

```
> restart: with (linalg):with(liesymm):with(diffforms):
  setup(x,y,z,t):deform(x=0,y=0,z=0,t=0,Vx=0,Vy=0,Vz=0,D1=0,D2=0,D3=0,Ax=0,Ay=0,Az=0,C=0,Phi=0,a=const,b=const,c=const,Lx=0,Ly=0,Lz=0);
```

```
>
>
>
```

```
> ParticleAFFINE:=array([[a11,a12,a13,Vx],[a21,a22,a23,Vy],[a31,a32,a33,Vz],[0,0,0,1]]);
```

$$ParticleAFFINE = \begin{bmatrix} a11 & a12 & a13 & Vx \\ a21 & a22 & a23 & Vy \\ a31 & a32 & a33 & Vz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

There is another group of matrices, with n-1 zeros as constraints, and similar to the affine group, but not identical to it. The two types are distinguished herein by the words ParticleAFFINE and WaveAFFINE groups. The WaveAFFINE group has a representation given by the format:

```
> WaveAFFINE:=array([[a11,a12,a13,0],[a21,a22,a23,0],[a31,a32,a33,0],[Ax,Ay,Az,Phi]]);
```

$$WaveAFFINE = \begin{bmatrix} a11 & a12 & a13 & 0 \\ a21 & a22 & a23 & 0 \\ a31 & a32 & a33 & 0 \\ Ax & Ay & Az & \Phi \end{bmatrix}$$

A simple translation ParticleAFFINE matrix operates on the differential position vector to yield a Vierbein collection of 1 forms:

```
> TransAFFINE:=array([[1,0,0,Vx],[0,1,0,Vy],[0,0,1,Vz],[0,0,0,1]];PVierbein:=innerprod(TransAFFINE,[dx,dy,dz,dt]);DETR:=det(TransAFFINE):
```

$$TransAFFINE := \begin{bmatrix} 1 & 0 & 0 & Vx \\ 0 & 1 & 0 & Vy \\ 0 & 0 & 1 & Vz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$PVierbein := [dx + Vx dt, dy + Vy dt, dz + Vz dt, dt]$$

```
>
```

```
> PhaseAFFINE:=array([[1,0,0,0],[0,1,0,0],[0,0,1,0],[Ax,Ay,Az,Phi]];WVierbein:=innerprod(PhaseAFFINE,[dx,dy,dz,dt]);
```

$$PhaseAFFINE := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ Ax & Ay & Az & \Phi \end{bmatrix}$$

$$WVierbein := [dx, dy, dz, Ax dx + Ay dy + Az dz + \Phi dt]$$

```
>
```

```
> Action:=WVierbein[4];
```

$$Action := Ax dx + Ay dy + Az dz + \Phi dt$$

A more general matrix can be represented by the General Linear matrix in 4D

```
> GL:=array([[a11,a12,a13,-Vx],[a21,a22,a23,-Vy],[a31,a32,a33,-Vz],[Ax,Ay,Az,Phi]]);factor(det(GL)):
```

$$GL := \begin{bmatrix} a11 & a12 & a13 & -Vx \\ a21 & a22 & a23 & -Vy \\ a31 & a32 & a33 & -Vz \\ Ax & Ay & Az & \Phi \end{bmatrix}$$

THE INVERSE FRAME FFINVD and the Vierbein.

```
> FFINV:=array([[D2,Lz,0,0],[-Lz,D2,0,0],[0,0,-D1,-Vz],[0,0,0,-Phi]]);
```

$$FFINV := \begin{bmatrix} D2 & Lz & 0 & 0 \\ -Lz & D2 & 0 & 0 \\ 0 & 0 & -D1 & -Vz \\ 0 & 0 & 0 & -\Phi \end{bmatrix}$$

```
> Z:=innerprod(FFINV,[d(x),d(y),d(z),d(t)]):sigma1:=Z[1];sigma2:=Z[2];sigma3:=Z[3];
omega:=(Z[4]);
```

$$\sigma_1 := D2 d(x) + Lz d(y)$$

$$\sigma_2 := -Lz d(x) + D2 d(y)$$

$$\sigma_3 := -D1 d(z) - Vz d(t)$$

$$\omega := -\Phi d(t)$$

```
> Vol4:=wcollect(simplify(sigma1&^sigma2&^sigma3&^Z[4])):rho:=factor(getcoeff(Vol4));
```

$$\rho := \Phi (D2^2 + Lz^2) D1$$

```
> FF:=inverse(FFINV):Gun:=subs(innerprod(transpose(FF),FF));
```

$$Gun := \begin{bmatrix} \frac{1}{D2^2 + Lz^2} & 0 & 0 & 0 \\ 0 & \frac{1}{D2^2 + Lz^2} & 0 & 0 \\ 0 & 0 & \frac{1}{D1^2} & -\frac{Vz}{D1^2 \Phi} \\ 0 & 0 & -\frac{Vz}{D1^2 \Phi} & \frac{Vz^2 + D1^2}{D1^2 \Phi^2} \end{bmatrix}$$

```
> dFF:=array([[d(FF[1,1]),d(FF[1,2]),d(FF[1,3]),d(FF[1,4])],[d(FF[2,1]),d(FF[2,2]),
d(FF[2,3]),d(FF[2,4])],[d(FF[3,1]),d(FF[3,2]),d(FF[3,3]),d(FF[3,4])],[d(FF[4,1]),
d(FF[4,2]),d(FF[4,3]),d(FF[4,4])]]):
```

```
> cartan:=(evalm(FFINV*dFF));
```

The Interior (space-space) Connection 1 forms

```
> Gamma11:=factor(wcollect(cartan[1,1]));Gamma21:=factor(wcollect(cartan[2,1]));Gamma31:=factor(wcollect(cartan[3,1]));
```

$$\Gamma_{11} := -\frac{D2 d(D2) + Lz d(Lz)}{D2^2 + Lz^2}$$

$$\Gamma_{21} := \frac{-Lz \, d(D2) + D2 \, d(Lz)}{D2^2 + Lz^2}$$

$$\Gamma_{31} := 0$$

```
> Gamma12:=factor(wcollect(cartan[1,2]));Gamma22:=factor(wcollect(cartan[2,2]));Gamma32:=factor(wcollect(cartan[3,2]));
```

$$\Gamma_{12} := -\frac{-Lz \, d(D2) + D2 \, d(Lz)}{D2^2 + Lz^2}$$

$$\Gamma_{22} := -\frac{D2 \, d(D2) + Lz \, d(Lz)}{D2^2 + Lz^2}$$

$$\Gamma_{32} := 0$$

```
> Gamma13:=factor(wcollect(cartan[1,3]));Gamma23:=factor(wcollect(cartan[2,3]));Gamma33:=factor(wcollect(cartan[3,3]));
```

$$\Gamma_{13} := 0$$

$$\Gamma_{23} := 0$$

$$\Gamma_{33} := -\frac{d(D1)}{D1}$$

```
>
```

The "space-time" connection 1-forms are:

```
> hh1:=factor(wcollect(cartan[4,1]));hh2:=factor(wcollect(cartan[4,2]));hh3:=factor(wcollect(cartan[4,3]));
```

$$hh1 := 0$$

$$hh2 := 0$$

$$hh3 := 0$$

```
>
```

```
> gg1:=factor(wcollect(factor(wcollect(cartan[1,4]))));gg2:=factor(wcollect(cartan[2,4]));gg3:=factor(wcollect(cartan[3,4]));
```

$$gg1 := 0$$

$$gg2 := 0$$

$$gg3 := -\frac{-Vz \, d(D1) + d(Vz) \, D1}{D1 \, \Phi}$$

The abnormality (time-time) connection 1-form

```
> Omega:=wcollect(subs(A=a,simplify(wcollect(cartan[4,4]))));
```

$$\Omega := -\frac{d(\Phi)}{\Phi}$$

```
> L:=factor(wcollect(hh1^sigma1+hh2^sigma2+hh3^sigma3));
```

$$L := 0$$

```
> S:=(wcollect(factor(hh1^gg1+hh2^gg2+hh3^gg3)));
```

$$S := 0$$

```
>
```

```
> Sigma1:=simplify(subs(wcollect(factor(omega^gg1))));
Sigma2:=factor(simplify(wcollect(factor(omega^gg2))));Sigma3:=simplify(omega^g
g3);
```

$$\begin{aligned}\Sigma 1 &:= 0 \\ \Sigma 2 &:= 0 \\ \Sigma 3 &:= \frac{-Vz(d(t) \&^{\wedge} d(D1)) + D1(d(t) \&^{\wedge} d(Vz))}{D1}\end{aligned}$$

WA TORSION 2-forms

```
> Phi1:=wcollect(subs(factor(Omega^gg1)));Phi2:=wcollect(factor(Omega^gg2));Phi3
:=wcollect(factor(Omega^gg3));
```

$$\begin{aligned}\Phi 1 &:= 0 \\ \Phi 2 &:= 0 \\ \Phi 3 &:= -\frac{Vz(d(\Phi) \&^{\wedge} d(D1))}{\Phi^2 D1} + \frac{d(\Phi) \&^{\wedge} d(Vz)}{\Phi^2}\end{aligned}$$

```
>
```

```
> Theta:=array([[gg1^hh1,gg1^hh2,gg1^hh3],[gg2^hh1,gg2^hh2,gg2^hh3],[gg3^hh
1,gg3^hh2,(gg3^hh3)]]);
```

$$\Theta := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

```
>
```

Example 2: Constant rotation, variable velocity (Particle Affine)

```
> FFINV:=array([[1,c,-b,-Vx],[-c,1,a,-Vy],[b,-a,1,-Vz],[0,0,0,Phi]]);
```

$$FFINV := \begin{bmatrix} 1 & c & -b & -Vx \\ -c & 1 & a & -Vy \\ b & -a & 1 & -Vz \\ 0 & 0 & 0 & \Phi \end{bmatrix}$$

```
> Z:=innerprod(FFINV,[d(x),d(y),d(z),d(t)]):sigma1:=Z[1];sigma2:=Z[2];sigma3:=Z[3]
;omega:=(Z[4]);
```

$$\begin{aligned}\sigma 1 &:= d(x) + c d(y) - b d(z) - Vx d(t) \\ \sigma 2 &:= -c d(x) + d(y) + a d(z) - Vy d(t) \\ \sigma 3 &:= b d(x) - a d(y) + d(z) - Vz d(t) \\ \omega &:= \Phi d(t)\end{aligned}$$

```
> Vol4:=wcollect(simplify(sigma1^sigma2^sigma3^Z[4])):rho:=factor(getcoeff(Vol4
));
```

$$\rho := \Phi (1 + c^2 + a^2 + b^2)$$

```
>
```

```
> FF:=inverse(FFINV):Gun:=subs(innerprod(transpose(FF),FF));
```

Gun :=

$$\left[\frac{1+a^2}{1+c^2+a^2+b^2}, \frac{ba}{1+c^2+a^2+b^2}, \frac{ca}{1+c^2+a^2+b^2}, \frac{Vx a^2 + Vz c a + a b Vy + Vx}{\Phi(1+c^2+a^2+b^2)} \right]$$
$$\left[\frac{ba}{1+c^2+a^2+b^2}, \frac{1+b^2}{1+c^2+a^2+b^2}, \frac{bc}{1+c^2+a^2+b^2}, \frac{b Vx a + Vz b c + Vy + b^2 Vy}{\Phi(1+c^2+a^2+b^2)} \right]$$
$$\left[\frac{ca}{1+c^2+a^2+b^2}, \frac{bc}{1+c^2+a^2+b^2}, \frac{1+c^2}{1+c^2+a^2+b^2}, \frac{a Vx c + b c Vy + Vz + Vz c^2}{\Phi(1+c^2+a^2+b^2)} \right]$$
$$\left[\frac{Vx a^2 + Vz c a + a b Vy + Vx}{\Phi(1+c^2+a^2+b^2)}, \frac{b Vx a + Vz b c + Vy + b^2 Vy}{\Phi(1+c^2+a^2+b^2)}, \frac{a Vx c + b c Vy + Vz + Vz c^2}{\Phi(1+c^2+a^2+b^2)}, \right.$$
$$\left. \frac{Vx^2 a^2 + a^2 + 2 Vx a b Vy + 2 Vx Vz c a + Vz^2 c^2 + b^2 Vy^2 + 1 + Vx^2 + b^2 + 2 c Vy Vz b + Vy^2 + Vz^2 + c^2}{(1+c^2+a^2+b^2)\Phi^2} \right]$$

[>

> dFF:=array([[d(FF[1,1]),d(FF[1,2]),d(FF[1,3]),d(FF[1,4])],[d(FF[2,1]),d(FF[2,2]),d(FF[2,3]),d(FF[2,4])],[d(FF[3,1]),d(FF[3,2]),d(FF[3,3]),d(FF[3,4])],[d(FF[4,1]),d(FF[4,2]),d(FF[4,3]),d(FF[4,4])]]):

[> cartan:=(evalm(FFINV*dFF)):

[**The Interior (space-space) Connection 1 forms**

> Gamma11:=factor(wcollect(cartan[1,1]));Gamma21:=factor(wcollect(cartan[2,1]));Gamma31:=factor(wcollect(cartan[3,1]));

$\Gamma_{11} := 0$

$\Gamma_{21} := 0$

$\Gamma_{31} := 0$

> Gamma12:=factor(wcollect(cartan[1,2]));Gamma22:=factor(wcollect(cartan[2,2]));Gamma32:=factor(wcollect(cartan[3,2]));

$\Gamma_{12} := 0$

$\Gamma_{22} := 0$

$\Gamma_{32} := 0$

> Gamma13:=factor(wcollect(cartan[1,3]));Gamma23:=factor(wcollect(cartan[2,3]));Gamma33:=factor(wcollect(cartan[3,3]));

$\Gamma_{13} := 0$

$\Gamma_{23} := 0$

$\Gamma_{33} := 0$

[>

[**The "space-time" connection 1-forms are:**

> hh1:=factor(wcollect(cartan[4,1]));hh2:=factor(wcollect(cartan[4,2]));hh3:=factor(wcollect(cartan[4,3]));

$hh1 := 0$

$hh2 := 0$

$hh3 := 0$

[

The "time-space connection" 1-forms are

> gg1:=factor(wcollect(factor(wcollect(cartan[1,4]))));gg2:=factor(wcollect(cartan[2,4]));gg3:=factor(wcollect(cartan[3,4]));

$$gg1 := \frac{d(Vx)}{\Phi}$$

$$gg2 := \frac{d(Vy)}{\Phi}$$

$$gg3 := \frac{d(Vz)}{\Phi}$$

The abnormality (time-time) connection 1-form

```
> Omega:=wcollect(subs(A=a,simplify(wcollect(cartan[4,4]))));
```

$$\Omega := -\frac{d(\Phi)}{\Phi}$$

```
> L:=factor(wcollect(hh1^sigma1+hh2^sigma2+hh3^sigma3));
```

$$L := 0$$

```
> S:=(wcollect(factor(hh1^gg1+hh2^gg2+hh3^gg3)));
```

$$S := 0$$

>

```
> Sigma1:=simplify(subs(Lx=a,Ly=b,Lz=c,wcollect(factor(omega^gg1))));
Sigma2:=factor(simplify(wcollect(factor(omega^gg2))));Sigma3:=simplify(omega^g
g3);
```

$$\Sigma1 := d(t) \wedge d(Vx)$$

$$\Sigma2 := d(t) \wedge d(Vy)$$

$$\Sigma3 := d(t) \wedge d(Vz)$$

WA TORSION 2-forms

```
> Phi1:=simplify(wcollect(subs(Lx=a,Ly=b,Lz=c,factor(Omega^gg1))));Phi2:=wcollect
(factor(Omega^gg2));Phi3:=wcollect(factor(Omega^gg3));
```

$$\Phi1 := -\frac{d(\Phi) \wedge d(Vx)}{\Phi^2}$$

$$\Phi2 := -\frac{d(\Phi) \wedge d(Vy)}{\Phi^2}$$

$$\Phi3 := -\frac{d(\Phi) \wedge d(Vz)}{\Phi^2}$$

>

```
> Theta:=array([[gg1^hh1,gg1^hh2,gg1^hh3],[gg2^hh1,gg2^hh2,gg2^hh3],[gg3^hh
1,gg3^hh2,(gg3^hh3)]]);
```

$$\Theta := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

>

DISCUSSION:

The projective frame field does not induce any curvature on the interior set {x,y,z},

Both sets of torsion coefficients depend upon the accelerations. (d(Vx) etc.

If the Accelerations are kinematic and depend upon d(t) only, then the PA torsion coefficients are zero.

If the Accelerations are kinematic and the Abnormality is spatially dependent, then the WA torsion coefficients are not zero.

```

>
> FFINV:=array([[D2,Lz,0,0],[-Lz,D2,0,0],[0,0,-D1,0],[Ax,Ay,Az,-Phi]]);


$$FFINV := \begin{bmatrix} D2 & Lz & 0 & 0 \\ -Lz & D2 & 0 & 0 \\ 0 & 0 & -D1 & 0 \\ Ax & Ay & Az & -\Phi \end{bmatrix}$$


> Z:=innerprod(FFINV,[d(x),d(y),d(z),d(t)]);sigma1:=Z[1];sigma2:=Z[2];sigma3:=Z[3];omega:=(Z[4]);d(omega);


$$\begin{aligned} \sigma_1 &:= D2 d(x) + Lz d(y) \\ \sigma_2 &:= -Lz d(x) + D2 d(y) \\ \sigma_3 &:= -D1 d(z) \\ \omega &:= Ax d(x) + Ay d(y) + Az d(z) - \Phi d(t) \\ & (d(Ax) \&^{\wedge} d(x)) + (d(Ay) \&^{\wedge} d(y)) + (d(Az) \&^{\wedge} d(z)) - (d(\Phi) \&^{\wedge} d(t)) \end{aligned}$$


> Vol4:=wcollect(simplify(sigma1&^sigma2&^sigma3&^Z[4]));rho:=factor(getcoeff(Vol4));


$$\rho := \Phi (D2^2 + Lz^2) D1$$


>
> FF:=inverse(FFINV);Gun:=subs(innerprod(transpose(FF),FF));

Gun :=

$$\left[ \frac{D2^2 \Phi^2 + Lz^2 \Phi^2 + Lz^2 Ay^2 + 2 D2 Ay Ax Lz + Ax^2 D2^2}{\Phi^2 (D2^2 + Lz^2)^2}, -\frac{(Lz Ay + Ax D2) (-D2 Ay + Ax Lz)}{\Phi^2 (D2^2 + Lz^2)^2}, \right.$$


$$\left. -\frac{(Lz Ay + Ax D2) Az}{\Phi^2 (D2^2 + Lz^2) D1}, -\frac{Lz Ay + Ax D2}{\Phi^2 (D2^2 + Lz^2)} \right]$$


$$\left[ -\frac{(Lz Ay + Ax D2) (-D2 Ay + Ax Lz)}{\Phi^2 (D2^2 + Lz^2)^2}, \frac{Lz^2 \Phi^2 + D2^2 \Phi^2 + D2^2 Ay^2 - 2 D2 Ay Ax Lz + Ax^2 Lz^2}{\Phi^2 (D2^2 + Lz^2)^2}, \right.$$


$$\left. \frac{(-D2 Ay + Ax Lz) Az}{\Phi^2 (D2^2 + Lz^2) D1}, \frac{-D2 Ay + Ax Lz}{\Phi^2 (D2^2 + Lz^2)} \right]$$


$$\left[ -\frac{(Lz Ay + Ax D2) Az}{\Phi^2 (D2^2 + Lz^2) D1}, \frac{(-D2 Ay + Ax Lz) Az}{\Phi^2 (D2^2 + Lz^2) D1}, \frac{\Phi^2 + Az^2}{D1^2 \Phi^2}, \frac{Az}{D1 \Phi^2} \right]$$


$$\left[ -\frac{Lz Ay + Ax D2}{\Phi^2 (D2^2 + Lz^2)}, \frac{-D2 Ay + Ax Lz}{\Phi^2 (D2^2 + Lz^2)}, \frac{Az}{D1 \Phi^2}, \frac{1}{\Phi^2} \right]$$


>
> dFF:=array([[d(FF[1,1]),d(FF[1,2]),d(FF[1,3]),d(FF[1,4]),[d(FF[2,1]),d(FF[2,2]),d(FF[2,3]),d(FF[2,4]),[d(FF[3,1]),d(FF[3,2]),d(FF[3,3]),d(FF[3,4]),[d(FF[4,1]

```

[,d(FF[4,2]),d(FF[4,3]),d(FF[4,4])]]):

[> cartan:=(evalm(FFINV*dFF)):

[**The Interior (space-space) Connection 1 forms**

[> Gamma11:=factor(wcollect(cartan[1,1]));Gamma21:=factor(wcollect(cartan[2,1]));Gamma31:=factor(wcollect(cartan[3,1]));

$$\Gamma_{11} := -\frac{D2 d(D2) + Lz d(Lz)}{D2^2 + Lz^2}$$

$$\Gamma_{21} := \frac{-Lz d(D2) + D2 d(Lz)}{D2^2 + Lz^2}$$

$$\Gamma_{31} := 0$$

[> Gamma12:=factor(wcollect(cartan[1,2]));Gamma22:=factor(wcollect(cartan[2,2]));Gamma32:=factor(wcollect(cartan[3,2]));

$$\Gamma_{12} := -\frac{-Lz d(D2) + D2 d(Lz)}{D2^2 + Lz^2}$$

$$\Gamma_{22} := -\frac{D2 d(D2) + Lz d(Lz)}{D2^2 + Lz^2}$$

$$\Gamma_{32} := 0$$

[> Gamma13:=factor(wcollect(cartan[1,3]));Gamma23:=factor(wcollect(cartan[2,3]));Gamma33:=factor(wcollect(cartan[3,3]));

$$\Gamma_{13} := 0$$

$$\Gamma_{23} := 0$$

$$\Gamma_{33} := -\frac{d(D1)}{D1}$$

[>

[**The "space-time" connection 1-forms are:**

[> hh1:=factor(wcollect(cartan[4,1]));hh2:=factor(wcollect(cartan[4,2]));hh3:=factor(wcollect(cartan[4,3]));

$$hh1 := \frac{-D2 d(Ax) \Phi - Lz d(Ay) \Phi + d(\Phi) Lz Ay + d(\Phi) Ax D2}{\Phi (D2^2 + Lz^2)}$$

$$hh2 := -\frac{-Lz d(Ax) \Phi + D2 d(Ay) \Phi - d(\Phi) D2 Ay + d(\Phi) Ax Lz}{\Phi (D2^2 + Lz^2)}$$

$$hh3 := -\frac{-d(Az) \Phi + Az d(\Phi)}{D1 \Phi}$$

[**The "time-space connection" 1-forms are**

[> gg1:=factor(wcollect(factor(wcollect(cartan[1,4]))));gg2:=factor(wcollect(cartan[2,4]));gg3:=factor(wcollect(cartan[3,4]));

$$gg1 := 0$$

$$gg2 := 0$$

$$gg3 := 0$$

[**The abnormality (time-time) connection 1-form**

[> Omega:=wcollect(subs(A=a,simplify(wcollect(cartan[4,4]))));

$$\Omega := -\frac{d(\Phi)}{\Phi}$$

```
> L:=factor(wcollect(hh1^sigma1+hh2^sigma2+hh3^sigma3));d(omega);
```

$$L := \frac{-(d(Ax) \wedge d(x)) \Phi - (d(Ay) \wedge d(y)) \Phi - \Phi (d(Az) \wedge d(z)) + Ax (d(\Phi) \wedge d(x)) + Ay (d(\Phi) \wedge d(y)) + Az (d(\Phi) \wedge d(z))}{(d(Ax) \wedge d(x)) + (d(Ay) \wedge d(y)) + (d(Az) \wedge d(z)) - (d(\Phi) \wedge d(t))}$$

```
> S:=(wcollect(factor(hh1^gg1+hh2^gg2+hh3^gg3)));
```

$$S := 0$$

```
>
```

```
> Sigma1:=simplify(subs(wcollect(factor(omega^gg1))));
```

```
Sigma2:=factor(simplify(wcollect(factor(omega^gg2))));Sigma3:=simplify(omega^gg3);
```

$$\Sigma 1 := 0$$

$$\Sigma 2 := 0$$

$$\Sigma 3 := 0$$

WA TORSION 2-forms

```
> Phi1:=wcollect(subs(factor(Omega^gg1)));Phi2:=wcollect(factor(Omega^gg2));Phi3:=wcollect(factor(Omega^gg3));
```

$$\Phi 1 := 0$$

$$\Phi 2 := 0$$

$$\Phi 3 := 0$$

```
>
```

```
> Theta:=array([[gg1^hh1,gg1^hh2,gg1^hh3],[gg2^hh1,gg2^hh2,gg2^hh3],[gg3^hh1,gg3^hh2,(gg3^hh3)]]);
```

$$\Theta := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

```
>
```

```
>
```

```
>
```

```
>
```

```
>
```

```
>
```

```
> FFINV:=array([[D2,Lz,0,0],[-Lz,D2,0,0],[0,0,-D1,0],[Ax,Ay,Az,-Phi]]);
```

$$FFINV := \begin{bmatrix} D2 & Lz & 0 & 0 \\ -Lz & D2 & 0 & 0 \\ 0 & 0 & -D1 & 0 \\ Ax & Ay & Az & -\Phi \end{bmatrix}$$

```
> Z:=innerprod(FFINV,[d(x),d(y),d(z),d(t)]):sigma1:=Z[1];sigma2:=Z[2];sigma3:=Z[3]
;omega:=(Z[4]);d(omega);
```

$$\sigma_1 := D2 d(x) + Lz d(y)$$

$$\sigma_2 := -Lz d(x) + D2 d(y)$$

$$\sigma_3 := -D1 d(z)$$

$$\omega := Ax d(x) + Ay d(y) + Az d(z) - \Phi d(t)$$

$$(d(Ax) \wedge d(x)) + (d(Ay) \wedge d(y)) + (d(Az) \wedge d(z)) - (d(\Phi) \wedge d(t))$$

```
> Vol4:=wcollect(simplify(sigma1^sigma2^sigma3^Z[4])):rho:=factor(getcoeff(Vol4
));
```

$$\rho := \Phi (D2^2 + Lz^2) D1$$

```
>
```

```
> FF:=inverse(FFINV):Gun:=subs(innerprod(transpose(FF),FF));
```

```
Gun :=
```

$$\left[\frac{D2^2 \Phi^2 + Lz^2 \Phi^2 + Lz^2 Ay^2 + 2 D2 Ay Ax Lz + Ax^2 D2^2}{\Phi^2 (D2^2 + Lz^2)^2}, -\frac{(Lz Ay + Ax D2) (-D2 Ay + Ax Lz)}{\Phi^2 (D2^2 + Lz^2)^2}, \right.$$

$$\left. -\frac{(Lz Ay + Ax D2) Az}{\Phi^2 (D2^2 + Lz^2) D1}, -\frac{Lz Ay + Ax D2}{\Phi^2 (D2^2 + Lz^2)} \right]$$

$$\left[-\frac{(Lz Ay + Ax D2) (-D2 Ay + Ax Lz)}{\Phi^2 (D2^2 + Lz^2)^2}, \frac{Lz^2 \Phi^2 + D2^2 \Phi^2 + D2^2 Ay^2 - 2 D2 Ay Ax Lz + Ax^2 Lz^2}{\Phi^2 (D2^2 + Lz^2)^2}, \right.$$

$$\left. \frac{(-D2 Ay + Ax Lz) Az}{\Phi^2 (D2^2 + Lz^2) D1}, -\frac{-D2 Ay + Ax Lz}{\Phi^2 (D2^2 + Lz^2)} \right]$$

$$\left[-\frac{(Lz Ay + Ax D2) Az}{\Phi^2 (D2^2 + Lz^2) D1}, -\frac{(-D2 Ay + Ax Lz) Az}{\Phi^2 (D2^2 + Lz^2) D1}, \frac{\Phi^2 + Az^2}{D1^2 \Phi^2}, \frac{Az}{D1 \Phi^2} \right]$$

$$\left[-\frac{Lz Ay + Ax D2}{\Phi^2 (D2^2 + Lz^2)}, -\frac{-D2 Ay + Ax Lz}{\Phi^2 (D2^2 + Lz^2)}, \frac{Az}{D1 \Phi^2}, \frac{1}{\Phi^2} \right]$$

```
>
```

```
> dFF:=array([[d(FF[1,1]),d(FF[1,2]),d(FF[1,3]),d(FF[1,4])],[d(FF[2,1]),d(FF[2,2])
,d(FF[2,3]),d(FF[2,4])],[d(FF[3,1]),d(FF[3,2]),d(FF[3,3]),d(FF[3,4])],[d(FF[4,1]
),d(FF[4,2]),d(FF[4,3]),d(FF[4,4])]]):
```

```
> cartan:=(evalm(FFINV*dFF));
```

```
[ The Interior (space-space) Connection 1 forms
```

```
> Gamma11:=factor(wcollect(cartan[1,1]));Gamma21:=factor(wcollect(cartan[2,1]));Ga
mma31:=factor(wcollect(cartan[3,1]));
```

$$\Gamma_{11} := -\frac{D2 d(D2) + Lz d(Lz)}{D2^2 + Lz^2}$$

$$\Gamma_{21} := \frac{-Lz d(D2) + D2 d(Lz)}{D2^2 + Lz^2}$$

$$\Gamma_{31} := 0$$

```
> Gamma12:=factor(wcollect(cartan[1,2]));Gamma22:=factor(wcollect(cartan[2,2]));Gamma32:=factor(wcollect(cartan[3,2]));
```

$$\Gamma_{12} := -\frac{-Lz d(D2) + D2 d(Lz)}{D2^2 + Lz^2}$$

$$\Gamma_{22} := -\frac{D2 d(D2) + Lz d(Lz)}{D2^2 + Lz^2}$$

$$\Gamma_{32} := 0$$

```
> Gamma13:=factor(wcollect(cartan[1,3]));Gamma23:=factor(wcollect(cartan[2,3]));Gamma33:=factor(wcollect(cartan[3,3]));
```

$$\Gamma_{13} := 0$$

$$\Gamma_{23} := 0$$

$$\Gamma_{33} := -\frac{d(D1)}{D1}$$

```
>
```

The "space-time" connection 1-forms are:

```
> hh1:=factor(wcollect(cartan[4,1]));hh2:=factor(wcollect(cartan[4,2]));hh3:=factor(wcollect(cartan[4,3]));
```

$$hh1 := \frac{-D2 d(Ax) \Phi - Lz d(Ay) \Phi + d(\Phi) Lz Ay + d(\Phi) Ax D2}{\Phi (D2^2 + Lz^2)}$$

$$hh2 := -\frac{-Lz d(Ax) \Phi + D2 d(Ay) \Phi - d(\Phi) D2 Ay + d(\Phi) Ax Lz}{\Phi (D2^2 + Lz^2)}$$

$$hh3 := -\frac{-d(Az) \Phi + Az d(\Phi)}{D1 \Phi}$$

The "time-space connection" 1-forms are

```
> gg1:=factor(wcollect(factor(wcollect(cartan[1,4]))));gg2:=factor(wcollect(cartan[2,4]));gg3:=factor(wcollect(cartan[3,4]));
```

$$gg1 := 0$$

$$gg2 := 0$$

$$gg3 := 0$$

The abnormality (time-time) connection 1-form

```
> Omega:=wcollect(subs(A=a,simplify(wcollect(cartan[4,4]))));
```

$$\Omega := -\frac{d(\Phi)}{\Phi}$$

```
> L:=factor(wcollect(hh1&^sigma1+hh2&^sigma2+hh3&^sigma3));d(omega);
```

$$L := (-d(Ax) \&^d(x) \Phi - d(Ay) \&^d(y) \Phi - \Phi (d(Az) \&^d(z) + Ax (d(\Phi) \&^d(x) + Ay (d(\Phi) \&^d(y) + Az (d(\Phi) \&^d(z)))) / \Phi$$

$$(d(Ax) \&^d(x) + d(Ay) \&^d(y) + d(Az) \&^d(z) - d(\Phi) \&^d(t))$$

```
> S:=(wcollect(factor(hh1&^gg1+hh2&^gg2+hh3&^gg3)));
```


> `FF:=inverse(FFINV):Gun:=subs(innerprod(transpose(FF),FF));`

$$Gun := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\Phi^2 + Az^2}{(\Phi + Vz Az)^2} & -\frac{-\Phi Vz + Az}{(\Phi + Vz Az)^2} \\ 0 & 0 & -\frac{-\Phi Vz + Az}{(\Phi + Vz Az)^2} & \frac{Vz^2 + 1}{(\Phi + Vz Az)^2} \end{bmatrix}$$

>

> `dFF:=array([[d(FF[1,1]),d(FF[1,2]),d(FF[1,3]),d(FF[1,4])],[d(FF[2,1]),d(FF[2,2]),d(FF[2,3]),d(FF[2,4])],[d(FF[3,1]),d(FF[3,2]),d(FF[3,3]),d(FF[3,4])],[d(FF[4,1]),d(FF[4,2]),d(FF[4,3]),d(FF[4,4])]]):`

> `cartan:=(evalm(FFINV*dFF)):`

The Interior (space-space) Connection 1 forms

> `Gamma11:=factor(wcollect(cartan[1,1]));Gamma21:=factor(wcollect(cartan[2,1]));Gamma31:=factor(wcollect(cartan[3,1]));`

$$\Gamma_{11} := 0$$

$$\Gamma_{21} := 0$$

$$\Gamma_{31} := 0$$

> `Gamma12:=factor(wcollect(cartan[1,2]));Gamma22:=factor(wcollect(cartan[2,2]));Gamma32:=factor(wcollect(cartan[3,2]));`

$$\Gamma_{12} := 0$$

$$\Gamma_{22} := 0$$

$$\Gamma_{32} := 0$$

> `Gamma13:=factor(wcollect(cartan[1,3]));Gamma23:=factor(wcollect(cartan[2,3]));Gamma33:=factor(wcollect(cartan[3,3]));`

$$\Gamma_{13} := 0$$

$$\Gamma_{23} := 0$$

$$\Gamma_{33} := -\frac{Az d(Vz)}{\Phi + Vz Az}$$

>

The "space-time" connection 1-forms are:

> `hh1:=factor(wcollect(cartan[4,1]));hh2:=factor(wcollect(cartan[4,2]));hh3:=factor(wcollect(cartan[4,3]));`

$$hh_1 := 0$$

$$hh_2 := 0$$

$$hh_3 := \frac{-d(Az)\Phi + Az d(\Phi)}{\Phi + Vz Az}$$

The "time-space connection" 1-forms are

> `gg1:=factor(wcollect(factor(wcollect(cartan[1,4]))));gg2:=factor(wcollect(cartan[2,4]));gg3:=factor(wcollect(cartan[3,4]));`

$$gg_1 := 0$$

$$gg2 := 0$$

$$gg3 := \frac{d(Vz)}{\Phi + Vz Az}$$

The abnormality (time-time) connection 1-form

```
> Omega:=wcollect(subs(A=a,simplify(wcollect(cartan[4,4]))));
```

$$\Omega := -\frac{Vz d(Az)}{\Phi + Vz Az} - \frac{d(\Phi)}{\Phi + Vz Az}$$

```
> L:=factor(wcollect(hh1&^sigma1+hh2&^sigma2+hh3&^sigma3));d(omega);
```

$$L := \frac{-\Phi (d(Az) \&\wedge d(z)) - \Phi Vz (d(Az) \&\wedge d(t)) + Az (d(\Phi) \&\wedge d(z)) + Az Vz (d(\Phi) \&\wedge d(t))}{\Phi + Vz Az}$$

$$(d(Az) \&\wedge d(z)) - (d(\Phi) \&\wedge d(t))$$

```
> S:=(wcollect(factor(hh1&^gg1+hh2&^gg2+hh3&^gg3)));
```

$$S := -\frac{\Phi (d(Az) \&\wedge d(Vz))}{(\Phi + Vz Az)^2} + \frac{Az (d(\Phi) \&\wedge d(Vz))}{(\Phi + Vz Az)^2}$$

```
>
```

```
> Sigma1:=simplify(subs(wcollect(factor(omega&^gg1))));
```

```
Sigma2:=factor(simplify(wcollect(factor(omega&^gg2))));Sigma3:=simplify(omega&^g3);
```

$$\Sigma 1 := 0$$

$$\Sigma 2 := 0$$

$$\Sigma 3 := \frac{Az (d(z) \&\wedge d(Vz)) - \Phi (d(t) \&\wedge d(Vz))}{\Phi + Vz Az}$$

WA TORSION 2-forms

```
> Phi1:=wcollect(subs(factor(Omega&^gg1)));Phi2:=wcollect(factor(Omega&^gg2));Phi3:=wcollect(factor(Omega&^gg3));
```

$$\Phi 1 := 0$$

$$\Phi 2 := 0$$

$$\Phi 3 := -\frac{Vz (d(Az) \&\wedge d(Vz))}{(\Phi + Vz Az)^2} - \frac{d(\Phi) \&\wedge d(Vz)}{(\Phi + Vz Az)^2}$$

```
>
```

```
> Theta:=array([[wcollect(gg1&^hh1),wcollect(gg1&^hh2),wcollect(gg1&^hh3)],[gg2&^hh1,gg2&^hh2,gg2&^hh3],[gg3&^hh1,gg3&^hh2,(gg3&^hh3)]]);
```

$$\Theta := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\Phi (d(Vz) \&\wedge d(Az))}{(\Phi + Vz Az)^2} + \frac{Az (d(Vz) \&\wedge d(\Phi))}{(\Phi + Vz Az)^2} \end{bmatrix}$$

```
> AMAXWELL:=Az*d(z)-Phi*d(t);FMAXWELL:=d(AMAXWELL);HMAXWELL:=(AMAXWELL&^FMAXWELL);
Topological_Torsion:=(Phi*d(Az)-Az*d(Phi))*d(z)&^d(t);Curvature_2_forms:=(Phi*d(Az)-Az*d(Phi))*(d(Vz))/(Phi+Vz*Az)^2;PA_torison_2_forms:=AMAXWELL*d(Vz)/(Phi+Az*Vz);
```

$$\begin{aligned}
AMAXWELL &:= Az \, d(z) - \Phi \, d(t) \\
FMAXWELL &:= (d(Az) \wedge d(z)) - (d(\Phi) \wedge d(t)) \\
HMAXWELL &:= -Az \wedge (d(z), d(\Phi), d(t)) - \Phi \wedge (d(t), d(Az), d(z)) \\
Topological_Torsion &:= (d(Az) \Phi - Az \, d(\Phi)) (d(z) \wedge d(t)) \\
Curvature_2_forms &:= \frac{(d(Az) \Phi - Az \, d(\Phi)) d(Vz)}{(\Phi + Vz \, Az)^2} \\
PA_torison_2_forms &:= \frac{(Az \, d(z) - \Phi \, d(t)) d(Vz)}{\Phi + Vz \, Az}
\end{aligned}$$

Example 6: An asymmetric Projective transformation

```
> FFINV:=array([[1,0,0,Vx],[0,1,0,Vy],[0,0,1,Vz],[Ax,Ay,Az,-Phi]]);
```

$$FFINV := \begin{bmatrix} 1 & 0 & 0 & Vx \\ 0 & 1 & 0 & Vy \\ 0 & 0 & 1 & Vz \\ Ax & Ay & Az & -\Phi \end{bmatrix}$$

```
> Z:=innerprod(FFINV,[d(x),d(y),d(z),d(t)]):sigma1:=Z[1];sigma2:=Z[2];sigma3:=Z[3];
omega:=(Z[4]);d(omega);
```

$$\sigma_1 := d(x) + Vx \, d(t)$$

$$\sigma_2 := d(y) + Vy \, d(t)$$

$$\sigma_3 := d(z) + Vz \, d(t)$$

$$\omega := Ax \, d(x) + Ay \, d(y) + Az \, d(z) - \Phi \, d(t)$$

$$(d(Ax) \wedge d(x)) + (d(Ay) \wedge d(y)) + (d(Az) \wedge d(z)) - (d(\Phi) \wedge d(t))$$

```
> Vol4:=wcollect(simplify(sigma1&^sigma2&^sigma3&^Z[4])):rho:=factor(getcoeff(Vol4));
```

$$\rho := -\Phi - Vz \, Az - Vy \, Ay - Vx \, Ax$$

```
> FF:=inverse(FFINV):Gun:=subs(innerprod(transpose(FF),FF));
```

Gun :=

$$\begin{aligned}
& \left[\frac{\Phi^2 + 2 \, Az \, \Phi \, Vz + 2 \, \Phi \, Vy \, Ay + Vz^2 \, Az^2 + 2 \, Vz \, Az \, Vy \, Ay + Vy^2 \, Ay^2 + Ax^2 \, Vy^2 + Ax^2 \, Vz^2 + Ax^2}{(\Phi + Vz \, Az + Vy \, Ay + Vx \, Ax)^2}, \right. \\
& - \frac{Ay \, Vx \, \Phi + Ay \, Vx \, Vz \, Az + Ay^2 \, Vx \, Vy + Ax \, Vy \, \Phi + Ax \, Vy \, Vz \, Az + Ax^2 \, Vy \, Vx - Ax \, Vz^2 \, Ay - Ax \, Ay}{(\Phi + Vz \, Az + Vy \, Ay + Vx \, Ax)^2}, \\
& - \frac{Vx \, Az \, \Phi + Vx \, Az^2 \, Vz + Vx \, Az \, Vy \, Ay - Ax \, Vy^2 \, Az + Ax \, Vz \, \Phi + Ax \, Vz \, Vy \, Ay + Ax^2 \, Vz \, Vx - Ax \, Az}{(\Phi + Vz \, Az + Vy \, Ay + Vx \, Ax)^2}, \\
& \left. - \frac{-Vx \, \Phi - Vx \, Vz \, Az - Vx \, Vy \, Ay + Ax \, Vy^2 + Ax \, Vz^2 + Ax}{(\Phi + Vz \, Az + Vy \, Ay + Vx \, Ax)^2} \right] \\
& \left[- \frac{Ay \, Vx \, \Phi + Ay \, Vx \, Vz \, Az + Ay^2 \, Vx \, Vy + Ax \, Vy \, \Phi + Ax \, Vy \, Vz \, Az + Ax^2 \, Vy \, Vx - Ax \, Vz^2 \, Ay - Ax \, Ay}{(\Phi + Vz \, Az + Vy \, Ay + Vx \, Ax)^2}, \right.
\end{aligned}$$

$$\frac{Ay^2 Vx^2 + \Phi^2 + 2 Az \Phi Vz + 2 \Phi Vx Ax + Vz^2 Az^2 + 2 Vz Az Vx Ax + Vx^2 Ax^2 + Vz^2 Ay^2 + Ay^2}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2},$$

$$-\frac{-Ay Vx^2 Az + Vy Az \Phi + Vy Az^2 Vz + Vy Az Vx Ax + Vz Ay \Phi + Vz Ay^2 Vy + Vz Ay Vx Ax - Ay Az}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2},$$

$$\left. \frac{-Ay Vx^2 + Vy \Phi + Vy Vz Az + Vy Vx Ax - Vz^2 Ay - Ay}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} \right]$$

$$\left[-\frac{Vx Az \Phi + Vx Az^2 Vz + Vx Az Vy Ay - Ax Vy^2 Az + Ax Vz \Phi + Ax Vz Vy Ay + Ax^2 Vz Vx - Ax Az}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}, \right.$$

$$\left. -\frac{-Ay Vx^2 Az + Vy Az \Phi + Vy Az^2 Vz + Vy Az Vx Ax + Vz Ay \Phi + Vz Ay^2 Vy + Vz Ay Vx Ax - Ay Az}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}, \right.$$

$$\frac{Vx^2 Az^2 + Vy^2 Az^2 + \Phi^2 + 2 \Phi Vy Ay + 2 \Phi Vx Ax + Vy^2 Ay^2 + 2 Vy Ay Vx Ax + Vx^2 Ax^2 + Az^2}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2},$$

$$\left. \frac{-Vx^2 Az - Vy^2 Az + \Phi Vz + Vz Vy Ay + Vz Vx Ax - Az}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} \right]$$

$$\left[-\frac{-Vx \Phi - Vx Vz Az - Vx Vy Ay + Ax Vy^2 + Ax Vz^2 + Ax}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}, \frac{-Ay Vx^2 + Vy \Phi + Vy Vz Az + Vy Vx Ax - Vz^2 Ay - Ay}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}, \right.$$

$$\left. \frac{-Vx^2 Az - Vy^2 Az + \Phi Vz + Vz Vy Ay + Vz Vx Ax - Az}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}, \frac{Vx^2 + Vy^2 + Vz^2 + 1}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} \right]$$

>

```
> dFF:=array([[d(FF[1,1]),d(FF[1,2]),d(FF[1,3]),d(FF[1,4]),[d(FF[2,1]),d(FF[2,2]),
,d(FF[2,3]),d(FF[2,4])],[d(FF[3,1]),d(FF[3,2]),d(FF[3,3]),d(FF[3,4])],[d(FF[4,1])
,d(FF[4,2]),d(FF[4,3]),d(FF[4,4])]]):
```

```
> cartan:=(evalm(FFINV*dFF)):
```

The Interior (space-space) Connection 1 forms

```
> Gamma11:=factor(wcollect(cartan[1,1]));Gamma21:=factor(wcollect(cartan[2,1]));Ga
mma31:=factor(wcollect(cartan[3,1]));
```

$$\Gamma_{11} := -\frac{Ax d(Vx)}{\Phi + Vz Az + Vy Ay + Vx Ax}$$

$$\Gamma_{21} := -\frac{Ax d(Vy)}{\Phi + Vz Az + Vy Ay + Vx Ax}$$

$$\Gamma_{31} := -\frac{Ax d(Vz)}{\Phi + Vz Az + Vy Ay + Vx Ax}$$

```
> Gamma12:=factor(wcollect(cartan[1,2]));Gamma22:=factor(wcollect(cartan[2,2]));Ga
mma32:=factor(wcollect(cartan[3,2]));
```

$$\Gamma_{12} := -\frac{Ay d(Vx)}{\Phi + Vz Az + Vy Ay + Vx Ax}$$

$$\Gamma_{22} := -\frac{Ay d(Vy)}{\Phi + Vz Az + Vy Ay + Vx Ax}$$

$$\Gamma_{32} := -\frac{Ay d(Vz)}{\Phi + Vz Az + Vy Ay + Vx Ax}$$

```
> Gamma13:=factor(wcollect(cartan[1,3]));Gamma23:=factor(wcollect(cartan[2,3]));Ga
mma33:=factor(wcollect(cartan[3,3]));
```

$$\Gamma_{13} := -\frac{A_z d(V_x)}{\Phi + V_z A_z + V_y A_y + V_x A_x}$$

$$\Gamma_{23} := -\frac{A_z d(V_y)}{\Phi + V_z A_z + V_y A_y + V_x A_x}$$

$$\Gamma_{33} := -\frac{A_z d(V_z)}{\Phi + V_z A_z + V_y A_y + V_x A_x}$$

>

The "space-time" connection 1-forms are:

> `hh1:=factor(wcollect(cartan[4,1]));hh2:=factor(wcollect(cartan[4,2]));hh3:=factor(wcollect(cartan[4,3]));`

$$hh1 := \frac{-\Phi d(A_x) - d(A_x) V_z A_z - d(A_x) V_y A_y + A_x V_y d(A_y) + A_x V_z d(A_z) + d(\Phi) A_x}{\Phi + V_z A_z + V_y A_y + V_x A_x}$$

$$hh2 := -\frac{-A_y V_x d(A_x) + d(A_y) \Phi + d(A_y) V_z A_z + d(A_y) V_x A_x - V_z A_y d(A_z) - A_y d(\Phi)}{\Phi + V_z A_z + V_y A_y + V_x A_x}$$

$$hh3 := -\frac{-V_x A_z d(A_x) - V_y A_z d(A_y) + d(A_z) \Phi + d(A_z) V_y A_y + d(A_z) V_x A_x - A_z d(\Phi)}{\Phi + V_z A_z + V_y A_y + V_x A_x}$$

The "time-space connection" 1-forms are

> `gg1:=factor(wcollect(factor(wcollect(cartan[1,4]))));gg2:=factor(wcollect(cartan[2,4]));gg3:=factor(wcollect(cartan[3,4]));`

$$gg1 := \frac{d(V_x)}{\Phi + V_z A_z + V_y A_y + V_x A_x}$$

$$gg2 := \frac{d(V_y)}{\Phi + V_z A_z + V_y A_y + V_x A_x}$$

$$gg3 := \frac{d(V_z)}{\Phi + V_z A_z + V_y A_y + V_x A_x}$$

The abnormality (time-time) connection 1-form

> `Omega:=wcollect(subs(A=a,simplify(wcollect(cartan[4,4]))));`

$$\Omega := -\frac{V_x d(A_x)}{\Phi + V_z A_z + V_y A_y + V_x A_x} - \frac{V_y d(A_y)}{\Phi + V_z A_z + V_y A_y + V_x A_x} - \frac{V_z d(A_z)}{\Phi + V_z A_z + V_y A_y + V_x A_x} - \frac{d(\Phi)}{\Phi + V_z A_z + V_y A_y + V_x A_x}$$

> `L:=factor(wcollect(hh1&^sigma1+hh2&^sigma2+hh3&^sigma3));d(omega);`

$$L := -(\Phi V_x (d(A_x) \&\wedge d(t)) + (d(A_x) \&\wedge d(x)) \Phi + V_z A_z (d(A_x) \&\wedge d(x)) + V_y A_y (d(A_x) \&\wedge d(x)) - A_y V_x (d(A_x) \&\wedge d(y)) - V_x A_z (d(A_x) \&\wedge d(z)) + \Phi V_y (d(A_y) \&\wedge d(t)) - A_x V_y (d(A_y) \&\wedge d(x)) + (d(A_y) \&\wedge d(y)) \Phi + V_z A_z (d(A_y) \&\wedge d(y)) + V_x A_x (d(A_y) \&\wedge d(y)) - V_y A_z (d(A_y) \&\wedge d(z)) + \Phi V_z (d(A_z) \&\wedge d(t)) - A_x V_z (d(A_z) \&\wedge d(x)) - V_z A_y (d(A_z) \&\wedge d(y)) + \Phi (d(A_z) \&\wedge d(z)) + V_y A_y (d(A_z) \&\wedge d(z)) + V_x A_x (d(A_z) \&\wedge d(z)) - A_y V_y (d(\Phi) \&\wedge d(t)) - A_x V_x (d(\Phi) \&\wedge d(t)) - A_z V_z (d(\Phi) \&\wedge d(t)) - A_z (d(\Phi) \&\wedge d(z)) - A_y (d(\Phi) \&\wedge d(y)) - A_x (d(\Phi) \&\wedge d(x))) / ((\Phi + V_z A_z + V_y A_y + V_x A_x) (d(A_x) \&\wedge d(x)) + (d(A_y) \&\wedge d(y)) + (d(A_z) \&\wedge d(z)) - (d(\Phi) \&\wedge d(t)))$$

> `S:=(wcollect(factor(hh1&^gg1+hh2&^gg2+hh3&^gg3)));`

$$S := \frac{(-\Phi - Vz Az - Vy Ay) (d(Ax) \&^{\wedge} d(Vx))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} + \frac{Ay Vx (d(Ax) \&^{\wedge} d(Vy))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} + \frac{Vx Az (d(Ax) \&^{\wedge} d(Vz))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}$$

$$+ \frac{Ax Vy (d(Ay) \&^{\wedge} d(Vx))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} + \frac{(-\Phi - Vz Az - Vx Ax) (d(Ay) \&^{\wedge} d(Vy))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} + \frac{Vy Az (d(Ay) \&^{\wedge} d(Vz))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}$$

$$+ \frac{Ax Vz (d(Az) \&^{\wedge} d(Vx))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} + \frac{Vz Ay (d(Az) \&^{\wedge} d(Vy))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} + \frac{(-\Phi - Vy Ay - Vx Ax) (d(Az) \&^{\wedge} d(Vz))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}$$

$$+ \frac{Ax (d(\Phi) \&^{\wedge} d(Vx))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} + \frac{Ay (d(\Phi) \&^{\wedge} d(Vy))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} + \frac{Az (d(\Phi) \&^{\wedge} d(Vz))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}$$

>

> **Sigma1:=simplify(subs(wcollect(factor(omega^gg1))));**
Sigma2:=factor(simplify(wcollect(factor(omega^gg2))));**Sigma3:=simplify(omega^g**
g3);

$$\Sigma 1 := \frac{Ax (d(x) \&^{\wedge} d(Vx)) + Ay (d(y) \&^{\wedge} d(Vx)) + Az (d(z) \&^{\wedge} d(Vx)) - \Phi (d(t) \&^{\wedge} d(Vx))}{\Phi + Vz Az + Vy Ay + Vx Ax}$$

$$\Sigma 2 := \frac{Ax (d(x) \&^{\wedge} d(Vy)) + Ay (d(y) \&^{\wedge} d(Vy)) + Az (d(z) \&^{\wedge} d(Vy)) - \Phi (d(t) \&^{\wedge} d(Vy))}{\Phi + Vz Az + Vy Ay + Vx Ax}$$

$$\Sigma 3 := \frac{Ax (d(x) \&^{\wedge} d(Vz)) + Ay (d(y) \&^{\wedge} d(Vz)) + Az (d(z) \&^{\wedge} d(Vz)) - \Phi (d(t) \&^{\wedge} d(Vz))}{\Phi + Vz Az + Vy Ay + Vx Ax}$$

WA TORSION 2-forms

> **Phi1:=wcollect(subs(factor(Omega^gg1)));Phi2:=wcollect(factor(Omega^gg2));Phi3**
:=wcollect(factor(Omega^gg3));

$$\Phi 1 := -\frac{Vx (d(Ax) \&^{\wedge} d(Vx))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} - \frac{Vy (d(Ay) \&^{\wedge} d(Vx))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} - \frac{Vz (d(Az) \&^{\wedge} d(Vx))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}$$

$$- \frac{d(\Phi) \&^{\wedge} d(Vx)}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}$$

$$\Phi 2 := -\frac{Vx (d(Ax) \&^{\wedge} d(Vy))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} - \frac{Vy (d(Ay) \&^{\wedge} d(Vy))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} - \frac{Vz (d(Az) \&^{\wedge} d(Vy))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}$$

$$- \frac{d(\Phi) \&^{\wedge} d(Vy)}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}$$

$$\Phi 3 := -\frac{Vx (d(Ax) \&^{\wedge} d(Vz))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} - \frac{Vy (d(Ay) \&^{\wedge} d(Vz))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} - \frac{Vz (d(Az) \&^{\wedge} d(Vz))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}$$

$$- \frac{d(\Phi) \&^{\wedge} d(Vz)}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2}$$

>

> **Theta:=array([[wcollect(gg1^hh1),wcollect(gg1^hh2),wcollect(gg1^hh3)], [gg2^h**
h1,gg2^hh2,gg2^hh3],[gg3^hh1,gg3^hh2,(gg3^hh3)]]);

$$\Theta := \left[\frac{(-\Phi - Vz Az - Vy Ay) (d(Vx) \&^{\wedge} d(Ax))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} + \frac{Ax Vy (d(Vx) \&^{\wedge} d(Ay))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} + \frac{Ax Vz (d(Vx) \&^{\wedge} d(Az))}{(\Phi + Vz Az + Vy Ay + Vx Ax)^2} \right]$$

