

Non-Equilibrium Systems and Irreversible Processes

Adventures in Applied Topology

Vol. 2

Falaco Solitons, Cosmology, and the Arrow of Time

from a Perspective of Continuous Topological Evolution.

R. M. Kiehn

Emeritus Professor of Physics

University of Houston

© Copyright CSDC INC. 2004

December 29, 2004

CONTENTS

0.1	Preface	7
0.2	Points of Departure	9
0.3	Results	10
0.4	Monograph Site Map	12
1	Falaco Solitons	15
1.1	Cosmic Strings in a Swimming Pool	15
1.1.1	The Experiment	22
1.2	Falaco Solitons and Dynamical Systems	23
1.2.1	Bifurcations	24
1.2.2	Projective Fluctuations and dynamical systems	25
1.2.3	Clues from the Hopf Map and Hopf vectors.	30
1.2.4	The bifurcation to Falaco Solitons	39
1.3	Falaco Solitons in exact solutions to the Navier-Stokes equations.	42
1.3.1	Saddle-Node: Hopf and Falaco bifurcations	44
1.3.2	Hysteresis-Hopf and Falaco bifurcations	45
1.3.3	Transcritical Hopf and Falaco Bifurcations	47
1.3.4	Minimal Surface Hopf and Falaco Bifurcations	49
1.4	Falaco Solitons as Landau Ginsburg structures in micro and mesoscopic systems	51
1.5	Summary	54
1.6	Some Anecdotal History	54
2	Cosmology and the Non Equilibrium Van der Waals gas	59
2.1	Cosmic Strings and Wheeler Wormholes	59
2.2	A Cosmological Conjecture based on Continuous Topological Evolution	61
2.2.1	Landau's argument for interactions of fluctuations	64
2.2.2	The Universe as a Turbulent van der Waals Gas near the Critical Point.	64

2.2.3	Results	66
2.3	The Ubiquitous Universal van der Waals Gas	67
2.3.1	The Phase function for a van der Waals Gas	74
2.3.2	The Jacobian Matrix of the Action 1-form.	77
2.3.3	The Non Equilibrium Characteristic Phase Function	79
2.3.4	Oscillations and the Hopf bifurcation	88
2.3.5	Minimal surfaces	88
2.3.6	Singularities as defects of Pfaff dimension 3	97
2.3.7	The Adjoint Current and Topological Spin	98
2.3.8	Non Equilibrium Examples.	100
2.4	The Cosmological van der Waals Gas	104
2.5	The Hopf Map, Spinors, and Minimal Surfaces	109
2.5.1	Hopf Maps and Hopf Vectors	111
2.5.2	Isotropic Vectors and Minimal surfaces in 3D	113
2.5.3	Complex Curves	120
2.5.4	Spinors and the Hopf map	121
2.5.5	The Adjoint field to the Hopf Map	122
2.6	Interesting Cosmological Conjectures	129
2.6.1	Minimal Surfaces in Minkowski space	129
2.6.2	Point Particles as Real and Complex Spheres of "zero radii"	130
2.6.3	More on Minimal Surfaces	131
2.6.4	Bulk Viscosity and Cosmology	132
2.6.5	The Four Forces and Differential Topology	132
2.6.6	Signature Symmetry Breaking	134
2.6.7	Hedgehog fields, Rotating plasmas, Accretion discs	136
2.6.8	Chandrasekhar Black Holes and the Hopf Map	137
2.6.9	Dark matter, Dark energy (Negative Pressure), Energy Balance and Curvatures.	138
3	The Arrow of Time	143
3.1	Retrodiction vs. Prediction	143
3.1.1	Intrinsic Covariance and the Classification of Maps	143
3.1.2	Physical Applications	145
3.1.3	Transformational covariance	150
3.2	Irreversibility and Continuous Topological Evolution	153
3.3	Topological Tools	157
3.3.1	Topological Structure	157
3.3.2	Continuity	158

3.3.3	Cartan's Magic Formula	159
3.3.4	Thermodynamic Irreversibility $Q \hat{=} dQ \neq 0$	160
3.3.5	The Pfaff Topological Dimension, n , versus Geometrical Dimension, m :	161
3.3.6	Symplectic manifolds:	163
3.3.7	Cartan's development of Hamiltonian systems.	164
3.3.8	Thermodynamic Reversibility $Q \hat{=} dQ = 0$	165
3.4	The Pfaff Dimension of the 1-form of Virtual Work, Pfaff(W)	166
3.4.1	Reversible Case 1: Pfaff(W) = 0. Cyclic processes that are adiabatic and extremal.	167
3.4.2	Reversible Case 2: Pfaff(W) = 1. Symplectic processes.	170
3.4.3	Case 3: Reversible, Pfaff(W) = 2 or 3	171
3.4.4	Case 4 : Irreversible, Pfaff(W) = 4.	171
3.5	Anholonomic Fluctuations.	172
3.6	Dissipative Evolution to States Far from Equilibrium	176
3.6.1	Electromagnetic Irreversible Process in the direction of the Topological Torsion vector	176
3.6.2	Mechanical Irreversible Process in the direction of the Topological Torsion vector	177
3.7	An Irreversible Example: The Sliding Bowling Ball	178
3.7.1	The Observation	178
3.7.2	The Analysis	179
4	A Summary of Non Equilibrium Thermodynamics	183
4.1	From the Topological Perspective of Continuous Topological Evolution	183
4.2	Applied topology versus applied geometry	188
4.3	Topological Universality	190
4.4	Fundamental Axioms and Notable Results	191
4.4.1	Axioms	191
4.4.2	Notable Results	192
4.5	Topological properties vs. Geometrical properties	195
4.6	Pfaff Topological Dimension	198
4.7	Evolutionary Invariants	201
4.7.1	Deformation Invariants as Topological Properties	201
4.7.2	Absolute Integral Invariants	202
4.7.3	Relative Integral Invariants	203
4.7.4	Holder Norms, Period Integrals and Topological Quantization	204

4.8	Unique Continuous Evolutionary Processes	206
4.8.1	Physical Systems: Equilibrium, Isolated, Closed and Open	210
4.8.2	Equilibrium versus non Equilibrium Systems	212
4.8.3	Change of Pfaff Topological Dimension	214
4.8.4	Systems with Multiple Components	215
4.9	Thermodynamic Processes	216
4.9.1	Continuous Processes	216
4.9.2	Reversible and Irreversible Processes	218
4.9.3	Adiabatic Processes - Reversible and Irreversible . . .	220
4.9.4	Processes classified by connected topological constraints on the Work 1-form.	222
4.9.5	Planck's Harmonic Oscillator and Type B processes - How does energy get quantized ?	224
4.9.6	Locally Adiabatic Processes	225
4.9.7	Reversible processes when the Pfaff topological dimension of Work is 2 or 3	226
4.10	A Physical System with Topological Torsion	229
4.11	The Lie differential $L_{(V)}$ and the Covariant differential $\nabla_{(V)}$.	234
4.12	Topological Fluctuations.	236
4.12.1	The Cartan-Hilbert Action 1-form..	236
4.12.2	Thermodynamics and Topological Fluctuations of Work	238
4.12.3	Thermodynamic Potentials as Bernoulli evolutionary invariants.	244
4.13	Entropy of Continuous Topological Evolution and Equilibrium Submanifolds	246
4.13.1	Extensions of the Cartan-Hilbert Action 1-form . . .	247
5	Notes and References	255
5.1	Notes	255
5.2	About the Cover Picture	272
5.3	About the Author	273

0.1 Preface

This monograph is the second volume of a series in which topological methods are applied to the study of diverse Non Equilibrium Systems and Irreversible Processes. The three topics considered in this monograph are "Falaco Solitons, Cosmology and the Arrow of Time". These topics, although seemingly unrelated in terms of geometric properties of size, shape and continuous geometric dynamics, appear to have an extraordinary universal equivalence from the perspective of Continuous Topological Evolution (See Vol 1 of this Series, "Non-equilibrium Thermodynamics"). Non-equilibrium systems undergoing irreversible processes most often consist of a collection of diverse, but synergistic and topologically coherent, components of more than one species. The components can consist of atoms, or they may consist of galaxies. The topology of interest in this monograph does not depend upon geometric properties such as size or shape. If the number of components change then topological evolution has taken place. Condensation or merging together is one form of topological evolution, where the number of components changes; such evolutionary changes can be described by continuous processes. These dynamical systems often "self organize" by irreversibly evolving to collective long-lived states, far from equilibrium. They then sustain themselves by "feeding" and delivering "waste" to their environments, but (apparently) will ultimately decay to an equilibrium state of inactivity or death. From a topological point of view, these non-equilibrium thermodynamic systems have an underlying (topological, not geometrical in the sense of size and shape) dynamical theory that makes them appear to be universally equivalent.

Classic equilibrium thermodynamics utilizes statistical methods influenced by the predictable and observable properties of continuous geometric evolution. Historically, the theory of continuous geometric evolution can be used to describe the evolutionary dynamics of particles and fluids. There exist categories of continuous geometric processes of translation and rotation whereby geometric invariant properties of size and shape can be used to encode the "particles". There also exist other categories of continuous geometric processes of deformation (which do not preserve size and shape) but where by topological (deformation) invariant properties can be used to encode the "fluid". Continuous geometric evolution implies that the process can be described by a diffeomorphism (a C^1 differentiable map, with a C^1 inverse), a constraint which permits the deduction of a unique final state *neighborhood* from given initial data.

For Non Equilibrium Systems and Irreversible Processes, the concept

of continuous geometric evolution must be replaced by the concept of continuous topological evolution (See chapter 5 of Vol. 1). Topological change is a necessary feature of continuous irreversible processes. Irreversible processes can not be described by diffeomorphisms. Hence, the concept of tensor fields (which are defined with respect to diffeomorphisms) must be replaced by other mathematical objects which are functionally well behaved with respect to processes which are not diffeomorphisms and which can be used to describe topological change. In this series of monographs the objects which are used to encode the physical system are taken to be "exterior differential forms" as developed and exploited by E. Cartan. Exterior differential forms behave as scalars (or scalar densities) with respect to tensor diffeomorphisms, so they work well when the domain of interest is restricted to the equivalence class of diffeomorphisms and geometrical evolution. More importantly, exterior differential forms are well behaved, in a functional neighborhood sense, with respect to C^1 mappings that are *not* homeomorphic. Recall that most tensor fields are not well behaved in a predictive functional *neighborhood* sense, relative to non-homeomorphic maps. In this monograph it is demonstrated that fundamental thermodynamic principles can be extended to describe Non Equilibrium Systems and Irreversible Processes - when physical systems are encoded in terms of exterior differential forms, and subjected to continuous topological, not geometrical, evolution.

The historical use of a geometric diffeomorphic approach (tensor analysis), with emphasis on uniqueness, symmetries and conservation laws, to solve problems in physics has heretofore constrained, if not eliminated, the stated objective of understanding Non Equilibrium Systems and Irreversible Processes. However, geometric methods, borrowing the words of Eugene Wigner, have been "unreasonably effective" in understanding physical phenomena - at least for phenomena that can be approximated by isolated-equilibrium systems and statistical averages. The geometric methods developed historically (and based upon geometry) are time reversal invariant. Although the geometric dimension of such "isolated" systems can be much larger than 2, Caratheodory has demonstrated that the Pfaff Topological dimension is not greater than 2. However, non equilibrium thermodynamic systems undergoing irreversible continuous processes require that the Pfaff Topological dimension must be greater than 2. The topology of the initial state and the topology of the final state are not the same if the process is irreversible.

Paraphrasing Eddington, and due to the insistence of predictive uniqueness (Pfaff topological dimension equal to 2 or less):

The concepts of aging and the arrow of time have slipped through the net of geometric analysis.

Most of the references to my earlier publications have been compiled for convenience in Vol 7 "Selected Publications", which is available in paper back form, or in PDF file download format. See www.cartan.pair.com.

0.2 Points of Departure

From the outset, it is assumed that the presence of a physical system induces a topology on a set of base variables (say space time). The dynamics of the system refines the topology. This idea is similar to, but different from, the assumption that the presence of matter establishes a metric on a set of base variables.

In this monograph certain physical systems and processes will be studied in terms of a top down topological method, rather than a bottom up method. That is, the physical system will be presumed to have started as a non-equilibrium fluidic system in a turbulent state and subjected to irreversible processes. The Pfaff Topological dimension (See Vol. 1) of the initial state for such systems must be an even number equal to 4 or more. The turbulent system will irreversibly decay to produce topological defects, which are collective, observable, long lived states far from equilibrium (often with an odd Pfaff Topological dimension of 3 or more). In the sense of the Cartan topology, such long lived objects are represented by closed exterior differential forms, which are deformable integral invariants (hence topological properties independent from geometrical properties of size and shape. As the exterior derivative of such closed objects vanishes, they do not have limit points with respect to the Cartan topology. This method is the opposite of the bottom up technique, which assumes the system is in equilibrium (Pfaff Topological dimension of 2 or less), and then examines the possibility that observables are generated by perturbations of the equilibrium state to create defect structures.

The major difference is that the bottom up method starts with a connected topology (Pfaff Topological dimension of 2 or less), while the top down method starts with a disconnected topology (Pfaff Topological dimension of 4 or more). It is possible by continuous maps to evolve from a disconnected topology to a connected topology, but it is impossible to evolve from a connected topology to a disconnected topology in a continuous manner. It is here, via the axiom of topological continuity, where the arrow of time becomes well defined. From a cosmological point of view, the universe

will be presumed to be a dilute non-equilibrium turbulent gas (deformably equivalent to of the van der Waals gas and of Pfaff Topological dimension 4) near its critical point. Hence large fluctuations in density are to be expected. These fluctuations in density are presumed to be stars and galaxies that cause the night sky to be inhomogeneous. Certain universal classes of topological defects of odd Pfaff topological dimension will be investigated. One universal class of such objects, Falaco Solitons, can be easily created in a swimming pool. The Falaco Solitons are topologically coherent, but deformable structures, which appear to self organize themselves during thermodynamically irreversible processes of topological change into long lived states far from equilibrium.

As developed in Vol 1, the Cartan topology for such non-equilibrium systems, of Pfaff topological dimension greater than 2, is a disconnected topology, which can support many components (mixed phases). Another way of describing such a topologically disconnected system (of topological defects) is that if solutions exist, there may be more than one solution (non uniqueness) at any geometric point, leading to the notion of envelopes, Huygen wavelets, tangential discontinuities, and edges of regression representing stability limits and the possibility of thermodynamic phase change. Indeed, an important topological property is the number of disconnected parts, which in the treatment of non-equilibrium thermodynamics will be related to the mole number n .

0.3 Results

It is remarkable that by using a topological perspective and the axioms for continuous processes (given in detail in Vol 1. and summarized in the following chapters) non-equilibrium systems and irreversible processes can be studied without the use of probability or statistical methods, and without the use of geometric metric constraints and linear connections. The topological method, constructed on a Cartan system of exterior differential forms which are inherently anti-symmetric, emphasizes the anti-symmetric properties of a physical system, where the more geometric and statistical methods, based upon quadratic metric forms and symmetric averages, tend to obscure the anti-symmetry properties.

It is further remarkable that the Jacobian matrix of the coefficients of the 1-form of Action - for those non equilibrium turbulent physical systems of Pfaff topological dimension 4 - leads to a universal thermodynamic phase function represented by a polynomial equation of 4th degree. The universal-

ity is related to the singularity theory of non degenerate systems which are equivalent under (small) deformations. The Phase function is constructed in terms of the symmetric similarity invariants of the Jacobian matrix of the component functions that encode the 1-form of Action, A . The envelope of the universal Phase function is deformably equivalent to a van der Waals gas. This universal resultant Phase function brings attention to thermodynamic phases that have equivalent (symmetry) structures other than those depending upon size and shape. In general, the exterior differential form method focuses attention on thermodynamic phases that have equivalent deformable topological structures (equivalent Pfaff topological dimension), and which are the result of continuous topological evolution.

Indeed, this resultant universal fourth order Phase function result matches the concepts of Landau Ψ^4 mean field theory and phase transitions on one hand, and on the other hand makes contact with the non equilibrium expansion of the universe described by "inflation", and dark matter and dark energy concepts due to a "Higgs" quartic potential below the critical point of a deformable van der Waals gas. The concepts of surface tension (or string theory) can be related to the mean curvature (induced by the molar density) of the universal phase surface, the concepts of temperature and entropy are related to the quadratic or Gauss curvature (induced by the molar density), while the concepts of pressure (of either sign) and interactions are related to the cubic curvatures (induced by the molar density). The theory as presented herein is far from being complete, yet the methods offer a new perspective for analyzing thermodynamic problems. Moreover, the techniques appear to solve the problem of making a marriage between mechanical dynamics and thermodynamics; the methods can be quite useful in the design of new applications previous excluded by assumptions of equilibrium and uniqueness.

The historical limitations of geometric (metric-size-and-shape) and topological (deformation) invariance usually imposed upon theoretical descriptions of nature (especially in relativity theories) are abandoned herein in favor of studying those properties that are homeomorphic invariants, and yet permit description of topological, as well as geometric, change relative to continuous transformations. The methods which are presented herein are based upon Cartan's calculus of exterior differential forms [64], [35]. Exterior differential forms are objects, which, in contrast to tensors, are well behaved with respect to differentiable (continuous) mappings that do not have an inverse (and therefore do not preserve topological properties), and are also well behaved with respect to diffeomorphisms, which are differentiable invertible

continuous mappings (and which preserve topological properties). Evolutionary processes will be defined in terms of the action of the Lie differential with respect to vector direction fields acting on differential forms [133]. The Lie differential acting on differential forms is not confined by the diffeomorphic constraints of tensor analysis, and can treat problems of topological change. The method goes beyond the more standard "extremal" techniques based upon the calculus of variations. In most of that which follows, the functions used to define the physical systems will be assumed to be C2 differentiable. The functions that describe processes most often will be assumed to be C2 differentiable as well, but certain C1 processes (inducing tangential discontinuities and wakes) and C0 processes (inducing shocks and first order phase transitions) are of physical interest.

A fundamental result of non equilibrium thermodynamics can be expressed by the statement:

Topological change is a necessary condition for a continuous thermodynamic process to be irreversible. .

Irreversible processes, related to the arrow of time and the biological aging process, require topological evolution and topological change. Current physical theories that describe evolutionary processes (for example, Hamiltonian or Unitary dynamics) usually are formulated in terms of homeomorphisms that emphasize geometrical properties, but do not permit topological change. Hence all such homeomorphic continuous processes are thermodynamically reversible and are inappropriate for the study of continuous topological evolution.

0.4 Monograph Site Map

The monograph starts with an experimental observation that highly motivated and sustained the author's research interest in Non Equilibrium Systems and Irreversible Processes. The experiment is easily performed (and has won prizes at state fairs for science projects conducted by high school students in the USA). Chapter 1 goes directly to a discussion of the extraordinary topological defects (known as Falaco Solitons) that can be (and have been) created and studied in a swimming pool. The ability to create Falaco Solitons gives a high level of credence to the fundamental theory of continuous topological evolution. These Falaco Solitons turn out to be locally unstable, but globally stabilized, long lived objects, that are far from equilibrium. Both the experiment and the theory are developed in Chapter 1,

where it becomes evident that the Falaco Solitons appear universally among the dynamical system governed by equations of the Navier-Stokes type.

Several challenges were thrown to the "String Theorists" to solve the problem using their methods. There were no replies. Yet it appears now that the solutions given in Chapter 1, without use of String Theory, seem to give an understanding of the problem in terms of non equilibrium thermodynamics.

Chapter 2 begins with a top down model for the universe. The initial motivation came from an argument presented by Landau, in terms of correlations of fluctuations. Herein, the statistical method is overwritten in terms of a cosmology that presumes the universe is deformably equivalent to a non equilibrium van der Waals gas near its critical point. Most of the mathematical development is detailed in chapter 2. Certain mathematical terms and a few useful theorems may be new to some readers. They are introduced without apology or tutorial description, but are sufficiently detailed in Vol 1. For those who need to be brought up to speed with Cartan's concepts of exterior differential forms, a number of textbooks are available [64], [12], [124], [6].

Chapter 3 describes how the concept of (topological) continuity can be used to formulate what has been called the arrow of time. It is demonstrated that homeomorphic physical theories, with evolutionary results that preserve topology, can not describe the details of the arrow of time. No such orientational structure exists for Hamiltonian systems. Recall that non-diffeomorphic maps cannot be used predict *functional forms* for neighborhood of tensor fields. However exterior differential forms are functionally well defined with respect to continuous, but non-homeomorphic maps, in a retrodictive manner. There is a logical difference in continuously evolutionary processes. When topology changes continuously, tensor fields and exterior differential forms are not uniquely predictable in a functional neighborhood sense. However, differential forms are retrodictable in a functional neighborhood sense with respect to such non-homeomorphic processes.

Chapter 4 gives a summary of the basic ideas used to describe non-equilibrium thermodynamics (details appear in Vol 1.).

