PROLOGUE

The Geometrization of Thermodynamics
(particles via Caratheodory)

VS

The Topolization of Thermodynamics
(particles and waves via Grassmann)

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Warning

The Topology of Particles

is not the same as

The Topology of Waves

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PROLOGUE

Thermodynamic states <=> topological structures.

Equilibrium states <=> Pfaff Topological Dimension M $\leqslant$ 2

Non-Equilibrium states <=> Pfaff Topological Dimension M $\geqslant$ 3

Homotopic methods acting on thermodynamic topological structures can mathematically distinguish continuous topological evolution of Irreversible, Non-Deterministic, Non-Integrable, or Self-Organizing Emergent processes

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PROLOGUE

An exterior differential 1-form, A, defines a thermodynamic state of k geometric functions and differentials.

The neighborhood topology defined by A is composed of M functions and differentials such that $M \leqslant k$.

M is defined as the Pfaff Topological Dimension of A (M is easy to compute in terms of the Pfaff Sequence)

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The Pfaff Sequence and the Pfaff Topological Neighborhood Dimension

The Geometrical 1-form A = x dx -y dy+ (k dz-ω dt) has 4 functions, 2 constants and 4 differentials
But
PS(A) = [A, dA= 0, A^dA = 0, 0] = [A,0,0,0].
(The Pfaff Sequence has only 1 non-zero entry).

Hence the Pfaff Topological Dimension = PTD(A) = M = 1.
The topological neighborhood environment is an equilibrium thermodynamic state.

The topological minimum Pfaffian = A = dΨ has 1 differential
Ψ= $\{(x^2-y^2)/2 + (k\ z-\omega\ t)\}$

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PROLOGUE

This essay is a summary of work that was started in 1962 and led to topological non-equilibrium properties that include:

Topological Spin in non-equilibrium systems (1971),
Topological Torsion in turbulent systems (1977),
Topological Emergence of Falaco Solitons (1987)
Homotopic Continuous Topological Evolution (1987-1991),
Thermodynamic Irreversibility and the Arrow of Time (2003),
Topological Torsion and Spin in Plasmas (2004),
Thermodynamic Topologies from Pfaffian 1-forms (2009),
Thermodynamic Topologies from Current N-1-forms (2010)
Topologies of Waves versus Topologies of Particles (2012)

PROLOGUE

Particles vs. Waves

The T0 Topology of Particles

is not the same as

the Not T0 Topology for waves,

but the two topologies can coexist

Physical measurements are over finite space-time intervals.
Finite topologies are of **2** types: **T0 and Not-T0.**
The **T0** topology should be used for systems of **Particles**.
The **Not-T0** topology should be used for systems of **Waves**.

The T0 topologies can be generated from the functional properties of an exterior differential 1-form, A, of Action-potentials, creating a Particle topology with uniquely **distinguishable** singlet subsets.

The Not-T0 topologies can be generated in terms of the properties of an N-1 form, [C, ρ], or current density, representing a Complex-Wavelet, or ensemble, topology, where all of the subsets are **NOT** uniquely **distinguishable**.

The complete lattice structure for an extremal **Non T0** topology of 3 ingredients without closure axioms. All subsets have the same closure and cannot be distinguished. There are 4 different Non T0 topologies of 3 ingredients.

X := {a, b, c}

LS := {{ }, {a, b, c}}

Not-T0 N=3 Example 1,Indiscrete, Connected, as all sub sets are dense, Bnd of Bnd(S) = { }

Is LS a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

COMPLETE Lattice Structure

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c}	{a, b, c}	{b, c}	{a}	{ }
{b}	{ }	{ }	{a, b, c}	{a, b, c}	{a, c}	{b}	{ }
{c}	{ }	{ }	{a, b, c}	{a, b, c}	{a, b}	{c}	{ }
{a, b}	{ }	{ }	{a, b, c}	{a, b, c}	{a, b, c}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c}	{a, b, c}	{a, b, c}	{ }	{a, c}
{b, c}	{ }	{ }	{a, b, c}	{a, b, c}	{a, b, c}	{ }	{b, c}
{a, b, c}	{a, b, c}	{ }	{ }	{a, b, c}	{a, b, c}	{ }	{a, b, c}

Closed-Open subsets of LS are = {{ }, {a, b, c}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3} array of CLOSURE elements is = ({a, b, c}, {a, b, c}, {a, b, c})

The complete lattice structure for an extremal **T0** topology of 3 ingredients with the Kolmogorov closure axioms. The singlet subsets have distinguishable closures. There are 5 different Kolmogorov T0 topologies of 3 ingredients.

X := {*a*, *b*, *c*}

LS := {{ }, {*a*}, {*b*}, {*c*}, {*a*, *b*}, {*a*, *c*}, {*b*, *c*}, {*a*, *b*, *c*}}

N=3 T2 poset 1 Discrete, Bnd of Bnd(S) = { } as all Bnd(S) = { }

Is LS *a topology* = *true*, *connected* = *false*, *Kolmogorov.T0* = *true*, *Hausdorff.T2* = *true*

COMPLETE Lattice Structure

Subset S	*Int(S)*	*Ext(S)*	*Bnd(S)*	*Clo(S)*	*Lim(S)*	*IsoClo(S)*	*IsoCar(S)*
{*a*}	{*a*}	{*b*, *c*}	{ }	{*a*}	{ }	{*a*}.*Seg*	{ }
{*b*}	{*b*}	{*a*, *c*}	{ }	{*b*}	{ }	{*b*}.*Seg*	{ }
{*c*}	{*c*}	{*a*, *b*}	{ }	{*c*}	{ }	{*c*}.*Seg*	{ }
{*a*, *b*}	{*a*, *b*}	{*c*}	{ }	{*a*, *b*}	{ }	{*a*, *b*}.*Seg*	{ }
{*a*, *c*}	{*a*, *c*}	{*b*}	{ }	{*a*, *c*}	{ }	{*a*, *c*}.*Seg*	{ }
{*b*, *c*}	{*b*, *c*}	{*a*}	{ }	{*b*, *c*}	{ }	{*b*, *c*}.*Seg*	{ }
{*a*, *b*, *c*}	{*a*, *b*, *c*}	{ }	{ }	{*a*, *b*, *c*}	{ }	{*a*, *b*, *c*}.*Seg*	{ }

Closed-Open subsets of LS are = {{ }, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}

The {1,2,3} array of CLOSURE elements is = ({a}, {b}, {c})

This essay will describe both
equilibrium and non-equilibrium thermodynamics
from the basis of Continuous Topological Evolution,
resulting in

A UNIVERSAL Category Theory of Topological Thermodynamics

It is remarkable that the method introduces a
topological formalism useful to the description of
irreversible, non-deterministic dynamics
of both Particles and Waves,
in terms of **only two types** of finite coexistent topologies.

A Marriage between Particles and Waves
requires the use of the two different topologies.

For example, a **gravitational** metric theory
(of particles),
cannot be merged with a **quantum theory**
(of waves),
without the simultaneous use of the
two finite different topologies,
T0 and Not T0.

Geometric metric curvature is not a
Topological invariant

Basic Ideas 1.

1. The topology of **Particles** is not the same as the topology of **Waves**.

2. Only the discrete topology (Hausdorff T2) of **Particles** is metrizable. *Relativity theorists beware.*

3. The Kuratowski T0 poset 3 Particle topology is NOT metrizable.

Hence a theory of Gravity based on metric and distinguishable particles is not topologically complete.

Is the gravity of HOT bodies
different than the gravity of COLD bodies?

Basic Ideas 2.

4. The indiscrete Not T0 Wave topologies are not metrizable.

5. The particle topologies can coexist with the wave topologies.

6. Topological evolution can be described by the action of the Lie differential HOMOTOPIC Operator acting on differential forms.

The Homotopy of a particle topology based on a 1-form of Action is equivalent to the universal Topological First Law of Thermodynamics.

The Homotopic differential becomes the Covariant differential when the process is adiabatic.

Applications of

The Category Theory of Topological Thermodynamics

This work was motivated more than 40 years ago by the challenge of the Clay Institute regarding the properties of the Navier-Stokes equations and their relationship to hydrodynamic turbulence.

To replicate a statement made by the Clay Institute:

"The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations."

Applications of

The Category Theory of Topological Thermodynamics

can demonstrate

Irreversible Continuous Topological Evolution

adsabs.harvard.edu/abs/2007arXiv0704

Applications of

The Category Theory of

Topological Thermodynamics

can prove that the

Navier-Stokes equations have Turbulent solutions

http://www22.pair.com/csdc/pdf/topturb.pdf

The Category theory of Topological Thermodynamics and any exterior differential 1-form, A, of rank 4, can be used to

1. generate a disconnected Cartan Topology that defines a non-equilibrium thermodynamic system of particles.

Use the Grassmann algebra of Pfaffians to define

A = Topological 1-form of Action

F = dA = Topological 2-form of Vorticity

H = A^F = Topological 3-form of Torsion

K = F^F = Topological 4-form of Parity

Then use A,F,H,K to construct a topology.

The C

Kuratowsi Topology T4 $\approxeq$ Kolmogorv T0, poset 3

From any 1-form in 4D: $A = A_k(x^m)dx^k$

Pfaff Sequence= $\{A,\ F = dA,\ H = A \wedge F,\ K = F \wedge F\}$

Basis subsets $\{A,\ K_{Cl}(A),\ H,\ K_{Cl}(H)\} = \{A, A \cup F, H, H \cup K\}$

T4{Lattice Structure $\{X, \varnothing, A, H, A \cup F, H \cup K, A \cup H, A \cup H \cup K, A \cup F \cup H\}$

Complete Lattice Structure

Subset	LimPt(S)	Int(S)	Ext(S)	Bnd(S)	Clos(S)
$\varnothing$	$\varnothing$	$\varnothing$	X	$\varnothing$	$\varnothing$
A	F	A	$H \cup K$	F	$A \cup F$
F	$\varnothing$	$\varnothing$	$A \cup H \cup K$	F	F
H	K	H	$A \cup F$	K	$H \cup K$
K	$\varnothing$	$\varnothing$	$A \cup F \cup H$	K	K
$A \cup F$	F	$A \cup F$	$H \cup K$	$\varnothing$	$A \cup F$
$A \cup H$	F, K	$A \cup H$	$\varnothing$	$F \cup K$	X
$A \cup K$	F	A	H	$F \cup K$	$A \cup F \cup K$
$F \cup H$	K	H	A	$F \cup K$	$F \cup H \cup K$
$F \cup K$	$\varnothing$	$\varnothing$	$A \cup H$	$F \cup K$	$F \cup K$
$H \cup K$	K	$H \cup K$	$A \cup F$	$\varnothing$	$H \cup K$
$A \cup F \cup H$	F, K	$A \cup F \cup H$	$\varnothing$	K	X
$F \cup H \cup K$	K	$H \cup K$	A	F	$F \cup H \cup K$
$A \cup H \cup K$	F, K	$A \cup H \cup K$	$\varnothing$	F	X
$A \cup F \cup K$	F	$A \cup F$	H	K	$A \cup F \cup K$
X	F, K	X	$\varnothing$	$\varnothing$	X

The **Category theory of Topological Thermodynamics** and any exterior differential 1-form, **A**, of rank 4, can be used to

1. generate a disconnected Cartan Topology that defines a **non-equilibrium** thermodynamic system of particles.

2. generate a Jacobian correlation matrix, $[\partial A_k/\partial x^k]$ that has a singular characteristic set of rank 3, which is a morphism of a **universal van der Waals gas.**

3. generate a unique process current of **Topological Torsion**, $T = A \hat{} dA$, which describes an irreversible thermodynamic process in a non-equilibrium system.

Topological Torsion A^dA is a key design tool for controlling and understanding

Dissipative Structures and

TURBULENCE

Almost NO Engineers and Very Few Physicists Understand

TOPOLOGICAL TORSION

(pity)

As his students see him

By Professor R. M. Kiehn

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Prologue

I am not an algebraist.

I am not a topologist.

I am an applied engineering physicist, looking for a better, non-phenomenological, universal way to understand non-equilibrium thermodynamic systems and irreversible processes.

The goal is to find useful (and practical) applications of the universal topological methods in many different disciplines of science and engineering.

Starting in 1962, after leaving Los Alamos, the objective of my research has been to formulate an extension of tensor and variational techniques that would yield a universal description of non-equilibrium systems and irreversible turbulent processes, applicable to both particle and statistical physical disciplines.

The result has been the creation of a category theory of topological thermodynamics, which includes both particle and statistical features as distinct topological structures. The method has produced useful patents, a better understanding of turbulence in fluids and plasmas, and a proof of the Prigogine conjecture of emergence of metastable states far from equilibrium (in both quantum and macroscopic disciplines).

Mathematicians define a Category Theory, C, to be

1. **A collection of abstract objects, ob(C), with**
2. **morphisms that map an object X to another object Y.**

The Category of Topological Thermodynamics is

1. **A collection of the 2 finite types of topological spaces which are defined by the exterior differential 1-form, A, or the N-1 form current density, C, ρ and which are used to represent the 2 thermodynamic systems subject to**

2. **(possibly non-invertible but continuous) Homotopic morphisms that map exterior differential forms into other exterior differential forms , representing processes, which include topological or geometrical change .**

Theorem 1: Any exterior differential 1-form of Action A can be used to generate the topological structure suitable to describe thermodynamic systems of distinguishable particles.

Using the Grassmann algebra compute the Pfaff Sequence of A,

PS(A) = [A, dA, A^dA, dA^dA...]

A = Topological Action 1-form
dA = Topological Vorticity 2-form
A^dA = Topological Torsion 3-form
dA^dA = Topological Parity 4-form

Theorem 2: The minimum number M of functions required to define the topological neighborhood of A defines the Pfaff Topological Dimension, PTD(A), of the 1-form A. M is less than or equal to the geometric dimension, k, of A.

The number of non-zero entries in the Pfaff Sequence Determines the Pfaff Topological Dimension of A.

PS = [A,0,0,0] => PTD(A) = 1
PS = [A,dA,0,0] => PTD(A) = 2
PS = [A,dA,A^dA,0] => PTD(A) = 3
PS = [A,dA, A^dA, dA^dA] => PTD(A) = 4

Define the top Environment as a PTD(A) = 4 system

Equilibrium PTD(A) = 1 subsystems do not exchange heat or mass with the environment

Isolated Equilibrium PTD(A) = 2 subsystems do not exchange heat or mass with the environment

NON-Equilibrium Closed PTD(A) = 3 subsystems can exchange heat , but not mass, (or mass, but not heat), with the environment

NON-Equilibrium Open PTD(A) = 4 subsystems can exchange both heat and mass with the environment

Thermodynamic processes J are used to define topologically continuous, differentiable, morphisms (maps) between the initial state A and the final state, Q.

Map A => Q

The process direction field, J can be composed of vectors and macroscopic spinors

A **Homotopy = H d + d H can be formulated in terms of the Lie differential $L_{(J)}$ with respect to a current, J:**

$$L_{(J)} = i(J)\,d + d\,(i(J)).$$

If applied to differential forms, the definitions lead to:

Cartan's Magic Homotopy Formula 1899-1923:

$$L_{(J)} A = i(J)dA + d(i(J)A)$$

Theorem: The homotopy operator, $L_{(J)}A$, Cartan, operating on exterior differential forms that are C2 smooth (C2 differentiable)

is topologically continuous.

The 4 components of **C** can consist of both vector/spinor direction fields (~ flow) and can be used to define a 3-form Current density.

The Cartan homotopy maps the Limit points, $d\Sigma$, of the ingredients, Σ, of the initial topology into the Closure, $\Sigma \cup d\Sigma$, of the ingredients of the final topology

For 1-forms, the initial state topology of A is not necessarily the same as the final state topology of Q !!!

$$L_{(J)} A = i(J)dA + d(i(J)A) \Rightarrow Q$$

In fact, Topological Evolution is a necessary requirement for process irreversibility.

The fact that any differential 1-form can be used to define a topological space came to my attention about 1987, after some 20 years of studying Cartan's utilization of differential forms and their application to the theory of integral invariants. Early on I defined t*his topological space as the disconnected Cartan topology of exterior differential forms*.

However, it was not until 2009, when I attended (in attempt to learn more about modern topology) my first conference for professional topologists at Haceteppe, Turkey, that I realized that the Cartan topology of 4 differential form ingredients was exactly equivalent to a Kuratowski representation of the disconnected Kolmogorov T0 poset 3 topology, based upon the power set of 4 exterior differential forms as ingredients.

Sequence of Ideas 1987-2009

Following the lead of E. Cartan, the "laws of motion" that describe the continuous evolution of the T0 "particle" topologies will be generated by applying the homotopic Lie differential (not derivative) to the exterior differential 1-form of Action, A, that encodes the thermodynamic system..

Sequence of Ideas 1987-2009

The result is to homotopically map A => Q

$$L_{(J)} A = i(J)dA + d(i(J)A) \Rightarrow Q$$

By change of notation:

$W = i(J)dA$ **Work 1-form,**

$U = i(J)A$ **Internal energy,**

$Q = L_{(J)} A$ **Heat 1-form**

Cartan's Magic Formula becomes

(for all systems, A, and all processes, J)

universally and topologically equivalent to

$$L_{(J)} A = W + d(U) = Q$$

Which is a topological formulation of

The FIRST LAW of THERMODYNAMICS!!

Even if the process J induces irreversible topological change!

The 1st Cartan Magic formula represents a system of PDE's :

$$Q - W = d(U)$$

Hence , the

FIRST LAW of THERMODYNAMICS

goes beyond DeRham cohomology theory , which requires that both Q and W be closed and integrable, $dQ = dW = 0$.

The more general "cohomology" statement implies Q and W are not necessarily integrable, but their limit sets are related and not necessarily zero.

$$Q\wedge dQ = d(U)\wedge dW$$

RESULTS

DeRham cohomology can be applied to Equilibrium Thermodynamics,

$PTD(A) < 3$

but the Homotopic Cartan *extended* cohomology can be applied to NON-Equilibrium Thermodynamics

$PTD(A)>2$

The current, or flow, C = ρV_4, is defined as a "differential direction field" that is NOT necessarily a derivative, nor a generator of a 1-parameter group.

$$C = \rho V_4 = [C^x, C^y, C^z, C^t]$$

On a 4D variety, the contraction of **C** with a 4-form differential volume element yields a 3-form Current density of 4 coefficients which may or may not have a Zero divergence. The direction field, **C=ρV_4**, can be an element of a differential semi-group, or an element of a **complex macroscopic spinor** space.

Additional Ideas 1987-2009

Macroscopic Spinors S_k

Any anti-symmetric matrix, such as the 2-form $F = dA$, will have complex direction field eigenvectors, S_k, with zero norm (length) but finite area

$< S_k \; S_k >$ equals 0, but $< S_k \; S_j >$ does not equal 0.

Additional Ideas 1987-2009

Macroscopic Spinors of F=dA

The Work 1-form W is defined in terms process , ρV_4,
and the 2-form F = dA.

$$W = i(\rho V_4)F$$

The eigen values of the antisymmetric matrix F are
Two imaginary conjugate pairs, if PTD(A) = 4,
1 imaginary conjugate pair, and one 0, if PTD(A) = 3,
1 imaginary conjugate pair, if PTD(A) = 2,

Additional Ideas 1987-2009

Macroscopic Spinors are

Majorana Spinors !

Additional Ideas 1987-2009

The correlation matrix, $[\partial A_k/\partial x^k]$, defines a Caley-Hamilton characteristic polynomial, Θ, of 4th degree in terms of similarity invariants. The hypersurface

$$\Theta = 0,$$

can be interpreted as a thermodynamic phase function in the symplectic topological space PTD(A) = 4,

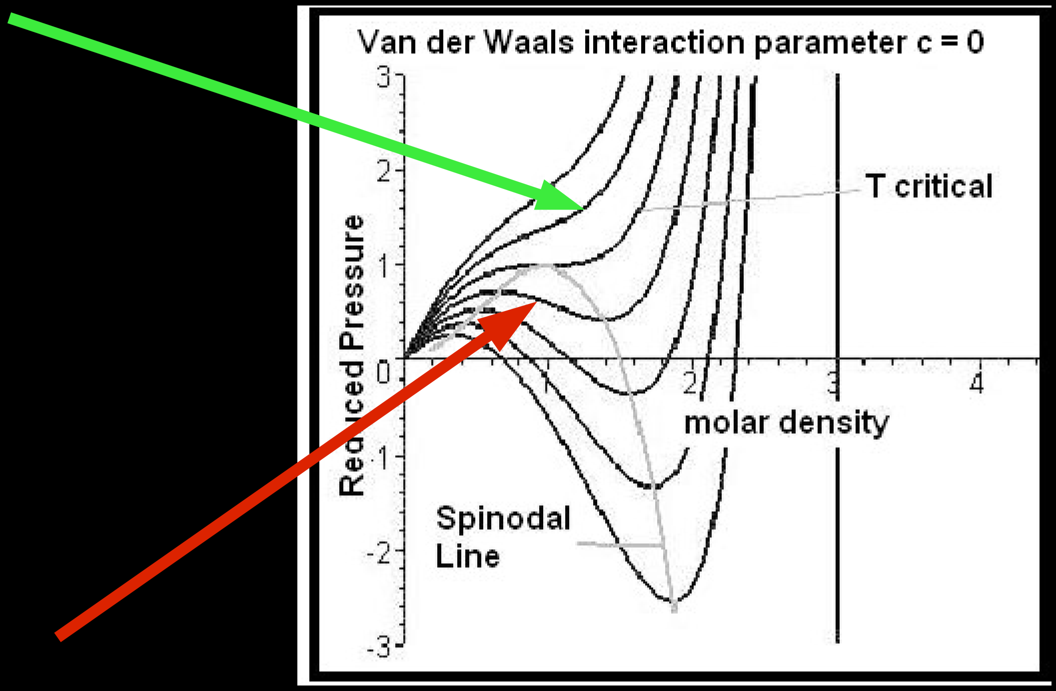
and has the ubiquitous (homoeomorphic) properties of a **van der Walls gas in the contact topological space, PTD(A) = 3.**

These particle aspects have been well documented with many examples in numerous publications over the years, starting in 1962. See http://www.cartan.pair.com

Additional Ideas 1987-2009

Complex eigenvectors represent anti-symmetric features of the Jacobian matrix for a van der Waals gas.

Above the critical isotherm there is 1 real eigenvector and 2 complex Macroscopic Spinor eigenvectors of zero norm.



Below T critical, there are 3 real eigenvectors.

Additional Ideas 1987-2009

Any 1-form of Action, A, can be used to define a unique non-zero 3-form process,

$$i(T_4)\Omega = i(T_4)\, dx\wedge dy\wedge dz\wedge dt = A\wedge dA,$$

with the properties such that

$$i(T_4)A = 0 = i(T_4)Q \quad \text{(transversely adiabatic)},$$

$$i(T_4)dA = W = \sigma A = Q,$$

T_4 REPRESENTS THE 4 COMPONENTS OF

TOPOLOGICAL TORSION

Additional Ideas 1987-2009

A projectively unique current 3-form is deduced from the functional properties that define the thermodynamic system.

$$i(T_4)\Omega = i(T_4)\, dx\wedge dy\wedge dz\wedge dt = A\wedge dA,$$

T_4 REPRESENTS THE 4 COMPONENTS OF TOPOLOGICAL TORSION

T_4 is an ARTIFACT of PTD(A) = 4

and non-equilibrium systems

(and vanishes for equilibrium systems.

Topological Torsion Properties

$$\mathbf{T}_4 \text{ on } \Omega_4 : \textbf{Properties of Topological Torsion}$$

$$i(\mathbf{T}_4)\Omega_4 = i(\mathbf{T}_4)dx\hat{}dy\hat{}dz\hat{}dt = A\hat{}dA,$$

$$i(\mathbf{T}_4)i(\mathbf{T}_4)\Omega_4 = 0, \text{ which implies that}$$

$$\text{Work 1-form} \quad W = i(\mathbf{T}_4)dA = \sigma\ A,$$

$$dW = d\sigma\hat{}A + \sigma dA = dQ,$$

$$\text{Internal Energy} \quad U = i(\mathbf{T}_4)A = 0, \quad \mathbf{T}_4 \text{ is associative,}$$

$$i(\mathbf{T}_4)dU = 0$$

$$i(\mathbf{T}_4)Q = 0 \qquad \mathbf{T}_4 \text{ is adiabatic}$$

$$L_{(\mathbf{T}_4)}A = \sigma\ A, \quad \mathbf{T}_4 \text{ is homogeneous and self-similar}$$

$$L_{(\mathbf{T}_4)}dA = d\sigma\hat{}A + \sigma dA = dQ,$$

$$Q\hat{}dQ = L_{(\mathbf{T}_4)}A\hat{}L_{(\mathbf{T}_4)}dA = \sigma^2 A\hat{}dA \neq 0, \ \mathbf{T}_4 \text{ is irreversible,}$$

$$dA\hat{}dA = d(A\hat{}dA) = d\{(i(\mathbf{T}_4)\Omega_4\} = (div_4\mathbf{T}_4)\Omega_4,$$

$$L_{(\mathbf{T}_4)}\Omega_4 = d\{(i(\mathbf{T}_4)\Omega_4\} = (2\sigma)\Omega_4, \ \mathbf{T}_4 \text{ causes } \Omega_4 \text{ expansion}$$

The 4 divergence of T_4

$$d(i(T_4)\Omega) = 4\mathrm{Div}(T_4)\Omega = \sigma\,\Omega$$

Determines the Space-Time

Irreversible Dissipation

Coefficient σ

And Now some heresy

Energy is conserved in classical processes of Equilibrium thermodynamics. **PTD(A) < 3**

But Energy is NOTconserved

by irreversible dissipative non-equilibrium processes

of PTD(Q) = 4

Application Requirements

1. Must choose the functional form for a 1-form of Action, **A**, to define a physical system.

2. Possibly constrain the PTD of the Work 1-form, W, to generate the desired constrained dynamical PDE's.

3. Evaluate the non-equilibrium properties encoded as

3-form of Topological Torsion, A^dA

4-form of Topological Parity, dA^dA.

A Universal Example

Use the 4D electromagnetic 1-form of Action:

$$A = A_k(x,y,z,t)dx^k - \phi\,(x,y,z,t)dt.$$

Construct the 2-form $F = dA = B_z\,dx\wedge dy \ldots - E_z\,dz\wedge dt \ldots$

Define: $E = -\,\partial(A_k(x)/\partial t - \text{grad}\,\phi\,;\qquad B = \text{curl}\,A_k$

Construct the Work 1-form:

$$W = i(V_4)dA = -\{E+V\times B\}_k dx^k + \{V\bullet E\}dt$$

Then, the coefficients of the 3-form $dF = ddA = 0$,

generate a set of Maxwell-Faraday PDE's.

UNIVERSAL FARADAY INDUCTION (no D or H!)

$$\text{Curl}\,E + \partial B/\partial t = 0,\quad \text{div}\,B = 0.$$

A Universal Example

Construct the 3-form A^F = A^dA (of Topological Torsion)

$$A \wedge dA = i(T_4)(dx \wedge dy \wedge dz \wedge dt)$$

with the 4 component Torsion Direction Field

$$T_4 = - [\, E \times A + \phi B, (A \cdot B)].$$

(The 4 divergence of the Torsion vector T_4 is equal to $-2(E \bullet B) = -2\,\sigma$.

Then, $L_{(T4)} A = (E \cdot B)\, A = \sigma A$

σ is a Conformality Factor,

and a Homogenity (including fractals) index of self similarity.

(Suggested as a dissipative extension to Hamilton's principle by RMK in 1974)

A Universal Example

Construct the 4-form F^F = dA^dA of Topological Parity:

$$dA \wedge dA = 2(E \bullet B)(dx \wedge dy \wedge dz \wedge dt).$$

On regions where $\sigma = (E \bullet B) > 0$,

the PTD(A) is 4.

The thermodynamic system is not in equilibrium. And as

$$L_{(T4)}A \wedge L_{(T4)}dA = Q \wedge dQ = (-E \bullet B)^2 A \wedge dA > 0.$$

The process in the direction of the Torsion vector T_4

is thermodynamically irreversible

“Entropy” production Rate
Bulk Viscosity

As $L_{(T)}\ A = (E \bullet B) A$,

$Q \wedge dQ = (-E \bullet B)^2\ A \wedge dA > 0.$

Then $(E \bullet B)^2 > 0$

Entropy Production Rate

$= (1/2 \text{ divergence of } T_4)^2$

For Engineers

To Reduce Irreversible Dissipation and Turbulence

For Engineers

To Reduce Irreversible Dissipation and Turbulence

In dynamical systems, minimize the co-linearity of Translation Acceleration and Vorticity

In Engineering terms

To Reduce Irreversible Dissipation and Turbulence

In a Plasma Minimize $\sigma = (E \bullet B)$

For Engineers

To Reduce Irreversible Dissipation and Turbulence

In a Navier Stokes fluid Minimize $(a \bullet \omega)$

$$\partial V/\partial t + \text{grad}\, V^2/2 - V \times \text{curl}\, V = -\, \text{grad}P/\rho + \lambda\, \text{grad div}\, V - \nu \nabla^2 V$$

Classically, ν = shear viscosity, and $\lambda = (\mu_B - \nu)$ where μ_B = Bulk viscosity

Turbulent solutions imply $\sigma = (a \bullet \omega) \neq 0$!

Additional Ideas 1987-2009

EMERGENCE of Topological Defects

CONJECTURE: The thermodynamic Cosmological Astronononomical Universe is best represented as a category of very low density radiation plasma of indistinguishable complex wavelets and macroscopic Spinor fields, combined with distinguishable sets of massive particles forming a non-equilibrium system of Pfaff Topological Dimension 4.

The Thermodynamic PDT4 environment is not a void.

Additional Ideas 1987-2009

EMERGENCE of Topological Defects

Claim: The Category theory of Topological Thermodynamics yields examples how continuous **irreversible** disspative processes can form emergent Closed topological defect structures, far from equilibrium, of PTD = 3 (such as stars), in an Open thermodynamic environment of PTD = 4.

These examples give formal justification to Progogine's conjectures.

Emergence of Stationary States far from equilibrium *by irreversible Processes*

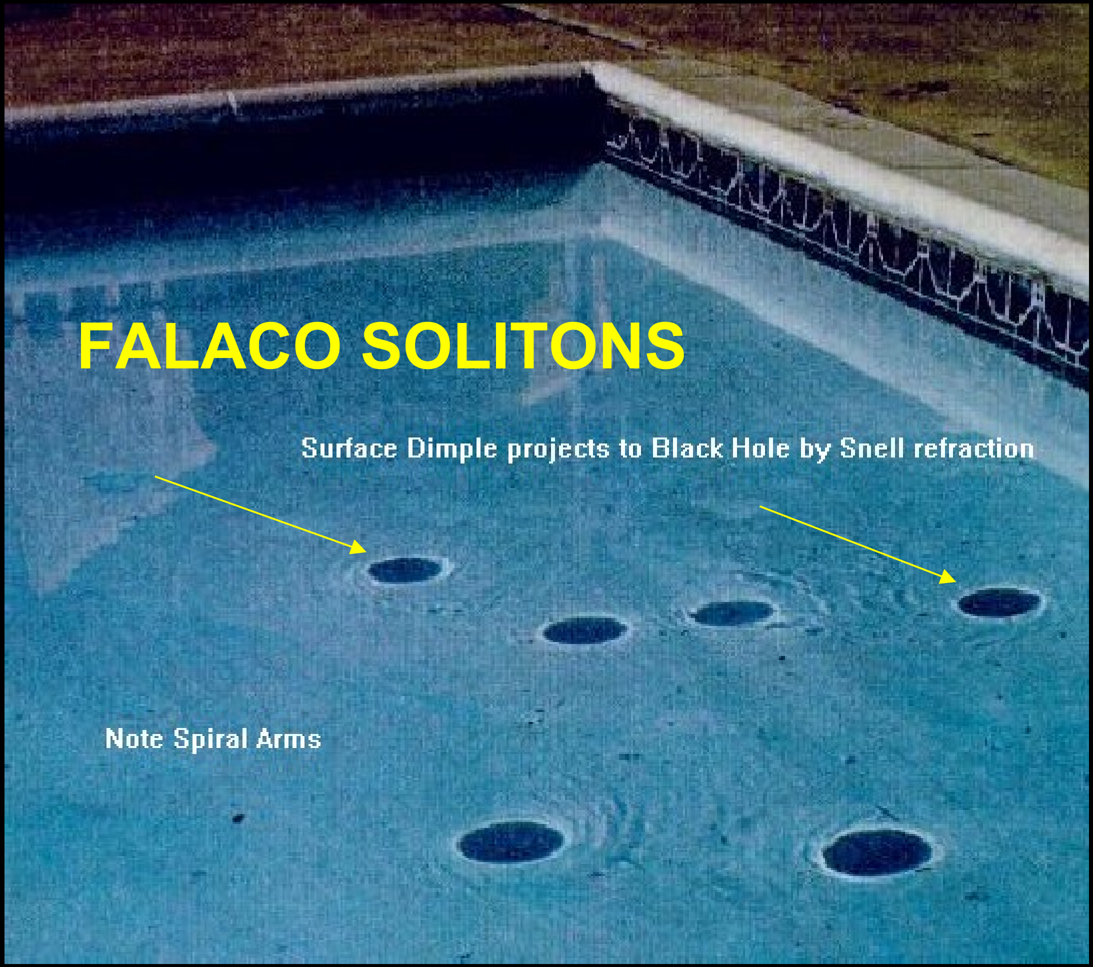
Example: **Continuous topological evolution can describe the irreversible evolution on an**

"Open" symplectic non-equilibrium domain of Pfaff dimension 4, with evolutionary irreversible orbits being attracted to a contact

"Closed" non-equilibrium domain of Pfaff dimension 3, with an ultimate decay to the

"Isolated-Equilibrium" domain of Pfaff dimension 2 or less (integrable Caratheodory surface).

The Experimental Emergence of Falaco Solitons



Long Lived Topological Defects in a Swimming Pool

Properties of 3-Form Currents
J = Processes

$L_{(J)} A \Rightarrow Q \qquad L_{(J)} dA \Rightarrow dQ$

1. Necessary condition for Reversible Processes

$$Q \wedge dQ = 0$$

2. Necessary condition for Irreversible Processes

$$Q \wedge dQ <> 0$$

Based on the Pfaff Topological Dimension of W

Thermodynamic Reversible Processes imply that the Heat 1-form, Q is integrable. $Q \wedge dQ = 0$

Extremal: PTD(W) = 0, W = 0,

Hamiltonian global

Bernoulli-Casimir: PTD(W) = 1, W exact

Hamiltonian local

Helmholtz: PTD(W) = 1, W closed

Conservation of Vorticity

Each of these flows are thermodynamically reversible, as

$dW = 0 = dQ$, implies $Q \wedge dQ = 0$.

All of the above processes are Reversible!

These theories make up the bulk of Classical Dynamics!!

THESE PROCESSES (classical theories) CAN NOT faithfully DESCRIBE TURBULENCE OR INTERACTION with the ENVIRONMENT

To be Irreversible the process and the thermodynamic system must be non-integrable, $Q \wedge dQ$ is not zero.

The Topological Torsion (based on Q) is NOT ZERO. PTD(Q) > 2.

Non-Equilibrium systems are non-integrable, in the sense that if there is a solution, it is not unique, or Not Deterministic.

A key artifact of non-equilibrium is the existence of

Topological Torsion current 3-forms, $J_{torsion} = A \wedge dA$,

Topological Spin current 3-forms, $J_{spin} = A \wedge G$

Topological Adjoint current 3-forms, $dJ_{adjoint} = A \wedge J_{spin}$

3-Form Currents = Processes

The Topological Torsion 3-form **A^F** is related to Helicity,

The Topological Spin 3-form **A^G** is related to Spin,

The Adjoint 3-form $\mathbf{J}_{\mathbf{adjoint}}$ is related to the interaction energy.

All three are related to different species of dissipative phenomena, which only occur in non-equilibrium systems.

The dissipation coefficients σ are equal to the non-zero divergences of the vector coefficients of each 3-form.

In electromagnetic systems, the 4D dissipation coefficient **σ** was shown to be equal to **E • B**; in hydrodynamics, the 4D dissipation coefficient is called **σ** "**Bulk viscosity**".

The combination of Continuous topological evolution and Parity decay determine

A Topological Arrow of Time.

You can describe the decay of turbulence continuously, but NOT the creation of turbulence.

New Ideas 2009-2011

The **Not-T0** topologies, without separation axioms, are such that some, if not all, of the subset closures are

Indistinguishable.

These topologies appear to describe statistical and quantum-like thermodynamical systems, whose parts, like Bosons and Fermions, are indistinguishable -- with equations of "evolution" described by complex diffusion or wave equations with source.

New Ideas 2009-2011

Stimulated by Cartan's successes for particle systems, the "laws of motion" that describe the continuous evolution of the Not-T0 topologies will be studied by applying the homotopic Lie differential to the exterior differential N-form density,

$$\mu = \rho \{dx \wedge dy \wedge dz \wedge dt\}.$$

The 2nd Cartan homotopic evolution formula becomes:

$$L_{(J)}\,\mu = i(J)\,d\mu + d(i(J)\,\mu) = \kappa\,\mu$$

where κ = the chaotic similarity, or 4D expansion coefficient

New Ideas 2009-2011

Stimulated by Cartan's successes for particle systems, the "laws of motion" that describe the continuous evolution of the Not-T0 topologies will be studied by applying the homotopic Lie differential to the exterior differential N-form density,

$$\mu = \rho \{dx\hat{}dy\hat{}dz\hat{}dt\}.$$

The 2nd Cartan homotopic evolution formula becomes:

$$L_{(J)}\, \mu = i(J)\, d\mu + d(i(J)\, \mu) = \kappa\, \mu$$

Leading to a possible statistical (wave) component of entropy, without counting particles.

New Ideas 2009-2011

Evaluate

$$L_{(J)}\, \mu = i(J)\, d\mu + d(i(J)\, \mu) = \kappa\, \mu$$

to obtain

$$L_{(J)}\, \mu = 0 + d(i(J)\, \mu) = \kappa\, \mu$$

or

$$\mathrm{div}_4\, J = \kappa - J^k\, \partial \ln\rho / \partial x^k$$

Cartan's Second Fundamental Equation

New Ideas 2009-2011

Solutions to the 2nd fundamental equation can be determined in terms of systems of PDE's that satisfy the fundamental formula:

$$\mathrm{div}_4\, \mathbf{J} = \kappa - \mathbf{J}^k\, \partial \ln\rho / \partial x^k$$

Different PDE Solutions can be recognized as

The Wave Equation.

The Wave equation with Sources.

The Diffusion equation.

The Schroedinger equation

The Minimal Surface equation.

The Dissipation-Interaction equation.

The Ginsburg-Landau equation.

The Gibbs entropy equation.

The Mandelbrot entropy equation for chaos.

Different PDE Solutions can be recognized as

The details of such system solutions of PDE's is given in the attached pdf file. A few simple examples are given in that which follows.

Solutions to the 2nd fundamental formula leads to systems of PDE's representing Not-T0 topologies of topological indistinguishable subsets.

$$\mathrm{div}_4\, \mathbf{J} = \kappa - \mathbf{J}^k\, \partial \ln\rho / \partial x^k$$

The term, $\mathbf{J}^k\, \partial \ln\rho / \partial x^k$, if equal to zero, is a generalization of the Eikonal Equation, which can represent propagating discontinuities. (The "discontinuous signals" of the Wave Equation)

Example 1 The 4D Wave Equation

Consider a set of topological coordinates

$(x^k) = [x,y,z,s=ict]$

with a density 4 form = $\rho(x^k)\, dx\wedge dy\wedge dz\wedge ds$

Identify the density coefficient $\rho(x^k)$ with the symbol $\Psi(x^k)$ and

grad4 (Ψ) = grad4 (ρ)

Consider the process J defined by grad4 (Ψ)

$$J(x^k) = (\partial\Psi/\partial x, \partial\Psi/\partial y, \partial\Psi/\partial z, \partial\Psi/\partial s)$$

$$div_4 J(x^k) = \partial^2\Psi/\partial x^2 + \partial^2\Psi/\partial y^2 + \partial^2\Psi/\partial z^2 - \partial^2\Psi/\partial s^2$$

Example 1 The 4D Wave Equation

THEN

If $\kappa - J^k \, \partial \ln\rho / \partial x^k \Rightarrow 0$

$\mathrm{div}_4 J(x^k) \Rightarrow 0$

Therefore

$$\partial^2\Psi/\partial x^2 + \partial^2\Psi/\partial y^2 + \partial^2\Psi/\partial z^2 - \partial^2\Psi/c^2\partial t^2 \Rightarrow 0$$

THE WAVE EQUATION !

Example 2 The Diffusion Equation

Consider a set of topological coordinates

$(x^k) = [x,y,z,t]$

with a density 4 form = $\rho(x^k)\, dx\hat{}dy\hat{}dz\hat{}dt$

Identify the density coefficient $\rho(x^k)$ with the symbol $\Psi(x^k)$ and

grad4 (Ψ) = grad4 (ρ)

Choose the process current format as:

$$J(x^k) = (\partial\Psi/\partial x,\ \partial\Psi/\partial y,\ \partial\Psi/\partial z,\ -D\Psi);$$

$$div_4 J(x^k) = \partial^2\Psi/\partial x^2 + \partial^2\Psi/\partial y^2 + \partial^2\Psi/\partial z^2 - D\partial\Psi/\partial t$$

Example 2 The Diffusion Equation

THEN

If $\kappa - J^k \partial \ln\rho / \partial x^k \Rightarrow 0$

$$\mathrm{div}_4 J(x^k) = > 0$$

Therefore

$$\partial^2\Psi/\partial x^2 + \partial^2\Psi/\partial y^2 + \partial^2\Psi/\partial z^2 - D\partial\Psi/\partial t \Rightarrow 0$$

THE DIFFUSION EQUATION !

Example 3 The Schroedinger Equation

Consider a set of topological coordinates

$(x^k) = [x,y,z,t]$

with a density 4 form = $\rho(x^k)\, dx\wedge dy\wedge dz\wedge dt$

Choose the process current as:

$$J(x^k) = (\partial\Psi/\partial x, \partial\Psi/\partial y, \partial\Psi/\partial z, -h/i\ \Psi);$$

$$div_4 J(x^k) = \partial^2\Psi/\partial x^2 + \partial^2\Psi/\partial y^2 + \partial^2\Psi/\partial z^2 - h/i\ \partial\Psi/\partial t$$

Example 3 The Schroedinger Equation

THEN

If $\kappa - J^k\, \partial \ln\rho / \partial x^k \Rightarrow V\Psi$

$$div_4 J(x^k) \Rightarrow V\Psi$$

Therefore

$$\partial^2\Psi/\partial x^2 + \partial^2\Psi/\partial y^2 + \partial^2\Psi/\partial z^2 - h/i\ \partial\Psi/\partial t = V\Psi$$

THE SCHROEDINGER EQUATION !

Summary:

THE CATEGORY THEORY OF T0 TOPOLOGY (PARTICLES) AND NON-T0 TOPOLOGY (WAVES), and THE POSSIBLE INTERACTIONS OF THE TW0 TOPOLOGICAL TYPES, HAS JUST BEGUN.

The two universal equations of Topological Evolution are:

$$L(J)\,A = i(J)dA + d(i(J)A) = W + d(U) = Q$$

$$\mathrm{div}_4\, J = \kappa - J^k\, \partial \ln \rho / \partial x^k$$

I AM CONVICED THAT THE BOSE – EINSTEIN AND THE FERMI DIRAC DISTRIBUTIONS CAN BE DEDUCED FROM

$$\mathrm{div}_4\, \mathbf{J} = \kappa - \mathbf{J}^k \partial \ln\rho / \partial x^k$$

CAN YOU HELP ME ???

Thanks for your interest

Contact Professor R. M. Kiehn at

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http://www.cartan.pair.com

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Non-Equilibrium Systems and Irreversible Processes

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Vol 1

Non-Equilibrium Thermodynamics

R. M. Kiehn

Vol 2

Falaco Solitons Cosmology and the Arrow of time

Photo courtesy David Radabaugh

R. M. Kiehn

Vol 3

Wakes Coherent Structures and Turbulence

Photo Courtesy Paul Bowen   www.airtoair.net

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Vol 4

Plasmas and Non-Equilibrium Electrodynamics

Long lived ionized plasma ring in the turbulence of a nuclear explosion.

R. M. Kiehn

Vol 5

Topological Torsion and Macroscopic Spinors

R. M. Kiehn

Vol 6

The Universal Effectiveness of Topological Thermodynamics

From the perspective of Continuous Topological Evolution

More to come

50 Significant Results

based on the

The Category of Topological Thermodynamics

Significant Results

- The method of Pfaff Topological Dimension explains topological differences between equilibrium PTD<3 (connected) and non-equilibrium PTD>2 (disconnected) thermodynamic systems of distinguishable "particles".

- The method explains the topological distinctions between thermodynamically reversible integrable ($Q \wedge dQ=0$) and irreversible non-integrable ($Q \wedge dQ \neq 0$) processes.

- The method explains topological distinctions between thermodynamic systems based on distinguishable sets (T0 topologies) and thermodynamic systems based upon statistical distributions of indistinguishable sets (Not T0 topologies).

Significant Results

- The method explains that Entropy can consist of two simultaneous components: one component is based on distinguishable sets (massive particles) and the other component is based upon indistinguishable sets (radiation).

- The method demonstrates that the First Law of Thermodynamics is a statement of cohomology, where the difference of the two, not necessarily exact, differential 1-forms of Heat Q and Work W are topologically equal to an exact differential of internal energy, dU.

- The method demonstrates that the decay of turbulence can be described in terms of a topologically continuous process, but the creation of turbulence can not.

Significant Results

- The method indicates that the top down nested sequence of Open, Closed, Isolated-Equilibrium and Equilibrium thermodynamic domains is in 1-1 correspondence with the Pfaff Topological dimension, [4,3,2,1] generated by any 1-form of Action A used to create the Cartan topological structure.

- The environment of the universe is not a vacuum (a void empty set PTD(A) = 0), but is best described as the disconnected Open system of PTD(A) = 4 in which sub-domains of PTD < 4 are distinguishable topological defects.

- The Equilibrium systems are of PTD(A) = 1. The Isolated Equilibrium systems are of PTD(A) = 2. The Closed systems are of PTD(A) = 3.

Significant Results

- The Pfaff Topological Dimension is equal to the number of non-zero entries in the Pfaff Sequence:

{A, dA, A^dA, dA^dA }

of the 1-form used to create the Cartan topological structure.

- Non-Equilibrium Open systems of Pfaff Topological Dimension 4 (PTD=4) can exhibit local decay (in finite time) to Closed non-equilibrium systems of PTD =3, thereby justifying Prigogine's conjecture of emergence to states far from equilibrium by means of dissipative processes. These metastable PTD=3 Closed states are topological defects in the Open systems of PTD=4.

Significant Results

· Topological change is a necessary condition for thermodynamic irreversibility.

· The combination of continuous, non-homeomorphic, processes of topological evolution establish a logical basis for thermodynamic irreversibility and the arrow of time.

· Any synergetic system of parts defines a topology such that the Category of Topological Thermodynamics is applicable to economic systems, political systems, as well as to biological systems. Such systems admit long-lived states far from equilibrium (adulthood). However, the theory can be used to design subtle perturbations to such systems such that their metastable lifetimes can be extended or destroyed.

Significant Results

- C2 Continuous Topological Evolution permits irreversible processes, $Q\wedge dQ\neq 0$. Segmented C1 processes, which approximate C2 processes, can be reversible, $Q\wedge dQ=0$.

- On odd-dimensional spaces, sequential C1 (translational) processes can be thermodynamically reversible, while intransitive C2 processes (rotation and expansion with a fixed point) can be thermodynamically irreversible.

- The relativistic Twin Paradox is resolved if the evolutionary paths of each twin are defined by processes that cause (unequal) topological change. If there is no topological change, or if each twin suffers the same topological change, there is no disparate biological aging.

Significant Results

- Adiabatic processes are transverse to the heat 1-form, $(i(\rho V_4)Q) = 0$. Adiabatic processes need not be quasi-static, and can be reversible or irreversible. Topological Transitions can occur in finite (often short) time.

- The work 1-form, W, is always transverse to the 4D process direction field, ρV_4, but the heat 1-form, Q, may or may not be transverse, The heat 1-form, Q can have longitudinal components in the direction of the process, corresponding to irreversible dissipation.

- Engineers should be guided by the universal concept of minimizing longitudinal heat in order to improve efficiency.

Significant Results

- **For fluids, this idea can be translated to reducing vorticity components in the direction of fluid accelerations (by using wing tip tabs on commercial jets).**

- **For non-equilibrium systems, the 3-form of Topological Torsion is not zero: $A\hat{}dA = i(T_4)dx\hat{}dy\hat{}dz\hat{}dt \neq 0$. T_4, is deduced intrinsically from the 1-form, A, that encodes the thermodynamic system, and can be used as special process current density, ρT_4, that defines an irreversible process.**

- **For PTD=3 "closed" thermodynamic systems, the process current has zero divergence, and the 4D volume element is a conformal invariant (any ρ). This result is the space-time extension of the Liouville theorem that preserves the phase-space volume element in classical theory.**

Significant Results

· For a PTD=4 "open" thermodynamic systems, the Topological Torsion vector does NOT have zero divergence, and so the process current ρT_4 may not be closed for arbitrary ρ (that is, the divergence of the process current is not zero).

· The combination of continuity and non-equilibrium, PTD(Q)>2, requires a causal direction to the arrow or time.

· PTD(A)=3 domains can have a local basis in terms of one complex Spinor pair with complex conjugate eigenvalues, and one real vector with eigenvalue zero. The eigenvalue 0 state can represent metastable (long-lived) configurations far from equilibrium. Such domains are locally "contact" manifolds.

Significant Results

- A key artifact of non-equilibrium is the existence of Topological Torsion current 3-forms, Topological Spin current 3-forms and Topological Adjoint current 3-forms.

All of these current 3-forms are similar to the Amperian charge current 3-form of electromagnetic theory, but are related to different species of dissipative phenomena, and occur only in non-equilibrium systems.

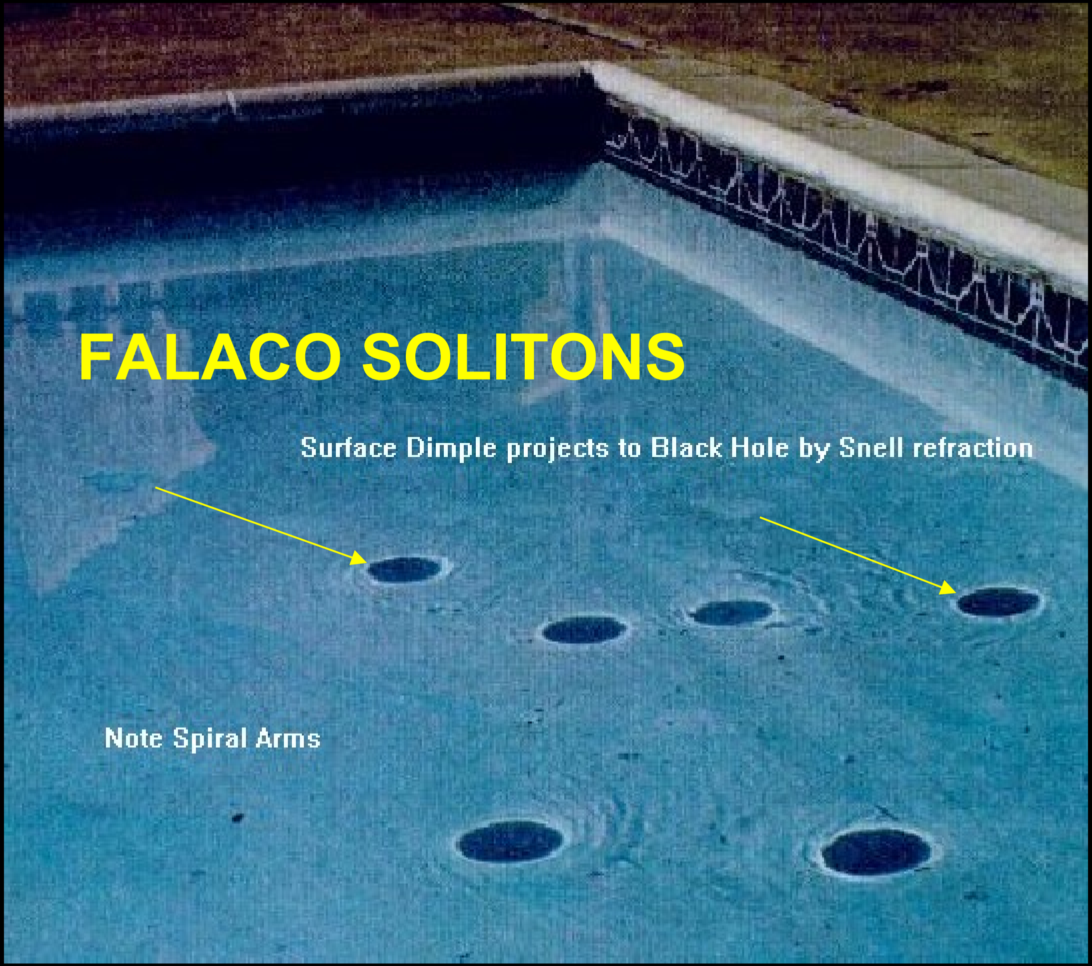
Dissipation coefficients are related to the non-zero divergences of the vector coefficients of the various 3-forms. For example, in electromagnetic systems, the dissipation coefficient is proportional to $E \circ B$ (see Vol 4) ; in hydrodynamics, the dissipation coefficient is called "Bulk Viscosity" (see Vol 3).

Significant Results

- PTD(A)=4 domains can have a local basis in terms of two complex Spinor pairs. Locally, such domains are symplectic manifolds. In such symplectic domains there exist local density distributions (subdomains), ρ, such that the divergence of any process current is zero in that subdomain.

Such subdomains are metastable contact manifolds. In other words, the symplectic domain can contain defect structures in the form of contact subdomains. It can be demonstrated in terms of continuous topological evolution that such local density distributions, which define a "stationary" state, can emerge as a topological defect in a PTD=4 system, by means of a dissipative processes.

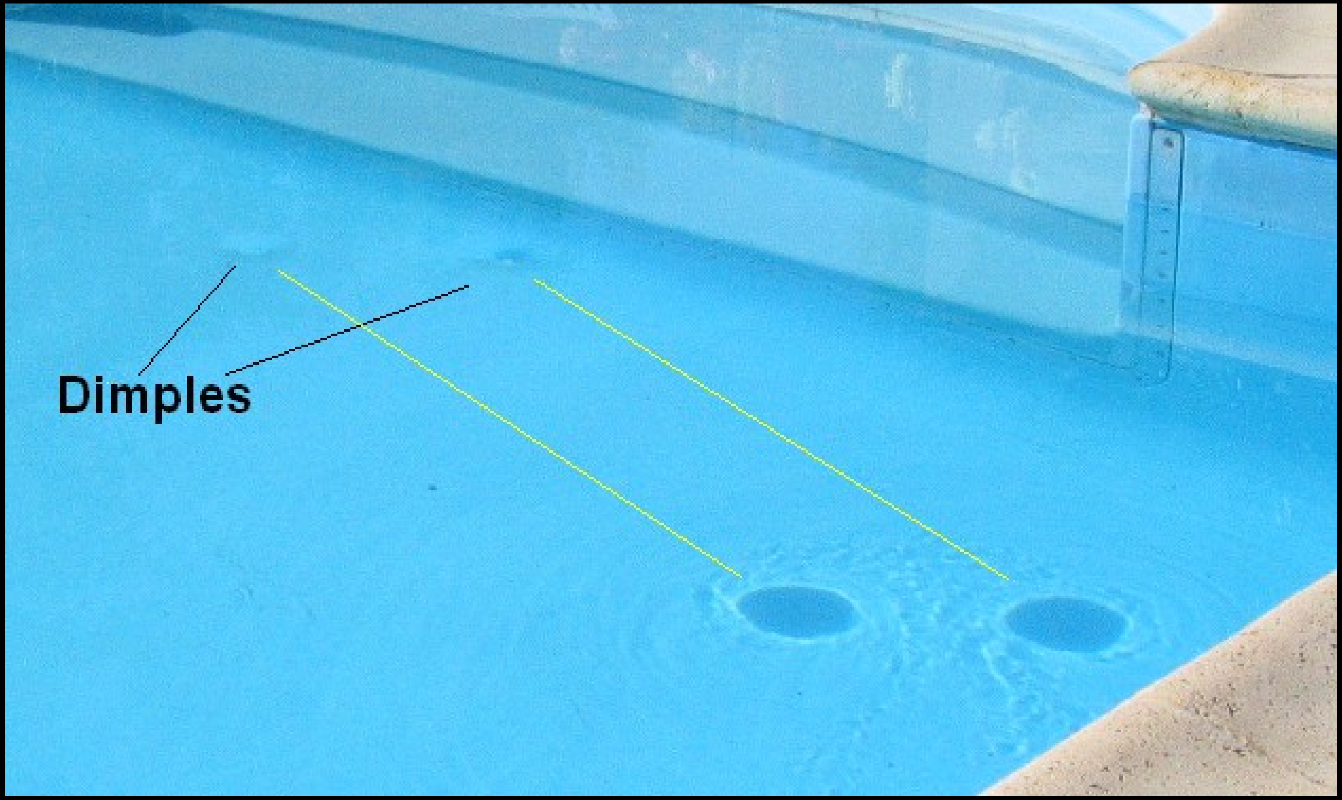
Emergence of Topological Defects



Long Lived Topological Defects in a Swimming Pool

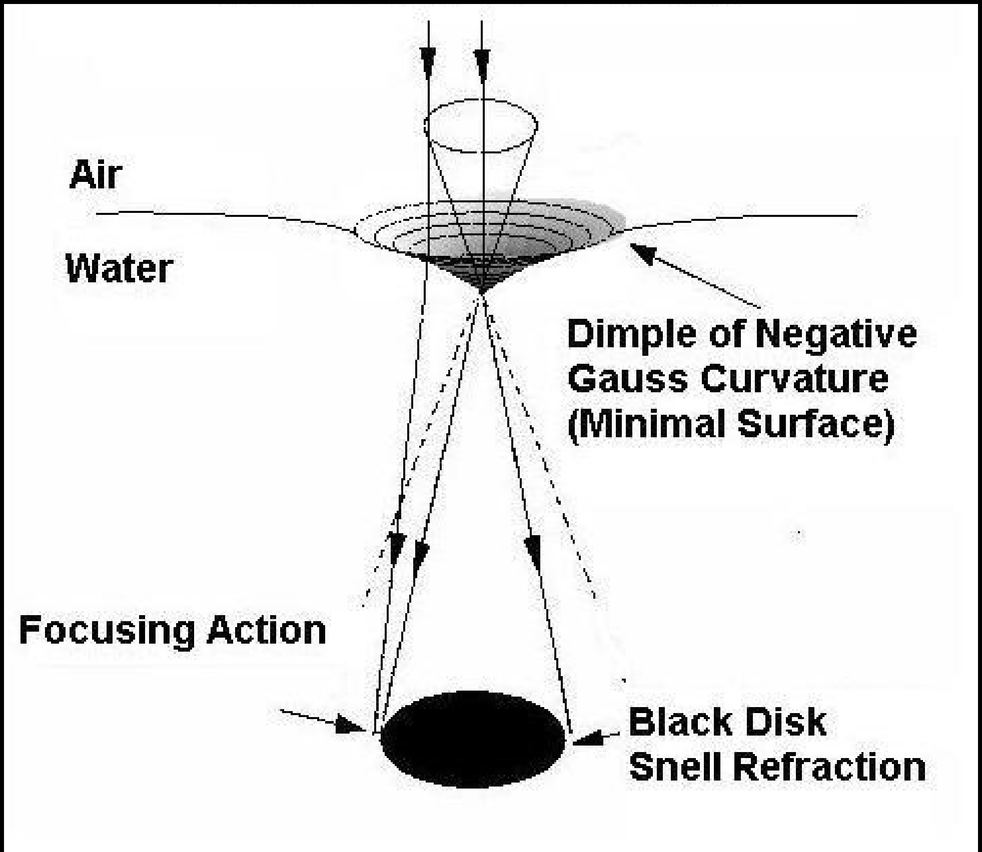
Emergence of Topological Defects

FALACO SOLITONS Movie by D. Radabaugh



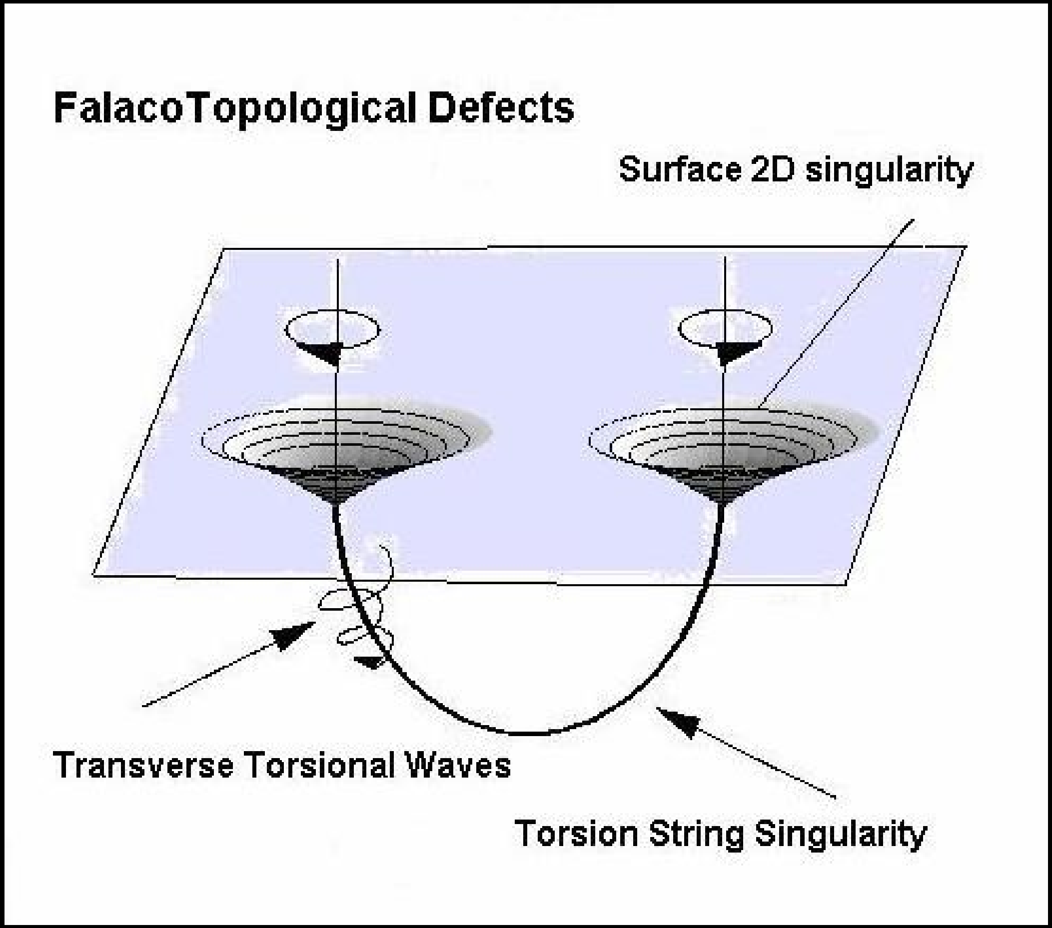
Solar Elevation about 30 degrees (See movie at http://www22.pair.com/csdc/download/spotsmovie.avi

Emergence of Topological Defects



Snell refraction of Falaco Soliton Spin Pairs

Emergence of Topological Defects



The first measurable Torsion String coupling between branes

This real world effect has been ignored by string theorists !!!

Significant Results

The 3-form of Adjoint Current, $\mathbf{J}_{adj}$, like the Torsion Current, can be constructed entirely from the topological features of the thermodynamic system as determined by the functional coefficients of the 1-form of Action, A. $\mathbf{J}_{adj}$ also admits an infinity of integrating factors. However, when the 1-form coefficients, A_k are divided by a Holder Norm, λ, of homogeneity index one (known as the Gauss map), then the vector components of $\mathbf{J}_{adj}$ are defined as $|\mathbf{J}_{adj}\rangle = [\text{Jacobian}(A_k/\lambda)]^{adj} \circ |A_k/\lambda\rangle$. The 3-form is defined in terms of vector components as $\mathbf{J}_{adj} = i(\mathbf{J}_{adj}\})dx\wedge dy\wedge dz\wedge dt$. It has zero divergence globally, and a pre-image 2-form G_{adj}, such that $dG_{adj} = \mathbf{J}_{adj}$. It can be demonstrated that $\mathbf{J}_{adj}$ is related to the cubic curvature of the shape matrix, leading to the idea that the source of electromagnetic charge is related to cubic curvatures, similar to the idea that mass is related to quadratic curvatures.

Significant Results

· Examples of thermodynamic systems can be given to demonstrate that the conjectured format of the London Current of superconductivity, where **J** = χ **A**, can be deduced as an emergent consequence of the Topological Theory of Thermodynamics (see Vol 5).

· Examples can generate a Spin Current 3-form, **S**=**A**^**G**, where, formally, the Spin current is proportional to the Lorentz force (the space-time components of the Work 1-form, W). This is a new interpretation of an old result, **J**=**σ**(**E**+**V**x**B**), which is Ohm's law. The new part is due to the idea that the dissipation is due to Spin Currents and the transport of collective spins, A^G.

Significant Results

· The topological structure of domains of Pfaff dimension 2 or less creates a connected, but not necessarily simply connected topology. Evolutionary predictive solution uniqueness is possible.

· The topological structure of domains of Pfaff dimension 3, or more, creates a disconnected topology of multiple components. If solutions to a particular evolutionary problem exist, then the solutions are not unique. Envelope solutions, such as Huygen wavelets and propagating tangential discontinuities (called signals, or wakes) are classic examples of solution non-uniqueness. **Topological Torsion is an artifact of non-uniqueness, and must be non-zero if a hydrodynamic system exhibits turbulence.**

Significant Results

· All Hamiltonian, Symplectic-Bernoulli and Helmholtz processes are thermodynamically reversible. In particular, the work 1-form, W, created by Hamiltonian processes is of Pfaff Topological Dimension 1 or less. In all reversible cases the Work 1-form is closed, dW=0. The evolutionary equations in such cases are time reversal invariant.

· The assumption of uniqueness of evolutionary solutions (which implies the Pfaff Topological Dimension of the thermodynamic system be equal to 2 or less), or the assumption of homeomorphic evolution, have imposed constraints upon classical mechanics that eliminate any time asymmetry, and preclude the existence of isotropic macroscopic spinors.

Significant Results

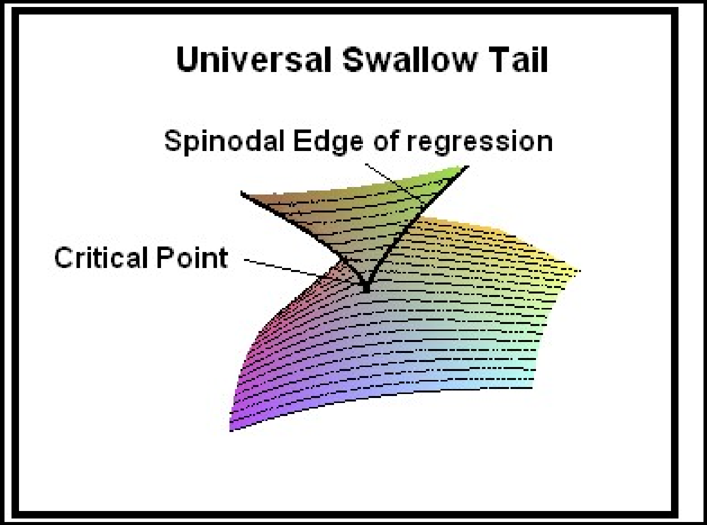
· The Lie differential acting on differential forms is not necessarily the same as a linear affine covariant differential acting on differential forms. The covariant differential always defines an adiabatic process, where the Lie differential does not.

· The particle view of thermodynamics is based upon the 1-form of action whose coefficients, $[A_m]$, admit a **correlation** Jacobian matrix $[\partial A_m/\partial x^n]$.

· The statistical point of view is based upon a current N-1 form whose coefficients, $[C^m]$, admit a collineation Jacobian matrix, $[\partial C^m/\partial x^n]$, with a trace = to a divergence of $[C^m]$.

Significant Results

· On spaces of PTD=4, the Correlation Jacobian matrix has a characteristic polynomial that defines an quartic equation of state in terms of Cayley-Hamilton similarity invariants. The characteristic polynomial produces an implicit hypersurface function that can have envelopes and edges of regression in the format of the Gibbs function for a (universal and deformable) **van der Waals gas.**

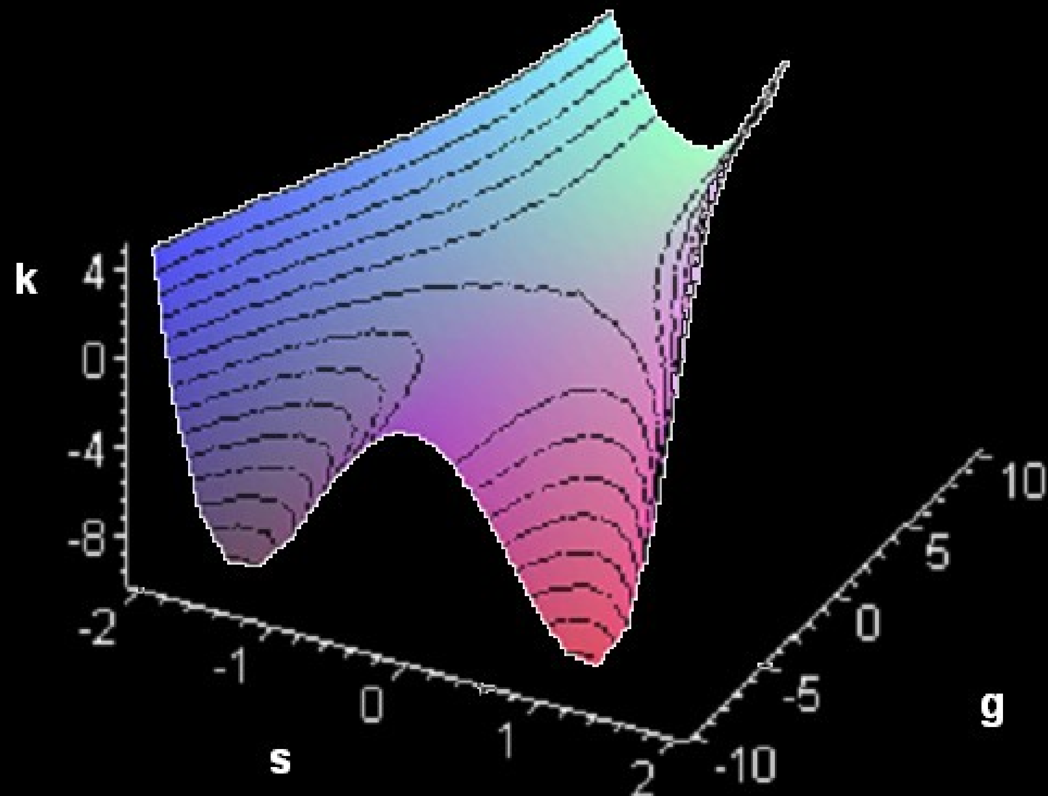


Significant Results

- The 4D Correlation Jacobian matrix can be mapped into a reduced characteristic polynomial representing a quartic Universal Thermodynamic Phase function, with an envelope, which, below the critical point, has the features of a Higgs potential, made famous by string theory.

The reduced thermodynamic phase function is a hypersurface with Zero mean curvature (a minimal surface) and many properties of a **van der Waals gas**.

Universal Topological Thermodynamic Phase Function



A van der Waals gas with a Higgs potential,

An Envelope of a 4D Cayley-Hamilton characteristic polynomial

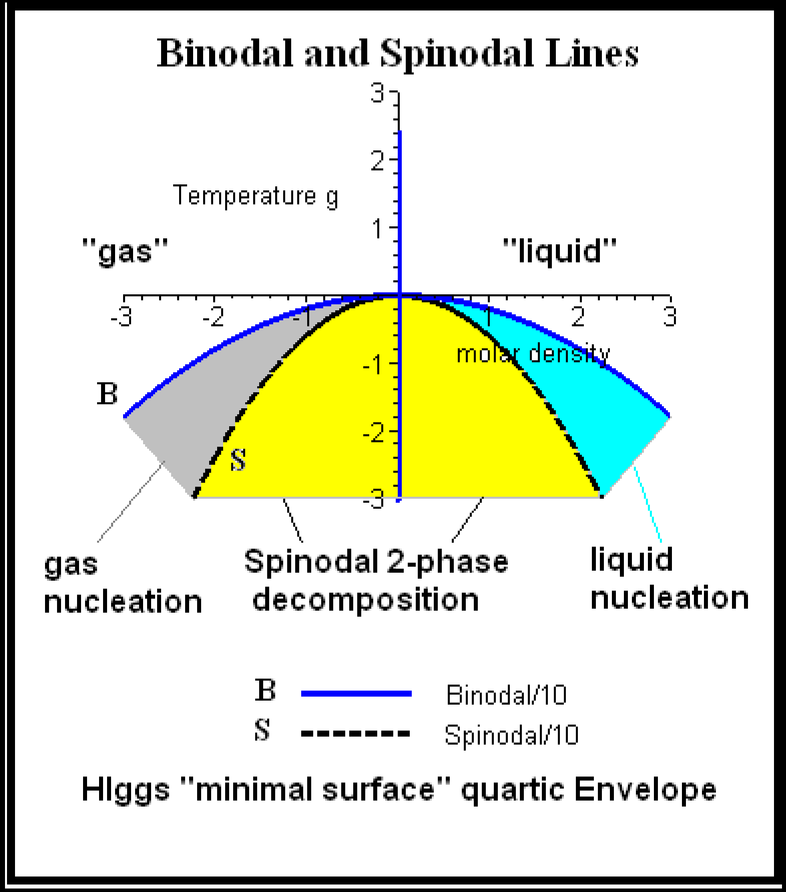
Significant Results

- The reduced phase function yields analytic expressions for the critical point and the binodal and spinodal lines in terms of the similarity invariants of the correlation matrix.

The same technique can be used to determine "phase transitions" and possible bifurcations in dynamical systems.

Significant Results

· The reduced phase function yields analytic expressions for the critical point and the binodal and spinodal lines in terms of the similarity invariants of the correlation matrix.



Significant Results

· The collineation matrix can have complex eigenvalues even though the maximal rank matrix elements are real. These complex eigenvectors have a zero norm, and are generators of conjugate minimal surfaces in 4D, that have both statistical and fractal components. Cartan defined such sets as Isotropic Spinors (Majorana, not Dirac) Spinors. Pairs of non-colinear Spinors define an area, but the norm of each Spinor is zero!

The hypersurface minimal surface can be generated by a holomorophic function that includes both the Gibbs entropy and a Mandelbrot fractal germ, $\theta = (z\ln z - z) + (a \pm bz^2)$. The third partial derivative leads to conjugate pairs of minimal surfaces, and the Mandelbrot term vanishes. All functional iterates remain holomorphic and generate minimal surfaces with fractal boundaries.

Significant Results

- For a **PTD=4** "open" thermodynamic systems, the Topological Torsion vector does not have zero divergence,

$$A\hat{}dA = i(T_4)dx\hat{}dy\hat{}dz\hat{}dt \neq 0.$$

but the process current ρT_4 may not be closed for arbitrary ρ (that is, the divergence of the process current is not zero).

$$d(A\hat{}dA) \neq 0$$

This result is an extension of the Vlasov equation; the 4D differential volume element is expanding or contracting. Such processes in the direction of T_4 are irreversible and dissipative.

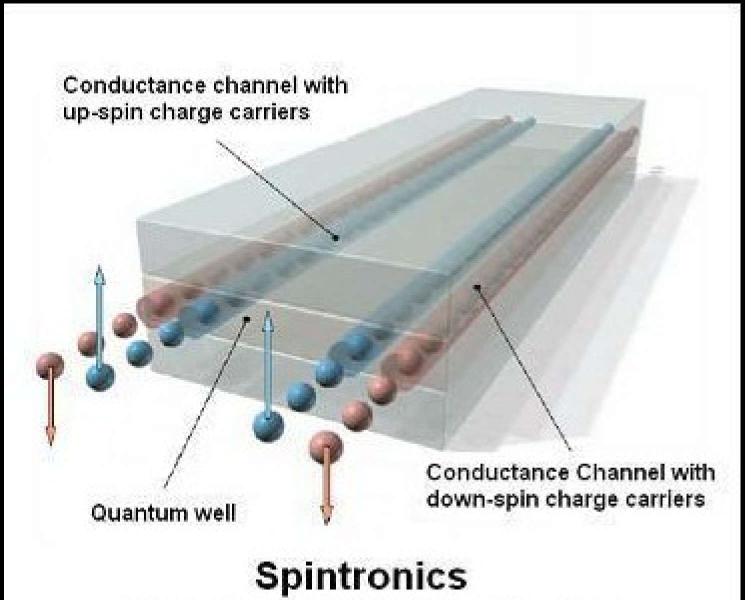
There exist many integrating factors that will produce zero divergence of the 3 form, such that $d(\rho A\hat{}dA)=0$.

Significant Results

- **Topological fluctuations** can be induced by processes that have components in terms of thermodynamic macroscopic Spinors. Thermodynamic **Macroscopic Spinors** are non-zero complex eigenvectors with complex eigenvalues (and zero quadratic form) of the antisymmetric 2-form (or matrix) representing the "Limit Points", dA, of the 1-form of Action, A. Such Macroscopic Spinors are capable of representing minimal surface conjugate pairs.

Significant Results

- **Topological Insulators** correspond to Impedances defined in terms of quantized Topological Spin, and Spin pairs coupled via Falaco Solitons. If the divergence of the Spin 4vector vanishes, the **Spin Current** is time reversal invariant.



Picture with permission of Shoucheng Zhang

Significant Results

- The fact that any synergetic system of parts in effect defines a topology implies that the universal method of topological thermodynamics is **applicable to economic systems, political systems, as well as to biological systems**. Such systems admit long-lived states far from equilibrium (adulthood). However, the theory can be used to design subtle perturbations to such systems such that their metastable lifetimes can be extended or destroyed.

Significant Results

- The thermodynamic processes that lead to self-similarity of a Current 3-form $L_{(J)}C=\sigma\, C$ can generate **fractals** and **holographic** effects where small neighborhoods replicate the whole, approximately. The homogeneity coefficient is the trace of the Jacobian Collineation: $\sigma = \mathrm{Trace}[\partial C^m/\partial x^n]$**,** or the divergence of the Process vector field.

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- A turbulent thermodynamic **cosmology** can be constructed in terms of a dilute **non-equilibrium van der Waals** gas near its critical point.

Cosmology as a non-equilibrium Van der Waals Gas explains

a.) The granularity of the night sky as exhibited by stars and galaxies due to density fluctuations near the critical point, and the Newtonian law of gravitational attraction proportional to $1/r^2$ as a correlation between fluctuations (due to Lev Landau).

Cosmology as a non-equilibrium Van der Waals Gas explains

a.) The granularity of the night sky as exhibited by stars and galaxies due to density fluctuations near the critical point, and the Newtonian law of gravitational attraction proportional to $1/r^2$ as a correlation between fluctuations (due to Lev Landau).

b.) The conformal expansion of the universe is an irreversible phenomenon-associated with Quartic similarity invariants in the thermodynamic phase function, and dissipation related to conformal deformations of the 4D volume element.

Cosmology as a non-equilibrium Van der Waals Gas explains

c.) The possibility of domains of negative pressure (explaining what has recently been called "**dark energy**") are due to a classical "Higgs" mechanism for aggregates below the critical temperature.

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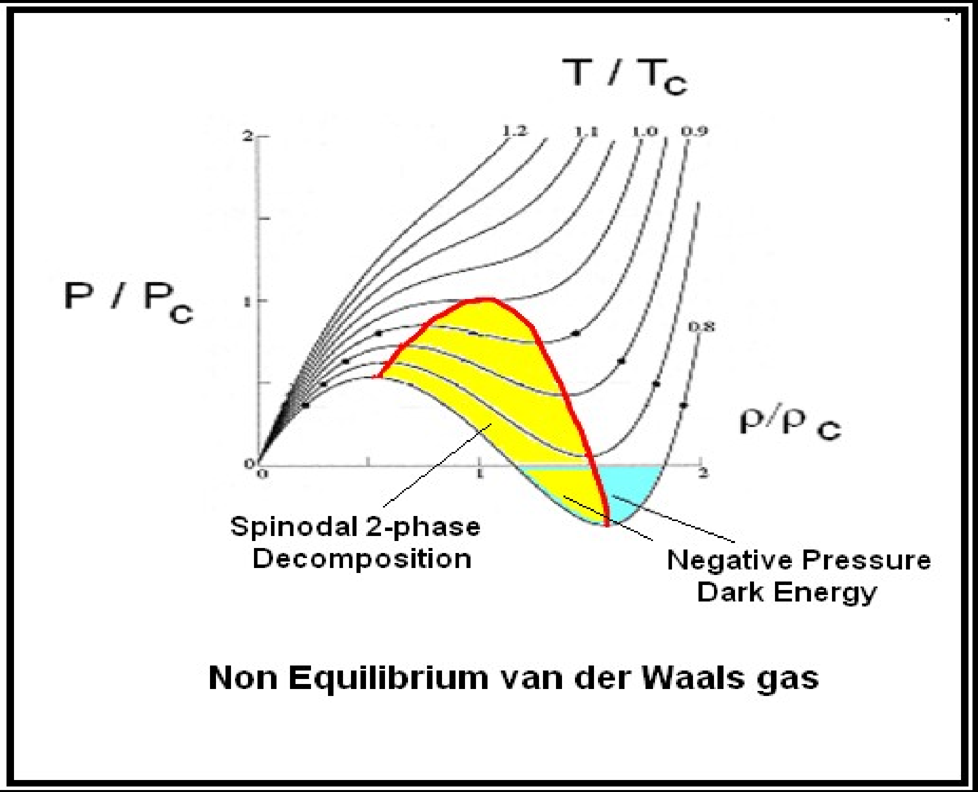

Cosmology as a non-equilibrium Van der Waals Gas explains

d.) The possibility of domains of negative **temperature** (explaining what has recently been called "**dark matter**") are due to macroscopic collective states of ordered spins. The conjecture is that **Positive temperature radiates, Negative temperature does not**.

Conjecture: Black holes could be negative temperature states of collective spins.

Cosmology as a non-equilibrium Van der Waals Gas explains

e.) The possibility of domains where gravitational effects (quadratic similarity invariants, or 2nd order Gauss curvature effects) appear to be related to entropy and temperature properties of the thermodynamic system, and where **cubic curvature effects due to Spin and Adjoint current 3-forms could impede gravitational collapse.**

Cosmology as a non-equilibrium Van der Waals Gas explains

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f.) Black Holes (generated by Petrov Type D solutions in gravitational theory) are to be related to **Minimal Surface** solutions to the Universal thermodynamic 4th order Phase function.

Thanks for your interest

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