

# The Universal Effectiveness of Topological Thermodynamics

From the point of view of Continuous Topological Evolution

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My presentation is somewhat low-key to an audience of topologists, but remember, I interact with engineers and scientists that have a **limited** (if any) training in **topology**.

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Long ago (1964) I rejected any attempts to describe the dynamics of irreversible processes in terms of equivalence classes of geometric diffeomorphisms (**tensors**).

Early on, I concluded that **topological change** is a necessary condition for **thermodynamic irreversibility**.

# The Universal Effectiveness of Topological Thermodynamics

It now appears that the topological perspective of thermodynamics gives universal insight into many non-equilibrium concepts associated with the emergence of metastable states, digital topology, plasmas and turbulent flows, biology, chemistry, metallurgy, fuzzy logic, holography and even the cognitive sciences.

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I suggest that those fluent in topology have a lot to offer to the practical world. A topological perspective gives a formal, non-phenomenological, foundation for understanding physical systems and non-equilibrium processes.



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As I told my students, there are two motivations and opportunities for such a discipline:

**WNP** or **EBB**

# The Universal Effectiveness of Topological Thermodynamics

**WNP** = Win Nobel Prize

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**EBB** = Earn Big Bucks (\$)

# Fundamental Ideas

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A Thermodynamic **system** can be encoded in terms of an exterior differentiable 1-form of Action, **A**, per unit "mole".

A Thermodynamic **Process** can be encoded by an ordered array (a vector direction field) of functions that form the coefficients ,  
**J**, of an  $N-1=3$ -form Current.

# Fundamental Ideas

The next step (more than 30 years ago) was to suspect that any exterior differential 1-form,  $A$ , contains topological information in its Pfaff Sequence in 4D:

$$\text{Pfaff Sequence} = \{A, dA, A \wedge dA, dA \wedge dA\} = \{A, F, H, K\}$$

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Then in a moment of **epiphany**, without any knowledge of order theory, I chose the set of exterior differential forms as a Topological basis =  $\{A, A \wedge F, H, H \wedge K\}$

# Fundamental Ideas

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$$\text{Pfaff Sequence} = \{A, dA, A \wedge dA, dA \wedge dA\} = \{A, F, H, K\}$$

Then in a moment of epiphany, without any knowledge of order theory, I chose the set of exterior differential forms as a

$$\text{Topological basis} = \{A, AF, H, HUK\} \Leftrightarrow \{a, b, c, d\},$$

and built the **Causal Cartan** topological structure of 4 “points”,  
a Kolomogorov topology based upon a **specialization order**:



# Cartan Topology of 4 “points”

Table 1. The CT4 Topology of 4 points

$$X = \{a, b, c, d\}$$

Basis subsets  $\{a\}, \{a, b\}, \{c\}, \{c, d\}$

$CT4\{open\} : \emptyset, \{a\}, \{c\}, \{a, b\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, X$

$CT4\{closed\} : X, \{b, c, d\}, \{a, b, d\}, \{c, d\}, \{a, b\}, \{b, d\}, \{d\}, \{b\}, \emptyset$

Subset S	Limit Pts	Interior-Exterior		Boundary	Closure
$\emptyset^*$	$\emptyset$	$\emptyset$	$X$	$\emptyset$	$\emptyset$
$\{a\}$	$\{b\}$	$\{a\}$	$\{c, d\}$	$\{b\}$	$\{a, b\}$
$\{b\}$	$\emptyset$	$\emptyset$	$\{a, c, d\}$	$\{b\}$	$\{b\}$
$\{c\}$	$\{d\}$	$\{c\}$	$\{a, b\}$	$\{d\}$	$\{c, d\}$
$\{d\}$	$\emptyset$	$\emptyset$	$\{a, b, c\}$	$\{d\}$	$\{d\}$
$\{a, b\}^*$	$\{b\}$	$\{a, b\}$	$\{c, d\}$	$\emptyset$	$\{a, b\}$
$\{a, c\}$	$\{b\}, \{d\}$	$\{a, c\}$	$\emptyset$	$\{b, d\}$	$X$
$\{a, d\}$	$\{b\}$	$\{a\}$	$\{c\}$	$\{b, d\}$	$\{a, b, d\}$
$\{b, c\}$	$\{d\}$	$\{c\}$	$\{a\}$	$\{b, d\}$	$\{b, c, d\}$
$\{b, d\}$	$\emptyset$	$\emptyset$	$\{a, c\}$	$\{b, d\}$	$\{b, d\}$
$\{c, d\}^*$	$\{d\}$	$\{c, d\}$	$\{a, b\}$	$\emptyset$	$\{c, d\}$
$\{a, b, c\}$	$\{b\}, \{d\}$	$\{a, b, c\}$	$\emptyset$	$\{d\}$	$X$
$\{b, c, d\}$	$\{d\}$	$\{c, d\}$	$\{a\}$	$\{b\}$	$\{b, c, d\}$
$\{a, c, d\}$	$\{b\}, \{d\}$	$\{a, c, d\}$	$\emptyset$	$\{b\}$	$X$
$\{a, b, d\}$	$\{b\}$	$\{a, b\}$	$\{c\}$	$\{d\}$	$\{b, c, d\}$
$\{a, b, c, d\}^*$	$\{b\}, \{d\}$	$X$	$\emptyset$	$\emptyset$	$X$

# Fundamental Ideas

**Topological Thermodynamics** is defined in terms of **exterior differential forms** evaluated on **ordered** classes of **differential varieties**  $\{x,y,z,t; dx,dy,dz,dt\}$ .

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The **ordered class** is defined in terms of  $C^1$  maps,

$$\phi, d\phi$$

$$\text{from } \{x^k, dx^k\} \text{ to } \{y^k, dy^k\}$$

These maps are not diffeomorphisms, and do not require the **geometric constraints of an inverse**,

$$\text{for either } \phi, \text{ or } d\phi.$$

# Fundamental Ideas

Then, just recently, I became aware that the formal CT4 topological structure that leads to a universal theory of **Thermodynamics** on Exterior Differential forms, is one of 16

## $T_0$ Topologies

**The  $T_0$  topology of interest is defined as the Kolmogorov-Cartan  $KCT_0$  topology**

*Formally, this topology is quite interesting for many demonstrable reasons. All of the singletons of the topology are not closed. **Warning**: the  $T_0$  topology is NOT a metric topology, NOT a Hausdorff topology, and even does NOT satisfy the separation axioms that define a  $T_1$  topology*

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with a specialization order,

{ $A$ , Closure of  $A$ ,  $A^{\perp}A$ , Closure of  $A^{\perp}A$  }.

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*The **KCT**<sub>0</sub> Topology has a*

**DUAL Topology  $T^*_0$**



# Kolmogorov–Cartan dual Topology dual, $T_0^*$ , of 4 “points”

Table 2. A DUAL CT4\* Topology of 4 points

$$X = \{a, b, c, d\}$$

Basis subsets  $\{b\}, \{a, b\}, \{d\}, \{c, d\}$

Dual CT4{open} :  $X, \{b, c, d\}, \{a, b, d\}, \{c, d\}, \{a, b\}, \{b, d\}, \{d\}, \{b\}, \emptyset$

Dual CT4{closed} :  $\emptyset, \{a\}, \{c\}, \{a, b\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, X$

Subset S	Limit Pts	Interior-Exterior	Boundary	Closure
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$\{c\}$	$\emptyset$	$\emptyset$	$\{a, b, d\}$	$\{c\}$
$\{d\}$	$\{c\}$	$\{d\}$	$\{a, b\}$	$\{c, d\}$
$\{a, b\}^*$	$\{a\}$	$\{a, b\}$	$\{c, d\}$	$\{a, b\}$
$\{a, c\}$	$\emptyset$	$\emptyset$	$\{b, d\}$	$\{a, c\}$
$\{a, d\}$	$\{c\}$	$\{d\}$	$\{b\}$	$\{a, c, d\}$
$\{b, c\}$	$\{a\}$	$\{b\}$	$\{d\}$	$\{a, b, c\}$
$\{b, d\}$	$\{a\}, \{c\}$	$\{b, d\}$	$\emptyset$	$X$
$\{c, d\}^*$	$\{c\}$	$\{c, d\}$	$\{a, b\}$	$\{c, d\}$
$\{a, b, c\}$	$\{a\}$	$\{a, b\}$	$\{d\}$	$\{a, b, c\}$
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$\{a, b, c, d\}$	$\{a\}, \{c\}$	$X$	$\emptyset$	$X$

# Fundamental Ideas

**K-CT4:**     $KCT_0$  ,  $KCT^*_0$  = partitions  
of quasi-discrete Alexandroff  
 $AlexT0$  topology of 4 points

Quasi discrete Alexandroff  $T0 \Leftrightarrow \{T0 \vee T^*0\}$

$\{\text{Partitioned Particles}\} \Leftrightarrow \{\text{Interaction Fields}\}$

The Kolmogorov Topology (and its dual)  $T_0$  and  $T_0^*$   
are the topologies of **Continuous Fields**

The Alexandroff quasi-discrete topology  $T_1$   
is the topology of **Discrete Quanta**  
(countable additive BITS)

Thermodynamic **field intensity** variables are based upon the continuous Kolmogorov topologies.

Thermodynamic **additive extensive** variables are based upon the discrete Alexandroff topologies.

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The topological evolution of **ENERGY** is based upon C2 **continuous** infinitesimal segments (Kolmogorov T0).

### The First Law

The topological evolution of **ENTROPY** is based upon C1 **discrete** finite line segments (Alexandroff T1).

### The Second Law

# Continuous Topological Evolution

Let the Topology of initial state be **T1**

Let the Topology of the final state be **T2**

# Continuous Topological Evolution

Let the Topology of initial state be **T1**

Let the Topology of the final state be **T2**

Topological change is continuous

iff for the map  $\phi: \mathbf{T1} \Rightarrow \mathbf{T2}$

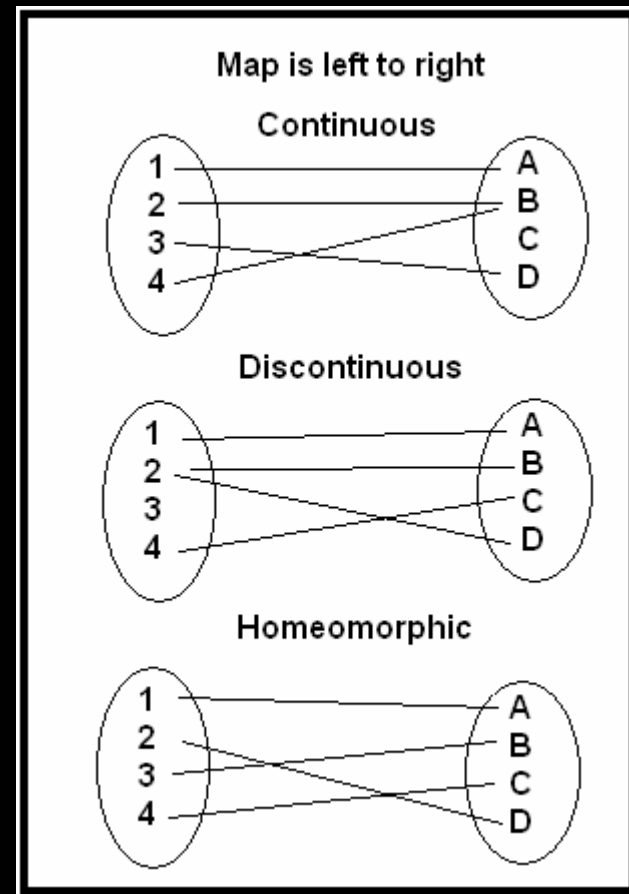
the Limit points of **T1** are included  
in the Closure of **T2**.

# Continuous Topological Evolution

Topological change

Topological change

No Topological change





# **Cartan's** Magic Formula of Continuous Topological Evolution of differential forms

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is the

**LIE DIFFERENTIAL**

acting on a **system** 1-form of Action, **A**,  
with respect to a **process** direction field, **J**

# Cartan's Magic Formula

$$L_{(J)} A = i(J) dA + d(i(J)A)$$

# Cartan's Magic Formula

$$L_{(\mathbf{J})} \mathbf{A} = i(\mathbf{J})d\mathbf{A} + d(i(\mathbf{J})\mathbf{A})$$

Is the Generator of Continuous  
Topological Evolution if  $\mathbf{J}=\mathbf{C}^2$

# Cartan's Magic Formula

$$L_{(\mathbf{J})} \mathbf{A} = i(\mathbf{J})d\mathbf{A} + d(i(\mathbf{J})\mathbf{A})$$

Change notation to yield

$$L_{(\mathbf{J})} \mathbf{A} = \mathbf{W} + d(\mathbf{U}) = \mathbf{Q}$$

$\mathbf{W}$  = Work 1-form,  $\mathbf{U}$  = Internal energy,  $\mathbf{Q}$  = Heat 1-form

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A UNIVERSAL cohomological formulation of the

# FIRST LAW of THERMODYNAMICS!!

# Kuratowski's Magic Formula

Relative to a Kolmogorov-Cartan T0 topology,

the exterior differential is a

**Limit Point generator.**

# Kuratowski's Magic Formula

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**Limit Point generator.**

For a differential form  $\Sigma$

$$\text{Limit Points of } \Sigma = d\Sigma$$

This result focuses attention on Cohomology



# Pfaff Topological Dimension

The KC T0 topology is generated by the elements of the Pfaff Sequence of  $A$  .

The Pfaff Topological Dimension = the # of non-zero elements in the Pfaff Sequence.

**Pfaff Sequence** :  $\{A, dA, A^{\wedge}dA, dA^{\wedge}dA\}$

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**Pfaff Sequence** :  $\{A, dA, A^{\wedge}dA, dA^{\wedge}dA\}$

The Pfaff Topological Dimension **PTD**( $A$ )

$$\text{PTD}(A) = 1 : \{A, 0, 0, 0\}$$

$$\text{PTD}(A) = 2 : \{A, dA, 0, 0\}$$

$$\text{PTD}(A) = 3 : \{A, dA, A^{\wedge}dA, 0\}$$

$$\text{PTD}(A) = 4 : \{A, dA, A^{\wedge}dA, dA^{\wedge}dA\}$$

# Continuous Topological Evolution and PTD(A)

can describe the irreversible evolution on an

**Open non-equilibrium** Symplectic domain, **PTD 4**, with  
evolutionary orbits being irreversibly attracted to a

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**Closed non-equilibrium** Contact domain, **PTD 3**, with emergent topological defects (stationary states and coherent structures), and a possible ultimate decay to the

**Isolated-Equilibrium** Caratheodory (integrable) domain of **PTD 2** or less.

# Significant Results

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2. Continuous non-homeomorphic processes of topological evolution establish a logical basis for thermodynamic irreversibility and the arrow of time without the use of statistics.

# Continuous Topological Evolution and $\text{PTD}(\mathbf{A})$

Regions: where  $\text{PTD}(\mathbf{A}) \leq 2$  generate a **connected** topology;  
 $\text{PTD}(\mathbf{A}) \geq 3$  generate a **disconnected** topology.



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Continuous Processes can represent the evolution from a **disconnected** topology ( $\geq 3$ ) to a **connected** topology ( $\leq 2$ ) .

Continuous Processes can **NOT** represent the evolution from a **connected** topology ( $\leq 2$ ) to a **disconnected** topology ( $\geq 3$ ) .

Therefore, **Connectivity and Continuity** determine

# **A Topological Arrow of Time.**

You can describe the decay of turbulence **continuously**,  
but NOT the creation of turbulence.

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# **A Topological Arrow of Time.**

You can describe the decay of turbulence **continuously**,  
but **NOT** the creation of turbulence.

Kiehn, R. M. (2003), Thermodynamic Irreversibility and the Arrow of Time, in "The Nature of Time: Geometry, Physics and Perception", R. Bucher et al. (eds.), Kluwer, Dordrecht, Netherlands, 243-250. (<http://www22.pair.com/csdc/pdf/arwfinal.pdf>)

# Significant Results

1. Topological change is a necessary condition for thermodynamic irreversibility.
2. Continuous non-homeomorphic processes of topological evolution establish a logical basis for thermodynamic irreversibility and the arrow of time without the use of statistics.
3. Evolution from a disconnected **KCT<sub>0</sub>** topology to a connected topology can be continuous and irreversible, but it is a theorem of topology that a map from a connected topology to a disconnected topology cannot be C2 continuous.

# Significant Results

4. C2 Continuous Topological Evolution **permits irreversible processes, for which,  $Q^dQ \neq 0$** . Segmented C1 processes approximating smooth C2 processes can be **reversible,  $Q^dQ = 0$** , while the C2 smooth processes are **irreversible,  $Q^dQ \neq 0$** .

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5. On odd-dimensional spaces, sequential C1 (**translational**) processes can be thermodynamically reversible, while intransitive C2 processes (**rotation and expansion with a fixed point**) can be thermodynamically irreversible.

# Significant Results

4. C2 Continuous Topological Evolution permits irreversible processes, for which,  $\dot{Q}dQ \neq 0$ . Segmented C1 processes approximating smooth C2 processes can be reversible,  $\dot{Q}dQ = 0$ , while the C2 smooth processes are irreversible,  $\dot{Q}dQ \neq 0$ .
5. On odd-dimensional spaces, sequential C1 (translational) processes can be thermodynamically reversible, while intransitive C2 processes (rotation and expansion with a fixed point) can be thermodynamically irreversible.

**Engineering Motto for Minimizing energy loss:**

**Translational Acceleration dot Angular Momentum  $\Rightarrow 0$**



# Significant Results

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5. On odd-dimensional spaces, sequential C1 (translational) processes can be thermodynamically reversible, while intransitive C2 processes (rotation and expansion with a fixed point) can be thermodynamically irreversible.
6. The Twin Paradox is resolved if the process paths indicate NO topological change – hence no disparate aging. Disparate aging requires topological change.

# Significant Results

7. Adiabatic processes are transverse to the Heat 1-form,  $(i(\rho V_4)Q)=0$ . Adiabatic processes need not be quasi-static, and can be reversible or irreversible. Topological Transitions can occur in finite (often short) time.

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Kiehn, R. M. (2008) **Topological Torsion and Macroscopic Spinors**, "Non-Equilibrium Systems and Irreversible Processes Vol 5", Lulu Enterprises, Inc., 3131 RDU Center, Suite 210, Morrisville, NC 27560, see (<http://www.lulu.com/kiehn>).

# Significant Results

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8. A fundamental difference between Work and Heat is that  $i(\rho V_4)W=0$ , always; but it is not true that  $i(\rho V_4)Q=0$ , always. The Work 1-form,  $W$ , is always transverse to the process,  $\rho V_4$ , but the Heat 1-form,  $Q$ , may or may not be transverse. The Heat 1-form,  $Q$  can have **longitudinal components** in the direction of the process. Such is the subtle topological difference between Work and Heat.

# Significant Results

9. For non-equilibrium systems, the 3-form of **Topological Torsion** (an  $N-1=3$ -form current) is not zero:

$$A^d A = i(T_4) dx^d dy^d dz^d dt \neq 0.$$

The **Topological Torsion vector**,  $T_4$ , is deduced intrinsically from the 1-form that encodes the thermodynamic system.

It can be used as a direction field for a process current,  $\rho T_4$ .

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It can be used as a direction field for a process current,  $\rho T_4$ .

10. For **PTD=3** "closed" thermodynamic systems, the process current has zero divergence, and the 4D volume element is a conformal invariant (any  $\rho$ ). This result is the space-time extension of the Liouville theorem that preserves the phase-space volume element in classical theory



# Significant Results

11. For a **PTD=4** "open" thermodynamic systems, the **Topological Torsion** vector does **not** have zero divergence, and so the process current  $\rho \mathbf{T}_4$  may not be closed for arbitrary  $\rho$  (that is, the divergence of the process current is not zero).

# Significant Results

11. For a **PTD=4** "open" thermodynamic systems, the **Topological Torsion** vector does **not** have zero divergence, and so the process current  $\rho \mathbf{T}_4$  may not be closed for arbitrary  $\rho$  (that is, the divergence of the process current is not zero).

This result is the extension of the Vlasov equation; the 4D differential volume element is expanding or contracting. Such processes in the direction of  $\mathbf{T}_4$  are **irreversible and dissipative**.

# Significant Results

12. A major result is that the Kolmogorov-Cartan  $T_0$  topology is a disconnected topology for non-equilibrium systems (**PTD=4,PTD=3**) and is a connected topology for equilibrium systems (**PTD=2,PTD=1**).

# Significant Results

12. A major result is that the Kolmogorov-Cartan  $T_0$  topology is a disconnected topology for non-equilibrium systems ( $\text{PTD}=4, \text{PTD}=3$ ) and is a connected topology for equilibrium systems ( $\text{PTD}=2, \text{PTD}=1$ ).

13. A key artifact of non-equilibrium is the existence of

**Topological Torsion current 3-forms,  $J_{\text{Torsion}}$ ,**

**Topological Spin current 3-forms,  $J_{\text{Spin}}$ ,**

**Topological Adjoint current 3-forms,  $J_{\text{adjoint}}$ .**

# 3-Form Currents

These 3-forms are similar to the  
Ampere current 3-form,  $J_{\text{Ampere}}$ ,

**BUT**

where  $d J_{\text{Ampere}} = 0$ , always,

the other current 3-forms are not closed unless  
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Ampere current 3-form,  $J_{\text{Ampere}}$ ,

**BUT**

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the other current 3-forms are not closed  
unless they are  
homogeneous of degree zero.

**NOTE:** Any 3-form admits (many) integrating factors that  
will make the 3-form homogenous of degree zero.

# 3-Form Currents

- The **Topological Torsion 3-form** is related to **Helicity**,
- The **Topological Spin 3-form** is related to **Spin**,
- The **Adjoint 3-form** is related to the **interaction energy**.

# 3-Form Currents

The Topological Torsion 3-form is related to **Helicity**,

The Topological Spin 3-form is related to **Spin**,

The Adjoint 3-form is related to the **interaction energy**.

All three are related to different species of dissipative phenomena, which only occur in non-equilibrium systems.



# 3-Form Currents

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The Topological Spin 3-form is related to **Spin**,

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**The dissipation coefficients are related to the non-zero divergences of the vector coefficients of each 3-form.**

For example, in electromagnetic systems, the dissipation coefficient is proportional to  $\mathbf{E} \circ \mathbf{B}$ ; in hydrodynamics, the dissipation coefficient is called "**Bulk viscosity**".

# Significant Results

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15. Examples can generate a Spin Current 3-form, where, formally, the Spin current is proportional to the Lorentz force (the space-time components of the Work 1-form,  $W$ ).

This is a new interpretation of an old result,  $\mathbf{J}=\sigma(\mathbf{E}+\mathbf{V}\times\mathbf{B})$ , which is Ohm's law. The new part is due to the idea that the dissipation is due to Spin Currents and the transport of collective spins,  $\mathbf{A}^{\wedge}\mathbf{G}$ .

# Significant Results

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**Such a result gives formal credence to Prigogine's conjectures.**

# Emergence of Topological Defects

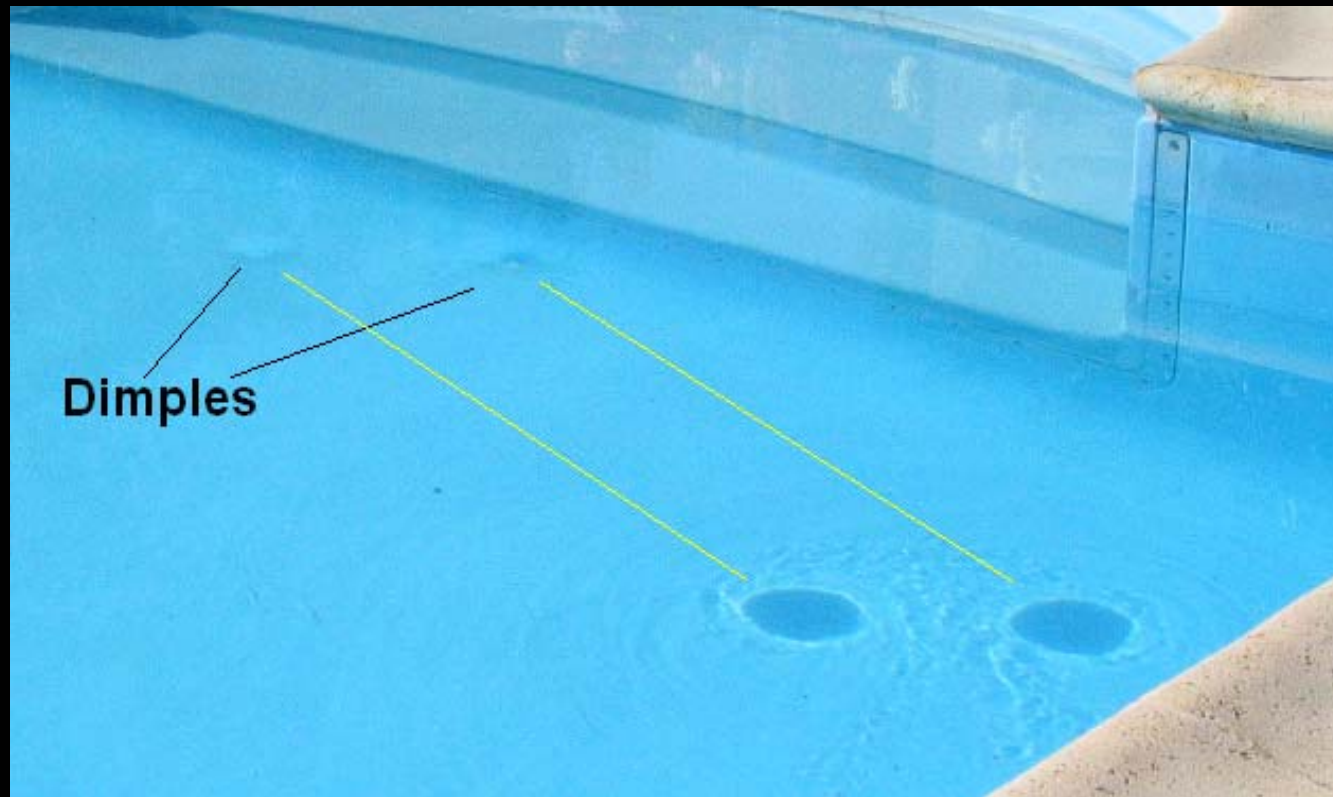


Long Lived Topological Defects in a Swimming Pool

Creation time < 5 seconds. Lifetime > 15 minutes

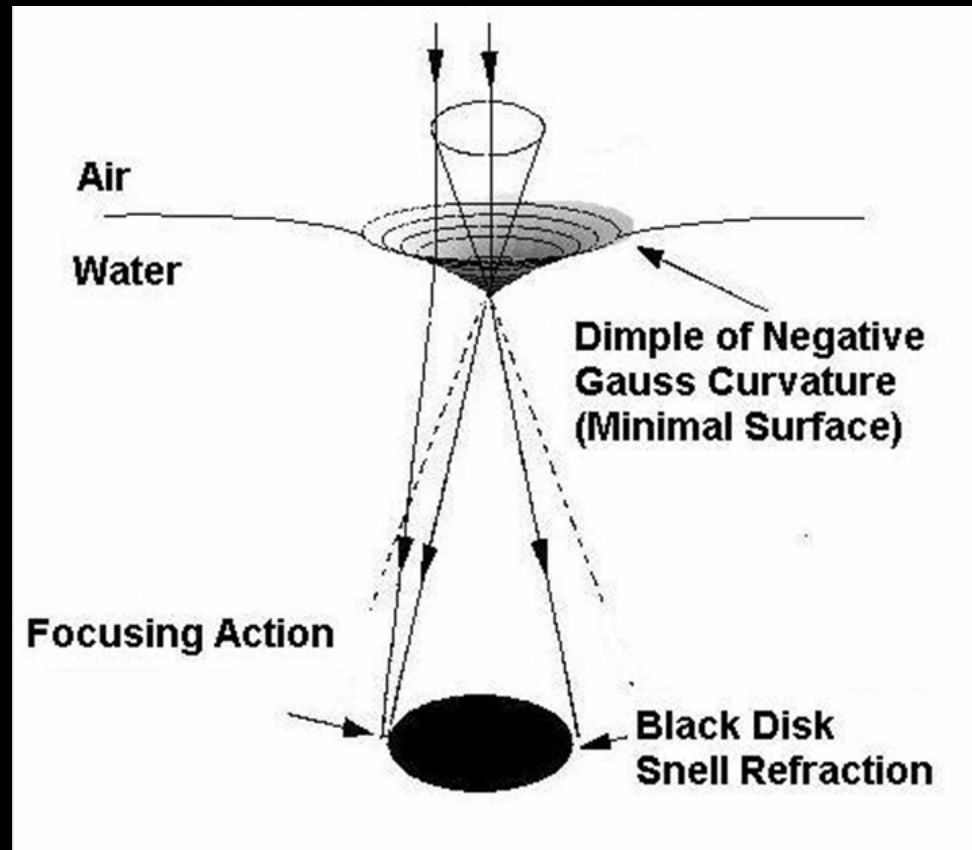
# Emergence of Topological Defects

FALACO SOLITONS Movie by D. Radabaugh



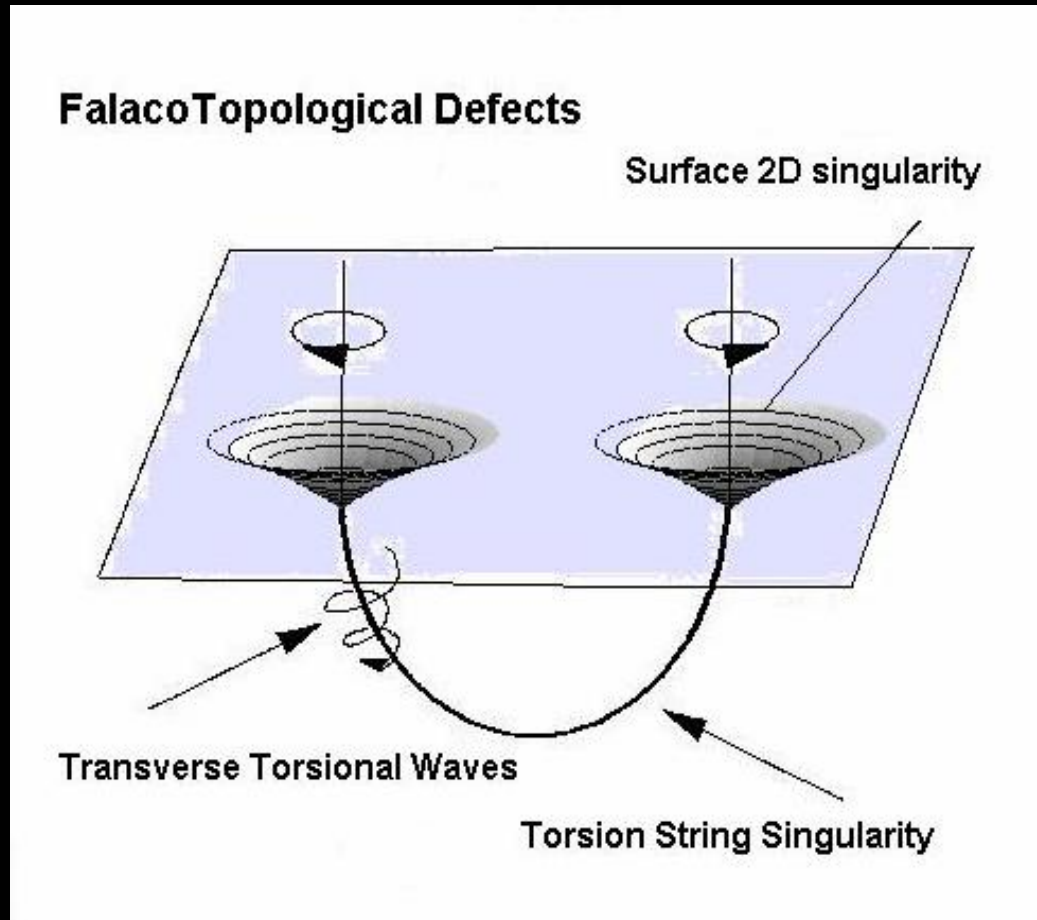
**Solar Elevation about 30 degrees (See movie at**  
**<http://www22.pair.com/csdcdownload/spotsmovie.avi>**

# Emergence of Topological Defects



Snell refraction of Falaco Soliton Spin Pairs

# Emergence of Topological Defects



The first measurable Torsion String coupling between branes

This real world effect has been ignored by string theorists !!!

# Significant Results

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A **PTD>2** non-equilibrium thermodynamic system always has a non-zero Topological Torsion 3-form, **A<sup>3</sup>F**.

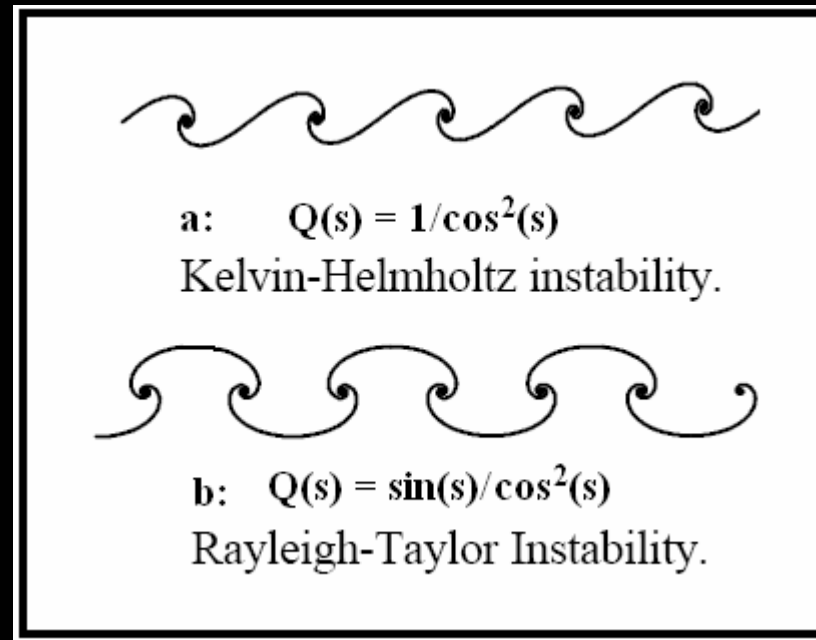
Envelope solutions, such as Huygen wavelets and propagating tangential discontinuities (called signals, or wakes) are classic examples of solution non-uniqueness.

**Topological Torsion** is an artifact of non-uniqueness, and of Turbulence.



# Significant Results

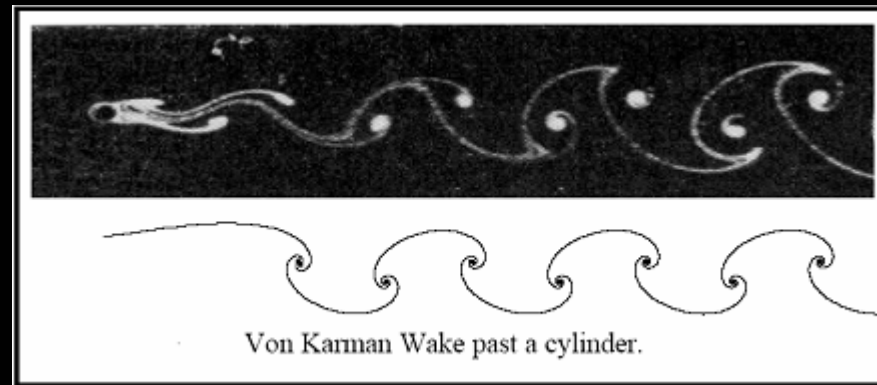
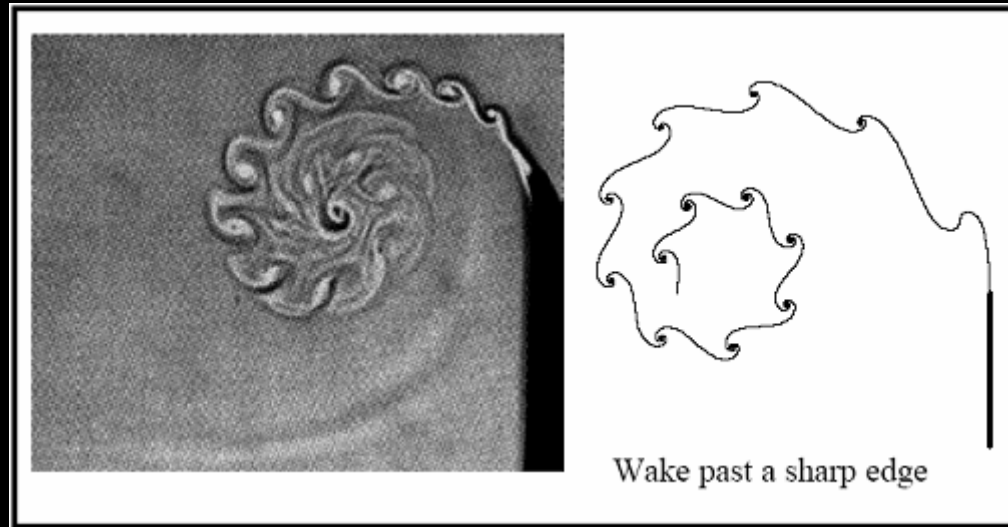
Hydrodynamic Wakes as topological “limit points”



Exact solutions of Hydrodynamic instabilities !!!

$$dx/ds = \cos(Q), dy/ds = \sin(Q)$$

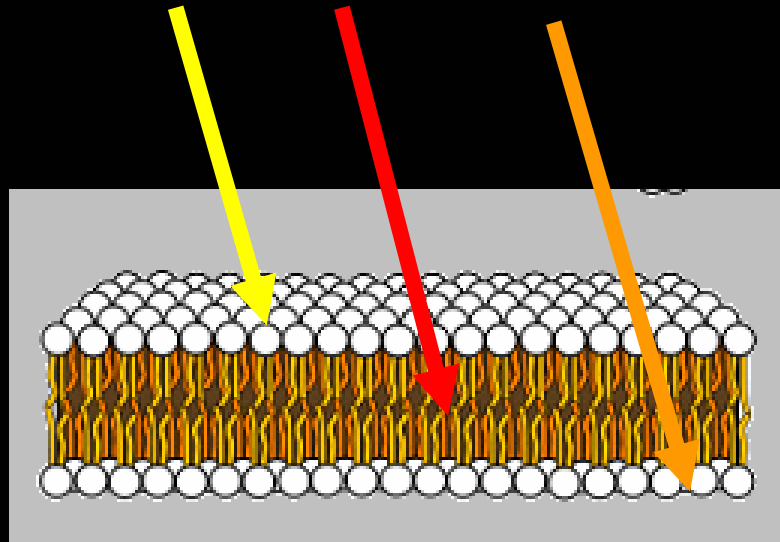
# Significant Results



# Significant Results

The Topological double layer Membrane

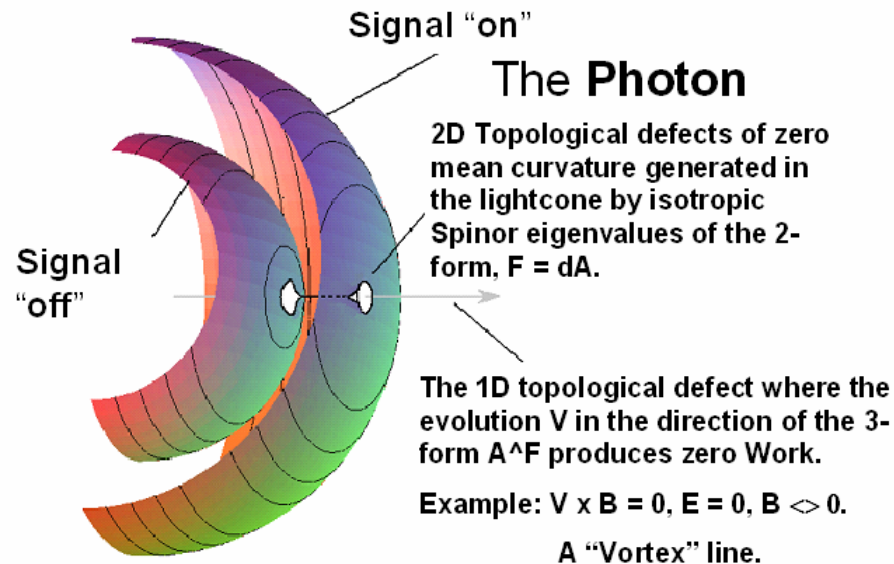
$$KCT_0 - ALEXT_0 - KCT^*_0$$



# Significant Results

## A Model for the Topological Photon

Spherical Shells of propagating discontinuity in E B field intensities  
(3D topological defects where  $F \wedge F = 0$ ; the spatial part of the 4D Light Cone)



The EM signal (the Photon) as a Topological double layer Membrane ... **T0-T1-T\*0**

# Significant Results

18. All Hamiltonian, Symplectic-Bernoulli and Helmholtz processes are thermodynamically reversible. In particular, the work 1-form,  $W$ , created by Hamiltonian processes is of Pfaff Topological Dimension 1 or less. In all reversible cases the Work 1-form is closed,  $dW=0$ .

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19. The assumption of uniqueness of evolutionary solutions (which implies the Pfaff Topological Dimension of the thermodynamic system be equal to 2 or less), or the assumption of homeomorphic evolution, have imposed constraints upon classical mechanics that eliminate any time asymmetry.

# Significant Results

20. The Lie differential acting on differential forms is not necessarily the same as a linear affine covariant differential acting on tensors. If the process is locally adiabatic (no heat flow in the direction of the evolutionary process), then the Lie differential and the covariant differential can be made to coincide, as they both satisfy the **Koszul** axioms for an affine connection.

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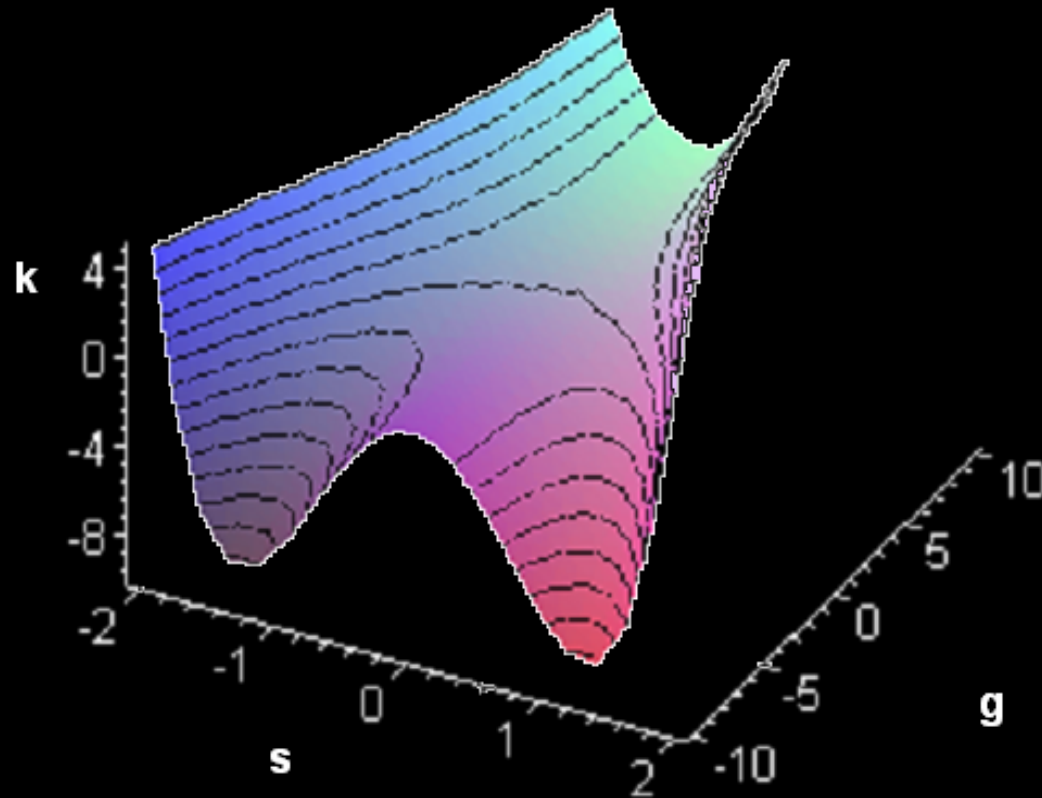
This is a surprising result, for, when the argument is reversed, the theorem implies that the ubiquitous affine covariant differential of tensor analysis, acting on a 1-form of Action, can always be cast into a form representing an adiabatic process. **Warning:** Restrictions of processes which satisfy the constraints of tensor analysis, and use an affine integrable connection to define **Covariant derivatives**, **are always adiabatic.**



# Significant Results

21. On spaces of  $\text{PTD}=4$ , the Jacobian of the components of the 1-form of Action,  $A$ , define a correlation matrix, which has a characteristic polynomial that defines an equation of state in terms of Cayley-Hamilton similarity invariants.

# Universal Topological Thermodynamic Phase Function



A van der Waals gas with a **Higgs potential**,  
An Envelope of a 4D Cayley-Hamilton characteristic polynomial

# Universal Topological Thermodynamic Phase Function

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The 4D universal topological phase function also can be used in dimensions greater than 4 in order to represent **multi-component potentials** in Chemistry Reactions. The results then can be pulled back to the 4D differential variety of measurement.

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The Cayley-Hamilton theorem produces an implicit hypersurface function that can have envelopes and edges of regression in the format of the Gibbs function for a (universal and deformable) van der Waals gas.

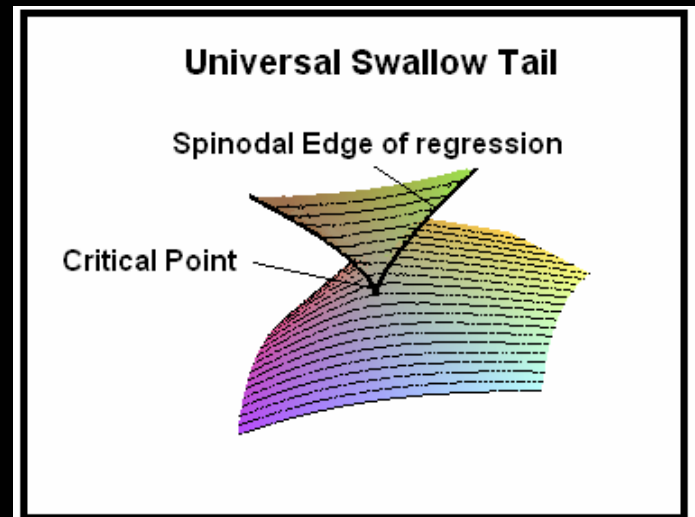
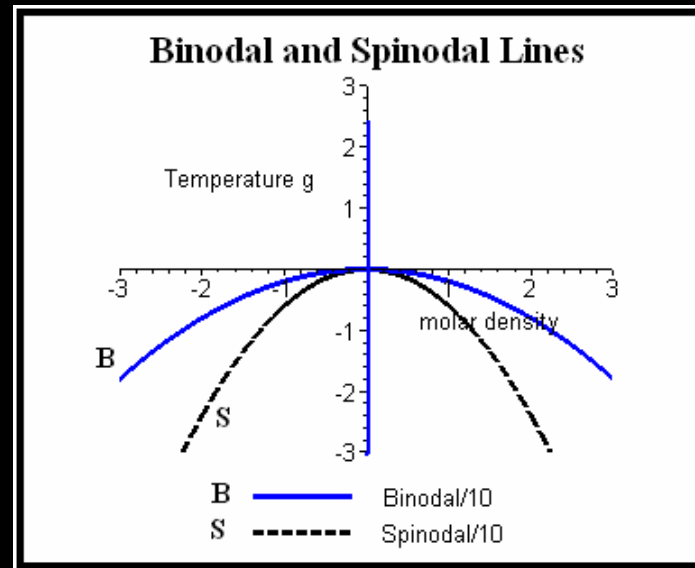
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The method yields analytic expressions for the critical point, and the **binodal and spinodal** lines, in terms of the similarity invariants. The same technique can be applied to dynamical systems.

# Significant Results



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22. **Cartan's Magic formula**, in terms of the Lie differential acting on exterior differential 1-forms, establishes the long sought for combination of dynamics and thermodynamics, enabling non-equilibrium systems and many irreversible processes to be computed in terms of **continuous topological evolution**, without resort to probability theory and statistics.



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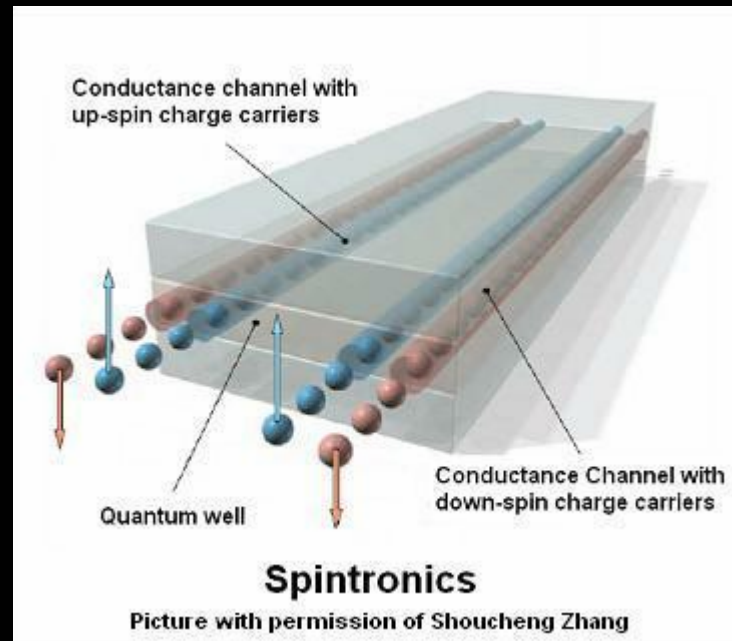
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23. **Topological fluctuations** can be induced by processes that have components in terms of thermodynamic macroscopic Spinors. Thermodynamic **Macroscopic Spinors** are non-zero complex eigenvectors with complex eigenvalues (and zero quadratic form) of the antisymmetric 2-form (or matrix) representing the "Limit Points",  $dA$ , of the 1-form of Action,  $A$ . Such Macroscopic Spinors are capable of representing minimal surface conjugate pairs.

# Significant Results

24. **Topological Insulators** correspond to Impedances defined in terms of quantized Topological Spin, and Spin pairs coupled via Falaco Solitons. If the divergence of the Spin 4vector vanishes, the **Spin Current** is time reversal invariant.

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25. The fact that any synergetic system of parts in effect defines a topology implies that the universal method of topological thermodynamics is **applicable to economic systems, political systems, as well as to biological systems**. Such systems admit long-lived states far from equilibrium (adulthood). However, the theory can be used to design subtle perturbations to such systems such that their metastable lifetimes can be extended or destroyed.

# Significant Results

26. The thermodynamic processes that lead to self-similarity of a Current 3-form  $L_{(j)}C = \sigma C$  can generate **fractals** and **holographic** effects where small neighborhoods replicate the whole, approximately. The homogeneity coefficient is the trace of the Jacobian Collineation:  $\sigma = \text{Trace}[\partial C^m / \partial x^n]$ , or the divergence of the Process vector field.

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27. A turbulent thermodynamic **cosmology** can be constructed in terms of a dilute **non-equilibrium van der Waals gas** near its critical point.

## Cosmology as a non-equilibrium Van der Waals Gas explains

- a.) The **granularity** of the night sky as exhibited by stars and galaxies due to density fluctuations near the critical point, and the Newtonian law of gravitational attraction proportional to  $1/r^2$  as a correlation between fluctuations (due to Lev Landau ).

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## Cosmology as a non-equilibrium Van der Waals Gas explains

- d.) The possibility of domains of negative temperature (explaining what has recently been called "**dark matter**") are due to macroscopic collective states of ordered spins. The conjecture is that **Positive temperature radiates, Negative temperature does not**. The conjecture is that black holes could be negative temperature states of collective spins.

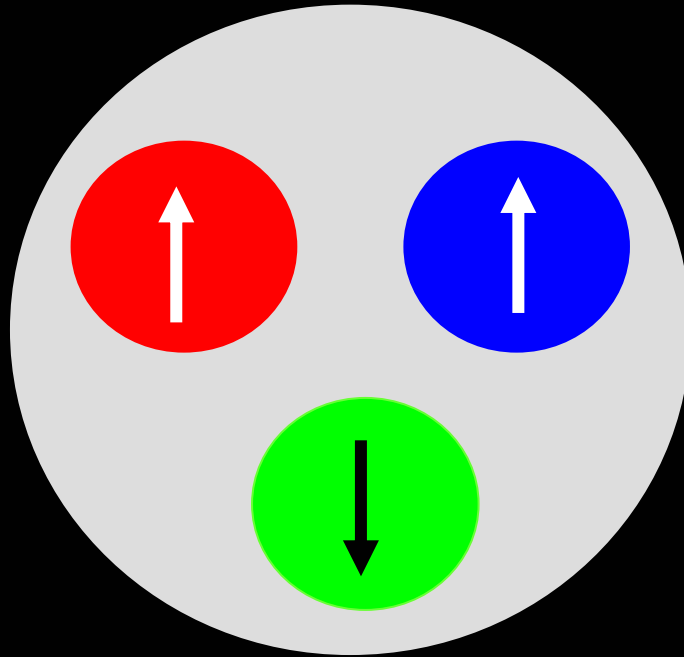
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- e.) The possibility of domains where gravitational effects (quadratic similarity invariants, or 2nd order Gauss curvature effects) appear to be related to entropy and temperature properties of the thermodynamic system, and where **cubic curvature effects due to Spin and Adjoint current 3-forms could impede gravitational collapse.**

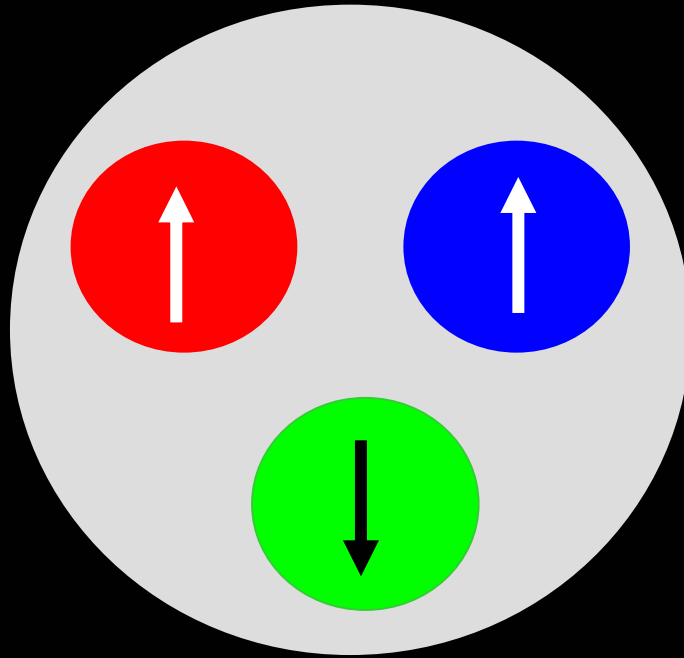
## Cosmology as a non-equilibrium Van der Waals Gas explains

- f.) Black Holes (generated by Petrov Type D solutions in gravitational theory) are to be related to **Minimal Surface** solutions to the Universal thermodynamic 4th order Phase function. .

**What does this symbol mean to you?**

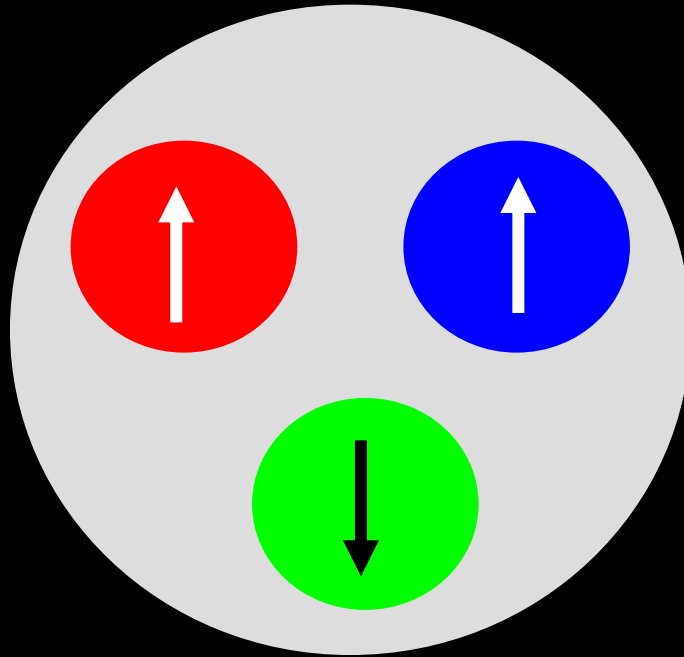


What does this symbol mean to you?



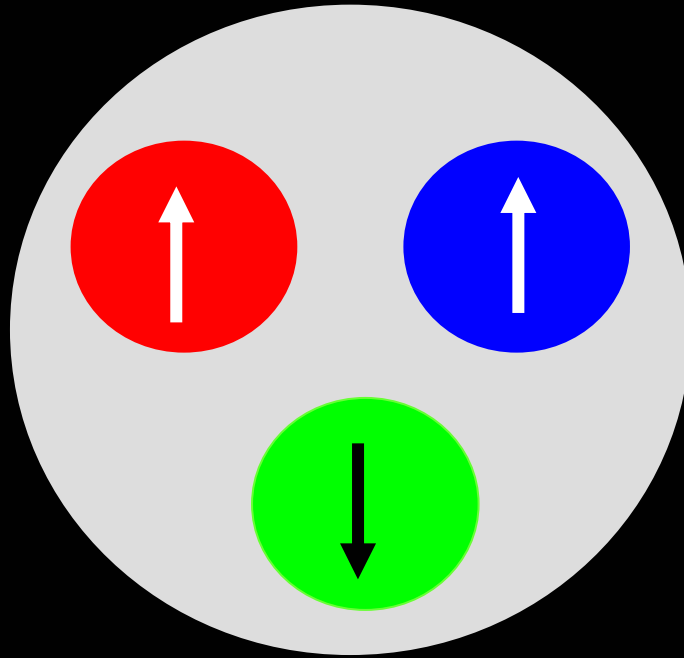
Two **Upper** sets and One **Lower** Set?

What does this symbol mean to you?



A **specialization** preorder system

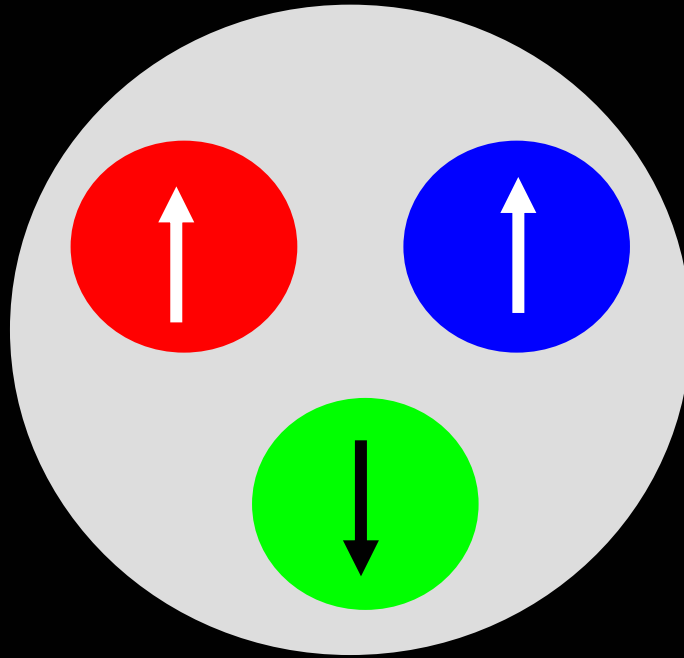
What does this symbol mean to you?



Or a **PROTON**





What does this symbol mean to you?



Or a **PROTON**    Made up from **QUARKS**

# Significant Results

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Both Kolmogorov topologies are partitions of a DISCRETE Alexandroff  $T_0$  topology. The  $T_0$  topology is inherent in the concept of **DISCRETE PARTICLES**.

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Is this the

## Universality of Topological Thermodynamics

acting as the Causal foundation for Quarks?

# Thanks for your interest

Contact Professor R. M. Kiehn at

rkiehn2352@ aol.com

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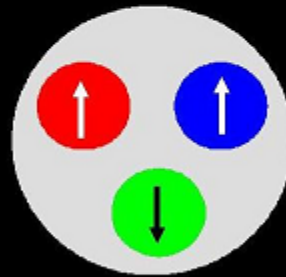


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Adventures in Applied Topology Vol. 6

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From the Perspective of Continuous Topological Evolution



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Sliding Ball => Rolling Ball



Irreversible Continuous Topological Evolution of Plat Topological dimension 1 to 2 to long lived states far from equilibrium and of Plat dimension 2 to 3

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### Falaco Solitons Cosmology and the Arrow of time



Photo courtesy David Radabaugh

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### Wakes Coherent Structures and Turbulence



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### Plasmas and Non-Equilibrium Electrodynamics



Long lived toroidal plasma ring in the aftermath of a nuclear explosion.

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## Vol 5

### Topological Torsion and Macroscopic Spinors

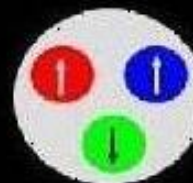


Helicity 6/10 5/0 and  
Macroscopic Spinors

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### The Universal Effectiveness of Topological Thermodynamics



Quantization Topological in Quantum

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From the perspective of Continuous Topological Evolution