3 - 4\,\varepsilon\,c^2\,z^6\,\mu\,x - 44\,y^2\,c^4\,t^4\,x + 4\,x\,y^6 - 8\,\varepsilon\,c^2\,\mu\,x^3\,y^4$

$-4\,x\,\varepsilon\,c^2\,\mu\,y^6 + 4\,c^5\,t^3\,\mu^2\,\varepsilon\,y^2 + c^3\,t\,\mu^2\,\varepsilon\,x^4 + 3\,c^3\,t\,\mu^2\,\varepsilon\,y^4 - 2\,c\,t\,\mu\,x^2\,y^2 + 2\,c^2\,t\,\mu\,x^2\,y^2 + 2\,c^2\,t\,\mu\,y^2\,z^2$

$+16\,z^2\,y^2\,x^3 + 12\,z^2\,y^4\,x + 12\,z^4\,y^2\,x + c^6\,t^5\,\mu - 4\,\varepsilon\,c^2\,\mu\,x^5\,y^2 - 2\,c^5\,t^3\,\mu^2\,\varepsilon\,x^2 + 2\,c^2\,t\,\mu\,x^2\,z^2$

$-44\,c^4\,t^4\,x\,z^2 + 40\,x^3\,y^2\,c^2\,t^2 + 40\,x\,y^4\,c^2\,t^2 + 40\,x^3\,z^2\,c^2\,t^2 + 40\,x\,z^4\,c^2\,t^2 + 4\,c^3\,t\,\mu^2\,\varepsilon\,x^2\,z^2$

$+6\,c^3\,t\,\mu^2\,\varepsilon\,y^2\,z^2 - 2\,c\,t\,\mu\,y^4 - 2\,c^4\,t^3\,\mu\,x^2 + c^2\,t\,\mu\,x^4 - 2\,c^4\,t^3\,\mu\,y^2 + c^2\,t\,\mu\,y^4 - 2\,c^4\,t^3\,\mu\,z^2 + c^2\,t\,\mu\,z^4$

$-80\,\varepsilon\,c^4\,\mu\,y^2\,z^2\,t^2\,x + c^7\,t^5\,\mu^2\,\varepsilon - 6\,c^3\,t^3\,\mu\,y^2 + 4\,x^5\,y^2 + 8\,x^3\,y^4 + 4\,c^3\,t\,\mu^2\,\varepsilon\,x^2\,y^2 - 6\,c^3\,t^3\,\mu\,z^2$

$-2\,c\,t\,\mu\,z^4)\,),\ -\dfrac{1}{(-x^2 - y^2 - z^2 + c^2\,t^2)^8\,\mu}\,(4\,(-14\,c^2\,t^2\,y\,x^4 + 12\,c^2\,t^2\,y^3\,x^2 + 52\,c^2\,t^2\,y^3\,z^2$

$+26\,c^2\,t^2\,y\,z^4 - 10\,c^4\,t^4\,y\,z^2 + 8\,y^2\,z^3\,c\,t\,x - 8\,y^2\,c^3\,t^3\,x\,z + 8\,x^3\,y^2\,c\,t\,z + 2\,x^4\,y^3\,\varepsilon\,c^2\,\mu + 4\,y^4\,c\,t\,x\,z$

$$-2\,y^5\,\varepsilon\,c^2\,\mu\,x^2+2\,y\,x^6\,\varepsilon\,c^2\,\mu-34\,c^6\,t^4\,y\,x^2\,\varepsilon\,\mu+10\,c^6\,t^4\,y\,z^2\,\varepsilon\,\mu-26\,c^4\,t^2\,y\,z^4\,\varepsilon\,\mu-52\,c^4\,t^2\,y^3\,\varepsilon\,z^2\,\mu$$

$$-8\,x^3\,z^3\,\varepsilon\,c^3\,\mu\,t-4\,z^5\,\varepsilon\,c^3\,\mu\,t\,x+8\,c^5\,t^3\,z^3\,\varepsilon\,\mu\,x-4\,c^3\,t\,x^5\,z\,\varepsilon\,\mu+8\,c^5\,t^3\,x^3\,z\,\varepsilon\,\mu-4\,c^7\,t^5\,x\,z\,\varepsilon\,\mu$$

$$-12\,c^4\,t^2\,y\,x^2\,\varepsilon\,z^2\,\mu+2\,y^7+10\,c^6\,t^4\,y^3\,\varepsilon\,\mu+18\,c^8\,t^6\,y\,\varepsilon\,\mu-2\,\varepsilon\,c^3\,\mu^2\,x^3\,t\,y-2\,\varepsilon\,c^3\,\mu^2\,x\,t\,y^3$$

$$+2\,\varepsilon\,c^5\,\mu^2\,x\,t^3\,y+2\,\varepsilon\,c^4\,\mu^2\,t^2\,x^2\,z-6\,\varepsilon\,c^4\,\mu^2\,t^2\,y^2\,z+2\,\mu\,c\,t\,y\,x\,z^2+34\,x^2\,c^4\,t^4\,y-4\,\varepsilon\,c^2\,z^2\,\mu\,y^3\,x^2$$

$$-2\,\varepsilon\,c^2\,z^4\,\mu\,y\,x^2-12\,\varepsilon\,c^4\,\mu\,t^2\,y^3\,x^2+2\,\varepsilon\,c^2\,z^2\,\mu\,y\,x^4+12\,c^2\,t^2\,y\,x^2\,z^2-6\,\varepsilon\,c^2\,z^2\,\mu\,y^5-6\,\varepsilon\,c^2\,z^4\,\mu\,y^3$$

$$-2\,\varepsilon\,c^3\,\mu^2\,x\,t\,y\,z^2+14\,\varepsilon\,c^4\,\mu\,t^2\,y\,x^4-2\,y\,x^6+\mu\,z^5-8\,y^2\,z^3\,\varepsilon\,c^3\,\mu\,t\,x+8\,y^2\,c^5\,t^3\,\varepsilon\,\mu\,x\,z$$

$$-8\,x^3\,y^2\,\varepsilon\,c^3\,\mu\,t\,z-4\,y^4\,\varepsilon\,c^3\,\mu\,t\,x\,z-10\,y^3\,c^4\,t^4-2\,\varepsilon\,c^2\,z^6\,\mu\,y-26\,\varepsilon\,c^4\,\mu\,t^2\,y^5+4\,z^2\,y^3\,x^2+2\,z^4\,y\,x^2$$

$$-2\,z^2\,y\,x^4+6\,z^2\,y^5+6\,z^4\,y^3+2\,z^6\,y+26\,c^2\,t^2\,y^5-18\,c^6\,t^6\,y+4\,c\,t\,x^5\,z-8\,c^3\,t^3\,x^3\,z+4\,c^5\,t^5\,x\,z$$

$$+2\,\mu\,y^2\,x^2\,z+4\,\mu\,c^2\,t^2\,z^3+3\,\mu\,z\,c^4\,t^4-2\,z^3\,c\,\mu\,x^2-z\,c\,\mu\,x^4-2\,z^3\,c\,\mu\,y^2-z\,c\,\mu\,y^4+2\,z^3\,c^3\,\mu\,t^2$$

$$-z\,c^5\,\mu\,t^4+2\,z\,c^3\,\mu\,x^2\,t^2+2\,\mu\,c\,t\,y\,x^3+2\,\mu\,c\,t\,y^3\,x-2\,\mu\,c^3\,t^3\,y\,x-6\,z^3\,\varepsilon\,c^4\,\mu^2\,t^2-2\,z\,\varepsilon\,c^6\,\mu^2\,t^4$$

$$+4\,\mu\,y^2\,c^2\,t^2\,z-4\,\mu\,c^2\,t^2\,x^2\,z-2\,z\,c\,\mu\,x^2\,y^2+2\,z\,c^3\,\mu\,y^2\,t^2+8\,x^3\,z^3\,c\,t+4\,z^5\,c\,t\,x-8\,c^3\,t^3\,z^3\,x$$

$$-2\,y^7\,\varepsilon\,c^2\,\mu-2\,x^4\,y^3+2\,x^2\,y^5+\mu\,y^4\,z+2\,\mu\,y^2\,z^3+\mu\,x^4\,z+2\,\mu\,x^2\,z^3-z^5\,c\,\mu)\,),$$

$$-\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^8\,\mu}\,(4\,(4\,c^7\,t^5\,y\,x\,\varepsilon\,\mu+2\,x^4\,z\,\varepsilon\,c^2\,\mu\,y^2-2\,y^4\,z\,\varepsilon\,c^2\,\mu\,x^2+4\,x^2\,z^3\,y^2-8\,c\,t\,y^3\,x^3$$

$$-4\,c\,t\,y^5\,x+8\,c^3\,t^3\,y^3\,x-4\,c\,t\,y\,x^5+8\,c^3\,t^3\,y\,x^3-4\,c^5\,t^5\,y\,x-4\,\mu\,c^2\,t^2\,y^3-3\,\mu\,c^4\,t^4\,y-2\,\mu\,y\,x^2\,z^2$$

$$+2\,y^3\,c\,\mu\,x^2+y\,c\,\mu\,x^4-2\,y^3\,c^3\,\mu\,t^2+2\,y^3\,c\,\mu\,z^2+y\,c\,\mu\,z^4+y\,c^5\,\mu\,t^4-2\,x^6\,z-\mu\,y^5-6\,\varepsilon\,c^2\,\mu\,y^4\,z^3$$

$$-6\,\varepsilon\,c^2\,\mu\,y^2\,z^5-26\,\varepsilon\,c^4\,\mu\,t^2\,z^5+10\,\varepsilon\,c^6\,\mu\,t^4\,z^3+2\,\varepsilon\,c^2\,\mu\,x^4\,z^3+18\,z\,\varepsilon\,c^8\,\mu\,t^6+12\,z\,x^2\,c^2\,t^2\,y^2$$

$$-2\,\varepsilon\,c^2\,\mu\,x^2\,z^5+8\,c^3\,t\,y\,x^3\,\varepsilon\,z^2\,\mu+8\,c^3\,t\,y^3\,x\,\varepsilon\,z^2\,\mu+4\,c^3\,t\,y\,x\,z^4\,\varepsilon\,\mu-8\,c^5\,t^3\,y\,x\,z^2\,\varepsilon\,\mu+4\,c^3\,t\,y\,x^5\,\varepsilon\,\mu$$

$$+8\,c^3\,t\,y^3\,x^3\,\varepsilon\,\mu-8\,c^5\,t^3\,y\,x^3\,\varepsilon\,\mu+4\,c^3\,t\,y^5\,x\,\varepsilon\,\mu-8\,c^5\,t^3\,y^3\,x\,\varepsilon\,\mu+2\,y^6\,z-12\,z\,\varepsilon\,c^4\,\mu\,y^2\,t^2\,x^2$$

$$-2\,\varepsilon\,c^3\,\mu^2\,t\,x\,z\,y^2-\mu\,y\,x^4-2\,\mu\,y^3\,x^2-2\,\mu\,y^3\,z^2-\mu\,y\,z^4+y^5\,c\,\mu-2\,y\,c^3\,\mu\,x^2\,t^2-4\,\varepsilon\,c^2\,\mu\,x^2\,z^3\,y^2$$

$$-12\,\varepsilon\,c^4\,\mu\,x^2\,z^3\,t^2-52\,\varepsilon\,c^4\,\mu\,y^2\,t^2\,z^3+14\,z\,\varepsilon\,c^4\,\mu\,x^4\,t^2-34\,z\,\varepsilon\,c^6\,\mu\,x^2\,t^4-26\,z\,\varepsilon\,c^4\,\mu\,y^4\,t^2$$

$$+10\,z\,\varepsilon\,c^6\,\mu\,y^2\,t^4-8\,c\,t\,y\,x^3\,z^2-8\,c\,t\,y^3\,x\,z^2-4\,c\,t\,y\,x\,z^4+8\,c^3\,t^3\,y\,x\,z^2+2\,x^6\,z\,\varepsilon\,c^2\,\mu-2\,y^6\,z\,\varepsilon\,c^2\,\mu$$

$$+2\,z^7+2\,\mu\,c\,t\,y^2\,z\,x-2\,\varepsilon\,c^4\,\mu^2\,t^2\,y\,x^2-2\,\varepsilon\,c^3\,\mu^2\,t\,x^3\,z-2\,\varepsilon\,c^3\,\mu^2\,t\,x\,z^3+2\,\varepsilon\,c^5\,\mu^2\,t^3\,x\,z$$

$$+6\,\varepsilon\,c^4\,\mu^2\,y\,z^2\,t^2-2\,\varepsilon\,c^2\,\mu\,z^7+12\,x^2\,c^2\,t^2\,z^3+52\,y^2\,c^2\,t^2\,z^3-14\,z\,x^4\,c^2\,t^2+34\,z\,x^2\,c^4\,t^4+26\,z\,y^4\,c^2\,t^2$$

$$-10\,z\,y^2\,c^4\,t^4-2\,x^4\,z\,y^2+2\,x^2\,y^4\,z+6\,y^2\,z^5+6\,y^4\,z^3+26\,c^2\,t^2\,z^5-10\,c^4\,t^4\,z^3-2\,x^4\,z^3-18\,z\,c^6\,t^6$$

$$+2\,x^2\,z^5+4\,\mu\,c^2\,t^2\,y\,x^2+2\,\mu\,c\,t\,x^3\,z-4\,\mu\,c^2\,t^2\,y\,z^2+2\,\mu\,c\,t\,x\,z^3-2\,\mu\,c^3\,t^3\,x\,z+6\,\varepsilon\,c^4\,\mu^2\,t^2\,y^3$$

$$+2\,\varepsilon\,c^6\,\mu^2\,t^4\,y+2\,y\,c\,\mu\,x^2\,z^2-2\,y\,c^3\,\mu\,t^2\,z^2)\,)\,]$$

$$Dissipation\ =\frac{2\,c\,(\mu\,\varepsilon\,c^3\,t^3-x^2\,c\,t\,\varepsilon\,\mu+3\,\varepsilon\,c\,\mu\,y^2\,t+3\,c\,t\,z^2\,\varepsilon\,\mu+x^3+x\,y^2+x\,z^2-c^2\,t^2\,x)}{(-x^2-y^2-z^2+c^2\,t^2)^5}$$

*****************************   *END PROCEDURE*   ***************************** **(33)**

Enter the name of the problem, and the components of the 4 potential.

p=2 n=4

```
> NAME:=`Example 9-- Index 1 Irreversible solution EdotB =0 Type 1 + Type 2`;

> Holder:=(x^2+y^2+z^2-(c*t)^2)^(4/2);
> Ax:=(c*t+y)*1/Holder;Ay:=(-z-x)*1/Holder;Az:=(c*t+y)*1/Holder;phi:=(+x+z)*c*1/Holder;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 9-- Index 1 Irreversible solution EdotB =0 Type 1 + Type 2

$$Holder := (x^2+y^2+z^2-c^2\,t^2)^2$$

$$Ax := \frac{c\,t+y}{(x^2+y^2+z^2-c^2\,t^2)^2}$$

$$Ay := \frac{-z-x}{(x^2+y^2+z^2-c^2\,t^2)^2}$$

$$Az := \frac{c\,t+y}{(x^2+y^2+z^2-c^2\,t^2)^2}$$

$$\phi := \frac{(x+z)\,c}{(x^2+y^2+z^2-c^2t^2)^2}$$

Example 9-- Index 1 Irreversible solution EdotB =0 Type 1 C Type 2

******************************** *Differential Form Format* ******************************
****

$$Action\ 1\text{-}form = \frac{(-x\,c-z\,c)\,d(t)}{(-x^2-y^2-z^2+c^2t^2)^2} + \frac{(c\,t+y)\,d(x)}{(-x^2-y^2-z^2+c^2t^2)^2} + \frac{(-z-x)\,d(y)}{(-x^2-y^2-z^2+c^2t^2)^2} + \frac{(c\,t+y)\,d(z)}{(-x^2-y^2-z^2+c^2t^2)^2}$$

$$Intensity\ 2\text{-}form\ F\!=\!dA\ = \left(-\frac{c(\%1)}{(-x^2-y^2-z^2+c^2t^2)^3}+\%3\right)(d(x))\ \&\hat{}\ (d(t)) + \left(-\frac{\%1}{(-x^2-y^2-z^2+c^2t^2)^3}-\frac{\%4}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(x))\ \&\hat{}\ ($$

$$Topological\ Torsion\ 3\text{-}form\quad A\hat{}F = \left(\frac{2\,(x+z)\,c\,(\%1)}{(-x^2-y^2-z^2+c^2t^2)^5}+\frac{2\,(x+z)\,c\,(\%2)}{(-x^2-y^2-z^2+c^2t^2)^5}-\%3\right)\&\hat{}(d(x),d(y),d(t)) + \left(-\frac{2\,(c\,t+y)\,(\%1)}{(-x^2-y^2-z^2+c^2t^2)^5}\right.$$

$$Topological\ Parity\ 4\text{-}form\qquad F\hat{}F = 0$$

******************************** *Using EM format* ********************************

$$E\,field = \left[\frac{2\,c\,(-x^2+y^2+z^2+c^2t^2+2\,c\,t\,y-2\,x\,z)}{(-x^2-y^2-z^2+c^2t^2)^3}, -\frac{4\,(x+z)\,c\,(c\,t+y)}{(-x^2-y^2-z^2+c^2t^2)^3}, \frac{2\,c\,(x^2+y^2-z^2+c^2t^2+2\,c\,t\,y-2\,x\,z)}{(-x^2-y^2-z^2+c^2t^2)^3}\right]$$

$$B\,field = \left[\frac{2\,(-x^2+y^2+z^2+c^2t^2+2\,c\,t\,y+2\,x\,z)}{(-x^2-y^2-z^2+c^2t^2)^3}, -\frac{4\,(c\,t+y)\,(-z+x)}{(-x^2-y^2-z^2+c^2t^2)^3},\right.$$

$$-\frac{2\left(x^2+y^2-z^2+c^2t^2+2xz+2cty\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}\Bigg]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension PTD $= 2$

******************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************
****

$$\text{Xm or linear (Mean) curvature} = \frac{4ct\left(1+c^2\right)(x+z)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}$$

Yg or quadratic (GAUSS) curvature =

$$-\frac{2\left(3c^4t^2+4c^3ty-c^2t^2+y^2c^2-x^2c^2-c^2z^2-4xc^2z-4cty-3y^2-4xz-x^2-z^2\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}$$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $= 0$

$$D\,field = \left[\frac{2\varepsilon c\left(-x^2+y^2+z^2+c^2t^2+2cty-2xz\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}, -\frac{4\varepsilon(x+z)c(ct+y)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}, \frac{2\varepsilon c\left(x^2+y^2-z^2+c^2t^2+2cty-2xz\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3}\right]$$

$$H\,field = \left[\frac{2\left(-x^2+y^2+z^2+c^2t^2+2cty+2xz\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3\mu}, -\frac{4(ct+y)(-z+x)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3\mu}, -\frac{2\left(x^2+y^2-z^2+c^2t^2+2xz+2cty\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^3\mu}\right]$$

$$\text{Poynting vector ExH} = \left[\frac{16c(ct+y)x(\%1)}{\left(-x^2-y^2-z^2+c^2t^2\right)^6\mu}\quad \frac{8c\left(y^4-z^4-2x^2z^2+c^4t^4+4cty^3+4c^3t^3y+6y^2c^2t^2-x^4\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^6\mu}\quad \frac{16c(ct+y)z(\%1)}{\left(-x^2-y^2-z^2+c^2t^2\right)^6\mu}\right]$$

$$\%1 = z^2+2cty+x^2+y^2+c^2t^2$$

Amperian Current 4Vector curlH-dD/dt=J4

$$= \left[\frac{4\left(-2ctx^2+4cty^2+4ctz^2+2c^3t^3+5yc^2t^2+yx^2+y^3+yz^2-6ctxz\right)\left(\varepsilon c^2\mu-1\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^4\mu},\right.$$

$$-\frac{4\,(x+z)\,(x^2+y^2+z^2+5\,c^2\,t^2+6\,c\,t\,y)\,(\varepsilon\,c^2\,\mu-1)}{(-x^2-y^2-z^2+c^2\,t^2)^4\,\mu},$$

$$\left.\frac{4\,(4\,c\,t\,x^2+4\,c\,t\,y^2-2\,c\,t\,z^2+2\,c^3\,t^3+5\,y\,c^2\,t^2+y\,x^2+y^3+y\,z^2-6\,c\,t\,x\,z)\,(\varepsilon\,c^2\,\mu-1)}{(-x^2-y^2-z^2+c^2\,t^2)^4\,\mu},0\right]$$

Amerian charge density *divD = rho* $=0$

divergence Lorentz Current 4Vector, *4div(J4)* $=0$

Topological SPIN 4 vector S4 $=\left[\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu}(2\,(-3\,\varepsilon\,c^2\,\mu\,x^2\,z+x^3-z^3-2\,c\,t\,y\,z\right.$

$+3\,c^2\,t^2\,x-z\,c^2\,t^2+x\,z^2+6\,x\,c\,t\,y-\varepsilon\,c^2\,\mu\,x\,z^2+2\,\varepsilon\,c^3\,\mu\,t\,y\,z+2\,\varepsilon\,c^3\,\mu\,x\,t\,y+3\,x\,y^2+3\,x^2\,z-y^2\,z$

$+\varepsilon\,c^4\,\mu\,z\,t^2+\varepsilon\,c^4\,\mu\,t^2\,x+\varepsilon\,c^2\,\mu\,y^2\,z+\varepsilon\,c^2\,\mu\,x\,y^2-\varepsilon\,c^2\,\mu\,x^3+\varepsilon\,c^2\,\mu\,z^3)),$

$$-\frac{4\,(c\,t+y)\,(-y^2-c^2\,t^2-2\,c\,t\,y-2\,x\,z+\varepsilon\,c^2\,\mu\,x^2+2\,\varepsilon\,c^2\,\mu\,x\,z+\varepsilon\,c^2\,z^2\,\mu)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu},$$

$\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu}(2\,(-\varepsilon\,c^2\,\mu\,x^2\,z-x^3+z^3+6\,c\,t\,y\,z-c^2\,t^2\,x+3\,z\,c^2\,t^2+3\,x\,z^2-2\,x\,c\,t\,y$

$-3\,\varepsilon\,c^2\,\mu\,x\,z^2+2\,\varepsilon\,c^3\,\mu\,t\,y\,z+2\,\varepsilon\,c^3\,\mu\,x\,t\,y-x\,y^2+x^2\,z+3\,y^2\,z+\varepsilon\,c^4\,\mu\,z\,t^2+\varepsilon\,c^4\,\mu\,t^2\,x+\varepsilon\,c^2\,\mu\,y^2\,z$

$\left.+\varepsilon\,c^2\,\mu\,x\,y^2+\varepsilon\,c^2\,\mu\,x^3-\varepsilon\,c^2\,\mu\,z^3)),\frac{4\,(c\,t+y)\,\varepsilon\,c\,(z^2+2\,c\,t\,y+x^2+y^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5}\right]$

Topological SPIN 3-form $=\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu}(2\,(-3\,\varepsilon\,c^2\,\mu\,x^2\,z+x^3-z^3-2\,c\,t\,y\,z+3\,c^2\,t^2\,x$

$-z\,c^2\,t^2+x\,z^2+6\,x\,c\,t\,y-\varepsilon\,c^2\,\mu\,x\,z^2+2\,\varepsilon\,c^3\,\mu\,t\,y\,z+2\,\varepsilon\,c^3\,\mu\,x\,t\,y+3\,x\,y^2+3\,x^2\,z-y^2\,z+\varepsilon\,c^4\,\mu\,z\,t^2$

$+\varepsilon\,c^4\,\mu\,t^2\,x+\varepsilon\,c^2\,\mu\,y^2\,z+\varepsilon\,c^2\,\mu\,x\,y^2-\varepsilon\,c^2\,\mu\,x^3+\varepsilon\,c^2\,\mu\,z^3)\ \&\hat{}(d(y),d(z),d(t)))$

$+\frac{4\,(c\,t+y)\,(-y^2-c^2\,t^2-2\,c\,t\,y-2\,x\,z+\varepsilon\,c^2\,\mu\,x^2+2\,\varepsilon\,c^2\,\mu\,x\,z+\varepsilon\,c^2\,z^2\,\mu)\ \&\hat{}(d(x),d(z),d(t))}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu}$

$+\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu}(2\,(-\varepsilon\,c^2\,\mu\,x^2\,z-x^3+z^3+6\,c\,t\,y\,z-c^2\,t^2\,x+3\,z\,c^2\,t^2+3\,x\,z^2$

$-2\,x\,c\,t\,y-3\,\varepsilon\,c^2\,\mu\,x\,z^2+2\,\varepsilon\,c^3\,\mu\,t\,y\,z+2\,\varepsilon\,c^3\,\mu\,x\,t\,y-x\,y^2+x^2\,z+3\,y^2\,z+\varepsilon\,c^4\,\mu\,z\,t^2+\varepsilon\,c^4\,\mu\,t^2\,x$

$+\varepsilon\,c^2\,\mu\,y^2\,z+\varepsilon\,c^2\,\mu\,x\,y^2+\varepsilon\,c^2\,\mu\,x^3-\varepsilon\,c^2\,\mu\,z^3)\ \&\hat{}(d(x),d(y),d(t)))$

$$-\frac{4\,(c\,t+y)\,\varepsilon\,c\,(z^2+2\,c\,t\,y+x^2+y^2+c^2\,t^2)\ \&\text{^}(d(x),d(y),d(z))}{(-x^2-y^2-z^2+c^2\,t^2)^5}$$

$$\textit{Spin density rho\_spin} = \frac{4\,(c\,t+y)\,\varepsilon\,c\,(z^2+2\,c\,t\,y+x^2+y^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5}$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{8\,(z^2+2\,c\,t\,y+x^2+y^2+c^2\,t^2)^2\,(\varepsilon\,c^2\,\mu-1)}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}$$

$$B.H = \frac{8\,(z^2+2\,c\,t\,y+x^2+y^2+c^2\,t^2)^2}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}$$

$$D.E = \frac{8\,\varepsilon\,c^2\,(z^2+2\,c\,t\,y+x^2+y^2+c^2\,t^2)^2}{(-x^2-y^2-z^2+c^2\,t^2)^6}$$

$$A.J = \frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}\,(4\,(\varepsilon\,c^2\,\mu-1)\,(3\,x^2\,y^2+2\,y^4+z^4+2\,x^2\,z^2+3\,y^2\,z^2+4\,c^4\,t^4$$
$$+10\,c\,t\,y\,x^2+10\,c\,t\,y\,z^2-2\,x\,z\,c^2\,t^2+10\,c\,t\,y^3+14\,c^3\,t^3\,y+2\,y^2\,z\,x+7\,c^2\,t^2\,z^2+7\,x^2\,c^2\,t^2$$
$$+18\,y^2\,c^2\,t^2+x^4+2\,x^3\,z+2\,x\,z^3))$$

$$\textit{-rho.phi} = 0$$

$$\textit{Poincare I}\qquad \textit{(B.H - D.E)-(A.J - rho.phi)} = -\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}\,(4\,(\varepsilon\,c^2\,\mu-1)\,(7\,x^2\,y^2+4\,y^4$$
$$+3\,z^4+6\,x^2\,z^2+7\,y^2\,z^2+6\,c^4\,t^4+18\,c\,t\,y\,x^2+18\,c\,t\,y\,z^2-2\,x\,z\,c^2\,t^2+18\,c\,t\,y^3+22\,c^3\,t^3\,y+2\,y^2\,z\,x$$
$$+11\,c^2\,t^2\,z^2+11\,x^2\,c^2\,t^2+30\,y^2\,c^2\,t^2+3\,x^4+2\,x^3\,z+2\,x\,z^3))$$

$$\textit{London Coefficient}\qquad LC = 0$$

$$\textit{PROCA coefficient curlcurlB} = \Bigg[-\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^5}\,(8\,(x^2\,y^2+2\,y^4+2\,z^4+x^2\,z^2+4\,y^2\,z^2+5\,c^4\,t^4$$
$$+9\,c\,t\,y\,x^2+9\,c\,t\,y\,z^2+21\,x\,z\,c^2\,t^2+9\,c\,t\,y^3+15\,c^3\,t^3\,y+3\,y^2\,z\,x+17\,c^2\,t^2\,z^2-4\,x^2\,c^2\,t^2+17\,y^2\,c^2\,t^2$$
$$-x^4+3\,x^3\,z+3\,x\,z^3)),$$

$$\frac{24\,(-z+x)\,(y\,x^2+3\,c\,t\,x^2+y\,z^2+3\,c\,t\,y^2+7\,y\,c^2\,t^2+y^3+5\,c^3\,t^3+3\,c\,t\,z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5},$$

$$\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^5}\,(8\,(4\,x^2\,y^2+2\,y^4-z^4+x^2\,z^2+y^2\,z^2+5\,c^4\,t^4+9\,c\,t\,y\,x^2+9\,c\,t\,y\,z^2$$
$$+21\,x\,z\,c^2\,t^2+9\,c\,t\,y^3+15\,c^3\,t^3\,y+3\,y^2\,z\,x-4\,c^2\,t^2\,z^2+17\,x^2\,c^2\,t^2+17\,y^2\,c^2\,t^2+2\,x^4+3\,x^3\,z$$

$+ 3\,x\,z^3))]$

Amperian Current 4Vector   curlH-dD/dt=J4

$$= \left[\frac{4\left(-2\,c\,t\,x^2 + 4\,c\,t\,y^2 + 4\,c\,t\,z^2 + 2\,c^3\,t^3 + 5\,y\,c^2\,t^2 + y\,x^2 + y^3 + y\,z^2 - 6\,c\,t\,x\,z\right)\left(\varepsilon\,c^2\,\mu - 1\right)}{\left(-x^2 - y^2 - z^2 + c^2\,t^2\right)^4\,\mu}, \right.$$

$$- \frac{4\,(x+z)\,\left(x^2 + y^2 + z^2 + 5\,c^2\,t^2 + 6\,c\,t\,y\right)\left(\varepsilon\,c^2\,\mu - 1\right)}{\left(-x^2 - y^2 - z^2 + c^2\,t^2\right)^4\,\mu},$$

$$\left. \frac{4\left(4\,c\,t\,x^2 + 4\,c\,t\,y^2 - 2\,c\,t\,z^2 + 2\,c^3\,t^3 + 5\,y\,c^2\,t^2 + y\,x^2 + y^3 + y\,z^2 - 6\,c\,t\,x\,z\right)\left(\varepsilon\,c^2\,\mu - 1\right)}{\left(-x^2 - y^2 - z^2 + c^2\,t^2\right)^4\,\mu}, 0 \right]$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B) $= \Big[$

$$- \frac{1}{\left(-x^2 - y^2 - z^2 + c^2\,t^2\right)^7\,\mu}\Big(8\left(\varepsilon\,c^2\,\mu - 1\right)\big(x^5 + 4\,x^3\,y^2 + 2\,x^3\,z^2 + 3\,x\,y^4 + x\,z^4 + 18\,c\,t\,y\,x^3$$

$$+ 18\,c\,t\,y^3\,x + 30\,c^3\,t^3\,y\,x + 4\,x\,y^2\,z^2 + 9\,c^4\,t^4\,x - y^4\,z - 2\,y^2\,z^3 + 3\,x^4\,z + 2\,x^2\,z^3 + 36\,x\,y^2\,c^2\,t^2$$

$$+ 14\,x\,z^2\,c^2\,t^2 - 2\,c\,t\,y^3\,z - 2\,c\,t\,y\,z^3 + 2\,c^3\,t^3\,y\,z - z^5 + 18\,c\,t\,y\,x\,z^2 + 2\,y^2\,x^2\,z + z\,c^4\,t^4 - 4\,c^2\,t^2\,x^2\,z$$

$$+ 14\,x^3\,c^2\,t^2 - 2\,c\,t\,y\,z\,x^2\big)\Big), - \frac{1}{\left(-x^2 - y^2 - z^2 + c^2\,t^2\right)^7\,\mu}\Big(16\left(\varepsilon\,c^2\,\mu - 1\right)\big(2\,c\,t\,y^2\,z\,x$$

$$+ 3\,c^2\,t^2\,y\,x^2 + 2\,c\,t\,x^3\,z + 3\,c^2\,t^2\,y\,z^2 + 2\,c\,t\,x\,z^3 - 2\,c^3\,t^3\,x\,z + y^5 + z^2\,c^3\,t^3 - 3\,z^4\,c\,t - 2\,y\,c^2\,t^2\,x\,z$$

$$+ 6\,y^4\,c\,t + 16\,y^2\,c^3\,t^3 + 14\,c^2\,t^2\,y^3 + 9\,c^4\,t^4\,y + y^3\,x^2 + y^3\,z^2 + 3\,z^2\,c\,t\,y^2 + 3\,c\,t\,x^2\,y^2 - 6\,c\,t\,x^2\,z^2$$

$$- 3\,c\,t\,x^4 + c^3\,t^3\,x^2 + 2\,y\,x^3\,z + 2\,y^3\,x\,z + 2\,y\,z^3\,x + 2\,c^5\,t^5\big)\Big),$$

$$- \frac{1}{\left(-x^2 - y^2 - z^2 + c^2\,t^2\right)^7\,\mu}\Big(8\left(\varepsilon\,c^2\,\mu - 1\right)\big(-x^5 - 2\,x^3\,y^2 + 2\,x^3\,z^2 - x\,y^4 + 3\,x\,z^4 - 2\,c\,t\,y\,x^3$$

$$- 2\,c\,t\,y^3\,x + 2\,c^3\,t^3\,y\,x + 2\,x\,y^2\,z^2 + c^4\,t^4\,x + 3\,y^4\,z + 4\,y^2\,z^3 + x^4\,z + 2\,x^2\,z^3 - 4\,x\,z^2\,c^2\,t^2 + 18\,c\,t\,y^3\,z$$

$$+ 18\,c\,t\,y\,z^3 + 30\,c^3\,t^3\,y\,z + z^5 - 2\,c\,t\,y\,x\,z^2 + 4\,y^2\,x^2\,z + 14\,c^2\,t^2\,z^3 + 9\,z\,c^4\,t^4 + 36\,y^2\,c^2\,t^2\,z$$

$$+ 14\,c^2\,t^2\,x^2\,z + 18\,c\,t\,y\,z\,x^2\big)\Big)\Big]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $=[0,0,0,0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $=[0,0,0]$

Torsion Dissipation Jtorsion dot E $=0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \Bigg[\frac{1}{(-x^2-y^2-z^2+c^2t^2)^5\,\mu} (2\,($

$-3\,\varepsilon\,c^2\,\mu\,x^2\,z+x^3-z^3-2\,c\,t\,y\,z+3\,c^2\,t^2\,x-z\,c^2\,t^2+x\,z^2+6\,x\,c\,t\,y-\varepsilon\,c^2\,\mu\,x\,z^2+2\,\varepsilon\,c^3\,\mu\,t\,y\,z$

$+2\,\varepsilon\,c^3\,\mu\,x\,t\,y+3\,x\,y^2+3\,x^2\,z-y^2\,z+\varepsilon\,c^4\,\mu\,z\,t^2+\varepsilon\,c^4\,\mu\,t^2\,x+\varepsilon\,c^2\,\mu\,y^2\,z+\varepsilon\,c^2\,\mu\,x\,y^2-\varepsilon\,c^2\,\mu\,x^3$

$+\varepsilon\,c^2\,\mu\,z^3)),\ -\frac{4\,(c\,t+y)\,(-y^2-c^2\,t^2-2\,c\,t\,y-2\,x\,z+\varepsilon\,c^2\,\mu\,x^2+2\,\varepsilon\,c^2\,\mu\,x\,z+\varepsilon\,c^2\,z^2\,\mu)}{(-x^2-y^2-z^2+c^2t^2)^5\,\mu},$

$\frac{1}{(-x^2-y^2-z^2+c^2t^2)^5\,\mu}(2\,(-\varepsilon\,c^2\,\mu\,x^2\,z-x^3+z^3+6\,c\,t\,y\,z-c^2\,t^2\,x+3\,z\,c^2\,t^2+3\,x\,z^2-2\,x\,c\,t\,y$

$-3\,\varepsilon\,c^2\,\mu\,x\,z^2+2\,\varepsilon\,c^3\,\mu\,t\,y\,z+2\,\varepsilon\,c^3\,\mu\,x\,t\,y-x\,y^2+x^2\,z+3\,y^2\,z+\varepsilon\,c^4\,\mu\,z\,t^2+\varepsilon\,c^4\,\mu\,t^2\,x+\varepsilon\,c^2\,\mu\,y^2\,z$

$+\varepsilon\,c^2\,\mu\,x\,y^2+\varepsilon\,c^2\,\mu\,x^3-\varepsilon\,c^2\,\mu\,z^3)),\ \frac{4\,(c\,t+y)\,\varepsilon\,c\,(z^2+2\,c\,t\,y+x^2+y^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2t^2)^5} \Bigg]$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= \left[-\%2,\ \frac{8\,(x+z)\,(\%1)^2\,(\varepsilon\,c^2\,\mu-1)}{(-x^2-y^2-z^2+c^2t^2)^8\,\mu},\ -\%2 \right]$

$\%1 = z^2+2\,c\,t\,y+x^2+y^2+c^2\,t^2$

$\%2 = \frac{8\,(\%1)^2\,(c\,t+y)\,(\varepsilon\,c^2\,\mu-1)}{(-x^2-y^2-z^2+c^2t^2)^8\,\mu}$

Spin Dissipation J_spin dot E $= \frac{8\,c\,(x+z)\,(z^2+2\,c\,t\,y+x^2+y^2+c^2\,t^2)^2\,(\varepsilon\,c^2\,\mu-1)}{(-x^2-y^2-z^2+c^2t^2)^8\,\mu}$

Disspative Force 3 vector $= \Bigg[-\frac{1}{(-x^2-y^2-z^2+c^2t^2)^8\,\mu}(8\,(\varepsilon\,c^2\,\mu-1)\,(-x^7-5\,x^5\,y^2-3\,x^5\,z^2$

$-7\,x^3\,y^4-3\,x^3\,z^4-3\,x\,y^6-x\,z^6+y^6\,z+3\,y^4\,z^3+3\,y^2\,z^5-3\,x^6\,z-5\,x^4\,z^3-x^2\,z^5+\mu\,y^5$

$-46\,x\,z^2\,y^2\,c^2\,t^2+4\,c\,t\,y^3\,z\,x^2+4\,c\,t\,y\,z^3\,x^2-4\,c^3\,t^3\,y\,z\,x^2-18\,c\,t\,y\,x\,z^4+6\,y^2\,x^2\,z\,c^2\,t^2+2\,c\,t\,y\,z\,x^4$

$+6\,\mu\,c^2\,t^2\,y\,x^2+6\,\mu\,c^2\,t^2\,y\,z^2+6\,\mu\,z^2\,c\,t\,y^2+6\,\mu\,c\,t\,x^2\,y^2+2\,\mu\,c\,t\,x^2\,z^2-5\,x\,z^4\,y^2-y^4\,z\,x^2-2\,y^2\,z^3\,x^2$

$-5\,x^4\,z\,y^2-z^5\,c^2\,t^2-z^3\,c^4\,t^4+z\,c^6\,t^6+\mu\,y\,x^4+2\,\mu\,y^3\,x^2+2\,\mu\,y^3\,z^2+\mu\,y\,z^4+\mu\,c^5\,t^5+9\,c^6\,t^6\,x$

$-10\,x^3\,y^2\,z^2-13\,x^5\,c^2\,t^2+5\,c^4\,t^4\,x^3-7\,x\,y^4\,z^2+z^7-36\,c\,t\,y\,x^3\,z^2-36\,c\,t\,y^3\,x\,z^2-12\,c^3\,t^3\,y\,x\,z^2$

$-2\,z^3\,y^2\,c^2\,t^2+5\,\mu\,y^4\,c\,t+10\,\mu\,y^2\,c^3\,t^3+10\,\mu\,c^2\,t^2\,y^3+5\,\mu\,c^4\,t^4\,y+2\,\mu\,y\,x^2\,z^2+\mu\,c\,t\,x^4+2\,\mu\,c^3\,t^3\,x^2$

$-46\,x^3\,y^2\,c^2\,t^2-26\,x^3\,z^2\,c^2\,t^2-33\,x\,y^4\,c^2\,t^2-13\,x\,z^4\,c^2\,t^2-18\,c\,t\,y\,x^5-36\,c\,t\,y^3\,x^3-12\,c^3\,t^3\,y\,x^3$

$-18\,c\,t\,y^5\,x-12\,c^3\,t^3\,y^3\,x+30\,c^5\,t^5\,y\,x+27\,c^4\,t^4\,x\,y^2+5\,c^4\,t^4\,x\,z^2-y^4\,z\,c^2\,t^2+7\,x^4\,z\,c^2\,t^2$

$+6\,x^2\,z^3\,c^2\,t^2+2\,c\,t\,y^5\,z+4\,c\,t\,y^3\,z^3-4\,c^3\,t^3\,y^3\,z+2\,c\,t\,y\,z^5-4\,c^3\,t^3\,y\,z^3+2\,c^5\,t^5\,y\,z-5\,z\,c^4\,t^4\,x^2$

$-z\,c^4\,t^4\,y^2+2\,\mu\,z^2\,c^3\,t^3+\mu\,z^4\,c\,t)),\ -\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^8\,\mu}(8\,(\varepsilon\,c^2\,\mu-1)\,(-2\,y^7$

$-20\,c^3\,t^3\,y^4-8\,z^4\,c^3\,t^3+10\,y^3\,c^4\,t^4-26\,y^5\,c^2\,t^2-2\,z^2\,c^5\,t^5+6\,z^6\,c\,t-12\,y^6\,c\,t+28\,y^2\,c^5\,t^5$

$-4\,\mu\,c\,t\,y\,x\,z^2-4\,\mu\,c\,t\,y\,z\,x^2-8\,c\,t\,y^2\,z\,x^3-4\,c\,t\,y^4\,z\,x-8\,c\,t\,y^2\,z^3\,x+8\,c^3\,t^3\,y^2\,z\,x-12\,c^2\,t^2\,y\,x^2\,z^2$

$+8\,y\,c^2\,t^2\,x^3\,z+8\,y^3\,c^2\,t^2\,x\,z+8\,y\,c^2\,t^2\,x\,z^3-4\,y\,c^4\,t^4\,x\,z-4\,\mu\,c\,t\,y\,x^3-4\,\mu\,c\,t\,y^3\,x-4\,\mu\,c^3\,t^3\,y\,x$

$-6\,\mu\,x\,y^2\,c^2\,t^2-2\,\mu\,x\,z^2\,c^2\,t^2-4\,\mu\,c\,t\,y^3\,z-4\,\mu\,c\,t\,y\,z^3-4\,\mu\,c^3\,t^3\,y\,z-6\,\mu\,y^2\,c^2\,t^2\,z-2\,\mu\,c^2\,t^2\,x^2\,z$

$-2\,y^3\,x^4-4\,y^5\,z^2+4\,c^7\,t^7-4\,y^5\,x^2-\mu\,x^5-\mu\,z^5-12\,c^4\,t^4\,y\,x^2-4\,c\,t\,x^5\,z-8\,c\,t\,x^3\,z^3+8\,c^3\,t^3\,x^3\,z$

$-12\,c^4\,t^4\,y\,z^2-4\,c\,t\,x\,z^5+8\,c^3\,t^3\,x\,z^3-4\,c^5\,t^5\,x\,z-16\,z^2\,c^3\,t^3\,x^2+18\,z^4\,c\,t\,x^2-18\,y^4\,c\,t\,x^2$

$$- 18 y^4 c t z^2 - 28 y^2 c^3 t^3 x^2 + 18 c t x^4 z^2 - 2 \mu x y^2 z^2 - \mu c^4 t^4 x - 2 \mu y^2 x^2 z - 2 \mu c^2 t^2 z^3 - \mu z c^4 t^4$$

$$- 2 \mu x^3 c^2 t^2 - 32 z^2 c^2 t^2 y^3 - 28 z^2 y^2 c^3 t^3 - 6 z^4 y c^2 t^2 - 2 z^4 y^3 + 18 c^6 t^6 y - 4 y^3 x^2 z^2 + 6 c t x^6$$

$$- 8 c^3 t^3 x^4 - 2 c^5 t^5 x^2 - 4 y x^5 z - 8 y^3 x^3 z - 8 y x^3 z^3 - 4 y^5 x z - 8 y^3 x z^3 - 4 y z^5 x - 2 \mu x^3 y^2$$

$$- 2 \mu x^3 z^2 - \mu x y^4 - \mu x z^4 - \mu y^4 z - 2 \mu y^2 z^3 - \mu x^4 z - 2 \mu x^2 z^3 - 6 c^2 t^2 y x^4 - 32 c^2 t^2 y^3 x^2)),$$

$$- \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^8 \mu} (8 (\varepsilon c^2 \mu - 1) (x^7 + 3 x^5 y^2 - x^5 z^2 + 3 x^3 y^4 - 5 x^3 z^4 + x y^6 - 3 x z^6$$

$$- 3 y^6 z - 7 y^4 z^3 - 5 y^2 z^5 - x^6 z - 3 x^4 z^3 - 3 x^2 z^5 + \mu y^5 + 6 x z^2 y^2 c^2 t^2 - 36 c t y^3 z x^2 - 36 c t y z^3 x^2$$

$$- 12 c^3 t^3 y z x^2 + 2 c t y x z^4 - 46 y^2 x^2 z c^2 t^2 - 18 c t y z x^4 + 6 \mu c^2 t^2 y x^2 + 6 \mu c^2 t^2 y z^2 + 6 \mu z^2 c t y^2$$

$$+ 6 \mu c t x^2 y^2 + 2 \mu c t x^2 z^2 - 5 x z^4 y^2 - 7 y^4 z x^2 - 10 y^2 z^3 x^2 - 5 x^4 z y^2 - 13 z^5 c^2 t^2 + 5 z^3 c^4 t^4$$

$$+ 9 z c^6 t^6 + \mu y x^4 + 2 \mu y^3 x^2 + 2 \mu y^3 z^2 + \mu y z^4 + \mu c^5 t^5 + c^6 t^6 x - 2 x^3 y^2 z^2 - x^5 c^2 t^2 - c^4 t^4 x^3$$

$$- x y^4 z^2 - z^7 + 4 c t y x^3 z^2 + 4 c t y^3 x z^2 - 4 c^3 t^3 y x z^2 - 46 z^3 y^2 c^2 t^2 + 5 \mu y^4 c t + 10 \mu y^2 c^3 t^3$$

$$+ 10 \mu c^2 t^2 y^3 + 5 \mu c^4 t^4 y + 2 \mu y x^2 z^2 + \mu c t x^4 + 2 \mu c^3 t^3 x^2 - 2 x^3 y^2 c^2 t^2 + 6 x^3 z^2 c^2 t^2 - x y^4 c^2 t^2$$

$$+ 7 x z^4 c^2 t^2 + 2 c t y x^5 + 4 c t y^3 x^3 - 4 c^3 t^3 y x^3 + 2 c t y^5 x - 4 c^3 t^3 y^3 x + 2 c^5 t^5 y x - c^4 t^4 x y^2$$

$$- 5 c^4 t^4 x z^2 - 33 y^4 z c^2 t^2 - 13 x^4 z c^2 t^2 - 26 x^2 z^3 c^2 t^2 - 18 c t y^5 z - 36 c t y^3 z^3 - 12 c^3 t^3 y^3 z$$

$$- 18 c t y z^5 - 12 c^3 t^3 y z^3 + 30 c^5 t^5 y z + 5 z c^4 t^4 x^2 + 27 z c^4 t^4 y^2 + 2 \mu z^2 c^3 t^3 + \mu z^4 c t))]$$

$$Dissipation = \frac{4 \mu (c t + y) \varepsilon c (z^2 + 2 c t y + x^2 + y^2 + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}$$

***************************** *END PROCEDURE* *************************** **(34)**

Example   Saturn's Rings   Plasma Accretion Disc   Hedge Hog Solution

****

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`Example 10a  Saturns Rings  -- a Plasma Accretion disc from a Hedge Hog
  solution.  p201 vol4`:
> Holder:=(1*x^2+1*y^2+1*c*z^2)^(1/2);Holder2:=(1*x^2+1*y^2);
> A1:=(alpha*z*m/Holder2/Holder*(-y));A2:=(alpha*z*m/Holder2/Holder*x);
> A3:=0; phi:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,c,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

$$Holder := \sqrt{x^2 + y^2 + z^2 c}$$

$$Holder2 := x^2 + y^2$$

$$A1 := -\frac{\alpha z m y}{(x^2+y^2)\sqrt{x^2+y^2+z^2 c}}$$

$$A2 := \frac{\alpha z m x}{(x^2+y^2)\sqrt{x^2+y^2+z^2 c}}$$

$$A3 := 0$$

$$\phi := 0$$

Example 10a Saturns Rings -- a Plasma Accretion disc from a Hedge Hog solution. p201 vol4

******************************** *Differential Form Format* ******************************

$$Action\ 1\text{-}form = -\frac{\alpha z m y\, d(x)}{(x^2+y^2)\sqrt{x^2+y^2+z^2 c}} + \frac{\alpha z m x\, d(y)}{(x^2+y^2)\sqrt{x^2+y^2+z^2 c}}$$

$$Intensity\ 2\text{-}form\ F{=}dA = \left(\frac{\alpha z m\,(-x^2 y^2 - 2 y^4 - z^2 c y^2 + x^4 + c x^2 z^2)}{(x^2+y^2)^2 (x^2+y^2+z^2 c)^{3/2}} - \frac{\alpha z m\,(2 x^4 + x^2 y^2 + c x^2 z^2 - y^4 - z^2 c y^2)}{(x^2+y^2)^2 (x^2+y^2+z^2 c)^{3/2}}\right) (d(x))\ \&\hat{}\ (d(y)) + \frac{\alpha m y\,(d(x))\ \&\hat{}\ (d(z))}{(x^2+y^2+z^2 c)^{3/2}} - \frac{\alpha m x\,(d(y))\ \&\hat{}\ (d(z))}{(x^2+y^2+z^2 c)^{3/2}}$$

$$Topological\ Torsion\ 3\text{-}form \quad A\hat{}F = 0$$

$$Topological\ Parity\ 4\text{-}form \quad F\hat{}F = 0$$

********************************* *Using EM format* *********************************

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[-\frac{\alpha\, m\, x}{(x^2+y^2+z^2 c)^{3/2}}, -\frac{\alpha\, m\, y}{(x^2+y^2+z^2 c)^{3/2}}, -\frac{\alpha\, z\, m}{(x^2+y^2+z^2 c)^{3/2}} \right]$$

Topological TORSION 4 vector   T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension   PTD = 2

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************************

Xm or linear (Mean) curvature = 0

$$Yg\ or\ quadratic\ (GAUSS)\ curvature = -\frac{(2x^2+2y^2+z^2 c)\,\alpha^2 z^2 m^2}{(x^2+y^2+z^2 c)^2 (x^2+y^2)^2}$$

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

**********   *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

D field = [0, 0, 0]

$$H\ field = \left[-\frac{\alpha\, m\, x}{(x^2+y^2+z^2 c)^{3/2}\mu}, -\frac{\alpha\, m\, y}{(x^2+y^2+z^2 c)^{3/2}\mu}, -\frac{\alpha\, z\, m}{(x^2+y^2+z^2 c)^{3/2}\mu} \right]$$

Poynting vector ExH = *EXH*

$$Amperian\ Current\ 4Vector\quad curlH\text{-}dD/dt=J4 = \left[-\frac{3\,\alpha\, z\, m\, y\,(-1+c)}{(x^2+y^2+z^2 c)^{5/2}\mu}, \frac{3\,\alpha\, z\, m\, x\,(-1+c)}{(x^2+y^2+z^2 c)^{5/2}\mu}, 0, 0 \right]$$

Amerian charge density   divD = rho = 0

divergence Lorentz Current 4Vector,   4div(J4) = 0

$$Topological\ SPIN\ 4\ vector\ S4 = \left[-\frac{\alpha^2 z^2 m^2 x}{(x^2+y^2)(x^2+y^2+z^2 c)^2\mu}, -\frac{\alpha^2 z^2 m^2 y}{(x^2+y^2)(x^2+y^2+z^2 c)^2\mu}, \frac{\alpha^2 z\, m^2}{(x^2+y^2+z^2 c)^2\mu}, 0 \right]$$

$$Topological\ SPIN\ 3\text{-}form = -\frac{\alpha^2 z^2 m^2 x\ \&\hat{}(d(y), d(z), d(t))}{(x^2+y^2)(x^2+y^2+z^2 c)^2\mu} + \frac{\alpha^2 z^2 m^2 y\ \&\hat{}(d(x), d(z), d(t))}{(x^2+y^2)(x^2+y^2+z^2 c)^2\mu} + \frac{\alpha^2 z\, m^2\ \&\hat{}(d(x), d(y), d(t))}{(x^2+y^2+z^2 c)^2\mu}$$

$$\textit{Spin density rho\_spin} = 0$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{\alpha^2 m^2 (x^2+y^2+z^2)}{(x^2+y^2+z^2 c)^3 \mu}$$

$$B.H = \frac{\alpha^2 m^2 (x^2+y^2+z^2)}{(x^2+y^2+z^2 c)^3 \mu}$$

$$D.E = 0$$

$$A.J = \frac{3\,\alpha^2 z^2 m^2 (-1+c)}{(x^2+y^2+z^2 c)^3 \mu}$$

$$\textit{-rho.phi} = 0$$

$$\textit{Poincare I} \qquad \textit{(B.H - D.E)-(A.J - rho.phi)} = -\frac{\alpha^2 m^2 (-x^2-y^2-4 z^2+3 z^2 c)}{(x^2+y^2+z^2 c)^3 \mu}$$

$$\textit{London Coefficient} \qquad LC = \frac{3\,(-1+c)\,(x^2+y^2)}{(x^2+y^2+z^2 c)^2 \mu}$$

$$\textit{PROCA coefficient curlcurlB} = \left[\frac{3\,\alpha\, m\, x\,(-1+c)\,(4 z^2 c-x^2-y^2)}{(x^2+y^2+z^2 c)^{7/2}}, \frac{3\,\alpha\, m\, y\,(-1+c)\,(4 z^2 c-x^2-y^2)}{(x^2+y^2+z^2 c)^{7/2}}, \frac{3\,\alpha\, z\, m\,(-1+c)\,(2 z^2 c-3 y^2-3 x^2)}{(x^2+y^2+z^2 c)^{7/2}} \right]$$

$$\textit{Amperian Current 4Vector} \qquad \textit{curlH-dD/dt=J4} = \left[-\frac{3\,\alpha\, z\, m\, y\,(-1+c)}{(x^2+y^2+z^2 c)^{5/2} \mu}, \frac{3\,\alpha\, z\, m\, x\,(-1+c)}{(x^2+y^2+z^2 c)^{5/2} \mu}, 0, 0 \right]$$

$$\textit{Lorentz Force 3 vector due to Ampere current} \quad \textit{FL = -(rho\_ampere E + J\_ampere x B)}$$

$$= \left[\frac{3\,\alpha^2 z^2 m^2 x\,(-1+c)}{(x^2+y^2+z^2 c)^4 \mu}, \frac{3\,\alpha^2 z^2 m^2 y\,(-1+c)}{(x^2+y^2+z^2 c)^4 \mu}, -\frac{3\,\alpha^2 z\, m^2 (-1+c)\,(x^2+y^2)}{(x^2+y^2+z^2 c)^4 \mu} \right]$$

$$\textit{Amperian Dissipation Jampere dot E} = 0$$

$$\textit{Lorentz Force Spin factor LFSPIN} = -\frac{1}{3} \frac{(x^2+y^2+z^2 c)^2}{(x^2+y^2)\,(-1+c)}$$

$$\textit{Topological Torsion current 4 vector} \quad \textit{T4 = -[ExA + B.phi,AdotB]} = [0, 0, 0, 0]$$

$$\textit{Lorentz Force 3 vector due to Torsion current} \quad \textit{TF = -(rho\_torsion E + J\_torsion x B)} = [0, 0, 0]$$

$$\textit{Torsion Dissipation Jtorsion dot E} = 0$$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \left[-\frac{\alpha^2 z^2 m^2 x}{(x^2+y^2)\,(x^2+y^2+z^2 c)^2 \mu}, -\frac{\alpha^2 z^2 m^2 y}{(x^2+y^2)\,(x^2+y^2+z^2 c)^2 \mu}, \frac{\alpha^2 z m^2}{(x^2+y^2+z^2 c)^2 \mu}, 0 \right]$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= \left[-\frac{\alpha^3 z m^3 y\,(x^2+y^2+z^2)}{(x^2+y^2+z^2 c)^{7/2}\,(x^2+y^2)\,\mu}, \frac{\alpha^3 z m^3 x\,(x^2+y^2+z^2)}{(x^2+y^2+z^2 c)^{7/2}\,(x^2+y^2)\,\mu}, 0 \right]$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \Big[\frac{1}{(x^2+y^2+z^2 c)^{15/2}\,\mu\,(x^2+y^2)} \big(\alpha^2 z m^2 \big(-3 z x\,(x^2+y^2+z^2 c)^{7/2} y^2$

$+ 3 z x^3 (x^2+y^2+z^2 c)^{7/2} c + 3 z x\,(x^2+y^2+z^2 c)^{7/2} c\, y^2 - 4\,\alpha\, m\, y^9 \mu\, c\, z^2 - 6\,\alpha\, m\, y^7 \mu\, c^2 z^4$

$- 4\,\alpha\, m\, y^5 \mu\, z^6 c^3 - \alpha\, m\, y^3 \mu\, z^8 c^4 - 4\,\alpha\, m\, y^3 \mu\, z^2 x^6 - 6\,\alpha\, m\, y^5 \mu\, z^2 x^4 - 4\,\alpha\, m\, y^7 \mu\, z^2 x^2 - 4\,\alpha\, m\, y^7 \mu\, z^4 c$

$- 6\,\alpha\, m\, y^5 \mu\, z^6 c^2 - 4\,\alpha\, m\, y^3 \mu\, z^8 c^3 - \alpha\, m\, y\, \mu\, z^2 x^8 - \alpha\, m\, y\, \mu\, z^{10} c^4 - 16\,\alpha\, m\, y^3 \mu\, x^6 z^2 c - \alpha\, m\, y^{11} \mu$

$- 3 z x^3 (x^2+y^2+z^2 c)^{7/2} - 24\,\alpha\, m\, y^5 \mu\, x^4 c\, z^2 - 18\,\alpha\, m\, y^3 \mu\, x^4 c^2 z^4 - 16\,\alpha\, m\, y^7 \mu\, x^2 c\, z^2$

$- 18\,\alpha\, m\, y^5 \mu\, x^2 c^2 z^4 - 8\,\alpha\, m\, y^3 \mu\, x^2 z^6 c^3 - 12\,\alpha\, m\, y^3 \mu\, z^4 x^4 c - 12\,\alpha\, m\, y^5 \mu\, z^4 x^2 c$

$- 12\,\alpha\, m\, y^3 \mu\, z^6 x^2 c^2 - 4\,\alpha\, m\, y\, \mu\, x^8 c\, z^2 - 6\,\alpha\, m\, y\, \mu\, x^6 c^2 z^4 - 4\,\alpha\, m\, y\, \mu\, x^4 z^6 c^3 - \alpha\, m\, y\, \mu\, x^2 z^8 c^4$

$- 4\,\alpha\, m\, y\, \mu\, z^4 x^6 c - 6\,\alpha\, m\, y\, \mu\, z^6 x^4 c^2 - 4\,\alpha\, m\, y\, \mu\, z^8 x^2 c^3 - \alpha\, m\, y\, \mu\, x^{10} - 5\,\alpha\, m\, y^9 \mu\, x^2 - 5\,\alpha\, m\, y^3 \mu\, x^8$

$- 10\,\alpha\, m\, y^5 \mu\, x^6 - 10\,\alpha\, m\, y^7 \mu\, x^4 - \alpha\, m\, y^9 \mu\, z^2 \big)\big),$

$\frac{1}{(x^2+y^2+z^2 c)^{15/2}\,\mu\,(x^2+y^2)} \big(\alpha^2 z m^2 \big(16\,\alpha\, m\, x^7 \mu\, y^2 z^2 c + \alpha\, m\, x\, \mu\, y^{10} + 5\,\alpha\, m\, x^3 \mu\, y^8$

$$+5\,\alpha\,m\,x^9\,\mu\,y^2+10\,\alpha\,m\,x^7\,\mu\,y^4+10\,\alpha\,m\,x^5\,\mu\,y^6+\alpha\,m\,x^9\,\mu\,z^2+24\,\alpha\,m\,x^5\,\mu\,c\,y^4\,z^2$$

$$+18\,\alpha\,m\,x^5\,\mu\,y^2\,c^2\,z^4+16\,\alpha\,m\,x^3\,\mu\,y^6\,c\,z^2+18\,\alpha\,m\,x^3\,\mu\,y^4\,c^2\,z^4+8\,\alpha\,m\,x^3\,\mu\,y^2\,z^6\,c^3$$

$$+12\,\alpha\,m\,x^5\,\mu\,z^4\,y^2\,c+12\,\alpha\,m\,x^3\,\mu\,z^4\,c\,y^4+12\,\alpha\,m\,x^3\,\mu\,z^6\,y^2\,c^2+4\,\alpha\,m\,x\,\mu\,y^8\,c\,z^2+6\,\alpha\,m\,x\,\mu\,y^6\,c^2\,z^4$$

$$+4\,\alpha\,m\,x\,\mu\,y^4\,z^6\,c^3+\alpha\,m\,x\,\mu\,y^2\,z^8\,c^4+4\,\alpha\,m\,x\,\mu\,z^4\,y^6\,c+6\,\alpha\,m\,x\,\mu\,z^6\,y^4\,c^2+4\,\alpha\,m\,x\,\mu\,z^8\,y^2\,c^3$$

$$+4\,\alpha\,m\,x^9\,\mu\,c\,z^2+6\,\alpha\,m\,x^7\,\mu\,c^2\,z^4+4\,\alpha\,m\,x^5\,\mu\,z^6\,c^3+\alpha\,m\,x^3\,\mu\,z^8\,c^4+4\,\alpha\,m\,x^7\,\mu\,z^2\,y^2+6\,\alpha\,m\,x^5\,\mu\,z^2\,y^4$$

$$+4\,\alpha\,m\,x^3\,\mu\,z^2\,y^6+4\,\alpha\,m\,x^7\,\mu\,z^4\,c+6\,\alpha\,m\,x^5\,\mu\,z^6\,c^2+4\,\alpha\,m\,x^3\,\mu\,z^8\,c^3+\alpha\,m\,x\,\mu\,z^2\,y^8+\alpha\,m\,x\,\mu\,z^{10}\,c^4$$

$$+3\,z\,y\,(x^2+y^2+z^2\,c)^{7/2}\,c\,x^2-3\,z\,y\,(x^2+y^2+z^2\,c)^{7/2}\,x^2+3\,z\,y^3\,(x^2+y^2+z^2\,c)^{7/2}\,c$$

$$\left.-3\,z\,y^3\,(x^2+y^2+z^2\,c)^{7/2}+\alpha\,m\,x^{11}\,\mu)\right),\ -\frac{3\,\alpha^2\,z\,m^2\,(-1+c)\,(x^2+y^2)}{(x^2+y^2+z^2\,c)^4\,\mu}\Bigg]$$

$$Dissipation\ =0$$

***************************** *END PROCEDURE* *************************** **(35)**

****

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`Example 10b  Dirac Type magnetic HedgeHog solution.  p201 vol4`:
> Holder:=(1*x^2+1*y^2+1*c*z^2)^(2/2);Holder2:=(1*x^2+1*y^2)^0;
> A1:=(alpha*m/Holder2/Holder*(-y));A2:=(alpha*m/Holder2/Holder*x);
> A3:=0; phi:=0;ee:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,c,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

$$Holder:=x^2+y^2+z^2\,c$$

$$Holder2:=1$$

$$A1 := -\frac{\alpha\, m\, y}{x^2+y^2+z^2 c}$$

$$A2 := \frac{\alpha\, m\, x}{x^2+y^2+z^2 c}$$

$$A3 := 0$$

$$\phi := 0$$

$$ee := 0$$

Example 10b Dirac Type magnetic HedgeHog solution. p201 vol4

******************************** *Differential Form Format* ********************************

$$\text{Action 1-form} = -\frac{\alpha\, m\, y\, d(x)}{x^2+y^2+z^2 c} + \frac{\alpha\, m\, x\, d(y)}{x^2+y^2+z^2 c}$$

$$\text{Intensity 2-form } F{=}dA = \left(\frac{m\,\alpha\,(-y^2+x^2+z^2 c)}{(x^2+y^2+z^2 c)^2} + \frac{m\,\alpha\,(-x^2+y^2+z^2 c)}{(x^2+y^2+z^2 c)^2}\right)(d(x))\ \&\text{^}\ (d(y))$$

$$-\frac{2\,\alpha\, m\, y\, z\, c\, (d(x))\ \&\text{^}\ (d(z))}{(x^2+y^2+z^2 c)^2} + \frac{2\,\alpha\, m\, x\, z\, c\, (d(y))\ \&\text{^}\ (d(z))}{(x^2+y^2+z^2 c)^2}$$

$$\text{Topological Torsion 3-form} \quad A\text{^}F = 0$$

$$\text{Topological Parity 4-form} \quad F\text{^}F = 0$$

******************************** *Using EM format* ********************************

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[\frac{2\,\alpha\, m\, x\, z\, c}{(x^2+y^2+z^2 c)^2}, \frac{2\,\alpha\, m\, y\, z\, c}{(x^2+y^2+z^2 c)^2}, \frac{2\, m\,\alpha\, z^2 c}{(x^2+y^2+z^2 c)^2}\right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension PTD $= 2$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

Xm or linear (Mean) curvature $= 0$

$$\text{Yg or quadratic (GAUSS) curvature} = \frac{\alpha^2 m^2 (z^2 c - x^2 - y^2)}{(x^2+y^2+z^2 c)^3}$$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\text{Chirality factor } CH = 0$$

$$D\ field = [0, 0, 0]$$

$$H\ field = \left[\frac{2\,\alpha\,m\,x\,z\,c}{(x^2+y^2+z^2\,c)^2\,\mu}, \frac{2\,\alpha\,m\,y\,z\,c}{(x^2+y^2+z^2\,c)^2\,\mu}, \frac{2\,m\,\alpha\,z^2\,c}{(x^2+y^2+z^2\,c)^2\,\mu}\right]$$

$$\text{Poynting vector } ExH = EXH$$

$$\text{Amperian Current 4Vector} \quad curlH\text{-}dD/dt\text{=}J4 \ = \left[\frac{2\,\alpha\,m\,y\,c\,(-x^2-y^2-4\,z^2+3\,z^2\,c)}{(x^2+y^2+z^2\,c)^3\,\mu}, -\frac{2\,\alpha\,m\,x\,c\,(-x^2-y^2-4\,z^2+3\,z^2\,c)}{(x^2+y^2+z^2\,c)^3\,\mu}, 0, 0\right]$$

$$\text{Amerian charge density} \quad divD = rho = 0$$

$$\text{divergence Lorentz Current 4Vector,} \quad 4div(J4) \ = 0$$

$$\text{Topological SPIN 4 vector } S4 = \left[\frac{2\,\alpha^2\,m^2\,x\,z^2\,c}{(x^2+y^2+z^2\,c)^3\,\mu}, \frac{2\,\alpha^2\,m^2\,y\,z^2\,c}{(x^2+y^2+z^2\,c)^3\,\mu}, -\frac{2\,\alpha^2\,m^2\,z\,c\,(x^2+y^2)}{(x^2+y^2+z^2\,c)^3\,\mu}, 0\right]$$

$$\text{Topological SPIN 3-form} = \frac{2\,\alpha^2\,m^2\,x\,z^2\,c\,\&\hat{}(d(y),d(z),d(t))}{(x^2+y^2+z^2\,c)^3\,\mu} - \frac{2\,\alpha^2\,m^2\,y\,z^2\,c\,\&\hat{}(d(x),d(z),d(t))}{(x^2+y^2+z^2\,c)^3\,\mu} - \frac{2\,\alpha^2\,m^2\,z\,c\,(x^2+y^2)\,\&\hat{}(d(x),d(y),d(t))}{(x^2+y^2+z^2\,c)^3\,\mu}$$

$$\text{Spin density } rho\_spin = 0$$

$$\text{LaGrange field energy density (B.H-D.E)} = \frac{4\,\alpha^2\,m^2\,z^2\,c^2\,(x^2+y^2+z^2)}{(x^2+y^2+z^2\,c)^4\,\mu}$$

$$B.H = \frac{4\,\alpha^2\,m^2\,z^2\,c^2\,(x^2+y^2+z^2)}{(x^2+y^2+z^2\,c)^4\,\mu}$$

$$D.E = 0$$

$$A.J = -\frac{2\,\alpha^2\,m^2\,c\,(-x^2-y^2-4\,z^2+3\,z^2\,c)\,(x^2+y^2)}{(x^2+y^2+z^2\,c)^4\,\mu}$$

$$-rho.phi = 0$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)*

$$= \frac{2\,c\,\alpha^2\,m^2\,(5\,c\,x^2\,z^2+5\,z^2\,c\,y^2+2\,z^4\,c-x^4-2\,x^2\,y^2-4\,x^2\,z^2-y^4-4\,y^2\,z^2)}{(x^2+y^2+z^2\,c)^4\,\mu}$$

London Coefficient $\quad LC = -\dfrac{2\,c\,(-x^2 - y^2 - 4\,z^2 + 3\,z^2\,c)}{(x^2 + y^2 + z^2\,c)^2\,\mu}$

PROCA coefficient curlcurlB $= \left[-\dfrac{8\,\alpha\,m\,x\,z\,c\,(\%1)}{(x^2+y^2+z^2\,c)^4}, -\dfrac{8\,\alpha\,m\,y\,z\,c\,(\%1)}{(x^2+y^2+z^2\,c)^4}, -\dfrac{4\,\alpha\,m\,c\,(-8\,c\,x^2\,z^2 - 8\,z^2\,c\,y^2 + 3\,c^2\,z^4 + x^4 + 2\,x^2\,y^2 + y^4 + 8\,x^2\,z^2 + 8\,y^2\,z^2 -}{(x^2+y^2+z^2\,c)^4} \right.$

$$\%1 = 3\,c^2\,z^2 - 3\,c\,x^2 - 3\,c\,y^2 + 2\,x^2 + 2\,y^2 - 4\,z^2\,c$$

Amperian Current 4Vector $\quad$ *curlH-dD/dt=J4* $= \left[\dfrac{2\,\alpha\,m\,y\,c\,(-x^2 - y^2 - 4\,z^2 + 3\,z^2\,c)}{(x^2+y^2+z^2\,c)^3\,\mu}, -\dfrac{2\,\alpha\,m\,x\,c\,(-x^2 - y^2 - 4\,z^2 + 3\,z^2\,c)}{(x^2+y^2+z^2\,c)^3\,\mu}, 0, 0 \right]$

Lorentz Force 3 vector due to Ampere current $\quad$ *FL = -(rho_ampere E + J_ampere x B)*

$$= \left[\frac{4\,\alpha^2\,m^2\,x\,c^2\,(-x^2 - y^2 - 4\,z^2 + 3\,z^2\,c)\,z^2}{(x^2+y^2+z^2\,c)^5\,\mu}, \frac{4\,\alpha^2\,m^2\,y\,c^2\,(-x^2 - y^2 - 4\,z^2 + 3\,z^2\,c)\,z^2}{(x^2+y^2+z^2\,c)^5\,\mu}, -\frac{4\,\alpha^2\,m^2\,c^2\,(-x^2 - y^2 - 4\,z^2 + 3\,z^2\,c)\,z\,(x^2+y^2)}{(x^2+y^2+z^2\,c)^5\,\mu} \right]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= \dfrac{1}{2}\,\dfrac{(x^2+y^2+z^2\,c)^2}{c\,(-x^2 - y^2 - 4\,z^2 + 3\,z^2\,c)}$

Topological Torsion current 4 vector $\quad$ *T4 = -[ExA + B.phi,AdotB]* $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current $\quad$ *TF = -(rho_torsion E + J_torsion x B)* $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector $\quad$ *TS4 = -[A x H + D.phi,AdotD]* $= \left[\dfrac{2\,\alpha^2\,m^2\,x\,z^2\,c}{(x^2+y^2+z^2\,c)^3\,\mu}, \dfrac{2\,\alpha^2\,m^2\,y\,z^2\,c}{(x^2+y^2+z^2\,c)^3\,\mu}, -\dfrac{2\,\alpha^2\,m^2\,z\,c\,(x^2+y^2)}{(x^2+y^2+z^2\,c)^3\,\mu}, 0 \right]$

Lorentz Force 3 vector due to Spin current $\quad$ *SF = --(rho_spin E + J_spin x B)* $= \left[-\dfrac{4\,\alpha^3\,m^3\,y\,z^2\,c^2\,(x^2+y^2+z^2)}{(x^2+y^2+z^2\,c)^5\,\mu}, \dfrac{4\,\alpha^3\,m^3\,z^2\,c^2\,x\,(x^2+y^2+z^2)}{(x^2+y^2+z^2\,c)^5\,\mu}, 0 \right]$

$$Spin\ Dissipation\ \ J\_spin\ dot\ E\ \ = 0$$

Disspative Force 3 vector

$$= \left[\frac{4\,\alpha^2\,m^2\,z^2\,c^2\,(3\,c\,x\,z^2 - 4\,x\,z^2 - x^3 - x\,y^2 - \alpha\,m\,y\,\mu\,x^2 - \alpha\,m\,y^3\,\mu - \alpha\,m\,y\,\mu\,z^2)}{(x^2+y^2+z^2\,c)^5\,\mu}, \right.$$

$$\frac{4\,\alpha^2\,m^2\,z^2\,c^2\,(3\,z^2\,c\,y - 4\,y\,z^2 - y\,x^2 - y^3 + \alpha\,m\,x^3\,\mu + \alpha\,m\,x\,\mu\,y^2 + \alpha\,m\,x\,\mu\,z^2)}{(x^2+y^2+z^2\,c)^5\,\mu},$$

$$\left. -\frac{4\,\alpha^2\,m^2\,c^2\,(-x^2 - y^2 - 4\,z^2 + 3\,z^2\,c)\,z\,(x^2+y^2)}{(x^2+y^2+z^2\,c)^5\,\mu} \right]$$

$$Dissipation\ \ = 0$$

******************************* *END PROCEDURE* ****************************** **(36)**

****
Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`Example 10c  Dirac Type magnetic HedgeHog solution.  p201 vol4`:
> Holder:=(1*x^2+1*y^2+1*c*z^2)^(4/2);Holder2:=(1*x^2+1*y^2)^0;
> A1:=(alpha*m*z^2/Holder2/Holder*(-y));A2:=(alpha*m*z^2/Holder2/Holder*x);
> A3:=0; phi:=0;ee:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,c,1,2,0,0,0*alpha*(g+I*gamma),0):
```

$$Holder := (x^2+y^2+z^2\,c)^2$$

$$Holder2 := 1$$

$$A1 := -\frac{\alpha\,m\,z^2\,y}{(x^2+y^2+z^2\,c)^2}$$

$$A2 := \frac{\alpha\,m\,z^2\,x}{(x^2+y^2+z^2\,c)^2}$$

$$A3 := 0$$

$$\phi := 0$$

$$ee := 0$$

Example 10c Dirac Type magnetic HedgeHog solution. p201 vol4

*********************************** *Differential Form Format* **********************************

$$Action\ 1\text{-}form = -\frac{\alpha\,m\,z^2\,y\,d(x)}{(x^2+y^2+z^2\,c)^2} + \frac{\alpha\,m\,z^2\,x\,d(y)}{(x^2+y^2+z^2\,c)^2}$$

Intensity 2-form F=dA $= \left(\frac{m\,\alpha\,z^2\,(-3\,y^2+x^2+z^2\,c)}{(x^2+y^2+z^2\,c)^3} + \frac{m\,\alpha\,z^2\,(-3\,x^2+y^2+z^2\,c)}{(x^2+y^2+z^2\,c)^3} \right) (d(x))\ \&\hat{}\ (d(y))$

$$- \frac{2\,\alpha\,z\,m\,y\,(z^2\,c-x^2-y^2)\,(d(x))\ \&\hat{}\ (d(z))}{(x^2+y^2+z^2\,c)^3} + \frac{2\,\alpha\,z\,m\,x\,(z^2\,c-x^2-y^2)\,(d(y))\ \&\hat{}\ (d(z))}{(x^2+y^2+z^2\,c)^3}$$

Topological Torsion 3-form *A^F* = 0

Topological Parity 4-form *F^F* = 0

******************************** *Using EM format* ********************************

E field = [0, 0, 0]

B field $= \left[\frac{2\,\alpha\,z\,m\,x\,(z^2\,c-x^2-y^2)}{(x^2+y^2+z^2\,c)^3}, \frac{2\,\alpha\,z\,m\,y\,(z^2\,c-x^2-y^2)}{(x^2+y^2+z^2\,c)^3}, \frac{2\,m\,\alpha\,z^2\,(z^2\,c-x^2-y^2)}{(x^2+y^2+z^2\,c)^3} \right]$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension *PTD* = 2

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************
****

Xm or linear (Mean) curvature = 0

Yg or quadratic (GAUSS) curvature $= \frac{\alpha^2\,m^2\,z^4\,(-3\,x^2-3\,y^2+z^2\,c)}{(x^2+y^2+z^2\,c)^5}$

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

D field = [0, 0, 0]

H field $= \left[\frac{2\,\alpha\,z\,m\,x\,(z^2\,c-x^2-y^2)}{(x^2+y^2+z^2\,c)^3\,\mu}, \frac{2\,\alpha\,z\,m\,y\,(z^2\,c-x^2-y^2)}{(x^2+y^2+z^2\,c)^3\,\mu}, \frac{2\,m\,\alpha\,z^2\,(z^2\,c-x^2-y^2)}{(x^2+y^2+z^2\,c)^3\,\mu} \right]$

Poynting vector ExH = *EXH*

Amperian Current 4Vector curlH-dD/dt=J4 $= \left[\frac{2\,\alpha\,m\,y\,(\%1)}{(x^2+y^2+z^2\,c)^4\,\mu}, -\frac{2\,\alpha\,m\,x\,(\%1)}{(x^2+y^2+z^2\,c)^4\,\mu}, 0, 0 \right]$

$\%1 = 4\,x^2\,z^2+4\,y^2\,z^2-8\,z^4\,c+2\,x^2\,y^2+y^4+x^4+3\,c^2\,z^4-8\,c\,x^2\,z^2-8\,z^2\,c\,y^2$

Amerian charge density *divD = rho* = 0

divergence Lorentz Current 4Vector, *4div(J4)* = 0

$$\text{Topological SPIN 4 vector S4} = \left[\frac{2\,\alpha^2\,m^2\,z^4\,x\,(z^2\,c - x^2 - y^2)}{(x^2+y^2+z^2\,c)^5\,\mu}, \frac{2\,\alpha^2\,m^2\,z^4\,y\,(z^2\,c - x^2 - y^2)}{(x^2+y^2+z^2\,c)^5\,\mu}, -\frac{2\,\alpha^2\,m^2\,z^3\,(z^2\,c - x^2 - y^2)\,(x^2+y^2)}{(x^2+y^2+z^2\,c)^5\,\mu}, 0 \right]$$

$$\text{Topological SPIN 3-form} = \frac{2\,\alpha^2\,m^2\,z^4\,x\,(z^2\,c - x^2 - y^2)\ \&\hat{}(d(y), d(z), d(t))}{(x^2+y^2+z^2\,c)^5\,\mu} - \frac{2\,\alpha^2\,m^2\,z^4\,y\,(z^2\,c - x^2 - y^2)\ \&\hat{}(d(x), d(z), d(t))}{(x^2+y^2+z^2\,c)^5\,\mu} - \frac{2\,\alpha^2\,m^2\,z^3\,(z^2\,c - x^2 - y^2)\,(x^2+y^2)\ \&\hat{}(d(x), d(y), d(t))}{(x^2+y^2+z^2\,c)^5\,\mu}$$

$$\text{Spin density rho\_spin} = 0$$

$$\text{LaGrange field energy density (B.H-D.E)} = \frac{4\,\alpha^2\,z^2\,m^2\,(z^2\,c - x^2 - y^2)^2\,(x^2+y^2+z^2)}{(x^2+y^2+z^2\,c)^6\,\mu}$$

$$B.H = \frac{4\,\alpha^2\,z^2\,m^2\,(z^2\,c - x^2 - y^2)^2\,(x^2+y^2+z^2)}{(x^2+y^2+z^2\,c)^6\,\mu}$$

$$D.E = 0$$

$$A.J = -\frac{2\,\alpha^2\,m^2\,z^2\,(4\,x^2\,z^2 + 4\,y^2\,z^2 - 8\,z^4\,c + 2\,x^2\,y^2 + y^4 + x^4 + 3\,c^2\,z^4 - 8\,c\,x^2\,z^2 - 8\,z^2\,c\,y^2)\,(x^2+y^2)}{(x^2+y^2+z^2\,c)^6\,\mu}$$

$$\text{-rho.phi} = 0$$

$$\text{Poincare I} \quad \text{(B.H - D.E)-(A.J - rho.phi)} = \frac{1}{(x^2+y^2+z^2\,c)^6\,\mu}\big(2\,\alpha^2\,z^2\,m^2\,(9\,x^4\,y^2 + 6\,x^4\,z^2 + 6\,y^4\,z^2 + 9\,x^2\,y^4 + 3\,y^6 + 3\,x^6 + 5\,x^2\,c^2\,z^4 + 5\,y^2\,c^2\,z^4 + 12\,y^2\,z^2\,x^2 - 24\,x^2\,y^2\,z^2\,c - 12\,z^4\,c\,y^2 + 2\,z^6\,c^2 - 12\,c\,z^4\,x^2 - 12\,c\,x^4\,z^2 - 12\,c\,y^4\,z^2)\big)$$

$$\text{London Coefficient} \quad LC = -\frac{2\,(4\,x^2\,z^2 + 4\,y^2\,z^2 - 8\,z^4\,c + 2\,x^2\,y^2 + y^4 + x^4 + 3\,c^2\,z^4 - 8\,c\,x^2\,z^2 - 8\,z^2\,c\,y^2)}{(x^2+y^2+z^2\,c)^2\,\mu\,z^2}$$

$$\text{PROCA coefficient curlcurlB} = \left[-\frac{8\,\alpha\,z\,m\,x\,(\%1)}{(x^2+y^2+z^2\,c)^5}, -\frac{8\,\alpha\,z\,m\,y\,(\%1)}{(x^2+y^2+z^2\,c)^5}, -\frac{4\,m\,\alpha\,(-3\,x^4\,y^2 - 8\,x^4\,z^2 - 8\,y^4\,z^2 - 3\,x^2\,y^4 - y^6 - x^6 - 25\,x^2\,c^2\,z^4 - 25\,y^2\,c^2\,z}{}\right.$$

$$\%1 = -4\,x^2\,y^2 - 2\,y^4 - 15\,c^2\,z^2\,y^2 + 3\,c^3\,z^4 - 8\,c^2\,z^4 - 15\,c^2\,z^2\,x^2 + 14\,z^2\,c\,y^2 + 12\,c$$

Amperian Current 4Vector   curlH-dD/dt=J4 $= \left[\frac{2\,\alpha\, m\, y\, (\%1)}{(x^2+y^2+z^2 c)^4\,\mu},\ -\frac{2\,\alpha\, m\, x\, (\%1)}{(x^2+y^2+z^2 c)^4\,\mu},\ 0,\ 0 \right]$

$$\%1 = 4\,x^2 z^2 + 4\,y^2 z^2 - 8\,z^4 c + 2\,x^2 y^2 + y^4 + x^4 + 3\,c^2 z^4 - 8\,c\,x^2 z^2 - 8\,z^2 c\,y^2$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B) $= \left[\frac{4\,\alpha^2 m^2 x\,(\%1)\, z^2\,(z^2 c - x^2 - y^2)}{(x^2+y^2+z^2 c)^7\,\mu},\ \frac{4\,\alpha^2 m^2 y\,(\%1)\, z^2\,(z^2 c -}{(x^2+y^2+z^2 c)^7\,\mu}\right.$

$$\%1 = 4\,x^2 z^2 + 4\,y^2 z^2 - 8\,z^4 c + 2\,x^2 y^2 + y^4 + x^4 + 3\,c^2 z^4 - 8\,c\,x^2 z^2 - 8\,z^2 c$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN

$$= \frac{1}{2}\,\frac{z^2\,(x^2+y^2+z^2 c)^2}{4\,x^2 z^2 + 4\,y^2 z^2 - 8\,z^4 c + 2\,x^2 y^2 + y^4 + x^4 + 3\,c^2 z^4 - 8\,c\,x^2 z^2 - 8\,z^2 c\,y^2}$$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] $= \left[\frac{2\,\alpha^2 m^2 z^4 x\,(z^2 c - x^2 - y^2)}{(x^2+y^2+z^2 c)^5\,\mu},\right.$

$$\left.\frac{2\,\alpha^2 m^2 z^4 y\,(z^2 c - x^2 - y^2)}{(x^2+y^2+z^2 c)^5\,\mu},\ -\frac{2\,\alpha^2 m^2 z^3\,(z^2 c - x^2 - y^2)\,(x^2+y^2)}{(x^2+y^2+z^2 c)^5\,\mu},\ 0\right]$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $= \Bigg[$

$$-\frac{4\,\alpha^3 m^3 z^4 y\,(z^2 c - x^2 - y^2)^2\,(x^2+y^2+z^2)}{(x^2+y^2+z^2 c)^8\,\mu},\ \frac{4\,\alpha^3 m^3 z^4\,(z^2 c - x^2 - y^2)^2\, x\,(x^2+y^2+z^2)}{(x^2+y^2+z^2 c)^8\,\mu},\ 0\Bigg]$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \left[\frac{1}{(x^2+y^2+z^2 c)^8\,\mu} \left(4\,\alpha^2 m^2 z^2\,(z^2 c - x^2 - y^2)\,\left(3\,x^5 y^2 + 4\,x^5 z^2 + 4\,x\,y^4 z^2\right.\right.\right.$

$$+ 3\,x^3 y^4 + x\,y^6 + x^7 - 5\,x^3 c^2 z^4 - 5\,x\,y^2 c^2 z^4 + 8\,x^3 y^2 z^2 - 14\,c\,z^2 x^3 y^2 + 3\,c^3 z^6 x - 4\,z^4 x\,c\,y^2$$

$$- 8\,c^2 z^6 x - 4\,z^4 x^3 c - 7\,c\,z^2 x^5 - 7\,c\,z^2 x\,y^4 - \alpha\, m\, y\,\mu\, z^4 c\,x^2 - \alpha\, m\, y^3\,\mu\, z^4 c - \alpha\, m\, y\,\mu\, z^6 c$$

$$+\alpha m y \mu z^2 x^4 + 2\alpha m y^3 \mu z^2 x^2 + \alpha m y \mu z^4 x^2 + \alpha m y^5 \mu z^2 + \alpha m y^3 \mu z^4)),$$

$$\frac{1}{(x^2+y^2+z^2 c)^8 \mu}\left(4\alpha^2 m^2 z^2 (z^2 c - x^2 - y^2)\, (3 y^3 x^4 + 4 z^2 y x^4 + 4 y^5 z^2 + 3 y^5 x^2 + y^7 + y x^6\right.$$

$$- 5 x^2 y c^2 z^4 - 5 y^3 c^2 z^4 + 8 y^3 x^2 z^2 - 14 c z^2 y^3 x^2 + 3 c^3 z^6 y - 4 z^4 y^3 c - 8 c^2 z^6 y - 4 z^4 x^2 y c$$

$$- 7 c z^2 y x^4 - 7 c z^2 y^5 + \alpha m x^3 \mu z^4 c + \alpha m x \mu z^4 c y^2 + \alpha m x \mu z^6 c - \alpha m x^5 \mu z^2 - 2\alpha m x^3 \mu z^2 y^2$$

$$\left. - \alpha m x^3 \mu z^4 - \alpha m x \mu z^2 y^4 - \alpha m x \mu z^4 y^2)\right), -\frac{1}{(x^2+y^2+z^2 c)^7 \mu}\left(4\alpha^2 m^2 (4 x^2 z^2 + 4 y^2 z^2\right.$$

$$\left. - 8 z^4 c + 2 x^2 y^2 + y^4 + x^4 + 3 c^2 z^4 - 8 c x^2 z^2 - 8 z^2 c y^2)\, z\, (z^2 c - x^2 - y^2)\, (x^2 + y^2)\right)\Big]$$

Dissipation $= 0$

******************************* *END PROCEDURE* ****************************** **(37)**

Enter the name of the problem, and the components of the 4 potential
Q = charge, Omega = strength and sign of rotation

```
> NAME:=`Example 11a - An Electromagnetic Pump   vol6 p. 117 `;

> A1:=0;A2:=-1/2*Bx*z;
> A3:=+1/2*Bx*y+Bx*z/2; phi:=+0*Ex*x+0*Ez*z+Ey*y;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := *Example 11a - An Electromagnetic Pump   vol6 p. 117*

$$A1 := 0$$

$$A2 := -\frac{1}{2} Bx\, z$$

$$A3 := \frac{1}{2} Bx\, y + \frac{1}{2} Bx\, z$$

$\phi := Ey\, y$

Example 11a - An Electromagnetic Pump   vol6 p. 117

************************************* *Differential Form Format* ************************************

$$\text{Action 1-form} = -\frac{1}{2}\, Bx\, z\, d(y) + \left(\frac{1}{2}\, Bx\, y + \frac{1}{2}\, Bx\, z\right) d(z) - Ey\, y\, d(t)$$

$$\text{Intensity 2-form } F{=}dA \quad = Bx\,(d(y))\ \&\text{^}\,(d(z)) - Ey\,(d(y))\ \&\text{^}\,(d(t))$$

$$\text{Topological Torsion 3-form} \quad A\text{^}F = \left(-Ey\, y\, Bx + \frac{1}{2}\, Bx\,(y+z)\, Ey\right) \&\text{^}(d(y), d(z), d(t))$$

$$\text{Topological Parity 4-form} \quad F\text{^}F = 0$$

*************************************** *Using EM format* ***************************************

$$E\ field = [0, -Ey, 0]$$

$$B\ field = [Bx, 0, 0]$$

$$\text{Topological TORSION 4 vector} \quad T4 = \text{-[ExA + Bphi,AdotB]} = \left[-\frac{1}{2}\, Bx\, Ey\,(y-z), 0, 0, 0\right]$$

$$\text{Helicity } AdotB = 0$$

$$\text{Poincare II =2(E.B)} = 0$$

$$\text{coefficient of Topological Parity 4-form} = 0$$

$$\text{Pfaff Topological Dimension} \quad PTD = 3$$

************************ *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***********************************

$$\text{Xm or linear (Mean) curvature} = \frac{1}{2}\, Bx$$

$$\text{Yg or quadratic (GAUSS) curvature} = \frac{1}{4}\, Bx^2$$

$$\text{Za or Cubic (Interaction internal energy) curvature} = 0$$

$$\text{Tk or quartic (4D expansion) curvature} = 0$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\text{Chirality factor } CH = 0$$

$$D\ field = [0, -\varepsilon\, Ey, 0]$$

$$H\ field = \left[\frac{Bx}{\mu}, 0, 0\right]$$

$$\text{Poynting vector } ExH = EXH$$

$$\text{Amperian Current 4Vector} \quad curlH\text{-}dD/dt{=}J4 = [0, 0, 0, 0]$$

$$\text{Amerian charge density} \quad divD = rho = 0$$

$$\text{divergence Lorentz Current 4Vector,} \quad 4div(J4) = 0$$

$$\textit{Topological SPIN 4 vector S4} = \left[0, \frac{1}{2}\,\frac{Bx^2\,y + Bx^2\,z - 2\,\varepsilon\,Ey^2\,y\,\mu}{\mu}, \frac{1}{2}\,\frac{Bx^2\,z}{\mu}, \frac{1}{2}\,Bx\,z\,\varepsilon\,Ey\right]$$

$$\textit{Topological SPIN 3-form} = -\frac{1}{2}\,\frac{\left(Bx^2\,y + Bx^2\,z - 2\,\varepsilon\,Ey^2\,y\,\mu\right)\,\&\text{^}(d(x), d(z), d(t))}{\mu}$$

$$+\frac{1}{2}\,\frac{Bx^2\,z\,\&\text{^}(d(x), d(y), d(t))}{\mu} - \frac{1}{2}\,Bx\,z\,\varepsilon\,Ey\,\&\text{^}(d(x), d(y), d(z))$$

$$\textit{Spin density rho\_spin} = \frac{1}{2}\,Bx\,z\,\varepsilon\,Ey$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{Bx^2 - \varepsilon\,Ey^2\,\mu}{\mu}$$

$$B.H = \frac{Bx^2}{\mu}$$

$$D.E = \varepsilon\,Ey^2$$

$$A.J = 0$$

$$\textit{-rho.phi} = 0$$

$$\textit{Poincare I} \quad \textit{(B.H - D.E)-(A.J - rho.phi)} = \frac{Bx^2 - \varepsilon\,Ey^2\,\mu}{\mu}$$

$$\textit{London Coefficient} \quad LC =$$

$$-\frac{2\left(4\,x^2\,z^2 + 4\,y^2\,z^2 - 8\,z^4\,c + 2\,x^2\,y^2 + y^4 + x^4 + 3\,c^2\,z^4 - 8\,c\,x^2\,z^2 - 8\,z^2\,c\,y^2\right)}{\left(x^2 + y^2 + z^2\,c\right)^2\,\mu\,z^2}$$

$$\textit{PROCA coefficient curlcurlB} = [0, 0, 0]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} = [0, 0, 0, 0]$$

$$\textit{Lorentz Force 3 vector due to Ampere current} \quad \textit{FL = -(rho\_ampere E + J\_ampere x B)} = [0, 0, 0]$$

$$\textit{Amperian Dissipation Jampere dot E} = 0$$

$$\textit{Lorentz Force Spin factor LFSPIN} = 0$$

$$\textit{Topological Torsion current 4 vector} \quad \textit{T4 = -[ExA + B.phi,AdotB]} = \left[-\frac{1}{2}\,Bx\,Ey\,(y - z), 0, 0, 0\right]$$

$$\textit{Lorentz Force 3 vector due to Torsion current} \quad \textit{TF = -(rho\_torsion E + J\_torsion x B)} = [0, 0, 0]$$

$$\textit{Torsion Dissipation Jtorsion dot E} = 0$$

$$\textit{Topological Spin current 4 vector} \quad \textit{TS4 = -[A x H + D.phi,AdotD]} = \left[0, \frac{1}{2}\,\frac{Bx^2\,y + Bx^2\,z - 2\,\varepsilon\,Ey^2\,y\,\mu}{\mu},\right.$$

$$\frac{1}{2}\frac{Bx^2 z}{\mu}, \frac{1}{2} Bx\, z\, \varepsilon\, Ey\Bigg]$$

$$\textit{Lorentz Force 3 vector due to Spin current} \quad \textit{SF = --(rho\_spin E + J\_spin x B)} \quad = \Bigg[0,$$

$$-\frac{1}{2}\frac{Bx\, z\left(Bx^2 - \varepsilon\, Ey^2 \mu\right)}{\mu}, \frac{1}{2}\frac{\left(Bx^2 y + Bx^2 z - 2\,\varepsilon\, Ey^2 y\, \mu\right) Bx}{\mu}\Bigg]$$

$$\textit{Spin Dissipation J\_spin dot E} \quad = -\frac{1}{2}\frac{\left(Bx^2 y + Bx^2 z - 2\,\varepsilon\, Ey^2 y\, \mu\right) Ey}{\mu}$$

$$\textit{Disspative Force 3 vector} = \left[0, -\frac{1}{2} Bx\, z\left(Bx^2 - \varepsilon\, Ey^2 \mu\right), \frac{1}{2}\left(Bx^2 y + Bx^2 z - 2\,\varepsilon\, Ey^2 y\, \mu\right) Bx\right]$$

$$\textit{Dissipation} \quad = \frac{1}{2} Bx\, Ey\left(\varepsilon\, \mu\, z - y + z\right)$$

****************************** *END PROCEDURE* **************************** **(38)**

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

```
> NAME:=`Example 10a -- Plasma Accretion disc -- complex Hedge Hog solution.  `;
> Gamma:=-z*I/(x^2+y^2)^1*m/(a*x^2+a*y^2+c*z^2)^(1/2);
> Ax:=Gamma*(-y);Ay:=Gamma*x;
> Az:=0; phi:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 10a -- Plasma Accretion disc -- complex Hedge Hog solution.

$$\Gamma := -\frac{\mathrm{I}\, z\, m}{\left(x^2 + y^2\right)\sqrt{x^2 a + a\, y^2 + z^2 c}}$$

$$Ax := \frac{\mathrm{I}\, z\, m\, y}{\left(x^2 + y^2\right)\sqrt{x^2 a + a\, y^2 + z^2 c}}$$

$$Ay := -\frac{\mathrm{I}\, z\, m\, x}{\left(x^2 + y^2\right)\sqrt{x^2 a + a\, y^2 + z^2 c}}$$

$$Az := 0$$

$$\phi := 0$$

Example 10a -- Plasma Accretion disc -- complex Hedge Hog solution.

******************************** *Differential Form Format* ********************************

$$Action\ 1\text{-}form = \frac{I\,z\,m\,y\,d(x)}{(x^2+y^2)\sqrt{x^2\,a+a\,y^2+z^2\,c}} - \frac{I\,z\,m\,x\,d(y)}{(x^2+y^2)\sqrt{x^2\,a+a\,y^2+z^2\,c}}$$

$$Intensity\ 2\text{-}form\ F{=}dA\ = \left(-\frac{I\,z\,m\,(-x^2\,a\,y^2-2\,a\,y^4-z^2\,c\,y^2+x^4\,a+c\,x^2\,z^2)}{(x^2+y^2)^2\,(x^2\,a+a\,y^2+z^2\,c)^{3/2}} + \frac{I\,z\,m\,(2\,x^4\,a+x^2\,a\,y^2+c\,x^2\,z^2-a\,y^4-z^2\,c\,y^2)}{(x^2+y^2)^2\,(x^2\,a+a\,y^2+z^2\,c)^{3/2}} \right)(d(x))\ \&\text{^}\ (d(y)) - \frac{I\,a\,m\,y\,(d(x))\ \&\text{^}\ (d(z))}{(x^2\,a+a\,y^2+z^2\,c)^{3/2}} + \frac{I\,a\,m\,x\,(d(y))\ \&\text{^}\ (d(z))}{(x^2\,a+a\,y^2+z^2\,c)^{3/2}}$$

Topological Torsion 3-form $A\text{^}F = 0$

Topological Parity 4-form $F\text{^}F = 0$

******************************** *Using EM format* ********************************

$E\ field = [0, 0, 0]$

$$B\ field = \left[\frac{I\,a\,m\,x}{(x^2\,a+a\,y^2+z^2\,c)^{3/2}}, \frac{I\,a\,m\,y}{(x^2\,a+a\,y^2+z^2\,c)^{3/2}}, \frac{I\,a\,z\,m}{(x^2\,a+a\,y^2+z^2\,c)^{3/2}} \right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension PTD $= 2$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************************

Xm or linear (Mean) curvature $= 0$

$$Yg\ or\ quadratic\ (GAUSS)\ curvature\ = \frac{(2\,x^2\,a+2\,a\,y^2+z^2\,c)\,z^2\,m^2}{(x^2\,a+a\,y^2+z^2\,c)^2\,(x^2+y^2)^2}$$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $= 0$

$D\ field = [0, 0, 0]$

$$H\,field = \left[\frac{I\,a\,m\,x}{(x^2 a + a y^2 + z^2 c)^{3/2}\,\mu}, \frac{I\,a\,m\,y}{(x^2 a + a y^2 + z^2 c)^{3/2}\,\mu}, \frac{I\,a\,z\,m}{(x^2 a + a y^2 + z^2 c)^{3/2}\,\mu} \right]$$

$$Poynting\ vector\ ExH = EXH$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt\text{=}J4 \quad = \left[-\frac{3\,I\,a\,z\,m\,y\,(a-c)}{(x^2 a + a y^2 + z^2 c)^{5/2}\,\mu}, \frac{3\,I\,a\,m\,x\,z\,(a-c)}{(x^2 a + a y^2 + z^2 c)^{5/2}\,\mu}, 0, 0 \right]$$

$$Amerian\ charge\ density \quad divD = rho = 0$$

$$divergence\ Lorentz\ Current\ 4Vector, \quad 4div(J4) = 0$$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[\frac{z^2 m^2 x\,a}{(x^2+y^2)\,(x^2 a + a y^2 + z^2 c)^2\,\mu}, \frac{z^2 m^2 y\,a}{(x^2+y^2)\,(x^2 a + a y^2 + z^2 c)^2\,\mu}, -\frac{z\,m^2 a}{(x^2 a + a y^2 + z^2 c)^2\,\mu}, 0 \right]$$

$$Topological\ SPIN\ 3\text{-}form = \frac{z^2 m^2 x\,a\,\&\hat{}(d(y), d(z), d(t))}{(x^2+y^2)\,(x^2 a + a y^2 + z^2 c)^2\,\mu} - \frac{z^2 m^2 y\,a\,\&\hat{}(d(x), d(z), d(t))}{(x^2+y^2)\,(x^2 a + a y^2 + z^2 c)^2\,\mu} - \frac{z\,m^2 a\,\&\hat{}(d(x), d(y), d(t))}{(x^2 a + a y^2 + z^2 c)^2\,\mu}$$

$$Spin\ density\ rho\_spin = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{a^2 m^2 (x^2+y^2+z^2)}{(x^2 a + a y^2 + z^2 c)^3\,\mu}$$

$$B.H = -\frac{a^2 m^2 (x^2+y^2+z^2)}{(x^2 a + a y^2 + z^2 c)^3\,\mu}$$

$$D.E = 0$$

$$A.J = \frac{3\,z^2 m^2 a\,(a-c)}{(x^2 a + a y^2 + z^2 c)^3\,\mu}$$

$$\text{-}rho.phi = 0$$

$$Poincare\ I \quad (B.H - D.E)\text{-}(A.J - rho.phi) = -\frac{a\,m^2 (x^2 a + a y^2 + 4\,a\,z^2 - 3\,z^2 c)}{(x^2 a + a y^2 + z^2 c)^3\,\mu}$$

$$London\ Coefficient \quad LC = -\frac{3\,a\,(a-c)\,(x^2+y^2)}{(x^2 a + a y^2 + z^2 c)^2\,\mu}$$

$$PROCA\ coefficient\ curlcurlB = \left[-\frac{3\,I\,(x^2 a + a y^2 - 4\,z^2 c)\,(a-c)\,a\,m\,x}{(x^2 a + a y^2 + z^2 c)^{7/2}}, \right.$$

$$-\frac{3\,I\left(x^2 a + a y^2 - 4 z^2 c\right)(a-c)\,a\,m\,y}{\left(x^2 a + a y^2 + z^2 c\right)^{7/2}},\ -\frac{3\,I\left(3 x^2 a + 3 a y^2 - 2 z^2 c\right)(a-c)\,a\,m\,z}{\left(x^2 a + a y^2 + z^2 c\right)^{7/2}}\Bigg]$$

Amperian Current 4Vector curlH-dD/dt=J4 $= \left[-\frac{3\,I\,a\,z\,m\,y\,(a-c)}{\left(x^2 a + a y^2 + z^2 c\right)^{5/2}\mu},\ \frac{3\,I\,a\,m\,x\,z\,(a-c)}{\left(x^2 a + a y^2 + z^2 c\right)^{5/2}\mu},\ 0,\ 0\right]$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B)

$$= \left[\frac{3\,a^2 m^2 x z^2 (a-c)}{\left(x^2 a + a y^2 + z^2 c\right)^4 \mu},\ \frac{3\,a^2 z^2 m^2 y\,(a-c)}{\left(x^2 a + a y^2 + z^2 c\right)^4 \mu},\ -\frac{3\,a^2 z\,m^2 (a-c)\left(x^2+y^2\right)}{\left(x^2 a + a y^2 + z^2 c\right)^4 \mu}\right]$$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $= \frac{1}{3}\,\frac{\left(x^2 a + a y^2 + z^2 c\right)^2}{\left(x^2+y^2\right) a\,(a-c)}$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $=0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \left[\frac{z^2 m^2 x\,a}{\left(x^2+y^2\right)\left(x^2 a + a y^2 + z^2 c\right)^2 \mu},\ \frac{z^2 m^2 y\,a}{\left(x^2+y^2\right)\left(x^2 a + a y^2 + z^2 c\right)^2 \mu},\ -\frac{z\,m^2 a}{\left(x^2 a + a y^2 + z^2 c\right)^2 \mu},\ 0\right]$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B)

$$= \left[\frac{I\left(x^2+y^2+z^2\right) y\,z\,a^2 m^3}{\left(x^2 a + a y^2 + z^2 c\right)^{7/2} \mu\,(-x+I y)(x+I y)},\ -\frac{I\left(x^2+y^2+z^2\right) x\,z\,a^2 m^3}{\left(x^2 a + a y^2 + z^2 c\right)^{7/2} \mu\,(-x+I y)(x+I y)},\ 0\right]$$

Spin Dissipation J_spin dot E $=0$

Disspative Force 3 vector $= \Bigg[\frac{1}{\left(x^2 a + a y^2 + z^2 c\right)^{15/2} \mu\,(-x+I y)(x+I y)}\Big(I\big(4 y\,m\,\mu\,z^8 x^2 a\,c^3$

$+ 6 y\,m\,\mu\,x^6 a^2 c^2 z^4 + 8 y^3 m\,\mu\,x^2 a\,z^6 c^3 + 12 y^3 m\,\mu\,z^4 x^4 a^3 c + 6 y\,m\,\mu\,z^6 x^4 a^2 c^2 + 4 y\,m\,\mu\,z^4 x^6 a^3 c$

$+ 4 y\,m\,\mu\,x^8 a^3 z^2 c + 4 y\,m\,\mu\,x^4 a\,z^6 c^3 + 12 y^5 m\,\mu\,z^4 x^2 a^3 c + 12 y^3 m\,\mu\,z^6 x^2 a^2 c^2 + 18 y^5 m\,\mu\,x^2 a^2 c^2 z^4$

$+ 16\, y^3\, m\, \mu\, x^6\, a^3\, z^2\, c + 16\, y^7\, m\, \mu\, x^2\, a^3\, z^2\, c + 18\, y^3\, m\, \mu\, x^4\, a^2\, c^2\, z^4 + 24\, y^5\, m\, \mu\, x^4\, a^3\, z^2\, c + 5\, y^3\, m\, \mu\, x^8\, a^4$

$+ 10\, y^5\, m\, \mu\, x^6\, a^4 + 10\, y^7\, m\, \mu\, x^4\, a^4 + y^9\, m\, \mu\, z^2\, a^4 + 5\, y^9\, m\, \mu\, x^2\, a^4 + y^3\, m\, \mu\, z^8\, c^4 + y\, m\, \mu\, x^{10}\, a^4$

$+ y\, m\, \mu\, z^{10}\, c^4 + 3\, \mathrm{I}\, x\, z\, \left(x^2\, a + a\, y^2 + z^2\, c\right)^{7/2}\, a\, y^2 - 3\, \mathrm{I}\, x\, z\, \left(x^2\, a + a\, y^2 + z^2\, c\right)^{7/2}\, c\, y^2 + 4\, y^3\, m\, \mu\, z^2\, x^6\, a^4$

$+ y\, m\, \mu\, z^2\, x^8\, a^4 + 6\, y^7\, m\, \mu\, a^2\, c^2\, z^4 + 4\, y^9\, m\, \mu\, a^3\, z^2\, c + 4\, y^7\, m\, \mu\, z^4\, a^3\, c + 4\, y^5\, m\, \mu\, a\, z^6\, c^3$

$+ 6\, y^5\, m\, \mu\, z^6\, a^2\, c^2 + 4\, y^3\, m\, \mu\, z^8\, a\, c^3 + 4\, y^7\, m\, \mu\, z^2\, x^2\, a^4 + 6\, y^5\, m\, \mu\, z^2\, x^4\, a^4 + y\, m\, \mu\, x^2\, z^8\, c^4 + y^{11}\, m\, \mu\, a^4$

$+ 3\, \mathrm{I}\, x^3\, z\, \left(x^2\, a + a\, y^2 + z^2\, c\right)^{7/2}\, a - 3\, \mathrm{I}\, x^3\, z\, \left(x^2\, a + a\, y^2 + z^2\, c\right)^{7/2}\, c \big)\, z\, a^2\, m^2 \Big),$

$\frac{1}{\left(x^2\, a + a\, y^2 + z^2\, c\right)^{15/2}\, \mu\, (-x + \mathrm{I}\, y)\, (x + \mathrm{I}\, y)} \Big(\mathrm{I}\, \big(-12\, y^2\, m\, \mu\, z^4\, x^5\, a^3\, c - 18\, y^4\, m\, \mu\, x^3\, a^2\, c^2\, z^4$

$- 16\, y^2\, m\, \mu\, x^7\, a^3\, z^2\, c - 8\, y^2\, m\, \mu\, x^3\, a\, z^6\, c^3 - 12\, y^4\, m\, \mu\, z^4\, x^3\, a^3\, c - 24\, y^4\, m\, \mu\, x^5\, a^3\, z^2\, c$

$- 12\, y^2\, m\, \mu\, z^6\, x^3\, a^2\, c^2 - 18\, y^2\, m\, \mu\, x^5\, a^2\, c^2\, z^4 - 4\, x\, y^8\, m\, \mu\, a^3\, z^2\, c - 4\, x\, y^4\, m\, \mu\, a\, z^6\, c^3 - 6\, x\, y^6\, m\, \mu\, a^2\, c^2\, z^4$

$- 4\, x\, y^6\, m\, \mu\, z^4\, a^3\, c - 4\, x\, y^2\, m\, \mu\, z^8\, a\, c^3 - 6\, x\, y^4\, m\, \mu\, z^6\, a^2\, c^2 - 16\, y^6\, m\, \mu\, x^3\, a^3\, z^2\, c - x\, m\, \mu\, z^{10}\, c^4$

$- 10\, y^6\, m\, \mu\, x^5\, a^4 - 10\, y^4\, m\, \mu\, x^7\, a^4 - 5\, y^2\, m\, \mu\, x^9\, a^4 - 5\, y^8\, m\, \mu\, x^3\, a^4 - m\, \mu\, x^3\, z^8\, c^4 - m\, \mu\, z^2\, x^9\, a^4$

$- x\, m\, \mu\, a^4\, y^{10} + 3\, \mathrm{I}\, z\, y^3\, \left(x^2\, a + a\, y^2 + z^2\, c\right)^{7/2}\, a - 3\, \mathrm{I}\, z\, y^3\, \left(x^2\, a + a\, y^2 + z^2\, c\right)^{7/2}\, c + 3\, \mathrm{I}\, y\, x^2\, z\, \big(x^2\, a$

$+ a\, y^2 + z^2\, c\big)^{7/2}\, a - 3\, \mathrm{I}\, y\, x^2\, z\, \left(x^2\, a + a\, y^2 + z^2\, c\right)^{7/2}\, c - 6\, m\, \mu\, x^7\, a^2\, c^2\, z^4 - 4\, m\, \mu\, z^8\, x^3\, a\, c^3$

$- x\, y^2\, m\, \mu\, z^8\, c^4 - x\, y^8\, m\, \mu\, z^2\, a^4 - 6\, m\, \mu\, z^6\, x^5\, a^2\, c^2 - 4\, y^6\, m\, \mu\, z^2\, x^3\, a^4 - 4\, m\, \mu\, x^5\, a\, z^6\, c^3 - 4\, m\, \mu\, z^4\, x^7\, a^3\, c$

$- 4\, m\, \mu\, x^9\, a^3\, z^2\, c - 4\, y^2\, m\, \mu\, z^2\, x^7\, a^4 - 6\, y^4\, m\, \mu\, z^2\, x^5\, a^4 - m\, \mu\, x^{11}\, a^4 \big)\, z\, a^2\, m^2 \Big),$

$$- \frac{3\, a^2\, z\, m^2\, (a - c)\, (x^2 + y^2)}{(x^2\, a + a\, y^2 + z^2\, c)^4\, \mu} \Bigg]$$

$$Dissipation \ = 0$$

******************************   *END PROCEDURE*   **************************** **(39)**

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`Example 12 -- Black Hole 2 singular vortex ring `;
```

```
> phi := 1; A1:=a*y/(x^2+y^2+z^2);A2 := -a*x/(x^2+y^2+z^2);A3:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 12 -- Black Hole 2 singular vortex ring

$$\phi := 1$$

$$A1 := \frac{a\, y}{x^2 + y^2 + z^2}$$

$$A2 := - \frac{a\, x}{x^2 + y^2 + z^2}$$

$$A3 := 0$$

Example 12 -- Black Hole 2 singular vortex ring

*********************************   *Differential Form Format*   *****************************
****

$$Action\ 1\text{-}form = \frac{(-x^2 - y^2 - z^2)\, d(t)}{x^2 + y^2 + z^2} + \frac{a\, y\, d(x)}{x^2 + y^2 + z^2} - \frac{a\, x\, d(y)}{x^2 + y^2 + z^2}$$

$$Intensity\ 2\text{-}form\ F{=}dA \ = \left(- \frac{a\, (x^2 - y^2 + z^2)}{(x^2 + y^2 + z^2)^2} + \frac{a\, (x^2 - y^2 - z^2)}{(x^2 + y^2 + z^2)^2} \right) (d(x))\ \&\text{^}\ (d(y)) + \frac{2\, a\, y\, z\, (d(x))\ \&\text{^}\ (d(z))}{(x^2 + y^2 + z^2)^2} - \frac{2\, a\, x\, z\, (d(y))\ \&\text{^}\ (d(z))}{(x^2 + y^2 + z^2)^2}$$

$$Topological\ Torsion\ 3\text{-}form \quad A\text{^}F = \frac{2\, a\, z^2\, \&\text{^}(d(x), d(y), d(t))}{(x^2 + y^2 + z^2)^2} - \frac{2\, a\, y\, z\, \&\text{^}(d(x), d(z), d(t))}{(x^2 + y^2 + z^2)^2} + \frac{2\, a\, x\, z\, \&\text{^}(d(y), d(z), d(t))}{(x^2 + y^2 + z^2)^2}$$

$$Topological\ Parity\ 4\text{-}form \quad F\text{^}F = 0$$

*********************************   *Using EM format*   *********************************

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[-\frac{2\,a\,x\,z}{(x^2+y^2+z^2)^2}, -\frac{2\,a\,y\,z}{(x^2+y^2+z^2)^2}, -\frac{2\,a\,z^2}{(x^2+y^2+z^2)^2} \right]$$

$$\text{Topological TORSION 4 vector} \quad T4 = -[ExA + Bphi, AdotB] = \left[\frac{2\,a\,x\,z}{(x^2+y^2+z^2)^2}, \frac{2\,a\,y\,z}{(x^2+y^2+z^2)^2}, \frac{2\,a\,z^2}{(x^2+y^2+z^2)^2}, 0 \right]$$

$$\text{Helicity } AdotB = 0$$

$$\text{Poincare II } = 2(E.B) = 0$$

$$\text{coefficient of Topological Parity 4-form } = 0$$

$$\text{Pfaff Topological Dimension} \quad PTD = 3$$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************************

$$\text{Xm or linear (Mean) curvature } = 0$$

$$\text{Yg or quadratic (GAUSS) curvature } = -\frac{a^2\,(x^2+y^2-z^2)}{(x^2+y^2+z^2)^3}$$

$$\text{Za or Cubic (Interaction internal energy) curvature } = 0$$

$$\text{Tk or quartic (4D expansion) curvature } = 0$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\text{Chirality factor } CH = 0$$

$$D\ field = [0, 0, 0]$$

$$H\ field = \left[-\frac{2\,a\,x\,z}{(x^2+y^2+z^2)^2\,\mu}, -\frac{2\,a\,y\,z}{(x^2+y^2+z^2)^2\,\mu}, -\frac{2\,a\,z^2}{(x^2+y^2+z^2)^2\,\mu} \right]$$

$$\text{Poynting vector } ExH = EXH$$

$$\text{Amperian Current 4Vector} \quad curlH\text{-}dD/dt = J4 \; = \left[\frac{2\,a\,y}{(x^2+y^2+z^2)^2\,\mu}, -\frac{2\,a\,x}{(x^2+y^2+z^2)^2\,\mu}, 0, 0 \right]$$

$$\text{Amerian charge density} \quad divD = rho = 0$$

$$\text{divergence Lorentz Current 4Vector,} \quad 4div(J4) \; = 0$$

$$\text{Topological SPIN 4 vector } S4 = \left[\frac{2\,a^2\,x\,z^2}{(x^2+y^2+z^2)^3\,\mu}, \frac{2\,a^2\,y\,z^2}{(x^2+y^2+z^2)^3\,\mu}, -\frac{2\,z\,a^2\,(x^2+y^2)}{\mu\,(x^2+y^2+z^2)^3}, 0 \right]$$

$$\text{Topological SPIN 3-form} = \frac{2\,a^2\,x\,z^2\,\&\hat{}(d(y), d(z), d(t))}{(x^2+y^2+z^2)^3\,\mu} - \frac{2\,a^2\,y\,z^2\,\&\hat{}(d(x), d(z), d(t))}{(x^2+y^2+z^2)^3\,\mu}$$

$$- \frac{2 z a^2 (x^2+y^2) \&\text{^}(d(x), d(y), d(t))}{\mu (x^2+y^2+z^2)^3}$$

Spin density rho_spin $= 0$

LaGrange field energy density (B.H-D.E) $= \frac{4 a^2 z^2}{\mu (x^2+y^2+z^2)^3}$

$B.H = \frac{4 a^2 z^2}{\mu (x^2+y^2+z^2)^3}$

$D.E = 0$

$A.J = \frac{2 a^2 (x^2+y^2)}{\mu (x^2+y^2+z^2)^3}$

$-rho.phi = 0$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* $= - \frac{2 a^2 (-2 z^2 + x^2 + y^2)}{\mu (x^2+y^2+z^2)^3}$

London Coefficient $LC = \frac{2}{(x^2+y^2+z^2) \mu}$

PROCA coefficient curlcurlB $= \left[- \frac{8 a x z}{(x^2+y^2+z^2)^3}, - \frac{8 a y z}{(x^2+y^2+z^2)^3}, \frac{4 a (x^2+y^2-z^2)}{(x^2+y^2+z^2)^3} \right]$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \left[\frac{2 a y}{(x^2+y^2+z^2)^2 \mu}, - \frac{2 a x}{(x^2+y^2+z^2)^2 \mu}, 0, 0 \right]$

Lorentz Force 3 vector due to Ampere current *FL = -(rho_ampere E + J_ampere x B)* $= \left[- \frac{4 a^2 x z^2}{(x^2+y^2+z^2)^4 \mu}, - \frac{4 a^2 y z^2}{(x^2+y^2+z^2)^4 \mu}, \frac{4 z a^2 (x^2+y^2)}{(x^2+y^2+z^2)^4 \mu} \right]$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= - \frac{1}{2} x^2 - \frac{1}{2} y^2 - \frac{1}{2} z^2$

Topological Torsion current 4 vector *T4 = -[ExA + B.phi,AdotB]* $= \left[\frac{2 a x z}{(x^2+y^2+z^2)^2}, \frac{2 a y z}{(x^2+y^2+z^2)^2}, \frac{2 a z^2}{(x^2+y^2+z^2)^2}, 0 \right]$

Lorentz Force 3 vector due to Torsion current *TF = -(rho_torsion E + J_torsion x B)* $= [0, 0, 0]$

$$\textit{Torsion Dissipation Jtorsion dot E} \quad = 0$$

$$\textit{Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD]} \; = \left[\frac{2\,a^2\,x\,z^2}{(x^2+y^2+z^2)^3\,\mu}, \frac{2\,a^2\,y\,z^2}{(x^2+y^2+z^2)^3\,\mu}, -\frac{2\,z\,a^2\,(x^2+y^2)}{\mu\,(x^2+y^2+z^2)^3}, 0 \right]$$

$$\textit{Lorentz Force 3 vector due to Spin current SF = --(rho\_spin E + J\_spin x B)} \; = \left[\frac{4\,a^3\,y\,z^2}{(x^2+y^2+z^2)^4\,\mu}, -\frac{4\,z^2\,a^3\,x}{(x^2+y^2+z^2)^4\,\mu}, 0 \right]$$

$$\textit{Spin Dissipation J\_spin dot E} \quad = 0$$

$$\textit{Disspative Force 3 vector} = \left[\frac{4\,a^2\,z^2\,(-x+a\,y\,\mu)}{(x^2+y^2+z^2)^4\,\mu}, -\frac{4\,a^2\,z^2\,(y+a\,x\,\mu)}{(x^2+y^2+z^2)^4\,\mu}, \frac{4\,z\,a^2\,(x^2+y^2)}{(x^2+y^2+z^2)^4\,\mu} \right]$$

$$\textit{Dissipation} \quad = \frac{2\,a\,x\,z}{(x^2+y^2+z^2)^2}$$

****************************** *END PROCEDURE* **************************** **(40)**

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`Example 10c -- complex Dirac Hedge Hog solution.
  J ~ A      S ~ Lorentz Force `;
> Gamma:=I*m*(1/(2*(x^2+y^2+z^2)^(1/2)));
> Ax:=Gamma*(-y);Ay:=Gamma*x;
> Az:=0; phi:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

NAME := Example 10c -- complex Dirac Hedge Hog solution.

J ~ A S ~ Lorentz Force

$$\Gamma := \frac{\frac{1}{2}\,\mathrm{I}\,m}{\sqrt{x^2+y^2+z^2}}$$

$$Ax := -\frac{\frac{1}{2}\,\mathrm{I}\,m\,y}{\sqrt{x^2+y^2+z^2}}$$

$$Ay := \frac{\frac{1}{2}\,\mathrm{I}\,m\,x}{\sqrt{x^2+y^2+z^2}}$$

$$Az := 0$$

$$\phi := 0$$

Example 10c -- complex Dirac Hedge Hog solution.

J ~ A   S ~ Lorentz Force

********************************* *Differential Form Format* *********************************

$$\text{Action 1-form} = -\frac{\frac{1}{2}\,\mathrm{I}\,m\,y\,d(x)}{\sqrt{x^2+y^2+z^2}} + \frac{\frac{1}{2}\,\mathrm{I}\,m\,x\,d(y)}{\sqrt{x^2+y^2+z^2}}$$

$$\text{Intensity 2-form F=dA} = \left(\frac{\frac{1}{2}\,\mathrm{I}\,m\,(x^2+z^2)}{(x^2+y^2+z^2)^{3/2}} + \frac{\frac{1}{2}\,\mathrm{I}\,m\,(y^2+z^2)}{(x^2+y^2+z^2)^{3/2}}\right)(d(x))\ \&\hat{}\ (d(y))$$

$$-\frac{\frac{1}{2}\,\mathrm{I}\,m\,y\,z\,(d(x))\ \&\hat{}\ (d(z))}{(x^2+y^2+z^2)^{3/2}} + \frac{\frac{1}{2}\,\mathrm{I}\,m\,x\,z\,(d(y))\ \&\hat{}\ (d(z))}{(x^2+y^2+z^2)^{3/2}}$$

Topological Torsion 3-form $A\hat{}F = 0$

Topological Parity 4-form $F\hat{}F = 0$

********************************* *Using EM format* *********************************

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[\frac{\frac{1}{2}\,\mathrm{I}\,m\,x\,z}{(x^2+y^2+z^2)^{3/2}}, \frac{\frac{1}{2}\,\mathrm{I}\,m\,y\,z}{(x^2+y^2+z^2)^{3/2}}, \frac{\frac{1}{2}\,\mathrm{I}\,m\,(x^2+y^2+2\,z^2)}{(x^2+y^2+z^2)^{3/2}}\right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension *PTD* $= 2$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

Xm or linear (Mean) curvature $= 0$

$$Yg\ or\ quadratic\ (GAUSS)\ curvature\ = -\frac{1}{4}\frac{m^2 z^2}{\left(x^2+y^2+z^2\right)^2}$$

$$Za\ or\ Cubic\ (Interaction\ internal\ energy)\ curvature\ = 0$$

$$Tk\ or\ quartic\ (4D\ expansion)\ curvature\ = 0$$

***********   Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH ************

$$Chirality\ factor\ CH = 0$$

$$D\ field = [0, 0, 0]$$

$$H\ field = \left[\frac{\frac{1}{2}\,\mathrm{I}\,m\,x\,z}{\left(x^2+y^2+z^2\right)^{3/2}\mu}, \frac{\frac{1}{2}\,\mathrm{I}\,m\,y\,z}{\left(x^2+y^2+z^2\right)^{3/2}\mu}, \frac{\frac{1}{2}\,\mathrm{I}\,m\left(x^2+y^2+2z^2\right)}{\left(x^2+y^2+z^2\right)^{3/2}\mu}\right]$$

$$Poynting\ vector\ ExH = EXH$$

$$Amperian\ Current\ 4Vector\quad curlH\text{-}dD/dt{=}J4\ = \left[-\frac{\mathrm{I}\,m\,y}{\left(x^2+y^2+z^2\right)^{3/2}\mu}, \frac{\mathrm{I}\,m\,x}{\left(x^2+y^2+z^2\right)^{3/2}\mu}, 0, 0\right]$$

$$Amerian\ charge\ density\quad divD\ =\ rho = 0$$

$$divergence\ Lorentz\ Current\ 4Vector,\quad 4div(J4)\ = 0$$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[-\frac{1}{4}\frac{m^2 x\left(x^2+y^2+2z^2\right)}{\left(x^2+y^2+z^2\right)^2\mu}, -\frac{1}{4}\frac{m^2 y\left(x^2+y^2+2z^2\right)}{\left(x^2+y^2+z^2\right)^2\mu},\right.$$
$$\left.\frac{1}{4}\frac{m^2 z\left(x^2+y^2\right)}{\left(x^2+y^2+z^2\right)^2\mu}, 0\right]$$

$$Topological\ SPIN\ 3\text{-}form = -\frac{1}{4}\frac{m^2 x\left(x^2+y^2+2z^2\right)\ \&\hat{}(d(y), d(z), d(t))}{\left(x^2+y^2+z^2\right)^2\mu}$$
$$+\frac{1}{4}\frac{m^2 y\left(x^2+y^2+2z^2\right)\ \&\hat{}(d(x), d(z), d(t))}{\left(x^2+y^2+z^2\right)^2\mu}+\frac{1}{4}\frac{m^2 z\left(x^2+y^2\right)\ \&\hat{}(d(x), d(y), d(t))}{\left(x^2+y^2+z^2\right)^2\mu}$$

$$Spin\ density\ rho\_spin = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{1}{4}\frac{\left(x^2+y^2+4z^2\right)m^2}{\left(x^2+y^2+z^2\right)^2\mu}$$

$$B.H = -\frac{1}{4}\frac{\left(x^2+y^2+4z^2\right)m^2}{\left(x^2+y^2+z^2\right)^2\mu}$$

$$D.E = 0$$

$$A.J = -\frac{1}{2}\frac{m^2\left(x^2+y^2\right)}{\left(x^2+y^2+z^2\right)^2\mu}$$

$$-rho.phi = 0$$

$$\textit{Poincare I} \quad \textit{(B.H - D.E)-(A.J - rho.phi)} = \frac{1}{4} \frac{m^2 (x^2+y^2-4z^2)}{(x^2+y^2+z^2)^2 \mu}$$

$$\textit{London Coefficient} \quad LC = \frac{2}{(x^2+y^2+z^2)\,\mu}$$

$$\textit{PROCA coefficient curlcurlB} = \left[\frac{3\,\mathrm{I}\,m\,x\,z}{(x^2+y^2+z^2)^{5/2}}, \frac{3\,\mathrm{I}\,m\,y\,z}{(x^2+y^2+z^2)^{5/2}}, -\frac{\mathrm{I}\,(-2z^2+x^2+y^2)\,m}{(x^2+y^2+z^2)^{5/2}} \right]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} = \left[-\frac{\mathrm{I}\,m\,y}{(x^2+y^2+z^2)^{3/2}\,\mu}, \frac{\mathrm{I}\,m\,x}{(x^2+y^2+z^2)^{3/2}\,\mu}, 0, 0 \right]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B)

$$= \left[\frac{1}{2} \frac{m^2 x (x^2+y^2+2z^2)}{(x^2+y^2+z^2)^3 \mu}, \frac{1}{2} \frac{m^2 y (x^2+y^2+2z^2)}{(x^2+y^2+z^2)^3 \mu}, -\frac{1}{2} \frac{m^2 z (x^2+y^2)}{\mu (x^2+y^2+z^2)^3} \right]$$

$$\textit{Amperian Dissipation Jampere dot E} = 0$$

$$\textit{Lorentz Force Spin factor LFSPIN} = -\frac{1}{2} x^2 - \frac{1}{2} y^2 - \frac{1}{2} z^2$$

$$\textit{Topological Torsion current 4 vector} \quad \textit{T4 = -[ExA + B.phi,AdotB]} = [0, 0, 0, 0]$$

$$\textit{Lorentz Force 3 vector due to Torsion current} \quad \textit{TF = -(rho\_torsion E + J\_torsion x B)} = [0, 0, 0]$$

$$\textit{Torsion Dissipation Jtorsion dot E} = 0$$

$$\textit{Topological Spin current 4 vector} \quad \textit{TS4 = -[A x H + D.phi,AdotD]} = \left[-\frac{1}{4} \frac{m^2 x (x^2+y^2+2z^2)}{(x^2+y^2+z^2)^2 \mu}, \right.$$

$$\left. -\frac{1}{4} \frac{m^2 y (x^2+y^2+2z^2)}{(x^2+y^2+z^2)^2 \mu}, \frac{1}{4} \frac{m^2 z (x^2+y^2)}{(x^2+y^2+z^2)^2 \mu}, 0 \right]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B)

$$= \left[\frac{\frac{1}{8}\,\mathrm{I}\,(x^2+y^2+4z^2)\,y\,m^3}{(x^2+y^2+z^2)^{5/2}\,\mu}, -\frac{\frac{1}{8}\,\mathrm{I}\,(x^2+y^2+4z^2)\,x\,m^3}{(x^2+y^2+z^2)^{5/2}\,\mu}, 0 \right]$$

$$\textit{Spin Dissipation J\_spin dot E} = 0$$

$$\textit{Disspative Force 3 vector} = \left[\frac{1}{8} \frac{1}{\mu (x^2+y^2+z^2)^{11/2}} \left(m^2 \left(4x^3 (x^2+y^2+z^2)^{5/2} + 4x (x^2+y^2+z^2)^5 \right.\right.\right.$$

$$^{/2} y^2 + 8x (x^2+y^2+z^2)^{5/2} z^2 + 7\,\mathrm{I}\,y^7 m \mu z^2 + 15\,\mathrm{I}\,y^5 m \mu z^4 + 7\,\mathrm{I}\,x^6 y\, m \mu z^2 + 15\,\mathrm{I}\,y\, m \mu x^4 z^4 + \mathrm{I}\,y^9 m \mu$$

$$+4\,\mathrm{I}\,x^2 y^7 m\,\mu+21\,\mathrm{I}\,x^2 y^5 m\,\mu\,z^2+4\,\mathrm{I}\,x^6 y^3 m\,\mu+6\,\mathrm{I}\,x^4 y^5 m\,\mu+\mathrm{I}\,y\,m\,\mu\,x^8+4\,\mathrm{I}\,y\,m\,\mu\,z^8+13\,\mathrm{I}\,y^3 m\,\mu\,z^6$$

$$+13\,\mathrm{I}\,y\,m\,\mu\,z^6 x^2+21\,\mathrm{I}\,x^4 y^3 m\,\mu\,z^2+30\,\mathrm{I}\,y^3 m\,\mu\,x^2 z^4\big)\big),\ -\frac{1}{8}\ \frac{1}{\left(x^2+y^2+z^2\right)^{11/2}\mu}\Big(m^2\Big($$

$$-4\,y\left(x^2+y^2+z^2\right)^{5/2}x^2-4\,y^3\left(x^2+y^2+z^2\right)^{5/2}-8\,y\left(x^2+y^2+z^2\right)^{5/2}z^2+15\,\mathrm{I}\,x^5 m\,\mu\,z^4+\mathrm{I}\,x\,m\,\mu\,y^8$$

$$+7\,\mathrm{I}\,x\,m\,\mu\,z^2 y^6+15\,\mathrm{I}\,x\,m\,\mu\,y^4 z^4+30\,\mathrm{I}\,x^3 m\,\mu\,z^4 y^2+4\,\mathrm{I}\,x^7 m\,\mu\,y^2+21\,\mathrm{I}\,x^5 m\,\mu\,z^2 y^2+\mathrm{I}\,m\,\mu\,x^9$$

$$+7\,\mathrm{I}\,x^7 m\,\mu\,z^2+6\,\mathrm{I}\,x^5 m\,\mu\,y^4+21\,\mathrm{I}\,x^3 m\,\mu\,z^2 y^4+4\,\mathrm{I}\,x^3 m\,\mu\,y^6+13\,\mathrm{I}\,x\,m\,\mu\,y^2 z^6+13\,\mathrm{I}\,x^3 m\,\mu\,z^6$$

$$+4\,\mathrm{I}\,x\,m\,\mu\,z^8\Big)\Big),\ -\frac{1}{2}\ \frac{m^2 z\left(x^2+y^2\right)}{\mu\left(x^2+y^2+z^2\right)^3}\Bigg]$$

Dissipation $=0$

****************************** *END PROCEDURE* ****************************** **(41)**

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`Example 13 -- Bateman`;
```

```
> phi:=AAa(x,y,z,t); Az:=AAb(x,y,z,t);Ax:=0;Ay:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+i*gamma),0);
```

****

NAME := Example 13 -- Bateman

$\phi := AAa(x, y, z, t)$

$Az := AAb(x, y, z, t)$

$Ax := 0$

$Ay := 0$

Example 13 -- Bateman

********************************* *Differential Form Format* *****************************
****

$$Action\ 1\text{-}form = AAb(x,y,z,t)\,d(z) - AAa(x,y,z,t)\,d(t)$$

$$Intensity\ 2\text{-}form\ F{=}dA \quad = -\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right)(d(x))\ \&\wedge\ (d(t)) + \left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)(d(x))\ \&\wedge\ (d(z)) - \left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)(d(y))\ \&\wedge\ (d(t)) + \left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)(d(y))\ \&\wedge\ (d(z)) + \left(-\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right) - \left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right)\right)(d(z))\ \&\wedge\ (d(t))$$

$$Topological\ Torsion\ 3\text{-}form \quad A\wedge F = \left(AAb(x,y,z,t)\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right) - AAa(x,y,z,t)\left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)\right)\&\wedge(d(x),d(z),d(t)) + \left(AAb(x,y,z,t)\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right) - AAa(x,y,z,t)\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)\right)\&\wedge(d(y),d(z),d(t))$$

$$Topological\ Parity\ 4\text{-}form \quad F\wedge F = 2\left(-\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right) + \left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)\right)\&\wedge(d(x),d(y),d(z),d(t))$$

******************************** *Using EM format* ********************************

$$E\ field = \left[-\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right), -\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right), -\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right) - \left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right)\right]$$

$$B\ field = \left[\frac{\partial}{\partial y} AAb(x,y,z,t), -\left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right), 0\right]$$

$$Topological\ TORSION\ 4\ vector \quad T4 = \text{-}[ExA + Bphi, AdotB] = \left[\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)AAb(x,y,z,t) - \left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)AAa(x,y,z,t), -\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right)AAb(x,y,z,t) + \left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)AAa(x,y,z,t), 0, 0\right]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ II = 2(E.B) = 2\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right) - 2\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)$$

$$coefficient\ of\ Topological\ Parity\ 4\text{-}form = 2\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right) - 2\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)$$

$$Pfaff\ Topological\ Dimension \quad PTD = 4$$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************

****

$$\textit{Xm or linear (Mean) curvature} = \frac{\partial}{\partial z} AAb(x,y,z,t) - \left(\frac{\partial}{\partial t} AAa(x,y,z,t)\right)$$

$$\textit{Yg or quadratic (GAUSS) curvature} = -\left(\frac{\partial}{\partial z} AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial t} AAa(x,y,z,t)\right) + \left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right)$$

$$\textit{Za or Cubic (Interaction internal energy) curvature} = 0$$

$$\textit{Tk or quartic (4D expansion) curvature} = 0$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\textit{Chirality factor } CH = 0$$

$$D\,field = \left[-\varepsilon\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right), -\varepsilon\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right), -\varepsilon\left(\frac{\partial}{\partial t} AAb(x,y,z,t) + \frac{\partial}{\partial z} AAa(x,y,z,t)\right)\right]$$

$$H\,field = \left[\frac{\frac{\partial}{\partial y} AAb(x,y,z,t)}{\mu}, -\frac{\frac{\partial}{\partial x} AAb(x,y,z,t)}{\mu}, 0\right]$$

$$\textit{Poynting vector ExH} = \left[-\frac{(\%1)\left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)}{\mu}, -\frac{(\%1)\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)}{\mu}, \frac{\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right) + \left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)}{\mu}\right.$$

$$\%1 = \frac{\partial}{\partial t} AAb(x,y,z,t) + \frac{\partial}{\partial z} AAa(x,y,z,t)$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} = \left[\frac{\frac{\partial^2}{\partial z\,\partial x} AAb(x,y,z,t) + \varepsilon\left(\frac{\partial^2}{\partial x\,\partial t} AAa(x,y,z,t)\right)\mu}{\mu}, \frac{\frac{\partial^2}{\partial z\,\partial y} AAb(x,y,z,t) + \varepsilon\left(\frac{\partial^2}{\partial y\,\partial t} AAa(x,y,z,t)\right)\mu}{\mu}, \frac{1}{\mu}\left(-\left(\frac{\partial^2}{\partial x^2} AAb(x,y,z,t)\right) - \left(\frac{\partial^2}{\partial y^2} AAb(x,y,z,t)\right) + \varepsilon\mu\left(\frac{\partial^2}{\partial t^2} AAb(x,y,z,t)\right) + \varepsilon\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAa(x,y,z,t)\right)\right), -\varepsilon\left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t) + \frac{\partial^2}{\partial y^2} AAa(x,y,z,t) + \frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,t) + \frac{\partial^2}{\partial z^2} AAa(x,y,z,t)\right)\right]$$

$$\textit{Amerian charge density} \quad divD = rho = -\varepsilon\left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t) + \frac{\partial^2}{\partial y^2} AAa(x,y,z,t) + \frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,t) + \frac{\partial^2}{\partial z^2} AAa(x,y,z,t)\right)$$

divergence Lorentz Current 4Vector, 4div(J4) = 0

$$\textit{Topological SPIN 4 vector S4} = \left[-\frac{-AAb(x,y,z,t)\left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right) + \varepsilon\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right) AAa(x,y,z,t)\,\mu}{\mu}, \frac{AAb(x,y,z,t)\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right) - \varepsilon\left(\right.}{\mu} \right.$$

$$\%1 = \frac{\partial}{\partial t} AAb(x,y,z,t) + \frac{\partial}{\partial z} AAa(x,y,z,t)$$

$$\textit{Topological SPIN 3-form} = -\frac{\left(-AAb(x,y,z,t)\left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right) + \varepsilon\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right) AAa(x,y,z,t)\,\mu\right) \&\text{^}(d(y), d(z), d(t))}{\mu} - \frac{\left(AAb(x,y,z,t)\left(\frac{\partial}{\partial y} A\right.\right.}{}$$

$$\%1 = \frac{\partial}{\partial t} AAb(x,y$$

$$\textit{Spin density rho\_spin} = -AAb(x,y,z,t)\,\varepsilon\left(\frac{\partial}{\partial t} AAb(x,y,z,t) + \frac{\partial}{\partial z} AAa(x,y,z,t)\right)$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{1}{\mu}\left(-\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)^2 - \left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)^2 + \varepsilon\mu\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right)^2 + \varepsilon\mu\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)^2 + \varepsilon\mu\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)^2 + 2\,\varepsilon\mu\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right) + \varepsilon\mu\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right)^2\right)$$

$$B.H = \frac{\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)^2 + \left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)^2}{\mu}$$

$$D.E = \varepsilon\left(\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right)^2 + \left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)^2 + \left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)^2 + 2\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right) + \left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right)^2\right)$$

$$A.J = \frac{1}{\mu}\left(AAb(x,y,z,t)\left(-\left(\frac{\partial^2}{\partial x^2} AAb(x,y,z,t)\right) - \left(\frac{\partial^2}{\partial y^2} AAb(x,y,z,t)\right) + \varepsilon\mu\left(\frac{\partial^2}{\partial t^2} AAb(x,y,z,t)\right) + \varepsilon\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAa(x,y,z,t)\right)\right)\right)$$

$$\textit{-rho.phi} = -AAa(x,y,z,t)\,\varepsilon\left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t) + \frac{\partial^2}{\partial y^2} AAa(x,y,z,t) + \frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,t) + \frac{\partial^2}{\partial z^2} AAa(x,y,z,t)\right)$$

$$\textit{Poincare I} \quad \textit{(B.H - D.E)-(A.J - rho.phi)} = -\frac{1}{\mu}\left(-\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)^2 - \left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)^2 + \varepsilon\mu\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right)^2 + \varepsilon\mu\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)^2 + \varepsilon\mu\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)^2\right.$$

$$+2\,\varepsilon\,\mu\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right)+\varepsilon\,\mu\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right)^2 - AAb(x,y,z,t)\left(\frac{\partial^2}{\partial x^2} AAb(x,y,z,t)\right) - AAb(x,y,z,t)\left(\frac{\partial^2}{\partial y^2} AAb(x,y,z,t)\right) + AAb(x,y,z,t)\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial t^2} AAb(x,y,z,t)\right) + AAb(x,y,z,t)\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAa(x,y,z,t)\right) + AAa(x,y,z,t)\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t)\right) + AAa(x,y,z,t)\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial y^2} AAa(x,y,z,t)\right) + AAa(x,y,z,t)\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,t)\right) + AAa(x,y,z,t)\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial z^2} AAa(x,y,z,t)\right)\Big)\Big)$$

London Coefficient $\quad LC = \dfrac{2}{(x^2+y^2+z^2)\,\mu}$

PROCA coefficient $curlcurlB = \left[-\left(\frac{\partial^3}{\partial y\,\partial x^2} AAb(x,y,z,t)\right) - \left(\frac{\partial^3}{\partial y^3} AAb(x,y,z,t)\right) - \left(\frac{\partial^3}{\partial z^2\,\partial y} AAb(x,y,z,t)\right), \frac{\partial^3}{\partial z^2\,\partial x} AAb(x,y,z,t) + \frac{\partial^3}{\partial x^3} AAb(x,y,z,t) + \frac{\partial^3}{\partial y^2\,\partial x} AAb(x,y,z,t), 0\right]$

Amperian Current 4Vector $\quad curlH\text{-}dD/dt=J4 = \left[\frac{\frac{\partial^2}{\partial z\,\partial x} AAb(x,y,z,t) + \varepsilon\left(\frac{\partial^2}{\partial x\,\partial t} AAa(x,y,z,t)\right)\mu}{\mu}, \frac{\frac{\partial^2}{\partial z\,\partial y} AAb(x,y,z,t) + \varepsilon\left(\frac{\partial^2}{\partial y\,\partial t} AAa(x,y,z,t)\right)\mu}{\mu}, \frac{1}{\mu}\left(-\left(\frac{\partial^2}{\partial x^2} AAb(x,y,z,t)\right) - \left(\frac{\partial^2}{\partial y^2} AAb(x,y,z,t)\right) + \varepsilon\,\mu\left(\frac{\partial^2}{\partial t^2} AAb(x,y,z,t)\right) + \varepsilon\,\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAa(x,y,z,t)\right)\right), -\varepsilon\left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t) + \frac{\partial^2}{\partial y^2} AAa(x,y,z,t) + \frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,t) + \frac{\partial^2}{\partial z^2} AAa(x,y,z,t)\right)\right]$

Lorentz Force 3 vector due to Ampere current $\quad FL = -(rho\_ampere\ E + J\_ampere\ x\ B) = \Bigg[$

$$-\frac{1}{\mu}\left(\varepsilon\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right)\mu\left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t)\right) + \varepsilon\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right)\mu\left(\frac{\partial^2}{\partial y^2} AAa(x,y,z,t)\right) + \varepsilon\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right)\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,t)\right) + \varepsilon\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right)\mu\left(\frac{\partial^2}{\partial z^2} AAa(x,y,z,\right.\right.$$

$$t)\Big) - \left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial x^2} AAb(x,y,z,t)\right) - \left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial y^2} AAb(x,y,z,t)\right)$$

$$+ \left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)\varepsilon\,\mu\left(\frac{\partial^2}{\partial t^2} AAb(x,y,z,t)\right) + \left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)\varepsilon\,\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAa(x,y,z,$$

$$t)\Big)\Big), -\frac{1}{\mu}\Big(\varepsilon\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)\mu\left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t)\right) + \varepsilon\Big(\frac{\partial}{\partial y} AAa(x,y,z,$$

$$t)\Big)\mu\left(\frac{\partial^2}{\partial y^2} AAa(x,y,z,t)\right) + \varepsilon\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,t)\right) + \varepsilon\Big(\frac{\partial}{\partial y} AAa(x,y,z,$$

$$t)\Big)\mu\left(\frac{\partial^2}{\partial z^2} AAa(x,y,z,t)\right) - \left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial x^2} AAb(x,y,z,t)\right) - \Big(\frac{\partial}{\partial y} AAb(x,y,z,$$

$$t)\Big)\left(\frac{\partial^2}{\partial y^2} AAb(x,y,z,t)\right) + \left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)\varepsilon\,\mu\left(\frac{\partial^2}{\partial t^2} AAb(x,y,z,t)\right) + \Big(\frac{\partial}{\partial y} AAb(x,y,z,$$

$$t)\Big)\varepsilon\,\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAa(x,y,z,t)\right)\Big), \frac{1}{\mu}\Big(-\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)$$

$$-\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right) - \varepsilon\,\mu\left(\frac{\partial^2}{\partial y^2} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)$$

$$-\varepsilon\,\mu\left(\frac{\partial^2}{\partial y^2} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right) - \varepsilon\,\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)$$

$$-\varepsilon\,\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right) - \varepsilon\,\mu\left(\frac{\partial^2}{\partial z^2} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)$$

$$-\varepsilon\,\mu\left(\frac{\partial^2}{\partial z^2} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right) + \left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial z\,\partial x} AAb(x,y,z,t)\right)$$

$$+ \left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)\varepsilon\left(\frac{\partial^2}{\partial x\,\partial t} AAa(x,y,z,t)\right)\mu + \left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial z\,\partial y} AAb(x,y,z,t)\right)$$

$$+ \left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)\varepsilon\left(\frac{\partial^2}{\partial y\,\partial t} AAa(x,y,z,t)\right)\mu\Big)\Big]$$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

$$\textit{Topological Torsion current 4 vector}\quad T4 = \text{-[ExA + B.phi,AdotB]} = \left[\left(\frac{\partial}{\partial y} AAa(x,y,z,t) \right) AAb(x,y,z,t) - \left(\frac{\partial}{\partial y} AAb(x,y,z,t) \right) AAa(x,y,z,t),\ -\left(\frac{\partial}{\partial x} AAa(x,y,z,t) \right) AAb(x,y,z,t) + \left(\frac{\partial}{\partial x} AAb(x,y,z,t) \right) AAa(x,y,z,t),\ 0,\ 0 \right]$$

$$\textit{Lorentz Force 3 vector due to Torsion current}\quad TF = \text{-(rho\_torsion E + J\_torsion x B)} = \left[0,\ 0,\ AAb(x,y,z,t) \left(\left(\frac{\partial}{\partial x} AAa(x,y,z,t) \right) \left(\frac{\partial}{\partial y} AAb(x,y,z,t) \right) - \left(\frac{\partial}{\partial y} AAa(x,y,z,t) \right) \left(\frac{\partial}{\partial x} AAb(x,y,z,t) \right) \right) \right]$$

$$\textit{Torsion Dissipation Jtorsion dot E} = -AAa(x,y,z,t) \left(\left(\frac{\partial}{\partial x} AAa(x,y,z,t) \right) \left(\frac{\partial}{\partial y} AAb(x,y,z,t) \right) - \left(\frac{\partial}{\partial y} AAa(x,y,z,t) \right) \left(\frac{\partial}{\partial x} AAb(x,y,z,t) \right) \right)$$

$$\textit{Topological Spin current 4 vector}\quad TS4 = \text{-[A x H + D.phi,AdotD]} = \left[-\frac{-AAb(x,y,z,t) \left(\frac{\partial}{\partial x} AAb(x,y,z,t) \right) + \varepsilon \left(\frac{\partial}{\partial x} AAa(x,y,z,t) \right) AAa(x,y,z,t)\, \mu}{\mu},\ \frac{AAb \ldots}{} \right.$$

$$\%1 = \frac{\partial}{\partial t} AAb(x,y,z,t) + \frac{\partial}{\partial z} AAa$$

$$\textit{Lorentz Force 3 vector due to Spin current}\quad SF = \text{--(rho\_spin E + J\_spin x B)} = \left[-\varepsilon (\%1) \left(\left(\frac{\partial}{\partial x} AAa(x,y,z,t) \right) AAb(x,y,z,t) - \left(\frac{\partial}{\partial x} AAb(x,y,z,t) \right) AA \ldots \right. \right.$$

$$\textit{Spin Dissipation J\_spin dot E} = \frac{1}{\mu} \left(-\left(\frac{\partial}{\partial x} AAa(x,y,z,t) \right) AAb(x,y,z,t) \left(\frac{\partial}{\partial x} AAb(x,y,z,t) \right) + \varepsilon \left(\frac{\partial}{\partial x} AAa(x,y,z,t) \right)^2 AAa(x,y,z,t)\, \mu - \left(\frac{\partial}{\partial y} AAa(x,y,z,t) \right) AAb(x,y,z,t) \left(\frac{\partial}{\partial y} AAb(x,y,z,t) \right) + \varepsilon \left(\frac{\partial}{\partial y} AAa(x,y,z,t) \right)^2 AAa(x,y,z,t)\, \mu + AAa(x,y,z,t)\, \varepsilon\, \mu \left(\frac{\partial}{\partial t} AAb(x,y,z,t) \right)^2 + 2\, AAa(x,y,z,t)\, \varepsilon\, \mu \left(\frac{\partial}{\partial t} AAb(x,y,z,t) \right) \left(\frac{\partial}{\partial z} AAa(x,y,z,t) \right) + AAa(x,y,z,t)\, \varepsilon\, \mu \left(\frac{\partial}{\partial z} AAa(x,y,z,t) \right)^2 \right)$$

$$\textit{Disspative Force 3 vector} = \left[-\frac{1}{\mu} \left(\varepsilon \left(\frac{\partial}{\partial x} AAa(x,y,z,t) \right) \mu \left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t) \right) + \varepsilon \left(\frac{\partial}{\partial x} AAa(x,y,z,t) \right) \mu \left(\frac{\partial^2}{\partial y^2} AAa(x,y,z,t) \right) + \varepsilon \left(\frac{\partial}{\partial x} AAa(x,y,z,t) \right) \mu \left(\frac{\partial^2}{\partial z\, \partial t} AAb(x,y,z,t) \right) + \varepsilon \left(\frac{\partial}{\partial x} AAa(x,y,z, \right. \right. \right.$$

$$t)\Big)\mu\left(\frac{\partial^2}{\partial z^2}AAa(x,y,z,t)\right)-\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial x^2}AAb(x,y,z,t)\right)-\left(\frac{\partial}{\partial x}AAb(x,y,z,\right.$$

$$t)\Big)\left(\frac{\partial^2}{\partial y^2}AAb(x,y,z,t)\right)+\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)\varepsilon\,\mu\left(\frac{\partial^2}{\partial t^2}AAb(x,y,z,t)\right)+\left(\frac{\partial}{\partial x}AAb(x,y,z,\right.$$

$$t)\Big)\varepsilon\,\mu\left(\frac{\partial^2}{\partial z\,\partial t}AAa(x,y,z,t)\right)+\mu^2\varepsilon\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right)AAb(x,y,z,t)$$

$$-\mu^2\varepsilon\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)AAa(x,y,z,t)+\mu^2\varepsilon\left(\frac{\partial}{\partial z}AAa(x,y,z,\right.$$

$$t)\Big)\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right)AAb(x,y,z,t)-\mu^2\varepsilon\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)AAa(x,y,z,$$

$$t)\Big),\ -\frac{1}{\mu}\Bigg(\varepsilon\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)\mu\left(\frac{\partial^2}{\partial x^2}AAa(x,y,z,t)\right)+\varepsilon\left(\frac{\partial}{\partial y}AAa(x,y,z,\right.$$

$$t)\Big)\mu\left(\frac{\partial^2}{\partial y^2}AAa(x,y,z,t)\right)+\varepsilon\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)\mu\left(\frac{\partial^2}{\partial z\,\partial t}AAb(x,y,z,t)\right)+\varepsilon\left(\frac{\partial}{\partial y}AAa(x,y,z,\right.$$

$$t)\Big)\mu\left(\frac{\partial^2}{\partial z^2}AAa(x,y,z,t)\right)-\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial x^2}AAb(x,y,z,t)\right)-\left(\frac{\partial}{\partial y}AAb(x,y,z,\right.$$

$$t)\Big)\left(\frac{\partial^2}{\partial y^2}AAb(x,y,z,t)\right)+\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\varepsilon\,\mu\left(\frac{\partial^2}{\partial t^2}AAb(x,y,z,t)\right)+\left(\frac{\partial}{\partial y}AAb(x,y,z,\right.$$

$$t)\Big)\varepsilon\,\mu\left(\frac{\partial^2}{\partial z\,\partial t}AAa(x,y,z,t)\right)+\mu^2\varepsilon\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)AAb(x,y,z,t)$$

$$-\mu^2\varepsilon\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)AAa(x,y,z,t)+\mu^2\varepsilon\left(\frac{\partial}{\partial z}AAa(x,y,z,\right.$$

$$t)\Big)\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)AAb(x,y,z,t)-\mu^2\varepsilon\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)AAa(x,y,z,$$

$$t)\Big),\ \frac{1}{\mu}\Big(-\varepsilon\mu\left(\frac{\partial^2}{\partial x^2}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)-\varepsilon\mu\Big(\frac{\partial^2}{\partial x^2}AAa(x,y,z,$$

$$t)\Big)\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)-\varepsilon\mu\left(\frac{\partial^2}{\partial y^2}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)-\varepsilon\mu\Big(\frac{\partial^2}{\partial y^2}AAa(x,y,z,$$

$$t)\Big)\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)-\varepsilon\mu\left(\frac{\partial^2}{\partial z\,\partial t}AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)-\varepsilon\mu\Big(\frac{\partial^2}{\partial z\,\partial t}AAb(x,y,z,$$

$$t)\Big)\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)-\varepsilon\mu\left(\frac{\partial^2}{\partial z^2}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)-\varepsilon\mu\Big(\frac{\partial^2}{\partial z^2}AAa(x,y,z,$$

$$t)\Big)\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)+\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial z\,\partial x}AAb(x,y,z,t)\right)+\Big(\frac{\partial}{\partial x}AAb(x,y,z,$$

$$t)\Big)\varepsilon\left(\frac{\partial^2}{\partial x\,\partial t}AAa(x,y,z,t)\right)\mu+\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial z\,\partial y}AAb(x,y,z,t)\right)+\Big(\frac{\partial}{\partial y}AAb(x,y,z,$$

$$t)\Big)\varepsilon\left(\frac{\partial^2}{\partial y\,\partial t}AAa(x,y,z,t)\right)\mu-AAb(x,y,z,t)\,\varepsilon\mu^2\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)^2-2\,AAb(x,y,z,$$

$$t)\,\varepsilon\mu^2\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)-AAb(x,y,z,t)\,\varepsilon\mu^2\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)^2$$

$$+AAb(x,y,z,t)\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)^2\mu-\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)\varepsilon\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right)AAa(x,y,z,$$

$$t)\,\mu^2+AAb(x,y,z,t)\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)^2\mu-\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\varepsilon\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)AAa(x,$$

$$y,z,t)\,\mu^2+AAb(x,y,z,t)\,\mu\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)-AAb(x,y,z,$$

$$t)\,\mu\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)\Big)\Big]$$

$$Dissipation\ =-\varepsilon\left(\frac{\partial^2}{\partial x^2}AAa(x,y,z,t)\right)-\varepsilon\left(\frac{\partial^2}{\partial y^2}AAa(x,y,z,t)\right)-\varepsilon\left(\frac{\partial^2}{\partial z\,\partial t}AAb(x,y,z,t)\right)$$

$$-\varepsilon\left(\frac{\partial^2}{\partial z^2}AAa(x,y,z,t)\right)-AAb(x,y,z,t)\,\varepsilon\mu\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)-AAb(x,y,z,t)\,\varepsilon\mu\Big(\frac{\partial}{\partial z}AAa(x,$$

$$y,z,t)\Big)+\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)AAb(x,y,z,t)-\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)AAa(x,y,z,t)$$

****************************** *END PROCEDURE* ****************************** **(42)**