```
> restart:
```

PARAMETRIC ZERO MEAN CURVATURE SURFACES Generated by Hyperbolic and Elliptic Spinor direction fields, for Euclidean, Lorentz, and Majorana Signatures

R. M. Kiehn.
rkiehn2352@aol.com

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COMPLEX HYPERBOLIC SPINOR Direction Fields

The Euclidean Spinor has the format,
Hyperbolic $\mathit{Euclidean\ Spinor} := [-(a + I\,b)\sinh(z), I\,(a + I\,b)\cosh(z), a + I\,b]$
where a and b are constants. The Euclidean signature matrix is (+,+,+).
****
The Lorentz Spinor has the format
Hyperbolic $\mathit{Lorentz\ Spinor} := [-(a + I\,b)\sinh(z), I\,(a + I\,b)\cosh(z), I\,(a + I\,b)]$
with the Lorentz signature matrix (+,+,-).
****
The Majorana spinor has the format
Hyperbolic $\mathit{Majorana\ Spinor} := [-I\,(a + I\,b)\sinh(z), -(a + I\,b)\cosh(z), a + I\,b]$
with the Majorana signature matrix (-,-,+).
****

COMPLEX ELLIPTIC SPINOR Direction Fields

The Euclidean Spinor has the format,
Elliptic $\mathit{Euclidean\ Spinor} := [(a + I\,b)\sin(z), (a + I\,b)\cos(z), I\,(a + I\,b)]$
where a and b are constants. The Euclidean signature matrix is (+,+,+).
****
The Lorentz Spinor has the format
Elliptic $\mathit{Lorentz\ Spinor} := [(a + I\,b)\sin(z), (a + I\,b)\cos(z), a + I\,b]$
with the Lorentz signature matrix (+,+,-).
****
The Majorana spinor has the format
Elliptic $\mathit{Majorana\ Spinor} := [(a + I\,b)\sin(z), (a + I\,b)\cos(z), a + I\,b]$
with the Majorana signature matrix (-,-,+).
****

The Spinors are integrated with respect to z to give a complex position vector, R,
The real and imaginary parts of R are considered to be maps from the real surface variables, u and v,
to the 3D position vector R = [X,Y,Z].
The standard immersion techniques are used to produce the Mean and Gauss curvatures,
taking into account the effects of different metric signatures.
The procedures are

PARAsurf(R) for a Euclidean metric,
PARAsurfLOR for a Lorentz metric,
and
PARAsurfMAJ for a Majorana metric.
****

```
> with(linalg):with(plots):with(liesymm):with(difforms): setup(u,v):
  defform(Z=0,u=0,v=0,p=const,q=const);
```

```
Warning, the protected names norm and trace have been redefined and unprotected

Warning, the name changecoords has been redefined

Warning, the protected name close has been redefined and unprotected

Warning, the names &^, d and wdegree have been redefined
```

```
> PARAsurf:=proc(R)
       local Yu,Yv,FFF,EE,FF,GG,Yuu,Yvv,Yuv,NNU:
      global GUN,Q,HH,KK,gun,DET,W,WI,NN,magn,b12,b22,b11:
        GUN:=matrix([[1,0,0],[0,1,0],[0,0,1]]):
      Yu:=[diff(R[1],u),diff(R[2],u),diff(R[3],u)]:
      Yv:=[diff(R[1],v),diff(R[2],v),diff(R[3],v)]:
     Yuu:=[diff(Yu[1],u),diff(Yu[2],u),diff(Yu[3],u)]:
      Yvv:=[diff(Yv[1],v),diff(Yv[2],v),diff(Yv[3],v)]:
      Yuv:=[diff(Yu[1],v),diff(Yu[2],v),diff(Yu[3],v)]:
      NN:=(simplify(crossprod(Yu,Yv))):
magn:=(factor(simplify(innerprod(NN,GUN,NN))^(1/2))):
       NNU:=(NN/magn):
      FFF:=transpose(array([Yu,Yv])):
      gun:=simplify(innerprod(transpose(FFF),GUN,(FFF))):DET:=simplify(det(gun)):
      gun:=evalm(gun):
      EE:=factor(gun[1,1]):
      FF:=gun[1,2]:
      GG:=gun[2,2]:
      Q:=EE*GG-FF*FF:
      b11:=simplify(innerprod(Yuu,GUN,NNU)):
      b12:=simplify(innerprod(Yuv,GUN,NNU)):
      b22:=simplify(innerprod(Yvv,GUN,NNU)):
      HH:=factor(simplify(gun[2,2]*b11+gun[1,1]*b22)-2*gun[1,2]*b12) /(2*DET):
      KK:=simplify((b11*b22-b12*b12))/DET:Print(`Radius`=R); print(`Mean
Curvature`=HH);
      print(`Gauss Curvature`=simplify(KK));
      print(`Metric Det Q `=simplify(Q));

>    end:
```

```
> PARAsurfLOR:=proc(R)
       local Yu,Yv,FFF,EE,FF,GG,Yuu,Yvv,Yuv,NNU:
      global GUN,Q,HH,KK,gun,DET,W,WI,NN,magn,b12,b22,b11:
```

```
        GUN:=matrix([[1,0,0],[0,1,0],[0,0,-1]]):
      Yu:=[diff(R[1],u),diff(R[2],u),diff(R[3],u)]:
      Yv:=[diff(R[1],v),diff(R[2],v),diff(R[3],v)]:
     Yuu:=[diff(Yu[1],u),diff(Yu[2],u),diff(Yu[3],u)]:
      Yvv:=[diff(Yv[1],v),diff(Yv[2],v),diff(Yv[3],v)]:
      Yuv:=[diff(Yu[1],v),diff(Yu[2],v),diff(Yu[3],v)]:
      NN:=(simplify(crossprod(Yu,Yv))):
magn:=(factor(simplify(innerprod(NN,GUN,NN))^(1/2))):
       NNU:=(NN/magn):
      FFF:=transpose(array([Yu,Yv])):
      gun:=simplify(innerprod(transpose(FFF),GUN,(FFF))):DET:=simplify(det(gun)):
      gun:=evalm(gun):
      EE:=factor(gun[1,1]):
      FF:=gun[1,2]:
      GG:=gun[2,2]:
      Q:=EE*GG-FF*FF:
      b11:=simplify(innerprod(Yuu,GUN,NNU)):
      b12:=simplify(innerprod(Yuv,GUN,NNU)):
      b22:=simplify(innerprod(Yvv,GUN,NNU)):
      HH:=factor(simplify(gun[2,2]*b11+gun[1,1]*b22)-2*gun[1,2]*b12) /(2*DET):
      KK:=simplify((b11*b22-b12*b12))/DET:Print(`Radius`=R); print(`Mean
Curvature`=HH);
      print(`Gauss Curvature`=simplify(KK));
      print(`Metric Det Q `=simplify(Q));

>    end:

> PARAsurfMAJ:=proc(R)
       local Yu,Yv,FFF,EE,FF,GG,Yuu,Yvv,Yuv,NNU:
      global GUN,Q,HH,KK,gun,DET,W,WI,NN,magn,b12,b22,b11:
        GUN:=matrix([[-1,0,0],[0,-1,0],[0,0,1]]):
      Yu:=[diff(R[1],u),diff(R[2],u),diff(R[3],u)]:
      Yv:=[diff(R[1],v),diff(R[2],v),diff(R[3],v)]:
     Yuu:=[diff(Yu[1],u),diff(Yu[2],u),diff(Yu[3],u)]:
      Yvv:=[diff(Yv[1],v),diff(Yv[2],v),diff(Yv[3],v)]:
      Yuv:=[diff(Yu[1],v),diff(Yu[2],v),diff(Yu[3],v)]:
      NN:=(simplify(crossprod(Yu,Yv))):
magn:=(factor(simplify(innerprod(NN,GUN,NN))^(1/2))):
       NNU:=(NN/magn):
      FFF:=transpose(array([Yu,Yv])):
      gun:=simplify(innerprod(transpose(FFF),GUN,(FFF))):DET:=simplify(det(gun)):
      gun:=evalm(gun):
      EE:=factor(gun[1,1]):
      FF:=gun[1,2]:
      GG:=gun[2,2]:
      Q:=EE*GG-FF*FF:
      b11:=simplify(innerprod(Yuu,GUN,NNU)):
      b12:=simplify(innerprod(Yuv,GUN,NNU)):
      b22:=simplify(innerprod(Yvv,GUN,NNU)):
      HH:=factor(simplify(gun[2,2]*b11+gun[1,1]*b22)-2*gun[1,2]*b12) /(2*DET):
      KK:=simplify((b11*b22-b12*b12))/DET:Print(`Radius`=R); print(`Mean
```

```
Curvature`=HH);
      print(`Gauss Curvature`=simplify(KK));
      print(`Metric Det Q `=simplify(Q));

>    end:
```

```
> GUN:=matrix([[1,0,0],[0,1,0],[0,0,-1]]);
```

$$GUN := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

```
> Sigma0:=[sin(z),cos(z),1];simplify(innerprod(Sigma0,GUN,Sigma0));
```

$$\Sigma 0 := [\sin(z), \cos(z), 1]$$

$$0$$

```
> X:=int(Sigma0[1],z);Y:=int(Sigma0[2],z);Z:=int(Sigma0[3],z);
```

$$X := -\cos(z)$$

$$Y := \sin(z)$$

$$Z := z$$

```
> R:=evalc(subs(z=(b*u+I*a*v),[X,Y,Z]));
```

$$R := [-\cos(b\,u)\cosh(a\,v) + I\sin(b\,u)\sinh(a\,v), \sin(b\,u)\cosh(a\,v) + I\cos(b\,u)\sinh(a\,v), b\,u + I\,a\,v]$$

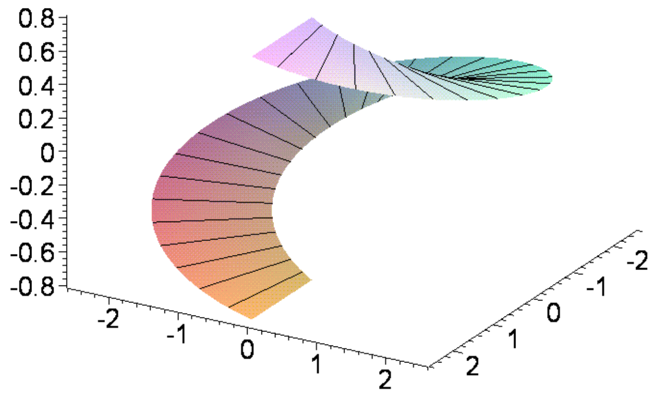
```
> Rreal:=[evalc(Re(R[1])),evalc(Re(R[2])),evalc(Re(-a*u))];Rimag:=[evalc(Im(R[1]))
,evalc(Im(R[2])),(-b*v)];
```

$$Rreal := [-\cos(b\,u)\cosh(a\,v), \sin(b\,u)\cosh(a\,v), -a\,u]$$

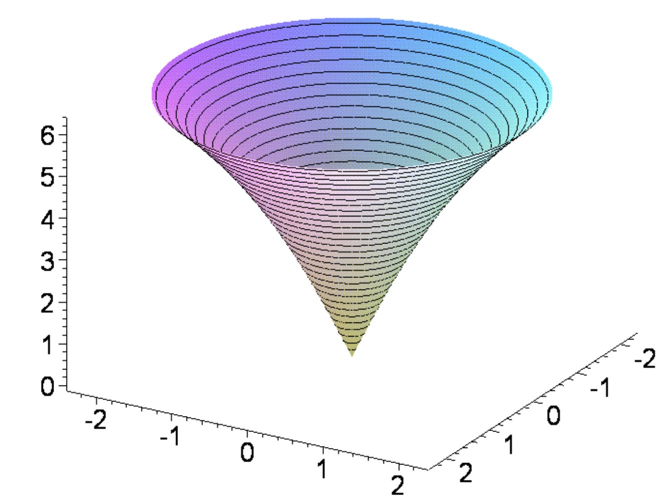
$$Rimag := [\sin(b\,u)\sinh(a\,v), \cos(b\,u)\sinh(a\,v), -b\,v]$$

```
> plot3d(subs(a=-1/4,b=1,Rreal),u=-Pi..Pi,v=-0*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Majorana \n R Real, factor a = 1/4, b =
1`);plot3d(subs(a=-1/4,b=-1,Rimag),u=-Pi..Pi,v=-0*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Majorana \n R Rimag, factor a = 1/4, b = 1`);
```

Majorana
R Real, factor a = 1/4, b = 1



Majorana
R Rimag, factor a = 1/4, b = 1



>

>

>

```
> PARAsurfLOR(Rreal);PARAsurfLOR(Rimag);
```

$$Mean\ Curvature = 0$$

$$Gauss\ Curvature = \frac{b^2 a^2}{\left(-a^2 + b^2 \cosh(a\,v)^2\right)^2}$$

$$Metric\ Det\ Q\ = a^2 \left(\cosh(a\,v)^2 - 1\right) \left(-a^2 + b^2 \cosh(a\,v)^2\right)$$

$$Mean\ Curvature = \frac{1}{2}$$

$$\frac{b^2 (\cosh(a\,v) - 1) (\cosh(a\,v) + 1) (b - a) (b + a)}{\sqrt{-b^2 (\cosh(a\,v) - 1) (\cosh(a\,v) + 1) (\cosh(a\,v)\,a - b) (\cosh(a\,v)\,a + b)}\ (\cosh(a\,v)^2 - 1) (-b^2 + \cosh(a\,v)^2 a^2)}$$

$$Gauss\ Curvature = \frac{b^2 a^2}{\left(-b^2 + \cosh(a\,v)^2 a^2\right)^2}$$

$$Metric\ Det\ Q\ = b^2 \left(\cosh(a\,v)^2 - 1\right) \left(-b^2 + \cosh(a\,v)^2 a^2\right)$$

```
>
>
>
>
>
```

Minimal Surfaces from Hyperbolic Euclidean Spinors

```
> Sigma:=[-(a+I*b)*sinh(z),(a+I*b)*I*cosh(z),(a+I*b)];RR:=0:R:=0:
```

$$\Sigma := [-(a + I\,b) \sinh(z),\ I\,(a + I\,b) \cosh(z),\ a + I\,b]$$

```
>
> X:=int(Sigma[1],z):Y:=int(Sigma[2],z):Z:=int(Sigma[3],z):
> R:=[X,Y,Z];RR:=evalc(subs(z=(u+I*v),R)):
```

$$R := [(-a - I\,b) \cosh(z),\ I\,(a + I\,b) \sinh(z),\ (a + I\,b)\,z]$$

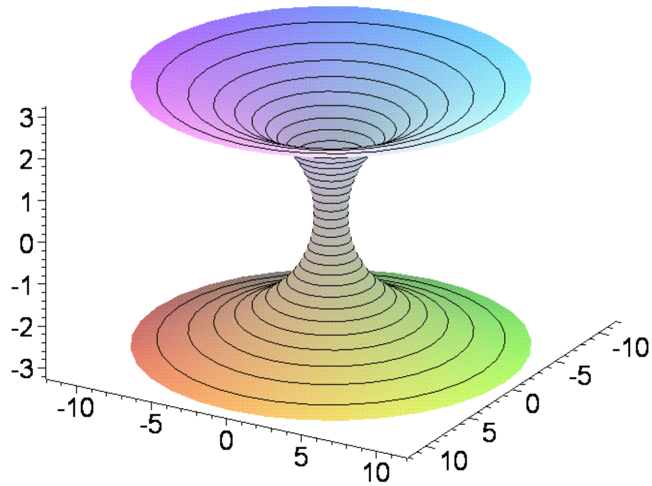
```
> RXreal:=evalc(Re(RR[1])):RXimag:=evalc(Im(RR[1])):
> RYreal:=evalc(Re(RR[2])):RYimag:=evalc(Im(RR[2])):
> RZreal:=evalc(Re(RR[3])):RZimag:=evalc(Im(RR[3])):
> Rreal := subs(a=a,b=b,[RXreal,RYreal,RZreal]);Rimag :=
  subs(a=a,b=b,[RXimag,RYimag,RZimag]);
```

$$Rreal := [-a \cosh(u) \cos(v) + b \sinh(u) \sin(v),\ -b \sinh(u) \cos(v) - a \cosh(u) \sin(v),\ a\,u - b\,v]$$

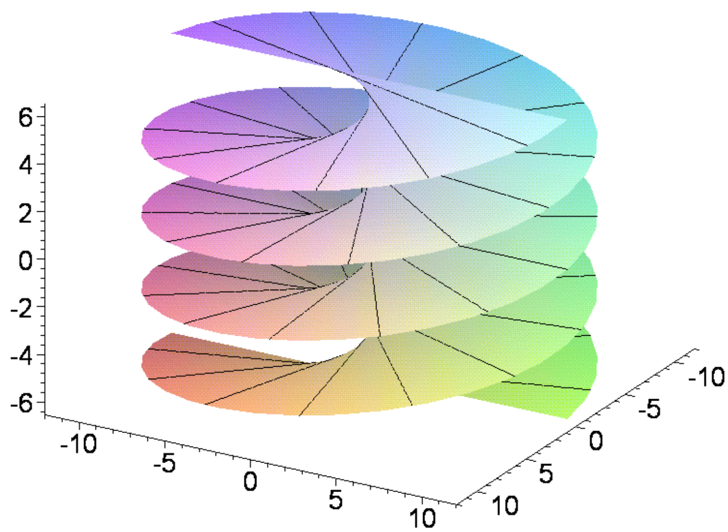
$$Rimag := [-b \cosh(u) \cos(v) - a \sinh(u) \sin(v),\ a \sinh(u) \cos(v) - b \cosh(u) \sin(v),\ a\,v + b\,u]$$

```
> plot3d(subs(a=1,b=0,Rreal),u=-Pi..Pi,v=-2*Pi..2*Pi,
  numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`Euclidean \n R Real, factor a = 1, b =
  0`);plot3d(subs(a=1,b=0,Rimag),u=-Pi..Pi,v=-2*Pi..2*Pi,
  numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`R Imag, factor a = 1, b =
  0`);plot3d(subs(a=0,b=1,Rreal),u=-Pi..Pi,v=-2*Pi..2*Pi,
  numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`R Real, factor a = 0, b =
  1`);plot3d(subs(a=0,b=1,Rimag),u=-Pi..Pi,v=-2*Pi..2*Pi,
  numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`R Imag, factor a = 0, b = 1`);
```

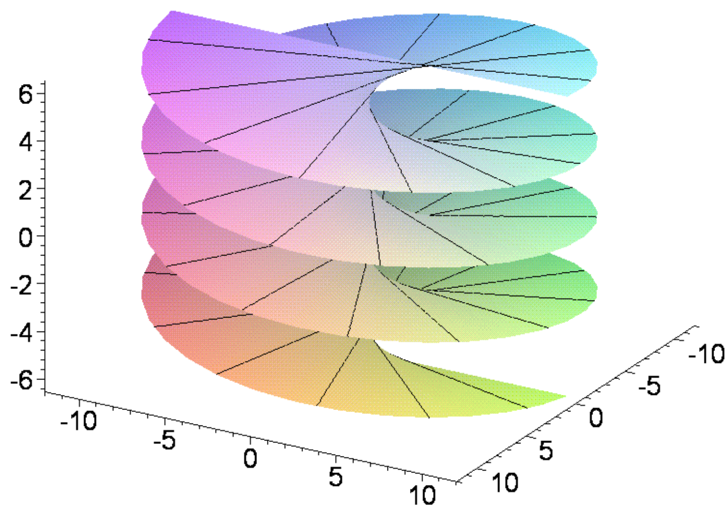
Euclidean
R Real, factor a = 1, b = 0



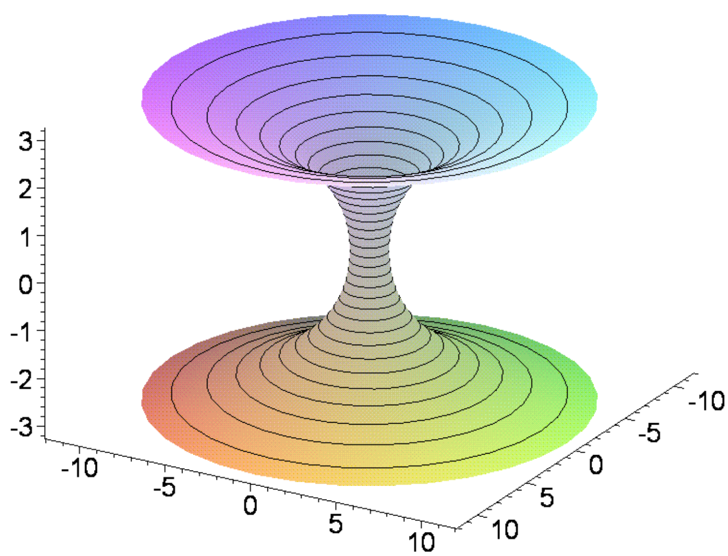
R Imag, factor a = 1, b = 0



R Real, factor a = 0, b = 1



R Imag, factor a = 0, b = 1




```
> R:=0;
```

$R := 0$

```
> `Real-component`:=Rreal;PARAsurf(Rreal);`Imag-component`:=Rimag;PARAsurf(Rimag);
  `Metric`=evalm(GUN);`Euclidean Spinor`:=Sigma;`Spinor quadratic form using
```

```
metric`:=simplify(subs(a=a,b=b,innerprod((Sigma,GUN,Sigma))));
>
```

$$Real\text{-}component := [-a\cosh(u)\cos(v)+b\sinh(u)\sin(v), -b\sinh(u)\cos(v)-a\cosh(u)\sin(v), a\,u-b\,v]$$

$$Mean\ Curvature = 0$$

$$Gauss\ Curvature = -\frac{1}{(b^2+a^2)\cosh(u)^4}$$

$$Metric\ Det\ Q\ = \cosh(u)^4\,(b^2+a^2)^2$$

$$Imag\text{-}component := [-b\cosh(u)\cos(v)-a\sinh(u)\sin(v), a\sinh(u)\cos(v)-b\cosh(u)\sin(v), a\,v+b\,u]$$

$$Mean\ Curvature = 0$$

$$Gauss\ Curvature = -\frac{1}{(b^2+a^2)\cosh(u)^4}$$

$$Metric\ Det\ Q\ = \cosh(u)^4\,(b^2+a^2)^2$$

$$Metric = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$Euclidean\ Spinor := [-(a+I\,b)\sinh(z), I\,(a+I\,b)\cosh(z), a+I\,b]$$

$$Spinor\ quadratic\ form\ using\ metric := 0$$

Minimal mean curvature surfaces from Hyperbolic Lorentzian Spinors

```
> Sigma:=[-(a+I*b)*sinh(z),(a+I*b)*I*cosh(z),(a+I*b)*I];
```

$$\Sigma := [-(a+I\,b)\sinh(z), I\,(a+I\,b)\cosh(z), I\,(a+I\,b)]$$

```
> X:=int(Sigma[1],z):Y:=int(Sigma[2],z):Z:=int(Sigma[3],z):
> R:=[X,Y,Z];RR:=evalc(subs(z=u+I*v,R)):
```

$$R := [(-a-I\,b)\cosh(z), I\,(a+I\,b)\sinh(z), I\,(a+I\,b)\,z]$$

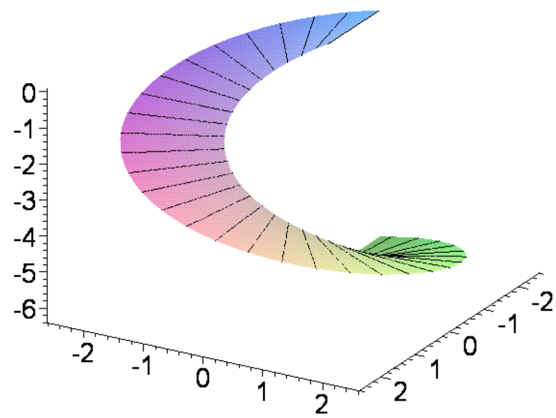
```
> RXreal:=evalc(Re(RR[1])):RXimag:=evalc(Im(RR[1])):
> RYreal:=evalc(Re(RR[2])):RYimag:=evalc(Im(RR[2])):
> RZreal:=evalc(Re(RR[3])):RZimag:=evalc(Im(RR[3])):
> Rreal  := subs(a=a,b=b,[RXreal,RYreal,RZreal]);Rimag  :=
  subs(a=a,b=b,[RXimag,RYimag,RZimag]);
```

$$Rreal := [-a\cosh(u)\cos(v)+b\sinh(u)\sin(v), -b\sinh(u)\cos(v)-a\cosh(u)\sin(v), -a\,v-b\,u]$$

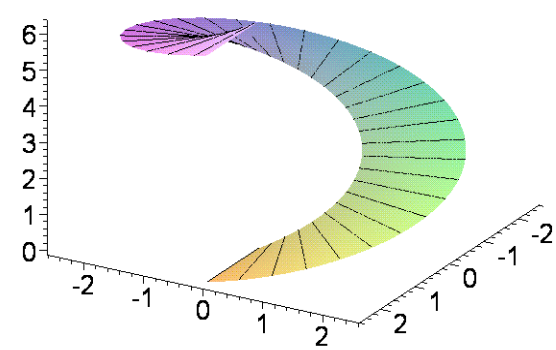
$$Rimag := [-b\cosh(u)\cos(v)-a\sinh(u)\sin(v), a\sinh(u)\cos(v)-b\cosh(u)\sin(v), a\,u-b\,v]$$

```
>
> plot3d(subs(a=1,b=0,Rreal),u=-0..Pi/2,v=-0*Pi..2*Pi,
  numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`Lorentz Helix \n R Real, left-handed) \n factor a
  = 1, b = 0`);plot3d(subs(a=-1,b=0,Rreal),u=-0..Pi/2,v=-0*Pi..2*Pi,
  numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`Lorentz Helix \n R Real, Right-handed) \n factor
  a = -1, b = 0`);
```

Lorentz Helix
R Real, left-handed)
factor a = 1, b = 0

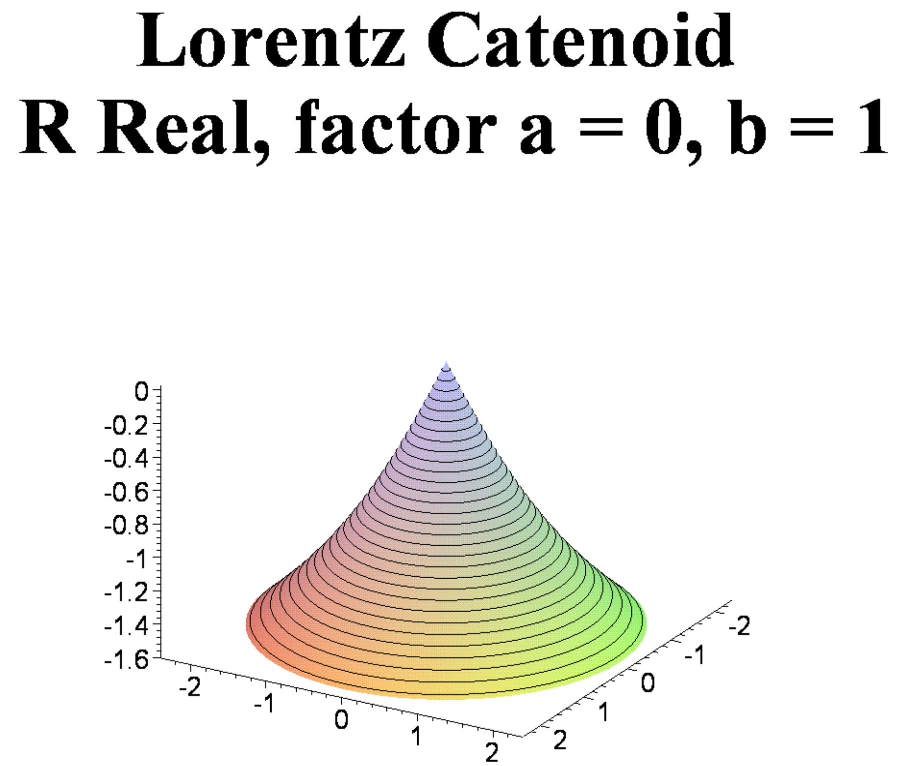


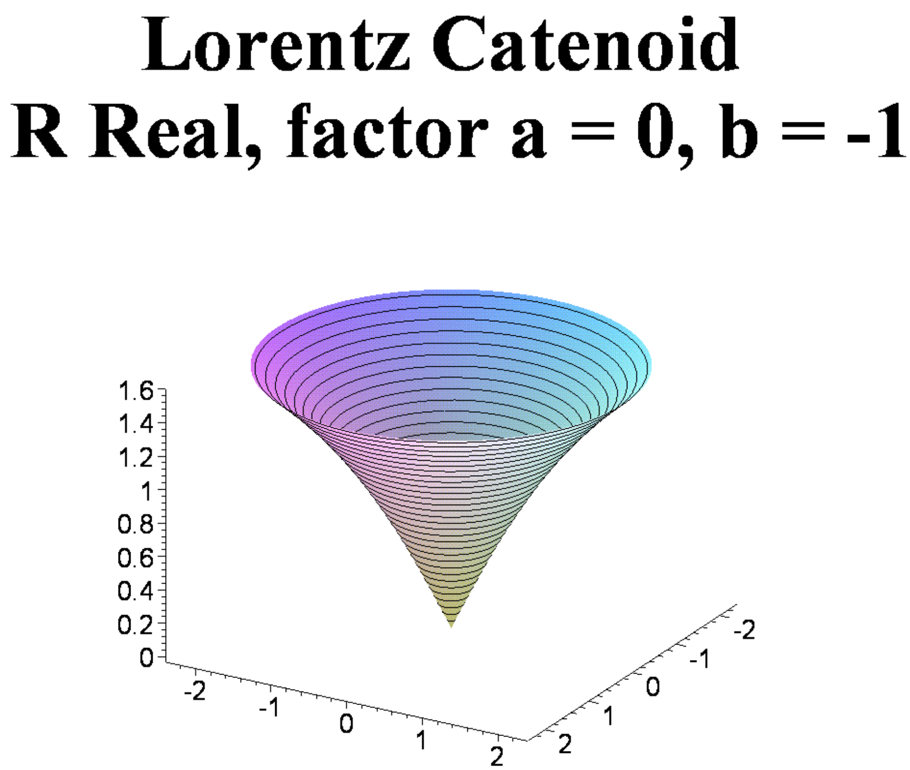
Lorentz Helix
R Real, Right-handed)
factor a = -1, b = 0




```
> plot3d(subs(a=0,b=1,Rreal),u=-0..Pi/2,v=-0..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Lorentz Catenoid \n R Real, factor a = 0, b =
1`);plot3d(subs(a=0,b=-1,Rreal),u=-0..Pi/2,v=-0..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
```

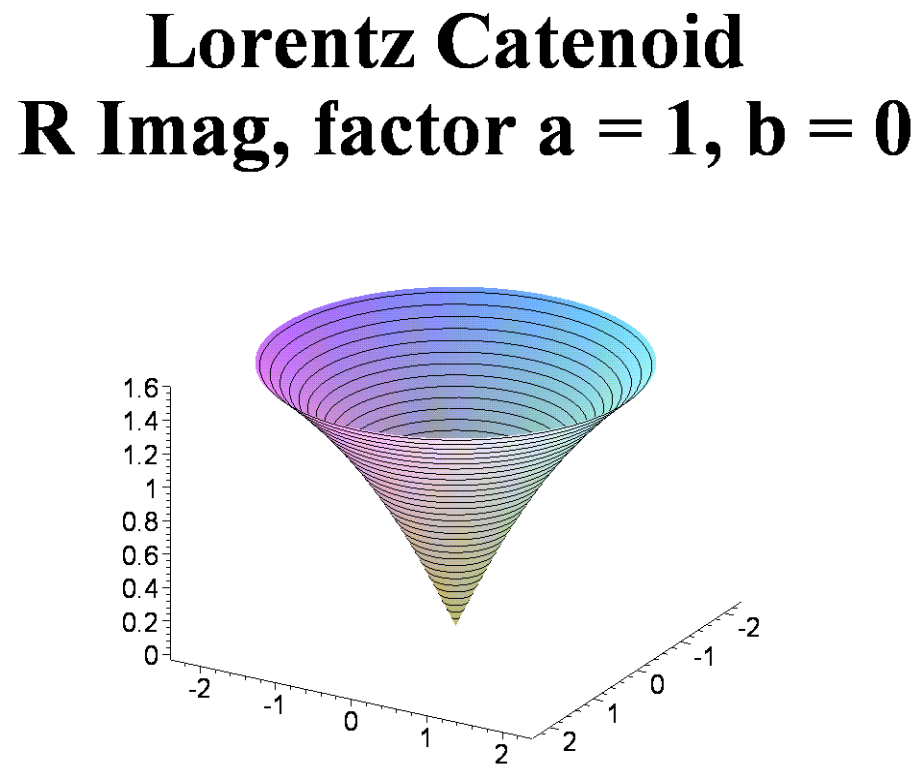
```
tlefont=[TIMES,BOLD,30],title=`Lorentz Catenoid \n R Real, factor a = 0, b =
-1`);
```

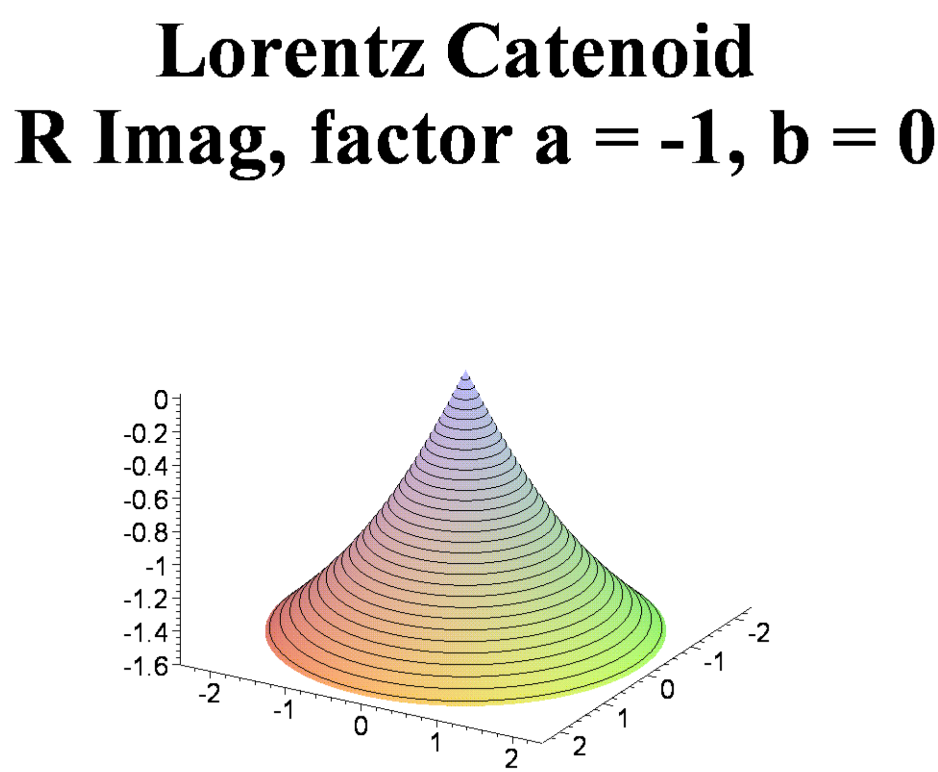





```
> plot3d(subs(a=1,b=0,Rimag),u=-0..Pi/2,v=-0*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Lorentz Catenoid \n R Imag, factor a = 1, b =
```

```
0`);plot3d(subs(a=-1,b=0,Rimag),u=-0..Pi/2,v=-0*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Lorentz Catenoid \n R Imag, factor a = -1, b =
0`);
```

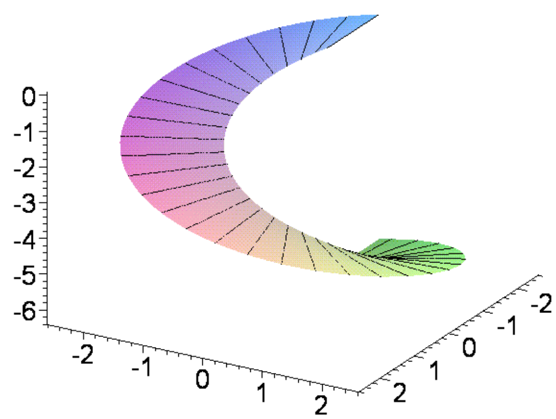




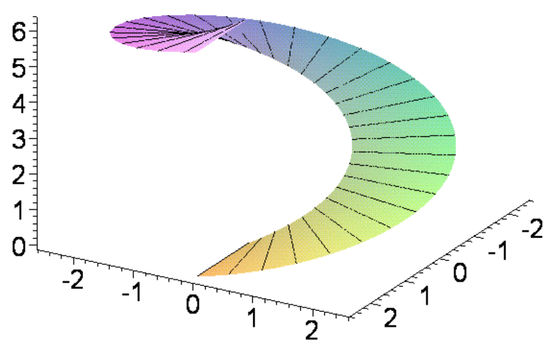
>

```
>
> plot3d(subs(a=0,b=1,Rimag),u=-0..Pi/2,v=-0..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Lorenz Helix \n R Imag, Right-handed) \n  factor
a = 0, b = = 1`);plot3d(subs(a=0,b =-1,Rimag),u=-0..Pi/2,v=-0..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Lorenz Helix \n R Imag, Right-handed) \n  factor
a = 0, b = = -1`);
```

Lorenz Helix
R Imag, Right-handed)
factor a = 0, b = = 1



Lorenz Helix
R Imag, Right-handed)
factor a = 0, b = = -1




```
> R:=0;
>
>
```

$$R := 0$$

```
> `Real-component`:=Rreal;PARAsurfLOR(Rreal);`Imag-component`:=Rimag;PARAsurfLOR(R
imag);`Metric`=evalm(GUN);`Lorentz Spinor`:=Sigma;`Spinor quadratic form using
metric`:=simplify(subs(a=a,b=b,innerprod((Sigma,evalm(GUN),Sigma))));
>
>
```

$$\mathit{Real\text{-}component} := [-a\cosh(u)\cos(v) + b\sinh(u)\sin(v), -b\sinh(u)\cos(v) - a\cosh(u)\sin(v), -a\,v - b\,u]$$

$$\mathit{Mean\ Curvature} = 0$$

$$\mathit{Gauss\ Curvature} = \frac{1}{(\cosh(u)^2 - 1)\,(a^2\cosh(u)^2 + b^2\cosh(u)^2 - b^2 - a^2)}$$

$$\mathit{Metric\ Det\ Q} = (b^2 + a^2)\,(a^2\cosh(u)^2 + b^2\cosh(u)^2 - b^2 - a^2)\,(\cosh(u)^2 - 1)$$

$$\mathit{Imag\text{-}component} := [-b\cosh(u)\cos(v) - a\sinh(u)\sin(v), a\sinh(u)\cos(v) - b\cosh(u)\sin(v), a\,u - b\,v]$$

$$\mathit{Mean\ Curvature} = 0$$

$$\mathit{Gauss\ Curvature} = \frac{1}{(\cosh(u)^2 - 1)\,(a^2\cosh(u)^2 + b^2\cosh(u)^2 - b^2 - a^2)}$$

$$\mathit{Metric\ Det\ Q} = (b^2 + a^2)\,(a^2\cosh(u)^2 + b^2\cosh(u)^2 - b^2 - a^2)\,(\cosh(u)^2 - 1)$$

$$\mathit{Metric} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\mathit{Lorentz\ Spinor} := [-(a + I\,b)\sinh(z), I\,(a + I\,b)\cosh(z), I\,(a + I\,b)]$$

$$\mathit{Spinor\ quadratic\ form\ using\ metric} := 0$$

Zero Mean Curvature surfaces from Hyperbolic Majorana Spinors.

```
> Sigma:=[-(a+I*b)*I*sinh(z),-(a+I*b)*cosh(z),(a+I*b)];evalc(subs(z=u+I*v,Sigma)):
  RR:=0:R:=0:
```

$$\Sigma := [-I\,(a+I\,b)\sinh(z),\ -(a+I\,b)\cosh(z),\ a+I\,b]$$

```
> X:=int(Sigma[1],z):Y:=int(Sigma[2],z):Z:=int(Sigma[3],z):
> R:=[X,Y,Z];RR:=evalc(subs(z=u+I*v,R)):
```

$$R := [-I\,(a+I\,b)\cosh(z),\ (-a-I\,b)\sinh(z),\ (a+I\,b)\,z]$$

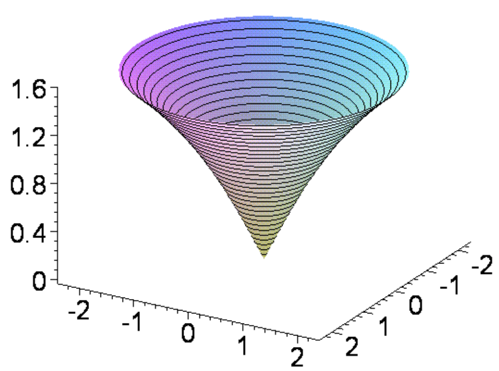
```
> RXreal:=evalc(Re(RR[1])):RXimag:=evalc(Im(RR[1])):
> RYreal:=evalc(Re(RR[2])):RYimag:=evalc(Im(RR[2])):
> RZreal:=evalc(Re(RR[3])):RZimag:=evalc(Im(RR[3])):
> Rreal  := subs(a=a,b=b,[RXreal,RYreal,RZreal]);Rimag  :=
  subs(a=a,b=b,[RXimag,RYimag,RZimag]);
```

$$Rreal := [a\sinh(u)\sin(v)+b\cosh(u)\cos(v),\ -a\sinh(u)\cos(v)+b\cosh(u)\sin(v),\ a\,u-b\,v]$$

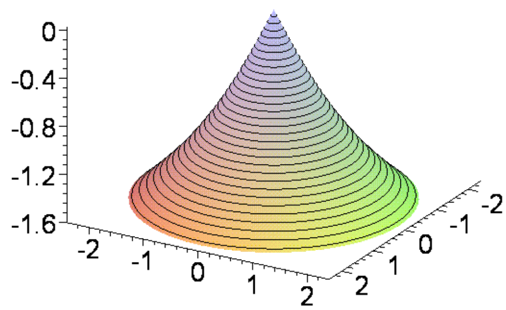
$$Rimag := [-a\cosh(u)\cos(v)+b\sinh(u)\sin(v),\ -b\sinh(u)\cos(v)-a\cosh(u)\sin(v),\ a\,v+b\,u]$$

```
> plot3d(subs(a=1,b=0,Rreal),u=-0..Pi/2,v=-0..2*Pi,
  numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`Majorana 2Pi symmetry \n R Real, Catenoid \n
  factor a = 1, b = 0 \n Increasing
  Z`);plot3d(subs(a=-1,b=0,Rreal),u=-0..Pi/2,v=-0..2*Pi,
  numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`Majorana 2Pi symmetry \n R Real, Catenoid \n
  factor a = -1, b = 0 \n Decreasing Z`);
```

Majorana 2Pi symmetry
R Real, Catenoid
factor a = 1, b = 0
Increasing Z

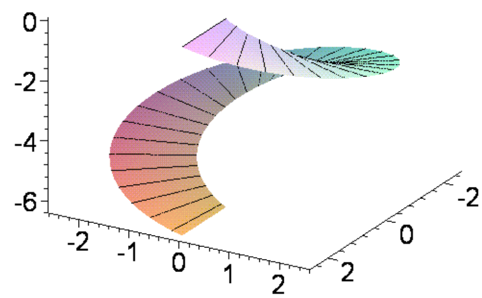


Majorana 2Pi symmetry
R Real, Catenoid
factor a = -1, b = 0
Decreasing Z

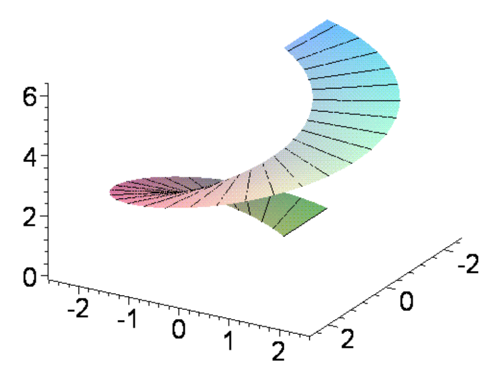



```
> plot3d(subs(a=0,b=1,Rreal),u=-0..Pi/2,v=-0..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Majorana Helix \n R real, Left handed \n factor a
= 0, b = 1 \n Decreasing Z`);plot3d(subs(a=0,b=-1,Rreal),u=-0..Pi/2,v=-0..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Majorana Helix \n R real, Right handed \n factor
a = 0, b = -1 \n Increasing Z`);
```

Majorana Helix
R real, Left handed
factor a = 0, b = 1
Decreasing Z



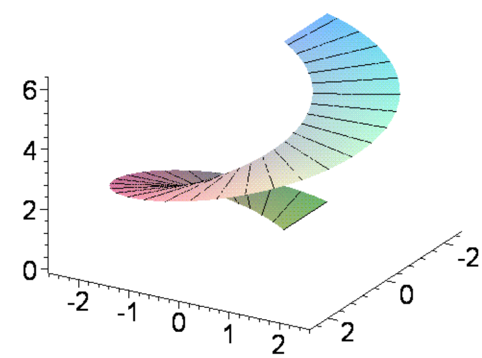
Majorana Helix
R real, Right handed
factor a = 0, b = -1
Increasing Z



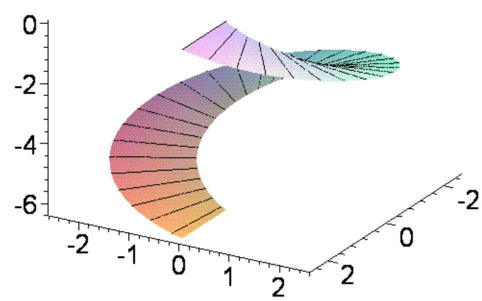

```
> plot3d(subs(a=1,b=0,Rimag),u=-0..Pi/2,v=-0..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Majorana Helix \n R Imaginary, Right-handed \n
factor a = 1, b = 0 \n Increasing Z`
);plot3d(subs(a=-1,b=0,Rimag),u=-0..Pi/2,v=-0..2*Pi,
```

```
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Majorana Helix \n R Imaginary, Left handed \n
factor a = -1, b = 0 \n Decreasing Z`);
```

Majorana Helix
R Imaginary, Right-handed
factor a = 1, b = 0
Increasing Z



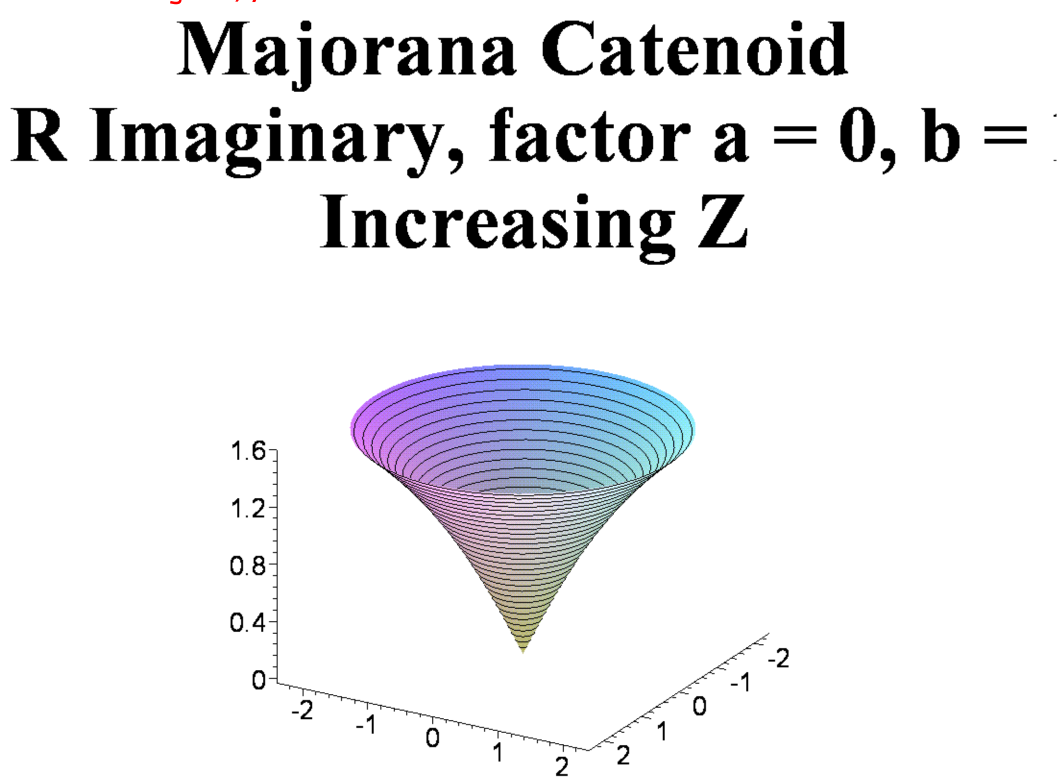
Majorana Helix
R Imaginary, Left handed
factor a = -1, b = 0
Decreasing Z



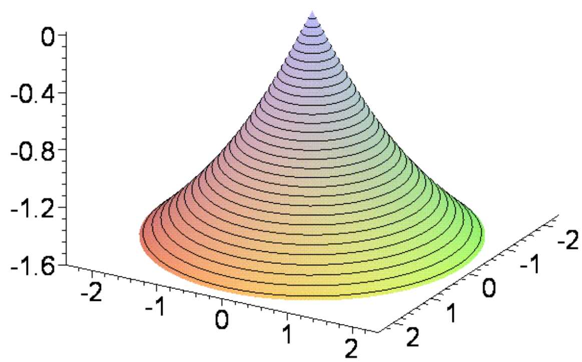
\>

\>

```
>
> plot3d(subs(a=0,b=1,Rimag),u=-0..Pi/2,v=-0..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Majorana Catenoid \n R Imaginary, factor a = 0, b
= 1 \n Increasing Z`);plot3d(subs(a=0,b=-1,Rimag),u=-0..Pi/2,v=-0..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Majorana Catenoid \n R Imaginary, factor a = 0, b
= -1 \n Decreasing Z`);
```



Majorana Catenoid
R Imaginary, factor a = 0, b = ·
Decreasing Z




```
> R:=0;
```

$$R := 0$$

```
> `Real-component`:=Rreal;PARAsurfMAJ(Rreal);`Imag-component`:=Rimag;PARAsurfMAJ(Rimag);`Metric`=evalm(GUN);`Majorana Spinor`:=Sigma;`Spinor quadratic form using metric`:=simplify(subs(a=a,b=b,innerprod((Sigma,GUN,Sigma))));
>
>
```

$$\textit{Real-component} := [a\sinh(u)\sin(v) + b\cosh(u)\cos(v), -a\sinh(u)\cos(v) + b\cosh(u)\sin(v), a\,u - b\,v]$$

$$\textit{Mean Curvature} = 0$$

$$\textit{Gauss Curvature} = -\frac{1}{(\cosh(u)^2 - 1)(a^2\cosh(u)^2 + b^2\cosh(u)^2 - b^2 - a^2)}$$

$$\textit{Metric Det Q } = (b^2 + a^2)(a^2\cosh(u)^2 + b^2\cosh(u)^2 - b^2 - a^2)(\cosh(u)^2 - 1)$$

$$\textit{Imag-component} := [-a\cosh(u)\cos(v) + b\sinh(u)\sin(v), -b\sinh(u)\cos(v) - a\cosh(u)\sin(v), a\,v + b\,u]$$

$$\textit{Mean Curvature} = 0$$

$$\textit{Gauss Curvature} = -\frac{1}{(\cosh(u)^2 - 1)(a^2\cosh(u)^2 + b^2\cosh(u)^2 - b^2 - a^2)}$$

$$\textit{Metric Det Q } = (b^2 + a^2)(a^2\cosh(u)^2 + b^2\cosh(u)^2 - b^2 - a^2)(\cosh(u)^2 - 1)$$

$$\textit{Metric} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\textit{Majorana Spinor} := [-I(a + I\,b)\sinh(z), -(a + I\,b)\cosh(z), a + I\,b]$$

$$\textit{Spinor quadratic form using metric} := 0$$

Note that the Zero mean curvature Lorentz surface

generated from Hyperbolic Spinor direction fields has a positive Gauss curvature,

where the Zero mean curvature Euclidean and Majorana sufaces generated from Hyperbolic Spinor direction fields have a Negative Gauss curvature.

Also note that the Lorentz Zero mean curvature surface generated from Hyperbolic Spinor direction fields has only left handed helices,

where the Majorana Zero mean curvature surface generated from Hyperbolic Spinor direction fields has both left and right helicities.

The Euclidean helices do not have a hole. The Lorentz and Majorana helices, generated from Hyperbolic Spinor direction fields, have a "hole"

****

Zero Mean Curvature surfaces from 4Pi (Non-Orientable?) Majorana Spinors. z=u+I*v/2

```
> Sigma:=[-(a+I*b)*I*sinh(z),-(a+I*b)*cosh(z),(a+I*b)];evalc(subs(z=u+I*v/2,Sigma)
):RR:=0:R:=0:
```

$$\Sigma := [-I\,(a+I\,b)\sinh(z), -(a+I\,b)\cosh(z), a+I\,b]$$

```
> X:=int(Sigma[1],z):Y:=int(Sigma[2],z):Z:=int(Sigma[3],z):
> R:=[X,Y,Z];RR:=evalc(subs(z=u+I*v/2,R)):
```

$$R := [-I\,(a+I\,b)\cosh(z), (-a-I\,b)\sinh(z), (a+I\,b)\,z]$$

```
> RXreal:=evalc(Re(RR[1])):RXimag:=evalc(Im(RR[1])):
> RYreal:=evalc(Re(RR[2])):RYimag:=evalc(Im(RR[2])):
> RZreal:=evalc(Re(RR[3])):RZimag:=evalc(Im(RR[3])):
> Rreal := subs(a=a,b=b,[RXreal,RYreal,RZreal]);Rimag :=
subs(a=a,b=b,[RXimag,RYimag,RZimag]);
```

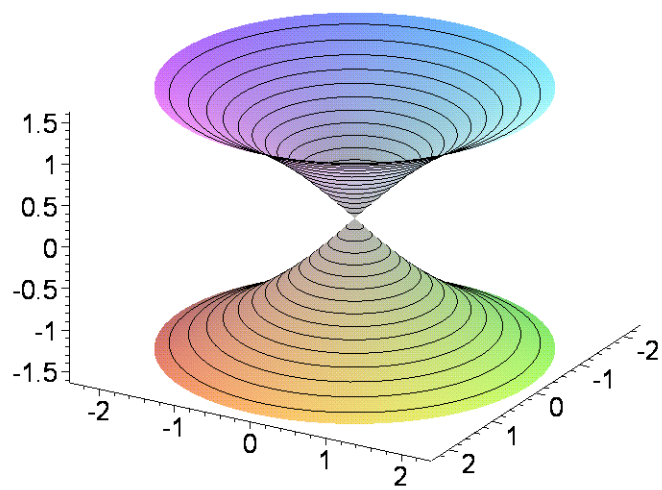
$$Rreal := \left[b\cosh(u)\cos\left(\frac{1}{2}v\right)+a\sinh(u)\sin\left(\frac{1}{2}v\right), -a\sinh(u)\cos\left(\frac{1}{2}v\right)+b\cosh(u)\sin\left(\frac{1}{2}v\right), a\,u-\frac{1}{2}b\,v\right]$$

$$Rimag := \left[-a\cosh(u)\cos\left(\frac{1}{2}v\right)+b\sinh(u)\sin\left(\frac{1}{2}v\right), -b\sinh(u)\cos\left(\frac{1}{2}v\right)-a\cosh(u)\sin\left(\frac{1}{2}v\right), \frac{1}{2}a\,v+b\,u\right]$$

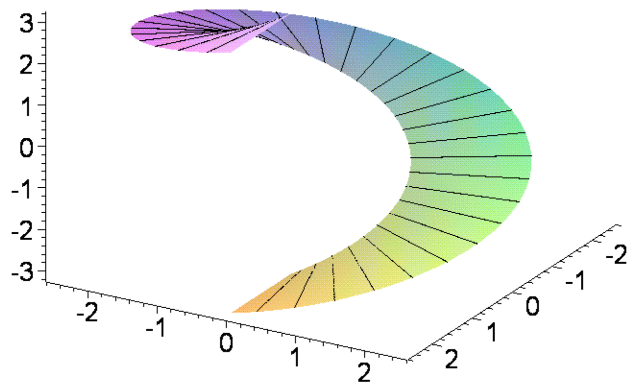
```
> plot3d(subs(a=1,b=0,Rreal),u=-Pi/2..Pi/2,v=-2*Pi..2*Pi,
```

```
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Majorana 4Pi symmetry \n R Real, factor a = 1, b
= 0`);plot3d(subs(a=1,b=0,Rimag),u=-Pi/2..Pi/2,v=-2*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`R Imag (right-handed) \n factor a = 1, b =
0`);plot3d(subs(a=0,b=1,Rreal),u=-Pi/2..Pi/2,v=-2*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`R Real, left-handed) \n factor a = 0, b =
1`);plot3d(subs(a=0,b=1,Rimag),u=-Pi/2..Pi/2,v=-2*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`R Imag, factor a = 0, b = 1`);
```

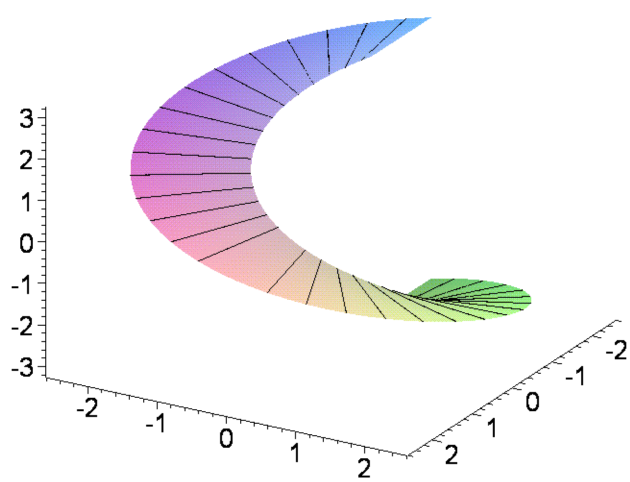
Majorana 4Pi symmetry
R Real, factor a = 1, b = 0



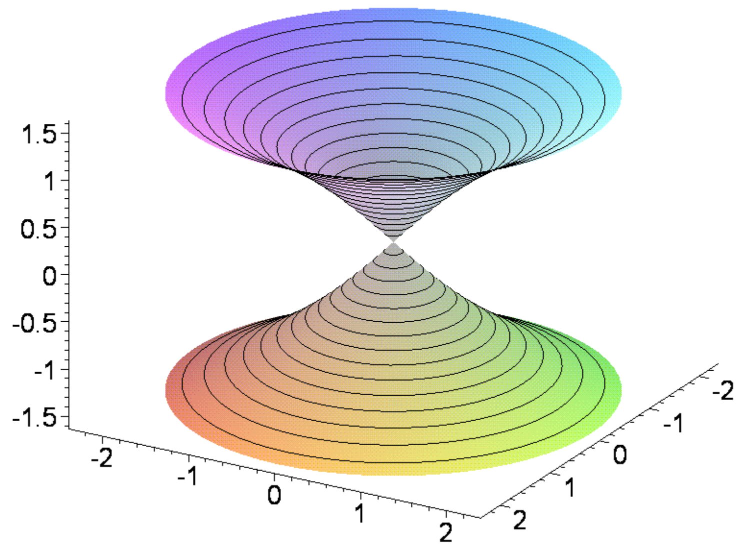
R Imag (right-handed) factor a = 1, b = 0



R Real, left-handed) factor a = 0, b = 1



R Imag, factor a = 0, b = 1




```
> R:=0;
> `Real-component`:=Rreal;PARAsurfMAJ(Rreal);`Imag-component`:=Rimag;PARAsurfMAJ(Rimag);`Metric`=evalm(GUN);`Majorana Spinor`:=Sigma;`Spinor quadratic form using metric`:=simplify(subs(a=a,b=b,innerprod((Sigma,GUN,Sigma))));
```

$$R := 0$$

Real-component :=

$$\left[b\cosh(u)\cos\left(\frac{1}{2}v\right)+a\sinh(u)\sin\left(\frac{1}{2}v\right), -a\sinh(u)\cos\left(\frac{1}{2}v\right)+b\cosh(u)\sin\left(\frac{1}{2}v\right), a\,u-\frac{1}{2}b\,v\right]$$

$$\textit{Mean Curvature} = 0$$

$$\textit{Gauss Curvature} = -\frac{1}{(\cosh(u)^2-1)\,(a^2\cosh(u)^2+b^2\cosh(u)^2-b^2-a^2)}$$

$$\textit{Metric Det Q} = \frac{1}{4}(b^2+a^2)\,(a^2\cosh(u)^2+b^2\cosh(u)^2-b^2-a^2)\,(\cosh(u)^2-1)$$

Imag-component :=

$$\left[-a\cosh(u)\cos\left(\frac{1}{2}v\right)+b\sinh(u)\sin\left(\frac{1}{2}v\right), -b\sinh(u)\cos\left(\frac{1}{2}v\right)-a\cosh(u)\sin\left(\frac{1}{2}v\right), \frac{1}{2}a\,v+b\,u\right]$$

$$\textit{Mean Curvature} = 0$$

$$\textit{Gauss Curvature} = -\frac{1}{(\cosh(u)^2-1)\,(a^2\cosh(u)^2+b^2\cosh(u)^2-b^2-a^2)}$$

$$\textit{Metric Det Q} = \frac{1}{4}(b^2+a^2)\,(a^2\cosh(u)^2+b^2\cosh(u)^2-b^2-a^2)\,(\cosh(u)^2-1)$$

$$\textit{Metric} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Majorana\ Spinor := [-I\,(a+I\,b)\sinh(z), -(a+I\,b)\cosh(z), a+I\,b]$$

$$Spinor\ quadratic\ form\ using\ metric := 0$$

```
>
>
```

****

CONICAL MINIMAL SURFACE of TWO PATCHES, WITH CONNECTION THREAD

```
> R:=[sinh(u)*cos(v),sinh(u)*sin(v),A*(u+B)];R1:=subs(A=1,B=2,R):R2:=subs(A=0,B=2,R):R3:=subs(A=-1,B=2,R):RZP:=[.05*cos(v),.05*sin(v),u]:RZM:=[.05*cos(v),.05*sin(v),-u]:
>
```

$$R := [\sinh(u)\cos(v), \sinh(u)\sin(v), A\,(u+B)]$$

Immersion is minimal only for A = 1, 0, -1 in Majorana or Lorentz space.

```
> PARAsurfLOR(R);
```

Mean Curvature =

$$-\frac{1}{2}\frac{(\cosh(u)-1)(\cosh(u)+1)A(A-1)(A+1)}{\sqrt{-(\cosh(u)-1)(\cosh(u)+1)(\cosh(u)-A)(\cosh(u)+A)}\,(\cosh(u)^2-1)(-A^2+\cosh(u)^2)}$$

$$Gauss\ Curvature = \frac{A^2}{(-A^2+\cosh(u)^2)^2}$$

$$Metric\ Det\ Q\ = (\cosh(u)^2-1)(-A^2+\cosh(u)^2)$$

```
> PARAsurfMAJ(R);
```

Mean Curvature =

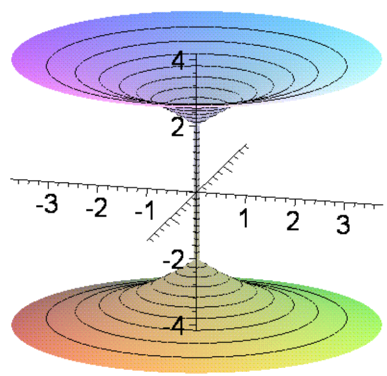
$$-\frac{1}{2}\frac{(\cosh(u)-1)(\cosh(u)+1)(A-1)(A+1)A}{\sqrt{(\cosh(u)-1)(\cosh(u)+1)(\cosh(u)-A)(\cosh(u)+A)}\,(\cosh(u)^2-1)(-A^2+\cosh(u)^2)}$$

$$Gauss\ Curvature = -\frac{A^2}{(-A^2+\cosh(u)^2)^2}$$

$$Metric\ Det\ Q\ = (\cosh(u)^2-1)(-A^2+\cosh(u)^2)$$

```
> plot3d({R1,RZP,RZM,R3},v=-0*Pi..2*Pi,u=-0..2,orientation=[15,70],numpoints=5000,axes=normal,style=PATCHCONTOUR,contours=20,titlefont=[TIMES,BOLD,20],title=`Mean curvature is zero in Majorana space \n for A = -1, 0, +1 \n Top endcap:  A = +1, B = 2 \n Bottom endcap:  A = -1, B = 2 \n Connecting String is for A = 0 \n Gauss curvature is negative `);
```

Mean curvature is zero in Majorana space
for A = -1, 0, +1
Top endcap: A = +1, B = 2
Bottom endcap: A = -1, B = 2
Connecting String is for A = 0
Gauss curvature is negative



Connect two dimpled patches with a singular string (thread) to create a deformed Falaco Soliton. See http://www22.pair.com/csdc/pdf/

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Minimal Surfaces from Elliptic Euclidean Spinors

```
> Sigma:=[(a+I*b)*sin(z),(a+I*b)*cos(z),(a*I-b)];RR:=0:R:=0:
```

$$\Sigma := [(a + I\,b)\sin(z),\ (a + I\,b)\cos(z),\ I\,a - b]$$

```
> 
> X:=int(Sigma[1],z):Y:=int(Sigma[2],z):Z:=int(Sigma[3],z):
> R:=[X,Y,Z];RR:=evalc(subs(z=(v+I*u),R)):
```

$$R := [-(a + I\,b)\cos(z),\ (a + I\,b)\sin(z),\ (I\,a - b)\,z]$$

```
> RXreal:=evalc(Re(RR[1])):RXimag:=evalc(Im(RR[1])):
> RYreal:=evalc(Re(RR[2])):RYimag:=evalc(Im(RR[2])):
> RZreal:=evalc(Re(RR[3])):RZimag:=evalc(Im(RR[3])):
> Rreal := subs(a=a,b=b,[RXreal,RYreal,RZreal]);Rimag :=
  subs(a=a,b=b,[RXimag,RYimag,RZimag]);
```

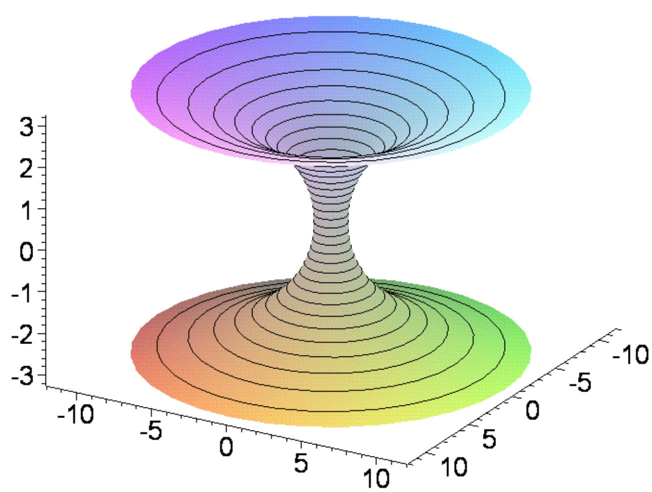
$$Rreal := [-a\cos(v)\cosh(u) - b\sinh(u)\sin(v),\ a\sin(v)\cosh(u) - b\sinh(u)\cos(v),\ -a\,u - b\,v]$$

$$Rimag := [-b\cos(v)\cosh(u) + a\sinh(u)\sin(v),\ b\sin(v)\cosh(u) + a\sinh(u)\cos(v),\ a\,v - b\,u]$$

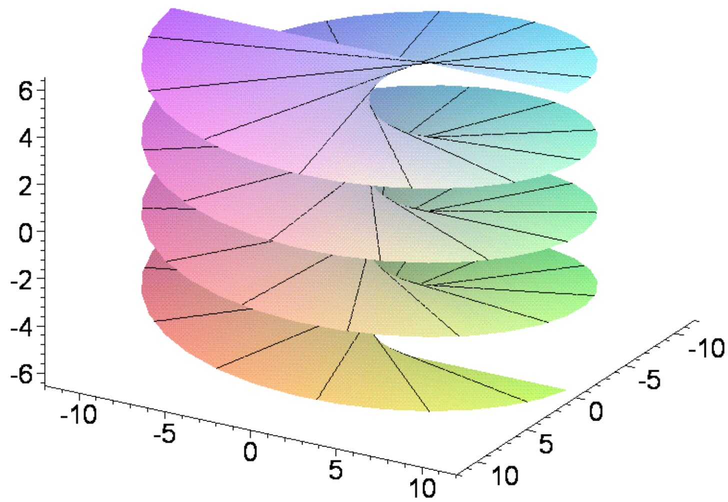
```
> plot3d(subs(a=1,b=0,Rreal),u=-Pi..Pi,v=-2*Pi..2*Pi,
  numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`Euclidean \n R Real, factor a = 1, b =
  0`);plot3d(subs(a=1,b=0,Rimag),u=-Pi..Pi,v=-2*Pi..2*Pi,
  numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`R Imag, factor a = 1, b =
  0`);plot3d(subs(a=0,b=1,Rreal),u=-Pi..Pi,v=-2*Pi..2*Pi,
  numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`R Real, factor a = 0, b =
  1`);plot3d(subs(a=0,b=1,Rimag),u=-Pi..Pi,v=-2*Pi..2*Pi,
  numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`R Imag, factor a = 0, b = 1`);
```

Euclidean

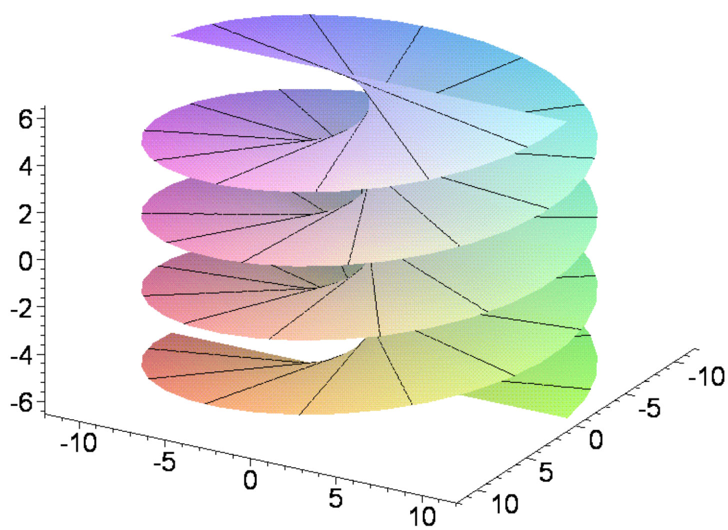
R Real, factor a = 1, b = 0



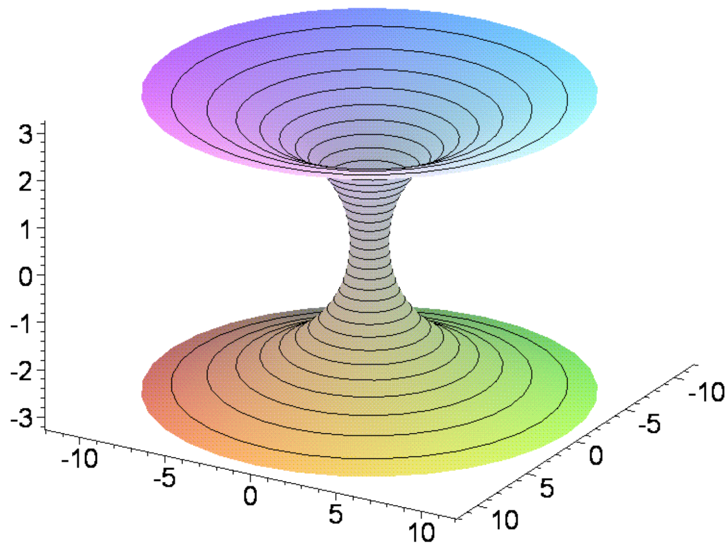
R Imag, factor a = 1, b = 0



R Real, factor a = 0, b = 1



R Imag, factor a = 0, b = 1




```
> R:=0;
```

$$R := 0$$

```
> `Real-component`:=Rreal;PARAsurf(Rreal);`Imag-component`:=Rimag;PARAsurf(Rimag);
  `Metric`=evalm(GUN);`Euclidean Spinor`:=Sigma;`Spinor quadratic form using
  metric`:=simplify(subs(a=a,b=b,innerprod((Sigma,GUN,Sigma))));
>
```

$$\textit{Real-component} := [-a\cos(v)\cosh(u) - b\sinh(u)\sin(v),\ a\sin(v)\cosh(u) - b\sinh(u)\cos(v),\ -a\,u - b\,v]$$

$$\textit{Mean Curvature} = 0$$

$$\textit{Gauss Curvature} = -\frac{1}{(b^2+a^2)\cosh(u)^4}$$

$$\textit{Metric Det Q} = \cosh(u)^4\,(b^2+a^2)^2$$

$$\textit{Imag-component} := [-b\cos(v)\cosh(u) + a\sinh(u)\sin(v),\ b\sin(v)\cosh(u) + a\sinh(u)\cos(v),\ a\,v - b\,u]$$

$$\textit{Mean Curvature} = 0$$

$$\textit{Gauss Curvature} = -\frac{1}{(b^2+a^2)\cosh(u)^4}$$

$$\textit{Metric Det Q} = \cosh(u)^4\,(b^2+a^2)^2$$

$$\textit{Metric} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\textit{Euclidean Spinor} := [(a + I\,b)\sin(z),\ (a + I\,b)\cos(z),\ I\,a - b]$$

$$\textit{Spinor quadratic form using metric} := 0$$

Minimal mean curvature surfaces from Elliptic Lorentzian Spinors

```
> Sigma:=[(a+I*b)*sin(z),(a+I*b)*cos(z),(a+I*b)];
```

$$\Sigma := [(a + I\,b)\sin(z),\ (a + I\,b)\cos(z),\ a + I\,b]$$

```
> X:=int(Sigma[1],z):Y:=int(Sigma[2],z):Z:=int(Sigma[3],z):
> R:=[X,Y,Z];RR:=evalc(subs(z=v+I*u,R)):
```

$$R := [-(a + I\,b)\cos(z),\ (a + I\,b)\sin(z),\ (a + I\,b)\,z]$$

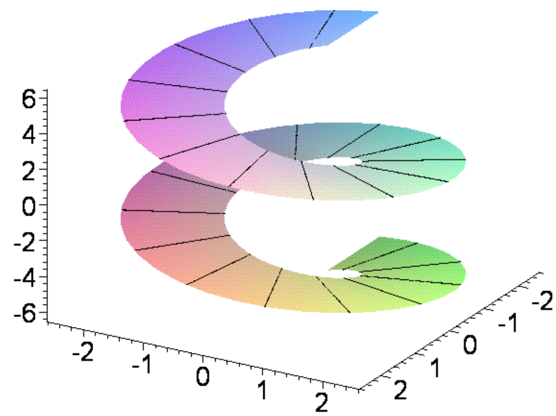
```
> RXreal:=evalc(Re(RR[1])):RXimag:=evalc(Im(RR[1])):
> RYreal:=evalc(Re(RR[2])):RYimag:=evalc(Im(RR[2])):
> RZreal:=evalc(Re(RR[3])):RZimag:=evalc(Im(RR[3])):
> Rreal := subs(a=a,b=b,[RXreal,RYreal,RZreal]);Rimag :=
subs(a=a,b=b,[RXimag,RYimag,RZimag]);
```

$$Rreal := [-a\cos(v)\cosh(u) - b\sinh(u)\sin(v),\ a\sin(v)\cosh(u) - b\sinh(u)\cos(v),\ a\,v - b\,u]$$

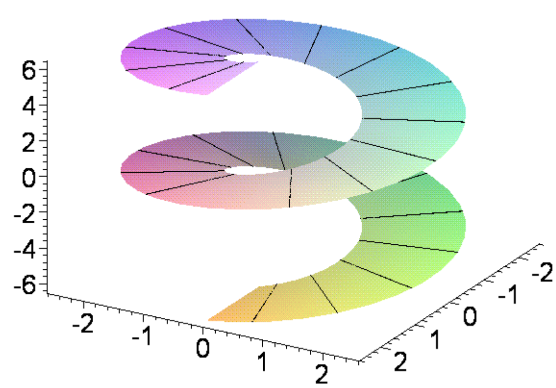
$$Rimag := [-b\cos(v)\cosh(u) + a\sinh(u)\sin(v),\ b\sin(v)\cosh(u) + a\sinh(u)\cos(v),\ a\,u + b\,v]$$

```
> plot3d(subs(a=1,b=0,Rreal),u=-Pi/2..Pi/2,v=-2*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Lorentz Helix \n R Real, left-handed \n factor a
= 1, b = 0`);plot3d(subs(a=-1,b=0,Rreal),u=-Pi/2..Pi/2,v=-2*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Lorentz Helix \n R Real, Right-handed \n factor a
= -1, b = 0`);plot3d(subs(a=1,b=0,Rimag),u=-Pi/2..Pi/2,v=-2*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Lorentz Catenoid \n R Imag, factor a = 1, b =
0`);plot3d(subs(a=0,b=1,Rreal),u=-Pi/2..Pi/2,v=-2*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Lorentz Catenoid \n R Real, factor a = 0, b =
1`);plot3d(subs(a=0,b=1,Rimag),u=-Pi/2..Pi/2,v=-2*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Lorentz Helix \n R Imag, left-handed) \n  factor
a = 0, b = 1`);plot3d(subs(a=0,b=-1,Rimag),u=-Pi/2..Pi/2,v=-2*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Lorentz Helix \n R Imag, Right-handed) \n  factor
a = 0, b = -1`);
```

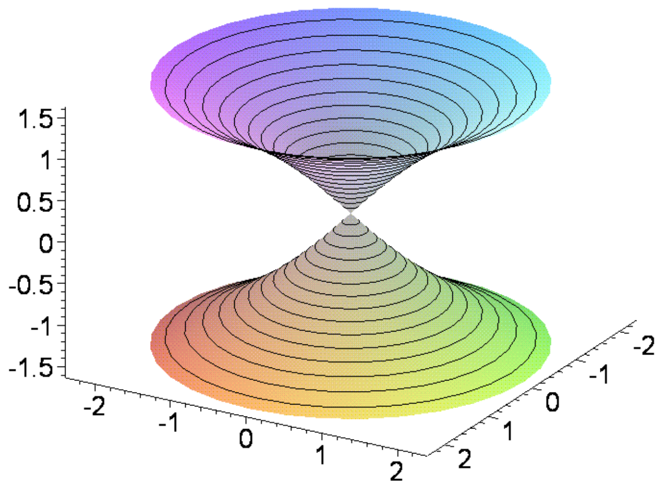
Lorentz Helix
R Real, left-handed
factor a = 1, b = 0



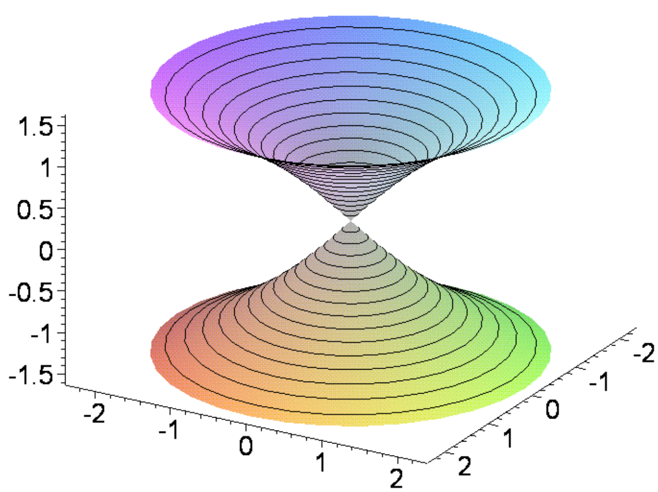
Lorentz Helix
R Real, Right-handed
factor a = -1, b = 0



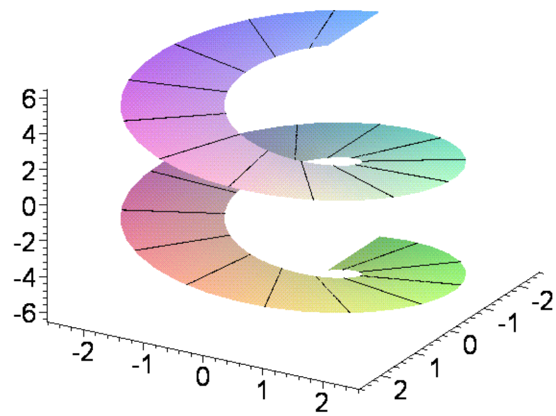
Lorentz Catenoid
R Imag, factor a = 1, b = 0



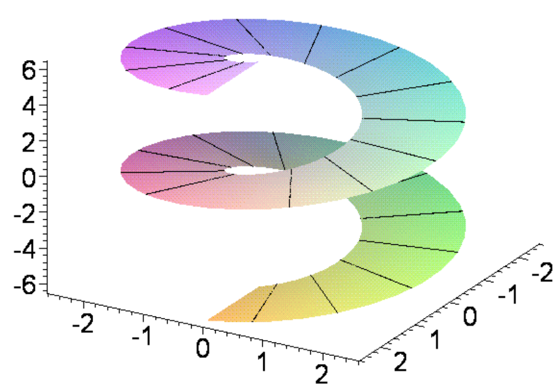
Lorentz Catenoid
R Real, factor a = 0, b = 1



Lorentz Helix
R Imag, left-handed)
factor a = 0, b = 1



Lorentz Helix
R Imag, Right-handed)
factor a = 0, b = -1




```
> R:=0;
>
>
```

$$R := 0$$

```
> `Real-component`:=Rreal;PARAsurfLOR(Rreal);`Imag-component`:=Rimag;PARAsurfLOR(R
imag);`Metric`=evalm(GUN);`Lorentz Spinor`:=Sigma;`Spinor quadratic form using
metric`:=simplify(subs(a=a,b=b,innerprod((Sigma,evalm(GUN),Sigma))));
>
>
```

$$\textit{Real-component} := [-a\cos(v)\cosh(u) - b\sinh(u)\sin(v),\ a\sin(v)\cosh(u) - b\sinh(u)\cos(v),\ a\,v - b\,u]$$

$$\textit{Mean Curvature} = 0$$

$$\textit{Gauss Curvature} = \frac{1}{(\cosh(u)^2-1)\,(b^2\cosh(u)^2 + a^2\cosh(u)^2 - a^2 - b^2)}$$

$$\textit{Metric Det Q} = (b^2+a^2)\,(b^2\cosh(u)^2 + a^2\cosh(u)^2 - a^2 - b^2)\,(\cosh(u)^2-1)$$

$$\textit{Imag-component} := [-b\cos(v)\cosh(u) + a\sinh(u)\sin(v),\ b\sin(v)\cosh(u) + a\sinh(u)\cos(v),\ a\,u + b\,v]$$

$$\textit{Mean Curvature} = 0$$

$$\textit{Gauss Curvature} = \frac{1}{(\cosh(u)^2-1)\,(b^2\cosh(u)^2 + a^2\cosh(u)^2 - a^2 - b^2)}$$

$$\textit{Metric Det Q} = (b^2+a^2)\,(b^2\cosh(u)^2 + a^2\cosh(u)^2 - a^2 - b^2)\,(\cosh(u)^2-1)$$

$$\textit{Metric} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\textit{Lorentz Spinor} := [(a+I\,b)\sin(z),\ (a+I\,b)\cos(z),\ a+I\,b]$$

$$\textit{Spinor quadratic form using metric} := 0$$

Zero Mean Curvature surfaces from Elliptic Majorana Spinors.

```
> Sigma:=[-(a+I*b)*sin(z),-(a+I*b)*cos(z),+(a+I*b)];evalc(subs(z=u+I*v,Sigma)):RR:
=0:R:=0:
```

$$\Sigma := [-(a+I\,b)\sin(z),\ -(a+I\,b)\cos(z),\ a+I\,b]$$

```
> X:=int(Sigma[1],z):Y:=int(Sigma[2],z):Z:=int(Sigma[3],z):
> R:=[X,Y,Z];RR:=evalc(subs(z=u+I*v,R)):
```

$$R := [-(-a-I\,b)\cos(z),\ (-a-I\,b)\sin(z),\ (a+I\,b)\,z]$$

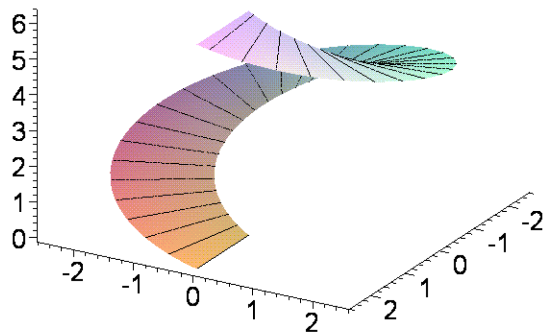
```
> RXreal:=evalc(Re(RR[1])):RXimag:=evalc(Im(RR[1])):
> RYreal:=evalc(Re(RR[2])):RYimag:=evalc(Im(RR[2])):
> RZreal:=evalc(Re(RR[3])):RZimag:=evalc(Im(RR[3])):
> Rreal := subs(a=a,b=b,[RXreal,RYreal,RZreal]);Rimag :=
subs(a=a,b=b,[RXimag,RYimag,RZimag]);
```

$$\textit{Rreal} := [a\cos(u)\cosh(v) + b\sin(u)\sinh(v),\ -a\sin(u)\cosh(v) + b\cos(u)\sinh(v),\ a\,u - b\,v]$$

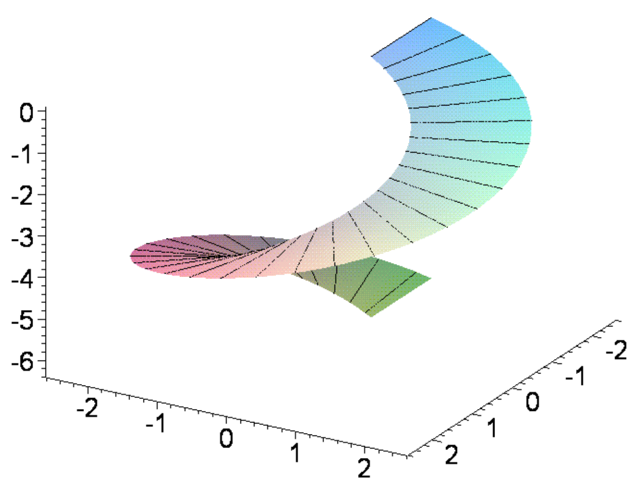
$$\textit{Rimag} := [b\cos(u)\cosh(v) - a\sin(u)\sinh(v),\ -b\sin(u)\cosh(v) - a\cos(u)\sinh(v),\ a\,v + b\,u]$$

```
> plot3d(subs(a=1,b=0,Rreal),v=-0..Pi/2,u=-0*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Majorana 2Pi symmetry \n R Real, factor a = 1, b
= 0 \n Left handed  Increasing
Z`);plot3d(subs(a=-1,b=0,Rreal),v=-0..Pi/2,u=-0*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=` R Real, factor a = -1, b = 0 \n Right handed
Decreasing Z`);
```

Majorana 2Pi symmetry
R Real, factor a = 1, b = 0
Left handed Increasing Z



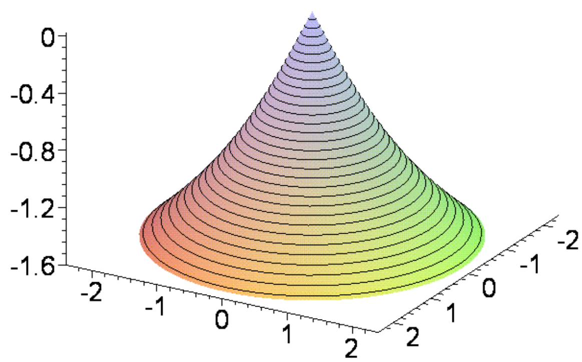
R Real, factor a = -1, b = 0
Right handed Decreasing Z



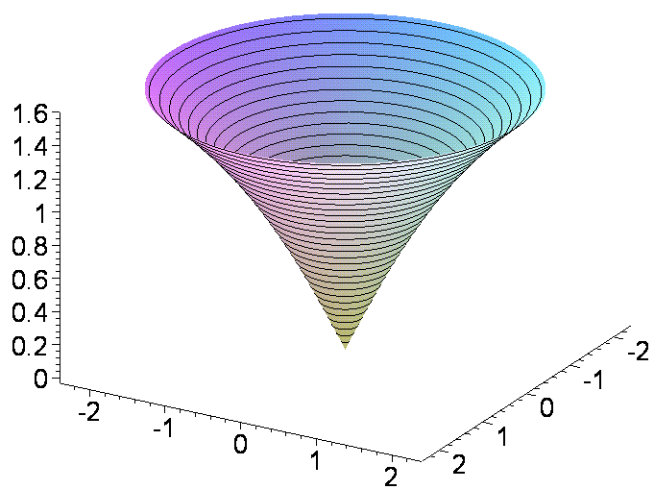

```
> plot3d(subs(a=0,b=1,Rreal),v=-0..Pi/2,u=-0*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Majorana 2Pi symmetry \n R Real, factor a = 0, b
= 1 \n Decreasing Z`);plot3d(subs(a=0,b=-1,Rreal),v=-0..Pi/2,u=-0*Pi..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
```

```
tlefont=[TIMES,BOLD,30],title=` R Real, factor a = 0, b = -1 \n Increasing Z`);
```

Majorana 2Pi symmetry
R Real, factor a = 0, b = 1
Decreasing Z



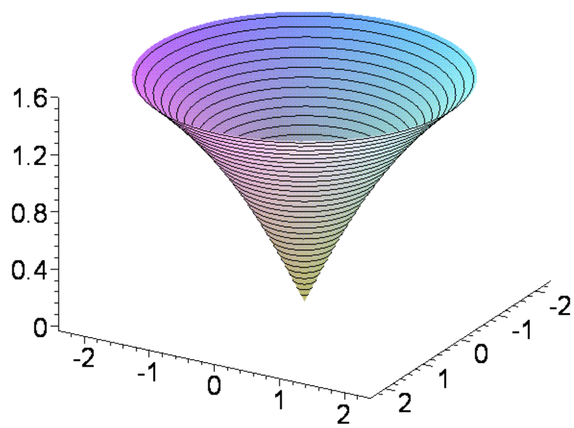
R Real, factor a = 0, b = -1
Increasing Z



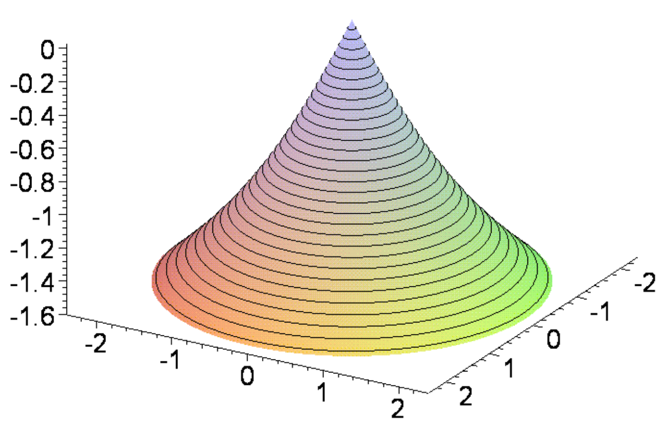

```
>
> plot3d(subs(a=1,b=0,Rimag),v=-0..Pi/2,u=-0..2*Pi,
  numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`Majorana 2Pi symmetry \n R Imag, factor a = 1, b
```

```
= 0 \n Increasing Z`);plot3d(subs(a=-1,b=0,Rimag),v=-0..Pi/2,u=-0..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=` R Imag, factor a = -1, b = 0 \n Decreasing Z`);
>
```

Majorana 2Pi symmetry
R Imag, factor a = 1, b = 0
Increasing Z

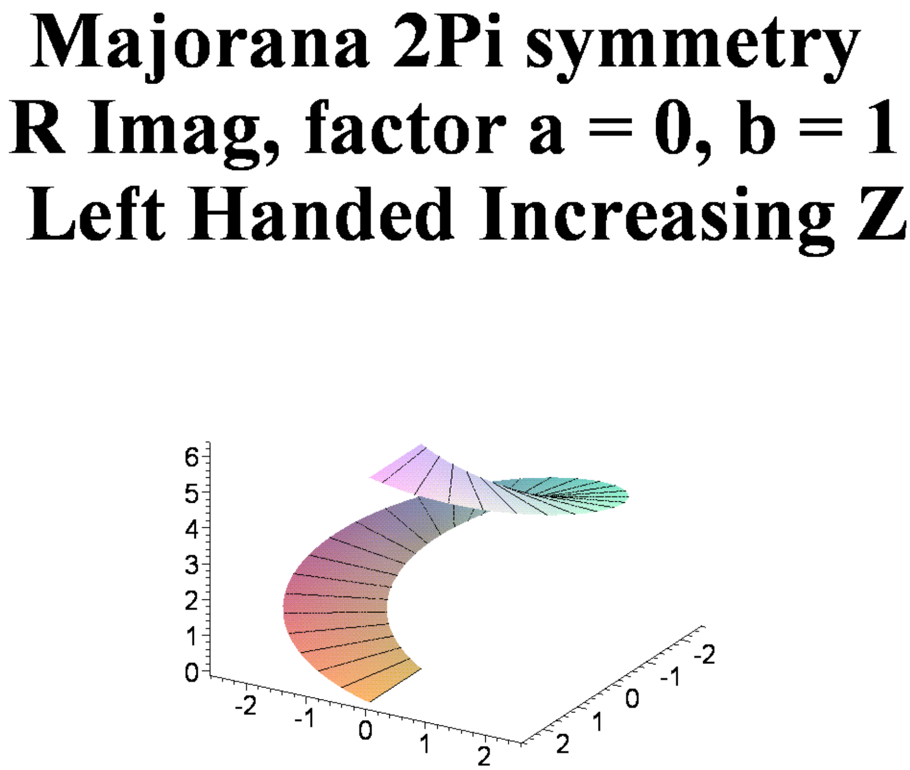


R Imag, factor a = -1, b = 0
Decreasing Z

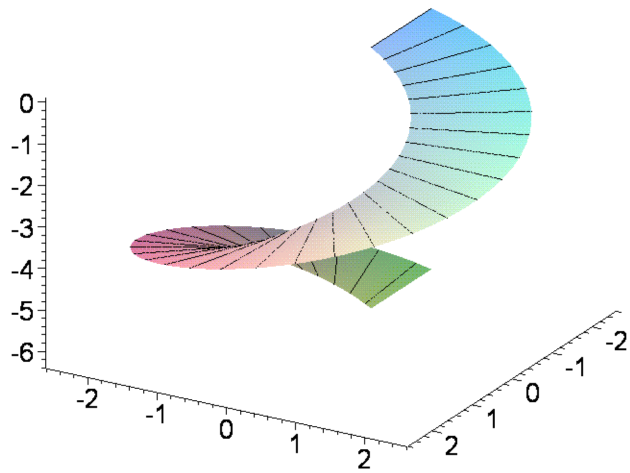



```
>
```

```
> plot3d(subs(a=0,b=1,Rimag),v=-0..Pi/2,u=-0..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=`Majorana 2Pi symmetry \n R Imag, factor a = 0, b
= 1 \n Left Handed Increasing
Z`);plot3d(subs(a=0,b=-1,Rimag),v=-0..Pi/2,u=-0..2*Pi,
numpoints=4096,axes=framed,orientation=[30,60],contours=20,style=patchcontour,ti
tlefont=[TIMES,BOLD,30],title=` R Imag, factor a = 0, b = -1 \n Right Handed
Decreasing Z`);
>
```



R Imag, factor a = 0, b = -1
Right Handed Decreasing Z

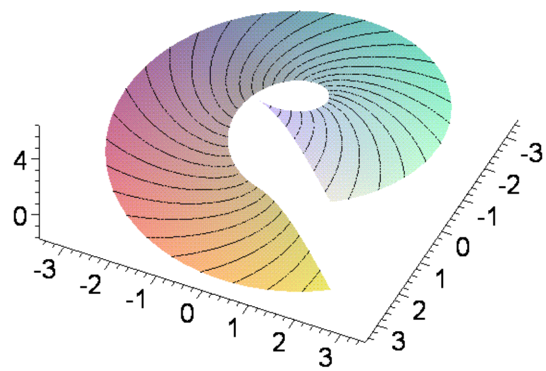



```
>
>
>
>
```

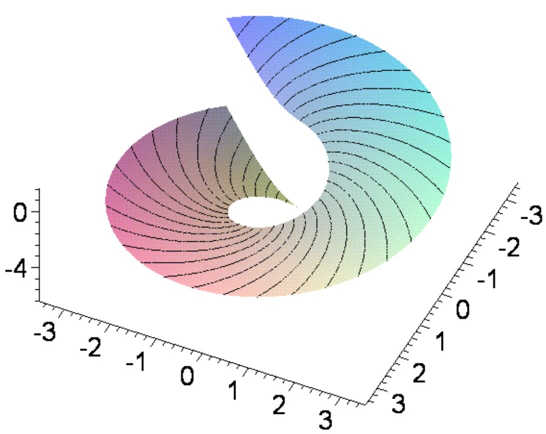
HELICOIDS

```
> plot3d(subs(a=1,b=1,Rreal),v=-0..Pi/2,u=-0..2*Pi,
  numpoints=4096,axes=framed,orientation=[25,25],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`R Real \n Left-handed Helicoids \n factor a = 1,
  b = 1`);plot3d(subs(a=1,b=-1,Rimag),v=-0..Pi/2,u=-0..2*Pi,
  numpoints=4096,axes=framed,orientation=[25,25],contours=20,style=patchcontour,ti
  tlefont=[TIMES,BOLD,30],title=`R Imag \n Right Handed Helicoids \n factor a = 1,
  b = -1`);
```

R Real
Left-handed Helicoids
factor a = 1, b = 1



R Imag
Right Handed Helicoids
factor a = 1, b = -1




```
> R:=0;
```

$R := 0$

```
> `Real-component`:=Rreal;PARAsurfMAJ(Rreal);`Imag-component`:=Rimag;PARAsurfMAJ(R
  imag);`Metric`=evalm(GUN);`Majorana Spinor`:=Sigma;`Spinor quadratic form using
```

```
metric`:=simplify(subs(a=a,b=b,innerprod((Sigma,GUN,Sigma))));
>
>
```

$$Real\text{-}component := [a\cos(u)\cosh(v) + b\sin(u)\sinh(v), -a\sin(u)\cosh(v) + b\cos(u)\sinh(v), a\,u - b\,v]$$

$$Mean\ Curvature = 0$$

$$Gauss\ Curvature = -\frac{1}{(\cosh(v)^2 - 1)(\cosh(v)^2 a^2 + b^2\cosh(v)^2 - b^2 - a^2)}$$

$$Metric\ Det\ Q\ = (b^2 + a^2)(\cosh(v)^2 a^2 + b^2\cosh(v)^2 - b^2 - a^2)(\cosh(v)^2 - 1)$$

$$Imag\text{-}component := [b\cos(u)\cosh(v) - a\sin(u)\sinh(v), -b\sin(u)\cosh(v) - a\cos(u)\sinh(v), a\,v + b\,u]$$

$$Mean\ Curvature = 0$$

$$Gauss\ Curvature = -\frac{1}{(\cosh(v)^2 - 1)(\cosh(v)^2 a^2 + b^2\cosh(v)^2 - b^2 - a^2)}$$

$$Metric\ Det\ Q\ = (b^2 + a^2)(\cosh(v)^2 a^2 + b^2\cosh(v)^2 - b^2 - a^2)(\cosh(v)^2 - 1)$$

$$Metric = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Majorana\ Spinor := [-(a + I\,b)\sin(z), -(a + I\,b)\cos(z), a + I\,b]$$

$$Spinor\ quadratic\ form\ using\ metric := 0$$

```
>
>
>
```