Prigogine's Thermodynamic Emergence and Continuous Topological Evolution

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Abstract: Irreversible processes can be described in Open non-equilibrium thermodynamic systems, of topological dimension 4. By means of Continuous Topological evolution, such processes can cause local decay to Closed non-equilibrium thermodynamic states, of topological dimension 3. These topologically coherent, perhaps deformable, regions or states of one or more components appear to "emerge" as compact 3D Contact submanifolds that can be defined as topological defects in the 4D Symplectic manifold. These emergent states are still far from equilibrium, as their topological (not geometrical) dimension is greater than 2. The 3D Contact submanifold admits evolutionary processes with a unique extremal Hamiltonian vector component, as well as fluctuation spinor components. If the subsequent evolution is dominated by the Hamiltonian component, the emergent topological defects will maintain a relatively long-lived, topologically coherent, approximately non-dissipative structure. These topologically coherent, "stationary states" far from equilibrium ultimately will decay, but only after a substantial "lifetime". Analytic solutions and examples of these processes of Continuous Topological Evolution give credence, and a deeper understanding, to the general theory of self-organized states far from equilibrium, as conjectured by I. Prigogine. Moreover, in an applied sense, universal engineering design criteria can be developed to minimize irreversible dissipation and to improve system efficiency in general non-equilibrium situations. As the methods are based on universal topological, not geometrical, ideas, the general thermodynamic results apply to all synergetic topological systems. It may come as a surprise, but ecological applications of thermodynamics need not be limited to the design of specific hardware devices, but apply to all synergetic systems, be they mechanical, biological, economical or political.

1 THE POINT OF DEPARTURE

1.1 Topological Evolution vs. Geometrical Evolution

This essay represents a brief summary of the features of a topological theory of thermodynamics. The point of departure starts with a topological (not statistical) formulation of dynamics, which can furnish a universal foundation for the Partial Differential Equations of non-equilibrium hydrodynamics and electrodynamics. The thermodynamic system is defined in terms of an exterior differential 1-form, A, constructed on a pre-geometric domain of base variables (coordinate functions). The topology of significance is defined in terms of Cartan's topological structure [6], constructed in terms of the functions used to define the 1-form of Action, A. The objective is to study the evolution of topological properties (topological change is the source of Heat), and ignore the more classical emphasis on geometrical properties of size and shape. Based upon Cartan's theory of exterior differential forms, many of the mysteries of non-equilibrium thermodynamics, irreversible processes, and turbulent flows, can be resolved. In addition, the non-equilibrium methods can lead to many new processes and patentable devices and concepts.

However, a more striking implication of the thermodynamic topological method is that it can be applied to any different complex systems of synergetic components obeying logical rules of union and intersection that define a differentiable topological structure. Such systems need not be constrained to plasmas and fluids, but apply universally to all topological systems that can be encoded in terms of exterior differential forms. Such non-equilibrium systems may be biological systems, political systems, economic systems, as well as mechanical systems. They all have common characteristics of emergence (conception and growth to maturity), and irreversible non-equilibrium interactions with their Open environment (by feeding and excrementing waste). Maturity implies that the system has reached a slowly decaying, almost stationary, Closed state of relatively long lifetime, often followed by a rapid decay to Equilibrium state, or death. The topological methods give a formal basis to Prigogine's conjectures of self organization by means of dissipative processes.

2 Topological Thermodynamics

2.1 The Axioms of Topological Thermodynamics

The topological methods used herein are based upon Cartan's theory of exterior differential forms. The thermodynamic view assumes that the physical systems to be studied can be encoded in terms of a 1-form of Action Potentials (per unit source, or, per mole), A, on a four-dimensional variety of ordered independent variables, $\{\xi^1, \xi^2, \xi^3, \xi^4\}$. The variety supports a differential volume element $\Omega_4 = d\xi^1\hat{}d\xi^2\hat{}d\xi^3\hat{}d\xi^4$. This statement implies that the differentials of the $\mu = 4$ base variables are functionally independent. No metric, no connection, no constraint of gauge symmetry is imposed upon the four-dimensional pre-geometric variety. Topological constraints can be expressed in terms of exterior differential systems placed upon this set of base variables [1].

In order to make the equations more suggestive to the reader, the symbolism for the variety of independent pre-geometric variables will be changed to the format $\{x, y, z, t\}$, but be aware that no constraints of metric or connection are imposed upon this variety, at this, thermodynamic, level. For instance, it is NOT assumed that the variety is spatially Euclidean.

With this notation, the Axioms of Topological Thermodynamics can be summarized as:

Axiom 1. *Thermodynamic physical systems can be encoded in terms of a 1-form* *of covariant Action Potentials,* $A_\mu(x, y, z, t...)$*, on a four-dimensional abstract variety of ordered independent variables,* $\{x, y, z, t\}$*. The variety supports differential volume element* $\Omega_4 = dx\hat{}dy\hat{}dz\hat{}dt$*.*

Axiom 2. *Thermodynamic processes are assumed to be encoded, to within a factor,* $\rho(x, y, z, t...)$*, in terms of a contravariant Vector and/or complex Spinor directionfields, symbolized as* $V_4(x, y, z, t)$*.*

Axiom 3. *Continuous Topological Evolution of the thermodynamic system can be encoded in terms of Cartan's magic formula (see p. 122 in [5]). The Lie differential with respect to the process,* ρV_4*, when applied to an exterior differential 1-form of Action,* $A = A_\mu dx^\mu$*, is equivalent, abstractly, to the first law of thermodynamics.*

$$\textbf{Cartan's Magic Formula} \tag{1}$$

$$L_{(\rho\mathbf{V}_4)}A = i(\rho\mathbf{V}_4)dA + d(i(\rho\mathbf{V}_4)A) = W + dU = Q, \tag{2}$$

$$\text{Heat 1-form } Q = W + dU = L_{(\rho\mathbf{V}_4)}A, \tag{3}$$

$$\text{Work 1-form } W = i(\rho\mathbf{V}_4)dA, \quad \text{Internal Energy } U = i(\rho\mathbf{V}_4)A. \tag{4}$$

Axiom 4. *Equivalence classes of systems and continuous processes can be defined in terms of the Pfaff Topological Dimension and topological structure generated by the possibly non-exact 1-forms of Action,* A, Work, W, and Heat, Q.

Axiom 5. If $Q\hat{}dQ \neq 0$, *then the thermodynamic process is irreversible.*

In effect, Cartan's magic formula leads to a topological basis of thermodynamics, where the thermodynamic Work, W, thermodynamic Heat, Q, and the thermodynamic internal energy, U, are defined *dynamically* in terms of Continuous Topological Evolution. In effect, the First Law is a statement of deRham cohomology theory: the difference between two non-exact differential forms is equal to an exact differential, $Q - W = dU$. My recognition (some 30 years ago) of this correspondence between the Lie *differential* and the First Law of thermodynamics has been the corner stone of my research efforts in applied topology.

2.1.1 The Pfaff Sequence

It is important to realize that the Pfaff Topological (not geometrical) Dimension of the system 1-form of Action, A, determines whether the thermodynamic system is Open, Closed, Isolated or Equilibrium. Also, it is important to realize that the Pfaff Topological Dimension (PTD) of the thermodynamic Work 1-form, W, determines a specific category of reversible and/or irreversible processes. It is therefore of some importance to understand the meaning of the PTD of a 1-form. Given the functional format of a general 1-form, A, on a 4D variety it is an easy step to compute the Pfaff Sequence, using one exterior differential operation, and several algebraic exterior products.

For a differential 1-form, A, defined on a geometric domain of 4 base variables, the Pfaff Sequence is defined as:

$$\textbf{Pfaff Sequence}\ \ \{A, dA, A\hat{}dA, dA\hat{}dA...\} \tag{5}$$

2.1.2 Pfaff Topological Dimension of the System 1-form, A

It is possible that over some domains, as the elements of the sequence are computed, one of the elements (and subsequent elements) of the Pfaff Sequence will vanish. The number of non-zero elements in the Pfaff Sequence (PS) defines the Pfaff Topological Dimension (PTD) of the specified 1-form[1]. Modulo singularities, the PTD determines the minimum number M of N functions of base (geometric) variables ($N \geq M$) required to define the topological properties of the connected component of the 1-form A.

The PTD of the 1-form of Action, A, can be put into correspondence with the four classic topological structures of thermodynamics. Equilibrium, Isolated, Closed, and Open systems. The classic thermodynamic interpretation is that the first two structures do not exchange mass (mole numbers) or radiation with their environment. The Closed structure ($PTD = 3$) can exchange radiation with its environment, but not mass (mole numbers). The Open structure ($PTD = 4$) can exchange both mass and radiation with its environment. The following table summarizes these properties. For reference purposes, I have given the various elements of the Pfaff sequence specific names:

Topological p-form name	**PS** element	**Nulls**	**PTD**	Thermodynamic system
Action	A	$dA = 0$	1	**Equilibrium**
Vorticity	dA	$A\hat{}dA = 0$	2	**Isolated**
Torsion	$A\hat{}dA$	$dA\hat{}dA = 0$	3	**Closed**
Parity	$dA\hat{}dA$	–	4	**Open**

Table 1 Applications of the Pfaff Topological Dimension.

The four thermodynamic systems can be placed into two disconnected topological categories. If the PTD of A is 2 or less, the first category is determined by the closure (or differential ideal) of the 1-form of Action, $A \cup dA$. This Cartan topology is a connected topology. In the case that the PTD is greater than 2, the Cartan topology is based on the union of two closures, $\{A \cup dA \cup A\hat{}dA \cup dA\hat{}dA\}$, and is a disconnected topology.

It is a topological fact that there exists a (topologically) continuous C2 process from a disconnected topology to a connected topology, but there does not exist a C2 continuous process from a connected topology to a disconnected topology. This fact implies that topological change can occur continuously by a "pasting" processes representing the *decay* of turbulence by "condensations" from

[1] The Pfaff Topological dimension has been called the "class" of a 1-form in the old literature. I prefer the more suggestive name.

non-equilibrium to equilibrium systems. On the other hand, the *creation* of Turbulence involves a discontinuous (non C2) process of "cutting" into parts. This warning was given long ago to prove that computer analyses that smoothly match value and slope will not replicate the *creation* of turbulence, but can faithfully replicate the *decay* of turbulence.

2.1.3 The Pfaff Topological Dimension of the Work 1-form, W

The PDE's that represent the system dynamics are determined by both the PTD of the 1-form of Work, W, and the PTD of the 1-form of Action, A, that encodes the physical system. The PTD of the thermodynamic Work 1-form depends upon both the physical system, A, and the process, $\mathbf{V}_4$. In particular if the PTD of the thermodynamic Work 1-form is zero, ($W = 0$), then system dynamics is generated by an extremal vector field which admits a Hamiltonian realization. However, such extremal direction fields can occur only when the PTD of the system encoded by A is odd, and equal or less than the geometric dimension of the base variables.

For example, if the geometric dimension is 3, and the PTD of A is 3, then there exists a unique extremal field on the Contact manifold defined by dA. This unique directionfield is the unique eigen directionfield of the 3x3 antisymmetric matrix (created by the 2-form $F = dA$) with eigenvalue equal to zero.

If the geometric dimension is 4, and the PTD of A is 3, then there exists a two extremal fields on the geometric manifold. These directionfields are those generated as the eigen directionfields of the 4x4 antisymmetric matrix (created by the 2-form $F = dA$) with eigenvalue equal to zero.

If the geometric dimension is 4, and the PTD of A is 4, then there do not exist extremal fields on the Symplectic manifold defined by dA. All of the eigen directionfields of the 4x4 antisymmetric matrix (created by the 2-form $F = dA$) are complex isotropic spinors with pure imaginary eigenvalues not equal to zero.

2.2 Topological Torsion and other Continuous Processes.

2.2.1 Reversible Processes

Physical Processes are determined by directionfields[2] with the symbol, $\mathbf{V}_4$, to within an arbitrary function, ρ. There are several classes of direction fields that are defined as follows: The **Associated Class**: $i(\rho\mathbf{V}_4)A = 0$. The **Extremal Class:** $i(\rho\mathbf{V}_4)dA = 0$. The **Characteristic Class:** $i(\rho\mathbf{V}_4)A = 0$, and $i(\rho\mathbf{V}_4)dA = 0$. The **Helmholtz Class:** $d(i(\rho\mathbf{V}_4)dA) = 0$. The maximal rank of the antisymmetric matrix, dA, determines the manifold structure, which is a Contact structure when the rank is odd, and a symplectic structure when the manifold is even. All odd dimensional Contact manifolds admit a Hamiltonian extremal field, but a an extremal field does not exist on a symplectic manifold.

The bottom line is that all such processes are thermodynamically reversible.

[2] Which include both vector and spinor fields.

2.2.2 Irreversible Processes

There is one directionfield that is uniquely defined by the coefficient functions of the 1-form, A, that encodes the thermodynamic system on a 4D geometric variety. This vector exists only in non-equilibrium systems, for which the PTD of A is 3 or 4. This 4-vector is defined herein as the topological Torsion vector, $\mathbf{T}_4$. To within a factor, this directionfield[3] has the four coefficients of the 3-form $A\hat{}dA$, with the following properties:

$$\begin{aligned}
\textbf{Properties of} &\quad : \quad \textbf{Topological Torsion } \mathbf{T}_4 \text{ on } \Omega_4 && (6)\\
i(\mathbf{T}_4)\Omega_4 &= i(\mathbf{T}_4)dx\hat{}dy\hat{}dz\hat{}dt = A\hat{}dA, && (7)\\
W &= i(\mathbf{T}_4)dA = \sigma\ A,\ dW = d\sigma\hat{}A + \sigma dA = dQ && (8)\\
U &= i(\mathbf{T}_4)A = 0,\quad \mathbf{T}_4 \text{ is associative} && (9)\\
i(\mathbf{T}_4)dU &= 0,\quad i(\mathbf{T}_4)Q = 0,\ \mathbf{T}_4 \text{ is adiabatic} && (10)\\
L_{(\mathbf{T}_4)}A &= \sigma\ A,\quad \mathbf{T}_4 \text{ is homogeneous} && (11)\\
L_{(\mathbf{T}_4)}dA &= d\sigma\hat{}A + \sigma dA = dQ, && (12)\\
Q\hat{}dQ &= L_{(\mathbf{T}_4)}A\hat{}L_{(\mathbf{T}_4)}dA = \sigma^2 A\hat{}dA \neq 0,\ \mathbf{T}_4 \text{ is irreversible} && (13)\\
dA\hat{}dA &= d(A\hat{}dA) = d\{(i(\mathbf{T}_4)\Omega_4\} = (div_4\mathbf{T}_4)\Omega_4, && (14)\\
L_{(\mathbf{T}_4)}\Omega_4 &= d\{(i(\mathbf{T}_4)\Omega_4\} = (2\sigma)\Omega_4,\ \sigma = \text{ irreversible dissipation function} && (15)
\end{aligned}$$

If the PTD of A is 4 (an Open thermodynamic system), then $\mathbf{T}_4$ has a non-zero 4 divergence, (2σ), representing an expansion or a contraction of the 4D volume element Ω_4. The Heat 1-form, Q, generated by the process, $\mathbf{T}_4$, is NOT integrable. Q is of PTD greater that 2, when $\sigma \neq 0$. Furthermore the $\mathbf{T}_4$ process is locally adiabatic as the change of internal energy in the direction of the process path is zero. Therefore, in the PTD 4 case, where $dA\hat{}dA \neq 0$, the $\mathbf{T}_4$ direction field represents an *irreversible, adiabatic process.*

When σ is zero *and* $d\sigma = 0$, but $A\hat{}dA \neq 0$, the PTD of the system is 3 (a Closed thermodynamic system). In this case, the $\mathbf{T}_4$ direction field becomes a characteristic vector field which is both extremal and associative, and induces a Hamilton-Jacobi representation (the ground state of the system for which $dQ = 0$).

For any process and any system, equation (13) can be used as a test for irreversibility.

It seems a pity, that the concept of the Topological Torsion vector and its association with non-equilibrium systems, where it can be used to establish design criteria to minimize energy dissipation, has been ignored by the engineering community.

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[3] A direction field is defined by the components of a vector field which establish the "line of action" of the vector in a projective sense. An arbitrary factor times the direction field defines the same projective line of action, just reparameterized. In metric based situations, the arbitrary factor can be interpreted as a renormalization factor.

2.3 Emergent Topological Defects

Suppose an evolutionary process starts in a domain of PTD 4, for which a process in the direction of the Topological Torsion vector, $\mathbf{T}_4$, is known to represent an irreversible process. Examples will demonstrate that the irreversible process can proceed to a domain of the geometric variety for which the dissipation coefficient, σ, becomes zero. The idea is that a subdomain of the original system of PTD 4 can evolve continuously with a change of topology to a region of PTD 3. The emergent subdomain of PTD 3 is a topological defect, with topological coherence, and often with an extended lifetime (as a soliton structure with a dominant Hamiltonian evolutionary path), embedded in the Pfaff dimension 4 turbulent background.

The Topological Torsion vector in a region of PTD 3 is an extremal vector direction field in systems of PTD 3; it then has a zero 4D divergence, and leaves the volume element invariant. Moreover the existence of an extremal direction field implies that the 1-form of Action can be given a Hamiltonian representation, $A = P_k dq^k + H(P, q, t)dt$. In the domain of Pfaff dimension 3 for the Action, A, the subsequent continuous evolution of the system, A, relative to the process $\mathbf{T}_4$, can proceed in an energy conserving, Hamiltonian manner, representing a "stationary" or "excited" state far from equilibrium (the ground state). This argument is based on the assumption that the Hamiltonian component of the direction field is dominant, and any Spinor components in the $PTD = 3$ domain, representing topological fluctuations, can be ignored. These excited states, far from equilibrium, can be interpreted as the evolutionary topological defects that emerge and self-organize due to irreversible processes in the turbulent dissipative system of Pfaff dimension 4.

The descriptive words of self-organized states far from equilibrium have been abstracted from the intuition and conjectures of I. Prigogine [4]. The methods of Continuous Topological Evolution correct the Prigogine conjecture that "dissipative structures" can be caused by dissipative processes and fluctuations. The long-lived excited state structures created by irreversible processes are non-equilibrium, deformable topological defects almost void of irreversible dissipation. The topological theory presented herein presents for the first time a solid, formal, mathematical justification (with examples) for the Prigogine conjectures. Precise definitions of equilibrium and non-equilibrium systems, as well as reversible and irreversible processes can be made in terms of the topological features of Cartan's exterior calculus. Using Cartan's methods of exterior differential systems, thermodynamic irreversibility and the arrow of time can be well defined in a topological sense, a technique that goes beyond (and without) statistical analysis [**?**]. Thermodynamic irreversibility and the arrow of time requires that the evolutionary process produce topological change.

2.3.1 Topological 3-forms and 4-forms in EM format

In engineering format, the components of the Topological Torsion vector, $\mathbf{T}_4$, can be determined as,

$$A\hat{}F = i(\mathbf{T}_4)\Omega_4 = i(\mathbf{T}_4)dx\hat{}dy\hat{}dz\hat{}dt \tag{16}$$

$$\mathbf{T}_4 \quad = \quad [\mathbf{T}, h] = -[\mathbf{E} \times \mathbf{A} + \mathbf{B}\phi, \mathbf{A} \circ \mathbf{B}]. \tag{17}$$

If $\mathbf{T}_4$ is used as to define the direction field of a process, then

$$L_{(\mathbf{T}_4)}A \quad = \quad \sigma A, \qquad i(\mathbf{T}_4)A = 0. \tag{18}$$

$$\text{where } \; 2\sigma \quad = \quad \{div_4(\mathbf{T}_4)\} = 2(\mathbf{E} \circ \mathbf{B}). \tag{19}$$

The important (universal) result is that if the "acceleration" associated with the direction field, **E**, is parallel to the "vorticity" associated with the direction field, **B,** then according to the equations starting with eq. (6) et. seq. the process is dissipatively irreversible. This result establishes the design criteria for engineering applications to minimize dissipation from turbulent processes.

The Topological Torsion vector has had almost no utilization in applications of classical electromagnetic theory.

3 Examples of Emergence

In this section, one of many possible examples is given describing the emergence of Closed, topologically coherent, domains of $PTD = 3$ (embedded as topological defects in an Open thermodynamic environment of $PTD = 4$). The emergent process in the example will use the direction field, $\mathbf{T}_4$, which is known to be thermodynamically irreversible. An electromagnetic notation will be used, but the results can be converted into hydrodynamic (and universal) format.

Zero Helicity case: PTD = 4 decays to PTD = 3 Start with the 4D thermodynamic domain, and consider the 1-form of Action, A, representing a thermodynamic system with the format:

$$A = A_x(x, y)dx + A_y(x, y)dy - \phi(x, y, z, t)dt. \tag{20}$$

The induced 2-form, $F = dA$, has components interpreted in terms of the EM field intensities:

$$F = dA = B_z(x, y)dx\hat{}dy + E_x(x, y, z, t)dx\hat{}dt + E_y(x, y, z, t)dy\hat{}dt + E_z(x, y, z, t)dy\hat{}dt. \tag{21}$$

The 3-form of Topological Torsion 3-form $A\hat{}F$ generates the 4-vector, $\mathbf{T}_4$, to be used as a process directionfield:

$$\mathbf{T}_4(x, y, z, t) \quad = \quad [-E_z A_y, +E_z A_x, (E_x A_y - E_y A_x) + \phi B_z, 0], \tag{22}$$

$$\text{with } div_4(\mathbf{T}_4(x, y, z, t)) \quad = \quad 2\{E_z(x, y, z, t)B_z(x, y)\} \neq 0. \tag{23}$$

In this example, the helicity ($\mathbf{A} \circ \mathbf{B} = \mathbf{0}$) is zero, such that the Topological Torsion vector, $\mathbf{T}_4(x,y,z,t)$, has only three spatial components. Moreover, the process generated by $\mathbf{T}_4(x,y,z,t)$ is thermodynamically irreversible, as $(\mathbf{E} \circ \mathbf{B}) \neq 0$. The example 1-form, A, is of $PTD\ = 4$.

To demonstrate the emergence of the $PTD = 3$ state, suppose the potential function in this example has the format,

$$\phi = \psi(x,y) + \varphi(z)e^{-\alpha t} \tag{24}$$

$$E_z(z,t) = -(\partial\varphi(z)/\partial z)e^{-\alpha t} = E_z(z)e^{-\alpha t}. \tag{25}$$

Then the irreversible dissipation function decays as $\{E_z(z)B_z\}e^{-\alpha t}$. By ignoring the Spinor fluctuation terms, at late times the process decays into an Hamiltonian extremal process on a system of $PTD = 3$.

$$\mathbf{T}_4(x,y) \Rightarrow [0, 0, (E_x A_y - E_y A_x) + \phi B_z, 0], \tag{26}$$

$$W \Rightarrow i(\mathbf{T}_4)dA = 0, \text{ Extremal} \tag{27}$$

$$\sigma \Rightarrow \{E_z(z)B_z\}e^{-\alpha t} \Rightarrow 0 \supset PTD = 3. \tag{28}$$

To demonstrate the emergence of the $PTD = 3$ state in finite time, suppose the potential function in this example has the format,

$$\phi = \psi(x,y) \pm \varphi(z)\sqrt{(-t-t_c)^3} \tag{29}$$

$$E_z(z,t) = \mp(\partial\varphi(z)/\partial z)\sqrt{(-t-t_c)^3} = \pm E_z(z)\sqrt{(-t-t_c)^3}. \tag{30}$$

Then the irreversible dissipation function decays in a cuspoidal way (typical of the approach to an edge of regression of an envelope function) according to the formula,

$$\sigma = \{E_z(z)B_z\}\sqrt{(-t-t_c)^3}. \tag{31}$$

The PTD of the system is 4 for $t < t_c$, and becomes equal to 3 for $t = t_c$. The process can be connected smoothly to an Extremal Hamiltonian process for $t > t_c$.

4 Epilogue: Topological Universality

This Section comments on the universality of the methods of Continuous Topological Evolution and topological thermodynamics, and the extension of thermodynamic concepts, to the theory of fiber bundles. Recall that complex topological systems can have domains that are defined in terms of exterior differential forms. Such complex systems can consist of many disconnected topological sub-domains, all cooperating in a synergetic way in terms of the many *simultaneous* topologies that can be imposed upon a geometric set. For example in the previous discussion it became apparent that the simultaneous, but different topologies, of A, W, and Q, imposed upon the 4D geometric variety, influenced the thermodynamic interpretations. Such ideas can be extended to fiber bundles.

All such complex systems admit the possibility of non-equilibrium emergent "birth" of primitive odd dimensional "stationary" states far from equilibrium, with gradual pubescence accumulation of matter resulting in a stage of "maturity". The maturity stage can be (relatively speaking) an almost stationary state with slow decay. Then at later stages the mature system dies.

Such descriptions appear to describe almost every observable synergetic system, including the classical thermodynamic models, but also such things as biological systems, economic systems and political systems. The hand of thermodynamics touches each of these interacting complexes. I conjecture that methods which can treat the ecological effects of synergetic political and economic systems can be developed in terms of continuous topological evolution and topological thermodynamics.

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