

[> restart:with(LinearAlgebra): with(GraphTheory):

For thermodynamic purposes based upon finite measurements, there are only two classes of finite topologies.

1. The Particle-like class of T0 topologies,
where all singlet subsets are distinguishable.

2. The Wave-like class of Not-T0 topologies, where some,
if not all, of the singlet subsets are indistinguishable.

For a set of $N = 2$ ingredients, $X=\{a,b\}$, there are 2 T0, and 1 Not-T0 topologies.
For a set of $N = 3$ ingredients, $X=\{a,b,c\}$, there are 5 T0, and 4 Not-T0 topologies.
For a set of $N = 4$ ingredients, $X=\{a,b,c,d\}$, there are 16 T0, and 17 Not-T0 topologies.
For a set of $N = 5$ ingredients, $X=\{a,b,c,d,s\}$, there are 63 T0, and 76 Not-T0 topologies.

In special cases the abstract symbols take on special meaning, especially
when the ingredients are presumed to be exterior differential forms .
If A is a 1-form, then $F=dA$ is a 2-form, $H=A\wedge F$ is a 3-form, and $K=F\wedge F$ is a 4-form.

For $N=4$ ingredients there is a special T0 poset 3 disconnected topology. This topology has a
Kuratowski representation in terms of the exterior differential, d , and the exterior product.
This special, particle-like, T0 topology was discovered about 1987, and led to the analysis
of non-equilibrium thermodynamic systems and irreversible processes based on the Grassman algebra.
For irreversible processes the three form $H = A\wedge F =$ Topological Torsion must not be zero.

A program has been created (using the Maple 14 symbolic calculator) to automatically compute
the properties of all finite Lattice Structures for $N = 3, 4,$ and 5 ingredients (hand computations can be tedious).
The computer program combines some of the techniques first developed by Didier Deses,
but then was augmented with numerous modifications, additions, and corrections.
This work was done R. M. Kiehn over the period November 20, 2009 until March 25, 2014.

When the Lattice Sets are presumed to be finite, the subsets of the power set can be identified as topological open sets. Then it
possible to compute the interior, exterior, boundary, closure, and limit points for every subset of the power set generated by a
lattice. The many topological solutions produced by the Maple program indicate that the finite lattice topologies are of only two
categories. The first category consists of those topologies that obey the Kolmogorov T0 separation axioms: All T0 singlet subsets
have topological closures that are distinguishable. These distinguishable subsets can be used to define topological particles. The
second category consists of those Not-T0 topologies where some, if not all, of the singlet subsets of the lattice are indistinguishable.
These indistinguishable subsets can be used to define topological waves.

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There are Lattice Subsets that define topologies, with separation axioms, T0 and T1, and yet these topologies are NOT metrizable.
If a topology is metrizable over the reals, the topology must be Hausdorff, T2, or stronger.

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In many instances the Lattice forms a planar graph, which means that the graph can be embedded by deformations into R^2 .
However, Kuratowski proved that certain lattices, those that contain $K_{3,3}$ or K_5 sub-lattices, cannot be embedded in R^2 as
planar graphs. Such Lattices require R^3 or more. A $K_{3,3}$ sub-lattice consists of 6 vertices and 9 connecting arcs (or edges). The
 K_5 sub-lattice consists of 5 vertices and 10 connecting arcs. These concepts have been used to understand chemical molecules,
and concepts of topological chirality.

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A visual aspect of an embedding into R^3 is given by a space curve that exhibits Affine torsion. Another, perhaps more general
idea, is defined in terms of Topological Torsion, which is related to the Pfaff Topological Dimension = 3 (or class) of an exterior
differential 1-form defined on a differential variety.

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Exterior differential 1-forms, A , that can not be embedded in R^2 are such that the 3-form $A\wedge dA$ is not zero.
This constraint requires that the Frobenius Unique integrability theorem fails! Unique initial data do NOT produce Unique final
data. Envelope solutions and edges of regression give visual examples of non-uniqueness.

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Exterior Differential 1-forms of Pfaff Topological Dimension 3 require a minimum of 3 independent functions (of an arbitrary geometric set) to capture their topological properties. An open question is: What Lattices cannot be embedded in R^3 ?"

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Geometric constraints based upon metric ideas imply that the underlying topology is Hausdorff T2. Finite Hausdorff spaces (the power set of X) are disconnected. Subsets of finite Hausdorff spaces can be simply connected, or not simply connected. If the subsets of the power set form a T0 topology, it can be a connected topology or a disconnected topology. The disconnected T0 topologies have subsets that do not have boundaries (the boundaries are empty). The connected T0 topologies do not have subsets without boundary,

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The study of the detailed lattice and topological structures gives an insight into physical concepts that may not be distinguished in terms of geometrical constraints. Finite Homological features focus attention on simply connected or not simply connected concepts which are not Hausdorff and therefore do not depend upon geometrical ideas of size and shape.

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It has been established that T0 topologies that are not metrizable can be put into correspondence with non-equilibrium thermodynamics, and irreversible processes. The arrow of time is explained in terms of topologically continuous processes from thermodynamic states of higher topological dimension to thermodynamic states of lesser topological dimension. In addition, certain topological quantum features can be put into correspondence with deRham integrals over closed exterior differential forms that are not exact.

See <http://www.lulu.com/kiehn> for 6 monographs that describe the "Unusual effectiveness of topological thermodynamics".

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Planck's Blackbody radiation formula describes physical phenomena that are independent of metric, size, shape, material composition, or even internal microscopic structure. It is conjectured that such concepts are related to both those Finite Lattice Structures that are particle-like T0 topologies and to those Lattice Structures that are wave-like Not-T0 topologies, but are Lattice Structures which do not obey the Kolmogorov T0 separation axioms.

Such Lattice Structures have singlet subsets of the power set that are indistinguishable. If Lattice Structures obey the Kolmogorov T0 separation axioms, then all subsets of the power set are distinguishable.

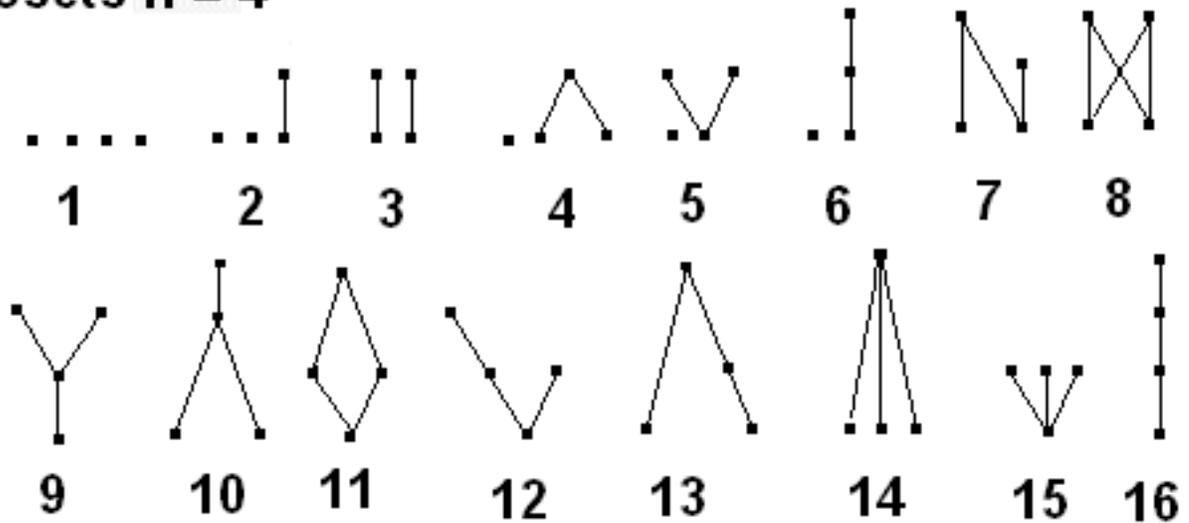
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The conjecture to be studied is :

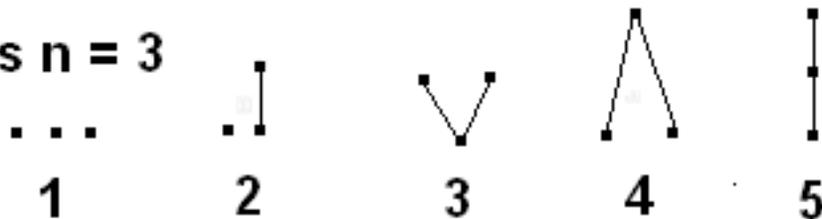
Do the Not-T0 non-metrizable wave-like topologies describe such effects as Planck's Black Body radiation formula, the salient features of indistinguishable Bosons and Fermions, and the theory of collective states, that are the foundations of Quantum Theory.

The T0 (poset) particle-like structures are based upon distinguishable singleton closures. Examples for $X=\{a,b,c,d\}$ and $X=\{a,b,c\}$ are given below:

Posets n = 4



Posets n = 3



For a subset A of a Lattice space,

$\text{Int}(S)$ = Interior of S = largest open set contained in S

$\text{Com}(S)$ = Complement of S = topset $X=\{a,b,c,d,\dots\}$ minus S = $X \text{ mod } S$.

$\text{Ext}(S)$ = Exterior of S = $\text{Int}(\text{Com}(S))$ = largest open set in the compliment of S.

$\text{Bnd}(S)$ = Boundary of S = $X \text{ mod } \text{Int}(S)$ mod $\text{Ext}(S)$.

$\text{Clo}(S)$ = Closure of S = $\text{Int}(S)$ union $\text{Bnd}(S)$ = S union $\text{Lim}(S)$

$\text{Lim}(S)$ = Limit points of S = points p such that $\{\text{Clo}(S \text{ mod } p) \text{ intersect } (p)\} \neq \text{the empty set}$.

Isolated points are of two types

$\text{Clo}(S)$ minus $\text{Lim}(S)$ = $\text{IsoClo}(S)$

and

$\{S\} \text{ intersection } \{\text{Lim}(S)\} = \text{IsoCar}(S)$, in the sense of Caratheodory

If S is an exterior differential 1-form, and the exterior differential, d, is a limit point generator then I call the non zero 3-form $A \wedge dA$: Topological Torsion. Topological Torsion = 0 implies the differential 1-form admits an integrating factor. For a Kuratowski topology Topological Torsion is related to IsoCar .

A closed set either has no limit sets or contains its limit sets. A closed set $S = \text{Clo}(S)$.

A set S with empty limit points and empty boundary points is defined as an isolated segregated point,

$\text{Bnd}(S) = \{\}$ and $\text{Lim}(S) = \{\} \Rightarrow \{S\}.\text{Seg}$.

The disconnected points in the closures of a set are described as adherent points.

Press Enter to load Program subroutines:

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#***** subroutines*****
> issubset:=proc(A,B)
> local a;
> for a in A do
>   if not(member(a,B)) then RETURN(false); fi;
> od;
> RETURN(true);
> end proc;
# Closure of Union
> clunion:=proc(T) local A,U;U:=T;
> for A in T do
>   U:= U union map(`union`,U,A);
> od;
> if U=T then U; else clunion(U); fi;
> end proc;

# Closure of intersections
> clintersect:=proc(T)
> local A,U;U:=T;
> for A in T do
>   U:= U union map(`intersect`,U,A);
> od;
> if U=T then U; else clintersect(U); fi;
> end proc;

# Is the lattice a Topological basis
> topbase:=proc(X,SB)
> clunion(clintersect(SB union {X,{})));
> end proc;

> kinterior:=proc(A,X,T)
> local i;global kINT;
> kINT:={};
> for i to nops(T) do
>   if issubset(T[i],A) then kINT:={} union T[i]; fi;
> od;
> end proc;

> kintext:=proc(A,X,T)
> local i;global kINTEXT;
> kINTEXT:={};
> for i to nops(T) do
>   if issubset(T[i],(X)minus(A)) then kINTEXT:={} union T[i]; fi;
> od;
> end proc;

> isTopo:=proc(X,T)
> clunion(T)=T and member(X,T) and member({},T) and clintersect(T)=T;
> end proc;

> akinterior:=proc(A,X,T)
> local i;global kINTEXT;
> kINTEXT:={0};
> for i to nops(T) do
>   if issubset(T[i],A) then kINTEXT:={0} union T[i]; fi;
> od;
> end proc;

# Is the Topology connected?

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> isConn:=proc(X,T)
>   evalb(CO(X,T)={X,{} });
> end proc;
> # Find the sets that are Open and Closed
> CO:=proc(X,T)
>   local A,W;W:={};
>   for A in T do
>     if member(X minus A,T) then W:=W union {A}; fi;
>   od;
>   W;
> end proc;
> #

> # Find the closure of T
> CLO:=proc(X,T)
>   {seq(X minus T[i],i=1..nops(T))};
> end proc;

> # The following are a set of procedures testing for some topological properties,
> # the used algorithms are simply exhaustive ones, all possibilities are checked.
> isT0:=proc(X,T)
>   local x,y,A,test;
>   for x in X do
>     for y in X minus {x} do
>       for A in T do
>         test:=evalb((member(x,A) and not(member(y,A))) or
>           (member(y,A) and not(member(x,A))));
>         if test then break; fi;
>       od;
>       if not(test) then break; fi;
>     od;
>     if not(test) then break; fi;
>   od;
>   test;
> end proc;
> isT1:=proc(X,T)
>   issubset({seq({X[i]},i=1..nops(X))},CLO(X,T));
> end proc;
> isHd:=proc(X,T)
>   local x,y,A,B,test;
>   for x in X do
>     for y in X minus {x} do
>       for A in T do
>         for B in T minus {A} do
>           test:=evalb(member(x,A) and member(y,B) and A intersect B={});
>           if test then break; fi;
>         od;
>         if test then break; fi;
>       od;
>       if not(test) then break; fi;
>     od;
>     if not(test) then break; fi;
>   od;
>   test;
> end proc;

> # The exterior of a set is the interior of its complement
> exteriorb:=proc(A,X,T)
>   local i,C,CIX;
>   CIX:=CLO(X,T);
>   C:=X;
>   for i to nops(CIX) do
>     if issubset(A,CIX[i]) then C:=C intersect CIX[i]; fi;
>   od;
>   print(`Exterior`=C);
> end proc;
> # Subspaces are found by taking intersections with the open sets.

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> subspace:=proc(A,X,T)
> map2( intersect ,A,T);
> end proc;
> # Given a list of sets this procedure finds all element in the cartesian product.
> expandcart:=proc(PX)
> local C,P,i;
> C:={};
> P:=cartprod([seq([op(PX[i]),i=1..nops(PX))]);
> while not P[finished] do C:=C union {P[nextvalue]()} od;
> C;
> end proc;
> # Implementation of the inverse projections, this is needed in later procedures.
> iprj:=proc(k,S,PX)
> local i,O;
> for i to nops(PX) do
>   if i=k then O[i]:=S else O[i]:=PX[i] fi;
> od;
> expandcart([seq(O[i],i=1..nops(PX))]);
> end proc;
> # Procedure to find a subbasis for a product topology using inverse projections.
> prodbase:=proc(PX,PS)
> local i,k,S;S:={};
> for k to nops(PX) do
>   for i to nops(PS[k]) do
>     S:=S union {iprj(k,PS[k][i],PX)};
>   od;
> od;
> S;
> end proc;
> # Given a list of sets and a list of topologies, the product topology is found.
> prodtop:=proc(PX,PT)
> topbase(expandcart(PX),prodbase(PX,PT));
> end proc;
> # A topology is weakly zero-dimensional iff the clopen sets form a base.
> isWzd:=proc(X,T)
> evalb(topbase(X,CO(X,T))=T)
> end proc;
> # A topology is connected iff there are no proper clopen sets.
> #
> isConn:=proc(X,T)
> evalb(CO(X,T)={X,{} });
> end proc;
> # A connection component is the largest connected subspace, containing a given point.
> K:=proc(x,X,T)
> local i,S,SK;
> SK:={};
> S:=map2( union`,{x},powerset(X));
> for i to nops(S) do
>   if isConn(S[i],subspace(S[i],X,T)) then SK:=SK union S[i]; fi;
> od;
> SK;
> end proc;
> # A topology is totally disconnected if all connections components are singletons.
> isTotDisc:=proc(X,T)
> local i;
> for i to nops(X) do
>   if not(K(X[i],X,T)={X[i]}) then RETURN(false) fi;
> od;
> RETURN(true);
> end proc;
> prLatpt3:=proc(GS,NBR)
> local VA,EA,De,i,j,k,LS3,LS;
> global GA,G,H,Dee,SS3,SSS3,XS,ZS,QS,TS,TSdual,LS33;
> LS:=GS;
> if NBR=3 then
>   LS:=(LS)union({{a,b,c}});fi;
> if NBR=4 then
>   LS33:=(LS)union({{a,b,c,d}});

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> fi;
> XS:={{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}};
> ZS:={a,b,c,ab,ac,bc,abc};
> j:=nops(LS)-1;QS:=LS[1];
> for i to j do QS:=QS,LS[i+1]; od;
> LS3:={QS}; k:=nops(LS3); TS:=0;
> for j to k do
>   for i to 7 do
>     if LS3[j]=XS[i] then TS:=TS,ZS[i]; fi;
>   od;
> od;
> if nops(LS3)=2 then
>   print('Nothing to draw');
> else
>   drawLat63pt([TS],NBR);
> fi;
> end proc:
> LEXorder:=proc(LX,NBR)
> local k,i,AA,BB,LEXsort:
> global AAA,BBB:
> k:=nops(LX); AA:=LX[1];
> for i to (k-1) do
>   AA:=AA,LX[i+1];
> od;
> AAA:={AA};
> k:=nops(LX); BB:=AAA[1];
> for i to (k-1) do BB:=BB,AAA[i+1]; od;
> BBB:={BB};
> end proc:
> drawLat63pt:=proc(TS,NBR)
> local EA,i,j,k;
> global GA,G,H,Dee,SS,SSS,XS,ZS,CLTop1,CLTop1dual,SS3,SSS3,VA,De,QS,LS7:
> GA:=Graph([0,a,b,c,ab,ac,bc,abc]);
> VA:=Vertices(GA);EA:=Edges(GA);
> LS7:=TS; De:={0,0}; k:=0;
> if member(a,LS7) then De:=De,[a,0]; fi;
> if member(b,LS7) then De:=De,[b,0]; fi;
> if member(c,LS7) then De:=De,[c,0]; fi;
> if member(ab,LS7) and member(b,LS7) then De:=De,[ab,b] fi;
> if member(ab,LS7) and member(a,LS7) then De:=De,[ab,a] fi;
> if member(ac,LS7) and member(a,LS7) then De:=De,[ac,a] fi;
> if member(ac,LS7) and member(c,LS7) then De:=De,[ac,c] fi;
> if member(bc,LS7) and member(b,LS7) then De:=De,[bc,b] fi;
> if member(bc,LS7) and member(c,LS7) then De:=De,[bc,c] fi;
> if member(abc,LS7) and member(ab,LS7) then De:=De,[abc,ab] fi;
> if member(abc,LS7) and member(ac,LS7) then De:=De,[abc,ac] fi;
> if member(abc,LS7) and member(bc,LS7) then De:=De,[abc,bc] fi;
> j:=nops([De])-1;
> Dee:=De[2]; for i to j do Dee:=Dee,De[i+1] od;
> if Dee=0 then H:=Graph(VA): else H:=Digraph(VA,{Dee}):fi;
> SetVertexPositions(H,[0,0],[-1,1],[0,1],[1,1],[-1,2],[0,2],[1,2],[0,3]);
> HighlightVertex(H, Vertices(H), white);
> if member(a,LS7) then HighlightVertex(H,a,red) fi;
> if member(b,LS7) then HighlightVertex(H,b,red) fi;
> if member(c,LS7) then HighlightVertex(H,c,red) fi;
> if member(ab,LS7) then HighlightVertex(H,ab,yellow) fi;
> if member(bc,LS7) then HighlightVertex(H,bc,yellow) fi;
> if member(ac,LS7) then HighlightVertex(H,ac,yellow) fi;
> HighlightVertex(H,abc,cyan);
> HighlightEdges(H, Edges(H), black);
> SSS3:=DrawGraph(H);
> end proc:

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> checkdual:=proc(GS,`title`,NBR) local i;
> global LSdual,CLS,CLL,RCL,TS,SSCLL,X,LS,LSTopdual,TTS,CILS,ClosLSd,ClosLatdual,ClosLS,SG;
    X:={a,b,c,d};
    CLS:={{},{a},{b},{c},{d},{a,b},{a,c},{a,
    d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d},{a,b,c,d}};
> LS:=GS; LSTopdual:=CLO(X,LS);
> LSdual:=(CLS)minus(LS)minus(LSTopdual)union({})union({X});
> SG:=Graph([a,b,c,d]);HighlightVertex(SG,[a,b,c,d],magenta);
> SetVertexPositions(SG,[[1,1],[2,1],[3,1],[4,1]]);
> print(`title`);print(`The input Lattice Structure, LS`=(LS));
> print(`Is LS a Topology ?`=isTopo(X,LS));
> print(`Is LS T0 ?`=isT0(X,LS));
> Closure(X,LS);
> SSCLL:=RCL[2,5]; for i to nops(X)-1 do SSCLL:=SSCLL,RCL[i+2,5] od;
> ClosLS:=SSCLL;
> print(`The Top Down Lattice Structure for LS is: `);
> print(prLatpt4(LS,4));
> print(`The singleton closure set of the lattice Structure, LS, is`=ClosLS);
> print({ClosLS});
> if nops({ClosLS})<4 then print(`NOT all singleton points are distinguishable`) else
> print(`Display the T0 poset`);print(getPO({ClosLS})) ;fi;
> LAT4abcd(LS,`title`,NBR); print(` `);
> print(`The dual (compliment) Lattice Structure, LS*`=(LSTopdual));
> print(`Is LS* a Topology ?`=isTopo(X,LSTopdual));
> print(`Is LS* T0 ?`=isT0(X,LS));
> Closure(X,LSTopdual);
> SSCLL:=RCL[2,5]; for i to nops(X)-1 do SSCLL:=SSCLL,RCL[i+2,5] od;
> ClosLSd:=SSCLL;
> print(`The DUAL Top Down Lattice Structure for LS* is: `);
> print(prLatpt4(LSTopdual,4)); print(` `);
> print(`The singleton closure set of the lattice Structure, LS*, is`=ClosLSd);
> print({ClosLSd});
> if nops({ClosLSd})<4 then print(`NOT all singleton points are distinguishable`) else
> print(`Display the T0 poset`);print(getPO({ClosLSd})) ;fi;
>
> print(` The Partition Dual: CLS-LS -LS*`=LSdual);
> print(`Is the Partition Dual a Topology ?`=isTopo(X,LSdual));
> print(`Is the :Partition Dual T0 ?`=isT0(X,LSdual));
> Closure(X,LSdual);
> SSCLL:=RCL[2,5]; for i to nops(X)-1 do SSCLL:=SSCLL,RCL[i+2,5] od;
> ClosLatdual:=SSCLL;
> if LSdual = {{},{a,b,c,d}} then
> print(`Hence, the Partition Dual is the Indiscrete Topology`=LSdual);
> print(`All singlet points {a},{b},{c},{d} are isolated`);
> else
> print(`The Top Down Lattice Structure for the Partition Dual is: `);
> print(prLatpt4(LSdual,4));
> fi; print(` `)
> end proc;
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> Closure:=proc(GS,NBR) local k,i,LS034,X; global RCL,SCLL,SCL;
> if NBR=3 then X:={a,b,c}; fi;
> if NBR=4 then X:={a,b,c,d}; fi;
> if NBR=5 then X:={a,b,c,d,s}; fi;
> k:=nops(X)+1;RCL:=Matrix(k,6);LS034:=GS;
> RCL[1,1]:=`Point`;
> RCL[1,2]:=`Interior`;
> RCL[1,3]:=`Exterior`;
> RCL[1,4]:=`Boundary`;
> RCL[1,5]:=`Closure`;RCL[1,6]:=`Limit Set`;
> for i to nops(X) do RCL[i+1,1]:={X[i]}; od;
> for i to nops(X) do kinterior({X[i]},X,LS034);
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> RCL[i+1,2]:={kINT}:kintext({X[i]},X,LS034):
> RCL[i+1,3]:={kINTEXT}:
> RCL[i+1,4]:=(X)minus(RCL[i+1,2])minus(RCL[i+1,3]):
> RCL[i+1,5]:=(RCL[i+1,4])union(RCL[i+1,2]):RCL[i+1,6]:=(RCL[i+1,4])union(RCL[i+1,2])minus(RCL[i+1,1]):od:

> RCL;
> if NBR=3 then SCLL:=(RCL[2,5],RCL[3,5],RCL[4,5]):fi;
> if NBR=4 then SCLL:=(RCL[2,5],RCL[3,5],RCL[4,5],RCL[5,5]):fi;
> if NBR=5 then SCLL:=(RCL[2,5],RCL[3,5],RCL[4,5],RCL[5,5],RCL[6,5]):fi;
> end proc:
*****

> LAT5abcd:=proc(GS,`title`,NBR)
> local i,j,LS09,LS07,CLS,V,G,VG: global RCL,LS00,k,X,Pos,Z,Link,LS,SCLL,CLOsLS,LSCT;
> LS:=GS;
> X:={a,b,c,d,s}; LS09:={{},{a},{b},{c},{d},{s},{a,b},{a,c},{a,d},{a,s},{b,c},{b,d},{b,s},{c,d},{c,s},{d,s},{a,b,c},{a,b,d},{a,b,s},{a,c,d},{a,c,s},{a,d,s},{b,c,d},{b,c,s},{b,d,s},{c,d,s},{a,b,c,d},{a,b,c,s},{a,b,d,s},{a,c,d,s},{b,c,d,s},{a,b,c,d,s}};

> interface(rtablesiz=60); Pos:=[0,0,0,0,0]; Link:=[0,0,0,0,0,0,0,0,0,0];
> Z:=Closure(LS,NBR);
> k:=31;
> RCL:=Matrix(32,8):
> RCL[1,1]:=`Subset S`:
> RCL[1,2]:=`Int(S)`:
> RCL[1,3]:=`Ext(S)`:
> RCL[1,4]:=`Bnd(S)`:
> RCL[1,5]:=`Clo(S)`:RCL[1,6]:=`Lim(S)`:RCL[1,7]:=`IsoClo(S)`:RCL[1,8]:=`IsoCar(S)`:

> i:=1;for i to k do RCL[i+1,1]:=LS09[i+1]: od:
> i:=1;for i to k do kinterior(LS09[i+1],X,GS):
> RCL[i+1,2]:=kINT:kintext(LS09[i+1],X,GS):
> RCL[i+1,3]:=kINTEXT:
> RCL[i+1,4]:=(X)minus(RCL[i+1,2])minus(RCL[i+1,3]):
> RCL[i+1,5]:=(RCL[i+1,4])union(RCL[i+1,2]):

> LIMPT(RCL[i+1,1],LS,NBR); RCL[i+1,6]:=SCLL:RCL[i+1,7]:=(RCL[i+1,5])minus(SCLL):RCL[i+1,8]:=(SCLL)intersect
(RCL[i+1,1]):

> od:
> interface(rtablesiz=60);

> Z:=(RCL[2,5],RCL[3,5],RCL[4,5],RCL[5,5],RCL[6,5]): print(` `);
> print(`Is LS5: a topology `isTopo(X,LS),` connected `isConn(X,LS),` Kolmogorov.T0`=isT0(X,LS),` Hausdorff.
T2`=isHd(X,LS));
> print(` `); print(RCL);
> if nops({Z}) < 5 then print(`Some singletons have the same closure and cannot be distinguished`);fi :

> end proc:

```

```

> #
> LAT4abcd:=proc(GS,`title`,NBR)
> local i,j,LS08,LS07,CLS,V,G,VG: global RCL,LS00,k,X,Pos,Z,Link,LS,LSD,LSP,PLSX,CLOsLS,LSU,LSP1,LSCT;

> LS:=GS;
> X:={a,b,c,d};LS08:={{},{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d},{a,b,c,d}}; LSD:=CLO
(X,LS); LSP:=(LS08)minus(LS)minus(LSD)union({})union({X}); LSU:=(LS)union(LSD); interface(rtablesiz=20);
LSP1:=(LS08)minus(LS)union({})union({X});
> Pos:=[0,0,0,0]; Link:=[0,0,0,0,0,0];
> k:=15; RCL:=Matrix(16,8):
> RCL[1,1]:=`Subset S`:
> RCL[1,2]:=`Int(S)`:
> RCL[1,3]:=`Ext(S)`:
> RCL[1,4]:=`Bnd(S)`:

```

```

> RCL[1,5]:=`Clo(S)` : RCL[1,6]:=`Lim(S)` : RCL[1,7]:= `IsoClo(S)` : RCL[1,8]:=`IsoCar(S)` :
> i:=1;for i to k do RCL[i+1,1]:=LS08[i+1]: od:
> i:=1;for i to k do kinterior(LS08[i+1],X,LS);
> RCL[i+1,2]:=kINT:kintext(LS08[i+1],X,LS):
> RCL[i+1,3]:=kINTEXT:
> RCL[i+1,4]:= (X)minus(RCL[i+1,2])minus(RCL[i+1,3]):
> RCL[i+1,5]:= (RCL[i+1,4])union(RCL[i+1,2]):
>
>
> LIMPT(RCL[i+1,1],LS,NBR); RCL[i+1,6]:=SCLL: RCL[i+1,8]:= (RCL[i+1,1])intersect(SCLL): RCL[i+1,7]:= (RCL[i+1,5])
minus(SCLL):
>
> od:
> Z:= (RCL[2,5],RCL[3,5],RCL[4,5],RCL[5,5]):k:=15:i:=1: for i to k do if (RCL[i+1,5])=(RCL[i+1,1]) then RCL[i+1,7]:= { }
:fi; od;
> #print(` `); print(`Clo(S)` =Z) ;
> #print(`Lattice subsets, LS`=LS); print(` `);print(` LS`=LS);
> i:=1; k:=15;for i to k do if (RCL[i+1,6]) = { } and (RCL[i+1,4])= { } then RCL[i+1,7]:=RCL[i+1,1].`Seg`; fi; od; print(` `);

> print(`Is LS4: a topology `=isTopo(X,LS),` connected `= isConn(X,LS),` Kolmogorov.T0`= isT0(X,LS),` Hausdorff.
T2`= isHd(X,LS));
>
>
>
> print(` `);
> print(RCL);
>
>
> print(`The Closed-Open subsets of LS are `=CO(X,LS)); if nops({Z})< 4 then print(`Some singletons have the same closure
and cannot be distinguished`);fi : print(`The {1,2,3,4} array of CLOSURE elements is`=Z);
>
>
> end proc:

```

```

> # ***** Lattice Structure 3 points
> LAT3abc:=proc(GS,`title`,NBR)
> local i,j,LS0,CLS,V,G,VG: global RCL,LS00,k,X,Pos,Z,Link,LS,CLoSLS,LSCT;
> LS:=GS; print(` `);
> X:={a,b,c};
> LS0:={{ },{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}};
> interface(rtables=20); Pos:=[0,0,0,0] ; Link:=[0,0,0,0,0,0];
> k:=7; RCL:=Matrix(8,8);
> RCL[1,1]:=`Subset S` :
> RCL[1,2]:=`Int(S)` :
> RCL[1,3]:=`Ext(S)` :
> RCL[1,4]:=`Bnd(S)` :
> RCL[1,5]:=`Clo(S)` : RCL[1,6]:=`Lim(S)` : RCL[1,7]:=`IsoClo(S)` : RCL[1,8]:=`IsoCar(S)` :

> i:=1;for i to k do RCL[i+1,1]:=LS0[i+1]: od:
> i:=1;for i to k do kinterior(LS0[i+1],X,LS);
> RCL[i+1,2]:=kINT:kintext(LS0[i+1],X,LS):
> RCL[i+1,3]:=kINTEXT:
> RCL[i+1,4]:= (X)minus(RCL[i+1,2])minus(RCL[i+1,3]):

```

```

> RCL[i+1,5]:=(RCL[i+1,4])union(RCL[i+1,2]);
>
> LIMPT(RCL[i+1,1],LS,NBR); RCL[i+1,6]:=SCLL;RCL[i+1,8]:=(RCL[i+1,1])intersect(SCLL);RCL[i+1,7]:=(RCL[i+1,5])
minus(SCLL);
>
> od:
> Z:=(RCL[2,5],RCL[3,5],RCL[4,5]);k:=7;i:=1: for i to k do if (RCL[i+1,5])=(RCL[i+1,1]) then RCL[i+1,7]:= { }:fi; od;
> #print( ` `); print( `Clo(S) `=Z): print( ` `);print( `LS `=LS); #print( `Lattice subsets, LS `=
LS);
> i:=1; k:=7;for i to k do if (RCL[i+1,6]) = { } and (RCL[i+1,4])= { } then RCL[i+1,7]:=RCL[i+1,1].`Seg`; fi; od; print( ` `);
> print( `Is LS3 a topology `=isTopo(X,LS), ` connected `= isConn(X,LS), ` Kolmogorov.T0 `= isT0(X,LS), `
Hausdorff.T2 `= isHd(X,LS));
> print( ` `);print(RCL);print( `Closed-Open subsets of LS are `=CO(X,LS));
>
>
> if nops({Z})< 3 then print( `Some singletons have the same closure and cannot be distinguished`);fi : print( `The {1,2,3,4}
array of CLOSURE elements is `=Z);
>
end proc:

```

limit point test subroutine for each subset

```

> LIMPT:=proc(GS,LS,NBR) local i,LSCL,k,LS0,X;global SCLL,ZZ,LRCL;
> if NBR=3 then X:={a,b,c}; LS0:={{},{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}}; fi;
> if NBR=4 then X:={a,b,c,d};LS0:={{},{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d},{a,b,c,d}};
fi;
> if NBR=5 then X:={a,b,c,d,s}; LS0:={{},{a},{b},{c},{d},{s},{a,b},{a,c},{a,d},{a,s},{b,c},{b,d},{b,s},{c,d},{c,s},{d,s},{a,b,c},{a,b,
d},{a,b,s},{a,c,d},{a,c,s},{a,d,s},{b,c,d},{b,c,s},{b,d,s},{c,d,s},{a,b,c,d},{a,b,c,s},{a,b,d,s},{a,c,d,s},{b,c,d,s},{a,b,c,d,s}}; fi;
> interface(rtablesize=20);LRCL:=Matrix(NBR+1,8):
> LRCL[1,1]:=`Point p`:LRCL[1,2]:=`(GS-p)`;
> LRCL[1,3]:=`Int(GS-p)`;
> LRCL[1,4]:=`Ext(GS-p)`;
> LRCL[1,5]:=`Bnd(GS-p)`;
> LRCL[1,6]:=`Clos(GS-p)`;LRCL[1,7]:=`Clos(GS-p)inter p`; LRCL[1,8]:=`GS`;
> for i to nops(X) do LRCL[i+1,1]:={X[i]}; LRCL[i+1,2]:=(GS)minus(LRCL[i+1,1]);LRCL[i+1,8]:=(GS):od;
>
> for i to nops(X) do kinterior(LRCL[i+1,2],X,LS);
> LRCL[i+1,3]:=kINT:kintext(LRCL[i+1,2],X,LS);
> LRCL[i+1,4]:=kINTEXT;
> LRCL[i+1,5]:=(X)minus(LRCL[i+1,3])minus(LRCL[i+1,4]);
> LRCL[i+1,6]:=(LRCL[i+1,5])union(LRCL[i+1,3]);LRCL[i+1,7]:=(LRCL[i+1,6])intersect(LRCL[i+1,1]):od;
>
>
> if NBR=3 then SCLL:=(LRCL[2,7])union(LRCL[3,7])union(LRCL[4,7]):fi;
> if NBR=4 then SCLL:=(LRCL[2,7])union(LRCL[3,7])union(LRCL[4,7])union(LRCL[5,7]):fi;
>
> if NBR=5 then SCLL:=(LRCL[2,7])union(LRCL[3,7])union(LRCL[4,7])union(LRCL[5,7])union(LRCL[6,7]):fi;
> #print(LRCL);

```

|> end proc:

**The FiniteT0 (particle-like) class of topologies
and the Finite Not-T0 (wave-like) class of topologies
are suitable for measuring**

Non-Equilibrium Thermodynamics

**R. M. Kiehn 2009-2014
www.cartan.pair.com**

**Maple code download available on email request
rkiehn2352@aol.com**

**1. Examples of the 5 finite $N=2$ (T0) particle-like topologies
embedded into $N=3$, $\{\{\}, X\} = \{\{\}, \{a,b,c\}\}$.
 $\text{Bnd}(X) = \{\}$**

All Singleton closures are distinguishable.


```
> print(` `);X:={a,b,c};LS2:={},{b},{c},{a,b},{b,c};LS3:={LS2,X};print(`LS3 = T0, particle-like topology, poset 2`);
LAT3abc(LS3,`Poset 2`,3):
```

```

X:={a,b,c}
LS2:={},{b},{c},{a,b},{b,c}
LS3:={{},{b},{c},{a,b},{b,c},{a,b,c}}
LS3 = T0, particle-like topology, poset 2
..
..

```

```
Is LS3 a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false
..

```

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c}	{a}	{a}	{}	{}	{}
{b}	{b}	{c}	{a}	{a, b}	{a}	{b}	{}
{c}	{c}	{a, b}	{}	{c}	{}	{c}.Seg	{}
{a, b}	{a, b}	{c}	{}	{a, b}	{a}	{}	{a}
{a, c}	{c}	{b}	{a}	{a, c}	{}	{}	{}
{b, c}	{b, c}	{}	{a}	{a, b, c}	{a}	{b, c}	{}
{a, b, c}	{a, b, c}	{}	{}	{a, b, c}	{a}	{}	{a}

Closed-Open subsets of LS are = {{}, {c}, {a, b}, {a, b, c}}

The {1,2,3,4} array of CLOSURE elements is= ({a}, {a, b}, {c})

(2)

```

> print(` `); X:={a,b,c};LS2:={},{},{b},{c},{a,b},{b,c}; LS3:={LS2,X};print('LS3 = T0, particle-like topology, poset 2')
;LAT3abc(LS3,` Poset 2, T0 `^,3): print('This Lattice Structure appears to be a KURATOWSKI T0POLOGY \n with a limit point
operator that can be formulated interms of the exterior derivative, d,\n acting on exterior differential forms, A, F = dA, H = A^F.
Substitute {A,F,H} for the symbol set {a,b,c}. \n Then a => A, a 1-form, b => F=dA, a 2-form with dF=0, c => A^F = H, a 3-form.
\n Note that d(d(S)) => 0 but Bnd(Bnd(S)) =/=> 0. d is an up operator: given A, F is determined as dA`):

```

```

X:={a, b, c}
LS2:={ }, { }, {b}, { }, {c}, { }, { }, {a, b}, {b, c}
LS3:={{ }, {b}, {c}, {a, b}, {b, c}, {a, b, c}}
LS3 = T0, particle-like topology, poset 2
` `
` `

```

Is LS3 a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c}	{a}	{a}	{ }	{ }	{ }
{b}	{b}	{c}	{a}	{a, b}	{a}	{b}	{ }
{c}	{c}	{a, b}	{ }	{c}	{ }	{c}.Seg	{ }
{a, b}	{a, b}	{c}	{ }	{a, b}	{a}	{ }	{a}
{a, c}	{c}	{b}	{a}	{a, c}	{ }	{ }	{ }
{b, c}	{b, c}	{ }	{a}	{a, b, c}	{a}	{b, c}	{ }
{a, b, c}	{a, b, c}	{ }	{ }	{a, b, c}	{a}	{ }	{a}

Closed-Open subsets of LS are = { { }, {c}, {a, b}, {a, b, c} }

The {1,2,3,4} array of CLOSURE elements is = ({a}, {a, b}, {c})

This Lattice Structure appears to be a KURATOWSKI T0POLOGY

with a limit point operator that can be formulated interms of the exterior derivative, d, acting on exterior differential forms, A, F = dA, H = A^F. Substitute {A,F,H} for the symbol set {a,b,c}.

Then a => A, a 1-form, b => F=dA, a 2-form with dF=0, c => A^F = H, a 3-form.

Note that d(d(S)) => 0 but Bnd(Bnd(S)) =/=> 0. d is an up operator: given A, F is determined as dA

(3)

```
> print(` `);X:={a,b,c};LS2:={},{a},{c},{ },{ },{a,c}; LS3:={LS2,X};print('LS3 = T0, particle-like topology, poset 3`);
LAT3abc(LS3,` Poset 3, T0 `^,3):
```

```

X:={a, b, c}
LS2:={ }, { }, {a}, { }, {c}, { }, { }, { }, {a, c}
LS3:={{ }, {a}, {c}, {a, c}, {a, b, c}}
LS3 = T0, particle-like topology, poset 3
..
..

```

```
Is LS3 a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false
..
```

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c}	{b}	{a, b}	{b}	{a}	{ }
{b}	{ }	{a, c}	{b}	{b}	{ }	{ }	{ }
{c}	{c}	{a}	{b}	{b, c}	{b}	{c}	{ }
{a, b}	{a}	{c}	{b}	{a, b}	{b}	{ }	{b}
{a, c}	{a, c}	{ }	{b}	{a, b, c}	{b}	{a, c}	{ }
{b, c}	{c}	{a}	{b}	{b, c}	{b}	{ }	{b}
{a, b, c}	{a, b, c}	{ }	{ }	{a, b, c}	{b}	{ }	{b}

Closed-Open subsets of LS are = {{ }, {a, b, c}}

The {1,2,3,4} array of CLOSURE elements is= ({a, b}, {b}, {b, c})

(4)


```
> print(` `);X:={a,b,c};LS2:={},{a},{a,b},{ },{}; LS3:={LS2,X};print(`LS3 = T0, particle-like topology, poset 5`);LAT3abc(LS3,`
N=3 Poset 5, T2 `^,3):
```

```

X:={a, b, c}
LS2:={ }, {a}, {a, b}, { }, { }
LS3:={{ }, {a}, {a, b}, {a, b, c}}
LS3 = T0, particle-like topology, poset 5
..
..

```

```
Is LS3 a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false
..
```

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c}	{a, b, c}	{b, c}	{a}	{ }
{b}	{ }	{a}	{b, c}	{b, c}	{c}	{b}	{ }
{c}	{ }	{a, b}	{c}	{c}	{ }	{ }	{ }
{a, b}	{a, b}	{ }	{c}	{a, b, c}	{b, c}	{a}	{b}
{a, c}	{a}	{ }	{b, c}	{a, b, c}	{b, c}	{a}	{c}
{b, c}	{ }	{a}	{b, c}	{b, c}	{c}	{ }	{c}
{a, b, c}	{a, b, c}	{ }	{ }	{a, b, c}	{b, c}	{ }	{b, c}

Closed-Open subsets of LS are = {{ }, {a, b, c}}

The {1,2,3,4} array of CLOSURE elements is= ({a, b, c}, {b, c}, {c})

(6)

L

2. **Examples of the 4 finite $N=2$ (Not- T_0) wave-like topologies embedded into $N=3$, $\{\{\}, X\} = \{\{\}, \{a,b,c\}\}$
 $\text{Bnd}(X) = \{\}$**

Some or All Singleton closures are indistinguishable.

```
> print(` `);X:={a,b,c};LS2:={ };LS3:={LS2,X};print(`LS3 = Not-T0, wave-like topology 1`);LAT3abc(LS3,` wave-like 1`,3):
```

```

X:={a,b,c}
LS2:={ }
LS3:={{ }, {a,b,c}}
LS3 = Not-T0, wave-like topology 1
` `
` `

```

Is LS3 a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c}	{a, b, c}	{b, c}	{a}	{ }
{b}	{ }	{ }	{a, b, c}	{a, b, c}	{a, c}	{b}	{ }
{c}	{ }	{ }	{a, b, c}	{a, b, c}	{a, b}	{c}	{ }
{a, b}	{ }	{ }	{a, b, c}	{a, b, c}	{a, b, c}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c}	{a, b, c}	{a, b, c}	{ }	{a, c}
{b, c}	{ }	{ }	{a, b, c}	{a, b, c}	{a, b, c}	{ }	{b, c}
{a, b, c}	{a, b, c}	{ }	{ }	{a, b, c}	{a, b, c}	{ }	{a, b, c}

Closed-Open subsets of LS are = {{ }, {a, b, c}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c}, {a, b, c}, {a, b, c})

(7)

```
> print(` `);X:={a,b,c};LS2:={},{c},{}; LS3:={LS2,X};print(`LS3 = Not-T0, wave-like topology 2`);LAT3abc(LS3,` wave-like topology`,3):
```

```

X:={a, b, c}
LS2:={ }, {c}, { }
LS3:={{ }, {c}, {a, b, c}}
LS3 = Not-T0, wave-like topology 2
..
..

```

Is LS3 a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{c}	{a, b}	{a, b}	{b}	{a}	{ }
{b}	{ }	{c}	{a, b}	{a, b}	{a}	{b}	{ }
{c}	{c}	{ }	{a, b}	{a, b, c}	{a, b}	{c}	{ }
{a, b}	{ }	{c}	{a, b}	{a, b}	{a, b}	{ }	{a, b}
{a, c}	{c}	{ }	{a, b}	{a, b, c}	{a, b}	{c}	{a}
{b, c}	{c}	{ }	{a, b}	{a, b, c}	{a, b}	{c}	{b}
{a, b, c}	{a, b, c}	{ }	{ }	{a, b, c}	{a, b}	{ }	{a, b}

Closed-Open subsets of LS are = {{ }, {a, b, c}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a, b}, {a, b}, {a, b, c})

(8)

> print(` `);X:={a,b,c};LS2:={},{a,b},{}; LS3:={LS2,X};print('LS3 = Not-T0, wave-like topology 3');LAT3abc(LS3,` wave-like topology`,3):

$X := \{a, b, c\}$
 $LS2 := \{ \}, \{a, b\}, \{ \}$
 $LS3 := \{ \{ \}, \{a, b\}, \{a, b, c\} \}$
 $LS3 = \text{Not-T0, wave-like topology 3}$

Is LS3 a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{}	{a, b, c}	{a, b, c}	{b, c}	{a}	{}
{b}	{}	{}	{a, b, c}	{a, b, c}	{a, c}	{b}	{}
{c}	{}	{a, b}	{c}	{c}	{}	{}	{}
{a, b}	{a, b}	{}	{c}	{a, b, c}	{a, b, c}	{}	{a, b}
{a, c}	{}	{}	{a, b, c}	{a, b, c}	{b, c}	{a}	{c}
{b, c}	{}	{}	{a, b, c}	{a, b, c}	{a, c}	{b}	{c}
{a, b, c}	{a, b, c}	{}	{}	{a, b, c}	{a, b, c}	{}	{a, b, c}

Closed-Open subsets of LS are = { {}, {a, b, c} }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a, b, c}, {a, b, c}, {c})

(9)

**3. Examples of the 16 (T0) particle-like topologies
embedded into N=4 $\{\{ \}, X\} = \{\{ \}, \{a,b,c,d\}\}$
 $\text{Bnd}(X) = \{ \}$**

```
> print( ` `);X:={a,b,c,d}; LS3:={},{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}; LS4:={LS3,
X};print("LS4 is a T0 particle-like topology, poset 1 ` `");LAT4abcd(LS4,` ``,4);
```

$X := \{a, b, c, d\}$

$LS3 := \{ \{ \}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\} \}$

$LS4 := \{ \{ \}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\} \}$

LS4 is a T0 particle-like topology, poset 1

Is LS4: a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = true

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d}	{ }	{a}	{ }	{a}.Seg	{ }
{b}	{b}	{a, c, d}	{ }	{b}	{ }	{b}.Seg	{ }
{c}	{c}	{a, b, d}	{ }	{c}	{ }	{c}.Seg	{ }
{d}	{d}	{a, b, c}	{ }	{d}	{ }	{d}.Seg	{ }
{a, b}	{a, b}	{c, d}	{ }	{a, b}	{ }	{a, b}.Seg	{ }
{a, c}	{a, c}	{b, d}	{ }	{a, c}	{ }	{a, c}.Seg	{ }
{a, d}	{a, d}	{b, c}	{ }	{a, d}	{ }	{a, d}.Seg	{ }
{b, c}	{b, c}	{a, d}	{ }	{b, c}	{ }	{b, c}.Seg	{ }
{b, d}	{b, d}	{a, c}	{ }	{b, d}	{ }	{b, d}.Seg	{ }
{c, d}	{c, d}	{a, b}	{ }	{c, d}	{ }	{c, d}.Seg	{ }
{a, b, c}	{a, b, c}	{d}	{ }	{a, b, c}	{ }	{a, b, c}.Seg	{ }
{a, b, d}	{a, b, d}	{c}	{ }	{a, b, d}	{ }	{a, b, d}.Seg	{ }
{a, c, d}	{a, c, d}	{b}	{ }	{a, c, d}	{ }	{a, c, d}.Seg	{ }
{b, c, d}	{b, c, d}	{a}	{ }	{b, c, d}	{ }	{b, c, d}.Seg	{ }
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{ }	{a, b, c, d}.Seg	{ }

The Closed-Open subsets of LS are = { { }, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a}, {b}, {c}, {d})

(11)

```
> print( );X:={a,b,c,d}; LS3:={},{b},{c},{d},{a,b},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{b,c,d}; LS4:={LS3,X};print(LS4 = T0
particle-like topology, poset 2`);LAT4abcd(LS4,`Poset 2`^,4);
```

$X := \{a, b, c, d\}$

$LS3 := \{ \{ \}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}$

$LS4 := \{ \{ \}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, b, c, d\} \}$

$LS4 = T0$ particle-like topology, poset 2

Is LS4: a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a}	{a}	{}	{}	{}
{b}	{b}	{c, d}	{a}	{a, b}	{a}	{b}	{}
{c}	{c}	{a, b, d}	{}	{c}	{}	{c}.Seg	{}
{d}	{d}	{a, b, c}	{}	{d}	{}	{d}.Seg	{}
{a, b}	{a, b}	{c, d}	{}	{a, b}	{a}	{}	{a}
{a, c}	{c}	{b, d}	{a}	{a, c}	{}	{}	{}
{a, d}	{d}	{b, c}	{a}	{a, d}	{}	{}	{}
{b, c}	{b, c}	{d}	{a}	{a, b, c}	{a}	{b, c}	{}
{b, d}	{b, d}	{c}	{a}	{a, b, d}	{a}	{b, d}	{}
{c, d}	{c, d}	{a, b}	{}	{c, d}	{}	{c, d}.Seg	{}
{a, b, c}	{a, b, c}	{d}	{}	{a, b, c}	{a}	{}	{a}
{a, b, d}	{a, b, d}	{c}	{}	{a, b, d}	{a}	{}	{a}
{a, c, d}	{c, d}	{b}	{a}	{a, c, d}	{}	{}	{}
{b, c, d}	{b, c, d}	{}	{a}	{a, b, c, d}	{a}	{b, c, d}	{}
{a, b, c, d}	{a, b, c, d}	{}	{}	{a, b, c, d}	{a}	{}	{a}

The Closed-Open subsets of LS are = { { }, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d}, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a}, {a, b}, {c}, {d})

(12)

> print(' ');X:={a,b,c,d};LS3:={},{a},{c},{a,b},{a,c},{c,d},{a,b,c},{a,c,d};LS4:={LS3,X};print('LS4 = T0 particle-like topology, poset 3 ');LAT4abcd(LS4,LS = N,4);print('This Lattice Structure appears to be a KURATOWSKI T0POLOGY \n with a limit point operator that can be formulated in terms of the exterior derivative, d,\n acting on exterior differential forms, A, F = dA, H = A^F, K = F^F. Substitute {A,F,H,K} for the symbol set {a,b,c,d}. \n Then a => A, a 1-form, b => F=dA, a 2-form with dF=0, c => A^F = H, a 3-form, F^F, a 4-form. \n Note that d(d(S)) => 0 but Bnd(Bnd(S)) !=> 0. d is an up operator: given A, F is determined as dA');

X := { a, b, c, d }
 LS3 := { {}, { a }, { c }, { a, b }, { a, c }, { c, d }, { a, b, c }, { a, c, d }
 LS4 := { { }, { a }, { c }, { a, b }, { a, c }, { c, d }, { a, b, c }, { a, c, d }, { a, b, c, d } }
 LS4 = T0 particle-like topology, poset 3

Is LS4: a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ c, d }	{ b }	{ a, b }	{ b }	{ a }	{ }
{ b }	{ }	{ a, c, d }	{ b }	{ b }	{ }	{ }	{ }
{ c }	{ c }	{ a, b }	{ d }	{ c, d }	{ d }	{ c }	{ }
{ d }	{ }	{ a, b, c }	{ d }	{ d }	{ }	{ }	{ }
{ a, b }	{ a, b }	{ c, d }	{ }	{ a, b }	{ b }	{ }	{ b }
{ a, c }	{ a, c }	{ }	{ b, d }	{ a, b, c, d }	{ b, d }	{ a, c }	{ }
{ a, d }	{ a }	{ c }	{ b, d }	{ a, b, d }	{ b }	{ a, d }	{ }
{ b, c }	{ c }	{ a }	{ b, d }	{ b, c, d }	{ d }	{ b, c }	{ }
{ b, d }	{ }	{ a, c }	{ b, d }	{ b, d }	{ }	{ }	{ }
{ c, d }	{ c, d }	{ a, b }	{ }	{ c, d }	{ d }	{ }	{ d }
{ a, b, c }	{ a, b, c }	{ }	{ d }	{ a, b, c, d }	{ b, d }	{ a, c }	{ b }
{ a, b, d }	{ a, b }	{ c }	{ d }	{ a, b, d }	{ b }	{ }	{ b }
{ a, c, d }	{ a, c, d }	{ }	{ b }	{ a, b, c, d }	{ b, d }	{ a, c }	{ d }
{ b, c, d }	{ c, d }	{ a }	{ b }	{ b, c, d }	{ d }	{ }	{ d }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ b, d }	{ }	{ b, d }

The Closed-Open subsets of LS are = { { }, { a, b }, { c, d }, { a, b, c, d } }

The {1,2,3,4} array of CLOSURE elements is = ({ a, b }, { b }, { c, d }, { d })

This Lattice Structure appears to be a KURATOWSKI T0POLOGY

(13)

with a limit point operator that can be formulated in terms of the exterior derivative, d, acting on exterior differential forms, A, F = dA, H = A^F, K = F^F. Substitute {A,F,H,K} for the symbol set {a,b,c,d}.

Then a => A, a 1-form, b => F=dA, a 2-form with dF=0, c => A^F = H, a 3-form, F^F, a 4-form.

Note that d(d(S)) => 0 but Bnd(Bnd(S)) !=> 0. d is an up operator: given A, F is determined as dA

```
> print(' ');X:={a,b,c,d};LS3:={},{c},{d},{a,c},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d};LS4:={LS3,X};print('LS4 is a T0 particle-like topology, poset 4. ');LAT4abcd(LS4,'N',4);
```

$X := \{a, b, c, d\}$

$LS3 := \{ \{ \}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\} \}$

$LS4 := \{ \{ \}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\} \}$

LS4 is a T0 particle-like topology, poset 4.

Is LS4: a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a}	{a}	{ }	{ }	{ }
{b}	{ }	{a, c, d}	{b}	{b}	{ }	{ }	{ }
{c}	{c}	{d}	{a, b}	{a, b, c}	{a, b}	{c}	{ }
{d}	{d}	{a, b, c}	{ }	{d}	{ }	{d}.Seg	{ }
{a, b}	{ }	{c, d}	{a, b}	{a, b}	{ }	{ }	{ }
{a, c}	{a, c}	{d}	{b}	{a, b, c}	{a, b}	{c}	{a}
{a, d}	{d}	{b, c}	{a}	{a, d}	{ }	{ }	{ }
{b, c}	{b, c}	{d}	{a}	{a, b, c}	{a, b}	{c}	{b}
{b, d}	{d}	{a, c}	{b}	{b, d}	{ }	{ }	{ }
{c, d}	{c, d}	{ }	{a, b}	{a, b, c, d}	{a, b}	{c, d}	{ }
{a, b, c}	{a, b, c}	{d}	{ }	{a, b, c}	{a, b}	{ }	{a, b}
{a, b, d}	{d}	{c}	{a, b}	{a, b, d}	{ }	{ }	{ }
{a, c, d}	{a, c, d}	{ }	{b}	{a, b, c, d}	{a, b}	{c, d}	{a}
{b, c, d}	{b, c, d}	{ }	{a}	{a, b, c, d}	{a, b}	{c, d}	{b}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b}	{ }	{a, b}

The Closed-Open subsets of LS are = { { }, {d}, {a, b, c}, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a}, {b}, {a, b, c}, {d})

(14)

```
> print( ` `);X:={a,b,c,d};LS3:={},{a},{c},{d},{a,c},{a,d},{c,d},{a,b,c},{a,c,d};LS4:={LS3,X};print( LS4 is a T0 particle-like topology, poset 5 ` );LAT4abcd(LS4, ` N,4);
```

$X := \{a, b, c, d\}$

$LS3 := \{ \{ \}, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\} \}$

$LS4 := \{ \{ \}, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, c, d\} \}$

LS4 is a T0 particle-like topology, poset 5

Is LS4: a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c, d}	{b}	{a, b}	{b}	{a}	{ }
{b}	{ }	{a, c, d}	{b}	{b}	{ }	{ }	{ }
{c}	{c}	{a, d}	{b}	{b, c}	{b}	{c}	{ }
{d}	{d}	{a, b, c}	{ }	{d}	{ }	{d}.Seg	{ }
{a, b}	{a}	{c, d}	{b}	{a, b}	{b}	{ }	{b}
{a, c}	{a, c}	{d}	{b}	{a, b, c}	{b}	{a, c}	{ }
{a, d}	{a, d}	{c}	{b}	{a, b, d}	{b}	{a, d}	{ }
{b, c}	{c}	{a, d}	{b}	{b, c}	{b}	{ }	{b}
{b, d}	{d}	{a, c}	{b}	{b, d}	{ }	{ }	{ }
{c, d}	{c, d}	{a}	{b}	{b, c, d}	{b}	{c, d}	{ }
{a, b, c}	{a, b, c}	{d}	{ }	{a, b, c}	{b}	{ }	{b}
{a, b, d}	{a, d}	{c}	{b}	{a, b, d}	{b}	{ }	{b}
{a, c, d}	{a, c, d}	{ }	{b}	{a, b, c, d}	{b}	{a, c, d}	{ }
{b, c, d}	{c, d}	{a}	{b}	{b, c, d}	{b}	{ }	{b}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b}	{ }	{b}

The Closed-Open subsets of LS are = { { }, {d}, {a, b, c}, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a, b}, {b}, {b, c}, {d})

(15)

```
> print( );X:={a,b,c,d};LS3:={},{c},{d},{b,c},{c,d},{a,b,c},{b,c,d};LS4:={LS3,X};print( LS4 is a T0, particle-like topology,
poset 6 );LAT4abcd(LS4,`N`,4);
```

```

X:={a, b, c, d}
LS3:={ }, { }, {c}, {d}, {b, c}, {c, d}, {a, b, c}, {b, c, d}
LS4:={{ }, {c}, {d}, {b, c}, {c, d}, {a, b, c}, {b, c, d}, {a, b, c, d}}
LS4 is a T0, particle-like topology, poset 6

```

Is LS4: a topology = true, connected = false, Kolmogorov.T0= true, Hausdorff.T2= false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a}	{a}	{ }	{ }	{ }
{b}	{ }	{c, d}	{a, b}	{a, b}	{a}	{b}	{ }
{c}	{c}	{d}	{a, b}	{a, b, c}	{a, b}	{c}	{ }
{d}	{d}	{a, b, c}	{ }	{d}	{ }	{d}.Seg	{ }
{a, b}	{ }	{c, d}	{a, b}	{a, b}	{a}	{ }	{a}
{a, c}	{c}	{d}	{a, b}	{a, b, c}	{a, b}	{c}	{a}
{a, d}	{d}	{b, c}	{a}	{a, d}	{ }	{ }	{ }
{b, c}	{b, c}	{d}	{a}	{a, b, c}	{a, b}	{c}	{b}
{b, d}	{d}	{c}	{a, b}	{a, b, d}	{a}	{b, d}	{ }
{c, d}	{c, d}	{ }	{a, b}	{a, b, c, d}	{a, b}	{c, d}	{ }
{a, b, c}	{a, b, c}	{d}	{ }	{a, b, c}	{a, b}	{ }	{a, b}
{a, b, d}	{d}	{c}	{a, b}	{a, b, d}	{a}	{ }	{a}
{a, c, d}	{c, d}	{ }	{a, b}	{a, b, c, d}	{a, b}	{c, d}	{a}
{b, c, d}	{b, c, d}	{ }	{a}	{a, b, c, d}	{a, b}	{c, d}	{b}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b}	{ }	{a, b}

The Closed-Open subsets of LS are = {{ }, {d}, {a, b, c}, {a, b, c, d}}
The {1,2,3,4} array of CLOSURE elements is= ({a}, {a, b}, {a, b, c}, {d})

(16)

```
> print(` `);X:={a,b,c,d};LS3:={},{c},{d},{a,d},{c,d},{a,c,d},{b,c,d};LS4:={LS3,X};print(` LS4 is a T0, particle-like topology,
poset 7,`);LAT4abcd(LS,` N`,4);
```

```

X:={a, b, c, d}
LS3:={ {}, {c}, {d}, {a, d}, {c, d}, {a, c, d}, {b, c, d}
LS4:={{ {}, {c}, {d}, {a, d}, {c, d}, {a, c, d}, {b, c, d}, {a, b, c, d}}
LS4 is a T0, particle-like topology, poset 7,
` `
` `

```

Is LS4: a topology = true, connected = false, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a}	{a}	{}	{}	{}
{b}	{}	{c, d}	{a, b}	{a, b}	{a}	{b}	{}
{c}	{c}	{d}	{a, b}	{a, b, c}	{a, b}	{c}	{}
{d}	{d}	{a, b, c}	{}	{d}	{}	{d}.Seg	{}
{a, b}	{}	{c, d}	{a, b}	{a, b}	{a}	{}	{a}
{a, c}	{c}	{d}	{a, b}	{a, b, c}	{a, b}	{c}	{a}
{a, d}	{d}	{b, c}	{a}	{a, d}	{}	{}	{}
{b, c}	{b, c}	{d}	{a}	{a, b, c}	{a, b}	{c}	{b}
{b, d}	{d}	{c}	{a, b}	{a, b, d}	{a}	{b, d}	{}
{c, d}	{c, d}	{}	{a, b}	{a, b, c, d}	{a, b}	{c, d}	{}
{a, b, c}	{a, b, c}	{d}	{}	{a, b, c}	{a, b}	{}	{a, b}
{a, b, d}	{d}	{c}	{a, b}	{a, b, d}	{a}	{}	{a}
{a, c, d}	{c, d}	{}	{a, b}	{a, b, c, d}	{a, b}	{c, d}	{a}
{b, c, d}	{b, c, d}	{}	{a}	{a, b, c, d}	{a, b}	{c, d}	{b}
{a, b, c, d}	{a, b, c, d}	{}	{}	{a, b, c, d}	{a, b}	{}	{a, b}

The Closed-Open subsets of LS are = {{ {}, {d}, {a, b, c}, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is= ({a}, {a, b}, {a, b, c}, {d})

(17)

```
> print(' ');X:={a,b,c,d};LS3:={},{b},{d},{b,d},{a,b,d},{b,c,d};LS4:={LS3,X};print(' LS4 is a T0, particle-like topology,
poset 8. ');LAT4abcd(LS4, N,4);
```

```

X:={a, b, c, d}
LS3:={ }, { }, {b}, {d}, {b, d}, {a, b, d}, {b, c, d}
LS4:={{ }, {b}, {d}, {b, d}, {a, b, d}, {b, c, d}, {a, b, c, d}}
LS4 is a T0, particle-like topology, poset 8.

```

```
Is LS4: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false
```

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a}	{a}	{ }	{ }	{ }
{b}	{b}	{d}	{a, c}	{a, b, c}	{a, c}	{b}	{ }
{c}	{ }	{a, b, d}	{c}	{c}	{ }	{ }	{ }
{d}	{d}	{b}	{a, c}	{a, c, d}	{a, c}	{d}	{ }
{a, b}	{b}	{d}	{a, c}	{a, b, c}	{a, c}	{b}	{a}
{a, c}	{ }	{b, d}	{a, c}	{a, c}	{ }	{ }	{ }
{a, d}	{d}	{b}	{a, c}	{a, c, d}	{a, c}	{d}	{a}
{b, c}	{b}	{d}	{a, c}	{a, b, c}	{a, c}	{b}	{c}
{b, d}	{b, d}	{ }	{a, c}	{a, b, c, d}	{a, c}	{b, d}	{ }
{c, d}	{d}	{b}	{a, c}	{a, c, d}	{a, c}	{d}	{c}
{a, b, c}	{b}	{d}	{a, c}	{a, b, c}	{a, c}	{ }	{a, c}
{a, b, d}	{a, b, d}	{ }	{c}	{a, b, c, d}	{a, c}	{b, d}	{a}
{a, c, d}	{d}	{b}	{a, c}	{a, c, d}	{a, c}	{ }	{a, c}
{b, c, d}	{b, c, d}	{ }	{a}	{a, b, c, d}	{a, c}	{b, d}	{c}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, c}	{ }	{a, c}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a}, {a, b, c}, {c}, {a, c, d})

(18)

```
> print( ` `);X:={a,b,c,d};LS3:={},{a},{c},{},{a,c},{},{a,b,c};LS4:={LS3,X};print( LS4 is a T0, particle-like topology, poset 9. `)
;LAT4abcd(LS4, `N`,4);
```

```

X:={a, b, c, d}
LS3:={ {}, {a}, {c}, {}, {a, c}, {}, {a, b, c}
LS4:={ { }, {a}, {c}, {a, c}, {a, b, c}, {a, b, c, d} }
LS4 is a T0, particle-like topology, poset 9.
` `
` `

```

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c}	{b, d}	{a, b, d}	{b, d}	{a}	{ }
{b}	{ }	{a, c}	{b, d}	{b, d}	{d}	{b}	{ }
{c}	{c}	{a}	{b, d}	{b, c, d}	{b, d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a}	{c}	{b, d}	{a, b, d}	{b, d}	{a}	{b}
{a, c}	{a, c}	{ }	{b, d}	{a, b, c, d}	{b, d}	{a, c}	{ }
{a, d}	{a}	{c}	{b, d}	{a, b, d}	{b, d}	{a}	{d}
{b, c}	{c}	{a}	{b, d}	{b, c, d}	{b, d}	{c}	{b}
{b, d}	{ }	{a, c}	{b, d}	{b, d}	{d}	{ }	{d}
{c, d}	{c}	{a}	{b, d}	{b, c, d}	{b, d}	{c}	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{b, d}	{a, c}	{b}
{a, b, d}	{a}	{c}	{b, d}	{a, b, d}	{b, d}	{ }	{b, d}
{a, c, d}	{a, c}	{ }	{b, d}	{a, b, c, d}	{b, d}	{a, c}	{d}
{b, c, d}	{c}	{a}	{b, d}	{b, c, d}	{b, d}	{ }	{b, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, d}	{ }	{b, d}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a, b, d}, {b, d}, {b, c, d}, {d})

(19)

```
> print( `);X:={a,b,c,d};LS3:={},{a},{a,c},{a,b,c},{a,c,d};LS4:={LS3,X};print( LS4 is a T0, particle-like topology, poset 10 `);
LAT4abcd(LS4,N^,4);
```

```
X:={a, b, c, d}
LS3:={ {}, {a}, {a, c}, {a, b, c}, {a, c, d}
LS4:={{ {}, {a}, {a, c}, {a, b, c}, {a, c, d}, {a, b, c, d}}
LS4 is a T0, particle-like topology, poset 10
```

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{}	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{}
{b}	{}	{a, c, d}	{b}	{b}	{}	{}	{}
{c}	{}	{a}	{b, c, d}	{b, c, d}	{b, d}	{c}	{}
{d}	{}	{a, b, c}	{d}	{d}	{}	{}	{}
{a, b}	{a}	{}	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{a, c}	{}	{b, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{a}	{}	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{}	{a}	{b, c, d}	{b, c, d}	{b, d}	{c}	{b}
{b, d}	{}	{a, c}	{b, d}	{b, d}	{}	{}	{}
{c, d}	{}	{a}	{b, c, d}	{b, c, d}	{b, d}	{c}	{d}
{a, b, c}	{a, b, c}	{}	{d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a}	{}	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, d}
{a, c, d}	{a, c, d}	{}	{b}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{}	{a}	{b, c, d}	{b, c, d}	{b, d}	{}	{b, d}
{a, b, c, d}	{a, b, c, d}	{}	{}	{a, b, c, d}	{b, c, d}	{}	{b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

The {1,2,3,4} array of CLOSURE elements is= ({a, b, c, d}, {b}, {b, c, d}, {d})

(20)

```
> print(' ');X:={a,b,c,d};LS3:={},{d},{b,d},{c,d},{b,c,d};LS4:={LS3,X};print('LS4 is a T0, particle-like topology, poset 11. ');
LAT4abcd(LS4,N^,4);
```

$X := \{a, b, c, d\}$
 $LS3 := \{ \{ \}, \{d\}, \{b, d\}, \{c, d\}, \{b, c, d\} \}$
 $LS4 := \{ \{ \}, \{d\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{a, b, c, d\} \}$
LS4 is a T0, particle-like topology, poset 11.

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a}	{a}	{ }	{ }	{ }
{b}	{ }	{c, d}	{a, b}	{a, b}	{a}	{b}	{ }
{c}	{ }	{b, d}	{a, c}	{a, c}	{a}	{c}	{ }
{d}	{d}	{ }	{a, b, c}	{a, b, c, d}	{a, b, c}	{d}	{ }
{a, b}	{ }	{c, d}	{a, b}	{a, b}	{a}	{ }	{a}
{a, c}	{ }	{b, d}	{a, c}	{a, c}	{a}	{ }	{a}
{a, d}	{d}	{ }	{a, b, c}	{a, b, c, d}	{a, b, c}	{d}	{a}
{b, c}	{ }	{d}	{a, b, c}	{a, b, c}	{a}	{b, c}	{ }
{b, d}	{b, d}	{ }	{a, c}	{a, b, c, d}	{a, b, c}	{d}	{b}
{c, d}	{c, d}	{ }	{a, b}	{a, b, c, d}	{a, b, c}	{d}	{c}
{a, b, c}	{ }	{d}	{a, b, c}	{a, b, c}	{a}	{ }	{a}
{a, b, d}	{b, d}	{ }	{a, c}	{a, b, c, d}	{a, b, c}	{d}	{a, b}
{a, c, d}	{c, d}	{ }	{a, b}	{a, b, c, d}	{a, b, c}	{d}	{a, c}
{b, c, d}	{b, c, d}	{ }	{a}	{a, b, c, d}	{a, b, c}	{d}	{b, c}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c}	{ }	{a, b, c}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a}, {a, b}, {a, c}, {a, b, c, d})

(21)

```
> print( );X:={a,b,c,d};LS3:={},{b},{d},{},{a,b},{b,d},{},{a,b,d};LS4:={LS3,X};print( LS4 is a T0, particle-like topology,
poset 12.);LAT4abcd(LS4,N,4);
```

```

X:={a, b, c, d}
LS3:={ }, {b}, {d}, { }, {a, b}, {b, d}, { }, {a, b, d}
LS4:={{ }, {b}, {d}, {a, b}, {b, d}, {a, b, d}, {a, b, c, d}}
LS4 is a T0, particle-like topology, poset 12.

```

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, d}	{a, c}	{a, c}	{c}	{a}	{ }
{b}	{b}	{d}	{a, c}	{a, b, c}	{a, c}	{b}	{ }
{c}	{ }	{a, b, d}	{c}	{c}	{ }	{ }	{ }
{d}	{d}	{a, b}	{c}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{d}	{c}	{a, b, c}	{a, c}	{b}	{a}
{a, c}	{ }	{b, d}	{a, c}	{a, c}	{c}	{ }	{c}
{a, d}	{d}	{b}	{a, c}	{a, c, d}	{c}	{a, d}	{ }
{b, c}	{b}	{d}	{a, c}	{a, b, c}	{a, c}	{b}	{c}
{b, d}	{b, d}	{ }	{a, c}	{a, b, c, d}	{a, c}	{b, d}	{ }
{c, d}	{d}	{a, b}	{c}	{c, d}	{c}	{ }	{c}
{a, b, c}	{a, b}	{d}	{c}	{a, b, c}	{a, c}	{ }	{a, c}
{a, b, d}	{a, b, d}	{ }	{c}	{a, b, c, d}	{a, c}	{b, d}	{a}
{a, c, d}	{d}	{b}	{a, c}	{a, c, d}	{c}	{ }	{c}
{b, c, d}	{b, d}	{ }	{a, c}	{a, b, c, d}	{a, c}	{b, d}	{c}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, c}	{ }	{a, c}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a, c}, {a, b, c}, {c}, {c, d})

(22)

```
> print( ` `);X:={a,b,c,d};LS3:={},{d},{},{},{b,d},{c,d},{},{},{a,b,d},{b,c,d};LS4:={LS3,X};print( ` LS4 is a T0, particle-like topology, poset 13. `);LAT4abcd(LS4, ` N`,4);
```

```

X:={a, b, c, d}
LS3:={ {}, {d}, {}, {}, {b, d}, {c, d}, {}, {}, {a, b, d}, {b, c, d}
LS4:={{ }, {d}, {b, d}, {c, d}, {a, b, d}, {b, c, d}, {a, b, c, d}}
LS4 is a T0, particle-like topology, poset 13.

```

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a}	{a}	{}	{}	{}
{b}	{}	{c, d}	{a, b}	{a, b}	{a}	{b}	{}
{c}	{}	{a, b, d}	{c}	{c}	{}	{}	{}
{d}	{d}	{}	{a, b, c}	{a, b, c, d}	{a, b, c}	{d}	{}
{a, b}	{}	{c, d}	{a, b}	{a, b}	{a}	{}	{a}
{a, c}	{}	{b, d}	{a, c}	{a, c}	{}	{}	{}
{a, d}	{d}	{}	{a, b, c}	{a, b, c, d}	{a, b, c}	{d}	{a}
{b, c}	{}	{d}	{a, b, c}	{a, b, c}	{a}	{b, c}	{}
{b, d}	{b, d}	{}	{a, c}	{a, b, c, d}	{a, b, c}	{d}	{b}
{c, d}	{c, d}	{}	{a, b}	{a, b, c, d}	{a, b, c}	{d}	{c}
{a, b, c}	{}	{d}	{a, b, c}	{a, b, c}	{a}	{}	{a}
{a, b, d}	{a, b, d}	{}	{c}	{a, b, c, d}	{a, b, c}	{d}	{a, b}
{a, c, d}	{c, d}	{}	{a, b}	{a, b, c, d}	{a, b, c}	{d}	{a, c}
{b, c, d}	{b, c, d}	{}	{a}	{a, b, c, d}	{a, b, c}	{d}	{b, c}
{a, b, c, d}	{a, b, c, d}	{}	{}	{a, b, c, d}	{a, b, c}	{}	{a, b, c}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

The {1,2,3,4} array of CLOSURE elements is= ({a}, {a, b}, {c}, {a, b, c, d})

(23)

```
> print( `);X:={a,b,c,d};LS3:={},{},{d},{a,d},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d};LS4:={LS3,X};print( LS4 is a T0, particle-like
topology, poset 14 `);LAT4abcd(LS4, ` N ` ,4);
```

$X := \{a, b, c, d\}$

$LS3 := \{ \{ \}, \{ \}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\} \}$

$LS4 := \{ \{ \}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\} \}$

LS4 is a T0, particle-like topology, poset 14

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a}	{a}	{ }	{ }	{ }
{b}	{ }	{a, c, d}	{b}	{b}	{ }	{ }	{ }
{c}	{ }	{a, b, d}	{c}	{c}	{ }	{ }	{ }
{d}	{d}	{ }	{a, b, c}	{a, b, c, d}	{a, b, c}	{d}	{ }
{a, b}	{ }	{c, d}	{a, b}	{a, b}	{ }	{ }	{ }
{a, c}	{ }	{b, d}	{a, c}	{a, c}	{ }	{ }	{ }
{a, d}	{a, d}	{ }	{b, c}	{a, b, c, d}	{a, b, c}	{d}	{a}
{b, c}	{ }	{a, d}	{b, c}	{b, c}	{ }	{ }	{ }
{b, d}	{b, d}	{ }	{a, c}	{a, b, c, d}	{a, b, c}	{d}	{b}
{c, d}	{c, d}	{ }	{a, b}	{a, b, c, d}	{a, b, c}	{d}	{c}
{a, b, c}	{ }	{d}	{a, b, c}	{a, b, c}	{ }	{ }	{ }
{a, b, d}	{a, b, d}	{ }	{c}	{a, b, c, d}	{a, b, c}	{d}	{a, b}
{a, c, d}	{a, c, d}	{ }	{b}	{a, b, c, d}	{a, b, c}	{d}	{a, c}
{b, c, d}	{b, c, d}	{ }	{a}	{a, b, c, d}	{a, b, c}	{d}	{b, c}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c}	{ }	{a, b, c}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is= ({a}, {b}, {c}, {a, b, c, d})

(24)

```
> print(` `);X:={a,b,c,d};LS3:={},{b},{c},{d},{b,c},{b,d},{c,d},{b,c,d};LS4:={LS3,X};print(` LS4 is a T0, particle-like topology, poset 15. `);LAT4abcd(LS4,` N`,4);
```

```

X:={a, b, c, d}
LS3:={ {}, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
LS4:={{ {}, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}, {a, b, c, d}}
LS4 is a T0, particle-like topology, poset 15.

```

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a}	{a}	{ }	{ }	{ }
{b}	{b}	{c, d}	{a}	{a, b}	{a}	{b}	{ }
{c}	{c}	{b, d}	{a}	{a, c}	{a}	{c}	{ }
{d}	{d}	{b, c}	{a}	{a, d}	{a}	{d}	{ }
{a, b}	{b}	{c, d}	{a}	{a, b}	{a}	{ }	{a}
{a, c}	{c}	{b, d}	{a}	{a, c}	{a}	{ }	{a}
{a, d}	{d}	{b, c}	{a}	{a, d}	{a}	{ }	{a}
{b, c}	{b, c}	{d}	{a}	{a, b, c}	{a}	{b, c}	{ }
{b, d}	{b, d}	{c}	{a}	{a, b, d}	{a}	{b, d}	{ }
{c, d}	{c, d}	{b}	{a}	{a, c, d}	{a}	{c, d}	{ }
{a, b, c}	{b, c}	{d}	{a}	{a, b, c}	{a}	{ }	{a}
{a, b, d}	{b, d}	{c}	{a}	{a, b, d}	{a}	{ }	{a}
{a, c, d}	{c, d}	{b}	{a}	{a, c, d}	{a}	{ }	{a}
{b, c, d}	{b, c, d}	{ }	{a}	{a, b, c, d}	{a}	{b, c, d}	{ }
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a}	{ }	{a}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }
The {1,2,3,4} array of CLOSURE elements is= ({a}, {a, b}, {a, c}, {a, d})

(25)

```
> print( );X:={a,b,c,d};LS3:={},{},{d},{c,d},{b,c,d};LS4:={LS3,X};print( LS4 = T0, particle-like topology, poset 16.);
LAT4abcd(LS4, N,4);
```

```

X:={a, b, c, d}
LS3:={ , { }, { }, {d}, {c, d}, {b, c, d}
LS4:={{ }, {d}, {c, d}, {b, c, d}, {a, b, c, d}}
LS4 = T0, particle-like topology, poset 16.

```

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a}	{a}	{ }	{ }	{ }
{b}	{ }	{c, d}	{a, b}	{a, b}	{a}	{b}	{ }
{c}	{ }	{d}	{a, b, c}	{a, b, c}	{a, b}	{c}	{ }
{d}	{d}	{ }	{a, b, c}	{a, b, c, d}	{a, b, c}	{d}	{ }
{a, b}	{ }	{c, d}	{a, b}	{a, b}	{a}	{ }	{a}
{a, c}	{ }	{d}	{a, b, c}	{a, b, c}	{a, b}	{c}	{a}
{a, d}	{d}	{ }	{a, b, c}	{a, b, c, d}	{a, b, c}	{d}	{a}
{b, c}	{ }	{d}	{a, b, c}	{a, b, c}	{a, b}	{c}	{b}
{b, d}	{d}	{ }	{a, b, c}	{a, b, c, d}	{a, b, c}	{d}	{b}
{c, d}	{c, d}	{ }	{a, b}	{a, b, c, d}	{a, b, c}	{d}	{c}
{a, b, c}	{ }	{d}	{a, b, c}	{a, b, c}	{a, b}	{ }	{a, b}
{a, b, d}	{d}	{ }	{a, b, c}	{a, b, c, d}	{a, b, c}	{d}	{a, b}
{a, c, d}	{c, d}	{ }	{a, b}	{a, b, c, d}	{a, b, c}	{d}	{a, c}
{b, c, d}	{b, c, d}	{ }	{a}	{a, b, c, d}	{a, b, c}	{d}	{b, c}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c}	{ }	{a, b, c}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a}, {a, b}, {a, b, c}, {a, b, c, d})

(26)

4. **Examples of the 17 Not-T0 wave-like topologies,
embedded in $\{\{\}, X\} = \{\{\}, \{a,b,c,d\}\}$.
 $X = \{a,b,c,d\}$**

**Some or All Singleton closures are not distinguishable.
Some indistinguishable subsets and their boundaries can be dense.**

> print(`);X:={a,b,c,d};LS3:={};LS4:={LS3,X};print(LS4 is a Not-T0, wave-like Indiscrete topology. All subsets are indistinguishable. Ex 1 `); LAT4abcd(LS4,`Not T0`,4);

$X := \{a, b, c, d\}$

$LS3 := \{ \}$

$LS4 := \{ \{ \}, \{a, b, c, d\} \}$

LS4 is a Not-T0, wave-like Indiscrete topology. All subsets are indistinguishable. Ex 1

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{}
{b}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{}
{c}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, b, d}	{c}	{}
{d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, b, c}	{d}	{}
{a, b}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{}	{a, b}
{a, c}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{}	{a, c}
{a, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{}	{a, d}
{b, c}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{}	{b, c}
{b, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{}	{b, d}
{c, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{}	{c, d}
{a, b, c}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{}	{a, b, c}
{a, b, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{}	{a, b, d}
{a, c, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{}	{a, c, d}
{b, c, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{}	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{}	{a, b, c, d}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {a, b, c, d}, {a, b, c, d}, {a, b, c, d})

(27)

```
> print(` `);X:={a,b,c,d};LS3:={},{a,b,c};LS4:={LS3,X};print(`LS4 is a Not-T0, wave-like topology. Ex 2 `);LAT4abcd(LS4,
`NOT-T0`,4);
```

$X := \{a, b, c, d\}$
 $LS3 := \{ \}, \{a, b, c\}$
 $LS4 := \{ \}, \{a, b, c\}, \{a, b, c, d\}$
LS4 is a Not-T0, wave-like topology. Ex 2

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{ }
{c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, c}
{a, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{b, c}
{b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{d}
{c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, d}	{c}	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c}
{a, b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, d}
{a, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, c, d}
{b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c, d}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a, b, c, d}, {a, b, c, d}, {a, b, c, d}, {d})

(28)

```
> print(' ');X:={a,b,c,d};LS3:={},{a,b}; LS4:={LS3,X};print(' LS4 is a Not-T0, wave-like topology Ex 3 ');LAT4abcd
(LS4, NOT-T0,4);
```

```

X:={a, b, c, d}
LS3:={ {}, {a, b}
LS4:={ {}, {a, b}, {a, b, c, d} }
LS4 is a Not-T0, wave-like topology Ex 3

```

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{}
{b}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{}
{c}	{}	{a, b}	{c, d}	{c, d}	{d}	{c}	{}
{d}	{}	{a, b}	{c, d}	{c, d}	{c}	{d}	{}
{a, b}	{a, b}	{}	{c, d}	{a, b, c, d}	{a, b, c, d}	{}	{a, b}
{a, c}	{}	{}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c}
{b, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{d}
{c, d}	{}	{a, b}	{c, d}	{c, d}	{c, d}	{}	{c, d}
{a, b, c}	{a, b}	{}	{c, d}	{a, b, c, d}	{a, b, c, d}	{}	{a, b, c}
{a, b, d}	{a, b}	{}	{c, d}	{a, b, c, d}	{a, b, c, d}	{}	{a, b, d}
{a, c, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c, d}
{a, b, c, d}	{a, b, c, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{}	{a, b, c, d}

The Closed-Open subsets of LS are = { {}, {a, b, c, d} }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a, b, c, d}, {a, b, c, d}, {c, d}, {c, d})

(29)

```
> print(' ');X:={a,b,c,d};LS3:={},{a};LS4:={LS3,X};print('LS4 is a Not-T0, wave-like topology Ex 4');LAT4abcd(LS4,`
NOT T0`,`4):
```

```

X:={a, b, c, d}
LS3:={ }, {a}
LS4:={{ }, {a}, {a, b, c, d}}
LS4 is a Not-T0, wave-like topology Ex 4
`
`
```

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{ }
{c}	{ }	{a}	{b, c, d}	{b, c, d}	{b, d}	{c}	{ }
{d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c}	{d}	{ }
{a, b}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, c}
{b, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, d}
{c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{c, d}
{a, b, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, d}
{a, c, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, c, d}	{ }	{b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {b, c, d}, {b, c, d}, {b, c, d})

(30)

```
> print( );X:={a,b,c,d};LS3:={},{a},{a,b};LS4:={LS3,X};print( LS4 is a Not-T0, wave-like topology, Ex 5`);LAT4abcd
(LS4, NOT T0,4);
```

$X := \{a, b, c, d\}$
 $LS3 := \{ \{ \}, \{a\}, \{a, b\} \}$
 $LS4 := \{ \{ \}, \{a\}, \{a, b\}, \{a, b, c, d\} \}$
LS4 is a Not-T0, wave-like topology, Ex 5

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{c}
{b, d}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{d}
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{c, d}	{ }	{c, d}
{a, b, c}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, d}
{a, c, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{ }	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, c, d}	{ }	{b, c, d}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a, b, c, d}, {b, c, d}, {c, d}, {c, d})

(31)

> print();X:={a,b,c,d};LS3:={},{a},{a,b,c};LS4:={LS3,X};print(LS4 is a Not-T0, wave-like topology. Ex 6);LAT4abcd (LS4,NOT-T0,4):

$X := \{a, b, c, d\}$
 $LS3 := \{ \{ \}, \{a\}, \{ \}, \{a, b, c\} \}$
 $LS4 := \{ \{ \}, \{a\}, \{a, b, c\}, \{a, b, c, d\} \}$
 LS4 is a Not-T0, wave-like topology. Ex 6

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{ }
{c}	{ }	{a}	{b, c, d}	{b, c, d}	{b, d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, c}
{b, d}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{d}
{c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, d}	{c}	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, d}
{a, c, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, c, d}	{ }	{b, c, d}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a, b, c, d}, {b, c, d}, {b, c, d}, {d})

(32)

```
> print(' ');X:={a,b,c,d};LS3:={},{a,b},{a,b,c};LS4:={LS3,X};print(' LS4 is a Not-T0, wave-like topology, Ex 7 ');LAT4abcd
(LS4,NOT-T0,4):
```

```

X:={a, b, c, d}
LS3:={ }, {a, b}, {a, b, c}
LS4:={{ }, {a, b}, {a, b, c}, {a, b, c, d}}
LS4 is a Not-T0, wave-like topology, Ex 7
..

```

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b, c}	{d}	{d}	{ }	{ }	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c}
{b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{d}
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{d}	{ }	{d}
{a, b, c}	{a, b, c}	{ }	{d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, d}
{a, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c, d}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a, b, c, d}, {a, b, c, d}, {c, d}, {d})

(33)

```
> print( ` `);X:={a,b,c,d};LS3:={},{a},{b},{a,b};LS4:={LS3,X};print( `LS4 is a Not-T0, wave-like topology, Ex 8 `);LAT4abcd
(LS4, NOT-T0,4):
```

```

X:={a, b, c, d}
LS3:={ }, {a}, {b}, {a, b}
LS4:={{ }, {a}, {b}, {a, b}, {a, b, c, d}}
LS4 is a Not-T0, wave-like topology, Ex 8
` `

```

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{ }
{b}	{b}	{a}	{c, d}	{b, c, d}	{c, d}	{b}	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{ }
{a, c}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{c}
{a, d}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{d}
{b, c}	{b}	{a}	{c, d}	{b, c, d}	{c, d}	{b}	{c}
{b, d}	{b}	{a}	{c, d}	{b, c, d}	{c, d}	{b}	{d}
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{c, d}	{ }	{c, d}
{a, b, c}	{a, b}	{ }	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{c}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{d}
{a, c, d}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{ }	{c, d}
{b, c, d}	{b}	{a}	{c, d}	{b, c, d}	{c, d}	{ }	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{c, d}	{ }	{c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a, c, d}, {b, c, d}, {c, d}, {c, d})

(34)

```
> print( ` `);X:={a,b,c,d};LS3:={},{a},{a,b},{a,c,d},{ };LS4:={LS3,X};print( `LS4 is a Not-T0, wave-like topology Ex 9 `);
LAT4abcd(LS4, `NOT-T0`,4):
```

```

X:={a, b, c, d}
LS3:={ {}, {a}, {a, b}, {a, c, d}, { }
LS4:={{ }, {a}, {a, b}, {a, c, d}, {a, b, c, d}}
LS4 is a Not-T0, wave-like topology Ex 9

```

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{a, c, d}	{b}	{b}	{ }	{ }	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{a}	{b, c, d}	{b, c, d}	{d}	{b, c}	{ }
{b, d}	{ }	{a}	{b, c, d}	{b, c, d}	{c}	{b, d}	{ }
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{c, d}	{ }	{c, d}
{a, b, c}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, d}
{a, c, d}	{a, c, d}	{ }	{b}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{ }	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, c, d}	{ }	{b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a, b, c, d}, {b}, {c, d}, {c, d})

(35)

```
> print( `);X:={a,b,c,d};LS3:={},{a},{b,c},{a,b,c};LS4:={LS3,X};print( LS4 is a Not-T0, wave-like topology Ex 10 `);
LAT4abcd(LS4, NOT-T0,4):
```

```

X:={a, b, c, d}
LS3:={ {}, {a}, {b, c}, {a, b, c}
LS4:={{ }, {a}, {b, c}, {a, b, c}, {a, b, c, d}}
LS4 is a Not-T0, wave-like topology Ex 10

```

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c}	{d}	{a, d}	{d}	{a}	{}
{b}	{}	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{}
{c}	{}	{a}	{b, c, d}	{b, c, d}	{b, d}	{c}	{}
{d}	{}	{a, b, c}	{d}	{d}	{}	{}	{}
{a, b}	{a}	{}	{b, c, d}	{a, b, c, d}	{c, d}	{a, b}	{}
{a, c}	{a}	{}	{b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{}
{a, d}	{a}	{b, c}	{d}	{a, d}	{d}	{}	{d}
{b, c}	{b, c}	{a}	{d}	{b, c, d}	{b, c, d}	{}	{b, c}
{b, d}	{}	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{d}
{c, d}	{}	{a}	{b, c, d}	{b, c, d}	{b, d}	{c}	{d}
{a, b, c}	{a, b, c}	{}	{d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a}	{}	{b, c, d}	{a, b, c, d}	{c, d}	{a, b}	{d}
{a, c, d}	{a}	{}	{b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{d}
{b, c, d}	{b, c}	{a}	{d}	{b, c, d}	{b, c, d}	{}	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{}	{}	{a, b, c, d}	{b, c, d}	{}	{b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a, d}, {b, c, d}, {b, c, d}, {d})

(36)

```
> print( ` `);X:={a,b,c,d};LS3:={},{a,b},{a,b,d},{a,b,c};LS4:={LS3,X};print( LS4 is a Not-T0, wave-like topology Ex 11 `);
LAT4abcd(LS4,`NOT-T0`,4):
```

```

X:={a, b, c, d}
LS3:={ {}, {a, b}, {a, b, d}, {a, b, c}
LS4:={ { }, {a, b}, {a, b, c}, {a, b, d}, {a, b, c, d} }
LS4 is a Not-T0, wave-like topology Ex 11
` `
` `
```

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{}
{b}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{}
{c}	{}	{a, b, d}	{c}	{c}	{}	{}	{}
{d}	{}	{a, b, c}	{d}	{d}	{}	{}	{}
{a, b}	{a, b}	{}	{c, d}	{a, b, c, d}	{a, b, c, d}	{}	{a, b}
{a, c}	{}	{}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c}
{b, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{d}
{c, d}	{}	{a, b}	{c, d}	{c, d}	{}	{}	{}
{a, b, c}	{a, b, c}	{}	{d}	{a, b, c, d}	{a, b, c, d}	{}	{a, b, c}
{a, b, d}	{a, b, d}	{}	{c}	{a, b, c, d}	{a, b, c, d}	{}	{a, b, d}
{a, c, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c, d}
{a, b, c, d}	{a, b, c, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{}	{a, b, c, d}

The Closed-Open subsets of LS are = { {}, {a, b, c, d} }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a, b, c, d}, {a, b, c, d}, {c}, {d})

(37)

```
> print( `);X:={a,b,c,d};LS3:={},{a},{}, {b,c,d};LS4:={LS3,X};print( LS4 is a Not-T0, wave-like topology Ex 12 `);
LAT4abcd(LS4,NOT-T0,4):
```

```

X:={a, b, c, d}
LS3:={ }, {a}, { }, {b, c, d}
LS4:={ { }, {a}, {b, c, d}, {a, b, c, d} }
LS4 is a Not-T0, wave-like topology Ex 12
`
`

```

Is LS4: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d}	{ }	{a}	{ }	{a}.Seg	{ }
{b}	{ }	{a}	{b, c, d}	{b, c, d}	{c, d}	{b}	{ }
{c}	{ }	{a}	{b, c, d}	{b, c, d}	{b, d}	{c}	{ }
{d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c}	{d}	{ }
{a, b}	{a}	{ }	{b, c, d}	{a, b, c, d}	{c, d}	{a, b}	{ }
{a, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{ }
{a, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{ }
{b, c}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, c}
{b, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, d}
{c, d}	{ }	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{c, d}
{a, b, c}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, d}
{a, c, d}	{a}	{ }	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{b, c, d}	{a}	{ }	{b, c, d}	{b, c, d}	{ }	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{b, c, d}	{ }	{b, c, d}

The Closed-Open subsets of LS are = ({ }, {a}, {b, c, d}, {a, b, c, d})

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a}, {b, c, d}, {b, c, d}, {b, c, d})

(38)

```
> print( `);X:={a,b,c,d};LS3:={},{a},{b},{a,b},{a,c,d};LS4:={LS3,X};print( LS4 is a Not-T0, wave-like topology Ex 13 `);
LAT4abcd(LS4,`NOT-T0`,4):
```

```

X:={a, b, c, d}
LS3:={ , {a}, {b}, {a, b}, {a, c, d}}
LS4:={{ }, {a}, {b}, {a, b}, {a, c, d}, {a, b, c, d}}
LS4 is a Not-T0, wave-like topology Ex 13

```

Is LS4: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{ }
{b}	{b}	{a, c, d}	{ }	{b}	{ }	{b}.Seg	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{ }
{a, c}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{c}
{a, d}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{d}
{b, c}	{b}	{a}	{c, d}	{b, c, d}	{d}	{b, c}	{ }
{b, d}	{b}	{a}	{c, d}	{b, c, d}	{c}	{b, d}	{ }
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{c, d}	{ }	{c, d}
{a, b, c}	{a, b}	{ }	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{c}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{d}
{a, c, d}	{a, c, d}	{b}	{ }	{a, c, d}	{c, d}	{ }	{c, d}
{b, c, d}	{b}	{a}	{c, d}	{b, c, d}	{c, d}	{ }	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{c, d}	{ }	{c, d}

The Closed-Open subsets of LS are = {{ }, {b}, {a, c, d}, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a, c, d}, {b}, {c, d}, {c, d})

(39)

```
> print(' ');X:={a,b,c,d};LS3={},{a},{b,c},{a,b,c},{b,c,d};LS4:={LS3,X};print(' LS4 is a Not-T0, wave-like topology Ex 14 ');
LAT4abcd(LS4,NOT-T0,4):
```

```

X:={a, b, c, d}
({{ }}, {{a}}, {{b}}, {{a, b}}, {{a, c, d}}) = {{ }}, {{a}}, {{b, c}}, {{a, b, c}}, {{b, c, d}}
LS4:={{ }, {a}, {b}, {a, b}, {a, c, d}, {a, b, c, d}}
LS4 is a Not-T0, wave-like topology Ex 14

```

Is LS4: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{ }
{b}	{b}	{a, c, d}	{ }	{b}	{ }	{b}.Seg	{ }
{c}	{ }	{a, b}	{c, d}	{c, d}	{d}	{c}	{ }
{d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{ }
{a, c}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{c}
{a, d}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{d}
{b, c}	{b}	{a}	{c, d}	{b, c, d}	{d}	{b, c}	{ }
{b, d}	{b}	{a}	{c, d}	{b, c, d}	{c}	{b, d}	{ }
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{c, d}	{ }	{c, d}
{a, b, c}	{a, b}	{ }	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{c}
{a, b, d}	{a, b}	{ }	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{d}
{a, c, d}	{a, c, d}	{b}	{ }	{a, c, d}	{c, d}	{ }	{c, d}
{b, c, d}	{b}	{a}	{c, d}	{b, c, d}	{c, d}	{ }	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{c, d}	{ }	{c, d}

The Closed-Open subsets of LS are = {{ }, {b}, {a, c, d}, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, c, d}, {b}, {c, d}, {c, d})

(40)

```
> print(' ');X:={a,b,c,d};LS3:={},{a,b},{c,d};LS4:={LS3,X};print(' LS4 is a Not-T0, wave-like topology Ex 15 ');LAT4abcd
(LS4,NOT-T0,4):
```

```

X:={a, b, c, d}
LS3:={ {}, {a, b}, {c, d}
LS4:={{ {}, {a, b}, {c, d}, {a, b, c, d} }
LS4 is a Not-T0, wave-like topology Ex 15

```

Is LS4: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{c, d}	{a, b}	{a, b}	{b}	{a}	{}
{b}	{}	{c, d}	{a, b}	{a, b}	{a}	{b}	{}
{c}	{}	{a, b}	{c, d}	{c, d}	{d}	{c}	{}
{d}	{}	{a, b}	{c, d}	{c, d}	{c}	{d}	{}
{a, b}	{a, b}	{c, d}	{}	{a, b}	{a, b}	{}	{a, b}
{a, c}	{}	{}	{a, b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{}
{a, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{}
{b, c}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, d}	{b, c}	{}
{b, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{a, c}	{b, d}	{}
{c, d}	{c, d}	{a, b}	{}	{c, d}	{c, d}	{}	{c, d}
{a, b, c}	{a, b}	{}	{c, d}	{a, b, c, d}	{a, b, d}	{c}	{a, b}
{a, b, d}	{a, b}	{}	{c, d}	{a, b, c, d}	{a, b, c}	{d}	{a, b}
{a, c, d}	{c, d}	{}	{a, b}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{c, d}	{}	{a, b}	{a, b, c, d}	{a, c, d}	{b}	{c, d}
{a, b, c, d}	{a, b, c, d}	{}	{}	{a, b, c, d}	{a, b, c, d}	{}	{a, b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b}, {c, d}, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, b}, {a, b}, {c, d}, {c, d})

(41)

```
> print(' ');X:={a,b,c,d};LS3:={},{a},{b},{a,b},{a,c,d};LS4:={LS3,X};print(' LS4 is a Not T0, wave-like topology Ex 16 ');
LAT4abcd(LS4,NOT-T0,4):
```

```

X:={a, b, c, d}
LS3:={ {}, {a}, {b}, {a, b}, {a, c, d}
LS4:={ { }, {a}, {b}, {a, b}, {a, c, d}, {a, b, c, d} }
LS4 is a Not T0, wave-like topology Ex 16

```

Is LS4: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{}
{b}	{b}	{a, c, d}	{}	{b}	{}	{b}.Seg	{}
{c}	{}	{a, b}	{c, d}	{c, d}	{d}	{c}	{}
{d}	{}	{a, b}	{c, d}	{c, d}	{c}	{d}	{}
{a, b}	{a, b}	{}	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{}
{a, c}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{c}
{a, d}	{a}	{b}	{c, d}	{a, c, d}	{c, d}	{a}	{d}
{b, c}	{b}	{a}	{c, d}	{b, c, d}	{d}	{b, c}	{}
{b, d}	{b}	{a}	{c, d}	{b, c, d}	{c}	{b, d}	{}
{c, d}	{}	{a, b}	{c, d}	{c, d}	{c, d}	{}	{c, d}
{a, b, c}	{a, b}	{}	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{c}
{a, b, d}	{a, b}	{}	{c, d}	{a, b, c, d}	{c, d}	{a, b}	{d}
{a, c, d}	{a, c, d}	{b}	{}	{a, c, d}	{c, d}	{}	{c, d}
{b, c, d}	{b}	{a}	{c, d}	{b, c, d}	{c, d}	{}	{c, d}
{a, b, c, d}	{a, b, c, d}	{}	{}	{a, b, c, d}	{c, d}	{}	{c, d}

The Closed-Open subsets of LS are = { {}, {b}, {a, c, d}, {a, b, c, d} }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({a, c, d}, {b}, {c, d}, {c, d})

(42)

```
> print(' ');X:={a,b,c,d};LS3:={},{a},{b},{a,b},{c,d},{a,c,d},{b,c,d};LS4:={LS3,X};print(' LS4 = Not-T0, wave-like topology
Ex 17 ');LAT4abcd(LS4,'NOT-T0',4):
```

```

X:={a, b, c, d}
LS3:={ {}, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}
LS4:={ {}, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}, {a, b, c, d} }
LS4 = Not-T0, wave-like topology Ex 17
```

Is LS4: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d}	{}	{a}	{}	{a}.Seg	{}
{b}	{b}	{a, c, d}	{}	{b}	{}	{b}.Seg	{}
{c}	{}	{a, b}	{c, d}	{c, d}	{d}	{c}	{}
{d}	{}	{a, b}	{c, d}	{c, d}	{c}	{d}	{}
{a, b}	{a, b}	{c, d}	{}	{a, b}	{}	{a, b}.Seg	{}
{a, c}	{a}	{b}	{c, d}	{a, c, d}	{d}	{a, c}	{}
{a, d}	{a}	{b}	{c, d}	{a, c, d}	{c}	{a, d}	{}
{b, c}	{b}	{a}	{c, d}	{b, c, d}	{d}	{b, c}	{}
{b, d}	{b}	{a}	{c, d}	{b, c, d}	{c}	{b, d}	{}
{c, d}	{c, d}	{a, b}	{}	{c, d}	{c, d}	{}	{c, d}
{a, b, c}	{a, b}	{}	{c, d}	{a, b, c, d}	{d}	{a, b, c}	{}
{a, b, d}	{a, b}	{}	{c, d}	{a, b, c, d}	{c}	{a, b, d}	{}
{a, c, d}	{a, c, d}	{b}	{}	{a, c, d}	{c, d}	{}	{c, d}
{b, c, d}	{b, c, d}	{a}	{}	{b, c, d}	{c, d}	{}	{c, d}
{a, b, c, d}	{a, b, c, d}	{}	{}	{a, b, c, d}	{c, d}	{}	{c, d}

The Closed-Open subsets of LS are = { {}, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}, {a, b, c, d} }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a}, {b}, {c, d}, {c, d})

(43)

**Special cases of the 139 different
N=5 finite topologies.**

5. Examples of Embedding $N=3$, T_0 , particle-like topologies into $N=5$, to produce $N=5$, NOT T_0 , wave-like, topologies.

```
> print(` `);X:={a,b,c,d,s};LS3:={},{},{b},{c},{},{b,c},{a,b,c};LS5:={LS3,X};print(`LS5 is a (Not-T0) wave-like topology `);
LAT5abcds(LS5, Embed N=3 to N =,5);
```

```

X:={ a, b, c, d, s}
LS3:={ {}, { }, {b}, {c}, { }, { }, {b, c}, {a, b, c}
LS5:={{ }, {b}, {c}, {b, c}, {a, b, c}, {a, b, c, d, s}}
LS5 is a (Not-T0) wave-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{ }
{b}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{ }
{c}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a}
{a, c}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a}
{a, d}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d}
{a, s}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{s}
{b, c}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{ }
{b, d}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d}
{b, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{s}
{c, d}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{d}
{c, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a}
{a, b, d}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d}
{a, b, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, s}
{a, c, d}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, d}
{a, c, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, s}
{a, d, s}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d, s}
{b, c, d}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d}
{b, c, s}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{s}
{b, d, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d, s}
{c, d, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, s}
{a, b, d, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d, s}
{a, c, d, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, d, s}
{b, c, d, s}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS3:={},{b},{c},{b,c},{a,b,c};LS5:={LS3,X};print(`LS5 is a (Not-T0) wave-like topology` );
LAT5abcds(LS5, Embed N=3 to N =,5);
```

```

X:={ a, b, c, d, s}
LS3:={ }, { }, { b }, { c }, { b, c }, { a, b, c }
LS5:={{ }, { b }, { c }, { b, c }, { a, b, c }, { a, b, c, d, s }}
LS5 is a (Not-T0) wave-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{ }
{b}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{ }
{c}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a}
{a, c}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a}
{a, d}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d}
{a, s}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{s}
{b, c}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{ }
{b, d}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d}
{b, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{s}
{c, d}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{d}
{c, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a}
{a, b, d}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d}
{a, b, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, s}
{a, c, d}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, d}
{a, c, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, s}
{a, d, s}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d, s}
{b, c, d}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d}
{b, c, s}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{s}
{b, d, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d, s}
{c, d, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, s}
{a, b, d, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d, s}
{a, c, d, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, d, s}
{b, c, d, s}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(`);X:={a,b,c,d,s};LS3:={},{b},{a,b},{b,c},{a,b,c};LS5:={LS3,X};print('LS5 is a (Not-T0) wave-like topology `)
:LAT5abcds(LS5, Embed N=3 to N=`5):
```

```

X:={ a, b, c, d, s}
LS3:={ {}, {b}, {a, b}, {b, c}, {a, b, c}
LS5:={ {}, {b}, {a, b}, {b, c}, {a, b, c}, {a, b, c, d, s}
LS5 is a (Not-T0) wave-like topology
```

```
Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false
```

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{}
{b}	{b}	{}	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{}
{c}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{}
{a, b}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a}
{a, c}	{}	{b}	{a, c, d, s}	{a, c, d, s}	{d, s}	{a, c}	{}
{a, d}	{}	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d}
{a, s}	{}	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{s}
{b, c}	{b, c}	{}	{a, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c}
{b, d}	{b}	{}	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{b}	{}	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{d, s}	{}	{d, s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c}
{a, b, d}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, d}
{a, b, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, s}
{a, c, d}	{}	{b}	{a, c, d, s}	{a, c, d, s}	{d, s}	{a, c}	{d}
{a, c, s}	{}	{b}	{a, c, d, s}	{a, c, d, s}	{d, s}	{a, c}	{s}
{a, d, s}	{}	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d, s}
{b, c, d}	{b, c}	{}	{a, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d}
{b, c, s}	{b, c}	{}	{a, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, s}
{b, d, s}	{b}	{}	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, s}
{a, b, d, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, d, s}
{a, c, d, s}	{}	{b}	{a, c, d, s}	{a, c, d, s}	{d, s}	{a, c}	{d, s}
{b, c, d, s}	{b, c}	{}	{a, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS3:={},{},{b},{},{},{a,b},{a,b,c};LS5:={LS3,X};print('LS5 is a (Not-T0) wave-like topology ` `)
:LAT5abcs(LS5, Embed N=3 to N=` ` ,5):
```

```

      X:={ a, b, c, d, s}
    LS3:={ {}, { }, { b }, { }, { }, { }, { a, b }, { a, b, c }
    LS5:={ { }, { b }, { a, b }, { a, b, c }, { a, b, c, d, s } }
    LS5 is a (Not-T0) wave-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{ }
{b}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{ }
{c}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a}
{a, c}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c}
{a, d}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d}
{a, s}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{s}
{b, c}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c}
{b, d}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c}
{a, b, d}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, d}
{a, b, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, s}
{a, c, d}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d}
{a, c, s}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, s}
{a, d, s}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d, s}
{b, c, d}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d}
{b, c, s}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, s}
{b, d, s}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, s}
{a, b, d, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, d, s}
{a, c, d, s}	{ }	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{b}	{ }	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, d, s}

Some singletons have the same closure and cannot be distinguished

> :

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6. **Examples of four $N=3$, NOT-T0, wave-like topologies which can be embedded into $N=5$, NOT-T0, wave-like topologies.**

```
> print(` `);X:={a,b,c,d,s};LS3:={},{a,b,c};LS5:={LS3,X};print( LS5 is a (Not-T0) wave-like topology`);LAT5abcds(LS5,` `5)
```

```

X:={ a, b, c, d, s}
LS3:={ }, {a, b, c}
LS5:={ { }, {a, b, c}, {a, b, c, d, s} }
LS5 is a (Not-T0) wave-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{ }
{b}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{ }
{c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b}
{a, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c}
{a, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c}
{b, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d}
{c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c}
{a, b, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d}
{a, b, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, s}
{a, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c, d}
{a, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c, s}
{a, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c, d}
{b, c, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c, s}
{b, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, s}
{a, b, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, d, s}
{a, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, c, d, s}
{b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d, s}	{ }	{a, b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS3:={},{c},{a,b,c};LS5:={LS3,X};print(` LS5 is a (Not-T0) wave-like topology`);LAT5abcds(LS5,5);
```

```

X:={ a, b, c, d, s}
LS3:={ {}, {c}, {a, b, c}
LS5:={{ }, {c}, {a, b, c}, {a, b, c, d, s}}
LS5 is a (Not-T0) wave-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{}
{b}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{}
{c}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{}
{a, b}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, b, d, s}	{}	{a, b}
{a, c}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a}
{a, d}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d}
{a, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{s}
{b, c}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{b}
{b, d}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d}
{b, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{s}
{c, d}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d}
{c, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{d, s}	{}	{d, s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b}
{a, b, d}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, b, d, s}	{}	{a, b, d}
{a, b, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, b, d, s}	{}	{a, b, s}
{a, c, d}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, d}
{a, c, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, s}
{a, d, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d, s}
{b, c, d}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{b, d}
{b, c, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{b, s}
{b, d, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d, s}
{c, d, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, s}
{a, b, d, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, b, d, s}	{}	{a, b, d, s}
{a, c, d, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, d, s}
{b, c, d, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{b, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS3:={},{},{a,b},{a,b,c};LS5:={LS3,X};print(`LS5 is a (Not-T0) wave-like topology`);LAT5abcds
(LS5, ` ` ,5);
```

```

X:={ a, b, c, d, s}
LS3:={ }, { }, { }, { a, b }, { a, b, c }
LS5:={ { }, { a, b }, { a, b, c }, { a, b, c, d, s } }
LS5 is a (Not-T0) wave-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ }
{b}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ a, c, d, s }	{ b }	{ }
{c}	{ }	{ a, b }	{ c, d, s }	{ c, d, s }	{ d, s }	{ c }	{ }
{d}	{ }	{ a, b, c }	{ d, s }	{ d, s }	{ s }	{ d }	{ }
{s}	{ }	{ a, b, c }	{ d, s }	{ d, s }	{ d }	{ s }	{ }
{a, b}	{ a, b }	{ }	{ c, d, s }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ }	{ a, b }
{a, c}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ c }
{a, d}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ d }
{a, s}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ s }
{b, c}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ a, c, d, s }	{ b }	{ c }
{b, d}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ a, c, d, s }	{ b }	{ d }
{b, s}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ a, c, d, s }	{ b }	{ s }
{c, d}	{ }	{ a, b }	{ c, d, s }	{ c, d, s }	{ d, s }	{ c }	{ d }
{c, s}	{ }	{ a, b }	{ c, d, s }	{ c, d, s }	{ d, s }	{ c }	{ s }
{d, s}	{ }	{ a, b, c }	{ d, s }	{ d, s }	{ d, s }	{ }	{ d, s }
{a, b, c}	{ a, b, c }	{ }	{ d, s }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ }	{ a, b, c }
{a, b, d}	{ a, b }	{ }	{ c, d, s }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ }	{ a, b, d }
{a, b, s}	{ a, b }	{ }	{ c, d, s }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ }	{ a, b, s }
{a, c, d}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ c, d }
{a, c, s}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ c, s }
{a, d, s}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ d, s }
{b, c, d}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ a, c, d, s }	{ b }	{ c, d }
{b, c, s}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ a, c, d, s }	{ b }	{ c, s }
{b, d, s}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ a, c, d, s }	{ b }	{ d, s }
{c, d, s}	{ }	{ a, b }	{ c, d, s }	{ c, d, s }	{ d, s }	{ c }	{ d, s }
{a, b, c, d}	{ a, b, c }	{ }	{ d, s }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ }	{ a, b, c, d }
{a, b, c, s}	{ a, b, c }	{ }	{ d, s }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ }	{ a, b, c, s }
{a, b, d, s}	{ a, b }	{ }	{ c, d, s }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ }	{ a, b, d, s }
{a, c, d, s}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ b, c, d, s }	{ a }	{ c, d, s }
{b, c, d, s}	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ a, c, d, s }	{ b }	{ c, d, s }
{a, b, c, d, s}	{ a, b, c, d, s }	{ }	{ }	{ a, b, c, d, s }	{ a, b, c, d, s }	{ }	{ a, b, c, d, s }

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS3:={},{},{c},{a,b},{a,b,c}; LS5:={LS3,X};print(` LS5 is a (Not-T0) wave-like topology`);
LAT5abcds(LS5, ` ` ,5);
```

```

X:={ a, b, c, d, s}
LS3:={ {}, { }, {c}, {a, b}, {a, b, c}
LS5:={{ {}, {c}, {a, b}, {a, b, c}, {a, b, c, d, s}}
LS5 is a (Not-T0) wave-like topology
```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{}
{b}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{}
{c}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{}
{a, b}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, b, d, s}	{}	{a, b}
{a, c}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{}
{a, d}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d}
{a, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{s}
{b, c}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{}
{b, d}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d}
{b, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{s}
{c, d}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{d, s}	{}	{d, s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b}
{a, b, d}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, b, d, s}	{}	{a, b, d}
{a, b, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, b, d, s}	{}	{a, b, s}
{a, c, d}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d}
{a, c, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{s}
{a, d, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d, s}
{b, c, d}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d}
{b, c, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{s}
{b, d, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d, s}
{c, d, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, s}
{a, b, d, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, b, d, s}	{}	{a, b, d, s}
{a, c, d, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d, s}
{b, c, d, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, d, s}

Some singletons have the same closure and cannot be distinguished

**7. Examples of $N=4$, $T0$, particle-like topologies
which can be embedded into $N=5$, $T0$, particle-like topologies.**

```
> print(``);X:={a,b,c,d,s};LS4:={},{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print('LS5 is a (T0) particle like topology');LAT5abcds(LS5,`,`,5):
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

LS5 is a (T0) particle like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d}	{s}	{a, s}	{s}	{a}	{}
{b}	{b}	{a, c, d}	{s}	{b, s}	{s}	{b}	{}
{c}	{c}	{a, b, d}	{s}	{c, s}	{s}	{c}	{}
{d}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{a, b}	{c, d}	{s}	{a, b, s}	{s}	{a, b}	{}
{a, c}	{a, c}	{b, d}	{s}	{a, c, s}	{s}	{a, c}	{}
{a, d}	{a, d}	{b, c}	{s}	{a, d, s}	{s}	{a, d}	{}
{a, s}	{a}	{b, c, d}	{s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{a, d}	{s}	{b, c, s}	{s}	{b, c}	{}
{b, d}	{b, d}	{a, c}	{s}	{b, d, s}	{s}	{b, d}	{}
{b, s}	{b}	{a, c, d}	{s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{a, b}	{s}	{c, d, s}	{s}	{c, d}	{}
{c, s}	{c}	{a, b, d}	{s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{d}	{s}	{a, b, c, s}	{s}	{a, b, c}	{}
{a, b, d}	{a, b, d}	{c}	{s}	{a, b, d, s}	{s}	{a, b, d}	{}
{a, b, s}	{a, b}	{c, d}	{s}	{a, b, s}	{s}	{a, b}	{s}
{a, c, d}	{a, c, d}	{b}	{s}	{a, c, d, s}	{s}	{a, c, d}	{}
{a, c, s}	{a, c}	{b, d}	{s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{a, d}	{b, c}	{s}	{a, d, s}	{s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{a}	{s}	{b, c, d, s}	{s}	{b, c, d}	{}
{b, c, s}	{b, c}	{a, d}	{s}	{b, c, s}	{s}	{b, c}	{s}
{b, d, s}	{b, d}	{a, c}	{s}	{b, d, s}	{s}	{b, d}	{s}
{c, d, s}	{c, d}	{a, b}	{s}	{c, d, s}	{s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{s}	{a, b, c, d}	{}
{a, b, c, s}	{a, b, c}	{d}	{s}	{a, b, c, s}	{s}	{a, b, c}	{s}
{a, b, d, s}	{a, b, d}	{c}	{s}	{a, b, d, s}	{s}	{a, b, d}	{s}
{a, c, d, s}	{a, c, d}	{b}	{s}	{a, c, d, s}	{s}	{a, c, d}	{s}
{b, c, d, s}	{b, c, d}	{a}	{s}	{b, c, d, s}	{s}	{b, c, d}	{s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{s}	{a, b, c, d}	{s}

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```
> print(` `);X:={a,b,c,d,s};LS4:={},{b},{c},{d},{a,b},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print( LS5
is a (T0) particle-like topology`);LAT5abcs(LS5, ` ` ,5):
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

LS5 is a (T0) particle-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{ }
{c}	{c}	{a, b, d}	{s}	{c, s}	{s}	{c}	{ }
{d}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{c, d}	{s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{c}	{b, d}	{a, s}	{a, c, s}	{s}	{a, c}	{ }
{a, d}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{ }
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{ }
{b, d}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{ }
{b, s}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{a, b}	{s}	{c, d, s}	{s}	{c, d}	{ }
{c, s}	{c}	{a, b, d}	{s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, s}	{b, c}	{a}
{a, b, d}	{a, b, d}	{c}	{s}	{a, b, d, s}	{a, s}	{b, d}	{a}
{a, b, s}	{a, b}	{c, d}	{s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{b}	{a, s}	{a, c, d, s}	{s}	{a, c, d}	{ }
{a, c, s}	{c}	{b, d}	{a, s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{ }
{b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{s}
{b, d, s}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{s}
{c, d, s}	{c, d}	{a, b}	{s}	{c, d, s}	{s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{a}
{a, b, c, s}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, s}	{b, c}	{a, s}
{a, b, d, s}	{a, b, d}	{c}	{s}	{a, b, d, s}	{a, s}	{b, d}	{a, s}
{a, c, d, s}	{c, d}	{b}	{a, s}	{a, c, d, s}	{s}	{a, c, d}	{s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, s}	{b, c, d}	{a, s}

```
> print(`X:={a,b,c,d,s};LS4:={},{b},{d},{a,b},{b,d},{c,d},{a,b,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(LS5 is a (T0)
partice-like topology`);LAT5abcds(LS5,`5):
```

$X := \{a, b, c, d, s\}$

$LS4 := \{\emptyset, \{b\}, \{d\}, \{a, b\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}, \{a, b, c, d\}\}$

$LS5 := \{\emptyset, \{b\}, \{d\}, \{a, b\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\}\}$

LS5 is a (T0) partice-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{}
{b}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{}
{c}	{}	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{}
{d}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{a, b}	{c, d}	{s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{}	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{}
{a, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{}
{a, s}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, s}	{b, c}	{}
{b, d}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{}
{b, s}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{a, b}	{s}	{c, d, s}	{c, s}	{d}	{c}
{c, s}	{}	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{s}
{a, b, c}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, s}	{b, c}	{a}
{a, b, d}	{a, b, d}	{}	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a}
{a, b, s}	{a, b}	{c, d}	{s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{b}	{a, s}	{a, c, d, s}	{c, s}	{a, d}	{c}
{a, c, s}	{}	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c}
{b, c, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, s}	{b, c}	{s}
{b, d, s}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{s}
{c, d, s}	{c, d}	{a, b}	{s}	{c, d, s}	{c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c}
{a, b, c, s}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, s}	{b, c}	{a, s}
{a, b, d, s}	{a, b, d}	{}	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, s}
{a, c, d, s}	{c, d}	{b}	{a, s}	{a, c, d, s}	{c, s}	{a, d}	{c, s}
{b, c, d, s}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c, s}

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```
> print(``);X:={a,b,c,d,s};LS4:={},{c},{d},{a,c},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print( LS5 is a (T0)
particle-like topology `);LAT5abcds(LS5,``,5):
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}$

$LS5 := \{ \}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\}$

LS5 is a (T0) particle-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{ }
{c}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{ }
{d}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{ }	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{ }
{a, c}	{a, c}	{d}	{b, s}	{a, b, c, s}	{a, b, s}	{c}	{a}
{a, d}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{ }
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, b, s}	{c}	{b}
{b, d}	{d}	{a, c}	{b, s}	{b, d, s}	{s}	{b, d}	{ }
{b, s}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{ }
{c, s}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{s}
{d, s}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, b, s}	{c}	{a, b}
{a, b, d}	{d}	{c}	{a, b, s}	{a, b, d, s}	{s}	{a, b, d}	{ }
{a, b, s}	{ }	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{s}
{a, c, d}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a}
{a, c, s}	{a, c}	{d}	{b, s}	{a, b, c, s}	{a, b, s}	{c}	{a, s}
{a, d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{b}
{b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, b, s}	{c}	{b, s}
{b, d, s}	{d}	{a, c}	{b, s}	{b, d, s}	{s}	{b, d}	{s}
{c, d, s}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, b}
{a, b, c, s}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, b, s}	{c}	{a, b, s}
{a, b, d, s}	{d}	{c}	{a, b, s}	{a, b, d, s}	{s}	{a, b, d}	{s}
{a, c, d, s}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{b, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, b, s}

```
> print(` `);X:={a,b,c,d,s};LS4:={},{a},{c},{d},{a,c},{a,d},{c,d},{a,b,c},{a,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a (T0)
partice-like topology`);LAT5abcds(LS5,`,5):
```

X := { a, b, c, d, s }

LS4 := { {}, { a }, { c }, { d }, { a, c }, { a, d }, { c, d }, { a, b, c }, { a, c, d }, { a, b, c, d } }

LS5 := { { }, { a }, { c }, { d }, { a, c }, { a, d }, { c, d }, { a, b, c }, { a, c, d }, { a, b, c, d }, { a, b, c, d, s } }

LS5 is a (T0) partice-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ c, d }	{ b, s }	{ a, b, s }	{ b, s }	{ a }	{ }
{ b }	{ }	{ a, c, d }	{ b, s }	{ b, s }	{ s }	{ b }	{ }
{ c }	{ c }	{ a, d }	{ b, s }	{ b, c, s }	{ b, s }	{ c }	{ }
{ d }	{ d }	{ a, b, c }	{ s }	{ d, s }	{ s }	{ d }	{ }
{ s }	{ }	{ a, b, c, d }	{ s }	{ s }	{ }	{ s }	{ }
{ a, b }	{ a }	{ c, d }	{ b, s }	{ a, b, s }	{ b, s }	{ a }	{ b }
{ a, c }	{ a, c }	{ d }	{ b, s }	{ a, b, c, s }	{ b, s }	{ a, c }	{ }
{ a, d }	{ a, d }	{ c }	{ b, s }	{ a, b, d, s }	{ b, s }	{ a, d }	{ }
{ a, s }	{ a }	{ c, d }	{ b, s }	{ a, b, s }	{ b, s }	{ a }	{ s }
{ b, c }	{ c }	{ a, d }	{ b, s }	{ b, c, s }	{ b, s }	{ c }	{ b }
{ b, d }	{ d }	{ a, c }	{ b, s }	{ b, d, s }	{ s }	{ b, d }	{ }
{ b, s }	{ }	{ a, c, d }	{ b, s }	{ b, s }	{ s }	{ b }	{ s }
{ c, d }	{ c, d }	{ a }	{ b, s }	{ b, c, d, s }	{ b, s }	{ c, d }	{ }
{ c, s }	{ c }	{ a, d }	{ b, s }	{ b, c, s }	{ b, s }	{ c }	{ s }
{ d, s }	{ d }	{ a, b, c }	{ s }	{ d, s }	{ s }	{ d }	{ s }
{ a, b, c }	{ a, b, c }	{ d }	{ s }	{ a, b, c, s }	{ b, s }	{ a, c }	{ b }
{ a, b, d }	{ a, d }	{ c }	{ b, s }	{ a, b, d, s }	{ b, s }	{ a, d }	{ b }
{ a, b, s }	{ a }	{ c, d }	{ b, s }	{ a, b, s }	{ b, s }	{ a }	{ b, s }
{ a, c, d }	{ a, c, d }	{ }	{ b, s }	{ a, b, c, d, s }	{ b, s }	{ a, c, d }	{ }
{ a, c, s }	{ a, c }	{ d }	{ b, s }	{ a, b, c, s }	{ b, s }	{ a, c }	{ s }
{ a, d, s }	{ a, d }	{ c }	{ b, s }	{ a, b, d, s }	{ b, s }	{ a, d }	{ s }
{ b, c, d }	{ c, d }	{ a }	{ b, s }	{ b, c, d, s }	{ b, s }	{ c, d }	{ b }
{ b, c, s }	{ c }	{ a, d }	{ b, s }	{ b, c, s }	{ b, s }	{ c }	{ b, s }
{ b, d, s }	{ d }	{ a, c }	{ b, s }	{ b, d, s }	{ s }	{ b, d }	{ s }
{ c, d, s }	{ c, d }	{ a }	{ b, s }	{ b, c, d, s }	{ b, s }	{ c, d }	{ s }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ s }	{ a, b, c, d, s }	{ b, s }	{ a, c, d }	{ b }
{ a, b, c, s }	{ a, b, c }	{ d }	{ s }	{ a, b, c, s }	{ b, s }	{ a, c }	{ b, s }
{ a, b, d, s }	{ a, d }	{ c }	{ b, s }	{ a, b, d, s }	{ b, s }	{ a, d }	{ b, s }
{ a, c, d, s }	{ a, c, d }	{ }	{ b, s }	{ a, b, c, d, s }	{ b, s }	{ a, c, d }	{ s }
{ b, c, d, s }	{ c, d }	{ a }	{ b, s }	{ b, c, d, s }	{ b, s }	{ c, d }	{ b, s }
{ a, b, c, d, s }	{ a, b, c, d, s }	{ }	{ }	{ a, b, c, d, s }	{ b, s }	{ a, c, d }	{ b, s }

```
> print(``);X:={a,b,c,d,s};LS4:={},{c},{d},{b,c},{c,d},{a,b,c},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(`SL5 is a (T0) partice-like topology`);LAT5abcs(LS5,`,5):
```

X := { a, b, c, d, s }

LS4 := { {}, {c}, {d}, {b, c}, {c, d}, {a, b, c}, {b, c, d}, {a, b, c, d} }

LS5 := { { }, {c}, {d}, {b, c}, {c, d}, {a, b, c}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s} }

SL5 is a (T0) partice-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{}
{b}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{}
{c}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{}
{d}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{a}
{a, d}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{}
{a, s}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, b, s}	{c}	{b}
{b, d}	{d}	{c}	{a, b, s}	{a, b, d, s}	{a, s}	{b, d}	{}
{b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{}
{c, s}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{s}
{d, s}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, b, s}	{c}	{a, b}
{a, b, d}	{d}	{c}	{a, b, s}	{a, b, d, s}	{a, s}	{b, d}	{a}
{a, b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a}
{a, c, s}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{a, s}
{a, d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{b}
{b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, b, s}	{c}	{b, s}
{b, d, s}	{d}	{c}	{a, b, s}	{a, b, d, s}	{a, s}	{b, d}	{s}
{c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, b}
{a, b, c, s}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, b, s}	{c}	{a, b, s}
{a, b, d, s}	{d}	{c}	{a, b, s}	{a, b, d, s}	{a, s}	{b, d}	{a, s}
{a, c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, s}
{b, c, d, s}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{b, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, b, s}

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```
> print(``);X:={a,b,c,d,s};LS4:={},{},{d},{a,d},{c,d},{a,c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a (T0) particle-like topology`);LAT5abcs(LS5,``,5):
```

X := { a, b, c, d, s }

LS4 := { {}, { }, { }, { d }, { a, d }, { c, d }, { a, c, d }, { b, c, d }, { a, b, c, d }

LS5 := { { }, { d }, { a, d }, { c, d }, { a, c, d }, { b, c, d }, { a, b, c, d }, { a, b, c, d, s } }

LS5 is a (T0) particle-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{}
{b}	{}	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{}
{c}	{}	{a, d}	{b, c, s}	{b, c, s}	{b, s}	{c}	{}
{d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{}	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{}
{a, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{b, s}	{a, c}	{}
{a, d}	{a, d}	{}	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a}
{a, s}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{}	{a, d}	{b, c, s}	{b, c, s}	{b, s}	{c}	{b}
{b, d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b}
{b, s}	{}	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c}
{c, s}	{}	{a, d}	{b, c, s}	{b, c, s}	{b, s}	{c}	{s}
{d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{b, s}	{a, c}	{b}
{a, b, d}	{a, d}	{}	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b}
{a, b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{s}
{a, c, d}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c}
{a, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{b, s}	{a, c}	{s}
{a, d, s}	{a, d}	{}	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c}
{b, c, s}	{}	{a, d}	{b, c, s}	{b, c, s}	{b, s}	{c}	{b, s}
{b, d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c}
{a, b, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{b, s}	{a, c}	{b, s}
{a, b, d, s}	{a, d}	{}	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, s}
{a, c, d, s}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c, s}

```
> print(` `);X:={a,b,c,d,s};LS4:={},{b},{d},{b,d},{a,b,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a (T0) particle-like topology `);LAT5abcs(LS5,` `5):
```

```

X:={a,b,c,d,s}
LS4:={},{b},{d},{b,d},{a,b,d},{b,c,d},{a,b,c,d}
LS5:={{},{b},{d},{b,d},{a,b,d},{b,c,d},{a,b,c,d},{a,b,c,d,s}}
LS5 is a (T0) particle-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{}
{b}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{}
{c}	{}	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{}
{d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{a}
{a, c}	{}	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{}
{a, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{a}
{a, s}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{c}
{b, d}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{}
{b, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{s}
{c, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{c}
{c, s}	{}	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{s}
{a, b, c}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, c}
{a, b, d}	{a, b, d}	{}	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a}
{a, b, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, s}
{a, c, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{a, c}
{a, c, s}	{}	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c}
{b, c, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{c, s}
{b, d, s}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{s}
{c, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c}
{a, b, c, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, c, s}
{a, b, d, s}	{a, b, d}	{}	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, s}
{a, c, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c, s}

```
> print(` `);X:={a,b,c,d,s};LS4:={},{a},{c},{a,c},{a,b,c},{a,b,c,d};LS5:={LS4,X};print('LS5 is a (T0) particle-like topology `');LAT5abcs(LS5,` `5):
```

..
 $X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{a\}, \{c\}, \{ \}, \{a, c\}, \{ \}, \{a, b, c\}, \{ \}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{a\}, \{c\}, \{a, c\}, \{a, b, c\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

LS5 is a (T0) particle-like topology
 ..

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false
 ..

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{ }
{b}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{ }
{c}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{b}
{a, c}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{ }
{a, d}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d}
{a, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{s}
{b, c}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b}
{b, d}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{d}
{b, s}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{s}
{c, d}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{d}
{c, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b}
{a, b, d}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, d}
{a, b, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, s}
{a, c, d}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d}
{a, c, s}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{s}
{a, d, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d, s}
{b, c, d}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, d}
{b, c, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, s}
{b, d, s}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{d, s}
{c, d, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, s}
{a, b, d, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d, s}
{b, c, d, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d, s}

```
> print(``);X:={a,b,c,d,s};LS4:={},{a},{a,c},{a,b,c},{a,c,d},{a,b,c,d};LS5:={LS4,X};print('LS5 is a (T0) particle-like topology');
LAT5abcds(LS5,``,5):
```

```

X:={ a, b, c, d, s}
LS4:={ {}, {a}, {a, c}, {a, b, c}, {a, c, d}, {a, b, c, d}
LS5:={{ {}, {a}, {a, c}, {a, b, c}, {a, c, d}, {a, b, c, d}, {a, b, c, d, s}}
LS5 is a (T0) particle-like topology

```

```
Is LS5: a topology = true,    connected = true,    Kolmogorov.T0 = true,    Hausdorff.T2 = false
```

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{}
{b}	{}	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{}
{c}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b}
{a, c}	{a, c}	{}	{b, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b}
{b, d}	{}	{a, c}	{b, d, s}	{b, d, s}	{s}	{b, d}	{}
{b, s}	{}	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{d}
{c, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c}
{a, b, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d}
{a, b, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, s}
{a, c, d}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a, c}	{}	{b, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, d}
{b, c, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, s}
{b, d, s}	{}	{a, c}	{b, d, s}	{b, d, s}	{s}	{b, d}	{s}
{c, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, s}
{a, b, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

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```
> print(``);X:={a,b,c,d,s};LS4:={},{d},{b,d},{c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a (T0) particle-like topology`);
LAT5abcds(LS5,``,5):
```

```

X:={ a, b, c, d, s}
LS4:={ {}, {d}, {b, d}, {c, d}, {b, c, d}, {a, b, c, d}
LS5:={{ {}, {d}, {b, d}, {c, d}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s}}
LS5 is a (T0) particle-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{}
{b}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{}
{c}	{}	{b, d}	{a, c, s}	{a, c, s}	{a, s}	{c}	{}
{d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{}	{b, d}	{a, c, s}	{a, c, s}	{a, s}	{c}	{a}
{a, d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a}
{a, s}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, s}	{b, c}	{}
{b, d}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b}
{b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c}
{c, s}	{}	{b, d}	{a, c, s}	{a, c, s}	{a, s}	{c}	{s}
{d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, s}	{b, c}	{a}
{a, b, d}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b}
{a, b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c}
{a, c, s}	{}	{b, d}	{a, c, s}	{a, c, s}	{a, s}	{c}	{a, s}
{a, d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c}
{b, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, s}	{b, c}	{s}
{b, d, s}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c}
{a, b, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, s}	{b, c}	{a, s}
{a, b, d, s}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, s}
{a, c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c, s}

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```
> print(``);X:={a,b,c,d,s};LS4:={},{a},{c},{},{a,c},{a,b},{},{},{a,b,c},{},{},{a,b,c,d};LS5:={LS4,X};print(LS5 is a (T0
particle-like topology`);LAT5abcs(LS5,``,5):
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{a\}, \{c\}, \{ \}, \{a, c\}, \{a, b\}, \{ \}, \{ \}, \{ \}, \{a, b, c\}, \{ \}, \{ \}, \{ \}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

LS5 is a (T0) particle-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{}
{b}	{}	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{}
{c}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, d, s}	{a}	{b}
{a, c}	{a, c}	{}	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{}
{a, d}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d}
{a, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{s}
{b, c}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{}
{b, d}	{}	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{d}
{b, s}	{}	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{s}
{c, d}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b}
{a, b, d}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, d}
{a, b, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, s}
{a, c, d}	{a, c}	{}	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d}
{a, c, s}	{a, c}	{}	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{s}
{a, d, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d, s}
{b, c, d}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{d}
{b, c, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{s}
{b, d, s}	{}	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{d, s}
{c, d, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, s}
{a, b, d, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a, c}	{}	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d, s}
{b, c, d, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d, s}

```
> print(``);X:={a,b,c,d,s};LS4:={},{d},{a,d},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print('LS5 is a (T0)
particle-like topology');LAT5abcds(LS5,``,5):
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

LS5 is a (T0) particle-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{}
{b}	{}	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{}
{c}	{}	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{}
{d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{}	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{}
{a, c}	{}	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{}
{a, d}	{a, d}	{}	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a}
{a, s}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{}	{a, d}	{b, c, s}	{b, c, s}	{s}	{b, c}	{}
{b, d}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b}
{b, s}	{}	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c}
{c, s}	{}	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{s}	{a, b, c}	{}
{a, b, d}	{a, b, d}	{}	{c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b}
{a, b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{s}
{a, c, d}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c}
{a, c, s}	{}	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{a, d}	{}	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c}
{b, c, s}	{}	{a, d}	{b, c, s}	{b, c, s}	{s}	{b, c}	{s}
{b, d, s}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c}
{a, b, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{s}	{a, b, c}	{s}
{a, b, d, s}	{a, b, d}	{}	{c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, s}
{a, c, d, s}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c, s}

```
> print(``);X:={a,b,c,d,s};LS4:={},{b},{c},{d},{b,c},{b,d},{c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a (T0) particle-like topology`);LAT5abcs(LS5,`,5);
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

LS5 is a (T0) particle-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{ }
{c}	{c}	{b, d}	{a, s}	{a, c, s}	{a, s}	{c}	{ }
{d}	{d}	{b, c}	{a, s}	{a, d, s}	{a, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{c}	{b, d}	{a, s}	{a, c, s}	{a, s}	{c}	{a}
{a, d}	{d}	{b, c}	{a, s}	{a, d, s}	{a, s}	{d}	{a}
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{ }
{b, d}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{ }
{b, s}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{b}	{a, s}	{a, c, d, s}	{a, s}	{c, d}	{ }
{c, s}	{c}	{b, d}	{a, s}	{a, c, s}	{a, s}	{c}	{s}
{d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{a, s}	{d}	{s}
{a, b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{a}
{a, b, d}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{a}
{a, b, s}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{b}	{a, s}	{a, c, d, s}	{a, s}	{c, d}	{a}
{a, c, s}	{c}	{b, d}	{a, s}	{a, c, s}	{a, s}	{c}	{a, s}
{a, d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{a, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{ }
{b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{s}
{b, d, s}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{s}
{c, d, s}	{c, d}	{b}	{a, s}	{a, c, d, s}	{a, s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{a}
{a, b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{a, s}
{a, b, d, s}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{a, s}
{a, c, d, s}	{c, d}	{b}	{a, s}	{a, c, d, s}	{a, s}	{c, d}	{a, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, s}	{b, c, d}	{a, s}

```
> print(``);X:={a,b,c,d,s};LS4:={},{d},{c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a (T0) particle-like topology`);
LAT5abcds(LS5,``,5);
```

```

X:={ a, b, c, d, s}
LS4:={ {}, {d}, {c, d}, {b, c, d}, {a, b, c, d}
LS5:={{ {}, {d}, {c, d}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s}}
LS5 is a (T0) particle-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{}
{b}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{}
{c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{}
{d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{a}
{a, d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a}
{a, s}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{b}
{b, d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b}
{b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c}
{c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{s}
{d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{a, b}
{a, b, d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b}
{a, b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c}
{a, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{a, s}
{a, d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c}
{b, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{b, s}
{b, d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c}
{a, b, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{a, b, s}
{a, b, d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, s}
{a, c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c, s}

7. Examples of Embedding $N=3$, T_0 , particle-like topologies into $N=5$, to produce $N=5$, NOT T_0 , wave-like, topologies.

```
> print(`);X:={a,b,c,d,s};LS3:={},{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c};LS5:={LS3,X};print('LS5 is a Not-T0 wave-like topology
');LAT5abcds(LS5,`,5):
```

```

X:={ a, b, c, d, s}
LS3:={ {}, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}
LS5:={ {}, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}, {a, b, c, d, s}}
LS5 is a Not-T0 wave-like topology
```

```
Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false
```

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c}	{d, s}	{a, d, s}	{d, s}	{a}	{}
{b}	{b}	{a, c}	{d, s}	{b, d, s}	{d, s}	{b}	{}
{c}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{}
{a, b}	{a, b}	{c}	{d, s}	{a, b, d, s}	{d, s}	{a, b}	{}
{a, c}	{a, c}	{b}	{d, s}	{a, c, d, s}	{d, s}	{a, c}	{}
{a, d}	{a}	{b, c}	{d, s}	{a, d, s}	{d, s}	{a}	{d}
{a, s}	{a}	{b, c}	{d, s}	{a, d, s}	{d, s}	{a}	{s}
{b, c}	{b, c}	{a}	{d, s}	{b, c, d, s}	{d, s}	{b, c}	{}
{b, d}	{b}	{a, c}	{d, s}	{b, d, s}	{d, s}	{b}	{d}
{b, s}	{b}	{a, c}	{d, s}	{b, d, s}	{d, s}	{b}	{s}
{c, d}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{d, s}	{}	{d, s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{d, s}	{a, b, c}	{}
{a, b, d}	{a, b}	{c}	{d, s}	{a, b, d, s}	{d, s}	{a, b}	{d}
{a, b, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{d, s}	{a, b}	{s}
{a, c, d}	{a, c}	{b}	{d, s}	{a, c, d, s}	{d, s}	{a, c}	{d}
{a, c, s}	{a, c}	{b}	{d, s}	{a, c, d, s}	{d, s}	{a, c}	{s}
{a, d, s}	{a}	{b, c}	{d, s}	{a, d, s}	{d, s}	{a}	{d, s}
{b, c, d}	{b, c}	{a}	{d, s}	{b, c, d, s}	{d, s}	{b, c}	{d}
{b, c, s}	{b, c}	{a}	{d, s}	{b, c, d, s}	{d, s}	{b, c}	{s}
{b, d, s}	{b}	{a, c}	{d, s}	{b, d, s}	{d, s}	{b}	{d, s}
{c, d, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{d, s}	{a, b, c}	{d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{d, s}	{a, b, c}	{s}
{a, b, d, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{d, s}	{a, b}	{d, s}
{a, c, d, s}	{a, c}	{b}	{d, s}	{a, c, d, s}	{d, s}	{a, c}	{d, s}
{b, c, d, s}	{b, c}	{a}	{d, s}	{b, c, d, s}	{d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{d, s}	{a, b, c}	{d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(``);X:={a,b,c,d,s};LS3:={},{b},{c},{a,b},{b,c},{a,b,c};LS5:={LS3,X};print(`LS5 is a Not-T0 wave-like topology`);
LAT5abcds(LS5,``,5):
```

```
LS3:={},{b},{c},{a,b},{b,c},{a,b,c}
LS5:={{},{b},{c},{a,b},{b,c},{a,b,c},{a,b,c,d,s}}
LS5 is a Not-T0 wave-like topology
```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{}
{b}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{}
{c}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{}
{a, b}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, d, s}	{b}	{a}
{a, c}	{c}	{b}	{a, d, s}	{a, c, d, s}	{d, s}	{a, c}	{}
{a, d}	{}	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d}
{a, s}	{}	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{s}
{b, c}	{b, c}	{}	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{}
{b, d}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d}
{b, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{s}
{c, d}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{d, s}	{}	{d, s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a}
{a, b, d}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d}
{a, b, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, s}
{a, c, d}	{c}	{b}	{a, d, s}	{a, c, d, s}	{d, s}	{a, c}	{d}
{a, c, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{d, s}	{a, c}	{s}
{a, d, s}	{}	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d, s}
{b, c, d}	{b, c}	{}	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d}
{b, c, s}	{b, c}	{}	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{s}
{b, d, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d, s}
{c, d, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, s}
{a, b, d, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d, s}
{a, c, d, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{d, s}	{a, c}	{d, s}
{b, c, d, s}	{b, c}	{}	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(`);X:={a,b,c,d,s};LS3:={},{b},{c},{},{};LS5:={LS3,X};print('LS5 is a Not-T0 wave-like topology `);
LAT5abcds(LS5,`,5):
```

```

X:={ a, b, c, d, s}
LS3:={ {}, { }, {b}, {c}, { }, { }, {b, c}, {a, b, c}
LS5:={{ {}, {b}, {c}, {b, c}, {a, b, c}, {a, b, c, d, s}}
LS5 is a Not-T0 wave-like topology
```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{}
{b}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{}
{c}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{}
{a, b}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a}
{a, c}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a}
{a, d}	{}	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d}
{a, s}	{}	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{s}
{b, c}	{b, c}	{}	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{}
{b, d}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d}
{b, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{s}
{c, d}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{d}
{c, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{d, s}	{}	{d, s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a}
{a, b, d}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d}
{a, b, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, s}
{a, c, d}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, d}
{a, c, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, s}
{a, d, s}	{}	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d, s}
{b, c, d}	{b, c}	{}	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d}
{b, c, s}	{b, c}	{}	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{s}
{b, d, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d, s}
{c, d, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, s}
{a, b, d, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d, s}
{a, c, d, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, d, s}
{b, c, d, s}	{b, c}	{}	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};L3:={},{b},{a,b},{b,c},{a,b,c};LS5:={LS3,X};print('LS5 is a Not-T0 wave-like topology `');
LAT5abcds(LS5,` `5):
```

```

      X:={ a, b, c, d, s}
      L3:={ }, { }, { b }, { }, { a, b }, { }, { b, c }, { a, b, c }
      LS5:={{ }, { b }, { c }, { b, c }, { a, b, c }, { a, b, c, d, s }}
      LS5 is a Not-T0 wave-like topology

```

```
Is LS5:  a topology = true,    connected = true,    Kolmogorov.T0 = false,    Hausdorff.T2 = false
```

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{ }
{b}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{ }
{c}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{ }
{a, b}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a}
{a, c}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a}
{a, d}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d}
{a, s}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{s}
{b, c}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{ }
{b, d}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d}
{b, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{s}
{c, d}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{d}
{c, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{d, s}	{ }	{d, s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a}
{a, b, d}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d}
{a, b, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, s}
{a, c, d}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, d}
{a, c, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, s}
{a, d, s}	{ }	{b, c}	{a, d, s}	{a, d, s}	{d, s}	{a}	{d, s}
{b, c, d}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d}
{b, c, s}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{s}
{b, d, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d, s}
{c, d, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, s}
{a, b, d, s}	{b}	{c}	{a, d, s}	{a, b, d, s}	{a, d, s}	{b}	{a, d, s}
{a, c, d, s}	{c}	{b}	{a, d, s}	{a, c, d, s}	{a, d, s}	{c}	{a, d, s}
{b, c, d, s}	{b, c}	{ }	{a, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, d, s}	{b, c}	{a, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS3:={},{b},{},{},{a,b},{a,b,c} ;LS5:={LS3,X};;print( LS5 is a Not-T0 wave-like topology `);
LAT5abcds(LS5,` ` ,5):
```

```

      X:={ a, b, c, d, s}
    LS3:={ {}, { }, { b }, { }, { }, { }, { a, b }, { a, b, c }
    LS5:={{ { }, { b }, { a, b }, { a, b, c }, { a, b, c, d, s } }
    LS5 is a Not-T0 wave-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{}
{b}	{b}	{}	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{}
{c}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{}
{a, b}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a}
{a, c}	{}	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c}
{a, d}	{}	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d}
{a, s}	{}	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{s}
{b, c}	{b}	{}	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c}
{b, d}	{b}	{}	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{b}	{}	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{d, s}	{}	{d, s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c}
{a, b, d}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, d}
{a, b, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, s}
{a, c, d}	{}	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d}
{a, c, s}	{}	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, s}
{a, d, s}	{}	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d, s}
{b, c, d}	{b}	{}	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d}
{b, c, s}	{b}	{}	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, s}
{b, d, s}	{b}	{}	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, s}
{a, b, d, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, d, s}
{a, c, d, s}	{}	{b}	{a, c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{b}	{}	{a, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, c, d, s}	{b}	{a, c, d, s}

Some singletons have the same closure and cannot be distinguished

> :

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8. **Examples of $N=3$, NOT T0, wave-like topologies
which can be embedded into $N=5$, NOT T0, wave-like topologies.**

```
> print(` `);X:={a,b,c,d,s};LS3:={},{},{},{},{},{},{a,b,c};LS5:={LS3,X};print( LS5 is a Not-T0 wave-like topology `);
LAT5abcds(LS5,` ,5):
```

```

X:={ a, b, c, d, s}
LS3:={ {}, { }, { }, { }, { }, { }, { }, { }, { a, b, c}
LS5:={ { }, { a, b, c}, { a, b, c, d, s} }
LS5 is a Not-T0 wave-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{}
{b}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{}
{c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{}
{a, b}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b}
{a, c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, c}
{a, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, c}
{b, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d}
{c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{d, s}	{}	{d, s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c}
{a, b, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, d}
{a, b, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, s}
{a, c, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, c, d}
{a, c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, c, s}
{a, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, c, d}
{b, c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, c, s}
{b, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, s}
{a, b, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, d, s}
{a, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, c, d, s}
{b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS3:={},{},{a,b},{},{a,b,c};LS5:={LS3,X};print(`LS5 is a Not-T0 wave-like topology `);
LAT5abcds(LS5,` ,5):
```

```

X:={ a, b, c, d, s}
LS3:={ {}, { }, { }, { a, b }, { }, { }, { }, { a, b, c }
LS5:={{ }, { a, b }, { a, b, c }, { a, b, c, d, s } }
LS5 is a Not-T0 wave-like topology

```

```
Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false
```

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{}
{b}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{}
{c}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{}
{a, b}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b}
{a, c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c}
{b, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{d, s}	{}	{d, s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c}
{a, b, d}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, d}
{a, b, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, s}
{a, c, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d}
{b, c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, s}
{b, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, s}
{a, b, d, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, d, s}
{a, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(`);X:={a,b,c,d,s};LS3:={},{},{c},{a,b},{},{},{a,b,c};LS5:={LS3,X};print('LS5 is a Not-T0 wave-like topology `);
LAT5abcds(LS5,`,5):
```

```

X:={ a, b, c, d, s}
LS3:={ {}, { }, { c}, { a, b}, { }, { }, { }, { a, b, c}
LS5:={{ { }, { c}, { a, b}, { a, b, c}, { a, b, c, d, s} }
LS5 is a Not-T0 wave-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{}
{b}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{}
{c}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{}
{a, b}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, b, d, s}	{}	{a, b}
{a, c}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{}
{a, d}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d}
{a, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{s}
{b, c}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{}
{b, d}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d}
{b, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{s}
{c, d}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{d, s}	{}	{d, s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b}
{a, b, d}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, b, d, s}	{}	{a, b, d}
{a, b, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, b, d, s}	{}	{a, b, s}
{a, c, d}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d}
{a, c, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{s}
{a, d, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d, s}
{b, c, d}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d}
{b, c, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{s}
{b, d, s}	{}	{c}	{a, b, d, s}	{a, b, d, s}	{a, d, s}	{b}	{d, s}
{c, d, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, s}
{a, b, d, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{a, b, d, s}	{}	{a, b, d, s}
{a, c, d, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d, s}
{b, c, d, s}	{c}	{}	{a, b, d, s}	{a, b, c, d, s}	{a, d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, d, s}	{c}	{a, b, d, s}

Some singletons have the same closure and cannot be distinguished

**9. Examples of $N=4$, $T0$, particle-like topologies
which can be embedded into $N=5$, $T0$, particle-like topologies.**

```
> print(``);X:={a,b,c,d,s};LS4:={},{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a T0 particle-like topology`);LAT5abcs(LS5,`,`,5):
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

LS5 is a T0 particle-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d}	{s}	{a, s}	{s}	{a}	{}
{b}	{b}	{a, c, d}	{s}	{b, s}	{s}	{b}	{}
{c}	{c}	{a, b, d}	{s}	{c, s}	{s}	{c}	{}
{d}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{a, b}	{c, d}	{s}	{a, b, s}	{s}	{a, b}	{}
{a, c}	{a, c}	{b, d}	{s}	{a, c, s}	{s}	{a, c}	{}
{a, d}	{a, d}	{b, c}	{s}	{a, d, s}	{s}	{a, d}	{}
{a, s}	{a}	{b, c, d}	{s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{a, d}	{s}	{b, c, s}	{s}	{b, c}	{}
{b, d}	{b, d}	{a, c}	{s}	{b, d, s}	{s}	{b, d}	{}
{b, s}	{b}	{a, c, d}	{s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{a, b}	{s}	{c, d, s}	{s}	{c, d}	{}
{c, s}	{c}	{a, b, d}	{s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{d}	{s}	{a, b, c, s}	{s}	{a, b, c}	{}
{a, b, d}	{a, b, d}	{c}	{s}	{a, b, d, s}	{s}	{a, b, d}	{}
{a, b, s}	{a, b}	{c, d}	{s}	{a, b, s}	{s}	{a, b}	{s}
{a, c, d}	{a, c, d}	{b}	{s}	{a, c, d, s}	{s}	{a, c, d}	{}
{a, c, s}	{a, c}	{b, d}	{s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{a, d}	{b, c}	{s}	{a, d, s}	{s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{a}	{s}	{b, c, d, s}	{s}	{b, c, d}	{}
{b, c, s}	{b, c}	{a, d}	{s}	{b, c, s}	{s}	{b, c}	{s}
{b, d, s}	{b, d}	{a, c}	{s}	{b, d, s}	{s}	{b, d}	{s}
{c, d, s}	{c, d}	{a, b}	{s}	{c, d, s}	{s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{s}	{a, b, c, d}	{}
{a, b, c, s}	{a, b, c}	{d}	{s}	{a, b, c, s}	{s}	{a, b, c}	{s}
{a, b, d, s}	{a, b, d}	{c}	{s}	{a, b, d, s}	{s}	{a, b, d}	{s}
{a, c, d, s}	{a, c, d}	{b}	{s}	{a, c, d, s}	{s}	{a, c, d}	{s}
{b, c, d, s}	{b, c, d}	{a}	{s}	{b, c, d, s}	{s}	{b, c, d}	{s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{s}	{a, b, c, d}	{s}

(75)

```
> print(``);X:={a,b,c,d,s};LS4={},{b},{c},{d},{a,b},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{b,c,d},{a,b,c,d};print('LS5 is a T0 particle-
like topology`);LS5:={LS4,X};LAT5abcds(LS5,`,5):
```

$X := \{a, b, c, d, s\}$

```
{{{}}, {{a}}, {{b}}, {{c}}, {{d}}, {{a, b}}, {{a, c}}, {{a, d}}, {{b, c}}, {{b, d}}, {{c, d}}, {{a, b, c}}, {{a,
b, d}}, {{a, c, d}}, {{b, c, d}}, {{a, b, c, d}} = {{}}, {{b}}, {{c}}, {{d}}, {{a, b}}, {{b, c}}, {{b, d}},
{{c, d}}, {{a, b, c}}, {{a, b, d}}, {{b, c, d}}, {{a, b, c, d}}
```

LS5 is a T0 particle-like topology

```
LS5:={{}, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d},
{a, b, c, d}, {a, b, c, d, s}}
```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d}	{s}	{a, s}	{s}	{a}	{}
{b}	{b}	{a, c, d}	{s}	{b, s}	{s}	{b}	{}
{c}	{c}	{a, b, d}	{s}	{c, s}	{s}	{c}	{}
{d}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{a, b}	{c, d}	{s}	{a, b, s}	{s}	{a, b}	{}
{a, c}	{a, c}	{b, d}	{s}	{a, c, s}	{s}	{a, c}	{}
{a, d}	{a, d}	{b, c}	{s}	{a, d, s}	{s}	{a, d}	{}
{a, s}	{a}	{b, c, d}	{s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{a, d}	{s}	{b, c, s}	{s}	{b, c}	{}
{b, d}	{b, d}	{a, c}	{s}	{b, d, s}	{s}	{b, d}	{}
{b, s}	{b}	{a, c, d}	{s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{a, b}	{s}	{c, d, s}	{s}	{c, d}	{}
{c, s}	{c}	{a, b, d}	{s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{d}	{s}	{a, b, c, s}	{s}	{a, b, c}	{}
{a, b, d}	{a, b, d}	{c}	{s}	{a, b, d, s}	{s}	{a, b, d}	{}
{a, b, s}	{a, b}	{c, d}	{s}	{a, b, s}	{s}	{a, b}	{s}
{a, c, d}	{a, c, d}	{b}	{s}	{a, c, d, s}	{s}	{a, c, d}	{}
{a, c, s}	{a, c}	{b, d}	{s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{a, d}	{b, c}	{s}	{a, d, s}	{s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{a}	{s}	{b, c, d, s}	{s}	{b, c, d}	{}
{b, c, s}	{b, c}	{a, d}	{s}	{b, c, s}	{s}	{b, c}	{s}
{b, d, s}	{b, d}	{a, c}	{s}	{b, d, s}	{s}	{b, d}	{s}
{c, d, s}	{c, d}	{a, b}	{s}	{c, d, s}	{s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{s}	{a, b, c, d}	{}
{a, b, c, s}	{a, b, c}	{d}	{s}	{a, b, c, s}	{s}	{a, b, c}	{s}
{a, b, d, s}	{a, b, d}	{c}	{s}	{a, b, d, s}	{s}	{a, b, d}	{s}
{a, c, d, s}	{a, c, d}	{b}	{s}	{a, c, d, s}	{s}	{a, c, d}	{s}
{b, c, d, s}	{b, c, d}	{a}	{s}	{b, c, d, s}	{s}	{b, c, d}	{s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{s}	{a, b, c, d}	{s}

```
> sprint(`);X:={a,b,c,d,s};LS4:={},{b},{d},{a,b},{b,d},{c,d},{a,b,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print('LS5 is a T0
particle-like topology');LAT5abcds(LS5,`5):
```

```
sprint(` `)
```

```
X:={ a, b, c, d, s}
```

```
LS4:={ }, { }, {b}, {d}, {a, b}, {b, d}, {c, d}, {a, b, d}, {b, c, d}, {a, b, c, d}
```

```
LS5:={{ }, {b}, {d}, {a, b}, {b, d}, {c, d}, {a, b, d}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s}}
```

```
LS5 is a T0 particle-like topology
```

```
Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false
```

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{ }
{c}	{ }	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{ }
{d}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{c, d}	{s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{ }	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{ }
{a, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{ }
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, s}	{b, c}	{ }
{b, d}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{ }
{b, s}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{a, b}	{s}	{c, d, s}	{c, s}	{d}	{c}
{c, s}	{ }	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{s}
{a, b, c}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, s}	{b, c}	{a}
{a, b, d}	{a, b, d}	{ }	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a}
{a, b, s}	{a, b}	{c, d}	{s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{b}	{a, s}	{a, c, d, s}	{c, s}	{a, d}	{c}
{a, c, s}	{ }	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c}
{b, c, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, s}	{b, c}	{s}
{b, d, s}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{s}
{c, d, s}	{c, d}	{a, b}	{s}	{c, d, s}	{c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c}
{a, b, c, s}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, s}	{b, c}	{a, s}
{a, b, d, s}	{a, b, d}	{ }	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, s}
{a, c, d, s}	{c, d}	{b}	{a, s}	{a, c, d, s}	{c, s}	{a, d}	{c, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c, s}

```
> print(``);X:={a,b,c,d,s};LS4:={},{c},{d},{a,c},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print( LS5 is a T0
particle-like topology `);LAT5abcds(LS5,`,5):
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}$

$LS5 := \{ \}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\}$

LS5 is a T0 particle-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{ }
{c}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{ }
{d}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{ }	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{ }
{a, c}	{a, c}	{d}	{b, s}	{a, b, c, s}	{a, b, s}	{c}	{a}
{a, d}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{ }
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, b, s}	{c}	{b}
{b, d}	{d}	{a, c}	{b, s}	{b, d, s}	{s}	{b, d}	{ }
{b, s}	{ }	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{ }
{c, s}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{s}
{d, s}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, b, s}	{c}	{a, b}
{a, b, d}	{d}	{c}	{a, b, s}	{a, b, d, s}	{s}	{a, b, d}	{ }
{a, b, s}	{ }	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{s}
{a, c, d}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a}
{a, c, s}	{a, c}	{d}	{b, s}	{a, b, c, s}	{a, b, s}	{c}	{a, s}
{a, d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{b}
{b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, b, s}	{c}	{b, s}
{b, d, s}	{d}	{a, c}	{b, s}	{b, d, s}	{s}	{b, d}	{s}
{c, d, s}	{c, d}	{ }	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, b}
{a, b, c, s}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, b, s}	{c}	{a, b, s}
{a, b, d, s}	{d}	{c}	{a, b, s}	{a, b, d, s}	{s}	{a, b, d}	{s}
{a, c, d, s}	{a, c, d}	{ }	{b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{b, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, b, s}

```
> print(``);X:={a,b,c,d,s};LS4:={},{a},{c},{d},{a,c},{a,d},{c,d},{a,b,c},{a,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a T0
particle-like topology`);LAT5abcds(LS5,``,5):
```

X := { a, b, c, d, s }

LS4 := { {}, { a }, { c }, { d }, { a, c }, { a, d }, { c, d }, { a, b, c }, { a, c, d }, { a, b, c, d } }

LS5 := { { }, { a }, { c }, { d }, { a, c }, { a, d }, { c, d }, { a, b, c }, { a, c, d }, { a, b, c, d }, { a, b, c, d, s } }

LS5 is a T0 particle-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{ a }	{ a }	{ c, d }	{ b, s }	{ a, b, s }	{ b, s }	{ a }	{ }
{ b }	{ }	{ a, c, d }	{ b, s }	{ b, s }	{ s }	{ b }	{ }
{ c }	{ c }	{ a, d }	{ b, s }	{ b, c, s }	{ b, s }	{ c }	{ }
{ d }	{ d }	{ a, b, c }	{ s }	{ d, s }	{ s }	{ d }	{ }
{ s }	{ }	{ a, b, c, d }	{ s }	{ s }	{ }	{ s }	{ }
{ a, b }	{ a }	{ c, d }	{ b, s }	{ a, b, s }	{ b, s }	{ a }	{ b }
{ a, c }	{ a, c }	{ d }	{ b, s }	{ a, b, c, s }	{ b, s }	{ a, c }	{ }
{ a, d }	{ a, d }	{ c }	{ b, s }	{ a, b, d, s }	{ b, s }	{ a, d }	{ }
{ a, s }	{ a }	{ c, d }	{ b, s }	{ a, b, s }	{ b, s }	{ a }	{ s }
{ b, c }	{ c }	{ a, d }	{ b, s }	{ b, c, s }	{ b, s }	{ c }	{ b }
{ b, d }	{ d }	{ a, c }	{ b, s }	{ b, d, s }	{ s }	{ b, d }	{ }
{ b, s }	{ }	{ a, c, d }	{ b, s }	{ b, s }	{ s }	{ b }	{ s }
{ c, d }	{ c, d }	{ a }	{ b, s }	{ b, c, d, s }	{ b, s }	{ c, d }	{ }
{ c, s }	{ c }	{ a, d }	{ b, s }	{ b, c, s }	{ b, s }	{ c }	{ s }
{ d, s }	{ d }	{ a, b, c }	{ s }	{ d, s }	{ s }	{ d }	{ s }
{ a, b, c }	{ a, b, c }	{ d }	{ s }	{ a, b, c, s }	{ b, s }	{ a, c }	{ b }
{ a, b, d }	{ a, d }	{ c }	{ b, s }	{ a, b, d, s }	{ b, s }	{ a, d }	{ b }
{ a, b, s }	{ a }	{ c, d }	{ b, s }	{ a, b, s }	{ b, s }	{ a }	{ b, s }
{ a, c, d }	{ a, c, d }	{ }	{ b, s }	{ a, b, c, d, s }	{ b, s }	{ a, c, d }	{ }
{ a, c, s }	{ a, c }	{ d }	{ b, s }	{ a, b, c, s }	{ b, s }	{ a, c }	{ s }
{ a, d, s }	{ a, d }	{ c }	{ b, s }	{ a, b, d, s }	{ b, s }	{ a, d }	{ s }
{ b, c, d }	{ c, d }	{ a }	{ b, s }	{ b, c, d, s }	{ b, s }	{ c, d }	{ b }
{ b, c, s }	{ c }	{ a, d }	{ b, s }	{ b, c, s }	{ b, s }	{ c }	{ b, s }
{ b, d, s }	{ d }	{ a, c }	{ b, s }	{ b, d, s }	{ s }	{ b, d }	{ s }
{ c, d, s }	{ c, d }	{ a }	{ b, s }	{ b, c, d, s }	{ b, s }	{ c, d }	{ s }
{ a, b, c, d }	{ a, b, c, d }	{ }	{ s }	{ a, b, c, d, s }	{ b, s }	{ a, c, d }	{ b }
{ a, b, c, s }	{ a, b, c }	{ d }	{ s }	{ a, b, c, s }	{ b, s }	{ a, c }	{ b, s }
{ a, b, d, s }	{ a, d }	{ c }	{ b, s }	{ a, b, d, s }	{ b, s }	{ a, d }	{ b, s }
{ a, c, d, s }	{ a, c, d }	{ }	{ b, s }	{ a, b, c, d, s }	{ b, s }	{ a, c, d }	{ s }
{ b, c, d, s }	{ c, d }	{ a }	{ b, s }	{ b, c, d, s }	{ b, s }	{ c, d }	{ b, s }
{ a, b, c, d, s }	{ a, b, c, d, s }	{ }	{ }	{ a, b, c, d, s }	{ b, s }	{ a, c, d }	{ b, s }

```
> print(``);X:={a,b,c,d,s};LS4:={},{c},{d},{b,c},{c,d},{a,b,c},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a T0 particle-like topology`);LAT5abcs(LS5,`,5):
```

X:={ a, b, c, d, s}

LS4:={ {}, {c}, {d}, {b, c}, {c, d}, {a, b, c}, {b, c, d}, {a, b, c, d}}

LS5:={{ {}, {c}, {d}, {b, c}, {c, d}, {a, b, c}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s}}

LS5 is a T0 particle-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{}
{b}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{}
{c}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{}
{d}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{a}
{a, d}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{}
{a, s}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, b, s}	{c}	{b}
{b, d}	{d}	{c}	{a, b, s}	{a, b, d, s}	{a, s}	{b, d}	{}
{b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{}
{c, s}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{s}
{d, s}	{d}	{a, b, c}	{s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, b, s}	{c}	{a, b}
{a, b, d}	{d}	{c}	{a, b, s}	{a, b, d, s}	{a, s}	{b, d}	{a}
{a, b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a}
{a, c, s}	{c}	{d}	{a, b, s}	{a, b, c, s}	{a, b, s}	{c}	{a, s}
{a, d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{s}	{a, d}	{s}
{b, c, d}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{b}
{b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, b, s}	{c}	{b, s}
{b, d, s}	{d}	{c}	{a, b, s}	{a, b, d, s}	{a, s}	{b, d}	{s}
{c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, b}
{a, b, c, s}	{a, b, c}	{d}	{s}	{a, b, c, s}	{a, b, s}	{c}	{a, b, s}
{a, b, d, s}	{d}	{c}	{a, b, s}	{a, b, d, s}	{a, s}	{b, d}	{a, s}
{a, c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, s}
{b, c, d, s}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, s}	{c, d}	{b, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, s}	{c, d}	{a, b, s}

```
> print(``);X:={a,b,c,d,s};LS4:={},{},{d},{a,d},{c,d},{a,c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a T0 particle-like topology`);LAT5abcs(LS5,``,5):
```

X:={ a, b, c, d, s }

LS4:={ {}, { }, { }, { d }, { a, d }, { c, d }, { a, c, d }, { b, c, d }, { a, b, c, d }

LS5:={ { }, { d }, { a, d }, { c, d }, { a, c, d }, { b, c, d }, { a, b, c, d }, { a, b, c, d, s } }

LS5 is a T0 particle-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{}
{b}	{}	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{}
{c}	{}	{a, d}	{b, c, s}	{b, c, s}	{b, s}	{c}	{}
{d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{}	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{}
{a, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{b, s}	{a, c}	{}
{a, d}	{a, d}	{}	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a}
{a, s}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{}	{a, d}	{b, c, s}	{b, c, s}	{b, s}	{c}	{b}
{b, d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b}
{b, s}	{}	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c}
{c, s}	{}	{a, d}	{b, c, s}	{b, c, s}	{b, s}	{c}	{s}
{d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{b, s}	{a, c}	{b}
{a, b, d}	{a, d}	{}	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b}
{a, b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{s}
{a, c, d}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c}
{a, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{b, s}	{a, c}	{s}
{a, d, s}	{a, d}	{}	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c}
{b, c, s}	{}	{a, d}	{b, c, s}	{b, c, s}	{b, s}	{c}	{b, s}
{b, d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c}
{a, b, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{b, s}	{a, c}	{b, s}
{a, b, d, s}	{a, d}	{}	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, s}
{a, c, d, s}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c, s}

```
> print(` `);X:={a,b,c,d,s};LS4:={},{b},{d},{b,d},{a,b,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a T0 particle-like topology `);LAT5abcds(LS5,` `5):
```

```

X:={a,b,c,d,s}
LS4:={},{b},{d},{b,d},{a,b,d},{b,c,d},{a,b,c,d}
LS5:={{},{b},{d},{b,d},{a,b,d},{b,c,d},{a,b,c,d},{a,b,c,d,s}}
LS5 is a T0 particle-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{}
{b}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{}
{c}	{}	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{}
{d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{a}
{a, c}	{}	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{}
{a, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{a}
{a, s}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{c}
{b, d}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{}
{b, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{s}
{c, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{c}
{c, s}	{}	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{s}
{a, b, c}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, c}
{a, b, d}	{a, b, d}	{}	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a}
{a, b, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, s}
{a, c, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{a, c}
{a, c, s}	{}	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c}
{b, c, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{c, s}
{b, d, s}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{s}
{c, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c}
{a, b, c, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, c, s}
{a, b, d, s}	{a, b, d}	{}	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, s}
{a, c, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{a, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c, s}

```
> print(`);X:={a,b,c,d,s};LS4:={},{a},{c},{a,c},{a,b,c},{a,b,c,d};LS5:={LS4,X};print('LS5 is a T0 particle-like topology
');LAT5abcs(LS5,`5):
```

..
 $X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{a\}, \{c\}, \{ \}, \{a, c\}, \{ \}, \{a, b, c\}, \{ \}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{a\}, \{c\}, \{a, c\}, \{a, b, c\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

..
LS5 is a T0 particle-like topology

..
Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{}
{b}	{}	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{}
{c}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{b}
{a, c}	{a, c}	{}	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{}
{a, d}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d}
{a, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{s}
{b, c}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b}
{b, d}	{}	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{d}
{b, s}	{}	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{s}
{c, d}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{d}
{c, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b}
{a, b, d}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, d}
{a, b, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, s}
{a, c, d}	{a, c}	{}	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d}
{a, c, s}	{a, c}	{}	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{s}
{a, d, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d, s}
{b, c, d}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, d}
{b, c, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, s}
{b, d, s}	{}	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{d, s}
{c, d, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, s}
{a, b, d, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a, c}	{}	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d, s}
{b, c, d, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d, s}

```
> print(``);X:={a,b,c,d,s};LS4:={},{a},{a,c},{a,b,c},{a,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a T0 particle-like topology `);
LAT5abcds(LS5,``,5):
```

```

X:={ a, b, c, d, s}
LS4:={ {}, {a}, {a, c}, {a, b, c}, {a, c, d}, {a, b, c, d}
LS5:={{ {}, {a}, {a, c}, {a, b, c}, {a, c, d}, {a, b, c, d}, {a, b, c, d, s}}
LS5 is a T0 particle-like topology

```

```
Is LS5: a topology = true,    connected = true,    Kolmogorov.T0 = true,    Hausdorff.T2 = false
```

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{}
{b}	{}	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{}
{c}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b}
{a, c}	{a, c}	{}	{b, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b}
{b, d}	{}	{a, c}	{b, d, s}	{b, d, s}	{s}	{b, d}	{}
{b, s}	{}	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{d}
{c, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c}
{a, b, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d}
{a, b, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, s}
{a, c, d}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a, c}	{}	{b, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, d}
{b, c, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, s}
{b, d, s}	{}	{a, c}	{b, d, s}	{b, d, s}	{s}	{b, d}	{s}
{c, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, s}
{a, b, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

```
> print(``);X:={a,b,c,d,s};LS4:={},{d},{b,d},{c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a T0 particle-like topology `);
LAT5abcds(LS5,`,5):
```

```

X:={ a, b, c, d, s}
LS4:={ {}, {d}, {b, d}, {c, d}, {b, c, d}, {a, b, c, d}
LS5:={{ {}, {d}, {b, d}, {c, d}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s}}
LS5 is a T0 particle-like topology
```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{}
{b}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{}
{c}	{}	{b, d}	{a, c, s}	{a, c, s}	{a, s}	{c}	{}
{d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{}	{b, d}	{a, c, s}	{a, c, s}	{a, s}	{c}	{a}
{a, d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a}
{a, s}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, s}	{b, c}	{}
{b, d}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b}
{b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c}
{c, s}	{}	{b, d}	{a, c, s}	{a, c, s}	{a, s}	{c}	{s}
{d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, s}	{b, c}	{a}
{a, b, d}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b}
{a, b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c}
{a, c, s}	{}	{b, d}	{a, c, s}	{a, c, s}	{a, s}	{c}	{a, s}
{a, d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c}
{b, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, s}	{b, c}	{s}
{b, d, s}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c}
{a, b, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, s}	{b, c}	{a, s}
{a, b, d, s}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, s}
{a, c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c, s}

```
> print(`);X:={a,b,c,d,s};LS4:={},{b},{d},{a,b},{b,d},{a,b,d},{a,b,c,d};LS5:={LS4,X};print( LS5 is a T0 particle-like topology`);LAT5abcs(LS5,`,5):
```

```

X:={ a, b, c, d, s}
LS4:={ }, {b}, {d}, { }, {a, b}, {b, d}, { }, {a, b, d}, {a, b, c, d}
LS5:={{ }, {b}, {d}, {a, b}, {b, d}, {a, b, d}, {a, b, c, d}, {a, b, c, d, s}}
LS5 is a T0 particle-like topology

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, d}	{a, c, s}	{a, c, s}	{c, s}	{a}	{ }
{b}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{ }
{c}	{ }	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{ }
{d}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, c, s}	{b}	{a}
{a, c}	{ }	{b, d}	{a, c, s}	{a, c, s}	{c, s}	{a}	{c}
{a, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{ }
{a, s}	{ }	{b, d}	{a, c, s}	{a, c, s}	{c, s}	{a}	{s}
{b, c}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{c}
{b, d}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{ }
{b, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{s}
{c, d}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{c}
{c, s}	{ }	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{s}
{a, b, c}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, c}
{a, b, d}	{a, b, d}	{ }	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a}
{a, b, s}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, s}
{a, c, d}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{c}
{a, c, s}	{ }	{b, d}	{a, c, s}	{a, c, s}	{c, s}	{a}	{c, s}
{a, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{s}
{b, c, d}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c}
{b, c, s}	{b}	{d}	{a, c, s}	{a, b, c, s}	{a, c, s}	{b}	{c, s}
{b, d, s}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{s}
{c, d, s}	{d}	{a, b}	{c, s}	{c, d, s}	{c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c}
{a, b, c, s}	{a, b}	{d}	{c, s}	{a, b, c, s}	{a, c, s}	{b}	{a, c, s}
{a, b, d, s}	{a, b, d}	{ }	{c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, s}
{a, c, d, s}	{d}	{b}	{a, c, s}	{a, c, d, s}	{c, s}	{a, d}	{c, s}
{b, c, d, s}	{b, d}	{ }	{a, c, s}	{a, b, c, d, s}	{a, c, s}	{b, d}	{c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, c, s}	{b, d}	{a, c, s}

```
> print(`);X:={a,b,c,d,s};LS4:={},{a},{c},{},{a,c},{a,b},{},{},{a,b,c},{},{},{a,b,c,d};LS5:={LS4,X};print('LS5 is a T0
particle-like topology');LAT5abcds(LS5,``,5):
```

X:={ a, b, c, d, s}

LS4:={ }, {a}, {c}, { }, {a, c}, {a, b}, { }, { }, { }, {a, b, c}, { }, { }, { }, {a, b, c, d}

LS5:={{ }, {a}, {c}, {a, b}, {a, c}, {a, b, c}, {a, b, c, d}, {a, b, c, d, s}}

LS5 is a T0 particle-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{ }
{b}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{ }
{c}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, d, s}	{a}	{b}
{a, c}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{ }
{a, d}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d}
{a, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{s}
{b, c}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{ }
{b, d}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{d}
{b, s}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{s}
{c, d}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b}
{a, b, d}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, d}
{a, b, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, s}
{a, c, d}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d}
{a, c, s}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{s}
{a, d, s}	{a}	{c}	{b, d, s}	{a, b, d, s}	{b, d, s}	{a}	{d, s}
{b, c, d}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{d}
{b, c, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{s}
{b, d, s}	{ }	{a, c}	{b, d, s}	{b, d, s}	{d, s}	{b}	{d, s}
{c, d, s}	{c}	{a, b}	{d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d}
{a, b, c, s}	{a, b, c}	{ }	{d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, s}
{a, b, d, s}	{a, b}	{c}	{d, s}	{a, b, d, s}	{b, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a, c}	{ }	{b, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{d, s}
{b, c, d, s}	{c}	{a}	{b, d, s}	{b, c, d, s}	{d, s}	{b, c}	{d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, d, s}

```
> print(`);X:={a,b,c,d,s};LS4:={},{d},{a,d},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(LS5 is a T0
particle-like topology`);LAT5abcds(LS5,`,5):
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

LS5 is a T0 particle-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{}
{b}	{}	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{}
{c}	{}	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{}
{d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{}	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{}
{a, c}	{}	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{}
{a, d}	{a, d}	{}	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a}
{a, s}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{}	{a, d}	{b, c, s}	{b, c, s}	{s}	{b, c}	{}
{b, d}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b}
{b, s}	{}	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{s}
{c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c}
{c, s}	{}	{a, b, d}	{c, s}	{c, s}	{s}	{c}	{s}
{d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{s}	{a, b, c}	{}
{a, b, d}	{a, b, d}	{}	{c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b}
{a, b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{s}	{a, b}	{s}
{a, c, d}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c}
{a, c, s}	{}	{b, d}	{a, c, s}	{a, c, s}	{s}	{a, c}	{s}
{a, d, s}	{a, d}	{}	{b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c}
{b, c, s}	{}	{a, d}	{b, c, s}	{b, c, s}	{s}	{b, c}	{s}
{b, d, s}	{b, d}	{}	{a, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c}
{a, b, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{s}	{a, b, c}	{s}
{a, b, d, s}	{a, b, d}	{}	{c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, s}
{a, c, d, s}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c, s}

```
> print(``);X:={a,b,c,d,s};LS4:={},{b},{c},{d},{b,c},{b,d},{c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a T0 particle-like topology`);LAT5abcds(LS5,`N`,5):
```

..
 $X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

LS5 is a T0 particle-like topology
 ..

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false
 ..

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{ }
{b}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{ }
{c}	{c}	{b, d}	{a, s}	{a, c, s}	{a, s}	{c}	{ }
{d}	{d}	{b, c}	{a, s}	{a, d, s}	{a, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{c}	{b, d}	{a, s}	{a, c, s}	{a, s}	{c}	{a}
{a, d}	{d}	{b, c}	{a, s}	{a, d, s}	{a, s}	{d}	{a}
{a, s}	{ }	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{ }
{b, d}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{ }
{b, s}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{b}	{a, s}	{a, c, d, s}	{a, s}	{c, d}	{ }
{c, s}	{c}	{b, d}	{a, s}	{a, c, s}	{a, s}	{c}	{s}
{d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{a, s}	{d}	{s}
{a, b, c}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{a}
{a, b, d}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{a}
{a, b, s}	{b}	{c, d}	{a, s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{b}	{a, s}	{a, c, d, s}	{a, s}	{c, d}	{a}
{a, c, s}	{c}	{b, d}	{a, s}	{a, c, s}	{a, s}	{c}	{a, s}
{a, d, s}	{d}	{b, c}	{a, s}	{a, d, s}	{a, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{ }
{b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{s}
{b, d, s}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{s}
{c, d, s}	{c, d}	{b}	{a, s}	{a, c, d, s}	{a, s}	{c, d}	{s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{a}
{a, b, c, s}	{b, c}	{d}	{a, s}	{a, b, c, s}	{a, s}	{b, c}	{a, s}
{a, b, d, s}	{b, d}	{c}	{a, s}	{a, b, d, s}	{a, s}	{b, d}	{a, s}
{a, c, d, s}	{c, d}	{b}	{a, s}	{a, c, d, s}	{a, s}	{c, d}	{a, s}
{b, c, d, s}	{b, c, d}	{ }	{a, s}	{a, b, c, d, s}	{a, s}	{b, c, d}	{s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, s}	{b, c, d}	{a, s}

```
> print(``);X:={a,b,c,d,s};LS4:={},{d},{c,d},{b,c,d},{a,b,c,d};LS5:={LS4,X};print(`LS5 is a T0 particle-like topology `);
LAT5abcds(LS5, N, 5):
```

```

..
X:={ a, b, c, d, s}
LS4:={ {}, {d}, {c, d}, {b, c, d}, {a, b, c, d}
LS5:={{ {}, {d}, {c, d}, {b, c, d}, {a, b, c, d}, {a, b, c, d, s}}
LS5 is a T0 particle-like topology
..
```

```
Is LS5: a topology = true, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false
..
```

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{}
{b}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{}
{c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{}
{d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a}
{a, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{a}
{a, d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a}
{a, s}	{}	{b, c, d}	{a, s}	{a, s}	{s}	{a}	{s}
{b, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{b}
{b, d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b}
{b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{s}
{c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c}
{c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{s}
{d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{a, b}
{a, b, d}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b}
{a, b, s}	{}	{c, d}	{a, b, s}	{a, b, s}	{a, s}	{b}	{a, s}
{a, c, d}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c}
{a, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{a, s}
{a, d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, s}
{b, c, d}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c}
{b, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{b, s}
{b, d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{c, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c}
{a, b, c, s}	{}	{d}	{a, b, c, s}	{a, b, c, s}	{a, b, s}	{c}	{a, b, s}
{a, b, d, s}	{d}	{}	{a, b, c, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, s}
{a, c, d, s}	{c, d}	{}	{a, b, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, c, s}
{b, c, d, s}	{b, c, d}	{}	{a, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{b, c, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, s}	{d}	{a, b, c, s}

**10. Examples of N=4, Not-T0, wave-like topologies
which can be embedded into N=5, Not-T0, wave-like topologies
 $X = \{a,b,c,d,s\}$**

Some or All Singleton closures are not distinguishable.

```
> print(` `);X:={a,b,c,d,s};LS4:={},{},{},{a,b,c,d};LS5:={LS4,X};print(` A NotT0 wavelike topology embeds into LS5, a N=
5, NOT T0, wave-like topology `);LAT5abcs(LS5, N,5);
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{ \}, \{ \}, \{ \}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

A NotT0 wavelike topology embeds into LS5, a N=5, NOT T0, wave-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{}
{b}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{}
{c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{}
{d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b}
{a, c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, c}
{a, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, d}
{a, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, c}
{b, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, d}
{b, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{c, d}
{c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{s}
{d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, s}	{d}	{s}
{a, b, c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c}
{a, b, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, d}
{a, b, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, s}
{a, c, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, c, d}
{a, c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, c, s}
{a, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, d, s}
{b, c, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, c, d}
{b, c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, c, s}
{b, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, d, s}
{c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{c, d, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, d}
{a, b, c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, s}
{a, b, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, d, s}
{a, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, c, d, s}
{b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS4:={},{},{a,b,c},{a,b,c,d};LS5:={LS4,X};print(` A NotT0 wavelike topology embeds into LS5, a N=5, NOT T0, wave-like topology `);LAT5abcds(LS5,`N`,5);
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{ \}, \{ \}, \{a, b, c\}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{a, b, c\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

A NotT0 wavelike topology embeds into LS5, a N=5, NOT T0, wave-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{}
{b}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{}
{c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b}
{a, c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, c}
{a, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, c}
{b, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d}
{c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c}
{a, b, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, d}
{a, b, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, s}
{a, c, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, c, d}
{a, c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, c, s}
{a, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, c, d}
{b, c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, c, s}
{b, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, s}
{a, b, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, d, s}
{a, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, c, d, s}
{b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS4:={},{},{a,b},{a,b,c,d};LS5:={LS4,X};print( A NotT0 wavelike topology embeds into LS5, a
N=5, NOT T0, wave-like topology ` `);LAT5abcds(LS5,`N`,5);
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{ \}, \{ \}, \{a, b\}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{a, b\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

A NotT0 wavelike topology embeds into LS5, a N=5, NOT T0, wave-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{}
{b}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{}
{c}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{}
{d}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b}
{a, c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c}
{b, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d}
{b, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{s}
{c, d}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{}	{c, d}
{c, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{s}
{a, b, c}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c}
{a, b, d}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, d}
{a, b, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, s}
{a, c, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d}
{b, c, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, s}
{b, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{d, s}
{c, d, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{}	{c, d, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, d}
{a, b, c, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, s}
{a, b, d, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, d, s}
{a, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{a, c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d, s}	{}	{a, b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(`);X:={a,b,c,d,s};LS4:={},{a},{},{};LS5:={LS4,X};print( A NotT0 wavelike topology embeds into LS5, a N=
5, NOT T0, wave-like topology `);LAT5abcds(LS5,N,5);
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{a\}, \{ \}, \{ \}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{a\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

A NotT0 wavelike topology embeds into LS5, a N=5, NOT T0, wave-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{}
{b}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{}
{c}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{}
{d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b}
{a, c}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{b, c}
{b, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{b, d}
{b, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{s}
{c, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{c, d}
{c, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{s}
{d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{s}
{a, b, c}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c}
{a, b, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d}
{a, b, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, s}
{a, c, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{b, c, d}
{b, c, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{b, c, s}
{b, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{b, d, s}
{c, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{c, d, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, s}
{a, b, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(`);X:={a,b,c,d,s};LS4:={},{a},{a,b},{a,b,c,d};LS5:={LS4,X};print( A N=4, NOT T0, wave-like topology embeds into
LS5, a N=5, NOT T0, wave-like topology.);LAT5abcds(LS5,N,5);;
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{a\}, \{a, b\}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{a\}, \{a, b\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

A N=4, NOT T0, wave-like topology embeds into LS5, a N=5, NOT T0, wave-like topology.

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{}
{b}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{}
{c}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{}
{d}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b}
{a, c}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c}
{b, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d}
{b, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{s}
{c, d}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{}	{c, d}
{c, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{s}
{a, b, c}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c}
{a, b, d}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d}
{a, b, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, s}
{a, c, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d}
{b, c, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, s}
{b, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d, s}
{c, d, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{}	{c, d, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, s}
{a, b, d, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(`);X:={a,b,c,d,s};LS4:={},{a},{b},{a,b},{a,b,c,d};;LS5:={LS4,X};print( A NotT0 wavelike topology embeds into LS5,
a N=5, NOT T0, wave-like topology `);;LAT5abcds(LS5,N,5);
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{a\}, \{b\}, \{a, b\}, \{a, b, c, d\} \}$

$LS5 := \{ \{ \}, \{a\}, \{b\}, \{a, b\}, \{a, b, c, d\}, \{a, b, c, d, s\} \}$

A NotT0 wavelike topology embeds into LS5, a N=5, NOT T0, wave-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{}
{b}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{}
{c}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{}
{d}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{}
{a, c}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c}
{a, d}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d}
{a, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{s}
{b, c}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c}
{b, d}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d}
{b, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{s}
{c, d}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{}	{c, d}
{c, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{s}
{a, b, c}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c}
{a, b, d}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{d}
{a, b, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{s}
{a, c, d}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d, s}
{b, c, d}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d}
{b, c, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, s}
{b, d, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d, s}
{c, d, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{}	{c, d, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c, d}
{a, b, c, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c, s}
{a, b, d, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{d, s}
{a, c, d, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(`);X:={a,b,c,d,s};LS4:={},{a},{a,b,c},{a,b,c,d};print( A NotT0 wavelike topology embeds into LS5, a N=5, NOT T0,
wave-like topology `);LAT5abcds(LS5,`N`,5);
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \}, \{a\}, \{a, b, c\}, \{a, b, c, d\}$

A NotT0 wavelike topology embeds into LS5, a N=5, NOT T0, wave-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{ }
{b}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{ }
{c}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{ }
{d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{ }
{s}	{ }	{a, b, c, d}	{s}	{s}	{ }	{s}	{ }
{a, b}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{ }
{a, c}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c}
{a, d}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d}
{a, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{s}
{b, c}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c}
{b, d}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d}
{b, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{s}
{c, d}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{ }	{c, d}
{c, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{s}
{a, b, c}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c}
{a, b, d}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{d}
{a, b, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{s}
{a, c, d}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d, s}
{b, c, d}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d}
{b, c, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, s}
{b, d, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d, s}
{c, d, s}	{ }	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{ }	{c, d, s}
{a, b, c, d}	{a, b, c, d}	{ }	{s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c, d}
{a, b, c, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c, s}
{a, b, d, s}	{a, b}	{ }	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{d, s}
{a, c, d, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{c, d, s}	{a, b}	{c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS4:={},{a,b},{a,b,c},{a,b,c,d};;print(`A N=4, NOT T0, wave-like topology embeds into LS5, a N=5, NOT T0,wave-like topology`);LAT5abcds(LS5,`N`,5);
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}$

A N=4, NOT T0, wave-like topology embeds into LS5, a N=5, NOT T0,wave-like topology

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{}
{b}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{}
{c}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{}
{d}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{}
{s}	{}	{a, b, c, d}	{s}	{s}	{}	{s}	{}
{a, b}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{}
{a, c}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c}
{a, d}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d}
{a, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{s}
{b, c}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c}
{b, d}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d}
{b, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{s}
{c, d}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{}	{c, d}
{c, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, s}	{d}	{s}
{a, b, c}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c}
{a, b, d}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{d}
{a, b, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{s}
{a, c, d}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{d, s}
{b, c, d}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d}
{b, c, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, s}
{b, d, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d, s}
{c, d, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{c, d, s}	{}	{c, d, s}
{a, b, c, d}	{a, b, c, d}	{}	{s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c, d}
{a, b, c, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c, s}
{a, b, d, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{d, s}
{a, c, d, s}	{a}	{b}	{c, d, s}	{a, c, d, s}	{c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{b}	{a}	{c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{c, d, s}	{a, b}	{c, d, s}

Some singletons have the same closure and cannot be distinguished



11. Some Examples that correspond to problems in Text Books

```
> print(``);X:={a,b,c,d,s};LS4:={},{s},{a,s},{c,d,s},{a,c,d,s},{b,c,d,s};LS5:={LS4,X};print('LS5 is a Not-T0 wave-lke topology');
print('Lipshitz, p 76 problem 14 and prob 30 p 79');LAT5abcs(LS5, 'Lipshitz, p 76 problem 14 and prob 30 p 79',5):
```

```

X:={ a, b, c, d, s}
LS4:={ {}, {s}, {a, s}, {c, d, s}, {a, c, d, s}, {b, c, d, s}
LS5:={ { }, {s}, {a, s}, {c, d, s}, {a, c, d, s}, {b, c, d, s}, {a, b, c, d, s}
LS5 is a Not-T0 wave-lke topology
Lipshitz, p 76 problem 14 and prob 30 p 79

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d, s}	{a}	{a}	{}	{a}	{}
{b}	{}	{a, c, d, s}	{b}	{b}	{}	{b}	{}
{c}	{}	{a, s}	{b, c, d}	{b, c, d}	{b, d}	{c}	{}
{d}	{}	{a, s}	{b, c, d}	{b, c, d}	{b, c}	{d}	{}
{s}	{s}	{}	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{}
{a, b}	{}	{c, d, s}	{a, b}	{a, b}	{}	{a, b}	{}
{a, c}	{}	{s}	{a, b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{}
{a, d}	{}	{s}	{a, b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{}
{a, s}	{a, s}	{}	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a}
{b, c}	{}	{a, s}	{b, c, d}	{b, c, d}	{b, d}	{c}	{b}
{b, d}	{}	{a, s}	{b, c, d}	{b, c, d}	{b, c}	{d}	{b}
{b, s}	{s}	{}	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b}
{c, d}	{}	{a, s}	{b, c, d}	{b, c, d}	{b, c, d}	{}	{c, d}
{c, s}	{s}	{}	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{c}
{d, s}	{s}	{}	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{d}
{a, b, c}	{}	{s}	{a, b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{b}
{a, b, d}	{}	{s}	{a, b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{b}
{a, b, s}	{a, s}	{}	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b}
{a, c, d}	{}	{s}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{a, c, s}	{a, s}	{}	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, c}
{a, d, s}	{a, s}	{}	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, d}
{b, c, d}	{}	{a, s}	{b, c, d}	{b, c, d}	{b, c, d}	{}	{b, c, d}
{b, c, s}	{s}	{}	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, c}
{b, d, s}	{s}	{}	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, d}
{c, d, s}	{c, d, s}	{}	{a, b}	{a, b, c, d, s}	{a, b, c, d}	{s}	{c, d}
{a, b, c, d}	{}	{s}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c, d}
{a, b, c, s}	{a, s}	{}	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, c}
{a, b, d, s}	{a, s}	{}	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, d}
{a, c, d, s}	{a, c, d, s}	{}	{b}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, c, d}
{b, c, d, s}	{b, c, d, s}	{}	{a}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, c, d}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, c, d}

Some singletons have the same closure and cannot be distinguished

> print(`);X:={a,b,c,d,s};LS3:={},{a},{a,b},{a,b,c};LS5:={LS4,X};print(LS5 is a Not-T0 wave-lke topology`);print(Lipshitz, p 76 problem 14 and prob 30 p 79`);LAT5abcs(LS5, Lipshitz, p 76 problem 14 and prob 30 p 79`,5):

$X := \{a, b, c, d, s\}$
 $LS3 := \{ \{ \}, \{a\}, \{a, b\}, \{a, b, c\} \}$
 $LS5 := \{ \{ \}, \{s\}, \{a, s\}, \{c, d, s\}, \{a, c, d, s\}, \{b, c, d, s\}, \{a, b, c, d, s\} \}$
LS5 is a Not-T0 wave-lke topology
Lipshitz, p 76 problem 14 and prob 30 p 79

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{b, c, d, s}	{a}	{a}	{ }	{a}	{ }
{b}	{ }	{a, c, d, s}	{b}	{b}	{ }	{b}	{ }
{c}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, d}	{c}	{ }
{d}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, c}	{d}	{ }
{s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{ }
{a, b}	{ }	{c, d, s}	{a, b}	{a, b}	{ }	{a, b}	{ }
{a, c}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{ }
{a, d}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{ }
{a, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a}
{b, c}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, d}	{c}	{b}
{b, d}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, c}	{d}	{b}
{b, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b}
{c, d}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{c, d}
{c, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{c}
{d, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{d}
{a, b, c}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{b}
{a, b, d}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{b}
{a, b, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b}
{a, c, d}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{a, c, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, c}
{a, d, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, d}
{b, c, d}	{ }	{a, s}	{b, c, d}	{b, c, d}	{b, c, d}	{ }	{b, c, d}
{b, c, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, c}
{b, d, s}	{s}	{ }	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, d}
{c, d, s}	{c, d, s}	{ }	{a, b}	{a, b, c, d, s}	{a, b, c, d}	{s}	{c, d}
{a, b, c, d}	{ }	{s}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c, d}
{a, b, c, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, c}
{a, b, d, s}	{a, s}	{ }	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, d}
{a, c, d, s}	{a, c, d, s}	{ }	{b}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, c, d}
{b, c, d, s}	{b, c, d, s}	{ }	{a}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, c, d}
{a, b, c, d, s}	{a, b, c, d, s}	{ }	{ }	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, c, d}

Some singletons have the same closure and cannot be distinguished

```
> print(` `);X:={a,b,c,d,s};LS4:={},{a},{c,d},{a,c,d},{b,c,d,s};LS5:={LS4,X};print('LS5 is a NOT-T0 wave-like TOPOLOGY ` `)
;LAT5abcs(LS5, Lipshitz, p 66 problem 14 and prob 30 p 79, 5):
```

```

X:={ a, b, c, d, s}
LS4:={ {}, {a}, {c, d}, {a, c, d}, {b, c, d, s}
LS5:={ { {}, {a}, {c, d}, {a, c, d}, {b, c, d, s}, {a, b, c, d, s} }
LS5 is a NOT-T0 wave-like TOPOLOGY
```

Is LS5: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d, s}	{}	{a}	{}	{a}	{}
{b}	{}	{a, c, d}	{b, s}	{b, s}	{s}	{b}	{}
{c}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{}
{d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{}
{s}	{}	{a, c, d}	{b, s}	{b, s}	{b}	{s}	{}
{a, b}	{a}	{c, d}	{b, s}	{a, b, s}	{s}	{a, b}	{}
{a, c}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{}
{a, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{}
{a, s}	{a}	{c, d}	{b, s}	{a, b, s}	{b}	{a, s}	{}
{b, c}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b}
{b, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{b}
{b, s}	{}	{a, c, d}	{b, s}	{b, s}	{b, s}	{}	{b, s}
{c, d}	{c, d}	{a}	{b, s}	{b, c, d, s}	{b, c, d, s}	{}	{c, d}
{c, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{s}
{d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{s}
{a, b, c}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b}
{a, b, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{b}
{a, b, s}	{a}	{c, d}	{b, s}	{a, b, s}	{b, s}	{a}	{b, s}
{a, c, d}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{s}
{a, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{s}
{b, c, d}	{c, d}	{a}	{b, s}	{b, c, d, s}	{b, c, d, s}	{}	{b, c, d}
{b, c, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{b, s}
{b, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{b, s}
{c, d, s}	{c, d}	{a}	{b, s}	{b, c, d, s}	{b, c, d, s}	{}	{c, d, s}
{a, b, c, d}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{b, s}
{a, b, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{b, s}
{a, c, d, s}	{a, c, d}	{}	{b, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{b, c, d, s}	{a}	{}	{b, c, d, s}	{b, c, d, s}	{}	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(``);X:={a,b,c,d};LS3:={},{},{},{a,b,d},{a,c,d};LS4:={LS3,X};print(`LS4 is NOT A TOPOLOGY !!!`);LAT4abcd
(LS4, Not T0 `4):
```

```

X:={a, b, c, d}
LS3:={ }, { }, { }, { }, {a, b, d}, {a, c, d}
LS4:={{ }, {a, b, d}, {a, c, d}, {a, b, c, d}}
LS4 is NOT A TOPOLOGY !!!

```

Is LS4: a topology = false, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{a, c, d}	{b}	{b}	{ }	{ }	{ }
{c}	{ }	{a, b, d}	{c}	{c}	{ }	{ }	{ }
{d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c}	{d}	{ }
{a, b}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, d}
{b, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, d}	{b, c}	{ }
{b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c}	{d}	{b}
{c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c}	{d}	{c}
{a, b, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c}
{a, b, d}	{a, b, d}	{ }	{c}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, d}
{a, c, d}	{a, c, d}	{ }	{b}	{a, b, c, d}	{a, b, c, d}	{ }	{a, c, d}
{b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c, d}

The Closed-Open subsets of LS are = {{ }, {a, b, c, d}}

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is= ({a, b, c, d}, {b}, {c}, {a, b, c, d})

(102)

```
> print(``);X:={a,b,c,d};LS3:={},{a,b},{a,b,d},{a,c,d};LS4:={LS3,X};print(`LS is NOT A TOPOLOGY, BUT satisfies
Kolmogorov T0 axioms !!!`);LAT4abcd(LS4, Case 2 vol6 ..LS 2 is Not a topology, but is T0 `4):
```

```

X:={a, b, c, d}
LS3:={ }, { }, {a, b}, { }, {a, b, d}, {a, c, d}
LS4:={{ }, {a, b}, {a, b, d}, {a, c, d}, {a, b, c, d}}
LS is NOT A TOPOLOGY, BUT satisfies Kolmogorov T0 axioms !!!
```

Is LS4: a topology = false, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{ }
{b}	{ }	{a, c, d}	{b}	{b}	{ }	{ }	{ }
{c}	{ }	{a, b, d}	{c}	{c}	{ }	{ }	{ }
{d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{d}	{ }
{a, b}	{a, b}	{ }	{c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, d}	{b, c}	{ }
{b, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c}	{b, d}	{ }
{c, d}	{ }	{a, b}	{c, d}	{c, d}	{c}	{ }	{c}
{a, b, c}	{a, b}	{ }	{c, d}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c}
{a, b, d}	{a, b, d}	{ }	{c}	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, d}
{a, c, d}	{a, c, d}	{ }	{b}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{a, c, d}	{b}	{c, d}
{a, b, c, d}	{a, b, c, d}	{ }	{ }	{a, b, c, d}	{a, b, c, d}	{ }	{a, b, c, d}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {b}, {c}, {c, d})

(103)

> print(`);X:={a,b,c,d};LS3:={},{a},{},{};LS4:={LS3,X};print(LS is NOT A TOPOLOGY, but satisfies Kolmogorov T0 axioms!!!`);LAT4abcd(LS4,` Case3 vol6 ..LS 3 is Not topology, but is Not T0 `4):

$X := \{a, b, c, d\}$
 $LS3 := \{ \{ \}, \{a\}, \{ \}, \{ \}, \{a, b, d\}, \{a, c, d\} \}$
 $LS4 := \{ \{ \}, \{a\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c, d\} \}$
LS is NOT A TOPOLOGY, but satisfies Kolmogorov T0 axioms!!!

Is LS4: a topology = false, connected = true, Kolmogorov.T0 = true, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{}	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{}
{b}	{}	{a, c, d}	{b}	{b}	{}	{}	{}
{c}	{}	{a, b, d}	{c}	{c}	{}	{}	{}
{d}	{}	{a}	{b, c, d}	{b, c, d}	{b, c}	{d}	{}
{a, b}	{a}	{}	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b}
{a, c}	{a}	{}	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c}
{a, d}	{a}	{}	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{d}
{b, c}	{}	{a}	{b, c, d}	{b, c, d}	{d}	{b, c}	{}
{b, d}	{}	{a}	{b, c, d}	{b, c, d}	{b, c}	{d}	{b}
{c, d}	{}	{a}	{b, c, d}	{b, c, d}	{b, c}	{d}	{c}
{a, b, c}	{a}	{}	{b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c}
{a, b, d}	{a, b, d}	{}	{c}	{a, b, c, d}	{b, c, d}	{a}	{b, d}
{a, c, d}	{a, c, d}	{}	{b}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{b, c, d}	{}	{a}	{b, c, d}	{b, c, d}	{b, c, d}	{}	{b, c, d}
{a, b, c, d}	{a, b, c, d}	{}	{}	{a, b, c, d}	{b, c, d}	{}	{b, c, d}

The Closed-Open subsets of LS are = { { }, {a, b, c, d} }

The {1,2,3,4} array of CLOSURE elements is = ({a, b, c, d}, {b}, {c}, {b, c, d})

```
> print(``);X:={a,b,c,d,s};LS4:={},{b,c,d,s},{a,c,d,s},{c,d,s},{a,s},{s};LS5:={LS4,X};print(LS5 is a N=5 Not-T0 wavelike TOPOLOGY);print( See Kiehn Vol 1 page 446 or Kiehn Vol 6 page 214`);LAT5abcs(LS5,`,5);
```

$X := \{a, b, c, d, s\}$

$LS4 := \{ \{ \}, \{b, c, d, s\}, \{a, c, d, s\}, \{c, d, s\}, \{a, s\}, \{s\}$

$LS5 := \{ \{ \}, \{s\}, \{a, s\}, \{c, d, s\}, \{a, c, d, s\}, \{b, c, d, s\}, \{a, b, c, d, s\} \}$

LS5 is a N=5 Not-T0 wavelike TOPOLOGY

See Kiehn Vol 1 page 446 or Kiehn Vol 6 page 214

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{}	{b, c, d, s}	{a}	{a}	{}	{a}	{}
{b}	{}	{a, c, d, s}	{b}	{b}	{}	{b}	{}
{c}	{}	{a, s}	{b, c, d}	{b, c, d}	{b, d}	{c}	{}
{d}	{}	{a, s}	{b, c, d}	{b, c, d}	{b, c}	{d}	{}
{s}	{s}	{}	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{}
{a, b}	{}	{c, d, s}	{a, b}	{a, b}	{}	{a, b}	{}
{a, c}	{}	{s}	{a, b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{}
{a, d}	{}	{s}	{a, b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{}
{a, s}	{a, s}	{}	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a}
{b, c}	{}	{a, s}	{b, c, d}	{b, c, d}	{b, d}	{c}	{b}
{b, d}	{}	{a, s}	{b, c, d}	{b, c, d}	{b, c}	{d}	{b}
{b, s}	{s}	{}	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b}
{c, d}	{}	{a, s}	{b, c, d}	{b, c, d}	{b, c, d}	{}	{c, d}
{c, s}	{s}	{}	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{c}
{d, s}	{s}	{}	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{d}
{a, b, c}	{}	{s}	{a, b, c, d}	{a, b, c, d}	{b, d}	{a, c}	{b}
{a, b, d}	{}	{s}	{a, b, c, d}	{a, b, c, d}	{b, c}	{a, d}	{b}
{a, b, s}	{a, s}	{}	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b}
{a, c, d}	{}	{s}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{c, d}
{a, c, s}	{a, s}	{}	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, c}
{a, d, s}	{a, s}	{}	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, d}
{b, c, d}	{}	{a, s}	{b, c, d}	{b, c, d}	{b, c, d}	{}	{b, c, d}
{b, c, s}	{s}	{}	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, c}
{b, d, s}	{s}	{}	{a, b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, d}
{c, d, s}	{c, d, s}	{}	{a, b}	{a, b, c, d, s}	{a, b, c, d}	{s}	{c, d}
{a, b, c, d}	{}	{s}	{a, b, c, d}	{a, b, c, d}	{b, c, d}	{a}	{b, c, d}
{a, b, c, s}	{a, s}	{}	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, c}
{a, b, d, s}	{a, s}	{}	{b, c, d}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, d}
{a, c, d, s}	{a, c, d, s}	{}	{b}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, c, d}
{b, c, d, s}	{b, c, d, s}	{}	{a}	{a, b, c, d, s}	{a, b, c, d}	{s}	{b, c, d}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{a, b, c, d}	{s}	{a, b, c, d}

Some singletons have the same closure and cannot be distinguished

```
> print(``);X:={a,b,c,d,s};LS3:={},{a,b,c},{a,b},{a};LS5:={LS3,X};print( LS5 is a N=5 Not-T0 wave-like topology !!! `);
LAT5abcds(LS5 , Topology 2, p 440, vol 1 ,5):
```

```

X:={ a, b, c, d, s}
LS3:={ }, {a, b, c}, {a, b}, {a}
LS5:={{ }, {a}, {a, b}, {a, b, c}, {a, b, c, d, s}}
LS5 is a N=5 Not-T0 wave-like topology !!!

```

Is LS5: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{}
{b}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{}
{c}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{}
{d}	{}	{a, b, c}	{d, s}	{d, s}	{s}	{d}	{}
{s}	{}	{a, b, c}	{d, s}	{d, s}	{d}	{s}	{}
{a, b}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b}
{a, c}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c}
{a, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d}
{a, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{s}
{b, c}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c}
{b, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d}
{b, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{s}
{c, d}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d}
{c, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{s}
{d, s}	{}	{a, b, c}	{d, s}	{d, s}	{d, s}	{}	{d, s}
{a, b, c}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c}
{a, b, d}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d}
{a, b, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, s}
{a, c, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d}
{b, c, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, s}
{b, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{d, s}
{c, d, s}	{}	{a, b}	{c, d, s}	{c, d, s}	{d, s}	{c}	{d, s}
{a, b, c, d}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a, b, c}	{}	{d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, s}
{a, b, d, s}	{a, b}	{}	{c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

Some singletons have the same closure and cannot be distinguished

```
> print(``);X:={a,b,c,d,s};LS4:={},{b,c,d,s},{a};LS5:={LS4,X};print(`LS5 is a N=5 Not-T0 wave=like topology `);LAT5abcds
(LS5, Topology 3, X:={a,b,c,d,s};,5):
```

```

X:={ a, b, c, d, s}
LS4:={ }, {b, c, d, s}, {a}
LS5:={{ }, {a}, {b, c, d, s}, {a, b, c, d, s}}
LS5 is a N=5 Not-T0 wave=like topology

```

Is LS5: a topology = true, connected = false, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{a}	{b, c, d, s}	{}	{a}	{}	{a}	{}
{b}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{c, d, s}	{b}	{}
{c}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, d, s}	{c}	{}
{d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, s}	{d}	{}
{s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d}	{s}	{}
{a, b}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{c, d, s}	{a, b}	{}
{a, c}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, d, s}	{a, c}	{}
{a, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, s}	{a, d}	{}
{a, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d}	{a, s}	{}
{b, c}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{b, c}
{b, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{b, d}
{b, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{b, s}
{c, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{c, d}
{c, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{c, s}
{d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{d, s}
{a, b, c}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c}
{a, b, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d}
{a, b, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, s}
{a, c, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d}
{a, c, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, s}
{a, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{d, s}
{b, c, d}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{b, c, d}
{b, c, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{b, c, s}
{b, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{b, d, s}
{c, d, s}	{}	{a}	{b, c, d, s}	{b, c, d, s}	{b, c, d, s}	{}	{c, d, s}
{a, b, c, d}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d}
{a, b, c, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, s}
{a, b, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, d, s}
{a, c, d, s}	{a}	{}	{b, c, d, s}	{a, b, c, d, s}	{b, c, d, s}	{a}	{c, d, s}
{b, c, d, s}	{b, c, d, s}	{a}	{}	{b, c, d, s}	{b, c, d, s}	{}	{b, c, d, s}
{a, b, c, d, s}	{a, b, c, d, s}	{}	{}	{a, b, c, d, s}	{b, c, d, s}	{a}	{b, c, d, s}

Some singletons have the same closure and cannot be distinguished

> print(`);X:={a,b,c,d,s};LS4:={},{b},{d},{a,b},{b,d},{c,d},{a,b,d},{b,c,d};print(` Let LSd be the dual of the preceding example X = {a,b,c,d} poset 3 `);LSd:=CLO({a,b,c,d},LS);print(` This dual Lattice Structure represents the marriage between Cartan's Closure of Exterior Differential Systems \n and the Kuratowski Closure based upon a limit point operator equivalent to the exterior differential, d`) :LAT4abcd(LSd,`LSd = poset 3, LSDual = poset 3, DISconnected T0 topologies EXCUSION DOWN INCLUSION UP`,4):

$$X := \{ a, b, c, d, s \}$$

$$LS4 := \{ \{ \}, \{ b \}, \{ d \}, \{ a, b \}, \{ b, d \}, \{ c, d \}, \{ a, b, d \}, \{ b, c, d \} \}$$

Let LSd be the dual of the preceding example X = {a,b,c,d} poset 3

$$LSd := \{ \{ \}, \{ a, b, c, d \} \}$$

This dual Lattice Structure represents the marriage between Cartan's Closure of Exterior Differential Systems and the Kuratowski Closure based upon a limit point operator equivalent to the exterior differential, d

Is LS4: a topology = true, connected = true, Kolmogorov.T0 = false, Hausdorff.T2 = false

Subset S	Int(S)	Ext(S)	Bnd(S)	Clo(S)	Lim(S)	IsoClo(S)	IsoCar(S)
{a}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ b, c, d }	{ a }	{ }
{b}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, c, d }	{ b }	{ }
{c}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, d }	{ c }	{ }
{d}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c }	{ d }	{ }
{a, b}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b }
{a, c}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, c }
{a, d}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, d }
{b, c}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ b, c }
{b, d}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ b, d }
{c, d}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ c, d }
{a, b, c}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, c }
{a, b, d}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, d }
{a, c, d}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, c, d }
{b, c, d}	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ b, c, d }
{a, b, c, d}	{ a, b, c, d }	{ }	{ }	{ a, b, c, d }	{ a, b, c, d }	{ }	{ a, b, c, d }

The Closed-Open subsets of LS are = { { }, { a, b, c, d } }

Some singletons have the same closure and cannot be distinguished

The {1,2,3,4} array of CLOSURE elements is = ({ a, b, c, d }, { a, b, c, d }, { a, b, c, d }, { a, b, c, d })

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