Applications of

The Category Theory of Topological Thermodynamics

R. M. Kiehn
Professor of Physics
University of Houston

Applications of

The Category Theory of Topological Thermodynamics

This article was motivated in part by the challenge of the Clay Institute regarding the properties of the Navier-Stokes equations and their relationship to hydrodynamic turbulence. To replicate a statement made by the Clay Institute:

"The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations."

Applications of

The Category Theory of Topological Thermodynamics

Applications of

The Category Theory of Topological Thermodynamics

will prove that the

Navier-Stokes equations have Turbulent solutions

The method will be to use the abstract Category Theory of Topological Thermodynamics for a non-equilibrium particle system, and show that there exists a homotopic evolution of the system topology for at least one specific process that is topologically equivalent to a thermodynamic irreversible process, T_4.

The method will be to use the abstract Category Theory of Topological Thermodynamics for a non-equilibrium particle system, and show that there exists a homotopic evolution of the system topology for at least one specific process that is topologically equivalent to a thermodynamic irreversible process, T_4.

Then it will be shown that there are specific choices of functions that permit the cohomological statement of the First Law of Thermodynamics to be put into 1-1 correspondence with the functions that define the Navier-Stokes equations.

The method will be to use the abstract Category Theory of Topological Thermodynamics for a non-equilibrium particle system, and show that there exists a homotopic evolution of the system topology for at least one specific process that is topologically equivalent to a thermodynamic irreversible process, T_4.

Then it will be shown that there are specific choices of functions that permit the cohomological statement of the First Law of Thermodynamics to be put into 1-1 correspondence with the functions that define the Navier-Stokes equations.

Finally, the abstract irreversible process, T_4, will be evaluated in terms of the Navier-Stokes functions, thereby proving that there is a least one solution which is irreversible, and describes a turbulent process .

What is Turbulence?

Unfortunately, no precise (universally accepted)

geometrical definition exists.

What is Turbulence?

Unfortunately, no precise (universally accepted)

geometrical definition exists.

Turbulence (more or less) is
(intuitively) a time dependent motion of a continuum fluid which is the antithesis of a streamline flow.

The fluid motion may or may not be chaotic or self-similar.

However there is a BOTTOM LINE:

What is Turbulence?

Unfortunately, no precise (universally accepted)

geometrical definition exists.

Turbulence (more or less) is
(intuitively) a time dependent motion of a continuum fluid
which is the antithesis of a streamline flow.

The fluid motion may or may not be chaotic or self-similar.

However there is a BOTTOM LINE:

Almost everyone will agree that Turbulence involves an

Irreversible Thermodynamic Process

How do you detect Turbulence?

Turbulence is a process in a topological space of

Pfaff Topological dimension 4 that involves

exchange of radiation (waves) and matter (particles).

How do you detect Turbulence?

Turbulence is a process in a topological space of

Pfaff Topological dimension 4 that involves

exchange of radiation (waves) and matter (particles).

However, turbulence is visually detected by particle-like

topologically coherent defects of

Pfaff Topological dimension 3 or less

embedded in the topological 4D environment

How do you detect Turbulence?

Turbulence is a process in a topological space of

Pfaff Topological dimension 4 that involves

exchange of radiation (waves) and matter (particles).

However, turbulence is visually detected by particle-like

topologically coherent defects of

Pfaff Topological dimension 3 or less

embedded in the topological 4D environment

If I have time I will display various topological defects

Embedded in the topological 4D environment.

The Category theory of Topological Thermodynamics and any exterior differential 1-form, A, of rank 4, can be used to

1. generate a disconnected Cartan Topology that defines a non-equilibrium thermodynamic system of particles.

The Category theory of Topological Thermodynamics and any exterior differential 1-form, A, of rank 4, can be used to

1. generate a disconnected Cartan Topology that defines a non-equilibrium thermodynamic system of particles.

2. generate a Jacobian correlation matrix, $[\partial A_k/\partial x^k]$ that has a singular set of rank 3, and which is a morphism of a universal van der Waals gas.

The Category theory of Topological Thermodynamics and any exterior differential 1-form, A, of rank 4, can be used to

1. generate a disconnected Cartan Topology that defines a non-equilibrium thermodynamic system of particles.

2. generate a Jacobian correlation matrix, [$\partial A_k/\partial x^k$] that has a singular set of rank 3, and which is a morphism of a universal van der Waals gas.

3. generate a unique process current of Topological Torsion, T = A^dA, which describes an irreversible thermodynamic process in a non-equilibrium system.

The Category theory of Topological Thermodynamics, with homotopic morphisms mapping topological structures A => Q produces a universal topological:

FIRST LAW OF THERMODYNAMICS

$$L(J)A = i(J)dA + d\{i(J)A\} \Rightarrow Q$$

The Category theory of Topological Thermodynamics, with homotopic morphisms mapping topological structures A => Q produces a universal topological:

FIRST LAW OF THERMODYNAMICS

$$L(J)A = i(J)dA + d\{i(J)A\} \Rightarrow Q$$

$$L(J)A = W + d\{U\} \Rightarrow Q$$

The Lie differential of the Action 1- form, A, (relative to the process, J) generates the inexact 1-form of Work, W, plus the differential of the Internal energy, dU, which is equal to the inexact 1-form of Heat = Q.

The Homotopy operator relative to a process J acting on exterior differential forms is given by Cartan's Magic formula

$$L_{(J)} = i(J)d + d\{i(J)\}$$

The Homotopy operator relative to a process J acting on exterior differential forms is given by Cartan's Magic formula

$$L_{(J)} = i(J)d + d\{i(J)\}$$

Consider a process J such that

$$J = A\hat{}dA = i(T_4)dx\hat{}dy\hat{}dz\hat{}dt$$

"The Topological Torsion 3-form"

Topological Torsion Properties

$$\mathbf{T}_4 \text{ on } \Omega_4 : \textbf{Properties of Topological Torsion}$$

$$i(\mathbf{T}_4)\Omega_4 = i(\mathbf{T}_4)dx\hat{}dy\hat{}dz\hat{}dt = A\hat{}dA,$$

$$i(\mathbf{T}_4)i(\mathbf{T}_4)\Omega_4 = 0, \text{ which implies that}$$

$$\text{Work 1-form} \quad W = i(\mathbf{T}_4)dA = \sigma\ A,$$

$$dW = d\sigma\hat{}A + \sigma dA = dQ,$$

$$\text{Internal Energy} \;\; U = i(\mathbf{T}_4)A = 0, \quad \mathbf{T}_4 \text{ is associative,}$$

$$i(\mathbf{T}_4)dU = 0$$

$$i(\mathbf{T}_4)Q = 0 \qquad \mathbf{T}_4 \text{ is adiabatic}$$

$$L_{(\mathbf{T}_4)}A = \sigma\ A, \quad \mathbf{T}_4 \text{ is homogeneous and self-similar}$$

$$L_{(\mathbf{T}_4)}dA = d\sigma\hat{}A + \sigma dA = dQ,$$

$$Q\hat{}dQ = L_{(\mathbf{T}_4)}A\hat{}L_{(\mathbf{T}_4)}dA = \sigma^2 A\hat{}dA \neq 0, \;\; \mathbf{T}_4 \text{ is irreversible,}$$

$$dA\hat{}dA = d(A\hat{}dA) = d\{(i(\mathbf{T}_4)\Omega_4\} = (div_4\mathbf{T}_4)\Omega_4,$$

$$L_{(\mathbf{T}_4)}\Omega_4 = d\{(i(\mathbf{T}_4)\Omega_4\} = (2\sigma)\Omega_4, \;\; \mathbf{T}_4 \text{ causes } \Omega_4 \text{ expansion}$$

To be thermodynamically irreversible, a process J must

1. Create a heat 1-form, Q, that (because of shear viscosity) is chaotic, not integrable and of PTD(Q)>2:

$$Q \wedge dQ \neq 0$$

2. and the 3-form A^dA should not be closed. (due to bulk viscosity of expansion-contraction) PTD(A) = 4

$$d(A \wedge dA) \neq 0$$

Topological Torsion A^dA is a key design tool for controlling and understanding

Dissipative Structures and

TURBULENCE

Topological Torsion A^dA is a key design tool for controlling and understanding

Dissipative Structures and

TURBULENCE

Almost NO Engineers and Very Few Physicists Understand

TOPOLOGICAL TORSION

(pity)

The next step is to use the Topological First Law

as an equation of homotopic evolution.

$$L_{(J)}A = W + d\{U\} = Q$$

The next step is to use the Topological First Law

as an equation of homotopic evolution.

$$L_{(J)}A = W + d\{U\} = Q$$

and deduce a functional choice of A

that replicates

the Navier-Stokes equations of motion.

Consider the exterior differential 1-form, A, of Action per unit source (in fluids, the unit source is mole number, or sometimes mass), constructed from a covariant 3D velocity field, $\mathbf{v} = \mathbf{v}_k(x,y,z,t)$, and a scalar potential function, ϕ :

$$A = \mathbf{v} \circ \mathbf{dr} - \phi dt = \mathbf{v}_k(x, y, z, t) dx^k - \phi dt.$$

Consider the exterior differential 1-form, A, of Action per unit source (in fluids, the unit source is mole number, or sometimes mass), constructed from a covariant 3D velocity field, $\mathbf{v} = \mathbf{v}_k(x,y,z,t)$, and a scalar potential function, ϕ :

$$A = \mathbf{v} \circ \mathbf{dr} - \phi dt = \mathbf{v}_k(x, y, z, t) dx^k - \phi dt.$$

Compute the exterior differential dA and define the following functions as:

$$\omega = curl\ \mathbf{v}, \qquad \mathbf{a} = +\{\partial \mathbf{v}/\partial t + grad(\phi)\},$$

$$F = dA = \omega_z dx\hat{\ }dy + \omega_x dy\hat{\ }dz + \omega_y dz\hat{\ }dx - \mathbf{a}_x dx\hat{\ }dt - \mathbf{a}_y dy\hat{\ }dt - \mathbf{a}_z dz\hat{\ }dt,$$

$$dF = 0 \supset curl\ (-\mathbf{a}) + \partial\omega/\partial t = 0, \qquad div\ \omega = 0. \text{ The Faraday induction PDE's.}$$

Consider the exterior differential 1-form, A, of Action per unit source (in fluids, the unit source is mole number, or sometimes mass), constructed from a covariant 3D velocity field, $\mathbf{v} = \mathbf{v}_k(x,y,z,t)$, and a scalar potential function, ϕ :

$$A = \mathbf{v} \circ \mathbf{dr} - \phi dt = \mathbf{v}_k(x, y, z, t)dx^k - \phi dt.$$

Compute the exterior differential dA and define the following functions as:

$$\omega = curl\ \mathbf{v}, \qquad \mathbf{a} = +\{\partial \mathbf{v}/\partial t + grad(\phi)\},$$
$$F = dA = \omega_z dx\hat{}dy + \omega_x dy\hat{}dz + \omega_y dz\hat{}dx - \mathbf{a}_x dx\hat{}dt - \mathbf{a}_y dy\hat{}dt - \mathbf{a}_z dz\hat{}dt,$$
$$dF = 0 \supset curl\ (-\mathbf{a}) + \partial\omega/\partial t = 0, \qquad div\ \omega = 0. \text{ The Faraday induction PDE's.}$$

In essence, the topological formulation of electrodynamic intensity fields and hydrodynamic intensity fields are identical except for notation.

Using the notational equivalences,

$$\mathbf{A} \Leftrightarrow \mathbf{v}, \qquad \phi \Leftrightarrow \mathbf{v} \cdot \mathbf{v}/2,$$
$$\mathbf{E} \Leftrightarrow -\mathbf{a}, \qquad \mathbf{B} \Leftrightarrow \omega.$$

permits the EM formats to be rewritten in hydrodynamic format. The Work 1-form for a fluid becomes,

$$-\rho\{-\mathbf{a}+\mathbf{v}\times\omega\}\circ d\mathbf{r}-\ \rho\{\mathbf{v}\circ\mathbf{a}\}dt=W.$$

permits the EM formats to be rewritten in hydrodynamic format. The Work 1-form for a fluid becomes,

$$-\rho\{-\mathbf{a}+\mathbf{v}\times\omega\}\circ d\mathbf{r}-\ \rho\{\mathbf{v}\circ\mathbf{a}\}dt=W.$$

Specialization of the topological properties of the Work 1-form lead to familiar formulations of hydrodynamics. For example, suppose that the PTD of W is 1; then W = - dP. With this topological constraint, the system of PDE's are recognized to be those that describe the classic Eulerian fluid:

$$-\rho\{-\mathbf{a}+\mathbf{v}\times\omega\}\circ d\mathbf{r}-\ \rho\{\mathbf{v}\circ\mathbf{a}\}dt=-dP,$$

$$\{\partial\mathbf{v}/\partial t+grad(\mathbf{v}\cdot\mathbf{v}/2)-\mathbf{v}\times\omega\}=-grad(P)/\rho,$$

$$\text{Power theorem}\quad \partial P/\partial t=\rho\mathbf{v}\circ\mathbf{a}.$$

permits the EM formats to be rewritten in hydrodynamic format. The Work 1-form for a fluid becomes,

$$-\rho\{-\mathbf{a}+\mathbf{v}\times\omega\}\circ d\mathbf{r}-\ \rho\{\mathbf{v}\circ\mathbf{a}\}dt=W.$$

Specialization of the topological properties of the Work 1-form lead to familiar formulations of hydrodynamics. For example, suppose that the PTD of W is 1; then W = - dP. With this topological constraint, the system of PDE's are recognized to be those that describe the classic Eulerian fluid:

$$\begin{gathered}-\rho\{-\mathbf{a}+\mathbf{v}\times\omega\}\circ d\mathbf{r}-\ \rho\{\mathbf{v}\circ\mathbf{a}\}dt=-dP,\\ \{\partial\mathbf{v}/\partial t+grad(\mathbf{v}\cdot\mathbf{v}/2)-\mathbf{v}\times\omega\}=-grad(P)/\rho,\\ \text{Power theorem}\quad \partial P/\partial t=\rho\mathbf{v}\circ\mathbf{a}.\end{gathered}$$

The "Bernoulli function (~Pressure)", P, if it exists, must be a first integral (a process invariant),

$$L(\rho\mathbf{V}_4)P=(i(\rho\mathbf{V}_4)dP=0.$$

Based on the Pfaff Topological Dimension of W

Thermodynamic Reversible Processes imply that the Heat 1-form, Q is integrable. $Q \wedge dQ = 0$

Extremal: PTD(W) = 0, W = 0,

Hamiltonian

Bernoulli-Casimir: PTD(W) = 1, W exact

Hamiltonian

Helmholtz: PTD(W) = 1, W closed

Conservation of Vorticity

Each of these flows are thermodynamically reversible, as

$$dW = 0 = dQ, \quad \text{implies} \quad Q \wedge dQ = 0.$$

In order to go beyond Hamiltonian or Bernoulli flows, it is necessary that the Pfaff Topological Dimension of the Work 1-form must be greater than 1. Recall that for any process, the Work done is transverse to the process trajectory,

$$(i(\rho \mathbf{V}_4)W = (i(\rho \mathbf{V}_4)(i(\rho \mathbf{V}_4)dA = 0.$$

In order to go beyond Hamiltonian or Bernoulli flows, it is necessary that the Pfaff Topological Dimension of the Work 1-form must be greater than 1. Recall that for any process, the Work done is transverse to the process trajectory,

$$(i(\rho \mathbf{V}_4)W = (i(\rho \mathbf{V}_4)(i(\rho \mathbf{V}_4)dA = 0.$$

Hence, if the **PTD** of the Work 1-form, **W**, is to be greater than 1, it must have the format,

$$W = i(\rho \mathbf{V}_4)dA = -dP + \varpi_j(d\mathbf{x}^j - \mathbf{v}^j dt) = -dP + \varpi_j \Delta \mathbf{x}^j,$$

where $\Delta \mathbf{x}^j$ represents the topological fluctuation about kinematic perfection. It is also important to remember that such non-zero contributions to the Work 1-form are due to the complex, isotropic Cartan Spinors, which are the eigen direction fields of the 2-form, F.

The coefficients, ϖ_j, of the topological fluctuations, Δx^j, act in the manner of Lagrange multipliers, and mimic the concept of system forces. If ϖ_j is defined (arbitrarily) as υ curl curlv, then the **spatial** components of the thermodynamic Work 1-form, W, are constrained to yield the partial differential equations for a constant density Navier-Stokes fluid:

$$\partial V/\partial t + \mathrm{grad}\ V^2/2 - V\times \mathrm{curl}\ V = -\ \mathrm{grad}P/\rho - \nu\nabla^2 V$$

The coefficients, ϖ_j, of the topological fluctuations, Δx^j, act in the manner of Lagrange multipliers, and mimic the concept of system forces. If ϖ_j is defined (arbitrarily) as υ curl curlv, then the **spatial** components of the thermodynamic Work 1-form, W, are constrained to yield the partial differential equations for a constant density Navier-Stokes fluid:

$$\partial \mathbf{V}/\partial t + \mathrm{grad}\ \mathbf{V}^2/2 - \mathbf{V} \times \mathrm{curl}\ \mathbf{V} = - \mathrm{grad}P/\rho - \nu \nabla^2 \mathbf{V}$$

This is one of many formal choices, but the choice demonstrates that the Navier-Stokes equations reside within the domain of non-equilibrium thermodynamics. **QED**

The coefficients, ϖ_j, of the topological fluctuations, Δx^j, act in the manner of Lagrange multipliers, and mimic the concept of system forces. If ϖ_j is defined (arbitrarily) as υ curl curlv, then the **spatial** components of the thermodynamic Work 1-form, W, are constrained to yield the partial differential equations for a constant density Navier-Stokes fluid:

$$\partial \mathbf{V}/\partial t + \mathrm{grad}\ \mathbf{V}^2/2 - \mathbf{V}\times \mathrm{curl}\ \mathbf{V} = -\ \mathrm{grad}P/\rho - \nu\nabla^2 \mathbf{V}$$

This is one of many formal choices, but the choice demonstrates that the Navier-Stokes equations reside within the domain of non-equilibrium thermodynamics. **QED**

Density variations can be included by adding a term λ div(**v**) to the potential {**v**∘**v**/2} to yield:

$$\partial \mathbf{V}/\partial t + \mathrm{grad}\ \mathbf{V}^2/2 - \mathbf{V}\times \mathrm{curl}\ \mathbf{V} = -\ \mathrm{grad}P/\rho + \lambda\, \mathrm{grad}\ \mathrm{div}\ \mathbf{V} - \nu\nabla^2 \mathbf{V}$$

Classically, **ν** = shear viscosity, and **λ** = (μ_B - ν) where μ_B = Bulk viscosity

The next step is to compute the abstract

Topological Torsion 3-form, T_4 = $A \wedge dA$

using the functions that replicate the

Navier-Stokes equations

It can be shown that the abstract

Topological Torsion 3-form, $T_4 = A \wedge dA$,

generates a thermodynamically irreversible process.

By inserting the functions that replicate the Navier-Stokes equations into the formula for T_4 , it is possible to derive an example of a solution to the Navier-Stokes equations that is non-integrable, chaotic, and thermodynamically irreversible

or

TURBULENT

The abstract formulation of the Topological Torsion leads to the 4 component functions:

$$\mathbf{T}_4 = [-\mathbf{a}\times\mathbf{v} + \{\mathbf{v}\circ\mathbf{v}/2\}\ curl\ \mathbf{v}, (\mathbf{v}\circ curl\ \mathbf{v})],$$
$$= [-\mathbf{a}\times\mathbf{v} + \{\mathbf{v}\circ\mathbf{v}/2\}\ \omega, (\mathbf{v}\circ\omega)] = [\mathbf{T}, h].$$

Note that the topological torsion axial vector current persists even for Euler flows with zero vorticity, $\omega=0$. Moreover if the flow is harmonic, the topological torsion axial vector still exists with a term proportional to the bulk viscosity, μ_B.

From the expression for the homotopic first law that replicates the Navier-Stokes equations, it is possible to solve for the acceleration, **a**.

$$
\begin{aligned}
\mathbf{a} &= [grad\{\mathbf{v} \circ \mathbf{v}/2\} + \partial \mathbf{v}/\partial t] \\
&= \mathbf{v} \times curl\ \mathbf{v} - gradP/\rho \\
&+ \lambda\{grad(div\ \mathbf{v})\} + \upsilon\{curl\ curl\ \mathbf{v}\},
\end{aligned}
$$

Substitute the expression for a into the equation for the components of the Topological Torsion 3-form,

$$\mathbf{T}_4 = \mathbf{A}\hat{}d\mathbf{A}$$

$$
\begin{aligned}
\mathbf{T} &= [h\mathbf{v} - (\mathbf{v} \circ \mathbf{v}/2)curl\ \mathbf{v} - \mathbf{v} \times (gradPl\rho) \\
&+ \lambda\{\mathbf{v} \times grad(div\ \mathbf{v})\} - \upsilon\{\mathbf{v} \times (curl\ curl\ \mathbf{v})\}], \\
h &= \mathbf{v} \circ curl\,\mathbf{v},
\end{aligned}
$$

For the Navier–Stokes fluid, the Topological Parity Dissipation Coefficient, K =dA^dA, if NOT zero, insures that flow-process V is thermodynamically irreversible,

or equivalently, Turbulent, with a dissipation factor K:

$$K = -2(a \cdot \omega) = 2\,\{\mathrm{grad}P/\rho - \mu_B\ \mathrm{grad\ div}\ V - \nu\ V \times \nabla^2 V\}\cdot\omega$$

For a Navier Stokes fluid, the universal Dissipation Coefficient, K, is the sum of Baroclinic forces, minus accelerations of expansion (Bulk viscosity) and accelerations of rotation (shear viscosity), times the flow vorticity, ω.

For A Plasma K= 2(E • B)

To minimize dissipation, the fluid acceleration and the fluid vorticity should be orthogonal .

Summing up, Category theory has shown

how certain solutions of the

Navier-Stokes Equations

$$\partial V/\partial t + \text{grad } V^2/2 - V\times \text{curl } V = - dP/\rho + \mu_B \text{ grad div } V - \nu\nabla^2 V$$

generate Thermodynamically Irreversible Processes

and how these solutions may be used to give insight into

Turbulence

In terms of a universal (topological) dissipation Coefficient, K

Part 2.

Topological, Non-Equilibrium Thermodynamics

And its Turbulent Artifacts

Examples of Turbulence

Turbulence recognized by entrapped disconnected bubbles

Laminar

Non-turbulent

Turbulent recognition in terms of bubble generation and froth.

Turbulence and Topological Defects

The topological defects produced by turbulence are often associated with the evolution of deformation invariants,

such as a evolutionary change

of phase, or the number of parts.

Or the condensation from vapor to droplets,

or the amalgamation of droplets into liquid,

or the creation of wakes, vortices and solitons.

Turbulence and Topological Defects

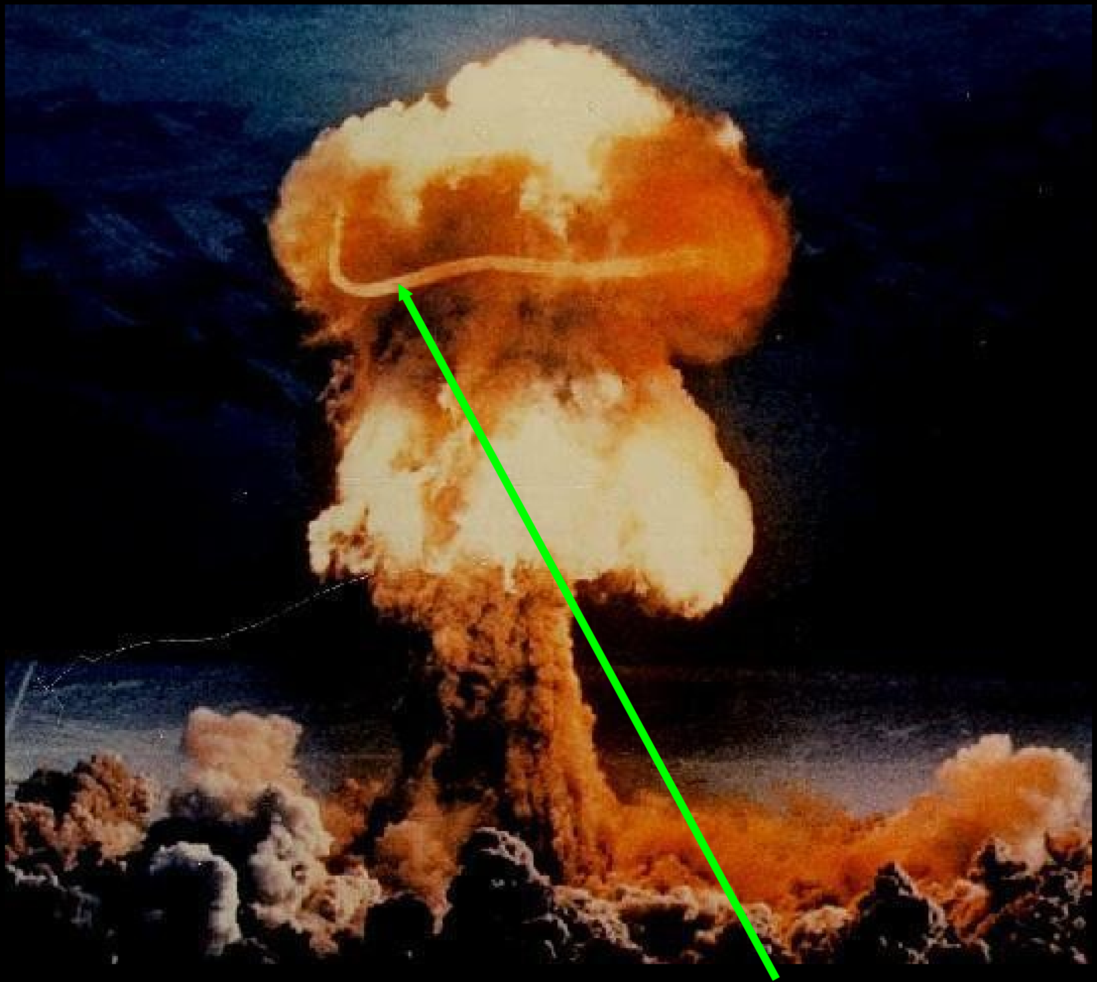
In this presentation I would like to emphasize a common occurrence,

which is the emergence caused by irreversible thermodynamic, turbulent processes,

# of Spiral Arms

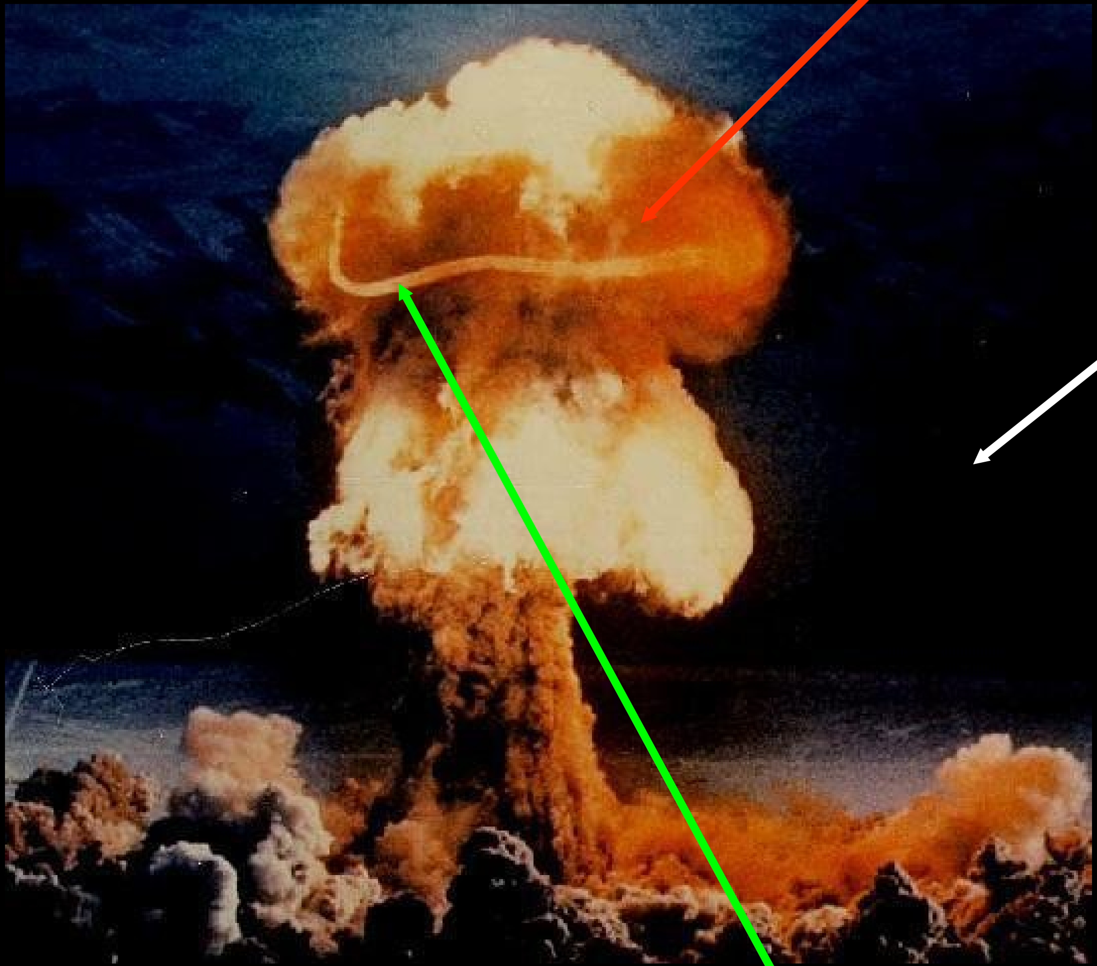
and a Tubular Vortex.

While detonating Atom Bombs in Nevada



I was puzzled by the persistent long-lived **Vortex Ring** emergent from the turbulence of a nuclear explosion

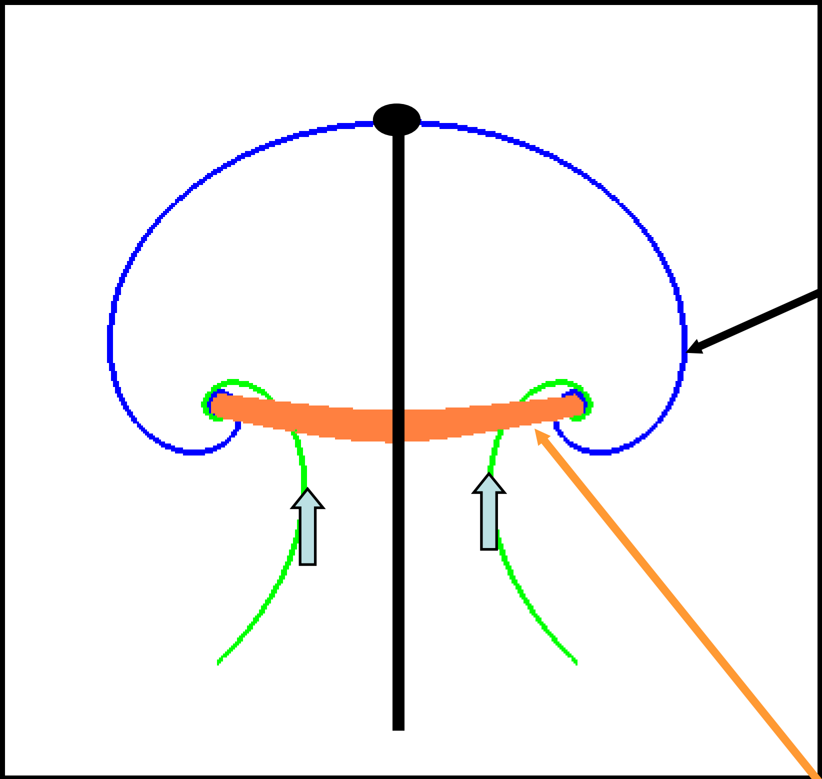
Vortex & Spiral Arm generation by turbulent flows Ex 1.



Non-Turbulent region disconnected by phase (mole number of parts)

Note the persistent long-lived **Vortex Ring** emergent from the turbulence of a nuclear explosion

Vortex & Spiral Arm generation by turbulent flows Ex 1.



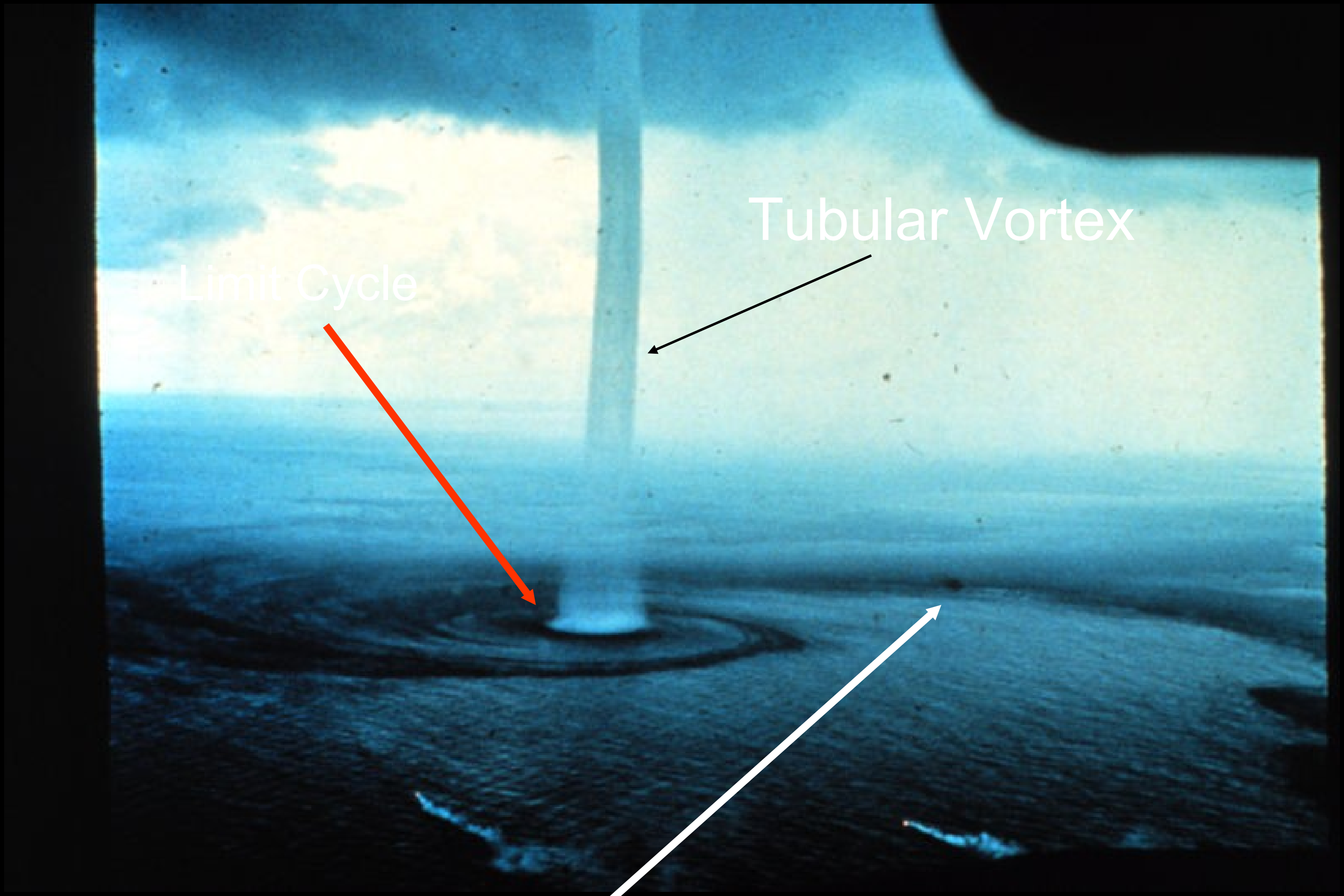
The Mushroom shape is an indicator of the Rayleigh-Taylor instability.

The spiral arms are rotated about z axis, and have Frenet Theory exact solutions

Note the persistent long-lived ionized **Vortex Ring** emergent from the turbulence of a nuclear explosion

Vortex & Spiral Arm generation by turbulent flows Ex 2.

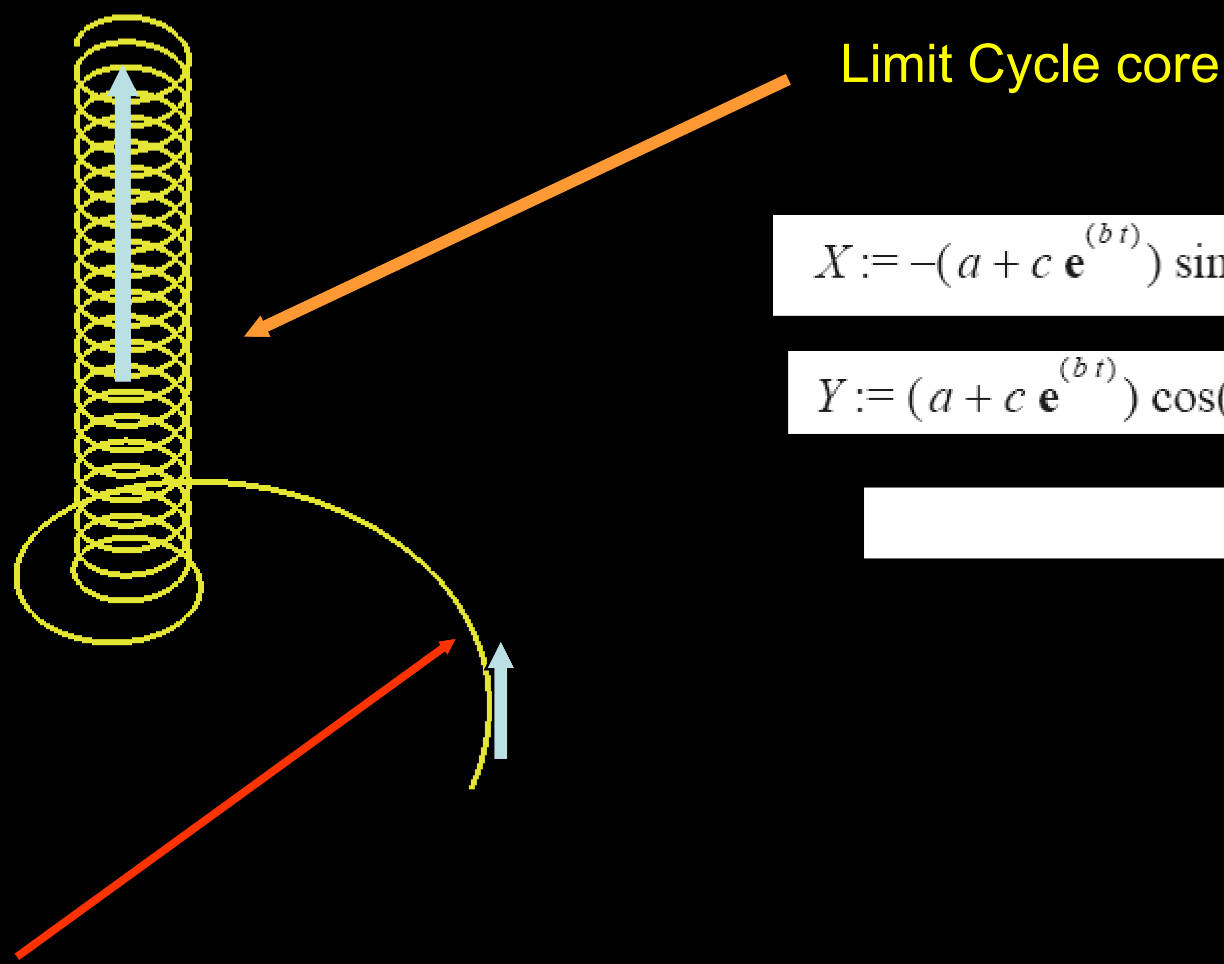
Vortex & Spiral Arm generation by turbulent flows Ex 2.



Note the spiral arms in the boundary layer, between the water and air, leading into the water spout vortex core.

Vortex and Spiral Arms

Generated by Solutions to the equations of Continuous Topological Evolution



$$X := -(a + c\,\mathbf{e}^{(b\,t)})\,\sin(\omega\,t)$$

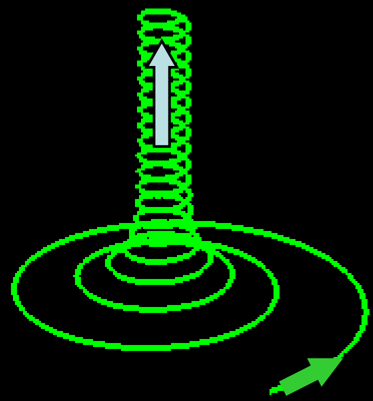
$$Y := (a + c\,\mathbf{e}^{(b\,t)})\,\cos(\omega\,t)$$

Note the spiral arms in the boundary layer, between the water and air, leading into the water spout vortex core.

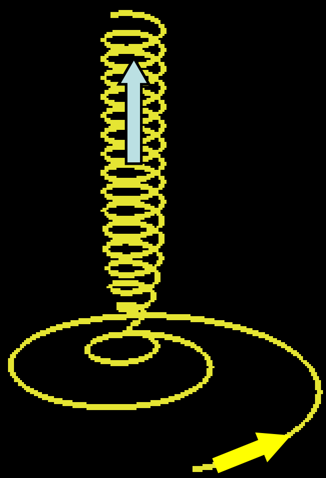
Vortex and Spiral Arms

Surprise Two Classes Of Solutions.

1. Those Solutions that decay by contraction from the exterior to the Limit Cycle.
2. A New Solution that decays by contraction from the exterior, Penetrates the Limit cycle, and then expands to the Limit Cycle from the interior.

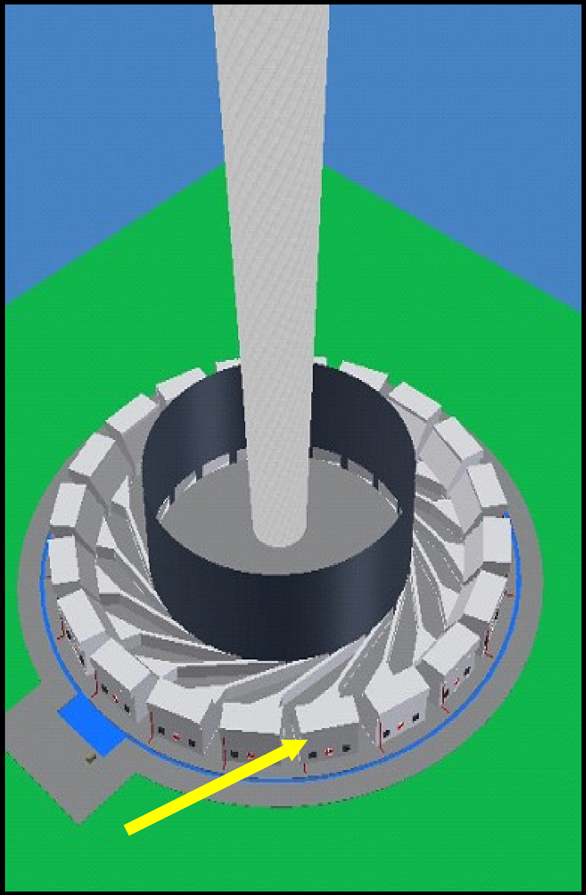


Solution 1



Solution 2

Vortex generation by spiral turbulent flows



Tangential entry of warm air creates a Vortex Limit Cycle

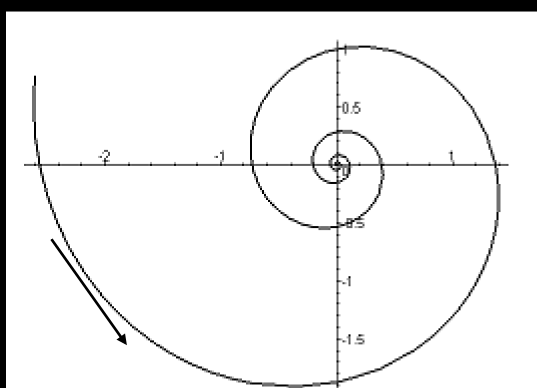
Compare this patented device to the water spout of the previous slide

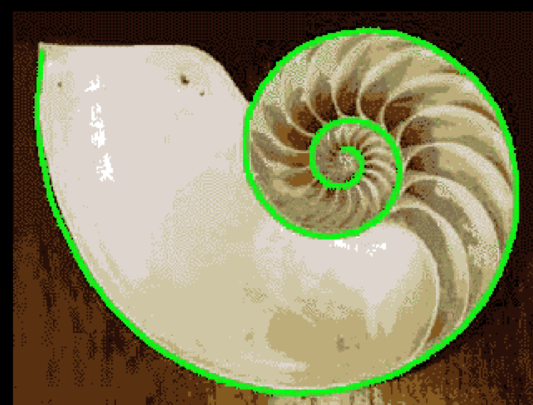
Consider a Dynamical System in a 2D Fluid

$$U := -b\,x(t) - \omega\,y(t)$$
$$V := -b\,y(t) + \omega\,x(t)$$

$$x(t) := e^{(-b\,t)}\cos(\omega\,t)$$
$$y(t) := e^{(-b\,t)}\sin(\omega\,t)$$

Generate Ubiquitous Logarithmic Spirals



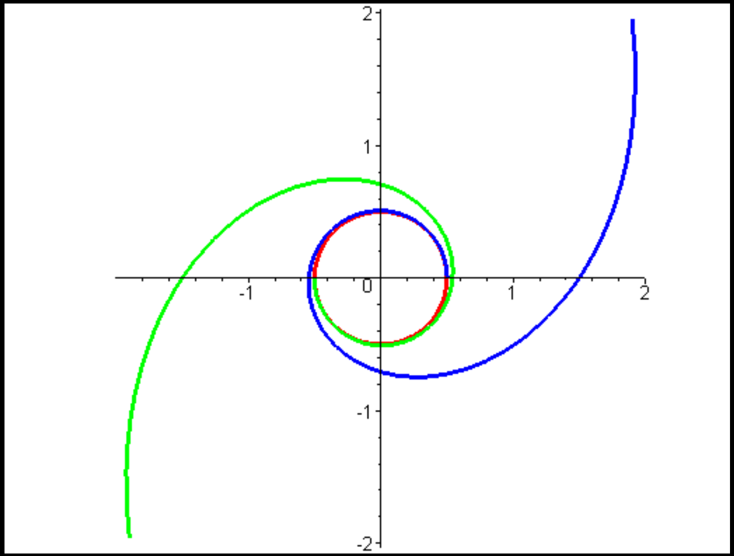


But no limit cycles

The equations

$$X := -(a + c\,\mathbf{e}^{(b\,t)})\sin(\omega\,t) \qquad Y := (a + c\,\mathbf{e}^{(b\,t)})\cos(\omega\,t)$$

Generate Logarithmic Spirals with Limit Cycles



Spiral arms generated by turbulent wakes

Limit Cycle Core

Vortex Cores and Spiral Wakes are artifacts of Turbulent Dissipation.

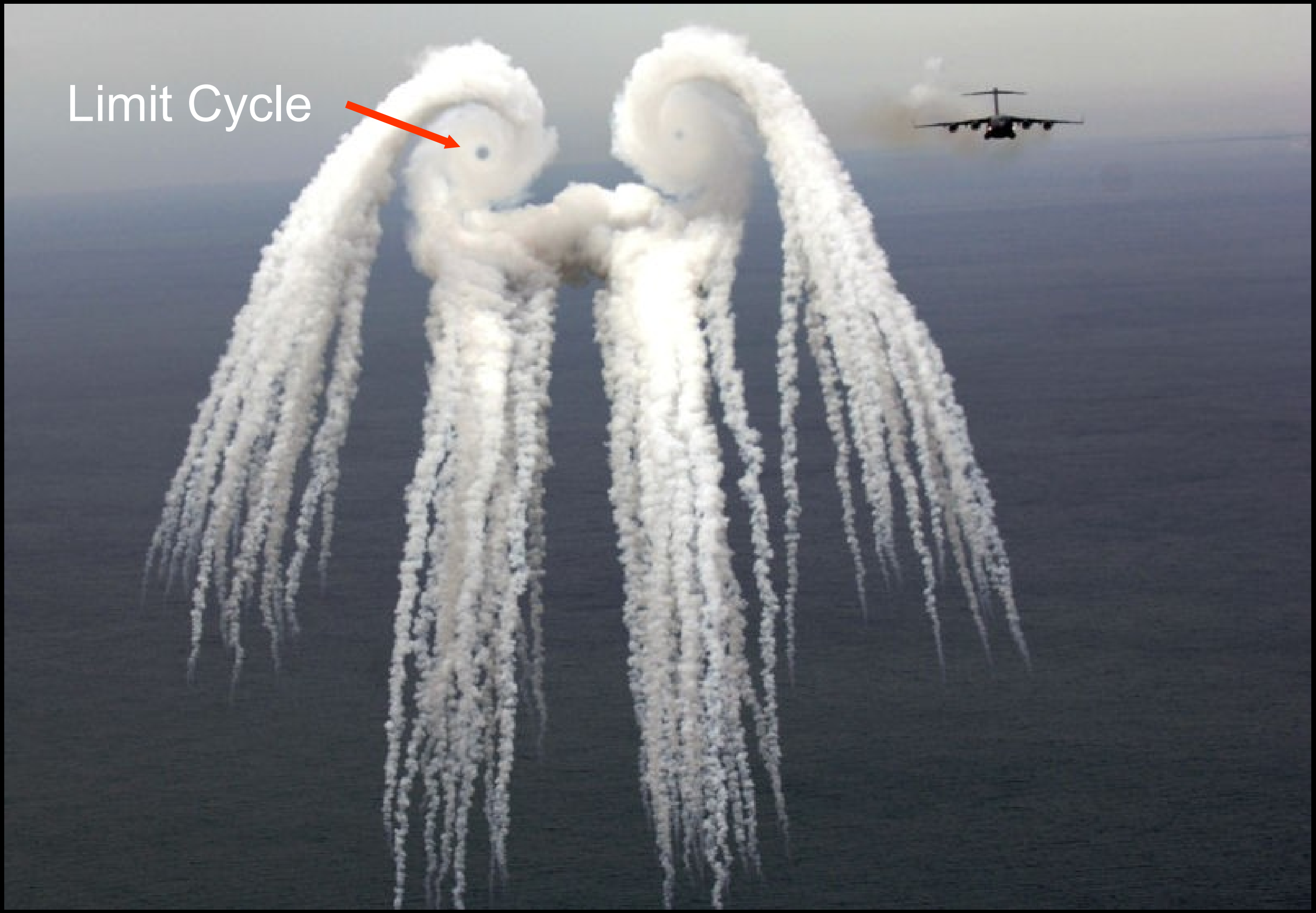
Vortex & Spiral Arm generation **by turbulent flows**

The bulk of energy loss for an aircraft is due to the turbulent generation of tip vortices – Save energy-preserve the ecology .

Vortex & Spiral Arm generation by turbulent flows

Wake turbulence is a severe, local, ecological problem.

Vortex & Spiral Arm generation by turbulent flows



Note the persistent long-lived eye of the storm about the vortex core generated in the wake of a C5a

Vortex & Spiral Arm generation by turbulent flows

Note the spiral arms in the turbulent wake. The Spirals appear to be precursors of the vortex cores.

Vortex & Spiral Arm generation **by turbulent flows Ex 3.**

The Basic idea is that expansion and rotation are associated with flow fixed points.

It can be shown that the 3D dynamical system
given be the equations

$$
\begin{aligned}
\mathbf{V}^x &= \{\mp\Omega y + (xg(r, z, a, b...\},\\
\mathbf{V}^y &= \{\pm\Omega x + (yg(r, z, a, b...)\},\\
\mathbf{V}^z &= f(r, z, \lambda, \alpha).
\end{aligned}
$$

are solutions to the Navier-Stokes Equations
in a Rotating Frame of reference,

"Some (new) closed form solutions to the Navier-Stokes equations"
http:www22.pair.com/csdc/pdf/nvsol2.pdf

Vortex & Spiral Arm generation by turbulent flows Ex 3.

A very simple subset of the formulas are given
by the equations of a dynamical system :

$$Vx := B\,x - \omega\,y$$
$$Vy := B\,y + \omega\,x$$
$$Vz := C\,z$$

B is the expansion parameter
ω is the rotation parameter

This dynamical system is a special case of a
3D Tertiary Hopf bifurcation (Langford).

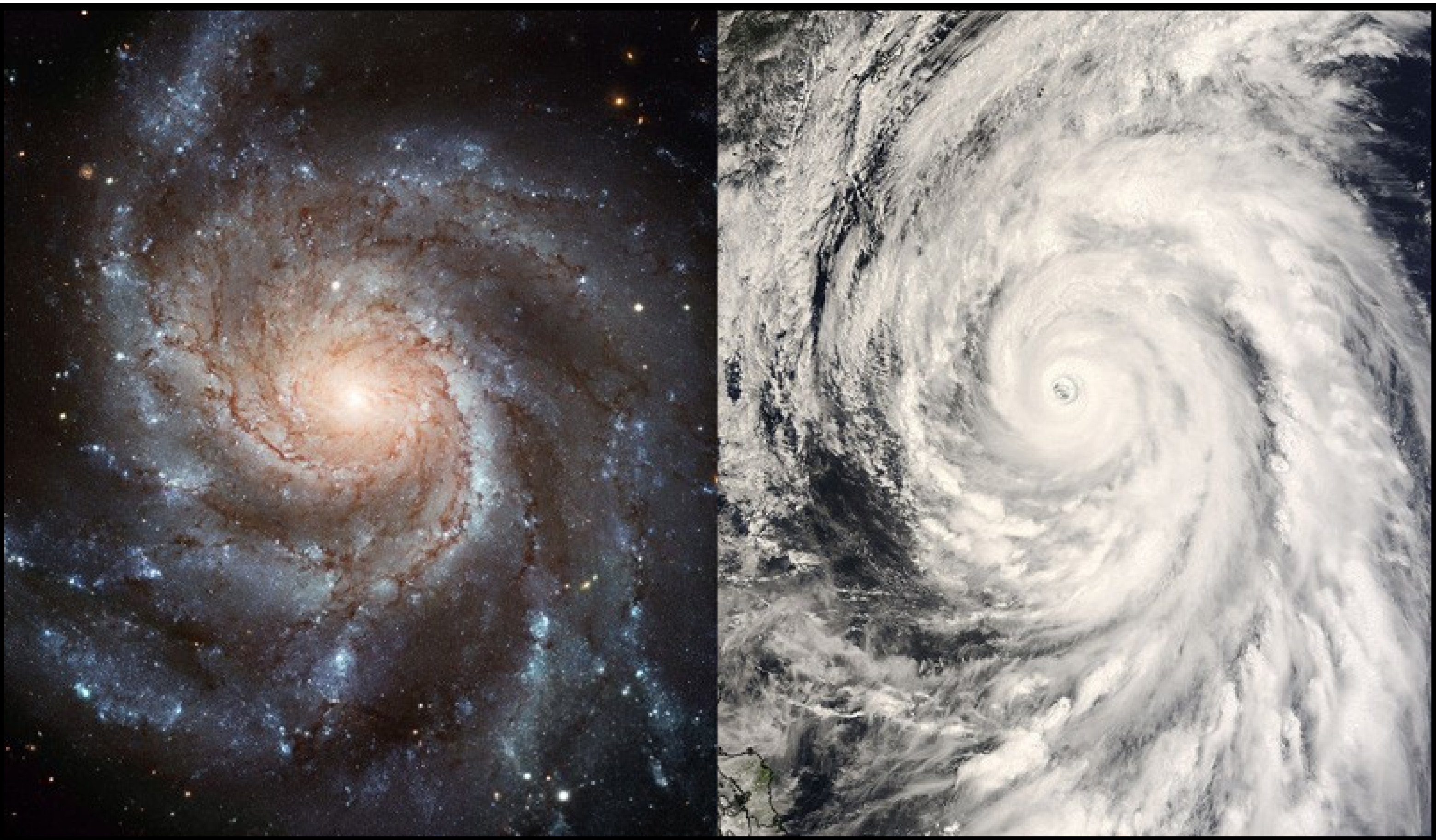
$$Vorticity := [0, 0, 2\,\omega]$$

$$Divergence := 2\,B + C$$
$$Helicity := 2\,C\,z\,\omega$$

This subset of solutions generates the Ubiquitous Logarithmic Spirals.

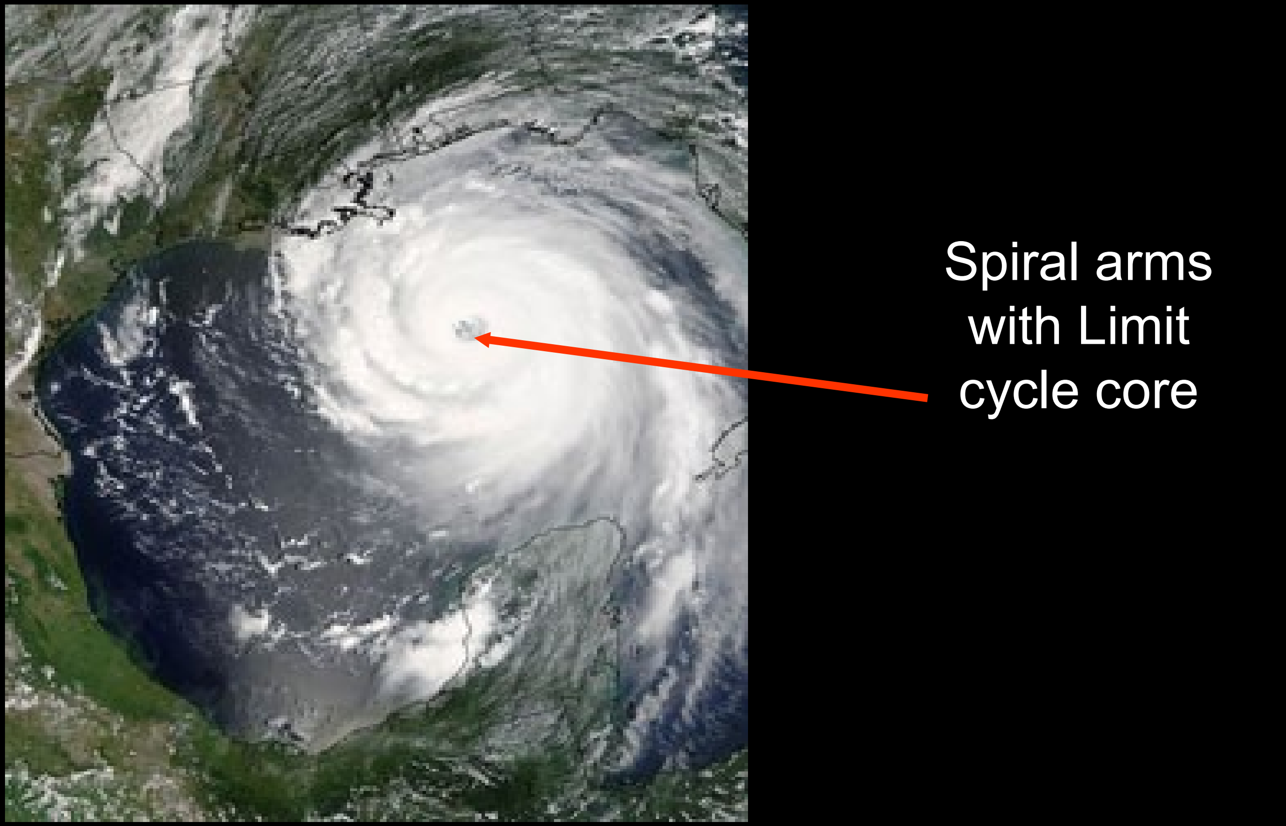
Is the Universe mostly a Turbulent Gas ??

With Stars and Galaxies as its defect condensates



Note the similarity between the spiral arm galaxy and the Hurricane. The independence from size is a topological property.

Ecological Impact of Hurricane Katrina

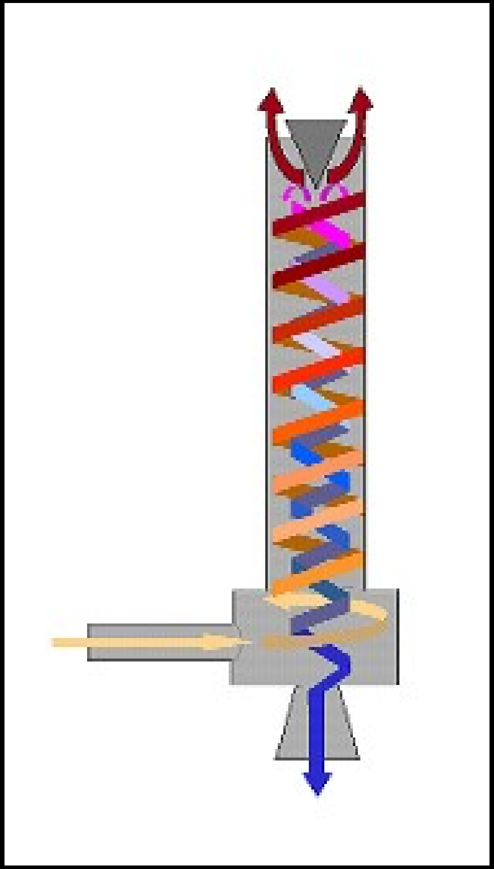


Hurricane Katrina, generated by turbulent vortex formation, killed or severely damaged 320 million large trees in Gulf Coast forests, which weakened the role the forests play in storing carbon from the atmosphere. The damage has led to these forests releasing large quantities of carbon dioxide into the atmosphere!

Vortex & Spiral Arm generation by turbulent flows Ex 5.

Dissipative turbulent generation of a persistent vortex. Ecological damage can be enormous.

Vortex & Spiral Arm generation **practical Apps**



Hot 100

Cold -30

Ranque-Hilsch tube

ALSO Checkout the Windhexe machine at
http://www.youtube.com/watch?v=xxuM7xWL5RQ

Summary of

Turbulence and its Visual Artifacts

Spiral Arms, Vortices, Wakes and Solitons

Are Residues of Turbulent Decay,

Which have EMERGED from a

Dissipative Open Thermodynamic Systems

Producing subsets of

Closed Non-Equilibrium Thermodynamic Systems

In a dynamical system

Expansions and Rotations

are associated with **Spiral Arms**

But a Topological Dimension of 3 is required for Irreducible Thermodynamic Dissipation

(hence 2D Turbulence is a Myth.)

The Existence of the Property of

TOPOLOGICAL TORSION

Insures the topological dimension is 3 or more.

Frenet Equations, Spinors and Wakes

Using ideas generated by Continuous Topological Evolution

$$dR/ds = \left| \begin{matrix} dx/ds \\ dy/ds \end{matrix} \right\rangle = \mathbf{t}(s) = \left| \begin{matrix} sin(Q(s)) \\ cos(Q(s)) \end{matrix} \right\rangle .$$

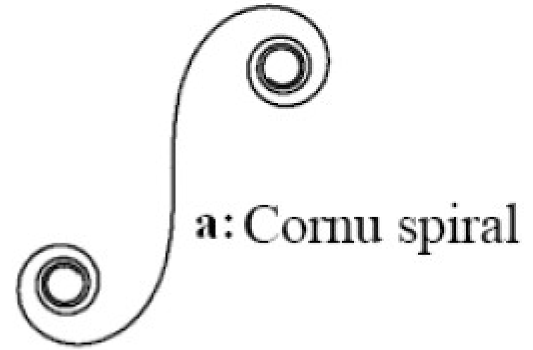
The unit tangent Vector, t

$$\begin{aligned} d\mathbf{t}(s)/ds &= \kappa\mathbf{n}(s) = \{dQ/ds\} \left| \begin{matrix} \cos(Q(s)) \\ -\sin(Q(s)) \end{matrix} \right\rangle , \\ \kappa &= \{dQ/ds\}. \end{aligned}$$

The unit normal Vector n

:	$\kappa = s^{-1}$,	$\kappa = s^{0}$,	$\kappa = s^{1}$...
:	Log spiral,	circle,	Cornu-Fresnel spiral...

Spiral shapes

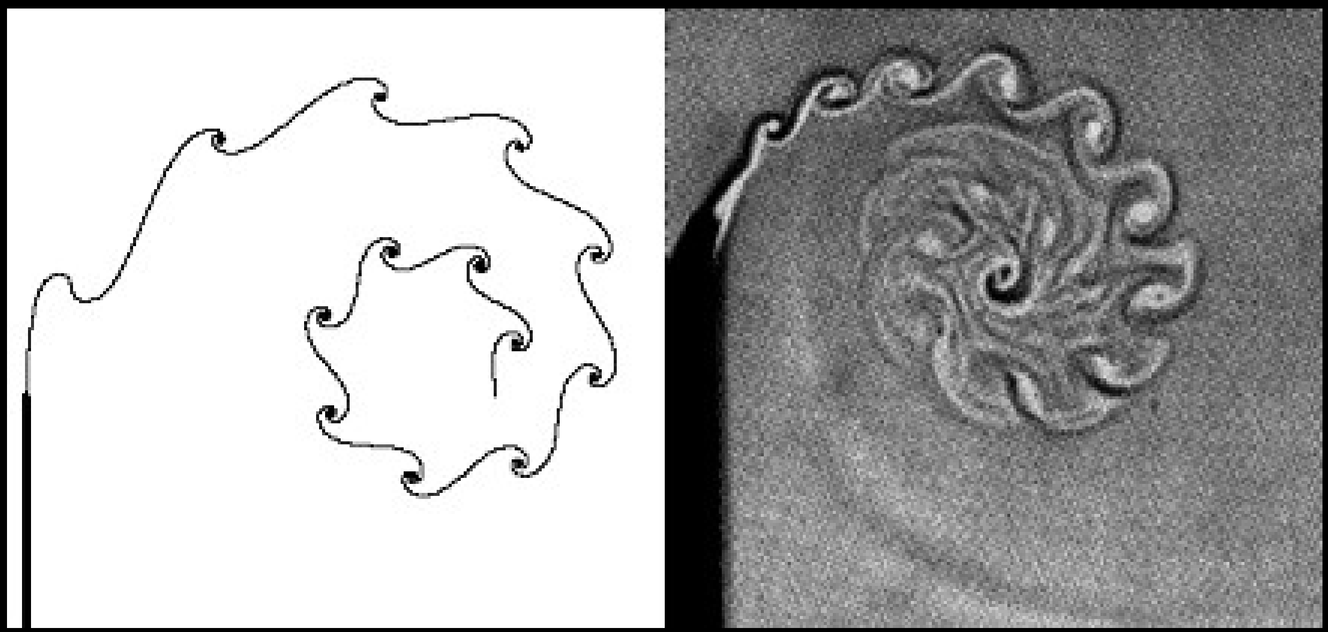


The phase function, $Q = s^2$.

The phase function, $Q = s^3$.

Frenet Equations, Spinors and Wakes

Using ideas generated by Continuous Topological Evolution



Flow past a sharp edge

Thanks for you ATTENTION

Contact Professor R. M. Kiehn

email: rkiehn2352@aol.com
WebSite: www.cartan.pair.com

Paperback Monographs available from
www.lulu.com/kiehn