Topological Torsion
versus
Geometric Torsion

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November 16, 2009

The mathematical ideas of torsion can be put into two general categories:

1. The category of geometric torsion produced by continuous deformation of a metric. The mathematical description has been called fiber bundle theory.
2. The category of topological torsion which does not depend upon metric. The mathematical description has been called twisted fiber bundle theory.

For example, geometric torsion is embeddable in R2 and is not stabile in R3. Topological Torsion is not embeddable in R2, but can be embedded, and is stable, in higher dimensions. Simple realizations of these ideas can be demonstrated visually .

The following sequence of short movies demonstrates the issues:

- Unstable Geometrical Torsion (untwisted fiber bundle) of continuous deformations in R2.

  http://www22.pair.com/csdc/download/R2torsion.wmv

- Stabile Topological Torsion (twisted fiber bundle) in R3.

  http://www22.pair.com/csdc/download/R3toptorsion.wmv

- Unstable Topological Torsion (twisted fiber bundle) in R3, but with topological defects in R2.

  http://www22.pair.com/csdc/download/R2toR3toptorsion.wmv

Visualizations in R4 are more difficult, mainly because the systems may be unstable and the processes may not be isentropic. The failure of the Second Poincare invariant in R4 is related to the unstable expanding universe. The failure of the First Poincare invariant in R4 corresponds to irreversible dissipation [8].

These ideas can be related to the Non-metrizable features of T0 lattices and topologies, and will make up the content of my forthcoming 6th monograph (which hopefully will be published before 2010 [9]). It is extraordinary that there are non-metrizable Quantum Features (cohomological period integrals [6]) that are excluded when geometric constraints are imposed. The Non-metrizable features of gravity appear to be related to the concept of the strong (nuclear) force, which is short range and parity preserving, and the weak (lepton) force which is short range, but not parity preserving [5]. The field theory of Classical Gravity and Electromagnetism have geometric metrizable descriptions (fiber bundle, metric, constraints) which exclude the particle–like (Quantum) properties of twisted fiber-bundles. These latter properties are related to chirality and non-orientability. Note that the Maxwell-Faraday field equations admit a continuous deformation to R2 [10], but the Maxwell-Ampere equations do not..

0.1 Preface to Chapter ?, Vol 6: Lattice Cohomology and Non-metrizable Physics

Physical theories of quantities and processes, defined in terms of geometric constraints of metric, and its attendant concepts of distance, size, or those linear connections associated with unique integrability, have formed the basis of (geometrical) physical theories for thousands of years. Topology teaches that all metric spaces satisfy the Hausdorff T2 separation axiom, hence T2 topological spaces have been at the foundation of geometric physical theories of measurement. The objective of this chapter is to demonstrate features of certain (simplified) Lattice Structures of sets (which may or may not be topologies) and certain non-metrizable Topological Structures that are (or should be) of interest to physics. In short, there exist non-metrizable methods of measurement, which can describe physical properties of our universe that have slipped through the net of geometric physics. These non-geometrical, non-metrizable, structures, and how they may be incorporated into physical theories to yield new understanding of the physical universe, are the main focus of this chapter.

Claim 1 *Finite non-metrizable non-geometric Topological (and Lattice) T0 structures which satisfy the separation axioms R0 and T0, but not the T2 axiom, must form the basis for non-metrizable non-geometric physical theories*

The point of departure in this chapter will be to limit the discussion to those finite lattice structures and finite topological structures of low (topological) dimension (4 or less) that satisfy the T0 separation axiom (or less), but which do not satisfy the Hausdorff T2 separation axiom. Such structures are NOT metrizable.

Why should physics take interest in such non-geometric structures?

1. There are concepts of physical measurement that do not depend upon geometry. As Bott ([1] p 1.) says "T*he most intuitively evident topological invariant of a space is the number of connected pieces* (parts) *into which it falls*". The number of holes in a piece of paper is a topological (deformation) invariant that does not depend upon metric (geometric) ideas.

2. It has become apparent that non-equilibrium thermodynamics can be based upon the Kolmogorov-Cartan T0 (not metrizable) topology. There are 16 types of T0 topologies in 4 topological dimensions. These 16 topologies have singleton closure sets of 4 ingredients. All ingredients are distinct in all 16 topologies. Five closure sets have isolated singletons, and 10 closure sets are composed of connected ingredients and thereby related to connected topological spaces. The only topology that is a disconnected topology in 4D is the Kolmogorov-Cartan T0 topology (Poset 3). One disconnected part is Torsion free (and can be embedded in the Euclidean plane) and the other disconnected part admits (what I have defined as) Topological Torsion (such structures cannot be embedded in the Euclidean plane). It is the latter part (the part that

admits Topological Torsion) that generates new, logical, mathematical solutions to, and a better understanding of, the long standing physical questions associated with the "Arrow of Time" and thermodynamic irreversibility. These theorems involve the continuous topological evolution of the non-metrizable features of physical systems.

3. It may come as a surprise, but the concepts of homology and cohomology can be applied to non-metrizable Lattice Structures, be they topologies or not. The cohomological concepts lead to the concept of period integrals whose ratios are rational. Period integrals exist for non-geometric structures and geometric structures. They can be used to describe, effectively, both microscopic and macroscopic quantum features ([4]) of the physical world. However there exist period integrals that are independent from geometrical constraints, and are not included in theories that are constrained to be geometrical. As will be explained in more detail, the period integrals (missing in a geometric formulation) are encoded by the 3-forms of topological torsion ($A\hat{}F$) and topological Spin ($A\hat{}G$) that do not obey the Kunneth factorization formula. The Kunneth formula fails if the structure is not Hausdorff T2; that is, the Kunneth formula is valid if and only if the structure is metrizable. Such non-metrizable period integrals lead to the quantum Hall impedance ($A\hat{}F/A\hat{}G$), Eckhart dissipation [2], ($d(A\hat{}F) = F\hat{}F$), in non-metrizable irreversible fluids, such as the expanding Universe, and Poincare invariants ($d(A\hat{}G) = F\hat{}G - A\hat{}J$) in irreversible non-metrizable plasmas, such as found in Spin insulators .

4. Quantum Mechanics has rational features in both the non-metrizable (non-geometrical) portions of the universe, and in the metrizable geometric portions of the universe. The total integration of gravity (classically based upon metric alone) and quantum mechanics (which has measurable features that do not require a metric) seems to be futile. Quantum features constrained to Hausdorff T2 topologies of compact sets without boundaries can be made to fit with gravitational theories based upon metric, but there are other non-metrizable quantum features (the 3-form period integrals that are not T2) that cannot be part of a geometric theory.

The study of topology is a formidable task for most engineers, for many physicists (including the author), and even for mathematicians. In this monograph, a highly selected set of "topological" concepts will be utilized in order to expedite the possibilities of a non-metrizable approach to physical sciences.

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