```
> restart:with(LinearAlgebra): with(GraphTheory):
```

Examples of Finite Lattice Structures,

useful to the study
of Low dimension Topological Structures, Partitions, Causality, Emergence,
and Non-equilibrium Systems and Irreversibe Processes.
Assembled 12/5/2011

The N ingredients of the lattice structure are represented abstractly as
N=3 X={a,b,c}
N=4 X={a,b,c,d}
N=5 X={a,b,c,d,s} .
In special cases the abstract symbols take on special meaning,
when the ingredients are presumed to be exterior differential forms.
If A is a 1-form, F=dA is a 2-form, H=A^F a 3-form, K=F^F a 4-form, then
for N= 4 there is a special T0 poset 3 disconnected topology
which has a Kuratowski representation in terms of the exterior differential, d.
This special topology was discovered about 1987, and permits the analysis
of non-equilibrium thermodynamic systems of particles and irreversible processes,
in terms of finite topological structures.
For irreversible processes H = A^F = Topological Torsion must not be zero.

This program uses some portions of a Maple program created by Didier Deses,
along with a number of modifications, additions, and corrections
by R. M. Kiehn Nov 20, 2009 and updated Sept 28, 2011 and Nov 20, 2011.

```
Vertex [a,b,c,d]
Edges {{a,ab},{d,bcd}}
Arcs    {[a,ab],[d,bcd]}
```

A primitive lattice is defined by a set of vertices, and a set of rules for connecting the vertices. When the Lattice Subsets of a Power Set are identified with "Open Sets", then it possible to compute the interior, exterior, boundary, closure, and limit points for every subset of the power set, relative to the Open sets are defined by the Lattice Subset, LS.
Note that the Lattice Subset, LS, need not define a topology!!!
*****
The computation of the Complete Lattice Structure for the 2^N elements of the power set of N ingredients can be very tedious. Hence this Maple program was constructed to reduce the labor.
*****
Note that there are Lattice Subsets that do not define topologies, and yet have T0 distinguishable elements of singleton closure. There are lattice subsets that do not define topologies, and have indistinguishable elements of closure. When elements have indistinguishable elements of closure, then the vertices of the same dimension singlet =1, doublets =2, ...etc.) can admit a connecting edge. This is an important concept when the arcs and edges connecting verticies are considered.
*****
There are Lattice Subsets that define topologies, and yet have indistinguishable elements of singleton closure. These topologies do not satisfy the simplest of the separation axioms. These topologies are Not T0.
*****

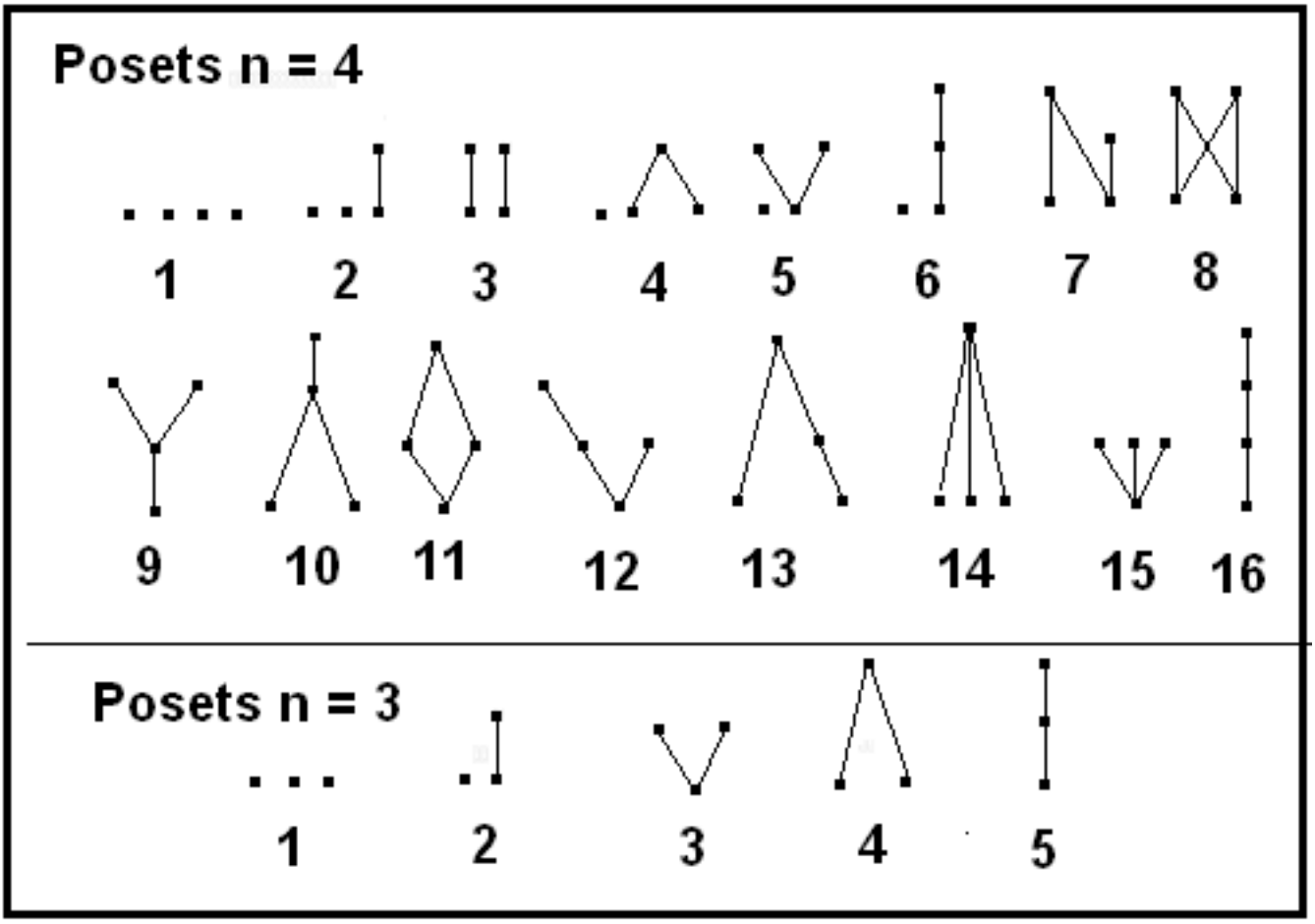
There are Lattice Subsets that define topologies, with separation axioms, T0 and T1, and yet these topologies are NOT metrizable. If a topology is metrizable, the topology must be Hausdorff, T2, or stronger.
*****
In many instances the Lattice forms a planar graph, which means that the graph can be embedded by deformations into R2. However, Kuratowski proved that certain lattices, those that contain K3,3 or K5 sub-lattices, cannot be embedded in R2 as planar graphs. Such Lattices require R3 or more.
A K3,3 sub-lattice consists of 6 vertices and 9 connecting arcs (or edges).
The K5 sub-lattice consists of 5 vertices and 10 connecting arcs.

These concepts have been used to understand chemical molecules, and concepts of topological chirality.
*****
A visual aspect of an embedding into R3 is given by a space curve that exhibits affine torsion. Another, perhaps more general idea, is defined in terms of Topological Torsion, which is related to the Pfaff Topological Dimension = 3 (or "finite" class) of an exterior differential 1-form defined on a differential variety.
*****
Exterior differential 1-forms, A, that can not be embedded in R2 are such that the 3-form A^dA is not zero. This constraint requires that the Frobenious Unique integrability theorem fails!
Unique initial data do NOT produce Unique final data, unless A^dA = 0. Envelope solutions and edges of regression give visual examples of non-uniqueness.
*****
Exterior Differential 1-forms of Pfaff Topological Dimension 3 require a minimum of 3 independent functions (of an arbitrary geometric set) to capture their topological properties. An open question is: What Lattices cannot be embedded in R3?"
*****
Geometric constraints based upon metric ideas imply that the underlying topology is Hausdorff T2. Finite Hausdorff spaces (the power set of X) are disconnected. Subsets of finite Hausdorff spaces can be simply onnected, or not simply connected. If the subsets of the power set form a T0 topology, it can be a connected topology or a disconnected topology. The disconnected T0 topologies have subsets that do not have boundaries (the boundaries are empty) . The connected T0 topologies do not have subsets without boundary,
*****
The study of the detailed lattice and topological structures gives an insite into physical concepts that may not be distinguished in terms of geometrical constraints. Finite Homological features focus attention on simply connected or not simply connected concepts which are not Hausdorff and therefor do not depend upon geometrical ideas of size and shape.
*****
It has been established that finite T0 topologies that are not metrizable can be put into correspondence with non-equilibrium thermodynamics, and irreversible processes. The arrow of time is explained in terms of topologically continuous processes from thermodynamic states of higher topological dimension to thermodynamic states of lessor topological dimension. In addition, certain topological quantum features can be put into correspondence with deRham integrals over closed exterior differential forms that are not exact.

See http://www.lulu.com/kiehn for 6 monographs that describe the "Unusual effectiveness of topological thermodynamics".
*****
Planck's Backbody radiation formula describes physical phenomena that are independent of metric, size, shape, material composition, or even internal microscopic structure.
It is conjectured that such concepts are related to both those Lattice Structures that are topologies and to those Lattice Structures that are not topologies, but are Lattice Structures which do not obey the

Kolmogorov T0 separation axiom. Such Lattice Structures have subsets of the power set that are indistinguishable. If finite Lattice Structures obey the Hausdorff T2 =>T0 separation axioms, then all subsets of the power set are distinguishable.
*****
The conjecture to be studied is :
Do the finite Not-T0 non-metrizable topologies, and the Lattice Structures that are not even T0 form the foundations for understanding such effects as Planck's Black Body radiation formula, the salient features of indistinguishable Bosons and Fermions, and the theory of collective states, that are the foundations of Quantum Theory.

****
The finite T0 closure Posets structures based upon singleton closure for X={a,b,c,d} and X={a,b,c} are



For a subset A of a Lattice space,
Int(S) = Interior of S = largest open set contained in S
Com(S) = Complement of S = topset X={a,b,c,d...} minus S = X mod S.
Ext(S) = Exterior of S = Int(Com(S)) = largest open set in the compliment of S.
Bnd(S) = Boundary of S = X mod Int(S) mod Ext(S).
Clo(S) = Closure of S = Int(S) union Bnd(S) = S union Lim(S)
Lim(S) = Limit points of S = points p such that {Clo(S mod p) intersect (p)} =/= the empty set.

Isolated points are of two types

Clo(S) minus Lim(S) = IsoClo(S)
and
{S}intersection{Lim(S)}=IsoCar(S), in the sense of Caratheodory

If S is an exterior differential 1-form, and the exterior differential, d, is a limit point generator then I call the non zero 3-form A^dA : "Topological Torsion". Topological Torsion = 0 implies the differential 1-form admits an integrating factor. For a Kuratowski topology Topological Torsion is related to IsoCar.

A closed set either has no limit sets or contains its limit sets. A closed set S = Clo(S).

A set S with empty limit points and empty boundary points is defined as a isolated segregated point, Bnd(S) = {} and Lim(S) = {} => {S}.Seg.

The disconnected points in the closures of a set are described as adherent points.

Examples of Low Dimension Lattice and Topological Structures

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Maple code download available on email request
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1. 5 Examples of T0 (Disconnected or Connected) topologies for N=3, X = {a,b,c} Bnd(X) = { }

```
> print(`    `);X:={a,b,c};LS:={{},{},{a},{b},{c},{a,b},{},{a,c},{b,c},X}
  ;print(` N=3   T2  Poset 1 \n Discrete, Disconnected, Bnd of Bnd(S) = {
  } as all Bnd(S) = { }`);LAT3abc(LS,` N=3 Poset 1, T2  `,3):
```

X := { a, b, c }

LS := { { }, { a }, { b }, { c }, { a, b }, { a, c }, { b, c }, { a, b, c } }

N=3 T2 Poset 1

Discrete, Disconnected, Bnd of Bnd(S) = { } as all Bnd(S) = { }

Is LS a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = true

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ b, c }	{ }	{ a }	{ }	{ a }.Seg	{ }
{ b }	{ b }	{ a, c }	{ }	{ b }	{ }	{ b }.Seg	{ }
{ c }	{ c }	{ a, b }	{ }	{ c }	{ }	{ c }.Seg	{ }
{ a, b }	{ a, b }	{ c }	{ }	{ a, b }	{ }	{ a, b }.Seg	{ }
{ a, c }	{ a, c }	{ b }	{ }	{ a, c }	{ }	{ a, c }.Seg	{ }
{ b, c }	{ b, c }	{ a }	{ }	{ b, c }	{ }	{ b, c }.Seg	{ }
{ a, b, c }	{ a, b, c }	{ }	{ }	{ a, b, c }	{ }	{ a, b, c }.Seg	{ }

Closed-Open subsets of LS are = { { }, { a }, { b }, { c }, { a, b }, { a, c }, { b, c }, { a, b, c } }

The {1,2,3,4} array of CLOSURE elements is = ({ a }, { b }, { c }) **(1)**

```
> print(`     `);X:={a,b,c};LS:={{},{},{b},{c},{a,b},{},{b,c},X};print(`
  N=3     T0   poset 2 \n ,  Disconnected, Bnd of Bnd(S) Not { } unless
  Bnd(S) = {}`);LAT3abc(LS,` N=3 Poset 2, T0  `,3):
```

X := {a, b, c}

LS := {{ }, {b}, {c}, {a, b}, {b, c}, {a, b, c}}

N=3 T0 poset 2

, Disconnected, Bnd of Bnd(S) Not { } unless Bnd(S) = {}

Is LS a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c}	{a}	{a}	{ }	{ }	{ }
{b}	{b}	{c}	{a}	{a, b}	{a}	{b}	{ }
{c}	{c}	{a, b}	{ }	{c}	{ }	{c}.Seg	{ }
{a, b}	{a, b}	{c}	{ }	{a, b}	{a}	{ }	{a}
{a, c}	{c}	{b}	{a}	{a, c}	{ }	{ }	{ }
{b, c}	{b, c}	{ }	{a}	{a, b, c}	{a}	{b, c}	{ }
{a, b, c}	{a, b, c}	{ }	{ }	{a, b, c}	{a}	{ }	{a}

Closed-Open subsets of LS are = {{ }, {c}, {a, b}, {a, b, c}}

The {1,2,3,4} array of CLOSURE elements is = ({a}, {a, b}, {c})

(2)

```
> print(`     `);X:={a,b,c};LS:={{},{},{a},{c},{a,b},{},{a,c},X};print (`
  N=3 T0 poset 2 \n DisConnected (Kuratowski format), Bnd of Bnd(S) Not {
  } unless Bnd(S) = {} `);LAT3abc(LS,` N=3 Poset 2, T0   `,3);print(`This
  Lattice Structure appears to be a KURATOWSKI T0POLOGY \n with a limit
  point operator that can be formulated interms of the exterior
  derivative, d,\n acting on exterior differential forms, A, F = dA, H =
  A^F. Substitute {A,F,H} for the symbol set {a,b,c}.  \n Then a = A, a 1
  -form, b => F=dA, a 2-form with dF=0,  c = A^F = H, a 3-form. \n Note
  that d(d(S)) = 0 but Bnd(Bnd(S)) =/= 0.   d is an up operator: given A,
  F is determined as dA`):
```

X := {a, b, c}

LS := {{ }, {a}, {c}, {a, b}, {a, c}, {a, b, c}}

N=3 T0 poset 2

DisConnected (Kuratowski format), Bnd of Bnd(S) Not { } unless Bnd(S) = {}

Is LS a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c}	{b}	{a, b}	{b}	{a}	{ }
{b}	{ }	{a, c}	{b}	{b}	{ }	{ }	{ }
{c}	{c}	{a, b}	{ }	{c}	{ }	{c}.Seg	{ }
{a, b}	{a, b}	{c}	{ }	{a, b}	{b}	{ }	{b}
{a, c}	{a, c}	{ }	{b}	{a, b, c}	{b}	{a, c}	{ }
{b, c}	{c}	{a}	{b}	{b, c}	{ }	{ }	{ }
{a, b, c}	{a, b, c}	{ }	{ }	{a, b, c}	{b}	{ }	{b}

Closed-Open subsets of LS are = {{ }, {c}, {a, b}, {a, b, c}}

The {1,2,3,4} array of CLOSURE elements is = ({a, b}, {b}, {c})

This Lattice Structure appears to be a KURATOWSKI T0POLOGY **(3)**

with a limit point operator that can be formulated interms of the exterior derivative, d,
acting on exterior differential forms, A, F = dA, H = A^F. Substitute {A,F,H} for the symbol set {a,b,c}.
Then a = A, a 1-form, b => F=dA, a 2-form with dF=0, c = A^F = H, a 3-form.
Note that d(d(S)) = 0 but Bnd(Bnd(S)) =/= 0. d is an up operator: given A, F is determined as dA

```
> print(`    `);X:={a,b,c};print(` N=3   T0  poset 3 \n Connected, Bnd of
  Bnd(S) Not { } unless Bnd(S)= {}`):LS:={{},{},{a},{c},{},{},{a,c},X};
  LAT3abc(LS,` N=3 Poset 3, T0   `,3);
```

X := { a, b, c }

N=3 T0 poset 3

Connected, Bnd of Bnd(S) Not { } unless Bnd(S)= {}

LS := { { }, { a }, { c }, { a, c }, { a, b, c } }

Is LS a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ c }	{ b }	{ a, b }	{ b }	{ a }	{ }
{ b }	{ }	{ a, c }	{ b }	{ b }	{ }	{ }	{ }
{ c }	{ c }	{ a }	{ b }	{ b, c }	{ b }	{ c }	{ }
{ a, b }	{ a }	{ c }	{ b }	{ a, b }	{ b }	{ }	{ b }
{ a, c }	{ a, c }	{ }	{ b }	{ a, b, c }	{ b }	{ a, c }	{ }
{ b, c }	{ c }	{ a }	{ b }	{ b, c }	{ b }	{ }	{ b }
{ a, b, c }	{ a, b, c }	{ }	{ }	{ a, b, c }	{ b }	{ }	{ b }

Closed-Open subsets of LS are = { { }, { a, b, c } }

The {1,2,3,4} array of CLOSURE elements is = ({ a, b }, { b }, { b, c }) **(4)**

```
> print(`    `);X:={a,b,c};print( ` N=3    T0   poset 4\n  Connected, Bnd
  of Bnd(S) Not { }, unless Bnd(S) is dense `);LS:={{},{c},{a,c},{b,c},{}
  ,{b,c},X}:LSd:=CLO({a,b,c},LS):print(` LS `=LS);LAT3abc(LS,` N=3 Poset
  4, T0  topology  `,3);
```

X := {a, b, c}

N=3 T0 poset 4

Connected, Bnd of Bnd(S) Not { }, unless Bnd(S) is dense

LS = { { }, {c}, {a, c}, {b, c}, {a, b, c} }

Is LS a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c}	{a}	{a}	{ }	{ }	{ }
{b}	{ }	{a, c}	{b}	{b}	{ }	{ }	{ }
{c}	{c}	{ }	{a, b}	{a, b, c}	{a, b}	{c}	{ }
{a, b}	{ }	{c}	{a, b}	{a, b}	{ }	{ }	{ }
{a, c}	{a, c}	{ }	{b}	{a, b, c}	{a, b}	{c}	{a}
{b, c}	{b, c}	{ }	{a}	{a, b, c}	{a, b}	{c}	{b}
{a, b, c}	{a, b, c}	{ }	{ }	{a, b, c}	{a, b}	{ }	{a, b}

Closed-Open subsets of LS are = { { }, {a, b, c} }

The {1,2,3,4} array of CLOSURE elements is = ({a}, {b}, {a, b, c}) **(5)**

```
> print(`     `);X:={a,b,c};print(` N=3   T0   poset 5 \n Connected, Bnd of
  Bnd(S) not { }, unless Bnd(S) =  {}` );LS:={{},{},{a},{},{},{},{a,b},X};
  LAT3abc(LS,`  N=3 Poset 5 ,T0  `,3):
```

X := {a, b, c}

N=3 T0 poset 5

Connected, Bnd of Bnd(S) not { }, unless Bnd(S) = {}

LS := { { }, {a}, {a, b}, {a, b, c} }

Is LS a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c}	{a, b, c}	{b, c}	{a}	{ }
{b}	{ }	{a}	{b, c}	{b, c}	{c}	{b}	{ }
{c}	{ }	{a, b}	{c}	{c}	{ }	{ }	{ }
{a, b}	{a, b}	{ }	{c}	{a, b, c}	{b, c}	{a}	{b}
{a, c}	{a}	{ }	{b, c}	{a, b, c}	{b, c}	{a}	{c}
{b, c}	{ }	{a}	{b, c}	{b, c}	{c}	{ }	{c}
{a, b, c}	{a, b, c}	{ }	{ }	{a, b, c}	{b, c}	{ }	{b, c}

Closed-Open subsets of LS are = { { }, {a, b, c} }

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c}, {b, c}, {c}) **(6)**

2. 4 Examples of NOT T0 topologies in N=3.

X = {a,b,c}

Some or All Singleton closures are not distinguishable.

```
> print(`     `); X := {a, b, c}; print(` Not-T0 N=3   Example 4 \n
  DisConnected, Bnd of Bnd(S) Not 0 unless Bnd(S) is dense or empty  `);
  LS:={{},{},{c},{a,b},{},{},{},X};LAT3abc(LS,`Not-T0   N=3   Ex 4`,3):
```

X := { a, b, c }

Not-T0 N=3 Example 4

DisConnected, Bnd of Bnd(S) Not 0 unless Bnd(S) is dense or empty

LS := { { }, { c }, { a, b }, { a, b, c } }

Is LS a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ c }	{ a, b }	{ a, b }	{ b }	{ a }	{ }
{ b }	{ }	{ c }	{ a, b }	{ a, b }	{ a }	{ b }	{ }
{ c }	{ c }	{ a, b }	{ }	{ c }	{ }	{ c }.Seg	{ }
{ a, b }	{ a, b }	{ c }	{ }	{ a, b }	{ a, b }	{ }	{ a, b }
{ a, c }	{ c }	{ }	{ a, b }	{ a, b, c }	{ b }	{ a, c }	{ }
{ b, c }	{ c }	{ }	{ a, b }	{ a, b, c }	{ a }	{ b, c }	{ }
{ a, b, c }	{ a, b, c }	{ }	{ }	{ a, b, c }	{ a, b }	{ }	{ a, b }

Closed-Open subsets of LS are = { { }, { c }, { a, b }, { a, b, c } }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({ a, b }, { a, b }, { c })

(7)

```
> print(`    `); X := {a, b, c};
      print(` Not-T0 N=3      Example 3 \n Connected, Bnd of Bnd(S) = { } if Bnd(S) is dense `); LS
      := {{ }, {a, b}, { }, { }, { }, { }, X}; LAT3abc(LS, `Not-T0   N=3  Ex 3`, 3) :
```

X := {a, b, c}

Not-T0 N=3 Example 3

Connected, Bnd of Bnd(S) = { } if Bnd(S) is dense

LS := {{ }, {a, b}, {a, b, c}}

Is LS a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c}	{a, b, c}	{b, c}	{a}	{ }
{b}	{ }	{ }	{a, b, c}	{a, b, c}	{a, c}	{b}	{ }
{c}	{ }	{a, b}	{c}	{c}	{ }	{ }	{ }
{a, b}	{a, b}	{ }	{c}	{a, b, c}	{a, b, c}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c}	{a, b, c}	{b, c}	{a}	{c}
{b, c}	{ }	{ }	{a, b, c}	{a, b, c}	{a, c}	{b}	{c}
{a, b, c}	{a, b, c}	{ }	{ }	{a, b, c}	{a, b, c}	{ }	{a, b, c}

Closed-Open subsets of LS are = {{ }, {a, b, c}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c}, {a, b, c}, {c}) **(8)**

```
> print(`     `); X := {a, b, c};
    print(` Not-T0 N=3      Example 2 \n Connected, Bnd of Bnd(S) not =
    { }   `); LS := {{ }, {c}, { }, { }, { }, { }, X}; LAT3abc(LS,
    `  Not-T0  N=3 Ex 2`, 3);
```

X := { a, b, c }

Not-T0 N=3 Example 2

Connected, Bnd of Bnd(S) not = { }

LS := { { }, { c }, { a, b, c } }

Is LS a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ c }	{ a, b }	{ a, b }	{ b }	{ a }	{ }
{ b }	{ }	{ c }	{ a, b }	{ a, b }	{ a }	{ b }	{ }
{ c }	{ c }	{ }	{ a, b }	{ a, b, c }	{ a, b }	{ c }	{ }
{ a, b }	{ }	{ c }	{ a, b }	{ a, b }	{ a, b }	{ }	{ a, b }
{ a, c }	{ c }	{ }	{ a, b }	{ a, b, c }	{ a, b }	{ c }	{ a }
{ b, c }	{ c }	{ }	{ a, b }	{ a, b, c }	{ a, b }	{ c }	{ b }
{ a, b, c }	{ a, b, c }	{ }	{ }	{ a, b, c }	{ a, b }	{ }	{ a, b }

Closed-Open subsets of LS are = { { }, { a, b, c } }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({ a, b }, { a, b }, { a, b, c }) **(9)**

```
> print(`     `);X := {a, b, c};
    print(` Not-T0 N=3 "extremal indiscrete"     Example 1 \n Connected,
    closures are dense, Bnd of Bnd(S) not = { }  `);LS := {{ }, X};
    LAT3abc(LS, `  Not-T0  N=3 `, 3);
```

X := { a, b, c }

Not-T0 N=3 "extremal indiscrete" Example 1

Connected, closures are dense, Bnd of Bnd(S) not = { }

LS := { { }, { a, b, c } }

Is LS a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ }	{ a, b, c }	{ a, b, c }	{ b, c }	{ a }	{ }
{ b }	{ }	{ }	{ a, b, c }	{ a, b, c }	{ a, c }	{ b }	{ }
{ c }	{ }	{ }	{ a, b, c }	{ a, b, c }	{ a, b }	{ c }	{ }
{ a, b }	{ }	{ }	{ a, b, c }	{ a, b, c }	{ a, b, c }	{ }	{ a, b }
{ a, c }	{ }	{ }	{ a, b, c }	{ a, b, c }	{ a, b, c }	{ }	{ a, c }
{ b, c }	{ }	{ }	{ a, b, c }	{ a, b, c }	{ a, b, c }	{ }	{ b, c }
{ a, b, c }	{ a, b, c }	{ }	{ }	{ a, b, c }	{ a, b, c }	{ }	{ a, b, c }

Closed-Open subsets of LS are = { { }, { a, b, c } }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({ a, b, c }, { a, b, c }, { a, b, c })

(10)

3. Examples of T0
(Disconnected or Connected) topologies for N=4,
X = {a,b,c,d}
Bnd(X) = { }

```
> print(`    `);X:={a,b,c,d};print(` N=4 T2 poset 1      "Extremal T0" ,
    Disconnected Discrete, Bnd of Bnd = { } for all subsets `);LS:={{},
  {a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,
  c,d},{b,c,d},{a,b,c,d}};LAT4abcd(LS,`LS =  N=4 Poset 1 Disconnected
  T2`,4):
```

X := { a, b, c, d }

N=4 T2 poset 1 "Extremal T0" , Disconnected Discrete, Bnd of Bnd = { } for all subsets

LS := { { }, { a }, { b }, { c }, { d }, { a, b }, { a, c }, { a, d }, { b, c }, { b, d }, { c, d }, { a, b, c }, { a, b, d }, { a, c, d }, { b, c, d }, { a, b, c, d } }

Is LS: a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = true

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ b, c, d }	{ }	{ a }	{ }	{ a }.Seg	{ }
{ b }	{ b }	{ a, c, d }	{ }	{ b }	{ }	{ b }.Seg	{ }
{ c }	{ c }	{ a, b, d }	{ }	{ c }	{ }	{ c }.Seg	{ }
{ d }	{ d }	{ a, b, c }	{ }	{ d }	{ }	{ d }.Seg	{ }
{ a, b }	{ a, b }	{ c, d }	{ }	{ a, b }	{ }	{ a, b }.Seg	{ }
{ a, c }	{ a, c }	{ b, d }	{ }	{ a, c }	{ }	{ a, c }.Seg	{ }
{ a, d }	{ a, d }	{ b, c }	{ }	{ a, d }	{ }	{ a, d }.Seg	{ }
{ b, c }	{ b, c }	{ a, d }	{ }	{ b, c }	{ }	{ b, c }.Seg	{ }
{ b, d }	{ b, d }	{ a, c }	{ }	{ b, d }	{ }	{ b, d }.Seg	{ }
{ c, d }	{ c, d }	{ a, b }	{ }	{ c, d }	{ }	{ c, d }.Seg	{ }
{ a, b, c }	{ a, b, c }	{ d }	{ }	{ a, b, c }	{ }	{ a, b, c }.Seg	{ }
{ a, b, d }	{ a, b, d }	{ c }	{ }	{ a, b, d }	{ }	{ a, b, d }.Seg	{ }
{ a, c, d }	{ a, c, d }	{ b }	{ }	{ a, c, d }	{ }	{ a, c, d }.Seg	{ }
{ b, c, d }	{ b, c, d }	{ a }	{ }	{ b, c, d }	{ }	{ b, c, d }.Seg	{ }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ }	{ a, b, c, d }.Seg	{ }

The Closed-Open subsets of LS are = { { }, { a }, { b }, { c }, { d }, { a, b }, { a, c }, { a, d }, { b, c }, { b, d }, { c, d }, { a, b, c }, { a, b, d }, { a, c, d }, { b, c, d }, { a, b, c, d } }

The {1,2,3,4} array of CLOSURE elements is = ({ a }, { b }, { c }, { d }) **(11)**

```
> print(`    `);X:={a,b,c,d};print(` N=4 T0 poset 2 ,    Disconnected,
  Bnd of Bnd(S) = { } if Bnd(S) = { } `);LS:={{},{b},{c},{d},{a,b},{b,c},
  {b,d},{c,d},{a,b,c},{a,b,d},{b,c,d},{a,b,c,d}};LAT4abcd(LS,`LS =  N=4
  Poset 2 DISconnected T0 topology`,4):
```

X := {a, b, c, d}

N=4 T0 poset 2 , Disconnected, Bnd of Bnd(S) = { } if Bnd(S) = { }

LS := { { }, {b}, {c}, {d}, {a, b}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {b, c, d}, {a, b, c, d} }

Is LS: a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a}	{a}	{ }	{ }	{ }
{b}	{b}	{c, d}	{a}	{a, b}	{a}	{b}	{ }
{c}	{c}	{a, b, d}	{ }	{c}	{ }	{c}.Seg	{ }
{d}	{d}	{a, b, c}	{ }	{d}	{ }	{d}.Seg	{ }
{a, b}	{a, b}	{c, d}	{ }	{a, b}	{a}	{ }	{a}
{a, c}	{c}	{b, d}	{a}	{a, c}	{ }	{ }	{ }
{a, d}	{d}	{b, c}	{a}	{a, d}	{ }	{ }	{ }
{b, c}	{b, c}	{d}	{a}	{a, b, c}	{a}	{b, c}	{ }
{b, d}	{b, d}	{c}	{a}	{a, b, d}	{a}	{b, d}	{ }
{c, d}	{c, d}	{a, b}	{ }	{c, d}	{ }	{c, d}.Seg	{ }
{a, b, c}	{a, b, c}	{d}	{ }	{a, b, c}	{a}	{ }	{a}
{a, b, d}	{a, b, d}	{c}	{ }	{a, b, d}	{a}	{ }	{a}
{a, c, d}	{c, d}	{b}	{a}	{a, c, d}	{ }	{ }	{ }
{b, c, d}	{b, c, d}	{ }	{a}	{a, b, c, d}	{a}	{b, c, d}	{ }
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a}	{ }	{a}

The Closed-Open subsets of LS are = { { }, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d}, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a}, {a, b}, {c}, {d})

```
> print(`    `);X:={a,b,c,d};print(` N=4 T0 poset 3     Kuratowski
  format,     DisConnected, Bnd of Bnd(S) usually not = { }`);LS:={{},{a}
  ,{c},{a,b},{a,c},{c,d},{a,b,c},{a,c,d},{a,b,c,d}};LAT4abcd(LS,`LS =  N=
  4 Poset 3 Disconnected T0`,4);print(`This Lattice Structure appears to
  be a KURATOWSKI T0POLOGY \n with a limit point operator that can be
  formulated interms of the exterior derivative, d,\n acting on exterior
  differential forms, A, F = dA, H = A^F, K = F^F. Substitute {A,F,H,K}
  for the symbol set {a,b,c,d}.  \n Then a = A, a 1-form, b = F=dA, a 2-
  form with dF=0,  c = A^F = H, a 3-form,   F^F, a 4-form. \n Note that d
  (d(S)) = 0 but Bnd(Bnd(S)) =/= 0.   d is an up operator: given A, F is
  determined as dA`):
```

X := {a, b, c, d}

N=4 T0 poset 3 Kuratowski format, DisConnected, Bnd of Bnd(S) usually not = { }

LS := { { }, {a}, {c}, {a, b}, {a, c}, {c, d}, {a, b, c}, {a, c, d}, {a, b, c, d} }

Is LS: a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c, d}	{b}	{a, b}	{b}	{a}	{ }
{b}	{ }	{a, c, d}	{b}	{b}	{ }	{ }	{ }
{c}	{c}	{a, b}	{d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a, b}	{c, d}	{ }	{a, b}	{b}	{ }	{b}
{a, c}	{a, c}	{ }	{b, d}	{a, b, c, d}	{b, d}	{a, c}	{ }
{a, d}	{a}	{c}	{b, d}	{a, b, d}	{b}	{a, d}	{ }
{b, c}	{c}	{a}	{b, d}	{b, c, d}	{d}	{b, c}	{ }
{b, d}	{ }	{a, c}	{b, d}	{b, d}	{ }	{ }	{ }
{c, d}	{c, d}	{a, b}	{ }	{c, d}	{d}	{ }	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{b, d}	{a, c}	{b}
{a, b, d}	{a, b}	{c}	{d}	{a, b, d}	{b}	{ }	{b}
{a, c, d}	{a, c, d}	{ }	{b}	{a, b, c, d}	{b, d}	{a, c}	{d}
{b, c, d}	{c, d}	{a}	{b}	{b, c, d}	{d}	{ }	{d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, d}	{ }	{b, d}

The Closed-Open subsets of LS are = { { }, {a, b}, {c, d}, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a, b}, {b}, {c, d}, {d})

This Lattice Structure appears to be a KURATOWSKI T0POLOGY (13)

with a limit point operator that can be formulated interms of the exterior derivative, d,
acting on exterior differential forms, A, F = dA, H = A^F, K = F^F. Substitute {A,F,H,K} for the symbol set {a,b,c,d}.
Then a = A, a 1-form, b = F=dA, a 2-form with dF=0, c = A^F = H, a 3-form, F^F, a 4-form.
Note that d(d(S)) = 0 but Bnd(Bnd(S)) =/= 0. d is an up operator: given A, F is determined as dA

```
> print(`    `);X:={a,b,c,d};print(` N=4 T0 poset 4 ,    Disconnected,
  Bnd of Bnd(S) = { } if Bnd(S)usually not = { }`);LS:={{},{a},{b},{a,b},
  {a,c},{a,d},{a,b,c},{a,b,d},{a,c,d},{a,b,c,d}};LAT4abcd(LS,`LS =  N=4
  Poset 3 Disconnected T0.`,4):
```

X := { a, b, c, d }

N=4 T0 poset 4 , Disconnected, Bnd of Bnd(S) = { } if Bnd(S)usually not = { }

LS := { { }, { a }, { b }, { a, b }, { a, c }, { a, d }, { a, b, c }, { a, b, d }, { a, c, d }, { a, b, c, d } }

Is LS: a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ b }	{ c, d }	{ a, c, d }	{ c, d }	{ a }	{ }
{ b }	{ b }	{ a, c, d }	{ }	{ b }	{ }	{ b }.Seg	{ }
{ c }	{ }	{ a, b, d }	{ c }	{ c }	{ }	{ }	{ }
{ d }	{ }	{ a, b, c }	{ d }	{ d }	{ }	{ }	{ }
{ a, b }	{ a, b }	{ }	{ c, d }	{ a, b, c, d }	{ c, d }	{ a, b }	{ }
{ a, c }	{ a, c }	{ b }	{ d }	{ a, c, d }	{ c, d }	{ a }	{ c }
{ a, d }	{ a, d }	{ b }	{ c }	{ a, c, d }	{ c, d }	{ a }	{ d }
{ b, c }	{ b }	{ a, d }	{ c }	{ b, c }	{ }	{ }	{ }
{ b, d }	{ b }	{ a, c }	{ d }	{ b, d }	{ }	{ }	{ }
{ c, d }	{ }	{ a, b }	{ c, d }	{ c, d }	{ }	{ }	{ }
{ a, b, c }	{ a, b, c }	{ }	{ d }	{ a, b, c, d }	{ c, d }	{ a, b }	{ c }
{ a, b, d }	{ a, b, d }	{ }	{ c }	{ a, b, c, d }	{ c, d }	{ a, b }	{ d }
{ a, c, d }	{ a, c, d }	{ b }	{ }	{ a, c, d }	{ c, d }	{ }	{ c, d }
{ b, c, d }	{ b }	{ a }	{ c, d }	{ b, c, d }	{ }	{ }	{ }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ c, d }	{ }	{ c, d }

The Closed-Open subsets of LS are = { { }, { b }, { a, c, d }, { a, b, c, d } }

The {1,2,3,4} array of CLOSURE elements is = ({ a, c, d }, { b }, { c }, { d })

(14)

```
> print(`    `);X:={a,b,c,d};print(` N=4 T0 poset 5 ,     Disconnected
  topology, Bnd of Bnd(S) usually not = { } `);LS:={{},{a},{b},{c},{a,b},
  {b,c},{a,b,c},{a,b,d},{a,b,c,d}};LAT4abcd(LS,`LS =  N=4 T0 but not a
  topology`,4):
```

X := {a, b, c, d}

N=4 T0 poset 5 , Disconnected topology, Bnd of Bnd(S) usually not = { }

LS := {{ }, {a}, {b}, {c}, {a, b}, {b, c}, {a, b, c}, {a, b, d}, {a, b, c, d}}

Is LS: a topology = false, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c}	{d}	{a, d}	{d}	{a}	{ }
{b}	{b}	{c}	{a, d}	{a, b, d}	{a, d}	{b}	{ }
{c}	{c}	{a, b, d}	{ }	{c}	{ }	{c}.Seg	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a, b}	{c}	{d}	{a, b, d}	{a, d}	{b}	{a}
{a, c}	{c}	{b}	{a, d}	{a, c, d}	{d}	{a, c}	{ }
{a, d}	{a}	{b, c}	{d}	{a, d}	{d}	{ }	{d}
{b, c}	{b, c}	{a}	{d}	{b, c, d}	{d}	{b, c}	{ }
{b, d}	{b}	{c}	{a, d}	{a, b, d}	{a, d}	{b}	{d}
{c, d}	{c}	{a, b}	{d}	{c, d}	{ }	{ }	{ }
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{d}	{a, b, c}	{ }
{a, b, d}	{a, b, d}	{c}	{ }	{a, b, d}	{a, d}	{ }	{a, d}
{a, c, d}	{c}	{b}	{a, d}	{a, c, d}	{d}	{ }	{d}
{b, c, d}	{b, c}	{a}	{d}	{b, c, d}	{d}	{ }	{d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{d}	{ }	{d}

The Closed-Open subsets of LS are = {{ }, {c}, {a, b, d}, {a, b, c, d}}

The {1,2,3,4} array of CLOSURE elements is = ({a, d}, {a, b, d}, {c}, {d})

(15)

```
> print(`    `);X:={a,b,c,d};print(` N=4 T0 poset 6 ,     Disconnected,
  Bnd of Bnd(S) = { } if Bnd(S) = { } `);LS:={{},{},{c},{d},{b,c},{c,d},
  {a,b,c},{b,c,d},{a,b,c,d}};LAT4abcd(LS,`LS =  N=4 Poset 6  T0`,4):
```

X := { a, b, c, d }

N=4 T0 poset 6 , Disconnected, Bnd of Bnd(S) = { } if Bnd(S) = { }

LS := { { }, { c }, { d }, { b, c }, { c, d }, { a, b, c }, { b, c, d }, { a, b, c, d } }

Is LS: a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ b, c, d }	{ a }	{ a }	{ }	{ }	{ }
{ b }	{ }	{ c, d }	{ a, b }	{ a, b }	{ a }	{ b }	{ }
{ c }	{ c }	{ d }	{ a, b }	{ a, b, c }	{ a, b }	{ c }	{ }
{ d }	{ d }	{ a, b, c }	{ }	{ d }	{ }	{ d }.Seg	{ }
{ a, b }	{ }	{ c, d }	{ a, b }	{ a, b }	{ a }	{ }	{ a }
{ a, c }	{ c }	{ d }	{ a, b }	{ a, b, c }	{ a, b }	{ c }	{ a }
{ a, d }	{ d }	{ b, c }	{ a }	{ a, d }	{ }	{ }	{ }
{ b, c }	{ b, c }	{ d }	{ a }	{ a, b, c }	{ a, b }	{ c }	{ b }
{ b, d }	{ d }	{ c }	{ a, b }	{ a, b, d }	{ a }	{ b, d }	{ }
{ c, d }	{ c, d }	{ }	{ a, b }	{ a, b, c, d }	{ a, b }	{ c, d }	{ }
{ a, b, c }	{ a, b, c }	{ d }	{ }	{ a, b, c }	{ a, b }	{ }	{ a, b }
{ a, b, d }	{ d }	{ c }	{ a, b }	{ a, b, d }	{ a }	{ }	{ a }
{ a, c, d }	{ c, d }	{ }	{ a, b }	{ a, b, c, d }	{ a, b }	{ c, d }	{ a }
{ b, c, d }	{ b, c, d }	{ }	{ a }	{ a, b, c, d }	{ a, b }	{ c, d }	{ b }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b }	{ }	{ a, b }

The Closed-Open subsets of LS are = { { }, { d }, { a, b, c }, { a, b, c, d } }

The {1,2,3,4} array of CLOSURE elements is = ({ a }, { a, b }, { a, b, c }, { d })

(16)

```
> print(`    `);X:={a,b,c,d};print(` N=4 T0 poset 7 ,    Connected, Bnd
  (X) = { }`);LS:={{},{},{c},{d},{a,d},{c,d},{a,c,d},{b,c,d},{a,b,c,d}};
  LAT4abcd(LS,`LS =  N=4 Poset 7  T0`,4):
```

X := { a, b, c, d }

N=4 T0 poset 7 , Connected, Bnd(X) = { }

LS := { { }, { c }, { d }, { a, d }, { c, d }, { a, c, d }, { b, c, d }, { a, b, c, d } }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ b, c, d }	{ a }	{ a }	{ }	{ }	{ }
{ b }	{ }	{ a, c, d }	{ b }	{ b }	{ }	{ }	{ }
{ c }	{ c }	{ a, d }	{ b }	{ b, c }	{ b }	{ c }	{ }
{ d }	{ d }	{ c }	{ a, b }	{ a, b, d }	{ a, b }	{ d }	{ }
{ a, b }	{ }	{ c, d }	{ a, b }	{ a, b }	{ }	{ }	{ }
{ a, c }	{ c }	{ d }	{ a, b }	{ a, b, c }	{ b }	{ a, c }	{ }
{ a, d }	{ a, d }	{ c }	{ b }	{ a, b, d }	{ a, b }	{ d }	{ a }
{ b, c }	{ c }	{ a, d }	{ b }	{ b, c }	{ b }	{ }	{ b }
{ b, d }	{ d }	{ c }	{ a, b }	{ a, b, d }	{ a, b }	{ d }	{ b }
{ c, d }	{ c, d }	{ }	{ a, b }	{ a, b, c, d }	{ a, b }	{ c, d }	{ }
{ a, b, c }	{ c }	{ d }	{ a, b }	{ a, b, c }	{ b }	{ }	{ b }
{ a, b, d }	{ a, d }	{ c }	{ b }	{ a, b, d }	{ a, b }	{ }	{ a, b }
{ a, c, d }	{ a, c, d }	{ }	{ b }	{ a, b, c, d }	{ a, b }	{ c, d }	{ a }
{ b, c, d }	{ b, c, d }	{ }	{ a }	{ a, b, c, d }	{ a, b }	{ c, d }	{ b }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b }	{ }	{ a, b }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

The {1,2,3,4} array of CLOSURE elements is = ({ a }, { b }, { b, c }, { a, b, d }) **(17)**

```
> print(`     `);X:={a,b,c,d};print(` N=4 T0 poset 8 ,     Connected, Bnd
  (X)= { }`);LS:={{},{},{b},{d},{b,d},{a,b,d},{b,c,d},{a,b,c,d}};LAT4abcd
  (LS,`LS =  N=4 Poset 8  Connected T0`,4);
```

X := { a, b, c, d }

N=4 T0 poset 8 , Connected, Bnd(X)= { }

LS := { { }, {b}, {d}, {b, d}, {a, b, d}, {b, c, d}, {a, b, c, d} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a}	{a}	{ }	{ }	{ }
{b}	{b}	{d}	{a, c}	{a, b, c}	{a, c}	{b}	{ }
{c}	{ }	{a, b, d}	{c}	{c}	{ }	{ }	{ }
{d}	{d}	{b}	{a, c}	{a, c, d}	{a, c}	{d}	{ }
{a, b}	{b}	{d}	{a, c}	{a, b, c}	{a, c}	{b}	{a}
{a, c}	{ }	{b, d}	{a, c}	{a, c}	{ }	{ }	{ }
{a, d}	{d}	{b}	{a, c}	{a, c, d}	{a, c}	{d}	{a}
{b, c}	{b}	{d}	{a, c}	{a, b, c}	{a, c}	{b}	{c}
{b, d}	{b, d}	{ }	{a, c}	{a, b, c, d}	{a, c}	{b, d}	{ }
{c, d}	{d}	{b}	{a, c}	{a, c, d}	{a, c}	{d}	{c}
{a, b, c}	{b}	{d}	{a, c}	{a, b, c}	{a, c}	{ }	{a, c}
{a, b, d}	{a, b, d}	{ }	{c}	{a, b, c, d}	{a, c}	{b, d}	{a}
{a, c, d}	{d}	{b}	{a, c}	{a, c, d}	{a, c}	{ }	{a, c}
{b, c, d}	{b, c, d}	{ }	{a}	{a, b, c, d}	{a, c}	{b, d}	{c}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, c}	{ }	{a, c}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a}, {a, b, c}, {c}, {a, c, d}) **(18)**

```
> print(`    `);X:={a,b,c,d};print(` N=4 T0 poset 9 ,     Connected, Bnd
  (X) = { }`);LS:={{},{a},{c},{},{a,c},{},{a,b,c},{},{a,b,c,d}};LAT4abcd
  (LS,`LS =  N=4 Poset 9,  Connected T0`,4):
```

X := { a, b, c, d }

N=4 T0 poset 9 , Connected, Bnd(X) = { }

LS := { { }, { a }, { c }, { a, c }, { a, b, c }, { a, b, c, d } }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ c }	{ b, d }	{ a, b, d }	{ b, d }	{ a }	{ }
{ b }	{ }	{ a, c }	{ b, d }	{ b, d }	{ d }	{ b }	{ }
{ c }	{ c }	{ a }	{ b, d }	{ b, c, d }	{ b, d }	{ c }	{ }
{ d }	{ }	{ a, b, c }	{ d }	{ d }	{ }	{ }	{ }
{ a, b }	{ a }	{ c }	{ b, d }	{ a, b, d }	{ b, d }	{ a }	{ b }
{ a, c }	{ a, c }	{ }	{ b, d }	{ a, b, c, d }	{ b, d }	{ a, c }	{ }
{ a, d }	{ a }	{ c }	{ b, d }	{ a, b, d }	{ b, d }	{ a }	{ d }
{ b, c }	{ c }	{ a }	{ b, d }	{ b, c, d }	{ b, d }	{ c }	{ b }
{ b, d }	{ }	{ a, c }	{ b, d }	{ b, d }	{ d }	{ }	{ d }
{ c, d }	{ c }	{ a }	{ b, d }	{ b, c, d }	{ b, d }	{ c }	{ d }
{ a, b, c }	{ a, b, c }	{ }	{ d }	{ a, b, c, d }	{ b, d }	{ a, c }	{ b }
{ a, b, d }	{ a }	{ c }	{ b, d }	{ a, b, d }	{ b, d }	{ }	{ b, d }
{ a, c, d }	{ a, c }	{ }	{ b, d }	{ a, b, c, d }	{ b, d }	{ a, c }	{ d }
{ b, c, d }	{ c }	{ a }	{ b, d }	{ b, c, d }	{ b, d }	{ }	{ b, d }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ b, d }	{ }	{ b, d }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

The {1,2,3,4} array of CLOSURE elements is = ({ a, b, d }, { b, d }, { b, c, d }, { d }) **(19)**

```
> print(`    `);X:={a,b,c,d};print(` N=4 T0 poset 10,    Connected, Bnd
  (X) = { }`);LS:={{},{a},{a,c},{a,b,c},{a,c,d},{a,b,c,d}};LAT4abcd(LS,
  `LS = N=4 poset 10 connected T0 `,4):
```

X := { a, b, c, d }

N=4 T0 poset 10, Connected, Bnd(X) = { }

LS := { { }, { a }, { a, c }, { a, b, c }, { a, c, d }, { a, b, c, d } }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ }
{ b }	{ }	{ a, c, d }	{ b }	{ b }	{ }	{ }	{ }
{ c }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ b, d }	{ c }	{ }
{ d }	{ }	{ a, b, c }	{ d }	{ d }	{ }	{ }	{ }
{ a, b }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ b }
{ a, c }	{ a, c }	{ }	{ b, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ c }
{ a, d }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ d }
{ b, c }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ b, d }	{ c }	{ b }
{ b, d }	{ }	{ a, c }	{ b, d }	{ b, d }	{ }	{ }	{ }
{ c, d }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ b, d }	{ c }	{ d }
{ a, b, c }	{ a, b, c }	{ }	{ d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ b, c }
{ a, b, d }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ b, d }
{ a, c, d }	{ a, c, d }	{ }	{ b }	{ a, b, c, d }	{ b, c, d }	{ a }	{ c, d }
{ b, c, d }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ b, d }	{ }	{ b, d }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ b, c, d }	{ }	{ b, c, d }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

The {1,2,3,4} array of CLOSURE elements is = ({ a, b, c, d }, { b }, { b, c, d }, { d })

(20)

```
> print(`    `);X:={a,b,c,d};print(` N=4 T0 poset 11 ,      Connected,
  Bnd(X)= { }`);LS:={{},{d},{b,d},{c,d},{b,c,d},{a,b,c,d}};LAT4abcd(LS,
  `LS = N=4 Poset 11 Connected T0 `,4):
```

X := { a, b, c, d }

N=4 T0 poset 11 , Connected, Bnd(X)= { }

LS := { { }, { d }, { b, d }, { c, d }, { b, c, d }, { a, b, c, d } }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ b, c, d }	{ a }	{ a }	{ }	{ }	{ }
{ b }	{ }	{ c, d }	{ a, b }	{ a, b }	{ a }	{ b }	{ }
{ c }	{ }	{ b, d }	{ a, c }	{ a, c }	{ a }	{ c }	{ }
{ d }	{ d }	{ }	{ a, b, c }	{ a, b, c, d }	{ a, b, c }	{ d }	{ }
{ a, b }	{ }	{ c, d }	{ a, b }	{ a, b }	{ a }	{ }	{ a }
{ a, c }	{ }	{ b, d }	{ a, c }	{ a, c }	{ a }	{ }	{ a }
{ a, d }	{ d }	{ }	{ a, b, c }	{ a, b, c, d }	{ a, b, c }	{ d }	{ a }
{ b, c }	{ }	{ d }	{ a, b, c }	{ a, b, c }	{ a }	{ b, c }	{ }
{ b, d }	{ b, d }	{ }	{ a, c }	{ a, b, c, d }	{ a, b, c }	{ d }	{ b }
{ c, d }	{ c, d }	{ }	{ a, b }	{ a, b, c, d }	{ a, b, c }	{ d }	{ c }
{ a, b, c }	{ }	{ d }	{ a, b, c }	{ a, b, c }	{ a }	{ }	{ a }
{ a, b, d }	{ b, d }	{ }	{ a, c }	{ a, b, c, d }	{ a, b, c }	{ d }	{ a, b }
{ a, c, d }	{ c, d }	{ }	{ a, b }	{ a, b, c, d }	{ a, b, c }	{ d }	{ a, c }
{ b, c, d }	{ b, c, d }	{ }	{ a }	{ a, b, c, d }	{ a, b, c }	{ d }	{ b, c }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c }	{ }	{ a, b, c }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

The {1,2,3,4} array of CLOSURE elements is = ({ a }, { a, b }, { a, c }, { a, b, c, d }) **(21)**

```
> print(`     `);X:={a,b,c,d};print(` N=4 T0 poset 12 ,       Connected,
  Bnd(X) = 0`);LS:={{},{b},{d},{},{a,b},{b,d},{},{a,b,d},{a,b,c,d}};
  LAT4abcd(LS,`LS =  N=4 Poset 12  Connected T0`,4):
```

X := { a, b, c, d }

N=4 T0 poset 12 , Connected, Bnd(X) = 0

LS := { { }, { b }, { d }, { a, b }, { b, d }, { a, b, d }, { a, b, c, d } }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ b, d }	{ a, c }	{ a, c }	{ c }	{ a }	{ }
{ b }	{ b }	{ d }	{ a, c }	{ a, b, c }	{ a, c }	{ b }	{ }
{ c }	{ }	{ a, b, d }	{ c }	{ c }	{ }	{ }	{ }
{ d }	{ d }	{ a, b }	{ c }	{ c, d }	{ c }	{ d }	{ }
{ a, b }	{ a, b }	{ d }	{ c }	{ a, b, c }	{ a, c }	{ b }	{ a }
{ a, c }	{ }	{ b, d }	{ a, c }	{ a, c }	{ c }	{ }	{ c }
{ a, d }	{ d }	{ b }	{ a, c }	{ a, c, d }	{ c }	{ a, d }	{ }
{ b, c }	{ b }	{ d }	{ a, c }	{ a, b, c }	{ a, c }	{ b }	{ c }
{ b, d }	{ b, d }	{ }	{ a, c }	{ a, b, c, d }	{ a, c }	{ b, d }	{ }
{ c, d }	{ d }	{ a, b }	{ c }	{ c, d }	{ c }	{ }	{ c }
{ a, b, c }	{ a, b }	{ d }	{ c }	{ a, b, c }	{ a, c }	{ }	{ a, c }
{ a, b, d }	{ a, b, d }	{ }	{ c }	{ a, b, c, d }	{ a, c }	{ b, d }	{ a }
{ a, c, d }	{ d }	{ b }	{ a, c }	{ a, c, d }	{ c }	{ }	{ c }
{ b, c, d }	{ b, d }	{ }	{ a, c }	{ a, b, c, d }	{ a, c }	{ b, d }	{ c }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, c }	{ }	{ a, c }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

The {1,2,3,4} array of CLOSURE elements is = ({ a, c }, { a, b, c }, { c }, { c, d })

(22)

```
> print(`    `);X:={a,b,c,d};print(` T0 poset 13 ,     Connected, Bnd(X)=
  { } `);LS:={{},{d},{},{ },{b,d},{c,d},{},{},{a,b,d},{b,c,d},{},{},{},
  {a,b,c,d}};LAT4abcd(LS,`LS =  N=4 Poset 13  Connected T0`,4):
```

X := { a, b, c, d }

T0 poset 13 , Connected, Bnd(X)= { }

LS := { { }, { d }, { b, d }, { c, d }, { a, b, d }, { b, c, d }, { a, b, c, d } }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ b, c, d }	{ a }	{ a }	{ }	{ }	{ }
{ b }	{ }	{ c, d }	{ a, b }	{ a, b }	{ a }	{ b }	{ }
{ c }	{ }	{ a, b, d }	{ c }	{ c }	{ }	{ }	{ }
{ d }	{ d }	{ }	{ a, b, c }	{ a, b, c, d }	{ a, b, c }	{ d }	{ }
{ a, b }	{ }	{ c, d }	{ a, b }	{ a, b }	{ a }	{ }	{ a }
{ a, c }	{ }	{ b, d }	{ a, c }	{ a, c }	{ }	{ }	{ }
{ a, d }	{ d }	{ }	{ a, b, c }	{ a, b, c, d }	{ a, b, c }	{ d }	{ a }
{ b, c }	{ }	{ d }	{ a, b, c }	{ a, b, c }	{ a }	{ b, c }	{ }
{ b, d }	{ b, d }	{ }	{ a, c }	{ a, b, c, d }	{ a, b, c }	{ d }	{ b }
{ c, d }	{ c, d }	{ }	{ a, b }	{ a, b, c, d }	{ a, b, c }	{ d }	{ c }
{ a, b, c }	{ }	{ d }	{ a, b, c }	{ a, b, c }	{ a }	{ }	{ a }
{ a, b, d }	{ a, b, d }	{ }	{ c }	{ a, b, c, d }	{ a, b, c }	{ d }	{ a, b }
{ a, c, d }	{ c, d }	{ }	{ a, b }	{ a, b, c, d }	{ a, b, c }	{ d }	{ a, c }
{ b, c, d }	{ b, c, d }	{ }	{ a }	{ a, b, c, d }	{ a, b, c }	{ d }	{ b, c }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c }	{ }	{ a, b, c }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

The {1,2,3,4} array of CLOSURE elements is = ({ a }, { a, b }, { c }, { a, b, c, d })

(23)

```
> print(`     `);X:={a,b,c,d};print(` T0 poset 14,     Connected, Bnd(X) =
  { } `);LS:={{},{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d},{a,b,c,d}}
  ;LAT4abcd(LS,`LS =  N=4 Poset 14   Connected T0`,4):
```

X := {a, b, c, d}

T0 poset 14, Connected, Bnd(X) = { }

LS := { { }, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}, {a, b, c, d} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{a, c, d}	{b}	{b}	{ }	{ }	{ }
{c}	{ }	{a, b, d}	{c}	{c}	{ }	{ }	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{a, c}	{ }	{b, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{a, d}	{ }	{b, c}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{a, d}	{b, c}	{b, c}	{ }	{ }	{ }
{b, d}	{ }	{a, c}	{b, d}	{b, d}	{ }	{ }	{ }
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{ }	{ }	{ }
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a, b, d}	{ }	{c}	{a, b, c, d}	{b, c, d}	{a}	{b, d}
{a, c, d}	{a, c, d}	{ }	{b}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{ }	{ }	{ }
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, c, d}	{ }	{b, c, d}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {b}, {c}, {d})

(24)

```
> print(`     `);X:={a,b,c,d};print(` T0 poset 15 ,     Connected, Bnd(X)
  = { }   `);LS:={{},{b},{c},{d},{b,c},{b,d},{c,d},{b,c,d},{a,b,c,d}};
  LAT4abcd(LS,`LS =  N=4 Poset 15, Connected T0 `,4):
```

X := {a, b, c, d}

T0 poset 15 , Connected, Bnd(X) = { }

LS := { { }, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}, {a, b, c, d} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a}	{a}	{ }	{ }	{ }
{b}	{b}	{c, d}	{a}	{a, b}	{a}	{b}	{ }
{c}	{c}	{b, d}	{a}	{a, c}	{a}	{c}	{ }
{d}	{d}	{b, c}	{a}	{a, d}	{a}	{d}	{ }
{a, b}	{b}	{c, d}	{a}	{a, b}	{a}	{ }	{a}
{a, c}	{c}	{b, d}	{a}	{a, c}	{a}	{ }	{a}
{a, d}	{d}	{b, c}	{a}	{a, d}	{a}	{ }	{a}
{b, c}	{b, c}	{d}	{a}	{a, b, c}	{a}	{b, c}	{ }
{b, d}	{b, d}	{c}	{a}	{a, b, d}	{a}	{b, d}	{ }
{c, d}	{c, d}	{b}	{a}	{a, c, d}	{a}	{c, d}	{ }
{a, b, c}	{b, c}	{d}	{a}	{a, b, c}	{a}	{ }	{a}
{a, b, d}	{b, d}	{c}	{a}	{a, b, d}	{a}	{ }	{a}
{a, c, d}	{c, d}	{b}	{a}	{a, c, d}	{a}	{ }	{a}
{b, c, d}	{b, c, d}	{ }	{a}	{a, b, c, d}	{a}	{b, c, d}	{ }
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a}	{ }	{a}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a}, {a, b}, {a, c}, {a, d})

```
> print(`    `);X:={a,b,c,d};print(` T0 poset 16 ,       Connected, Bnd
  (X) = { }  `);LS:={{},{},{},{d},{c,d},{b,c,d},{a,b,c,d}};LAT4abcd(LS,
  `LS = Poset 16  Connected T0`,4):
```

X := { a, b, c, d }

T0 poset 16 , Connected, Bnd(X) = { }

LS := { { }, { d }, { c, d }, { b, c, d }, { a, b, c, d } }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ b, c, d }	{ a }	{ a }	{ }	{ }	{ }
{ b }	{ }	{ c, d }	{ a, b }	{ a, b }	{ a }	{ b }	{ }
{ c }	{ }	{ d }	{ a, b, c }	{ a, b, c }	{ a, b }	{ c }	{ }
{ d }	{ d }	{ }	{ a, b, c }	{ a, b, c, d }	{ a, b, c }	{ d }	{ }
{ a, b }	{ }	{ c, d }	{ a, b }	{ a, b }	{ a }	{ }	{ a }
{ a, c }	{ }	{ d }	{ a, b, c }	{ a, b, c }	{ a, b }	{ c }	{ a }
{ a, d }	{ d }	{ }	{ a, b, c }	{ a, b, c, d }	{ a, b, c }	{ d }	{ a }
{ b, c }	{ }	{ d }	{ a, b, c }	{ a, b, c }	{ a, b }	{ c }	{ b }
{ b, d }	{ d }	{ }	{ a, b, c }	{ a, b, c, d }	{ a, b, c }	{ d }	{ b }
{ c, d }	{ c, d }	{ }	{ a, b }	{ a, b, c, d }	{ a, b, c }	{ d }	{ c }
{ a, b, c }	{ }	{ d }	{ a, b, c }	{ a, b, c }	{ a, b }	{ }	{ a, b }
{ a, b, d }	{ d }	{ }	{ a, b, c }	{ a, b, c, d }	{ a, b, c }	{ d }	{ a, b }
{ a, c, d }	{ c, d }	{ }	{ a, b }	{ a, b, c, d }	{ a, b, c }	{ d }	{ a, c }
{ b, c, d }	{ b, c, d }	{ }	{ a }	{ a, b, c, d }	{ a, b, c }	{ d }	{ b, c }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c }	{ }	{ a, b, c }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

The {1,2,3,4} array of CLOSURE elements is = ({ a }, { a, b }, { a, b, c }, { a, b, c, d }) **(26)**

4. 17 Examples of NOT T0 ,
Connected or Disconnected, topologies for N=4.
X = {a,b,c,d}
Some or All Singleton closures are not distinguishable.
Some indisinguishable subsets and their boundaries can be dense.

```
> print(`    `);X:={a,b,c,d};LS:={{},X};print(` Example 1  N=4  Not-T0
   "Extremal"\n InDiscrete, Connected, all subsets are dense and
  indistinguishable, Bnd of Bnd(S) = { }, `); LAT4abcd(LS,`Not T0` ,4)
  :print(`4 singletons have the same closures, Indiscrete  -  NoT-T0, but
  CONNECTED `):
```

X := { a, b, c, d }

LS := { { }, { a, b, c, d } }

Example 1 N=4 Not-T0 "Extremal"

InDiscrete, Connected, all subsets are dense and indistinguishable, Bnd of Bnd(S) = { },

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ }
{ b }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, c, d }	{ b }	{ }
{ c }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, d }	{ c }	{ }
{ d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c }	{ d }	{ }
{ a, b }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b }
{ a, c }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, c }
{ a, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, d }
{ b, c }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ b, c }
{ b, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ b, d }
{ c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ c, d }
{ a, b, c }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, c }
{ a, b, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, d }
{ a, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, c, d }
{ b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ b, c, d }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, c, d }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({ a, b, c, d }, { a, b, c, d }, { a, b, c, d }, { a, b, c, d })

4 singletons have the same closures, Indiscrete - NoT-T0, but CONNECTED

```
> print(`    `);X:={a,b,c,d};LS:={{},{a,b,c},X};print(` Example 2  N=4
  A Not-T0 topology \n     Connected,      Bnd of Bnd(S) = { } if Bnd(S)
  is dense or { }`);LAT4abcd(LS,` Example 2`,4):
```

X := {a, b, c, d}

LS := {{ }, {a, b, c}, {a, b, c, d}}

Example 2 N=4 A Not-T0 topology

Connected, Bnd of Bnd(S) = { } if Bnd(S) is dense or { }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{ }
{c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, c}
{a, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{b, c}
{b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{d}
{c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, d}	{c}	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c}
{a, b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, d}
{a, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, c, d}
{b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {a, b, c, d}, {a, b, c, d}, {d})

```
> print(`    `);X:={a,b,c,d};LS:={{},{a,b},X};print(`Example 3    N=4
  Not-T0 topology \n Connected , Bnd of Bnd(S) = { } if Bnd(S) is dense.
  `);LAT4abcd(LS,` NOT T0`,4):
```

X := {a, b, c, d}

LS := {{ }, {a, b}, {a, b, c, d}}

Example 3 N=4 Not-T0 topology

Connected , Bnd of Bnd(S) = { } if Bnd(S) is dense.

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c}
{b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{d}
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{c, d}	{ }	{c, d}
{a, b, c}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, d}
{a, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {a, b, c, d}, {c, d}, {c, d}) **(29)**

```
> print(`    `);X:={a,b,c,d};LS:={{},{a},X};print(`Example 4    A Not-T0
  topology \n Connected, Bnd of Bnd(S) is not { } , `);LAT4abcd(LS,` NOT
  T0,4`,4):
```

X := {a, b, c, d}

LS := {{ }, {a}, {a, b, c, d}}

Example 4 A Not-T0 topology

Connected, Bnd of Bnd(S) is not { } ,

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{ }
{c}	{ }	{a}	{b, c, d}	{b, c, d}	{b, d}	{c}	{ }
{d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c}	{d}	{ }
{a, b}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, c}
{b, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, d}
{c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{c, d}
{a, b, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, d}
{a, c, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, c, d}	{ }	{b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {b, c, d}, {b, c, d}, {b, c, d})

(30)

```
> print(`    `);X:={a,b,c,d};LS:={{},{a},{a,b},X};print(` Example 5    A
  Not-T0 topology \n Connected,    Bnd of Bnd(S) is not { }  `);LAT4abcd
  (LS,` NOT T0, but CONNECTED `,4):
```

X := {a, b, c, d}

LS := {{ }, {a}, {a, b}, {a, b, c, d}}

Example 5 A Not-T0 topology

Connected, Bnd of Bnd(S) is not { }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{c}
{b, d}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{d}
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{c, d}	{ }	{c, d}
{a, b, c}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, d}
{a, c, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{ }	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, c, d}	{ }	{b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {b, c, d}, {c, d}, {c, d})

(31)

```
> print(`    `);X:={a,b,c,d};LS:={{},{a},{a,b,c},X};print(` Example 6    A
  Not-T0 topology \n  Connected,    Bnd of Bnd(S) not = { }  `);LAT4abcd
  (LS,`2 singletons have duplicate closures. NOT T0, `,4):
```

X := { a, b, c, d }

LS := { { }, { a }, { a, b, c }, { a, b, c, d } }

Example 6 A Not-T0 topology

Connected, Bnd of Bnd(S) not = { }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ }
{ b }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ c, d }	{ b }	{ }
{ c }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ b, d }	{ c }	{ }
{ d }	{ }	{ a, b, c }	{ d }	{ d }	{ }	{ }	{ }
{ a, b }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ b }
{ a, c }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ c }
{ a, d }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ d }
{ b, c }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ b, c, d }	{ }	{ b, c }
{ b, d }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ c, d }	{ b }	{ d }
{ c, d }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ b, d }	{ c }	{ d }
{ a, b, c }	{ a, b, c }	{ }	{ d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ b, c }
{ a, b, d }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ b, d }
{ a, c, d }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ c, d }
{ b, c, d }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ b, c, d }	{ }	{ b, c, d }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ b, c, d }	{ }	{ b, c, d }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({ a, b, c, d }, { b, c, d }, { b, c, d }, { d })

```
> print(`    `);X:={a,b,c,d};LS:={{},{a,b},{a,b,c},X};print(` Example 7 A
  Not-T0 topology \n Connected Bnd of Bnd(S) not always= { }`);LAT4abcd
  (LS,` NOT T0 `,4):
```

X := { a, b, c, d }

LS := { { }, { a, b }, { a, b, c }, { a, b, c, d } }

Example 7 A Not-T0 topology

Connected Bnd of Bnd(S) not always= { }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ }
{ b }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, c, d }	{ b }	{ }
{ c }	{ }	{ a, b }	{ c, d }	{ c, d }	{ d }	{ c }	{ }
{ d }	{ }	{ a, b, c }	{ d }	{ d }	{ }	{ }	{ }
{ a, b }	{ a, b }	{ }	{ c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b }
{ a, c }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ c }
{ a, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ d }
{ b, c }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, c, d }	{ b }	{ c }
{ b, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, c, d }	{ b }	{ d }
{ c, d }	{ }	{ a, b }	{ c, d }	{ c, d }	{ d }	{ }	{ d }
{ a, b, c }	{ a, b, c }	{ }	{ d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, c }
{ a, b, d }	{ a, b }	{ }	{ c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, d }
{ a, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ c, d }
{ b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, c, d }	{ b }	{ c, d }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, c, d }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({ a, b, c, d }, { a, b, c, d }, { c, d }, { d }) **(33)**

```
> print(`    `);X:={a,b,c,d};LS:={{},{a},{b},{a,b},X};print(` Example 8
   A Not-T0 topology \n  Connected,    Bnd of Bnd(S) not = { }`);LAT4abcd
  (LS,` NOT T0, CONNECTED `,4):
```

X := {a, b, c, d}

LS := {{ }, {a}, {b}, {a, b}, {a, b, c, d}}

Example 8 A Not-T0 topology

Connected, Bnd of Bnd(S) not = { }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{ }
{b}	{b}	{a}	{c, d}	{b, c, d}	{c, d}	{b}	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{ }
{a, c}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{c}
{a, d}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{d}
{b, c}	{b}	{a}	{c, d}	{b, c, d}	{c, d}	{b}	{c}
{b, d}	{b}	{a}	{c, d}	{b, c, d}	{c, d}	{b}	{d}
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{c, d}	{ }	{c, d}
{a, b, c}	{a, b}	{ }	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{c}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{d}
{a, c, d}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{ }	{c, d}
{b, c, d}	{b}	{a}	{c, d}	{b, c, d}	{c, d}	{ }	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{c, d}	{ }	{c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, c, d}, {b, c, d}, {c, d}, {c, d})

```
> print(`    `);X:={a,b,c,d};LS:={{},{a},{a,b},{a,c,d},X};print(` Example
  9   A Not-T0 topology \n  Connected, Bnd of Bnd(S) not = {}  `);LAT4abcd
  (LS,` Not T0, `,4):
```

X := { a, b, c, d }

LS := { { }, { a }, { a, b }, { a, c, d }, { a, b, c, d } }

Example 9 A Not-T0 topology

Connected, Bnd of Bnd(S) not = {}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ }
{ b }	{ }	{ a, c, d }	{ b }	{ b }	{ }	{ }	{ }
{ c }	{ }	{ a, b }	{ c, d }	{ c, d }	{ d }	{ c }	{ }
{ d }	{ }	{ a, b }	{ c, d }	{ c, d }	{ c }	{ d }	{ }
{ a, b }	{ a, b }	{ }	{ c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ b }
{ a, c }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ c }
{ a, d }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ d }
{ b, c }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ d }	{ b, c }	{ }
{ b, d }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ c }	{ b, d }	{ }
{ c, d }	{ }	{ a, b }	{ c, d }	{ c, d }	{ c, d }	{ }	{ c, d }
{ a, b, c }	{ a, b }	{ }	{ c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ b, c }
{ a, b, d }	{ a, b }	{ }	{ c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ b, d }
{ a, c, d }	{ a, c, d }	{ }	{ b }	{ a, b, c, d }	{ b, c, d }	{ a }	{ c, d }
{ b, c, d }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ c, d }	{ }	{ c, d }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ b, c, d }	{ }	{ b, c, d }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({ a, b, c, d }, { b }, { c, d }, { c, d })

```
> print(`    `);X:={a,b,c,d};LS:={{},{a},{b,c},{a,b,c},X};print(` Example
  10    A Not T0topology \n Connected,    Bnd of Bnd(S) not = {} `);
  LAT4abcd(LS,`2 singletons have duplicate closures. NOT T0, from Not T0
  CONNECTED topology `,4):
```

X := { a, b, c, d }

LS := { { }, { a }, { b, c }, { a, b, c }, { a, b, c, d } }

Example 10 A Not T0topology

Connected, Bnd of Bnd(S) not = {}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ b, c }	{ d }	{ a, d }	{ d }	{ a }	{ }
{ b }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ c, d }	{ b }	{ }
{ c }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ b, d }	{ c }	{ }
{ d }	{ }	{ a, b, c }	{ d }	{ d }	{ }	{ }	{ }
{ a, b }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ c, d }	{ a, b }	{ }
{ a, c }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, d }	{ a, c }	{ }
{ a, d }	{ a }	{ b, c }	{ d }	{ a, d }	{ d }	{ }	{ d }
{ b, c }	{ b, c }	{ a }	{ d }	{ b, c, d }	{ b, c, d }	{ }	{ b, c }
{ b, d }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ c, d }	{ b }	{ d }
{ c, d }	{ }	{ a }	{ b, c, d }	{ b, c, d }	{ b, d }	{ c }	{ d }
{ a, b, c }	{ a, b, c }	{ }	{ d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ b, c }
{ a, b, d }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ c, d }	{ a, b }	{ d }
{ a, c, d }	{ a }	{ }	{ b, c, d }	{ a, b, c, d }	{ b, d }	{ a, c }	{ d }
{ b, c, d }	{ b, c }	{ a }	{ d }	{ b, c, d }	{ b, c, d }	{ }	{ b, c, d }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ b, c, d }	{ }	{ b, c, d }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({ a, d }, { b, c, d }, { b, c, d }, { d })

```
> print(`    `);X:={a,b,c,d};LS:={{},{a,b},{a,b,c},{a,b,d},X};print(`
  Example 11    A Not T0 topology \n Connected,    Bnd of Bnd(S) not = { }
  `);LAT4abcd(LS,` `,4):
```

X := { a, b, c, d }

LS := { { }, { a, b }, { a, b, c }, { a, b, d }, { a, b, c, d } }

Example 11 A Not T0 topology

Connected, Bnd of Bnd(S) not = { }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ }
{ b }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, c, d }	{ b }	{ }
{ c }	{ }	{ a, b, d }	{ c }	{ c }	{ }	{ }	{ }
{ d }	{ }	{ a, b, c }	{ d }	{ d }	{ }	{ }	{ }
{ a, b }	{ a, b }	{ }	{ c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b }
{ a, c }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ c }
{ a, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ d }
{ b, c }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, c, d }	{ b }	{ c }
{ b, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, c, d }	{ b }	{ d }
{ c, d }	{ }	{ a, b }	{ c, d }	{ c, d }	{ }	{ }	{ }
{ a, b, c }	{ a, b, c }	{ }	{ d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, c }
{ a, b, d }	{ a, b, d }	{ }	{ c }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, d }
{ a, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ c, d }
{ b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, c, d }	{ b }	{ c, d }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, c, d }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({ a, b, c, d }, { a, b, c, d }, { c }, { d }) **(37)**

```
[>
```

```
> print(`    `);X:={a,b,c,d};LS:={{},{a},{b,c,d},X};print(` Example 12
  A Not-T0 topology \n DISConnected,     Bnd of Bnd(S) = { }`);LAT4abcd
  (LS,` NOT T0`,4):
```

X := {a, b, c, d}

LS := {{ }, {a}, {b, c, d}, {a, b, c, d}}

Example 12 A Not-T0 topology

DISConnected, Bnd of Bnd(S) = { }

Is LS: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d}	{ }	{a}	{ }	{a}.Seg	{ }
{b}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{ }
{c}	{ }	{a}	{b, c, d}	{b, c, d}	{b, d}	{c}	{ }
{d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c}	{d}	{ }
{a, b}	{a}	{ }	{b, c, d}	{a, b, c, d}	{c, d}	{a, b}	{ }
{a, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{ }
{a, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{ }
{b, c}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, c}
{b, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, d}
{c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{c, d}
{a, b, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, d}
{a, c, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{b, c, d}	{a}	{ }	{b, c, d}	{b, c, d}	{ }	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, c, d}	{ }	{b, c, d}

The Closed-Open subsets of LS are = {{ }, {a}, {b, c, d}, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a}, {b, c, d}, {b, c, d}, {b, c, d})

```
> print(`    `);X:={a,b,c,d};LS:={{},{a},{b},{a,b}, {a,c,d},X};print
  (`Example 13    A Not-T0 topology \n   Disconnected,   Bnd of Bnd(S) not
  = { }`);LAT4abcd(LS,` `,4):
```

X := { a, b, c, d }

LS := { { }, { a }, { b }, { a, b }, { a, c, d }, { a, b, c, d } }

Example 13 A Not-T0 topology

Disconnected, Bnd of Bnd(S) not = { }

Is LS: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ b }	{ c, d }	{ a, c, d }	{ c, d }	{ a }	{ }
{ b }	{ b }	{ a, c, d }	{ }	{ b }	{ }	{ b }.Seg	{ }
{ c }	{ }	{ a, b }	{ c, d }	{ c, d }	{ d }	{ c }	{ }
{ d }	{ }	{ a, b }	{ c, d }	{ c, d }	{ c }	{ d }	{ }
{ a, b }	{ a, b }	{ }	{ c, d }	{ a, b, c, d }	{ c, d }	{ a, b }	{ }
{ a, c }	{ a }	{ b }	{ c, d }	{ a, c, d }	{ c, d }	{ a }	{ c }
{ a, d }	{ a }	{ b }	{ c, d }	{ a, c, d }	{ c, d }	{ a }	{ d }
{ b, c }	{ b }	{ a }	{ c, d }	{ b, c, d }	{ d }	{ b, c }	{ }
{ b, d }	{ b }	{ a }	{ c, d }	{ b, c, d }	{ c }	{ b, d }	{ }
{ c, d }	{ }	{ a, b }	{ c, d }	{ c, d }	{ c, d }	{ }	{ c, d }
{ a, b, c }	{ a, b }	{ }	{ c, d }	{ a, b, c, d }	{ c, d }	{ a, b }	{ c }
{ a, b, d }	{ a, b }	{ }	{ c, d }	{ a, b, c, d }	{ c, d }	{ a, b }	{ d }
{ a, c, d }	{ a, c, d }	{ b }	{ }	{ a, c, d }	{ c, d }	{ }	{ c, d }
{ b, c, d }	{ b }	{ a }	{ c, d }	{ b, c, d }	{ c, d }	{ }	{ c, d }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ c, d }	{ }	{ c, d }

The Closed-Open subsets of LS are = { { }, { b }, { a, c, d }, { a, b, c, d } }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({ a, c, d }, { b }, { c, d }, { c, d })

```
> print(`    `);X:={a,b,c,d};LS:={{},{a},{b,c},{a,b,c},{b,c,d},X};print(`
  Example 14   A Not-T0, topology \n Disconnected,     Bnd of Bnd(S) not
  = { }`);LAT4abcd(LS,` NOT T0`,4):
```

X := {a, b, c, d}

LS := {{ }, {a}, {b, c}, {a, b, c}, {b, c, d}, {a, b, c, d}}

Example 14 A Not-T0, topology
Disconnected, Bnd of Bnd(S) not = { }

Is LS: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d}	{ }	{a}	{ }	{a}.Seg	{ }
{b}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{ }
{c}	{ }	{a}	{b, c, d}	{b, c, d}	{b, d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a}	{ }	{b, c, d}	{a, b, c, d}	{c, d}	{a, b}	{ }
{a, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{ }
{a, d}	{a}	{b, c}	{d}	{a, d}	{ }	{ }	{ }
{b, c}	{b, c}	{a}	{d}	{b, c, d}	{b, c, d}	{ }	{b, c}
{b, d}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{d}
{c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, d}	{c}	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{c, d}	{a, b}	{d}
{a, c, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{d}
{b, c, d}	{b, c, d}	{a}	{ }	{b, c, d}	{b, c, d}	{ }	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, c, d}	{ }	{b, c, d}

The Closed-Open subsets of LS are = {{ }, {a}, {b, c, d}, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a}, {b, c, d}, {b, c, d}, {d}) **(40)**

```
>
>
```

```
> print(`    `); X := {a, b, c, d}; LS := {{ }, {a, b}, {c, d}, X};
      print(Example 15   A Not−T0 topology \n Disconnected ,      Bnd of  Bnd(S) = { });
      LAT4abcd(LS, ` `, 4) :
```

X := {a, b, c, d}

LS := {{ }, {a, b}, {c, d}, {a, b, c, d}}

Example 15 A Not-T0 topology

Disconnected , Bnd of Bnd(S) = { }

Is LS: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{c, d}	{a, b}	{a, b}	{b}	{a}	{ }
{b}	{ }	{c, d}	{a, b}	{a, b}	{a}	{b}	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{c, d}	{ }	{a, b}	{a, b}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{ }
{a, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{ }
{b, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, d}	{b, c}	{ }
{b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c}	{b, d}	{ }
{c, d}	{c, d}	{a, b}	{ }	{c, d}	{c, d}	{ }	{c, d}
{a, b, c}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, b, d}	{c}	{a, b}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, b, c}	{d}	{a, b}
{a, c, d}	{c, d}	{ }	{a, b}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{c, d}	{ }	{a, b}	{a, b, c, d}	{a, c, d}	{b}	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b}, {c, d}, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b}, {a, b}, {c, d}, {c, d}) **(41)**

```
>
>
```

```
> print(`  `); X := {a, b, c, d}; LS := {{ }, {a}, {a, b}, {c, d}, {a, c, d}, X};
    print(`Example 16      A Not-T0 topology \n Disconnected,      Bnd of Bnd(S) not = { }   `);
    LAT4abcd(LS, ` `, 4) :
```

X := {a, b, c, d}

LS := {{ }, {a}, {a, b}, {c, d}, {a, c, d}, {a, b, c, d}}

Example 16 A Not-T0 topology

Disconnected, Bnd of Bnd(S) not = { }

Is LS: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c, d}	{b}	{a, b}	{b}	{a}	{ }
{b}	{ }	{a, c, d}	{b}	{b}	{ }	{ }	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{c, d}	{ }	{a, b}	{b}	{ }	{b}
{a, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{ }
{a, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{ }
{b, c}	{ }	{a}	{b, c, d}	{b, c, d}	{d}	{b, c}	{ }
{b, d}	{ }	{a}	{b, c, d}	{b, c, d}	{c}	{b, d}	{ }
{c, d}	{c, d}	{a, b}	{ }	{c, d}	{c, d}	{ }	{c, d}
{a, b, c}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, d}	{a, c}	{b}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, c}	{a, d}	{b}
{a, c, d}	{a, c, d}	{ }	{b}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{c, d}	{a}	{b}	{b, c, d}	{c, d}	{ }	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, c, d}	{ }	{b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b}, {c, d}, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b}, {b}, {c, d}, {c, d})

(42)

```
>
```

```
> print(`  `); X := {a, b, c, d}; LS := {{ }, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}, X};
    print(`Example 17      A Not-T0 topology \n Disconnected,      Bnd of Bnd(S) = { }  `);
    LAT4abcd(LS, ` `, 4) :
```

$$X := \{a, b, c, d\}$$

$$LS := \{\{ \}, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$$

Example 17 A Not-T0 topology
Disconnected, Bnd of Bnd(S) = { }

Is LS: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d}	{ }	{a}	{ }	{a}.Seg	{ }
{b}	{b}	{a, c, d}	{ }	{b}	{ }	{b}.Seg	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{c, d}	{ }	{a, b}	{ }	{a, b}.Seg	{ }
{a, c}	{a}	{b}	{c, d}	{a, c, d}	{d}	{a, c}	{ }
{a, d}	{a}	{b}	{c, d}	{a, c, d}	{c}	{a, d}	{ }
{b, c}	{b}	{a}	{c, d}	{b, c, d}	{d}	{b, c}	{ }
{b, d}	{b}	{a}	{c, d}	{b, c, d}	{c}	{b, d}	{ }
{c, d}	{c, d}	{a, b}	{ }	{c, d}	{c, d}	{ }	{c, d}
{a, b, c}	{a, b}	{ }	{c, d}	{a, b, c, d}	{d}	{a, b, c}	{ }
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{c}	{a, b, d}	{ }
{a, c, d}	{a, c, d}	{b}	{ }	{a, c, d}	{c, d}	{ }	{c, d}
{b, c, d}	{b, c, d}	{a}	{ }	{b, c, d}	{c, d}	{ }	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{c, d}	{ }	{c, d}

The Closed-Open subsets of LS are = {{ }, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a}, {b}, {c, d}, {c, d}) **(43)**

```
>
```

5. Examples of Embedding N=3 T0 (Disconnected or Connected) topologies into N=4, to produce N=4 T0 Connected topologies.

```
> print(`    `);X:={a,b,c,d};LS:={{},{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c},
  X};LAT4abcd(LS,`Embed N=3 Disconnected T0 poset 1 into 4D to give N=4
  poset 15, T0 connected topology `,4):
```

$$X := \{a, b, c, d\}$$

$$LS := \{\{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, c, d\}\}$$

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c}	{d}	{a, d}	{d}	{a}	{ }
{b}	{b}	{a, c}	{d}	{b, d}	{d}	{b}	{ }
{c}	{c}	{a, b}	{d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a, b}	{c}	{d}	{a, b, d}	{d}	{a, b}	{ }
{a, c}	{a, c}	{b}	{d}	{a, c, d}	{d}	{a, c}	{ }
{a, d}	{a}	{b, c}	{d}	{a, d}	{d}	{ }	{d}
{b, c}	{b, c}	{a}	{d}	{b, c, d}	{d}	{b, c}	{ }
{b, d}	{b}	{a, c}	{d}	{b, d}	{d}	{ }	{d}
{c, d}	{c}	{a, b}	{d}	{c, d}	{d}	{ }	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{d}	{a, b, c}	{ }
{a, b, d}	{a, b}	{c}	{d}	{a, b, d}	{d}	{ }	{d}
{a, c, d}	{a, c}	{b}	{d}	{a, c, d}	{d}	{ }	{d}
{b, c, d}	{b, c}	{a}	{d}	{b, c, d}	{d}	{ }	{d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{d}	{ }	{d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

The {1,2,3,4} array of CLOSURE elements is = ({a, d}, {b, d}, {c, d}, {d})

(44)

```
> print(`  `);X:={a,b,c,d};LS:={{},{},{b},{c},{a,b},{},{b,c},{a,b,c},X};
  LAT4abcd(LS,`Embed N=3 Disconnected T0 poset 2 into 4D to give N=4 poset
  12;  T0 Connected topology `,4):
```

$$X := \{a, b, c, d\}$$

$$LS := \{\{\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, c, d\}\}$$

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c}	{a, d}	{a, d}	{d}	{a}	{ }
{b}	{b}	{c}	{a, d}	{a, b, d}	{a, d}	{b}	{ }
{c}	{c}	{a, b}	{d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a, b}	{c}	{d}	{a, b, d}	{a, d}	{b}	{a}
{a, c}	{c}	{b}	{a, d}	{a, c, d}	{d}	{a, c}	{ }
{a, d}	{ }	{b, c}	{a, d}	{a, d}	{d}	{ }	{d}
{b, c}	{b, c}	{ }	{a, d}	{a, b, c, d}	{a, d}	{b, c}	{ }
{b, d}	{b}	{c}	{a, d}	{a, b, d}	{a, d}	{b}	{d}
{c, d}	{c}	{a, b}	{d}	{c, d}	{d}	{ }	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{a, d}	{b, c}	{a}
{a, b, d}	{a, b}	{c}	{d}	{a, b, d}	{a, d}	{ }	{a, d}
{a, c, d}	{c}	{b}	{a, d}	{a, c, d}	{d}	{ }	{d}
{b, c, d}	{b, c}	{ }	{a, d}	{a, b, c, d}	{a, d}	{b, c}	{d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, d}	{ }	{a, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

The {1,2,3,4} array of CLOSURE elements is = ({a, d}, {a, b, d}, {c, d}, {d}) **(45)**

```
> print(`    `);X:={a,b,c,d};LS:={{},{},{b},{c},{},{},{b,c},{a,b,c},X};
  LAT4abcd(LS,` Embed N=3 Connected T0 poset3 into 4D to give N=4 poset 9;
   T0 Connected topology  `,4):
```

X := { a, b, c, d }

LS := { { }, { b }, { c }, { b, c }, { a, b, c }, { a, b, c, d } }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ b, c }	{ a, d }	{ a, d }	{ d }	{ a }	{ }
{ b }	{ b }	{ c }	{ a, d }	{ a, b, d }	{ a, d }	{ b }	{ }
{ c }	{ c }	{ b }	{ a, d }	{ a, c, d }	{ a, d }	{ c }	{ }
{ d }	{ }	{ a, b, c }	{ d }	{ d }	{ }	{ }	{ }
{ a, b }	{ b }	{ c }	{ a, d }	{ a, b, d }	{ a, d }	{ b }	{ a }
{ a, c }	{ c }	{ b }	{ a, d }	{ a, c, d }	{ a, d }	{ c }	{ a }
{ a, d }	{ }	{ b, c }	{ a, d }	{ a, d }	{ d }	{ }	{ d }
{ b, c }	{ b, c }	{ }	{ a, d }	{ a, b, c, d }	{ a, d }	{ b, c }	{ }
{ b, d }	{ b }	{ c }	{ a, d }	{ a, b, d }	{ a, d }	{ b }	{ d }
{ c, d }	{ c }	{ b }	{ a, d }	{ a, c, d }	{ a, d }	{ c }	{ d }
{ a, b, c }	{ a, b, c }	{ }	{ d }	{ a, b, c, d }	{ a, d }	{ b, c }	{ a }
{ a, b, d }	{ b }	{ c }	{ a, d }	{ a, b, d }	{ a, d }	{ }	{ a, d }
{ a, c, d }	{ c }	{ b }	{ a, d }	{ a, c, d }	{ a, d }	{ }	{ a, d }
{ b, c, d }	{ b, c }	{ }	{ a, d }	{ a, b, c, d }	{ a, d }	{ b, c }	{ d }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, d }	{ }	{ a, d }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

The {1,2,3,4} array of CLOSURE elements is = ({ a, d }, { a, b, d }, { a, c, d }, { d })

```
> print(`     `);X:={a,b,c,d};LS:={{},{},{b},{},{a,b},{},{b,c},{a,b,c},X};
  LAT4abcd(LS,` Embed N=3 Connected T0 poset 4 into 4D to give N=4 poset
  11 ,T0 Connected topology `,4):
```

X := {a, b, c, d}

LS := {{ }, {b}, {a, b}, {b, c}, {a, b, c}, {a, b, c, d}}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c}	{a, d}	{a, d}	{d}	{a}	{ }
{b}	{b}	{ }	{a, c, d}	{a, b, c, d}	{a, c, d}	{b}	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, c, d}	{b}	{a}
{a, c}	{ }	{b}	{a, c, d}	{a, c, d}	{d}	{a, c}	{ }
{a, d}	{ }	{b, c}	{a, d}	{a, d}	{d}	{ }	{d}
{b, c}	{b, c}	{ }	{a, d}	{a, b, c, d}	{a, c, d}	{b}	{c}
{b, d}	{b}	{ }	{a, c, d}	{a, b, c, d}	{a, c, d}	{b}	{d}
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{d}	{ }	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{a, c, d}	{b}	{a, c}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, c, d}	{b}	{a, d}
{a, c, d}	{ }	{b}	{a, c, d}	{a, c, d}	{d}	{ }	{d}
{b, c, d}	{b, c}	{ }	{a, d}	{a, b, c, d}	{a, c, d}	{b}	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, c, d}	{ }	{a, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

The {1,2,3,4} array of CLOSURE elements is = ({a, d}, {a, b, c, d}, {c, d}, {d})

(47)

```
> ;
```

```
> print(`    `);X:={a,b,c,d};LS:={{},{},{b},{},{},{},{a,b},{a,b,c},X};
  LAT4abcd(LS,` Embed N=3 Connected T0 poset 5 into 4D to give N=4 poset
  16 ,T0 Connected topology `,4):
```

X := {a, b, c, d}

LS := {{ }, {b}, {a, b}, {a, b, c}, {a, b, c, d}}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b}	{a, c, d}	{a, c, d}	{c, d}	{a}	{ }
{b}	{b}	{ }	{a, c, d}	{a, b, c, d}	{a, c, d}	{b}	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, c, d}	{b}	{a}
{a, c}	{ }	{b}	{a, c, d}	{a, c, d}	{c, d}	{a}	{c}
{a, d}	{ }	{b}	{a, c, d}	{a, c, d}	{c, d}	{a}	{d}
{b, c}	{b}	{ }	{a, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c}
{b, d}	{b}	{ }	{a, c, d}	{a, b, c, d}	{a, c, d}	{b}	{d}
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{d}	{ }	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{a, c, d}	{b}	{a, c}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, c, d}	{b}	{a, d}
{a, c, d}	{ }	{b}	{a, c, d}	{a, c, d}	{c, d}	{ }	{c, d}
{b, c, d}	{b}	{ }	{a, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, c, d}	{ }	{a, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

The {1,2,3,4} array of CLOSURE elements is = ({a, c, d}, {a, b, c, d}, {c, d}, {d}) **(48)**

6. Examples of N=3 NOT T0 connected or disconnected topologies which can be embedded in N=4 as Connected NOT T0 topologies

```
> print(`  `);X:={a,b,c,d};LS:={{},{},{},{},{},{},{},{a,b,c},X};LAT4abcd
  (LS,`The N=3 Connected Indiscrete Topology is NOT T0 - but embeds as a
  Connected NOT T0 topology in N=4  Ex #1`,4):
```

$$X := \{a, b, c, d\}$$

$$LS := \{\{\}, \{a, b, c\}, \{a, b, c, d\}\}$$

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{ }
{c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, c}
{a, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{b, c}
{b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{d}
{c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, d}	{c}	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c}
{a, b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, d}
{a, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, c, d}
{b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {a, b, c, d}, {a, b, c, d}, {d}) **(49)**

```
> print( `    `);X:={a,b,c,d};LS:={{},{},{c},{},{},{},{},{a,b,c},X};
  LAT4abcd(LS,`A N=3 Connected Topology that is not T0, - but embeds as a
  Connected NOT T0 topology in N=4, Ex #2`,4):
```

X := {a, b, c, d}

LS := {{ }, {c}, {a, b, c}, {a, b, c, d}}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{c}	{a, b, d}	{a, b, d}	{b, d}	{a}	{ }
{b}	{ }	{c}	{a, b, d}	{a, b, d}	{a, d}	{b}	{ }
{c}	{c}	{ }	{a, b, d}	{a, b, c, d}	{a, b, d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{ }	{c}	{a, b, d}	{a, b, d}	{a, b, d}	{ }	{a, b}
{a, c}	{c}	{ }	{a, b, d}	{a, b, c, d}	{a, b, d}	{c}	{a}
{a, d}	{ }	{c}	{a, b, d}	{a, b, d}	{b, d}	{a}	{d}
{b, c}	{c}	{ }	{a, b, d}	{a, b, c, d}	{a, b, d}	{c}	{b}
{b, d}	{ }	{c}	{a, b, d}	{a, b, d}	{a, d}	{b}	{d}
{c, d}	{c}	{ }	{a, b, d}	{a, b, c, d}	{a, b, d}	{c}	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{a, b, d}	{c}	{a, b}
{a, b, d}	{ }	{c}	{a, b, d}	{a, b, d}	{a, b, d}	{ }	{a, b, d}
{a, c, d}	{c}	{ }	{a, b, d}	{a, b, c, d}	{a, b, d}	{c}	{a, d}
{b, c, d}	{c}	{ }	{a, b, d}	{a, b, c, d}	{a, b, d}	{c}	{b, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, d}	{ }	{a, b, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b, d}, {a, b, d}, {a, b, c, d}, {d})

(50)

```
> print(`  `);X:={a,b,c,d};LS:={{},{},{},{a,b},{},{},{},{a,b,c},X};
  LAT4abcd(LS,`A N=3 Connected Topology that is not T0 - but embeds as a
  Connected NOT T0 topology in N=4, Ex #3`,4):
```

$$X := \{a, b, c, d\}$$

$$LS := \{\{\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\}$$

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c}
{b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{d}
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{d}	{ }	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, d}
{a, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {a, b, c, d}, {c, d}, {d}) **(51)**

```
> print(` `);X:={a,b,c,d};LS:={{},{},{c},{a,b},{},{},{},{a,b,c},X};
  LAT4abcd(LS,`A  N=3 Disconnected Topology that is not T0  - but embeds
  as a Connected NOT T0 topology in N=4, Ex #4`,4);
```

X := {a, b, c, d}

LS := {{ }, {c}, {a, b}, {a, b, c}, {a, b, c, d}}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{c}	{a, b, d}	{a, b, d}	{b, d}	{a}	{ }
{b}	{ }	{c}	{a, b, d}	{a, b, d}	{a, d}	{b}	{ }
{c}	{c}	{a, b}	{d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a, b}	{c}	{d}	{a, b, d}	{a, b, d}	{ }	{a, b}
{a, c}	{c}	{ }	{a, b, d}	{a, b, c, d}	{b, d}	{a, c}	{ }
{a, d}	{ }	{c}	{a, b, d}	{a, b, d}	{b, d}	{a}	{d}
{b, c}	{c}	{ }	{a, b, d}	{a, b, c, d}	{a, d}	{b, c}	{ }
{b, d}	{ }	{c}	{a, b, d}	{a, b, d}	{a, d}	{b}	{d}
{c, d}	{c}	{a, b}	{d}	{c, d}	{d}	{ }	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{a, b, d}	{c}	{a, b}
{a, b, d}	{a, b}	{c}	{d}	{a, b, d}	{a, b, d}	{ }	{a, b, d}
{a, c, d}	{c}	{ }	{a, b, d}	{a, b, c, d}	{b, d}	{a, c}	{d}
{b, c, d}	{c}	{ }	{a, b, d}	{a, b, c, d}	{a, d}	{b, c}	{d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, d}	{ }	{a, b, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b, d}, {a, b, d}, {c, d}, {d})

(52)

```
> :
```

7. Examples of Embedding N=3 T0 (Disconnected or Connected) topologies into N=5, to produce NOT T0 Connected topologies in N=5.

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c},
  X};LAT5abcd(LS,`Embed N=3 Disconnected T0 poset 1 into 5D to give a N=5
  Connected NOT T0 topology `,5):
```

X := {a, b, c, d, s}

LS := {{ }, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}, {a, b, c, d, s}}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c}	{d, s}	{a, d, s}	{d, s}	{a}	{ }
{b}	{b}	{a, c}	{d, s}	{b, d, s}	{d, s}	{b}	{ }
{c}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{a, b}	{c}	{d, s}	{a, b, d, s}	{d, s}	{a, b}	{ }
{a, c}	{a, c}	{b}	{d, s}	{a, c, d, s}	{d, s}	{a, c}	{ }
{a, d}	{a}	{b, c}	{d, s}	{a, d, s}	{d, s}	{a}	{d}
{a, s}	{a}	{b, c}	{d, s}	{a, d, s}	{d, s}	{a}	{s}
{b, c}	{b, c}	{a}	{d, s}	{b, c, d, s}	{d, s}	{b, c}	{ }
{b, d}	{b}	{a, c}	{d, s}	{b, d, s}	{d, s}	{b}	{d}
{b, s}	{b}	{a, c}	{d, s}	{b, d, s}	{d, s}	{b}	{s}
{c, d}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{d, s}	{a, b, c}	{ }
{a, b, d}	{a, b}	{c}	{d, s}	{a, b, d, s}	{d, s}	{a, b}	{d}
{a, b, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{d, s}	{a, b}	{s}
{a, c, d}	{a, c}	{b}	{d, s}	{a, c, d, s}	{d, s}	{a, c}	{d}
{a, c, s}	{a, c}	{b}	{d, s}	{a, c, d, s}	{d, s}	{a, c}	{s}
{a, d, s}	{a}	{b, c}	{d, s}	{a, d, s}	{d, s}	{a}	{d, s}
{b, c, d}	{b, c}	{a}	{d, s}	{b, c, d, s}	{d, s}	{b, c}	{d}
{b, c, s}	{b, c}	{a}	{d, s}	{b, c, d, s}	{d, s}	{b, c}	{s}
{b, d, s}	{b}	{a, c}	{d, s}	{b, d, s}	{d, s}	{b}	{d, s}
{c, d, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{d, s}	{a, b, c}	{d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{d, s}	{a, b, c}	{s}
{a, b, d, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{d, s}	{a, b}	{d, s}
{a, c, d, s}	{a, c}	{b}	{d, s}	{a, c, d, s}	{d, s}	{a, c}	{d, s}
{b, c, d, s}	{b, c}	{a}	{d, s}	{b, c, d, s}	{d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{d, s}	{a, b, c}	{d, s}

```
> print(` `);X:={a,b,c,d,s};X:={a,b,c,d,s};LS:={{},{},{b},{c},{a,b},{},
  {b,c},{a,b,c},X};LAT5abcd(LS,`Embed N=3 T0 Disconnected poset 2 into 5D
  to give a N=5 Connected NOT T0 topology `,5):
```

LS := { { }, {b}, {c}, {a, b}, {b, c}, {a, b, c}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{ }
{b}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{ }
{c}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, d, s}	{b}	{a}
{a, c}	{c}	{b}	{a, d, s}	{a, c, d, s}	{d, s}	{a, c}	{ }
{a, d}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d}
{a, s}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{s}
{b, c}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{ }
{b, d}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d}
{b, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{s}
{c, d}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a}
{a, b, d}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d}
{a, b, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, s}
{a, c, d}	{c}	{b}	{a, d, s}	{a, c, d, s}	{d, s}	{a, c}	{d}
{a, c, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{d, s}	{a, c}	{s}
{a, d, s}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d, s}
{b, c, d}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d}
{b, c, s}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{s}
{b, d, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d, s}
{c, d, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, s}
{a, b, d, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d, s}
{a, c, d, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{d, s}	{a, c}	{d, s}
{b, c, d, s}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS:={{},{},{b},{c},{},{},{b,c},{a,b,c},X};
  LAT5abcd(LS,` Embed N=3 Connected T0 poset3 into 5D to give a N=5
  Connected NOT T0 topology `,5):
```

X := {a, b, c, d, s}

LS := { { }, {b}, {c}, {b, c}, {a, b, c}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{ }
{b}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{ }
{c}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a}
{a, c}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a}
{a, d}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d}
{a, s}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{s}
{b, c}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{ }
{b, d}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d}
{b, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{s}
{c, d}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{d}
{c, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a}
{a, b, d}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d}
{a, b, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, s}
{a, c, d}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, d}
{a, c, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, s}
{a, d, s}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d, s}
{b, c, d}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d}
{b, c, s}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{s}
{b, d, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d, s}
{c, d, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, s}
{a, b, d, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d, s}
{a, c, d, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, d, s}
{b, c, d, s}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS:={{},{},{b},{},{a,b},{},{b,c},{a,b,c},X};
  LAT5abcd(LS,` Embed N=3 Connected T0 poset 4 into 5D to give a N=5
  Connected NOT T0 topology `,5):
```

X := {a, b, c, d, s}

LS := { { }, {b}, {a, b}, {b, c}, {a, b, c}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{ }
{b}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{ }
{c}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a}
{a, c}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{d, s}	{a, c}	{ }
{a, d}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d}
{a, s}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{s}
{b, c}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c}
{b, d}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c}
{a, b, d}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, d}
{a, b, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, s}
{a, c, d}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{d, s}	{a, c}	{d}
{a, c, s}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{d, s}	{a, c}	{s}
{a, d, s}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d, s}
{b, c, d}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d}
{b, c, s}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, s}
{b, d, s}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, s}
{a, b, d, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, d, s}
{a, c, d, s}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{d, s}	{a, c}	{d, s}
{b, c, d, s}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS:={{},{},{b},{},{},{},{a,b},{a,b,c},X};
  LAT5abcd(LS,` Embed N=3 T0 Connected poset 5 into 5D to give a N=5
  Connected NOT T0 topology `,5):
```

X := {a, b, c, d, s}

LS := { { }, {b}, {a, b}, {a, b, c}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{ }
{b}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{ }
{c}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a}
{a, c}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c}
{a, d}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d}
{a, s}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{s}
{b, c}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c}
{b, d}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c}
{a, b, d}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, d}
{a, b, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, s}
{a, c, d}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d}
{a, c, s}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, s}
{a, d, s}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d, s}
{b, c, d}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d}
{b, c, s}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, s}
{b, d, s}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, s}
{a, b, d, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, d, s}
{a, c, d, s}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, d, s}

Some singletons have the same closure and cannot be distinguished

L> :

8. Examples of N=3 NOT T0 connected or disconnected topologies which can be embedded in N=5 as Connected NOT T0 topologies

```
> print(` `);X:={a,b,c,d,s};LS:={{},{},{},{},{},{},{},{a,b,c},X};LAT5abcd
  (LS,`The Connected Indiscrete Topology is NOT T0 - but embeds as a
  Connected NOT T0 topology in N=5  Ex #1`,5):
```

X := {a, b, c, d, s}

LS := {{ }, {a, b, c}, {a, b, c, d, s}}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{ }
{b}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{ }
{c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c}
{a, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c}
{b, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d}
{c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c}
{a, b, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d}
{a, b, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, s}
{a, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c, d}
{a, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c, s}
{a, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c, d}
{b, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c, s}
{b, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, s}
{a, b, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d, s}
{a, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c, d, s}
{b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS:={{},{},{c},{},{},{},{},{a,b,c},X};LAT5abcd
  (LS,`A Connected Topology that is not T0, - but embeds as a Connected
  NOT T0 topology in N=5, Ex #2`,5);
```

X := {a, b, c, d, s}

LS := {{ }, {c}, {a, b, c}, {a, b, c, d, s}}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{ }
{b}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{ }
{c}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{a, b, d, s}	{ }	{a, b}
{a, c}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a}
{a, d}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d}
{a, s}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{s}
{b, c}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{b}
{b, d}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d}
{b, s}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{s}
{c, d}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d}
{c, s}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b}
{a, b, d}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{a, b, d, s}	{ }	{a, b, d}
{a, b, s}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{a, b, d, s}	{ }	{a, b, s}
{a, c, d}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, d}
{a, c, s}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, s}
{a, d, s}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d, s}
{b, c, d}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{b, d}
{b, c, s}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{b, s}
{b, d, s}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d, s}
{c, d, s}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, s}
{a, b, d, s}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{a, b, d, s}	{ }	{a, b, d, s}
{a, c, d, s}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, d, s}
{b, c, d, s}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{b, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS:={{},{},{},{a,b},{},{},{},{a,b,c},X};
  LAT5abcd(LS,`A Connected Topology that is not T0 - but embeds as a
  Connected NOT T0 topology in N=5, Ex #3`,5):
```

X := {a, b, c, d, s}

LS := { { }, {a, b}, {a, b, c}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{ }
{b}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{ }
{c}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c}
{b, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c}
{a, b, d}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d}
{a, b, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, s}
{a, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d}
{b, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, s}
{b, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, s}
{a, b, d, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d, s}
{a, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d, s}

```
> print(` `);X:={a,b,c,d,s};LS:={{},{},{c},{a,b},{},{},{},{a,b,c},X};
  LAT5abcd(LS,`A Disconnected Topology that is not T0 - but embeds as a
  Connected NOT T0 topology in N=5 Ex #4`,5):
```

X := {a, b, c, d, s}

LS := { { }, {c}, {a, b}, {a, b, c}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{ }
{b}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{ }
{c}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, b, d, s}	{ }	{a, b}
{a, c}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{ }
{a, d}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d}
{a, s}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{s}
{b, c}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{ }
{b, d}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d}
{b, s}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{s}
{c, d}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b}
{a, b, d}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, b, d, s}	{ }	{a, b, d}
{a, b, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, b, d, s}	{ }	{a, b, s}
{a, c, d}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d}
{a, c, s}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{s}
{a, d, s}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d, s}
{b, c, d}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d}
{b, c, s}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{s}
{b, d, s}	{ }	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d, s}
{c, d, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, s}
{a, b, d, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, b, d, s}	{ }	{a, b, d, s}
{a, c, d, s}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d, s}
{b, c, d, s}	{c}	{ }	{a, b, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, d, s}

Some singletons have the same closure and cannot be distinguished

9. Examples of Embedding N=4 T0, Disconnected or Connected topologies, into N=5 Connected T0 topologies

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,
  c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d},{a,b,c,d},X};LAT5abcd(LS,
  `LS = N=4 Poset 1 Disconnected T0 topology embedded into N=5 Connected
  T0 topology `,5):
```

X := {a, b, c, d, s}

LS := { { }, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

(62)

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d}	{s}	{a, s}	{s}	{a}	{ }
{b}	{b}	{a, c, d}	{s}	{b, s}	{s}	{b}	{ }
{c}	{c}	{a, b, d}	{s}	{c, s}	{s}	{c}	{ }
{d}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{c, d}	{s}	{a, b, s}	{s}	{a, b}	{ }
{a, c}	{a, c}	{b, d}	{s}	{a, c, s}	{s}	{a, c}	{ }
{a, d}	{a, d}	{b, c}	{s}	{a, d, s}	{s}	{a, d}	{ }
{a, s}	{a}	{b, c, d}	{s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{a, d}	{s}	{b, c, s}	{s}	{b, c}	{ }
{b, d}	{b, d}	{a, c}	{s}	{b, d, s}	{s}	{b, d}	{ }
{b, s}	{b}	{a, c, d}	{s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{a, b}	{s}	{c, d, s}	{s}	{c, d}	{ }
{c, s}	{c}	{a, b, d}	{s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{d}	{s}	{a, b, c, s}	{s}	{a, b, c}	{ }
{a, b, d}	{a, b, d}	{c}	{s}	{a, b, d, s}	{s}	{a, b, d}	{ }
{a, b, s}	{a, b}	{c, d}	{s}	{a, b, s}	{s}	{a, b}	{s}
{a, c, d}	{a, c, d}	{b}	{s}	{a, c, d, s}	{s}	{a, c, d}	{ }
{a, c, s}	{a, c}	{b, d}	{s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{a, d}	{b, c}	{s}	{a, d, s}	{s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{a}	{s}	{b, c, d, s}	{s}	{b, c, d}	{ }
{b, c, s}	{b, c}	{a, d}	{s}	{b, c, s}	{s}	{b, c}	{s}
{b, d, s}	{b, d}	{a, c}	{s}	{b, d, s}	{s}	{b, d}	{s}
{c, d, s}	{c, d}	{a, b}	{s}	{c, d, s}	{s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{s}	{a, b, c, d}	{ }
{a, b, c, s}	{a, b, c}	{d}	{s}	{a, b, c, s}	{s}	{a, b, c}	{s}
{a, b, d, s}	{a, b, d}	{c}	{s}	{a, b, d, s}	{s}	{a, b, d}	{s}
{a, c, d, s}	{a, c, d}	{b}	{s}	{a, c, d, s}	{s}	{a, c, d}	{s}
{b, c, d, s}	{b, c, d}	{a}	{s}	{b, c, d, s}	{s}	{b, c, d}	{s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{s}	{a, b, c, d}	{s}

(62)

```
> print(` `);X:={a,b,c,d,s};LS:={{},{b},{c},{d},{a,b},{b,c},{b,d},{c,d},
  {a,b,c},{a,b,d},{b,c,d},{a,b,c,d},X};LAT5abcd(LS,`LS = N=4 Poset 2
  DISconnected T0 topology embedded into N=5 Connected T0 topology `,5):
```

$$X := \{a, b, c, d, s\}$$

$$LS := \{\{\ \}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\}\}$$

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{ }
{c}	{c}	{a, b, d}	{s}	{c, s}	{s}	{c}	{ }
{d}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{c, d}	{s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{c}	{b, d}	{a, s}	{a, c, s}	{s}	{a, c}	{ }
{a, d}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{ }
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{ }
{b, d}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{ }
{b, s}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{a, b}	{s}	{c, d, s}	{s}	{c, d}	{ }
{c, s}	{c}	{a, b, d}	{s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, s}	{b, c}	{a}
{a, b, d}	{a, b, d}	{c}	{s}	{a, b, d, s}	{a, s}	{b, d}	{a}
{a, b, s}	{a, b}	{c, d}	{s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{b}	{a, s}	{a, c, d, s}	{s}	{a, c, d}	{ }
{a, c, s}	{c}	{b, d}	{a, s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{ }
{b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{s}
{b, d, s}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{s}
{c, d, s}	{c, d}	{a, b}	{s}	{c, d, s}	{s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{a}
{a, b, c, s}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, s}	{b, c}	{a, s}
{a, b, d, s}	{a, b, d}	{c}	{s}	{a, b, d, s}	{a, s}	{b, d}	{a, s}
{a, c, d, s}	{c, d}	{b}	{a, s}	{a, c, d, s}	{s}	{a, c, d}	{s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, s}	{b, c, d}	{a, s}

(63)

```
> print(` `);X:={a,b,c,d,s};LS:={{},{},{b},{d},{a,b},{b,d},{c,d},{a,b,d},
  {b,c,d},{a,b,c,d},X};LAT5abcd(LS,`LS =  N=4 Poset 3 Disconnected T0`,5):
```

X := {a, b, c, d, s}

LS := { { }, {b}, {d}, {a, b}, {b, d}, {c, d}, {a, b, d}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{ }
{c}	{ }	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{ }
{d}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{c, d}	{s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{ }	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{ }
{a, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{ }
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, s}	{b, c}	{ }
{b, d}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{ }
{b, s}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{a, b}	{s}	{c, d, s}	{c, s}	{d}	{c}
{c, s}	{ }	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{s}
{a, b, c}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, s}	{b, c}	{a}
{a, b, d}	{a, b, d}	{ }	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a}
{a, b, s}	{a, b}	{c, d}	{s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{b}	{a, s}	{a, c, d, s}	{c, s}	{a, d}	{c}
{a, c, s}	{ }	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c}
{b, c, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, s}	{b, c}	{s}
{b, d, s}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{s}
{c, d, s}	{c, d}	{a, b}	{s}	{c, d, s}	{c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c}
{a, b, c, s}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, s}	{b, c}	{a, s}
{a, b, d, s}	{a, b, d}	{ }	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, s}
{a, c, d, s}	{c, d}	{b}	{a, s}	{a, c, d, s}	{c, s}	{a, d}	{c, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c, s}

(64)

```
> print(` `);X:={a,b,c,d,s};LS:={{},{c},{d},{a,c},{b,c},{c,d},{a,b,c},{a,
  c,d},{b,c,d},{a,b,c,d},X};LAT5abcd(LS,`LS =  N=4 Poset 4 Disconnected T0
  topology embedded into N=5 Connected T0 topology `,5):
```

$$X := \{a, b, c, d, s\}$$

$$LS := \{\{\ \}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\}\}$$

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{ }
{c}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{ }
{d}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{ }	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{ }
{a, c}	{a, c}	{d}	{b, s}	{a, b, c, s}	{a, b, s}	{c}	{a}
{a, d}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{ }
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, b, s}	{c}	{b}
{b, d}	{d}	{a, c}	{b, s}	{b, d, s}	{s}	{b, d}	{ }
{b, s}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{ }
{c, s}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{s}
{d, s}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, b, s}	{c}	{a, b}
{a, b, d}	{d}	{c}	{a, b, s}	{a, b, d, s}	{s}	{a, b, d}	{ }
{a, b, s}	{ }	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{s}
{a, c, d}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a}
{a, c, s}	{a, c}	{d}	{b, s}	{a, b, c, s}	{a, b, s}	{c}	{a, s}
{a, d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{b}
{b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, b, s}	{c}	{b, s}
{b, d, s}	{d}	{a, c}	{b, s}	{b, d, s}	{s}	{b, d}	{s}
{c, d, s}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, b}
{a, b, c, s}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, b, s}	{c}	{a, b, s}
{a, b, d, s}	{d}	{c}	{a, b, s}	{a, b, d, s}	{s}	{a, b, d}	{s}
{a, c, d, s}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{b, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, b, s}

(65)

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a},{c},{d},{a,c},{a,d},{c,d},{a,b,c},
  {a,c,d},{a,b,c,d},X};LAT5abcd(LS,`LS =  N=4 Poset 5 Disconnected T0
  topology embedded into N=5 Connected T0 topology `,5):
```

X := {a, b, c, d, s}

LS := { { }, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {a, b, c}, {a, c, d}, {a, b, c, d}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c, d}	{b, s}	{a, b, s}	{b, s}	{a}	{ }
{b}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{ }
{c}	{c}	{a, d}	{b, s}	{b, c, s}	{b, s}	{c}	{ }
{d}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a}	{c, d}	{b, s}	{a, b, s}	{b, s}	{a}	{b}
{a, c}	{a, c}	{d}	{b, s}	{a, b, c, s}	{b, s}	{a, c}	{ }
{a, d}	{a, d}	{c}	{b, s}	{a, b, d, s}	{b, s}	{a, d}	{ }
{a, s}	{a}	{c, d}	{b, s}	{a, b, s}	{b, s}	{a}	{s}
{b, c}	{c}	{a, d}	{b, s}	{b, c, s}	{b, s}	{c}	{b}
{b, d}	{d}	{a, c}	{b, s}	{b, d, s}	{s}	{b, d}	{ }
{b, s}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{a}	{b, s}	{b, c, d, s}	{b, s}	{c, d}	{ }
{c, s}	{c}	{a, d}	{b, s}	{b, c, s}	{b, s}	{c}	{s}
{d, s}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{d}	{s}	{a, b, c, s}	{b, s}	{a, c}	{b}
{a, b, d}	{a, d}	{c}	{b, s}	{a, b, d, s}	{b, s}	{a, d}	{b}
{a, b, s}	{a}	{c, d}	{b, s}	{a, b, s}	{b, s}	{a}	{b, s}
{a, c, d}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{b, s}	{a, c, d}	{ }
{a, c, s}	{a, c}	{d}	{b, s}	{a, b, c, s}	{b, s}	{a, c}	{s}
{a, d, s}	{a, d}	{c}	{b, s}	{a, b, d, s}	{b, s}	{a, d}	{s}
{b, c, d}	{c, d}	{a}	{b, s}	{b, c, d, s}	{b, s}	{c, d}	{b}
{b, c, s}	{c}	{a, d}	{b, s}	{b, c, s}	{b, s}	{c}	{b, s}
{b, d, s}	{d}	{a, c}	{b, s}	{b, d, s}	{s}	{b, d}	{s}
{c, d, s}	{c, d}	{a}	{b, s}	{b, c, d, s}	{b, s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{b, s}	{a, c, d}	{b}
{a, b, c, s}	{a, b, c}	{d}	{s}	{a, b, c, s}	{b, s}	{a, c}	{b, s}
{a, b, d, s}	{a, d}	{c}	{b, s}	{a, b, d, s}	{b, s}	{a, d}	{b, s}
{a, c, d, s}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{b, s}	{a, c, d}	{s}
{b, c, d, s}	{c, d}	{a}	{b, s}	{b, c, d, s}	{b, s}	{c, d}	{b, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{b, s}	{a, c, d}	{b, s}

(66)

```
> print(` `);X:={a,b,c,d,s};LS:={{},{},{c},{d},{b,c},{c,d},{a,b,c},{b,c,d}
  ,{a,b,c,d},X};LAT5abcd(LS,`LS =  N=4 Poset 6 Disconnected T0 topology
  embedded into N=5 Connected T0 topology `,5):
```

X := {a, b, c, d, s}

LS := { { }, {c}, {d}, {b, c}, {c, d}, {a, b, c}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{ }	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{ }
{c}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{ }
{d}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{ }	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{a}
{a, d}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{ }
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, b, s}	{c}	{b}
{b, d}	{d}	{c}	{a, b, s}	{a, b, d, s}	{a, s}	{b, d}	{ }
{b, s}	{ }	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{ }
{c, s}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{s}
{d, s}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, b, s}	{c}	{a, b}
{a, b, d}	{d}	{c}	{a, b, s}	{a, b, d, s}	{a, s}	{b, d}	{a}
{a, b, s}	{ }	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a}
{a, c, s}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{a, s}
{a, d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{b}
{b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, b, s}	{c}	{b, s}
{b, d, s}	{d}	{c}	{a, b, s}	{a, b, d, s}	{a, s}	{b, d}	{s}
{c, d, s}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, b}
{a, b, c, s}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, b, s}	{c}	{a, b, s}
{a, b, d, s}	{d}	{c}	{a, b, s}	{a, b, d, s}	{a, s}	{b, d}	{a, s}
{a, c, d, s}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{b, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, b, s}

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```
> print(` `);X:={a,b,c,d,s};LS:={{},{},{},{d},{a,d},{c,d},{a,c,d},{b,c,d},
  {a,b,c,d},X};LAT5abcd(LS,`LS =  N=4 Poset 7  Connected T0 topology
  embedded into N=5 Connected T0 topology `,5):
```

X := {a, b, c, d, s}

LS := {{ }, {d}, {a, d}, {c, d}, {a, c, d}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s}}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{ }
{c}	{ }	{a, d}	{b, c, s}	{b, c, s}	{b, s}	{c}	{ }
{d}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{ }	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{ }
{a, c}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{b, s}	{a, c}	{ }
{a, d}	{a, d}	{ }	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a}
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{ }	{a, d}	{b, c, s}	{b, c, s}	{b, s}	{c}	{b}
{b, d}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b}
{b, s}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c}
{c, s}	{ }	{a, d}	{b, c, s}	{b, c, s}	{b, s}	{c}	{s}
{d, s}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{b, s}	{a, c}	{b}
{a, b, d}	{a, d}	{ }	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b}
{a, b, s}	{ }	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{s}
{a, c, d}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c}
{a, c, s}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{b, s}	{a, c}	{s}
{a, d, s}	{a, d}	{ }	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c}
{b, c, s}	{ }	{a, d}	{b, c, s}	{b, c, s}	{b, s}	{c}	{b, s}
{b, d, s}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c}
{a, b, c, s}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{b, s}	{a, c}	{b, s}
{a, b, d, s}	{a, d}	{ }	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, s}
{a, c, d, s}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c, s}

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```
> print(` `);X:={a,b,c,d,s};LS:={{},{},{b},{d},{b,d},{a,b,d},{b,c,d},{a,b,
  c,d},X};LAT5abcd(LS,`LS =  N=4 Poset 8  Connected T0 topology embedded
  into N=5 Connected T0 topology `,5):
```

$$X := \{a, b, c, d, s\}$$

$$LS := \{\{\}, \{b\}, \{d\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\}\}$$

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{ }
{c}	{ }	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{ }
{d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{a}
{a, c}	{ }	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{ }
{a, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{a}
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{c}
{b, d}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{ }
{b, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{s}
{c, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{c}
{c, s}	{ }	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{s}
{a, b, c}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, c}
{a, b, d}	{a, b, d}	{ }	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a}
{a, b, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, s}
{a, c, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{a, c}
{a, c, s}	{ }	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c}
{b, c, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{c, s}
{b, d, s}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{s}
{c, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c}
{a, b, c, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, c, s}
{a, b, d, s}	{a, b, d}	{ }	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, s}
{a, c, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c, s}

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```
> print(` `);X:={a,b,c,d,s};LS:={{},{a},{c},{},{a,c},{},{a,b,c},{},{a,b,c,
  d},X};LAT5abcd(LS,`LS =  N=4 Poset 9,  Connected T0`,5):
```

X := {a, b, c, d, s}

LS := { { }, {a}, {c}, {a, c}, {a, b, c}, {a, b, c, d}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{ }
{b}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{ }
{c}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{b}
{a, c}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{ }
{a, d}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d}
{a, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{s}
{b, c}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b}
{b, d}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{d}
{b, s}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{s}
{c, d}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{d}
{c, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b}
{a, b, d}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, d}
{a, b, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, s}
{a, c, d}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d}
{a, c, s}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{s}
{a, d, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d, s}
{b, c, d}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, d}
{b, c, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, s}
{b, d, s}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{d, s}
{c, d, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, s}
{a, b, d, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d, s}
{b, c, d, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d, s}

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```
> print(` `);X:={a,b,c,d,s};LS:={{},{a},{a,c},{a,b,c},{a,c,d},{a,b,c,d},X}
  ;LAT5abcd(LS,`LS =  N=4 Poset 10  Connected T0 topology embedded into N=
  5 Connected T0 topology `,5):
```

$$X := \{a, b, c, d, s\}$$

$$LS := \{\{\}, \{a\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\}\}$$

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{ }
{b}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{ }
{c}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b}
{a, c}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b}
{b, d}	{ }	{a, c}	{b, d, s}	{b, d, s}	{s}	{b, d}	{ }
{b, s}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{d}
{c, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c}
{a, b, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d}
{a, b, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, s}
{a, c, d}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, d}
{b, c, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, s}
{b, d, s}	{ }	{a, c}	{b, d, s}	{b, d, s}	{s}	{b, d}	{s}
{c, d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, s}
{a, b, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

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```
> print(` `);X:={a,b,c,d,s};LS:={{},{d},{b,d},{c,d},{b,c,d},{a,b,c,d},X};
  LAT5abcd(LS,`LS = N=4 Poset 11 Connected T0 topology embedded into N=5
  Connected T0 topology `,5):
```

X := {a, b, c, d, s}

LS := { { }, {d}, {b, d}, {c, d}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{ }	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{ }
{c}	{ }	{b, d}	{a, c, s}	{a, c, s}	{a, s}	{c}	{ }
{d}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{ }	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{ }	{b, d}	{a, c, s}	{a, c, s}	{a, s}	{c}	{a}
{a, d}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a}
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{a, s}	{b, c}	{ }
{b, d}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b}
{b, s}	{ }	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c}
{c, s}	{ }	{b, d}	{a, c, s}	{a, c, s}	{a, s}	{c}	{s}
{d, s}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{a, s}	{b, c}	{a}
{a, b, d}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b}
{a, b, s}	{ }	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c}
{a, c, s}	{ }	{b, d}	{a, c, s}	{a, c, s}	{a, s}	{c}	{a, s}
{a, d, s}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c}
{b, c, s}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{a, s}	{b, c}	{s}
{b, d, s}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c}
{a, b, c, s}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{a, s}	{b, c}	{a, s}
{a, b, d, s}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, s}
{a, c, d, s}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c, s}

(72)

```
> print(` `);X:={a,b,c,d,s};LS:={{},{b},{d},{},{a,b},{b,d},{},{a,b,d},{a,b,
  c,d},X};LAT5abcd(LS,`LS =  N=4 Poset 12  Connected T0 topology embedded
  into N=5 Connected T0 topology `,5):
```

$$X := \{a, b, c, d, s\}$$

$$LS := \{\{\}, \{b\}, \{d\}, \{a, b\}, \{b, d\}, \{a, b, d\}, \{a, b, c, d\}, \{a, b, c, d, s\}\}$$

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, d}	{a, c, s}	{a, c, s}	{c, s}	{a}	{ }
{b}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{ }
{c}	{ }	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{ }
{d}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, c, s}	{b}	{a}
{a, c}	{ }	{b, d}	{a, c, s}	{a, c, s}	{c, s}	{a}	{c}
{a, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{ }
{a, s}	{ }	{b, d}	{a, c, s}	{a, c, s}	{c, s}	{a}	{s}
{b, c}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{c}
{b, d}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{ }
{b, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{s}
{c, d}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{c}
{c, s}	{ }	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{s}
{a, b, c}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, c}
{a, b, d}	{a, b, d}	{ }	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a}
{a, b, s}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, s}
{a, c, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{c}
{a, c, s}	{ }	{b, d}	{a, c, s}	{a, c, s}	{c, s}	{a}	{c, s}
{a, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{s}
{b, c, d}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c}
{b, c, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{c, s}
{b, d, s}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{s}
{c, d, s}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c}
{a, b, c, s}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, c, s}
{a, b, d, s}	{a, b, d}	{ }	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, s}
{a, c, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{c, s}
{b, c, d, s}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c, s}

(73)

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a},{c},{ },{a,c},{a,b},{},{},{},{a,b,
  c},{},{},{},{a,b,c,d},X};LAT5abcd(LS,`LS =  N=4 Poset 13  Connected T0
  topology embedded into N=5 Connected T0 topology `,5):
```

$$X := \{a, b, c, d, s\}$$

$$LS := \{\{\ \}, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, c, d\}, \{a, b, c, d, s\}\}$$

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{ }
{b}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{ }
{c}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, d, s}	{a}	{b}
{a, c}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{ }
{a, d}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d}
{a, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{s}
{b, c}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{ }
{b, d}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{d}
{b, s}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{s}
{c, d}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b}
{a, b, d}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, d}
{a, b, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, s}
{a, c, d}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d}
{a, c, s}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{s}
{a, d, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d, s}
{b, c, d}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{d}
{b, c, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{s}
{b, d, s}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{d, s}
{c, d, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, s}
{a, b, d, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d, s}
{b, c, d, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d, s}

(74)

```
> print(` `);X:={a,b,c,d,s};LS:={{},{},{d},{a,d},{b,d},{c,d},{a,b,d},{a,c,
  d},{b,c,d},{a,b,c,d},X};LAT5abcd(LS,`LS =  N=4 Poset 14   Connected T0
  topology embedded into N=5 Connected T0 topology `,5):
```

X := {a, b, c, d, s}

LS := {{ }, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s}}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{ }
{c}	{ }	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{ }
{d}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{ }	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{ }
{a, c}	{ }	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{ }
{a, d}	{a, d}	{ }	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a}
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{ }	{a, d}	{b, c, s}	{b, c, s}	{s}	{b, c}	{ }
{b, d}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b}
{b, s}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c}
{c, s}	{ }	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{s}	{a, b, c}	{ }
{a, b, d}	{a, b, d}	{ }	{c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b}
{a, b, s}	{ }	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{s}
{a, c, d}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c}
{a, c, s}	{ }	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{a, d}	{ }	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c}
{b, c, s}	{ }	{a, d}	{b, c, s}	{b, c, s}	{s}	{b, c}	{s}
{b, d, s}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c}
{a, b, c, s}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{s}	{a, b, c}	{s}
{a, b, d, s}	{a, b, d}	{ }	{c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, s}
{a, c, d, s}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c, s}

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```
> print(` `);X:={a,b,c,d,s};LS:={{},{b},{c},{d},{b,c},{b,d},{c,d},{b,c,d},
  {a,b,c,d},X};LAT5abcd(LS,`LS =  N=4 Poset 15, Connected T0 topology
  embedded into N=5 Connected T0 topology `,5);
```

X := {a, b, c, d, s}

LS := {{ }, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s}}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{ }
{c}	{c}	{b, d}	{a, s}	{a, c, s}	{a, s}	{c}	{ }
{d}	{d}	{b, c}	{a, s}	{a, d, s}	{a, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{c}	{b, d}	{a, s}	{a, c, s}	{a, s}	{c}	{a}
{a, d}	{d}	{b, c}	{a, s}	{a, d, s}	{a, s}	{d}	{a}
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{ }
{b, d}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{ }
{b, s}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{b}	{a, s}	{a, c, d, s}	{a, s}	{c, d}	{ }
{c, s}	{c}	{b, d}	{a, s}	{a, c, s}	{a, s}	{c}	{s}
{d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{a, s}	{d}	{s}
{a, b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{a}
{a, b, d}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{a}
{a, b, s}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{b}	{a, s}	{a, c, d, s}	{a, s}	{c, d}	{a}
{a, c, s}	{c}	{b, d}	{a, s}	{a, c, s}	{a, s}	{c}	{a, s}
{a, d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{a, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{ }
{b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{s}
{b, d, s}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{s}
{c, d, s}	{c, d}	{b}	{a, s}	{a, c, d, s}	{a, s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{a}
{a, b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{a, s}
{a, b, d, s}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{a, s}
{a, c, d, s}	{c, d}	{b}	{a, s}	{a, c, d, s}	{a, s}	{c, d}	{a, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, s}	{b, c, d}	{a, s}

(76)

```
> print(` `);X:={a,b,c,d,s};LS:={{},{},{},{d},{c,d},{b,c,d},{a,b,c,d},X};
  LAT5abcd(LS,`LS = Poset 16  Connected T0 topology embedded into N=5
  Connected T0 topology `,5);
```

X := {a, b, c, d, s}

LS := { { }, {d}, {c, d}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{ }	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{ }
{c}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{ }
{d}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{ }	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{a}
{a, d}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a}
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{b}
{b, d}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b}
{b, s}	{ }	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c}
{c, s}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{s}
{d, s}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{a, b}
{a, b, d}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b}
{a, b, s}	{ }	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c}
{a, c, s}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{a, s}
{a, d, s}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c}
{b, c, s}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{b, s}
{b, d, s}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c}
{a, b, c, s}	{ }	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{a, b, s}
{a, b, d, s}	{d}	{ }	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, s}
{a, c, d, s}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c, s}

(77)

10. Examples of N=4 Not-T0 , but Connected, topologies Embedded in N=5. X = {a,b,c,d,E} Some or All Singleton closures are not distinguishable.

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a,b,c,d},X};LAT5abcd(LS,`N=4
  Indiscrete NOT T0 topology embeds into a N=5 NOT T0, but CONNECTED,
  topology where 4  N=5 singletons have the same closure. `,5):
```

X := {a, b, c, d, s}

LS := {{ }, {a, b, c, d}, {a, b, c, d, s}}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{ }
{b}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{ }
{c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{ }
{d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c}
{a, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, d}
{a, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c}
{b, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, d}
{b, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{c, d}
{c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{s}
{d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c}
{a, b, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d}
{a, b, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, s}
{a, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c, d}
{a, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c, s}
{a, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, d, s}
{b, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c, d}
{b, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c, s}
{b, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, d, s}
{c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{c, d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d}
{a, b, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, s}
{a, b, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d, s}
{a, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c, d, s}
{b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a,b,c},{a,b,c,d},X};LAT5abcd(LS,`N=4
  NOT T0 topology embeds into a N=5 NOT T0, but CONNECTED, topology, where
  3 singletons have the same closures. `,5):
```

X := {a, b, c, d, s}

LS := { { }, {a, b, c}, {a, b, c, d}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{ }
{b}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{ }
{c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c}
{a, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c}
{b, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d}
{c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c}
{a, b, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d}
{a, b, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, s}
{a, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c, d}
{a, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c, s}
{a, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c, d}
{b, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c, s}
{b, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, s}
{a, b, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d, s}
{a, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c, d, s}
{b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a,b},{a,b,c,d},X};LAT5abcd(LS,`N=4
  NOT T0 topology embeds into a N=5 NOT T0, but CONNECTED, topology, where
  2 N=5 singleton pairs have duplicate closures. NOT T0, but CONNECTED`,5)
  :
```

X := {a, b, c, d, s}

LS := { { }, {a, b}, {a, b, c, d}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{ }
{b}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{ }
{c}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c}
{b, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{ }	{c, d}
{c, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{s}
{a, b, c}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c}
{a, b, d}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d}
{a, b, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, s}
{a, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d}
{b, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, s}
{b, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{ }	{c, d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d}
{a, b, c, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, s}
{a, b, d, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d, s}
{a, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d, s}

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a},{a,b,c,d},X};LAT5abcd(LS,`N=4 NOT
  T0 topology embeds into a N=5 NOT T0, but CONNECTED, topology, where 3
  N=5 singletons have duplicate closures.`,5):
```

X := {a, b, c, d, s}

LS := {{ }, {a}, {a, b, c, d}, {a, b, c, d, s}}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{ }
{b}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{ }
{c}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{ }
{d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b}
{a, c}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, c}
{b, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, d}
{b, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{s}
{c, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{c, d}
{c, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{s}
{d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{s}
{a, b, c}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c}
{a, b, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d}
{a, b, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, s}
{a, c, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, c, d}
{b, c, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, c, s}
{b, d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, d, s}
{c, d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{c, d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, s}
{a, b, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a},{a,b},{a,b,c,d},X};LAT5abcd(LS,`N=
  4 NOT T0 topology embeds into a N=5 NOT T0, but CONNECTED, topology,
  where 2 N=5 singletons have duplicate closures.`,5):
```

X := {a, b, c, d, s}

LS := { { }, {a}, {a, b}, {a, b, c, d}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{ }
{b}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{ }
{c}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b}
{a, c}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c}
{b, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d}
{b, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{s}
{c, d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{ }	{c, d}
{c, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{s}
{a, b, c}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c}
{a, b, d}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d}
{a, b, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, s}
{a, c, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d}
{b, c, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, s}
{b, d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d, s}
{c, d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{ }	{c, d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, s}
{a, b, d, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a},{b},{a,b},{a,b,c,d},X};LAT5abcd
  (LS,`N=4 NOT T0 topology embeds into a N=5 NOT T0, but CONNECTED,
  topology, where 2 N=5 singletons have duplicate closures. NOT T0, but
  CONNECTED`,5):
```

X := {a, b, c, d, s}

LS := { { }, {a}, {b}, {a, b}, {a, b, c, d}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{ }
{b}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{ }
{c}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{ }
{a, c}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c}
{a, d}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d}
{a, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{s}
{b, c}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c}
{b, d}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d}
{b, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{s}
{c, d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{ }	{c, d}
{c, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{s}
{a, b, c}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c}
{a, b, d}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{d}
{a, b, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{s}
{a, c, d}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d, s}
{b, c, d}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d}
{b, c, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, s}
{b, d, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d, s}
{c, d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{ }	{c, d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c, d}
{a, b, c, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c, s}
{a, b, d, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{d, s}
{a, c, d, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{c, d, s}	{a, b}	{c, d, s}

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a},{a,b,c},{a,b,c,d},X};LAT5abcd(LS,
  `N=4 NOT T0 topology embeds into a N=5 NOT T0`,5);
```

X := { a, b, c, d, s }

LS := { { }, { a }, { a, b, c }, { a, b, c, d }, { a, b, c, d, s } }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ }
{ b }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ c, d, s }	{ b }	{ }
{ c }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ b, d, s }	{ c }	{ }
{ d }	{ }	{ a, b, c }	{ d, s }	{ d, s }	{ s }	{ d }	{ }
{ s }	{ }	{ a, b, c, d }	{ s }	{ s }	{ }	{ s }	{ }
{ a, b }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b }
{ a, c }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ c }
{ a, d }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ d }
{ a, s }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ s }
{ b, c }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ b, c, d, s }	{ }	{ b, c }
{ b, d }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ c, d, s }	{ b }	{ d }
{ b, s }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ c, d, s }	{ b }	{ s }
{ c, d }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ b, d, s }	{ c }	{ d }
{ c, s }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ b, d, s }	{ c }	{ s }
{ d, s }	{ }	{ a, b, c }	{ d, s }	{ d, s }	{ s }	{ d }	{ s }
{ a, b, c }	{ a, b, c }	{ }	{ d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, c }
{ a, b, d }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, d }
{ a, b, s }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, s }
{ a, c, d }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ c, d }
{ a, c, s }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ c, s }
{ a, d, s }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ d, s }
{ b, c, d }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ b, c, d, s }	{ }	{ b, c, d }
{ b, c, s }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ b, c, d, s }	{ }	{ b, c, s }
{ b, d, s }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ c, d, s }	{ b }	{ d, s }
{ c, d, s }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ b, d, s }	{ c }	{ d, s }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, c, d }
{ a, b, c, s }	{ a, b, c }	{ }	{ d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, c, s }
{ a, b, d, s }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, d, s }
{ a, c, d, s }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ c, d, s }
{ b, c, d, s }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ b, c, d, s }	{ }	{ b, c, d, s }
{ a, b, c, d, s }	{ a, b, c, d, s }	{ }	{ }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, c, d, s }

Some singletons have the same closure and cannot be distinguished

```
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  CONNECTED`,5):
```

X := {a, b, c, d, s}

LS := {{ }, {a, b}, {a, b, c}, {a, b, c, d}, {a, b, c, d, s}}

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{ }
{b}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{ }
{c}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c}
{b, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c}
{a, b, d}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d}
{a, b, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, s}
{a, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d}
{b, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, s}
{b, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, s}
{a, b, d, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d, s}
{a, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d, s}

Some Examples that correspond to problems in the Monographs

```
> print(` `);X:={a,b,c,d,s};LS:={{},{s},{a,s},{c,d,s},{a,c,d,s},{b,c,d,s}
  ,X};print(`Lipshitz, p 76 problem 14 and prob 30 p 79`);LAT5abcd(LS,
  `Lipshitz, p 76 problem 14 and prob 30 p 79`,5):
```

X := {a, b, c, d, s}

LS := { { }, {s}, {a, s}, {c, d, s}, {a, c, d, s}, {b, c, d, s}, {a, b, c, d, s} }

Lipshitz, p 76 problem 14 and prob 30 p 79

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d, s}	{a}	{a}	{ }	{a}	{ }
{b}	{ }	{a, c, d, s}	{b}	{b}	{ }	{b}	{ }
{c}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, d}	{c}	{ }
{d}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, c}	{d}	{ }
{s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{ }
{a, b}	{ }	{c, d, s}	{a, b}	{a, b}	{ }	{a, b}	{ }
{a, c}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{ }
{a, d}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{ }
{a, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a}
{b, c}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, d}	{c}	{b}
{b, d}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, c}	{d}	{b}
{b, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b}
{c, d}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{c, d}
{c, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{c}
{d, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{d}
{a, b, c}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{b}
{a, b, d}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{b}
{a, b, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b}
{a, c, d}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{a, c, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, c}
{a, d, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, d}
{b, c, d}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, c, d}
{b, c, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, c}
{b, d, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, d}
{c, d, s}	{c, d, s}	{ }	{a, b}	{a, b, c, d, s}	{a, b, c, d}	{s}	{c, d}
{a, b, c, d}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c, d}
{a, b, c, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, c}
{a, b, d, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, d}
{a, c, d, s}	{a, c, d, s}	{ }	{b}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, c, d}
{b, c, d, s}	{b, c, d, s}	{ }	{a}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, c, d}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, c, d}

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a},{a,b},{a,b,c},X};LAT5abcd(LS,
  `Lipshitz, p 66 problem 14 and prob 30 p 79`,5):
```

X := {a, b, c, d, s}

LS := { { }, {a}, {a, b}, {a, b, c}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{ }
{b}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{ }
{c}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b}
{a, c}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c}
{b, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d}
{b, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{s}
{c, d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c}
{a, b, d}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d}
{a, b, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, s}
{a, c, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d}
{b, c, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, s}
{b, d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d, s}
{c, d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, s}
{a, b, d, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a},{c,d},{a,c,d},{b,c,d,s},X};
  LAT5abcd(LS,`Lipshitz, p 66 problem 14 and prob 30 p 79`,5):
```

X := {a, b, c, d, s}

LS := {{ }, {a}, {c, d}, {a, c, d}, {b, c, d, s}, {a, b, c, d, s}}

Is LS: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d, s}	{ }	{a}	{ }	{a}	{ }
{b}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{ }
{c}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{ }
{d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{ }
{s}	{ }	{a, c, d}	{b, s}	{b, s}	{b}	{s}	{ }
{a, b}	{a}	{c, d}	{b, s}	{a, b, s}	{s}	{a, b}	{ }
{a, c}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{ }
{a, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{ }
{a, s}	{a}	{c, d}	{b, s}	{a, b, s}	{b}	{a, s}	{ }
{b, c}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b}
{b, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{b}
{b, s}	{ }	{a, c, d}	{b, s}	{b, s}	{b, s}	{ }	{b, s}
{c, d}	{c, d}	{a}	{b, s}	{b, c, d, s}	{b, c, d, s}	{ }	{c, d}
{c, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{s}
{d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{s}
{a, b, c}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b}
{a, b, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{b}
{a, b, s}	{a}	{c, d}	{b, s}	{a, b, s}	{b, s}	{a}	{b, s}
{a, c, d}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{s}
{a, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{s}
{b, c, d}	{c, d}	{a}	{b, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, c, d}
{b, c, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, s}
{b, d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{a}	{b, s}	{b, c, d, s}	{b, c, d, s}	{ }	{c, d, s}
{a, b, c, d}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, s}
{a, b, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{b, s}
{a, c, d, s}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{b, c, d, s}	{a}	{ }	{b, c, d, s}	{b, c, d, s}	{ }	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d};LS:={{},{},{},{},{a,b,d},{a,c,d},X};print(`NOT
  A TOPOLOGY !!! `);LAT4abcd(LS,`Case1 vol6.. LS 1 is Not a topology, Not
  T0  `,4):
```

X := {a, b, c, d}

LS := { { }, {a, b, d}, {a, c, d}, {a, b, c, d} }

NOT A TOPOLOGY !!!

Is LS: a topology = false, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{a, c, d}	{b}	{b}	{ }	{ }	{ }
{c}	{ }	{a, b, d}	{c}	{c}	{ }	{ }	{ }
{d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c}	{d}	{ }
{a, b}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, d}
{b, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, d}	{b, c}	{ }
{b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c}	{d}	{b}
{c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c}	{d}	{c}
{a, b, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c}
{a, b, d}	{a, b, d}	{ }	{c}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, d}
{a, c, d}	{a, c, d}	{ }	{b}	{a, b, c, d}	{a, b, c, d}	{ }	{a, c, d}
{b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c, d}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {b}, {c}, {a, b, c, d}) **(89)**

```
> print(` `);X:={a,b,c,d};LS:={{},{},{a,b},{},{a,b,d},{a,c,d},X};print
  (`NOT A TOPOLOGY BUT T0  !!! `);LAT4abcd(LS,`Case 2 vol6  ..LS 2 is Not
  a topology, but is T0  `,4):
```

X := {a, b, c, d}

LS := {{ }, {a, b}, {a, b, d}, {a, c, d}, {a, b, c, d}}

NOT A TOPOLOGY BUT T0 !!!

Is LS: a topology = false, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{a, c, d}	{b}	{b}	{ }	{ }	{ }
{c}	{ }	{a, b, d}	{c}	{c}	{ }	{ }	{ }
{d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, d}	{b, c}	{ }
{b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c}	{b, d}	{ }
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{ }	{c}
{a, b, c}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c}
{a, b, d}	{a, b, d}	{ }	{c}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, d}
{a, c, d}	{a, c, d}	{ }	{b}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {b}, {c}, {c, d})

(90)

```
> print(` `);X:={a,b,c,d};LS:={{},{a},{},{},{a,b,d},{a,c,d},X};print(`NOT
  A TOPOLOGY but T0 !!! `);LAT4abcd(LS,` Case3 vol6  ..LS 3 is Not
  topology, but is Not T0  `,4):
```

X := {a, b, c, d}

LS := {{ }, {a}, {a, b, d}, {a, c, d}, {a, b, c, d}}

NOT A TOPOLOGY but T0 !!!

Is LS: a topology = false, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{a, c, d}	{b}	{b}	{ }	{ }	{ }
{c}	{ }	{a, b, d}	{c}	{c}	{ }	{ }	{ }
{d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c}	{d}	{ }
{a, b}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{a}	{b, c, d}	{b, c, d}	{d}	{b, c}	{ }
{b, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c}	{d}	{b}
{c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c}	{d}	{c}
{a, b, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a, b, d}	{ }	{c}	{a, b, c, d}	{b, c, d}	{a}	{b, d}
{a, c, d}	{a, c, d}	{ }	{b}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, c, d}	{ }	{b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {b}, {c}, {b, c, d}) **(91)**

```
> print(` `);X:={a,b,c,d,s};LS:={{},{b,c,d,s},{a,c,d,s},{c,d,s},{a,s},{s}
  ,X};print(`A N=5  Not-T0 TOPOLOGY !!! `);LAT5abcd(LS,`Topology 1, p
  446, vol 1 and p 214 vol6 `,5):
```

X := {a, b, c, d, s}

LS := { { }, {s}, {a, s}, {c, d, s}, {a, c, d, s}, {b, c, d, s}, {a, b, c, d, s} }

A N=5 Not-T0 TOPOLOGY !!!

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d, s}	{a}	{a}	{ }	{a}	{ }
{b}	{ }	{a, c, d, s}	{b}	{b}	{ }	{b}	{ }
{c}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, d}	{c}	{ }
{d}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, c}	{d}	{ }
{s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{ }
{a, b}	{ }	{c, d, s}	{a, b}	{a, b}	{ }	{a, b}	{ }
{a, c}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{ }
{a, d}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{ }
{a, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a}
{b, c}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, d}	{c}	{b}
{b, d}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, c}	{d}	{b}
{b, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b}
{c, d}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{c, d}
{c, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{c}
{d, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{d}
{a, b, c}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{b}
{a, b, d}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{b}
{a, b, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b}
{a, c, d}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{a, c, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, c}
{a, d, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, d}
{b, c, d}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, c, d}
{b, c, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, c}
{b, d, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, d}
{c, d, s}	{c, d, s}	{ }	{a, b}	{a, b, c, d, s}	{a, b, c, d}	{s}	{c, d}
{a, b, c, d}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c, d}
{a, b, c, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, c}
{a, b, d, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, d}
{a, c, d, s}	{a, c, d, s}	{ }	{b}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, c, d}
{b, c, d, s}	{b, c, d, s}	{ }	{a}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, c, d}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, c, d}

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a,b,c},{a,b},{a},X};print(`A N=5
  Not-T0 TOPOLOGY !!! `);LAT5abcd(LS,`Topology 2, p 440, vol 1`,5):
```

X := { a, b, c, d, s }

LS := { { }, { a }, { a, b }, { a, b, c }, { a, b, c, d, s } }

A N=5 Not-T0 TOPOLOGY !!!

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ }
{ b }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ c, d, s }	{ b }	{ }
{ c }	{ }	{ a, b }	{ c, d, s }	{ c, d, s }	{ d, s }	{ c }	{ }
{ d }	{ }	{ a, b, c }	{ d, s }	{ d, s }	{ s }	{ d }	{ }
{ s }	{ }	{ a, b, c }	{ d, s }	{ d, s }	{ d }	{ s }	{ }
{ a, b }	{ a, b }	{ }	{ c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b }
{ a, c }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ c }
{ a, d }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ d }
{ a, s }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ s }
{ b, c }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ c, d, s }	{ b }	{ c }
{ b, d }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ c, d, s }	{ b }	{ d }
{ b, s }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ c, d, s }	{ b }	{ s }
{ c, d }	{ }	{ a, b }	{ c, d, s }	{ c, d, s }	{ d, s }	{ c }	{ d }
{ c, s }	{ }	{ a, b }	{ c, d, s }	{ c, d, s }	{ d, s }	{ c }	{ s }
{ d, s }	{ }	{ a, b, c }	{ d, s }	{ d, s }	{ d, s }	{ }	{ d, s }
{ a, b, c }	{ a, b, c }	{ }	{ d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, c }
{ a, b, d }	{ a, b }	{ }	{ c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, d }
{ a, b, s }	{ a, b }	{ }	{ c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, s }
{ a, c, d }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ c, d }
{ a, c, s }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ c, s }
{ a, d, s }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ d, s }
{ b, c, d }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ c, d, s }	{ b }	{ c, d }
{ b, c, s }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ c, d, s }	{ b }	{ c, s }
{ b, d, s }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ c, d, s }	{ b }	{ d, s }
{ c, d, s }	{ }	{ a, b }	{ c, d, s }	{ c, d, s }	{ d, s }	{ c }	{ d, s }
{ a, b, c, d }	{ a, b, c }	{ }	{ d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, c, d }
{ a, b, c, s }	{ a, b, c }	{ }	{ d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, c, s }
{ a, b, d, s }	{ a, b }	{ }	{ c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, d, s }
{ a, c, d, s }	{ a }	{ }	{ b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ c, d, s }
{ b, c, d, s }	{ }	{ a }	{ b, c, d, s }	{ b, c, d, s }	{ c, d, s }	{ b }	{ c, d, s }
{ a, b, c, d, s }	{ a, b, c, d, s }	{ }	{ }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ b, c, d, s }

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS:={{},{b,c,d,s},{a},X};print(`A N=5  Not-T0
  TOPOLOGY !!! `);LAT5abcd(LS,`Topology 3, p 440, vol 1`,5):
```

X := {a, b, c, d, s}

LS := {{ }, {a}, {b, c, d, s}, {a, b, c, d, s}}

A N=5 Not-T0 TOPOLOGY !!!

Is LS: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d, s}	{ }	{a}	{ }	{a}	{ }
{b}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{ }
{c}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{ }
{d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{ }
{s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d}	{s}	{ }
{a, b}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{ }
{a, c}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{ }
{a, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{ }
{a, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d}	{a, s}	{ }
{b, c}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, c}
{b, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, d}
{b, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, s}
{c, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{c, d}
{c, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{c, s}
{d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{d, s}
{a, b, c}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c}
{a, b, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d}
{a, b, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, s}
{a, c, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, c, d}
{b, c, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, c, s}
{b, d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, d, s}
{c, d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{ }	{c, d, s}
{a, b, c, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, s}
{a, b, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{b, c, d, s}	{a}	{ }	{b, c, d, s}	{b, c, d, s}	{ }	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS:={{},{a},{c,d},{a,c,d},{b,c,d,s},X};print
  (`A N=5  Not-T0 TOPOLOGY !!! `);LAT5abcd(LS,`Topology 4, p 440, vol 1
  ??????`,5):
```

X := {a, b, c, d, s}

LS := {{ }, {a}, {c, d}, {a, c, d}, {b, c, d, s}, {a, b, c, d, s}}

A N=5 Not-T0 TOPOLOGY !!!

Is LS: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d, s}	{ }	{a}	{ }	{a}	{ }
{b}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{ }
{c}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{ }
{d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{ }
{s}	{ }	{a, c, d}	{b, s}	{b, s}	{b}	{s}	{ }
{a, b}	{a}	{c, d}	{b, s}	{a, b, s}	{s}	{a, b}	{ }
{a, c}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{ }
{a, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{ }
{a, s}	{a}	{c, d}	{b, s}	{a, b, s}	{b}	{a, s}	{ }
{b, c}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b}
{b, d}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{b}
{b, s}	{ }	{a, c, d}	{b, s}	{b, s}	{b, s}	{ }	{b, s}
{c, d}	{c, d}	{a}	{b, s}	{b, c, d, s}	{b, c, d, s}	{ }	{c, d}
{c, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{s}
{d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{s}
{a, b, c}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b}
{a, b, d}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{b}
{a, b, s}	{a}	{c, d}	{b, s}	{a, b, s}	{b, s}	{a}	{b, s}
{a, c, d}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{s}
{a, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{s}
{b, c, d}	{c, d}	{a}	{b, s}	{b, c, d, s}	{b, c, d, s}	{ }	{b, c, d}
{b, c, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, s}
{b, d, s}	{ }	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{a}	{b, s}	{b, c, d, s}	{b, c, d, s}	{ }	{c, d, s}
{a, b, c, d}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, s}
{a, b, d, s}	{a}	{ }	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{b, s}
{a, c, d, s}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{b, c, d, s}	{a}	{ }	{b, c, d, s}	{b, c, d, s}	{ }	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

```
> print(` `);LS:={{},{},{},{},{},{},{},{},{a,b,c,d}};LAT4abcd(LS,`A
  Connected Topology that is not T0, - but embeds as a Connected NOT T0
  topology in N=4, Ex #2 page 207 vol 6`,4):
```

LS := { { }, { a, b, c, d } }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ }
{ b }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, c, d }	{ b }	{ }
{ c }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, d }	{ c }	{ }
{ d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c }	{ d }	{ }
{ a, b }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b }
{ a, c }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, c }
{ a, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, d }
{ b, c }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ b, c }
{ b, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ b, d }
{ c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ c, d }
{ a, b, c }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, c }
{ a, b, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, d }
{ a, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, c, d }
{ b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ b, c, d }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, c, d }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({ a, b, c, d }, { a, b, c, d }, { a, b, c, d }, { a, b, c, d })

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```
> X:={a,b,c,d,s};print(` `);print(` discrete N=5, {{ },X}  with N=4 poset
  3 partition`);LS:={{},{a},{c},{a,b},{a,c},{c,d},{a,b,c},{a,c,d},{a,b,c,
  d},X};LAT5abcd(LS,`     EXCUSION DOWN   INCLUSION UP`,5):
```

X := {a, b, c, d, s}

discrete N=5, {{ },X} with N=4 poset 3 partition

LS := { { }, {a}, {c}, {a, b}, {a, c}, {c, d}, {a, b, c}, {a, c, d}, {a, b, c, d}, {a, b, c, d, s} }

Is LS: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

LS COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c, d}	{b, s}	{a, b, s}	{b, s}	{a}	{ }
{b}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{ }
{c}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{c, d}	{s}	{a, b, s}	{b, s}	{a}	{b}
{a, c}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{ }
{a, d}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, s}	{a, d}	{ }
{a, s}	{a}	{c, d}	{b, s}	{a, b, s}	{b, s}	{a}	{s}
{b, c}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{ }
{b, d}	{ }	{a, c}	{b, d, s}	{b, d, s}	{s}	{b, d}	{ }
{b, s}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{a, b}	{s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b}
{a, b, d}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, s}	{a, d}	{b}
{a, b, s}	{a, b}	{c, d}	{s}	{a, b, s}	{b, s}	{a}	{b, s}
{a, c, d}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d}
{a, c, s}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{s}
{a, d, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, s}	{a, d}	{s}
{b, c, d}	{c, d}	{a}	{b, s}	{b, c, d, s}	{d, s}	{b, c}	{d}
{b, c, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{s}
{b, d, s}	{ }	{a, c}	{b, d, s}	{b, d, s}	{s}	{b, d}	{s}
{c, d, s}	{c, d}	{a, b}	{s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, s}
{a, b, d, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, s}	{a, d}	{b, s}
{a, c, d, s}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d, s}
{b, c, d, s}	{c, d}	{a}	{b, s}	{b, c, d, s}	{d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d, s}

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```
> print(` `);LS:={{},{b},{d},{a,b},{b,d},{c,d},{a,b,d},{b,c,d},X};print(
  ` Let LSd be the dual of the preceeding example  X = {a,b,c,d}   poset
  3 `);LSd:=CLO({a,b,c,d},LS);print(`This dual Lattice Structure
  represents the marriage between Cartan's Closure of Exterior
  Differential Systems \n and the Kuratowski Closure based upon a limit
  point operator equivalent to the exterior differential, d`) :LAT4abcd
  (LSd,`LSd =  poset 3, LSDual = poset 3, DISconnected T0 topologies
  EXCUSION DOWN   INCLUSION UP`,4):
```

LS := { { }, {b}, {d}, {a, b}, {b, d}, {c, d}, {a, b, d}, {b, c, d}, {a, b, c, d, s} }

Let LSd be the dual of the preceeding example X = {a,b,c,d} poset 3

LSd := { { }, {a}, {c}, {a, b}, {a, c}, {c, d}, {a, b, c}, {a, c, d}, {a, b, c, d} }

This dual Lattice Structure represents the marriage between Cartan's Closure of Exterior Differential Systems and the Kuratowski Closure based upon a limit point operator equivalent to the exterior differential, d

Is LS: a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c, d}	{b}	{a, b}	{b}	{a}	{ }
{b}	{ }	{a, c, d}	{b}	{b}	{ }	{ }	{ }
{c}	{c}	{a, b}	{d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a, b}	{c, d}	{ }	{a, b}	{b}	{ }	{b}
{a, c}	{a, c}	{ }	{b, d}	{a, b, c, d}	{b, d}	{a, c}	{ }
{a, d}	{a}	{c}	{b, d}	{a, b, d}	{b}	{a, d}	{ }
{b, c}	{c}	{a}	{b, d}	{b, c, d}	{d}	{b, c}	{ }
{b, d}	{ }	{a, c}	{b, d}	{b, d}	{ }	{ }	{ }
{c, d}	{c, d}	{a, b}	{ }	{c, d}	{d}	{ }	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{b, d}	{a, c}	{b}
{a, b, d}	{a, b}	{c}	{d}	{a, b, d}	{b}	{ }	{b}
{a, c, d}	{a, c, d}	{ }	{b}	{a, b, c, d}	{b, d}	{a, c}	{d}
{b, c, d}	{c, d}	{a}	{b}	{b, c, d}	{d}	{ }	{d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, d}	{ }	{b, d}

The Closed-Open subsets of LS are = { { }, {a, b}, {c, d}, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a, b}, {b}, {c, d}, {d})