```
> restart:with(linalg):with(difforms);with(liesymm):
> setup(x,y,z,t):defform(x=0,y=0,z=0,t=0,UU=0,VV=0,f=0,a=const,b=const,c=const,
  k=const,mu=const,m=const,alpha=const,beta=const,n=const,omega=const,kappa=
  const,epsilon=const,pi=const,p=const,e=const,N=const,H=const,Az=0,phi=0,Ax=0,
  Ay=0,Gamma=const,Omega=const,gamma=const,q=const,g=const,sigma=const);
```

$$[\&\wedge, d, defform, formpart, parity, scalarpart, simpform, wdegree] \quad (1)$$

Correlations, and closed Constitutive Currents

Maple Name: Constitutive Currents.mws

R. M. Kiehn

USES HOLDER NORMS, CONSTITUTIVE CURRENTS from D =epsilon E and B = Mu H in 4D

R. M. Kiehn

from maxwell/mws and maxwellplasma.mws and MapleEM.mws
Updated 12/12/97, 11/5/98, 10/24/2002 Correcting sign of T4 and d(F), 11/09/2003,

Last update: November 22, 2008

Overview

On a variety of independent variables (x,y,z,t), consider a 4 component 1-form, Ao, with coefficients Ak = Ax,Ay,Az,phi.
Define the HolderNormN, of signature (a,b,c,e) degree p and homogeneity index N, as the function:

$$HolderNormN := \left(a\,Ax(x,y,z,t)^p + b\,Ay(x,y,z,t)^p + c\,Az(x,y,z,t)^p + e\,\phi(x,y,z,t)^p\right)^{\frac{N}{p}}$$

Define a scaled 1-form AN by dividing the 1-form Ao by the HolderNorm with index N.
AN = Ao/HolderNormN
also define AH=Ao/HolderNormH
Define the Jacobian matrix [**J**(AN)] of the Coefficents of of the scaled Action 1-form AN
Define the Adjoint matrix [**ADJ**(AN)] as the matrix of cofactors transposed of the Jacobian matrix computed from [**J**(AN)]
Define the Adjoint Current as the vector equal |**C**> to the product of the Adoint matrix times the vector AH (not necessarily AN).
|**C**> = [**ADJ**(AN)]|AH>

If 4 - 3N - H = 0, then |C> => |J> AND divergence |J> =0. (A conserved adjoint current !!!)

IF N = 1, H=1, and a=b=c=e=1, p=2, then the JAcobian matrix has zero determinant , and the similarity invariants of the JAcobian matrix can be related to the curvatures of the associated implicit surface.

The Adjoint current method will be demonstrate in another pdf file.

The 1-form of Action potentials

$$A = \Sigma_{k=1}^{3} A_k(x,y,z,t)dx^k - \phi(x,y,z,t)dt = \mathbf{A}\circ d\mathbf{r} - \phi dt.$$

The Engineering vector format of the field intensities

$$\mathbf{E} = -\partial\mathbf{A}/\partial t - grad\phi, \qquad \mathbf{B} = curl\ \mathbf{A} \equiv \partial A_k/\partial x^j - \partial A_j/\partial x^k.$$

The 2-form of Field intensities

$$F = dA = \{\partial A_k/\partial x^j - \partial A_j/\partial x^k\} dx^j \hat{}\, dx^k = F_{jk} dx^j \hat{}\, dx^k$$
$$= \mathbf{B}_z dx\hat{}dy + \mathbf{B}_x dy\hat{}dz + \mathbf{B}_y dz\hat{}dx + \mathbf{E}_x dx\hat{}dt + \mathbf{E}_y dy\hat{}dt + \mathbf{E}_z dz\hat{}dt$$

The Topological Torsion vector, T4,
The 3-form of Topological Torsion (note the minus sign)
and the 4-form of Topological Parity.

$$\begin{aligned}
\mathbf{T}_4 &= -[\mathbf{E}\times\mathbf{A} + \mathbf{B}\phi,\ \mathbf{A}\circ\mathbf{B}],\\
A\hat{}dA &= i(\mathbf{T}_4)\Omega_4,\\
&= T_4^x dy\hat{}dz\hat{}dt - T_4^y dx\hat{}dz\hat{}dt\\
&\quad + T_4^z dx\hat{}dy\hat{}dt - T_4^t dx\hat{}dy\hat{}dz,\\
dA\hat{}dA &= 2(\mathbf{E}\circ\mathbf{B})\ \Omega_4 = K\Omega_4,\\
&= \{\partial T_4^x/\partial x + \partial T_4^y/\partial y + \partial T_4^z/\partial z + \partial T_4^t/\partial t\}\ \Omega_4.
\end{aligned}$$

Some additional formulas (note the signs)

The Work 1-form: $W = i(\rho\mathbf{V}_4)dA = i(J)F,$

$\Rightarrow -\{\rho\mathbf{E} + \mathbf{J}\times\mathbf{B}\}\circ d\mathbf{r} + \{\mathbf{J}\circ\mathbf{E}\}dt.$

The Lorentz force : $-\{\mathbf{f}_{Lorentz}\}\circ d\mathbf{r}$ **component.**

The dissipative power : $+\{\mathbf{J}\circ\mathbf{E}\}dt$ **component.**

Properties of the Topological Torsion vector T_4

$$\begin{aligned}
i(\mathbf{T}_4)\Omega_4 &= A\hat{}dA,\\
W &= i(\mathbf{T}_4)dA = \sigma\ A,\\
U &= i(\mathbf{T}_4)A = 0,\\
L_{(\mathbf{T}_4)}A &= \sigma\ A,\\
Q\hat{}dQ &= L_{(\mathbf{T}_4)}A\hat{}L_{(\mathbf{T}_4)}dA = \sigma^2 A\hat{}dA \neq 0,\\
dA\hat{}dA &= (2!)\ \sigma\ \Omega_4.
\end{aligned}$$

NOTES:

The fundamental references are my monographs Vol1 and Vol4, which can be found at http://www.lulu.com/kiehn
*
This Maple program computes Maxwell-Faraday formulas from the postulate of potentials F-dA=0.
Given a 1-form of Action on 4D space time, the E and B fields follow by exterior differentiation.
The 2-form F is the set of limit points for the 1-form, A.
The Maxwell Ampere equations are computed from the postulate of charge currents, J-dG=0.
The 2-form density, G, with components D and H, is constructed in several ways

1. The most simple assumption selects the Lorentz-Minkowski vacuum constitutive equations, D = epsilon E   B = mu H.
2. A more complicated procedure selects the complex 6x6 constitutive matrix formulated by Post (see Vol 4)
3. Another procedure selects a chiral formulation for a the constitutive matrix. (see Vol4).
   The chiral additions to the Lorentz involve adding terms alpha*(g+I*gamma)*sqrt(mu/epsion) The factor alpha is more than likely equal to 1/(2*fine structure constant), which makes the effective chiral impedance the Hall Impedance ~ 137/2   Free Space Impedance.

*
The 1-form of Action not only encodes the electromagnetic potentials,
but also topologically encodes a thermodynamic system. (see Vol1).
The Potentials, not the charge current densities, are used as the computational starting point,
with functions defined on a basis variety if 4 dimensions (x,y,z,t).
This topological approach is more useful for the constuction of field, not necessarily particle, properties
of Plasma systems, where charge currents can be associated with collective states, not individual particles
*
The program also permits the study of homogeneous systems of various degrees, through the use of
Holder Norm divisors.
*
The procedure starts with the functional input the 4 potential, and computes
E,B, then D,H using a constitutive map, ,Jamperian,Jdisplacement,Jtotal, and the Charge density, rho,
as well as
the Torsion vector = -[ExA+Bphi, AdotB]
the Spin Vector = A x H + Dphi, AdotD,
the First Poincare invariant = F^G -A^J = (BdotH-DdotE) -(AdotJ-rho.phi),
the second Poincare Invariant = F^F = +2EdotB,
the Lorentz Force = - rho E +J x B,
the dissipation = JdotE,
the Poynting vector ExH,
the Topological Torsion, A^F
the Topological Spin A^G,
the 4D interaction energy density, AdotJ - rho phi,
the Work 1-form W = i(J,rho) F,
the internal energy U = AdotJ - rho phi
the Heat 1-form Q = W + dU
The program checks to see if Q^dQ is zero (hence {J,rho} is a reversible process) or not zero (hence {J,rho} is an irreversible process.
and the similarity invariants of the correlation Jacobian matrix computed from the (possibly scaled and homogeneous) Action 1-form of Potentials.
If A^F = 0, then the thermodynamic system is of PTD 2 or less
If A^F is not zero, but F^F is zero, then the thermodynamic system is a Closed non-equilibrium system that can exchange mass/energy or radiation, but not particles with its environment.
If F^F is not zero, then the thermodynamic system is an Open non-equilibrium system that can exchange particles as well as mass/energy or radiation with its environment
(see vol4)
It is remarkable that a choice of vector and scalar potential functions can lead to charge current 3-forms whose coefficients are
proportional to the the vector and scalar potentials. J = chi A which is the form of a London current.
The are also cases where the Topological Spin current has coefficients that are proportional to the coefficients of the Lorentz force.

BE AWARE The algebra can be overpowering. Thank you Maple.

The main procedure

```
JCM:=proc(A1,A2,A3,phi,a,b,c,e,p,N,H,CH,sigma)
     local BFC,TFC,EF1,EF2,EF3,JAC,JDC,SFC,ExBC,S2:
global A1form,HEL,ExB,NAME,lambdaN,lambdaH,ACTIONN,ACTIONH,Action,Actionem,JACOB,
ADJACOB,ADJOINT,ADJOINTCURR,Xm,Yg,Za,Tk,EdotB,E,EXA,A,AA,BB,Ea,Eg,B,F,Fem,
Torsion3_form,Torsion3_formem,Parity4_form,Q,dQ,QdQ,dQdQ,QdQ4,DETJACOB,T3,T4,U,Uch,
Uadj,JXB,JXBch,JXBadj,AdotT4,DIVT4,DIVADJOINTCURR,ParityFFFF,ParityDIV_T4,
Parity2EdotB,AdotJ,AdotJch,AdotJadj,CHECK,CHECKch,CHECKadj,A4,DFcha,HFcha,JdotE,
JdotEch,JdotEadj,J,Jch,Jadj,rho,rho_ch,rho_adj,rho_E,rho_Ech,rho_Eadj,dWork,dWorkch,
dWorkadj,Work_1form,Work_1formch,Work_1formadj,Work,Workch,Workadj,DF,DFch,DFadj,HF,
HFch,HFadj,EXH,EXHch,EXHadj,CD,CDch,CDadj,JA,JAch,JAadj,JTOT,JTOTch,JTOTadj,DIVJTOT,
DIVJTOTch,DIFJTOTadj,JD,JDch,JDadj,SP3,SP3ch,SP3adj,SP4,SP4ch,SP4adj,dSP4,dSP4ch,
dSP4adj,OPT,OPTch,OPTadj,AXH,AXHch,AXHadj,LAGF,LAGFch,LADFadj,PI,PIch,PIadj,LF,LFcn,
LFadj,SPIN3_form,dSPIN3_form,BH,DE,AJ,rhophi,CCB,AJJJ,AAAA,LONFAC,PTD,AxJ,Zfs,Zfsm,
Zfse,LFSPIN,SF,TF:
```

Compute Holder Norms with N and H homogeneity index. Note N and H can both be zero, But if 4 - 3N - H = 0, then the adjoint current is closed, dJ = 0.

```
> lambdaN:=subs(a=a,b=b,c=b,e=e,p=p,(a*A1^p+b*A2^p+c*A3^p+e*(-phi)^p)^(N/p));
> lambdaH:=subs(a=a,b=b,c=c,e=e,p=p,(a*A1^p+b*A2^p+c*A3^p+e*(-phi)^p)^(H/p));
```

Scale Action 1-forms with Holder divisors, compute Jacobian Correlation matrix of scaled Action and its Adjoint
Compute Action 1-form of Potentials,
the 2-form of field intensities F = dA , giving the E and B fields as coefficients
the 3-form of Topological Torison, A^F,
and the 4-form of Topological Parity.
Compare differential form methods with vector methods for Maxwell-Faraday equations.

```
> ACTIONN:=[A1/lambdaN,A2/lambdaN,A3/lambdaN,-phi/lambdaN];
> ACTIONH:=[A1/lambdaH,A2/lambdaH,A3/lambdaH,-phi/lambdaH]:
> JACOB:=jacobian(ACTIONN,[x,y,z,t]):
> ADJOINT:=adjoint(JACOB):
> DETJACOB:=factor(det(JACOB));
> Action:=wcollect(innerprod(ACTIONN,[d(x),d(y),d(z),d(t)]));
> A:=([ACTIONN[1],ACTIONN[2],ACTIONN[3]]);
> A4:=phi/lambdaN;
> BB:=curl(A,[x,y,z]);B:=[factor(BB[1]),factor(BB[2]),factor(BB[3])];
> Ea:=(-diff(A,t));CCB:=simplify(curl(curl(B,[x,y,z]),[x,y,z]));
> Eg:=evalm(grad(-phi/lambdaN,[x,y,z]));
> E:=([factor(Ea[1]+Eg[1]),factor(Ea[2]+Eg[2]),factor(Ea[3]+Eg[3])]);
> EdotB:=factor(E[1]*B[1]+E[2]*B[2]+E[3]*B[3]);
> Actionem:=innerprod(A,[d(x),d(y),d(z)])-phi/lambdaN*d(t);
> F:=wcollect(d(Action)):
> Fem:=E[1]*d(x)&^d(t)+E[2]*d(y)&^d(t)+E[3]*d(z)&^d(t)+B[1]*d(y)&^d(z)+B[2]*d(z)
  &^d(x)+B[3]*d(x)&^d(y);
> Torsion3_form:=wcollect((Action&^F));
> Parity4_form:=factor(F&^F);
```

Compute the Topological Torsion vector T4 using T4 = - [ExA + B phi, AdotB]

Compare the Parity coefficients of the 4-form and vector methods.
Note that A^A^F = 0 so that the 4-potentials are orthogonal to T4.

```
> EXA:=crossprod(E,A);
> T3:=([EXA[1]+B[1]*phi/lambdaN,EXA[2]+B[2]*phi/lambdaN,EXA[3]+B[3]*phi/lambdaN]
  ):
> HEL:=factor(innerprod(A,B));
> T4:=-([factor(T3[1]),factor(T3[2]),factor(T3[3]),HEL]);
> AdotT4:=factor(A[1]*T4[1]+A[2]*T4[2]+A[3]*T4[3]-phi/lambdaN*T4[4]);
> DIVT4:=factor(diverge(T4,[x,y,z,t])):
> ParityDIV_T4:=factor(DIVT4);
> Parity2EdotB:=factor(2*EdotB);
> ParityFFFF:=factor(getcoeff(F&^F));
> ParityDIV_T4:=factor(DIVT4);
> Parity2EdotB:=factor(2*EdotB);
> ParityFFFF:=factor(getcoeff(F&^F));
> Torsion3_formem:=T4[1]&^d(y)&^d(z)&^d(t)-T4[2]&^d(x)&^d(z)&^d(t)+T4[3]&^d(x)
  &^d(y)&^d(t)-T4[4]&^d(x)&^d(y)&^d(z);

> if E = [0,0,0] and B = [0,0,0] then PTD:= 1 else PTD:=2 end if;
> if T4[1]<>0 or T4[2]<>0 or T4[3]<>0 or T4[4]<>0  then PTD:=3 else PTD:= PTD
  end if;
> if ParityFFFF <> 0 then PTD:=4 else PTD:=PTD end if;
```

Similarity Invariants of the (scaled) Jacobian matrix

```
> Xm:=factor(trace(JACOB));
> S2:=factor(trace(innerprod(JACOB,JACOB))):
> Yg:=factor((-(1/2)*((-trace(JACOB)*trace(JACOB)+S2))));
> Za:=factor((trace(adjoint(JACOB))));
> Tk:=factor(det(JACOB));
```

Print routines for E and B

```
> print(NAME);print(`*********************************    Differential Form
  Format    ********************************* `);
> print(`Action 1-form`= Action);
> print(`Intensity 2-form F=dA  `= F);
> print(`Topological Torsion 3-form    A^F` = (Torsion3_form));
> print(`Topological Parity 4-form     F^F` =  (Parity4_form));
> print(`*********************************  Using EM format  *******************
  **************`);
> print(`E field`= simplify(E));
> print(`B field`= simplify(B));

> print(`Topological TORSION 4 vector   T4 = -[ExA + Bphi,AdotB] `= T4);
> print(`Helicity AdotB`= HEL);
> print(`Poincare II =2(E.B)`= 2*EdotB);
```

```
> print(`coefficient of Topological Parity 4-form ` = factor(getcoeff(Fem&^Fem))
  );
> print(`  Pfaff Topological Dimension     PTD`=PTD);
```

```
> print(`******************** Correlation Similarity Invariants of Jacobian of
  (Ak/lambda_N) ***************************`);
> print(`                                Xm or linear (Mean) curvature ` = Xm);
> print(`                         Yg or quadratic (GAUSS) curvature ` = Yg);
> print(` Za or Cubic (Interaction internal energy) curvature  ` = Za);
> print(`                   Tk or quartic (4D expansion) curvature  ` = Tk);
```

CONSTITUTIVE CURRENTS FROM D=epsilon E and H = B/mu
LORENTZ CURRENTS   use free space impedance

Use the Lorentz constitutive format D = epsilon E,   H = B/mu to compute the field Excitations (D and H).
The use the Maxwell Ampere equations to compute a Closed Current
J = curl H - dD/dt. (partial derivatives).   rho = div D

```
> Zfsm:=(1/epsilon)^(1/2);Zfse:=CH*(mu/epsilon)^(1/2);
> DF:=[factor(epsilon*E[1]+Zfse*B[1]),factor(epsilon*E[2]+Zfse*B[2]),factor
  (epsilon*E[3]+Zfse*B[3])];
> HF:=[factor(B[1]/mu-Zfse*E[1]),factor(B[2]/mu-Zfse*E[2]),factor(B[3]/mu-Zfse*E
  [3])];BH:=factor(innerprod(B,HF));DE:=factor(innerprod(DF,E));
> EXH:=(crossprod(E,HF));
> CD:=factor(diverge(DF,[x,y,z]));
> JAC:=curl(HF,[x,y,z]);
> JA:=[factor(JAC[1]),factor(JAC[2]),factor(JAC[3])];
> JDC:=-[diff(DF[1],t),diff(DF[2],t),diff(DF[3],t)];
> JD:=[factor(JDC[1]),factor(JDC[2]),factor(JDC[3])];
> JTOT:=[factor(factor(JAC[1]+sigma*E[1])+factor(JDC[1])),factor(factor(JAC[2]+
  sigma*E[2])+factor(JDC[2])),factor(factor(JAC[3]+sigma*E[3])+factor(JDC[3])),
  factor(CD)];
> AdotJ:=factor(innerprod(ACTIONN,JTOT));
> JdotE:=factor(JTOT[1]*sigma*E[1]+JTOT[2]*sigma*E[2]+JTOT[3]*sigma*E[3]);
> J:=evalm([JTOT[1],JTOT[2],JTOT[3]]);AJ:=factor(innerprod(A,J));rhophi:=factor
  (-JTOT[4]*ACTIONN[4]);
> rho:=JTOT[4];
> rho_E:=simplify(rho*E);
> JXB:=(crossprod(J,B));
> LAGF:=factor(factor(innerprod(B,HF))-factor(innerprod(DF,E)));
> AdotJ:=factor(innerprod(JTOT,ACTIONN));
> PI:=factor(LAGF-AdotJ);
> DIVJTOT:=factor(diverge(JTOT,[x,y,z,t]));
> AAAA:=innerprod(A,A);AJJJ:=innerprod(A,J);
> AxJ:=simplify(crossprod([JTOT[1],JTOT[2],JTOT[3]],[A[1],A[2],A[3]]));
> if AxJ[1]<>0 and AxJ[2]<>0 and AxJ[3]<>0 then LONFAC:= 0 end if;
```

```
> if (AxJ[1]=0 and AxJ[2]=0 and AxJ[3]<>0) and A[1]<>0 then LONFAC:=factor(JTOT
  [1]/A[1]) end if;
> if AxJ[1]<>0 and AxJ[2]=0 and AxJ[3]=0 and A[2]<>0 then LONFAC:=factor(JTOT[2]
  /A[2])  end if;
> if AxJ[1]=0 and AxJ[2]<>0 and AxJ[3]=0 and A[1]<>0 then LONFAC:=factor(JTOT[1]
  /A[1]) end if;
> if AxJ[1]=0 and AxJ[2]=0 and AxJ[3]=0 and A[1]<>0 then LONFAC:=+factor(JTOT[1]
  /A[1]) end if;
```

compute Topological Spin

```
> AXH:=(crossprod(A,HF));
> SP3:=([AXH[1]+DF[1]*phi/lambdaN,AXH[2]+DF[2]*phi/lambdaN,AXH[3]+DF[3]*
  phi/lambdaN]):
> OPT:=factor(innerprod(evalm(A),DF));
> SP4:=([factor(SP3[1]),factor(SP3[2]),factor(SP3[3]),factor(OPT)]);
> dSP4:=factor(diverge(SP4,[x,y,z,t]));
> SPIN3_form:= SP4[1]*d(y)&^d(z)&^d(t)-SP4[2]*d(x)&^d(z)&^d(t)+SP4[3]*d(x)&^d(y)
  &^d(t)-SP4[4]*d(x)&^d(y)&^d(z);
> dSPIN3_form:=factor(wcollect(d(SPIN3_form)));

> print(`**********      Compute Current using from Maxell-Ampere equations for
  constitutive equations with chirality CH  ***********`);print(`Chirality
  factor CH`=Zfse);
> print(`D field`= DF);
> print(`H field`= HF);
> print(`Poynting vector ExH`= simplify(EXH));
> print(`Amperian Current 4Vector     curlH-dD/dt=J4  `= JTOT);
> print(`Amerian charge density       divD = rho`= CD);
> print(`divergence Lorentz Current 4Vector,    4div(J4) `= factor(DIVJTOT));
print(`Topological SPIN 4 vector S4`=SP4);
print(`Topological SPIN 3-form`=SPIN3_form);
print(`Spin density rho_spin`= factor(SP4[4]));


> print(`LaGrange field energy density (B.H-D.E)`= LAGF);
```

Compute the Work, Heat and Internal Energy for a thermodynamic systems defined by the 1-form of Action, and the Closed Current.
Determine if the process J is reversible or irreversible in a thermodynamic sense.

```
> Work:=-([factor(factor(rho*E[1])+factor(crossprod(J,B)[1])),factor(factor(rho*
  E[2])+factor(crossprod(J,B)[2])),factor(factor(rho*E[3])+factor(crossprod(J,B)
  [3])),-factor(JdotE)]);
> CHECK:=innerprod((Work),JTOT);
> Work_1form:=wcollect(innerprod(Work,[d(x),d(y),d(z),d(t)]));
> dWork:=wcollect(simplify(d(Work_1form)));
> LF:=-([factor(factor(rho*E[1])+factor(crossprod(J,B)[1])),factor(factor(rho*E
  [2])+factor(crossprod(J,B)[2])),factor(factor(rho*E[3])+factor(crossprod(J,B)
```

```
[3]))]);
> U:=AdotJ;LFSPIN:=simplify(crossprod(LF,[SP4[1],SP4[2],SP4[3]]));
> if LFSPIN[1]+LFSPIN[2]+LFSPIN[3]=0 and LF[1]<>0 then LFSPIN:=SP4[1]/LF[1] else
LFSPIN:=0  end if;
```

```
> SF:=-([factor(factor(SP4[4]*E[1])+factor(crossprod(SP3,B)[1])),factor(factor
(SP4[4]*E[2])+factor(crossprod(SP3,B)[2])),factor(factor(SP4[4]*E[3])+factor
(crossprod(SP3,B)[3]))]);
```

```
> TF:=-([factor(factor(HEL*E[1])+factor(crossprod(T3,B)[1])),factor(factor(HEL*E
[2])+factor(crossprod(T3,B)[2])),factor(factor(HEL*E[3])+factor(crossprod(T3,
B)[3]))]);
```

```
> print(`    B.H`=BH);print(`    D.E`=DE);
> print(`    A.J`=AJ);print(`    -rho.phi`=rhophi);
```

```
> print(` Poincare I       (B.H - D.E)-(A.J - rho.phi) `= PI );
> print(`   London Coefficient      LC`=LONFAC);
>  print(`PROCA coefficient curlcurlB`=[factor(CCB[1]),factor(CCB[2]),factor(CCB
[3])]); print(`  `);
> print(`Amperian Current 4Vector     curlH-dD/dt=J4  `= JTOT);

> print(` Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E +
J_ampere x B)   `= LF);

> print(`Amperian Dissipation  Jampere dot E   `= +JdotE );
> print(` Lorentz Force Spin factor  LFSPIN`=LFSPIN); print(`  `);
```

```
>
```

```
> print(`Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] `= T4)
  ;

> print(` Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E +
  J_torsion x B)   `= TF);
> print(`Torsion Dissipation  Jtorsion dot E   `= factor(innerprod(T3,E)));

>  print(`  `);print(`Topological Spin current 4 vector   TS4 = -[A x H + D.phi,
  AdotD] `= SP4);
> print(` Lorentz Force 3 vector due to Spin current    SF = --(rho_spin E +
  J_spin x B)   `= SF);
> print(`Spin Dissipation  J_spin dot E   `= factor(innerprod(SP3,E)));

> print(`  Disspative Force 3 vector`=[factor(LF[1]+mu*SF[1]+TF[1]),factor(LF[2]
  +mu*SF[2]+TF[2]),factor(LF[3]+mu*SF[3]+TF[3])]);
> print(`  Dissipation  `=factor(JTOT[4]+mu*SP4[4]+T4[1]));



> print(`******************************    END PROCEDURE   *********************
  *******`);
> end:
```

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`-  vol 1 p. 397  vol1 p.397- Hedgehog, accretion discs`;r1:=(x^2+y^2);
> G:=alpha*z/(x^2+y^2+e*z^2)^(1/2);A1:=-G*b*(y)/r1;A2:=G*b*(x)/r1;
> A3:=0; phi:=0/(4*pi*epsilon)*q/(x^2+y^2+z^2)^(1/2);
```

Then call the procedure JCM(A 1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

$$NAME := \text{- vol 1 p. 397 vol1 p.397- Hedgehog, accretion discs}$$

$$r1 := x^2 + y^2$$

$$G := \frac{\alpha z}{\sqrt{x^2 + y^2 + e z^2}}$$

$$A1 := -\frac{\alpha z b y}{\sqrt{x^2 + y^2 + e z^2}\,(x^2 + y^2)}$$

$$A2 := \frac{\alpha z b x}{\sqrt{x^2 + y^2 + e z^2}\,(x^2 + y^2)}$$

$$A3 := 0$$

$$\phi := 0$$

- *vol 1 p. 397 vol1 p.397- Hedgehog, accretion discs*

******************************** *Differential Form Format* ********************
****

$$\textit{Action 1-form} = -\frac{\alpha z b y\, d(x)}{\sqrt{x^2+y^2+e z^2}\,(x^2+y^2)} + \frac{\alpha z b x\, d(y)}{\sqrt{x^2+y^2+e z^2}\,(x^2+y^2)}$$

$$\textit{Intensity 2-form F=dA} = \left(\frac{\alpha z b\,(-x^2 y^2 - 2 y^4 + x^4 + x^2 e z^2 - y^2 e z^2)}{(x^2+y^2+e z^2)^{3/2}\,(x^2+y^2)^2} - \frac{\alpha z b\,(2 x^4 + x^2 y^2 + x^2 e z^2 - y^4 - y^2 e z^2)}{(x^2+y^2+e z^2)^{3/2}\,(x^2+y^2)^2}\right)(d(x))\ \&\wedge\ (d(y)) + \frac{\alpha b y\,(d(x))\ \&\wedge\ (d(z))}{(x^2+y^2+e z^2)^{3/2}} - \frac{\alpha b x\,(d(y))\ \&\wedge\ (d(z))}{(x^2+y^2+e z^2)^{3/2}}$$

Topological Torsion 3-form $A\wedge F = 0$

Topological Parity 4-form $F\wedge F = 0$

******************************** *Using EM format* ****************************
****

$$\textit{E field} = [0, 0, 0]$$

$$\textit{B field} = \left[-\frac{\alpha b x}{(x^2+y^2+e z^2)^{3/2}}, -\frac{\alpha b y}{(x^2+y^2+e z^2)^{3/2}}, -\frac{\alpha z b}{(x^2+y^2+e z^2)^{3/2}}\right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension PTD = 2

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

Xm or linear (Mean) curvature = 0

$$\textit{Yg or quadratic (GAUSS) curvature} = -\frac{(2 x^2 + 2 y^2 + e z^2)\, z^2 \alpha^2 b^2}{(x^2+y^2+e z^2)^2\,(x^2+y^2)^2}$$

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

D field = [0, 0, 0]

$$H\ field = \left[-\frac{\alpha\, b\, x}{(x^2+y^2+e\,z^2)^{3/2}\,\mu},\ -\frac{\alpha\, b\, y}{(x^2+y^2+e\,z^2)^{3/2}\,\mu},\ -\frac{\alpha\, z\, b}{(x^2+y^2+e\,z^2)^{3/2}\,\mu} \right]$$

Poynting vector ExH = EXH

Amperian Current 4Vector *curlH-dD/dt=J4*
$$= \left[-\frac{3\,\alpha\, z\, b\, y\,(-1+e)}{(x^2+y^2+e\,z^2)^{5/2}\,\mu},\ \frac{3\,\alpha\, z\, b\, x\,(-1+e)}{(x^2+y^2+e\,z^2)^{5/2}\,\mu},\ 0,\ 0 \right]$$

Amerian charge density *divD = rho* = 0

divergence Lorentz Current 4Vector, *4div(J4)* = 0

$$Topological\ SPIN\ 4\ vector\ S4 = \left[-\frac{\alpha^2 z^2 b^2 x}{(x^2+y^2+e\,z^2)^2\,(x^2+y^2)\,\mu},\ -\frac{\alpha^2 z^2 b^2 y}{(x^2+y^2+e\,z^2)^2\,(x^2+y^2)\,\mu},\ \frac{b^2\alpha^2 z}{(x^2+y^2+e\,z^2)^2\,\mu},\ 0 \right]$$

$$Topological\ SPIN\ 3\text{-}form = -\frac{\alpha^2 z^2 b^2 x\ \&\text{^}(d(y), d(z), d(t))}{(x^2+y^2+e\,z^2)^2\,(x^2+y^2)\,\mu} + \frac{\alpha^2 z^2 b^2 y\ \&\text{^}(d(x), d(z), d(t))}{(x^2+y^2+e\,z^2)^2\,(x^2+y^2)\,\mu} + \frac{b^2\alpha^2 z\ \&\text{^}(d(x), d(y), d(t))}{(x^2+y^2+e\,z^2)^2\,\mu}$$

Spin density rho_spin = 0

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = \frac{\alpha^2 b^2 (x^2+y^2+z^2)}{(x^2+y^2+e\,z^2)^3\,\mu}$$

$$B.H = \frac{\alpha^2 b^2 (x^2+y^2+z^2)}{(x^2+y^2+e\,z^2)^3\,\mu}$$

D.E = 0

$$A.J = \frac{3\,\alpha^2 z^2 b^2 (-1+e)}{(x^2+y^2+e\,z^2)^3\,\mu}$$

-rho.phi = 0

$$Poincare\ I \quad (B.H - D.E)\text{-}(A.J - rho.phi) = -\frac{\alpha^2 b^2 (-x^2-y^2-4\,z^2+3\,e\,z^2)}{(x^2+y^2+e\,z^2)^3\,\mu}$$

$$London\ Coefficient \quad LC = \frac{3\,(-1+e)\,(x^2+y^2)}{(x^2+y^2+e\,z^2)^2\,\mu}$$

PROCA coefficient curlcurlB $= \left[\frac{3\,\alpha\,b\,x\,(-1+e)\,(4\,e\,z^2-y^2-x^2)}{(x^2+y^2+e\,z^2)^{7/2}}, \right.$

$\left. \frac{3\,\alpha\,b\,y\,(-1+e)\,(4\,e\,z^2-y^2-x^2)}{(x^2+y^2+e\,z^2)^{7/2}}, \frac{3\,\alpha\,z\,b\,(-1+e)\,(2\,e\,z^2-3\,y^2-3\,x^2)}{(x^2+y^2+e\,z^2)^{7/2}} \right]$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \left[-\frac{3\,\alpha\,z\,b\,y\,(-1+e)}{(x^2+y^2+e\,z^2)^{5/2}\,\mu}, \right.$

$\left. \frac{3\,\alpha\,z\,b\,x\,(-1+e)}{(x^2+y^2+e\,z^2)^{5/2}\,\mu}, 0, 0 \right]$

Lorentz Force 3 vector due to Ampere current *FL = -(rho_ampere E + J_ampere x B)*

$= \left[\frac{3\,\alpha^2\,z^2\,b^2\,x\,(-1+e)}{(x^2+y^2+e\,z^2)^4\,\mu}, \frac{3\,\alpha^2\,z^2\,b^2\,y\,(-1+e)}{(x^2+y^2+e\,z^2)^4\,\mu}, -\frac{3\,\alpha^2\,z\,b^2\,(-1+e)\,(x^2+y^2)}{(x^2+y^2+e\,z^2)^4\,\mu} \right]$

Amperian Dissipation *Jampere dot E* $= 0$

Lorentz Force Spin factor *LFSPIN* $= -\frac{1}{3}\,\frac{(x^2+y^2+e\,z^2)^2}{(x^2+y^2)\,(-1+e)}$

Topological Torsion current 4 vector *T4 = -[ExA + B.phi,AdotB]* $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current *TF = -(rho_torsion E + J_torsion x B)* $= [0, 0, 0]$

Torsion Dissipation *Jtorsion dot E* $= 0$

Topological Spin current 4 vector *TS4 = -[A x H + D.phi,AdotD]* $= \left[-\frac{\alpha^2\,z^2\,b^2\,x}{(x^2+y^2+e\,z^2)^2\,(x^2+y^2)\,\mu}, -\frac{\alpha^2\,z^2\,b^2\,y}{(x^2+y^2+e\,z^2)^2\,(x^2+y^2)\,\mu}, \frac{b^2\,\alpha^2\,z}{(x^2+y^2+e\,z^2)^2\,\mu}, 0 \right]$

Lorentz Force 3 vector due to Spin current *SF = --(rho_spin E + J_spin x B)* $= \left[-\frac{\alpha^3\,z\,b^3\,y\,(x^2+y^2+z^2)}{(x^2+y^2+e\,z^2)^{7/2}\,(x^2+y^2)\,\mu}, \frac{b^3\,\alpha^3\,z\,x\,(x^2+y^2+z^2)}{(x^2+y^2+e\,z^2)^{7/2}\,(x^2+y^2)\,\mu}, 0 \right]$

Spin Dissipation *J_spin dot E* $= 0$

$$Disspative\ Force\ 3\ vector = \left[\frac{1}{\left(x^2+y^2+e z^2\right)^{15/2} \mu \left(x^2+y^2\right)} \left(b^2 \alpha^2 z \left(-3 z x^3 \left(x^2+y^2 + e z^2\right)^{7/2} - \alpha b y^{11} \mu - 3 z x \left(x^2+y^2+e z^2\right)^{7/2} y^2 + 3 z x^3 \left(x^2+y^2+e z^2\right)^{7/2} e - 10 \alpha b y^5 \mu x^6 - 10 \alpha b y^7 \mu x^4 - 5 \alpha b y^3 \mu x^8 - 5 \alpha b y^9 \mu x^2 - \alpha b y^9 \mu z^2 - \alpha b y \mu x^{10} + 3 z x \left(x^2+y^2+e z^2\right)^{7/2} e y^2 - 24 \alpha b y^5 \mu x^4 e z^2 - 18 \alpha b y^3 \mu x^4 e^2 z^4 - 16 \alpha b y^7 \mu x^2 e z^2 - 18 \alpha b y^5 \mu x^2 e^2 z^4 - 8 \alpha b y^3 \mu x^2 e^3 z^6 - 12 \alpha b y^3 \mu z^4 x^4 e - 12 \alpha b y^5 \mu z^4 x^2 e - 12 \alpha b y^3 \mu z^6 x^2 e^2 - 4 \alpha b y \mu x^8 e z^2 - 6 \alpha b y \mu x^6 e^2 z^4 - 4 \alpha b y \mu x^4 e^3 z^6 - \alpha b y \mu x^2 e^4 z^8 - 4 \alpha b y \mu z^4 x^6 e - 6 \alpha b y \mu z^6 x^4 e^2 - 4 \alpha b y \mu z^8 x^2 e^3 - 4 \alpha b y^9 \mu e z^2 - 6 \alpha b y^7 \mu e^2 z^4 - 4 \alpha b y^5 \mu e^3 z^6 - \alpha b y^3 \mu e^4 z^8 - 6 \alpha b y^5 \mu z^2 x^4 - 4 \alpha b y^7 \mu z^2 x^2 - 4 \alpha b y^3 \mu z^2 x^6 - 4 \alpha b y^7 \mu z^4 e - 6 \alpha b y^5 \mu z^6 e^2 - 4 \alpha b y^3 \mu z^8 e^3 - \alpha b y \mu z^{10} e^4 - \alpha b y \mu z^2 x^8 - 16 \alpha b y^3 \mu x^6 e z^2\right)\right),$$

$$\frac{1}{\left(x^2+y^2+e z^2\right)^{15/2} \mu \left(x^2+y^2\right)} \left(b^2 \alpha^2 z \left(10 b \alpha x^7 \mu y^4 + 10 b \alpha x^5 \mu y^6 + 5 b \alpha x^9 \mu y^2 + 5 b \alpha x^3 \mu y^8 + b \alpha x^9 \mu z^2 + b \alpha x \mu y^{10} - 3 z y^3 \left(x^2+y^2+e z^2\right)^{7/2} + 24 b \alpha x^5 \mu e z^2 y^4 + 18 b \alpha x^5 \mu y^2 e^2 z^4 + 16 b \alpha x^3 \mu y^6 e z^2 + 18 b \alpha x^3 \mu y^4 e^2 z^4 + 8 b \alpha x^3 \mu y^2 e^3 z^6 + 12 b \alpha x^5 \mu z^4 e y^2 + 12 b \alpha x^3 \mu z^4 e y^4 + 12 b \alpha x^3 \mu z^6 y^2 e^2 + 4 b \alpha x \mu y^8 e z^2\right.\right.$$

$$+6\,b\,\alpha\,x\,\mu\,y^6\,e^2\,z^4+4\,b\,\alpha\,x\,\mu\,y^4\,e^3\,z^6+b\,\alpha\,x\,\mu\,y^2\,e^4\,z^8+4\,b\,\alpha\,x\,\mu\,z^4\,y^6\,e+6\,b\,\alpha\,x\,\mu\,z^6\,y^4\,e^2$$

$$+4\,b\,\alpha\,x\,\mu\,z^8\,y^2\,e^3+4\,b\,\alpha\,x^9\,\mu\,e\,z^2+6\,b\,\alpha\,x^7\,\mu\,e^2\,z^4+4\,b\,\alpha\,x^5\,\mu\,e^3\,z^6+b\,\alpha\,x^3\,\mu\,e^4\,z^8$$

$$+6\,b\,\alpha\,x^5\,\mu\,z^2\,y^4+4\,b\,\alpha\,x^3\,\mu\,z^2\,y^6+4\,b\,\alpha\,x^7\,\mu\,z^2\,y^2+4\,b\,\alpha\,x^7\,\mu\,z^4\,e+6\,b\,\alpha\,x^5\,\mu\,z^6\,e^2$$

$$+4\,b\,\alpha\,x^3\,\mu\,z^8\,e^3+b\,\alpha\,x\,\mu\,z^{10}\,e^4+b\,\alpha\,x\,\mu\,z^2\,y^8-3\,z\,y\,(x^2+y^2+e\,z^2)^{7/2}\,x^2+3\,z\,y^3\,(x^2$$

$$+y^2+e\,z^2)^{7/2}\,e+b\,\alpha\,x^{11}\,\mu+16\,b\,\alpha\,x^7\,\mu\,e\,z^2\,y^2+3\,z\,y\,(x^2+y^2+e\,z^2)^{7/2}\,e\,x^2)\,),$$

$$-\frac{3\,\alpha^2\,z\,b^2\,(-1+e)\,(x^2+y^2)}{(x^2+y^2+e\,z^2)^4\,\mu}\Bigg]$$

$$Dissipation\ =0$$

****************************** *END PROCEDURE* ****************************** **(2)**

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`-  vol 1 p. 397  vol4 p.147- Abrikosov Falaco vortex singularities.`;
  f:=(x^2+y^2);
> r1:=(f)^(2/2);A1:=1*b*0*(y)/r1;A2:=1*b*(-x)*0/r1;
> A3:=0; phi:=1/(4*pi*epsilon)*q/(x^2+y^2+z^2)^(1/2);
```

Then call the procedure JCM(A 1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := - vol 1 p. 397 vol4 p.147- Abrikosov Falaco vortex singularities.

$$f:=x^2+y^2$$

$$r1:=x^2+y^2$$

$$A1 := 0$$

$$A2 := 0$$

$$A3 := 0$$

$$\phi := \frac{1}{4}\,\frac{q}{\pi\,\varepsilon\,\sqrt{x^2+y^2+z^2}}$$

- vol 1 p. 397 vol4 p.147- Abrikosov Falaco vortex singularities.

******************************* *Differential Form Format* *******************************

$$\text{Action 1-form} = -\frac{1}{4}\,\frac{q\,d(t)}{\pi\,\varepsilon\,\sqrt{x^2+y^2+z^2}}$$

$$\text{Intensity 2-form F=dA} = \frac{1}{4}\,\frac{q\,x\,(d(x))\ \&\wedge\ (d(t))}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}} + \frac{1}{4}\,\frac{q\,y\,(d(y))\ \&\wedge\ (d(t))}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}} + \frac{1}{4}\,\frac{q\,z\,(d(z))\ \&\wedge\ (d(t))}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}$$

Topological Torsion 3-form $A\wedge F = 0$

Topological Parity 4-form $F\wedge F = 0$

******************************* *Using EM format* *******************************

$$\text{E field} = \left[\frac{1}{4}\,\frac{q\,x}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}},\ \frac{1}{4}\,\frac{q\,y}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}},\ \frac{1}{4}\,\frac{q\,z}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}\right]$$

$$\text{B field} = [0, 0, 0]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension PTD $= 2$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= 0$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $= 0$

$$D\ field = \left[\frac{1}{4} \frac{q\,x}{\pi\,(x^2+y^2+z^2)^{3/2}},\ \frac{1}{4} \frac{q\,y}{\pi\,(x^2+y^2+z^2)^{3/2}},\ \frac{1}{4} \frac{q\,z}{\pi\,(x^2+y^2+z^2)^{3/2}} \right]$$

$$H\ field = [0, 0, 0]$$

$$Poynting\ vector\ ExH = EXH$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt=J4 \quad = [0, 0, 0, 0]$$

$$Amerian\ charge\ density \quad divD\ =\ rho = 0$$

$$divergence\ Lorentz\ Current\ 4Vector, \quad 4div(J4)\ = 0$$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[\frac{1}{16} \frac{q^2\,x}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon},\ \frac{1}{16} \frac{q^2\,y}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon},\ \frac{1}{16} \frac{q^2\,z}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon},\ 0 \right]$$

$$Topological\ SPIN\ 3\text{-}form = \frac{1}{16} \frac{q^2\,x\,\&\wedge(d(y), d(z), d(t))}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon} - \frac{1}{16} \frac{q^2\,y\,\&\wedge(d(x), d(z), d(t))}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon} + \frac{1}{16} \frac{q^2\,z\,\&\wedge(d(x), d(y), d(t))}{\pi^2\,(x^2+y^2+z^2)^2\,\varepsilon}$$

$$Spin\ density\ rho\_spin = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{1}{16} \frac{q^2}{\varepsilon\,(x^2+y^2+z^2)^2\,\pi^2}$$

$$B.H = 0$$

$$D.E = \frac{1}{16} \frac{q^2}{\varepsilon\,(x^2+y^2+z^2)^2\,\pi^2}$$

$$A.J = 0$$

$$-rho.phi = 0$$

$$Poincare\ I \quad (B.H\ -\ D.E)\text{-}(A.J\ -\ rho.phi)\ = -\frac{1}{16} \frac{q^2}{\varepsilon\,(x^2+y^2+z^2)^2\,\pi^2}$$

$$London\ Coefficient \quad LC = \frac{3\,(-1+e)\,(x^2+y^2)}{(x^2+y^2+e\,z^2)^2\,\mu}$$

$$PROCA\ coefficient\ curlcurlB = [0, 0, 0]$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt=J4 \quad = [0, 0, 0, 0]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Ampere\ current \quad FL\ =\ \text{-}(rho\_ampere\ E\ + J\_ampere\ x\ B) \quad = [0, 0, 0]$$

$$Amperian\ Dissipation \quad Jampere\ dot\ E \quad = 0$$

Lorentz Force Spin factor LFSPIN = 0

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] = [0, 0, 0, 0]

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) = [0, 0, 0]

Torsion Dissipation Jtorsion dot E = 0

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{1}{16} \frac{q^2 x}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, \frac{1}{16} \frac{q^2 y}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, \frac{1}{16} \frac{q^2 z}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, 0 \right]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) = [0, 0, 0]

$$\textit{Spin Dissipation J\_spin dot E} = \frac{1}{64} \frac{q^3}{(x^2+y^2+z^2)^{5/2} \pi^3 \varepsilon^2}$$

Disspative Force 3 vector = [0, 0, 0]

Dissipation = 0

***************************** *END PROCEDURE* *************************** **(3)**

Example Potentials from the Hopf map

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

```
NAME:=`Potentials from the Hopf Map   Chirality 0*(g+I*gamma)`:
> A1:=y;A2:=-x;A3:=t;phi:=z;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*(g+I*gamma),0);
```

****

$$A1 := y$$

$$A2 := -x$$

$$A3 := t$$

$$\phi := z$$

Potentials from the Hopf Map Chirality 0(g+I*gamma)*

******************************** *Differential Form Format* ********************
****

$$Action\ 1\text{-}form = y\,d(x) - x\,d(y) + t\,d(z) - z\,d(t)$$

$$Intensity\ 2\text{-}form\ F{=}dA\ = -2\,(d(x))\ \&\hat{}\ (d(y)) - 2\,(d(z))\ \&\hat{}\ (d(t))$$

$$Topological\ Torsion\ 3\text{-}form\quad A\hat{}F = -2\,t\,\&\hat{}(d(x), d(y), d(z)) + 2\,z\,\&\hat{}(d(x), d(y), d(t)) - 2\,y\,\&\hat{}(d(x), d(z), d(t)) + 2\,x\,\&\hat{}(d(y), d(z), d(t))$$

$$Topological\ Parity\ 4\text{-}form\quad F\hat{}F = 8\,\&\hat{}(d(x), d(y), d(z), d(t))$$

******************************** *Using EM format* ****************************
****

$$E\ field = [0, 0, -2]$$

$$B\ field = [0, 0, -2]$$

$$Topological\ TORSION\ 4\ vector\quad T4 = \text{-}[ExA + Bphi, AdotB] = [2\,x, 2\,y, 2\,z, 2\,t]$$

$$Helicity\ AdotB = -2\,t$$

$$Poincare\ II = 2(E.B) = 8$$

$$coefficient\ of\ Topological\ Parity\ 4\text{-}form = 8$$

$$Pfaff\ Topological\ Dimension\quad PTD = 4$$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

$$Xm\ or\ linear\ (Mean)\ curvature = 0$$

$$Yg\ or\ quadratic\ (GAUSS)\ curvature = 2$$

$$Za\ or\ Cubic\ (Interaction\ internal\ energy)\ curvature = 0$$

$$Tk\ or\ quartic\ (4D\ expansion)\ curvature = 1$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

D field = [0, 0, −2 ε]

$$H\ field = \left[0, 0, -\frac{2}{\mu}\right]$$

Poynting vector ExH = *EXH*

Amperian Current 4Vector curlH-dD/dt=J4 = [0, 0, 0, 0]

Amerian charge density divD = rho = 0

divergence Lorentz Current 4Vector, 4div(J4) = 0

$$Topological\ SPIN\ 4\ vector\ S4 = \left[\frac{2x}{\mu}, \frac{2y}{\mu}, -2\varepsilon z, -2t\varepsilon\right]$$

$$Topological\ SPIN\ 3\text{-}form = \frac{2x\ \&\hat{}(d(y), d(z), d(t))}{\mu} - \frac{2y\ \&\hat{}(d(x), d(z), d(t))}{\mu}$$

$$-2\varepsilon z\ \&\hat{}(d(x), d(y), d(t)) + 2t\varepsilon\ \&\hat{}(d(x), d(y), d(z))$$

Spin density rho_spin = −2 t ε

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{4(-1+\varepsilon\mu)}{\mu}$$

$$B.H = \frac{4}{\mu}$$

D.E = 4 ε

A.J = 0

-rho.phi = 0

$$Poincare\ I\quad (B.H - D.E)\text{-}(A.J - rho.phi) = -\frac{4(-1+\varepsilon\mu)}{\mu}$$

London Coefficient LC = 0

PROCA coefficient curlcurlB = [0, 0, 0]

Amperian Current 4Vector curlH-dD/dt=J4 = [0, 0, 0, 0]

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) = [0, 0, 0]

Amperian Dissipation Jampere dot E = 0

Lorentz Force Spin factor LFSPIN = 0

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] = [2 x, 2 y, 2 z, 2 t]

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) = [−4 y, 4 x, −4 t]

Torsion Dissipation Jtorsion dot E $= 4\,z$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] $= \left[\frac{2\,x}{\mu}, \frac{2\,y}{\mu}, -2\,\varepsilon\,z, -2\,t\,\varepsilon\right]$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $= \left[\frac{4\,y}{\mu}, -\frac{4\,x}{\mu}, -4\,t\,\varepsilon\right]$

Spin Dissipation J_spin dot E $= 4\,\varepsilon\,z$

Disspative Force 3 vector $= [0, 0, -4\,t\,(\varepsilon\,\mu + 1)]$

Dissipation $= -2\,\mu\,t\,\varepsilon + 2\,x$

****************************** *END PROCEDURE* **************************** **(4)**

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`-  vol 1 p. 397  vol4 p.147- Abrikosov Falaco vortex singularities.`;
  f:=(x^2+y^2);
> r2:=1;r1:=(((f+1*kappa*z+(a*(1))^(2/2))));
> A1:=1*b*m*(y)/r2/r1;A2:=1*b*(-x)*m/r2/r1;
> A3:=0; phi:=0/(4*pi*epsilon)*q/(x^2+y^2+z^2)^(1/2);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := - vol 1 p. 397 vol4 p.147- Abrikosov Falaco vortex singularities.

$$f := x^2 + y^2$$

$$r2 := 1$$

$$r1 := x^2 + y^2 + \kappa\,z + a$$

$$A1 := \frac{b\,m\,y}{x^2 + y^2 + \kappa\,z + a}$$

$$A2 := -\frac{b\,x\,m}{x^2 + y^2 + \kappa\,z + a}$$

$$A3 := 0$$

$$\phi := 0$$

- vol 1 p. 397 vol4 p.147- Abrikosov Falaco vortex singularities.

********************************* *Differential Form Format* ******************** *************

Action 1-form $= \frac{b\,m\,y\,d(x)}{x^2 + y^2 + \kappa\,z + a} - \frac{b\,x\,m\,d(y)}{x^2 + y^2 + \kappa\,z + a}$

$$\textit{Intensity 2-form F=dA} = \left(-\frac{b\,m\,(x^2 - y^2 + \kappa z + a)}{(x^2 + y^2 + \kappa z + a)^2} - \frac{b\,m\,(-x^2 + y^2 + \kappa z + a)}{(x^2 + y^2 + \kappa z + a)^2} \right) (d(x)) \;\&\wedge\; (d(y)) + \frac{b\,m\,y\,\kappa\,(d(x)) \;\&\wedge\; (d(z))}{(x^2 + y^2 + \kappa z + a)^2} - \frac{b\,x\,m\,\kappa\,(d(y)) \;\&\wedge\; (d(z))}{(x^2 + y^2 + \kappa z + a)^2}$$

Topological Torsion 3-form *A^F* = 0

Topological Parity 4-form *F^F* = 0

******************************** *Using EM format* ********************************

E field = [0, 0, 0]

$$\textit{B field} = \left[-\frac{b\,x\,m\,\kappa}{(x^2 + y^2 + \kappa z + a)^2},\; -\frac{b\,m\,y\,\kappa}{(x^2 + y^2 + \kappa z + a)^2},\; -\frac{2\,b\,m\,(\kappa z + a)}{(x^2 + y^2 + \kappa z + a)^2} \right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension *PTD* = 2

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

Xm or linear (Mean) curvature = 0

$$\textit{Yg or quadratic (GAUSS) curvature} = \frac{b^2\,m^2\,(a + \kappa z - x^2 - y^2)}{(x^2 + y^2 + \kappa z + a)^3}$$

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

D field = [0, 0, 0]

$$\textit{H field} = \left[-\frac{b\,x\,m\,\kappa}{(x^2 + y^2 + \kappa z + a)^2\,\mu},\; -\frac{b\,m\,y\,\kappa}{(x^2 + y^2 + \kappa z + a)^2\,\mu},\; -\frac{2\,b\,m\,(\kappa z + a)}{(x^2 + y^2 + \kappa z + a)^2\,\mu} \right]$$

Poynting vector ExH = *EXH*

Amperian Current 4Vector *curlH-dD/dt=J4* $= \left[\frac{2\, b\, m\, y\, (4\, \kappa\, z + 4\, a - \kappa^2)}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu}, \right.$

$\left. -\frac{2\, b\, x\, m\, (4\, \kappa\, z + 4\, a - \kappa^2)}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu}, 0, 0 \right]$

Amerian charge density *divD* $= rho = 0$

divergence Lorentz Current 4Vector, *4div(J4)* $= 0$

Topological SPIN 4 vector S4 $= \left[\frac{2\, b^2\, x\, m^2\, (\kappa\, z + a)}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu}, \frac{2\, b^2\, m^2\, y\, (\kappa\, z + a)}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu}, \right.$

$\left. -\frac{b^2\, m^2\, \kappa\, (x^2 + y^2)}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu}, 0 \right]$

Topological SPIN 3-form $= \frac{2\, b^2\, x\, m^2\, (\kappa\, z + a)\ \&\wedge(d(y), d(z), d(t))}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu}$

$- \frac{2\, b^2\, m^2\, y\, (\kappa\, z + a)\ \&\wedge(d(x), d(z), d(t))}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu} - \frac{b^2\, m^2\, \kappa\, (x^2 + y^2)\ \&\wedge(d(x), d(y), d(t))}{(x^2 + y^2 + \kappa\, z + a)^3\, \mu}$

Spin density rho_spin $= 0$

LaGrange field energy density (B.H-D.E) $= \frac{b^2\, m^2\, (x^2\, \kappa^2 + y^2\, \kappa^2 + 4\, \kappa^2\, z^2 + 8\, \kappa\, z\, a + 4\, a^2)}{(x^2 + y^2 + \kappa\, z + a)^4\, \mu}$

B.H $= \frac{b^2\, m^2\, (x^2\, \kappa^2 + y^2\, \kappa^2 + 4\, \kappa^2\, z^2 + 8\, \kappa\, z\, a + 4\, a^2)}{(x^2 + y^2 + \kappa\, z + a)^4\, \mu}$

D.E $= 0$

A.J $= \frac{2\, b^2\, m^2\, (4\, \kappa\, z + 4\, a - \kappa^2)\, (x^2 + y^2)}{(x^2 + y^2 + \kappa\, z + a)^4\, \mu}$

-rho.phi $= 0$

Poincare I *(B.H - D.E)-(A.J - rho.phi)*

$= \frac{b^2\, m^2\, (3\, x^2\, \kappa^2 + 3\, y^2\, \kappa^2 + 4\, \kappa^2\, z^2 + 8\, \kappa\, z\, a + 4\, a^2 - 8\, x^2\, \kappa\, z - 8\, \kappa\, z\, y^2 - 8\, x^2\, a - 8\, a\, y^2)}{(x^2 + y^2 + \kappa\, z + a)^4\, \mu}$

London Coefficient *LC* $= \frac{2\, (4\, \kappa\, z + 4\, a - \kappa^2)}{(x^2 + y^2 + \kappa\, z + a)^2\, \mu}$

PROCA coefficient curlcurlB $= \left[-\frac{2\, b\, x\, m\, \kappa\, (-3\, \kappa^2 - 4\, x^2 - 4\, y^2 + 8\, \kappa\, z + 8\, a)}{(x^2 + y^2 + \kappa\, z + a)^4}, \right.$

$$-\frac{2\,b\,m\,y\,\kappa\,(-3\,\kappa^2-4\,x^2-4\,y^2+8\,\kappa\,z+8\,a)}{(x^2+y^2+\kappa\,z+a)^4},$$

$$\left.-\frac{4\,b\,m\,(4\,\kappa\,z+4\,a-\kappa^2)\,(a-2\,x^2+\kappa\,z-2\,y^2)}{(x^2+y^2+\kappa\,z+a)^4}\right]$$

Amperian Current 4Vector curlH-dD/dt=J4 $=\left[\frac{2\,b\,m\,y\,(4\,\kappa\,z+4\,a-\kappa^2)}{(x^2+y^2+\kappa\,z+a)^3\,\mu},\right.$

$$\left.-\frac{2\,b\,x\,m\,(4\,\kappa\,z+4\,a-\kappa^2)}{(x^2+y^2+\kappa\,z+a)^3\,\mu},0,0\right]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $=\Bigg[$

$$-\frac{4\,b^2\,x\,m^2\,(4\,\kappa\,z+4\,a-\kappa^2)\,(\kappa\,z+a)}{(x^2+y^2+\kappa\,z+a)^5\,\mu},-\frac{4\,b^2\,m^2\,y\,(4\,\kappa\,z+4\,a-\kappa^2)\,(\kappa\,z+a)}{(x^2+y^2+\kappa\,z+a)^5\,\mu},$$

$$\left.\frac{2\,b^2\,m^2\,(4\,\kappa\,z+4\,a-\kappa^2)\,\kappa\,(x^2+y^2)}{(x^2+y^2+\kappa\,z+a)^5\,\mu}\right]$$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor $LFSPIN=-\frac{1}{2}\,\frac{(x^2+y^2+\kappa\,z+a)^2}{4\,\kappa\,z+4\,a-\kappa^2}$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $=[0,0,0,0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $=[0,0,0]$

Torsion Dissipation Jtorsion dot E $=0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $=\left[\frac{2\,b^2\,x\,m^2\,(\kappa\,z+a)}{(x^2+y^2+\kappa\,z+a)^3\,\mu},\right.$

$$\left.\frac{2\,b^2\,m^2\,y\,(\kappa\,z+a)}{(x^2+y^2+\kappa\,z+a)^3\,\mu},-\frac{b^2\,m^2\,\kappa\,(x^2+y^2)}{(x^2+y^2+\kappa\,z+a)^3\,\mu},0\right]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B)

$$=\left[\frac{b^3\,m^3\,y\,(x^2\,\kappa^2+y^2\,\kappa^2+4\,\kappa^2\,z^2+8\,\kappa\,z\,a+4\,a^2)}{(x^2+y^2+\kappa\,z+a)^5\,\mu},\right.$$

$$-\frac{b^3 m^3 x \left(x^2 \kappa^2 + y^2 \kappa^2 + 4 \kappa^2 z^2 + 8 \kappa z a + 4 a^2\right)}{\left(x^2 + y^2 + \kappa z + a\right)^5 \mu}, 0\Bigg]$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \Bigg[\frac{1}{\left(x^2 + y^2 + \kappa z + a\right)^5 \mu} \Big(b^2 m^2 \big(-16 x \kappa^2 z^2 - 32 x \kappa z a - 16 x a^2$

$+ 4 x \kappa^3 z + 4 x \kappa^2 a + b m y \mu x^2 \kappa^2 + b m y^3 \mu \kappa^2 + 4 b m y \mu \kappa^2 z^2 + 8 b m y \mu \kappa z a$

$+ 4 b m y \mu a^2\big)\Big), -\frac{1}{\left(x^2 + y^2 + \kappa z + a\right)^5 \mu} \Big(b^2 m^2 \big(16 y \kappa^2 z^2 + 32 y \kappa z a + 16 y a^2$

$- 4 y \kappa^3 z - 4 y \kappa^2 a + b m x^3 \mu \kappa^2 + b m x \mu y^2 \kappa^2 + 4 b m x \mu \kappa^2 z^2 + 8 b m x \mu \kappa z a$

$+ 4 b m x \mu a^2\big)\Big), \frac{2 b^2 m^2 \left(4 \kappa z + 4 a - \kappa^2\right) \kappa \left(x^2 + y^2\right)}{\left(x^2 + y^2 + \kappa z + a\right)^5 \mu} \Bigg]$

Dissipation $= 0$

******************************** *END PROCEDURE* ******************************** **(5)**

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`-  vol 1 p. 397  vol4 p.147- Abrikosov vortex singularities.`;f:=(x^2+
  y^2);
> r2:=1;r1:=(((f+0*kappa*z+(a*(0))^(2/2))));
> A1:=1*b*m*(y)/r2/r1;A2:=1*b*(-x)*m/r2/r1;
> A3:=0; phi:=1/(4*pi*epsilon)*q/(x^2+y^2+z^2)^(1/2);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := - vol 1 p. 397 vol4 p.147- Abrikosov vortex singularities.

$$f := x^2 + y^2$$

$$r2 := 1$$

$$r1 := x^2 + y^2$$

$$A1 := \frac{b\,m\,y}{x^2 + y^2}$$

$$A2 := -\frac{b\,x\,m}{x^2 + y^2}$$

$$A3 := 0$$

$$\phi := \frac{1}{4}\,\frac{q}{\pi\,\varepsilon\,\sqrt{x^2 + y^2 + z^2}}$$

- vol 1 p. 397 vol4 p.147- Abrikosov vortex singularities.

********************************* *Differential Form Format* ********************
****

$$\text{Action 1-form} = \frac{1}{4}\,\frac{(-q\,x^2 - q\,y^2)\,d(t)}{(x^2+y^2)\,\pi\,\varepsilon\,\sqrt{x^2+y^2+z^2}} + \frac{b\,m\,y\,d(x)}{x^2+y^2} - \frac{b\,x\,m\,d(y)}{x^2+y^2}$$

$$\text{Intensity 2-form } F=dA \;= \frac{1}{4}\,\frac{q\,x\,(d(x))\,\&\wedge\,(d(t))}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}} + \frac{1}{4}\,\frac{q\,y\,(d(y))\,\&\wedge\,(d(t))}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}$$
$$+ \frac{1}{4}\,\frac{q\,z\,(d(z))\,\&\wedge\,(d(t))}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}$$

$$\text{Topological Torsion 3-form} \quad A\wedge F = \left(\frac{1}{4}\,\frac{b\,x^2\,m\,q}{(x^2+y^2)\,\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}\right.$$
$$\left. + \frac{1}{4}\,\frac{b\,m\,y^2\,q}{(x^2+y^2)\,\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}\right)\&\wedge(d(x), d(y), d(t))$$
$$+ \frac{1}{4}\,\frac{b\,m\,y\,q\,z\,\&\wedge(d(x), d(z), d(t))}{(x^2+y^2)\,\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}} - \frac{1}{4}\,\frac{b\,x\,m\,q\,z\,\&\wedge(d(y), d(z), d(t))}{(x^2+y^2)\,\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}$$

Topological Parity 4-form $F\hat{}F = 0$

****************************** *Using EM format* ******************************
****

$$E\ field = \left[\frac{1}{4} \frac{q\,x}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4} \frac{q\,y}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4} \frac{q\,z}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}} \right]$$

$$B\ field = [0, 0, 0]$$

Topological TORSION 4 vector $T4$ = -[ExA + Bphi,AdotB] =

$$\left[-\frac{1}{4} \frac{b\,x\,m\,q\,z}{(x^2+y^2)\,\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, -\frac{1}{4} \frac{b\,m\,y\,q\,z}{(x^2+y^2)\,\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4} \frac{b\,m\,q}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, 0 \right]$$

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension *PTD* = 3

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

Xm or linear (Mean) curvature = 0

$$Yg \text{ or quadratic (GAUSS) curvature} = -\frac{b^2\,m^2}{(x^2+y^2)^2}$$

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

$$D\ field = \left[\frac{1}{4} \frac{q\,x}{\pi\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4} \frac{q\,y}{\pi\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4} \frac{q\,z}{\pi\,(x^2+y^2+z^2)^{3/2}} \right]$$

$$H\ field = [0, 0, 0]$$

Poynting vector ExH = *EXH*

Amperian Current 4Vector *curlH-dD/dt=J4* = [0, 0, 0, 0]

Amerian charge density *divD = rho* = 0

divergence Lorentz Current 4Vector, *4div(J4)* = 0

$$\textit{Topological SPIN 4 vector S4} = \left[\frac{1}{16} \frac{q^2 x}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, \frac{1}{16} \frac{q^2 y}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, \frac{1}{16} \frac{q^2 z}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, 0 \right]$$

$$\textit{Topological SPIN 3-form} = \frac{1}{16} \frac{q^2 x \,\&\hat{}(d(y), d(z), d(t))}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon} - \frac{1}{16} \frac{q^2 y \,\&\hat{}(d(x), d(z), d(t))}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon} + \frac{1}{16} \frac{q^2 z \,\&\hat{}(d(x), d(y), d(t))}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}$$

$$\textit{Spin density rho\_spin} = 0$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{1}{16} \frac{q^2}{\varepsilon (x^2+y^2+z^2)^2 \pi^2}$$

$$B.H = 0$$

$$D.E = \frac{1}{16} \frac{q^2}{\varepsilon (x^2+y^2+z^2)^2 \pi^2}$$

$$A.J = 0$$

$$\textit{-rho.phi} = 0$$

$$\textit{Poincare I} \quad \textit{(B.H - D.E)-(A.J - rho.phi)} = -\frac{1}{16} \frac{q^2}{\varepsilon (x^2+y^2+z^2)^2 \pi^2}$$

$$\textit{London Coefficient} \quad LC = 0$$

$$\textit{PROCA coefficient curlcurlB} = [0, 0, 0]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} = [0, 0, 0, 0]$$

$$\textit{Lorentz Force 3 vector due to Ampere current} \quad FL = -(\textit{rho\_ampere E} + \textit{J\_ampere x B}) = [0, 0, 0]$$

$$\textit{Amperian Dissipation Jampere dot E} = 0$$

$$\textit{Lorentz Force Spin factor LFSPIN} = 0$$

$$\textit{Topological Torsion current 4 vector} \quad T4 = -[ExA + B.phi, AdotB] = \left[-\frac{1}{4} \frac{b\,x\,m\,q\,z}{(x^2+y^2)\,\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, -\frac{1}{4} \frac{b\,m\,y\,q\,z}{(x^2+y^2)\,\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, \frac{1}{4} \frac{b\,m\,q}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}, 0 \right]$$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) = [0, 0, 0]

Torsion Dissipation Jtorsion dot E = 0

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{1}{16} \frac{q^2 x}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, \frac{1}{16} \frac{q^2 y}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, \frac{1}{16} \frac{q^2 z}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, 0 \right]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) = [0, 0, 0]

$$\textit{Spin Dissipation J\_spin dot E} = \frac{1}{64} \frac{q^3}{(x^2+y^2+z^2)^{5/2} \pi^3 \varepsilon^2}$$

Disspative Force 3 vector = [0, 0, 0]

$$\textit{Dissipation} = -\frac{1}{4} \frac{b\, x\, m\, q\, z}{(x^2+y^2)\, \pi\, \varepsilon\, (x^2+y^2+z^2)^{3/2}}$$

****************************** *END PROCEDURE* **************************** **(6)**

Note that there is a London current factor (an Amperian current proportional to A) and a Spin current with the Lorentz Force proportional to the Spin current, LF is proportional to Spin

---

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`-- wave rotation vol4 p159.`:

> Ax:=(y)*cos(-k*z+omega*t)/(1+x^2+y^2+z^2);Ay:=-x*cos(-k*z+omega*t)/(1+x^2+y^2+z^2);
> Az:=k*cos(-k*z+omega*t); phi:=omega*cos(-k*z+omega*t);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

$$Ax := \frac{y \cos(k z - \omega t)}{1 + x^2 + y^2 + z^2}$$

$$Ay := -\frac{x \cos(k z - \omega t)}{1 + x^2 + y^2 + z^2}$$

$$Az := k\cos(k z - \omega t)$$

$$\phi := \omega\cos(k z - \omega t)$$

-- wave rotation vol4 p159.

********************************* *Differential Form Format* *********************************

Action 1-form $= \dfrac{\cos(k z - \omega t)\,(-\omega - \omega x^2 - \omega y^2 - \omega z^2)\,d(t)}{1 + x^2 + y^2 + z^2} + \dfrac{y\cos(k z - \omega t)\,d(x)}{1 + x^2 + y^2 + z^2} - \dfrac{x\cos(k z - \omega t)\,d(y)}{1 + x^2 + y^2 + z^2} + \dfrac{\cos(k z - \omega t)\,(k + k x^2 + k y^2 + k z^2)\,d(z)}{1 + x^2 + y^2 + z^2}$

Intensity 2-form F=dA $= -\dfrac{y\sin(k z - \omega t)\,\omega\,(d(x))\ \&\wedge\ (d(t))}{1 + x^2 + y^2 + z^2} + \left(-\dfrac{\cos(k z - \omega t)\,(1 + x^2 - y^2 + z^2)}{(1 + x^2 + y^2 + z^2)^2} + \dfrac{\cos(k z - \omega t)\,(-1 + x^2 - y^2 - z^2)}{(1 + x^2 + y^2 + z^2)^2}\right)(d(x))\ \&\wedge\ (d(y)) - \dfrac{1}{(1 + x^2 + y^2 + z^2)^2}\big(x\,(\sin(k z - \omega t)\,k + \sin(k z - \omega t)\,k x^2 + \sin(k z - \omega t)\,k y^2 + \sin(k z - \omega t)\,k z^2 + 2\cos(k z - \omega t)\,z)\,(d(y))\ \&\wedge\ (d(z))\big) + \dfrac{1}{(1 + x^2 + y^2 + z^2)^2}\big(y\,(\sin(k z - \omega t)\,k + \sin(k z - \omega t)\,k x^2 + \sin(k z - \omega t)\,k y^2 + \sin(k z - \omega t)\,k z^2 + 2\cos(k z - \omega t)\,z)\,(d(x))\ \&\wedge\ (d(z))\big) + \dfrac{x\sin(k z - \omega t)\,\omega\,(d(y))\ \&\wedge\ (d(t))}{1 + x^2 + y^2 + z^2}$

Topological Torsion 3-form $A\wedge F = \dfrac{2\cos(k z - \omega t)^2\,\omega\,(1 + z^2)\ \&\wedge(d(x), d(y), d(t))}{(1 + x^2 + y^2 + z^2)^2} - \dfrac{2\cos(k z - \omega t)^2\,k\,(1 + z^2)\ \&\wedge(d(x), d(y), d(z))}{(1 + x^2 + y^2 + z^2)^2} + \left(\dfrac{\cos(k z - \omega t)\,k y\sin(k z - \omega t)\,\omega}{1 + x^2 + y^2 + z^2} - \dfrac{1}{(1 + x^2 + y^2 + z^2)^2}\big(\cos(k z - \omega t)\,\omega y\,(\sin(k z - \omega t)\,k + \sin(k z - \omega t)\,k x^2 + \sin(k z - \omega t)\,k y^2 + \sin(k z - \omega t)\,k z^2 + 2\cos(k z - \omega t)\,z)\big)\right)\&\wedge(d(x), d(z), d(t)) + \Bigg(-\dfrac{\cos(k z - \omega t)\,k x\sin(k z - \omega t)\,\omega}{1 + x^2 + y^2 + z^2} + \dfrac{1}{(1 + x^2 + y^2 + z^2)^2}\big(\cos(k z$

$-\omega t)\ \omega x\ (\sin(kz-\omega t)\ k+\sin(kz-\omega t)\ kx^2+\sin(kz-\omega t)\ ky^2+\sin(kz$

$-\omega t)\ kz^2+2\cos(kz-\omega t)\ z)))\ \&\hat{}(d(y),d(z),d(t))$

Topological Parity 4-form $\quad F\hat{}F=0$

******************************** *Using EM format* ******************************
****

$$E\ field=\left[-\frac{y\sin(kz-\omega t)\ \omega}{1+x^2+y^2+z^2},\frac{x\sin(kz-\omega t)\ \omega}{1+x^2+y^2+z^2},0\right]$$

$$B\ field=\left[-\frac{1}{(1+x^2+y^2+z^2)^2}(x\ (\sin(kz-\omega t)\ k+\sin(kz-\omega t)\ kx^2+\sin(kz-\omega t)\ ky^2\right.$$

$$+\sin(kz-\omega t)\ kz^2+2\cos(kz-\omega t)\ z)),-\frac{1}{(1+x^2+y^2+z^2)^2}(y\ (\sin(kz$$

$$-\omega t)\ k+\sin(kz-\omega t)\ kx^2+\sin(kz-\omega t)\ ky^2+\sin(kz-\omega t)\ kz^2+2\cos(kz$$

$$\left.-\omega t)\ z)),-\frac{2\cos(kz-\omega t)\ (1+z^2)}{(1+x^2+y^2+z^2)^2}\right]$$

Topological TORSION 4 vector $\quad T4$ *= -[ExA + Bphi,AdotB]* $=\left[\frac{2\cos(kz-\omega t)^2\ \omega xz}{(1+x^2+y^2+z^2)^2},\right.$

$$\left.\frac{2\cos(kz-\omega t)^2\ \omega yz}{(1+x^2+y^2+z^2)^2},\frac{2\cos(kz-\omega t)^2\ \omega\ (1+z^2)}{(1+x^2+y^2+z^2)^2},\frac{2\cos(kz-\omega t)^2\ k\ (1+z^2)}{(1+x^2+y^2+z^2)^2}\right]$$

$$Helicity\ AdotB=-\frac{2\cos(kz-\omega t)^2\ k\ (1+z^2)}{(1+x^2+y^2+z^2)^2}$$

Poincare II =2(E.B) $=0$

coefficient of Topological Parity 4-form $=0$

Pfaff Topological Dimension $\quad PTD=3$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

Xm or linear (Mean) curvature $=-\sin(kz-\omega t)\ (k^2+\omega^2)$

Yg or quadratic (GAUSS) curvature $=-\frac{\cos(kz-\omega t)^2\ (x^2+y^2-1-z^2)}{(1+x^2+y^2+z^2)^3}$

Za or Cubic (Interaction internal energy) curvature

$$=\frac{\cos(kz-\omega t)^2\sin(kz-\omega t)\ (x^2+y^2-1-z^2)\ (k^2+\omega^2)}{(1+x^2+y^2+z^2)^3}$$

Tk or quartic (4D expansion) curvature $= 0$

*********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $= 0$

$$D\ field = \left[-\frac{\varepsilon\, y \sin(k z - \omega t)\, \omega}{1 + x^2 + y^2 + z^2}, \frac{\varepsilon\, x \sin(k z - \omega t)\, \omega}{1 + x^2 + y^2 + z^2}, 0 \right]$$

$$H\ field = \left[-\frac{1}{(1 + x^2 + y^2 + z^2)^2 \mu} \left(x \left(\sin(k z - \omega t)\, k + \sin(k z - \omega t)\, k x^2 + \sin(k z - \omega t)\, k y^2 + \sin(k z - \omega t)\, k z^2 + 2 \cos(k z - \omega t)\, z\right)\right), -\frac{1}{(1 + x^2 + y^2 + z^2)^2 \mu} \left(y \left(\sin(k z - \omega t)\, k + \sin(k z - \omega t)\, k x^2 + \sin(k z - \omega t)\, k y^2 + \sin(k z - \omega t)\, k z^2 + 2 \cos(k z - \omega t)\, z\right)\right), -\frac{2 \cos(k z - \omega t)\, (1 + z^2)}{(1 + x^2 + y^2 + z^2)^2 \mu} \right]$$

$$Poynting\ vector\ ExH = \left[-\frac{2 x \sin(k z - \omega t)\, \omega \cos(k z - \omega t)\, (1 + z^2)}{(1 + x^2 + y^2 + z^2)^3 \mu}, -\frac{2 y \sin(k z - \omega t)\, \omega \cos(k z - \omega t)\, (1 + z^2)}{(1 + x^2 + y^2 + z^2)^3 \mu}, \frac{1}{(1 + x^2 + y^2 + z^2)^3 \mu} \left(\sin(k z - \omega t)\, \omega \left(\sin(k z - \omega t)\, k + \sin(k z - \omega t)\, k x^2 + \sin(k z - \omega t)\, k y^2 + \sin(k z - \omega t)\, k z^2 + 2 \cos(k z - \omega t)\, z\right) (x^2 + y^2)\right) \right]$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt\text{=}J4 = \left[-\frac{1}{(1 + x^2 + y^2 + z^2)^3 \mu} \left(y \left(-k^2 \cos(k z - \omega t) - 10 \cos(k z - \omega t) - 2 k^2 \cos(k z - \omega t)\, x^2 - 2 k^2 \cos(k z - \omega t)\, y^2 + 4 \sin(k z - \omega t)\, k z - 2 k^2 \cos(k z - \omega t)\, z^2 - 2 y^2 \cos(k z - \omega t) - 2 x^2 \cos(k z - \omega t) - 2 \cos(k z - \omega t)\, z^2 - \cos(k z - \omega t)\, k^2 x^4 - \cos(k z - \omega t)\, k^2 y^4 - \cos(k z - \omega t)\, k^2 z^4 + 4 \sin(k z - \omega t)\, k z^3 + 4 x^2 \sin(k z - \omega t)\, k z + 4 y^2 \sin(k z - \omega t)\, k z - 2 \cos(k z - \omega t)\, k^2 x^2 y^2\right.\right.$$

$$- 2\cos(kz - \omega t)\, k^2 x^2 z^2 - 2\cos(kz - \omega t)\, k^2 y^2 z^2 + \varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu$$

$$+ 2\varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu x^2 + 2\varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu y^2 + 2\varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu z^2$$

$$+ \varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu x^4 + \varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu y^4 + \varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu z^4$$

$$+ 2\varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu x^2 y^2 + 2\varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu x^2 z^2 + 2\varepsilon\,\omega^2 \cos(kz$$

$$- \omega t)\,\mu y^2 z^2)\,), \frac{1}{(1 + x^2 + y^2 + z^2)^3\,\mu} \left(x\,(\text{-}k^2 \cos(kz - \omega t) - 10\cos(kz - \omega t)\right.$$

$$- 2k^2 \cos(kz - \omega t)\, x^2 - 2k^2 \cos(kz - \omega t)\, y^2 + 4\sin(kz - \omega t)\, kz - 2k^2 \cos(kz$$

$$- \omega t)\, z^2 - 2y^2 \cos(kz - \omega t) - 2x^2 \cos(kz - \omega t) - 2\cos(kz - \omega t)\, z^2 - \cos(kz$$

$$- \omega t)\, k^2 x^4 - \cos(kz - \omega t)\, k^2 y^4 - \cos(kz - \omega t)\, k^2 z^4 + 4\sin(kz - \omega t)\, kz^3$$

$$+ 4x^2 \sin(kz - \omega t)\, kz + 4y^2 \sin(kz - \omega t)\, kz - 2\cos(kz - \omega t)\, k^2 x^2 y^2 - 2\cos(kz$$

$$- \omega t)\, k^2 x^2 z^2 - 2\cos(kz - \omega t)\, k^2 y^2 z^2 + \varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu + 2\varepsilon\,\omega^2 \cos(kz$$

$$- \omega t)\,\mu x^2 + 2\varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu y^2 + 2\varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu z^2 + \varepsilon\,\omega^2 \cos(kz$$

$$- \omega t)\,\mu x^4 + \varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu y^4 + \varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu z^4 + 2\varepsilon\,\omega^2 \cos(kz$$

$$\left.- \omega t)\,\mu x^2 y^2 + 2\varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu x^2 z^2 + 2\varepsilon\,\omega^2 \cos(kz - \omega t)\,\mu y^2 z^2)\,\right), 0, 0\Big]$$

Amerian charge density *divD = rho* $= 0$

divergence Lorentz Current 4Vector, *4div(J4)* $= 0$

$$\textit{Topological SPIN 4 vector S4} = \left[\frac{1}{(1 + x^2 + y^2 + z^2)^3\,\mu} \left(\cos(kz - \omega t)\,\left(2x\cos(kz - \omega t)\right.\right.\right.$$

$$+ 2k^2 y^3 \sin(kz - \omega t) + yk^2 \sin(kz - \omega t) + 2k^2 y \sin(kz - \omega t)\, x^2 + 2ky\cos(kz$$

$$- \omega t)\, z + 2k^2 y \sin(kz - \omega t)\, z^2 + k^2 y^5 \sin(kz - \omega t) - 2\varepsilon y \sin(kz - \omega t)\,\omega^2 \mu x^2 z^2$$

$$+ k^2 y \sin(kz - \omega t)\, x^4 + 2k^2 y^3 \sin(kz - \omega t)\, x^2 + 2k^2 y^3 \sin(kz - \omega t)\, z^2 + k^2 y \sin(kz$$

$$-\omega t)\, z^4+2\, k\, y^3 \cos(k\, z-\omega\, t)\, z+2\, k\, y \cos(k\, z-\omega\, t)\, z^3+2 \cos(k\, z-\omega\, t)\, x\, z^2$$

$$+2\, k\, y \cos(k\, z-\omega\, t)\, z\, x^2+2\, k^2\, y \sin(k\, z-\omega\, t)\, x^2\, z^2-\varepsilon\, y^5 \sin(k\, z-\omega\, t)\, \omega^2\, \mu$$

$$-\varepsilon\, y \sin(k\, z-\omega\, t)\, \omega^2\, \mu-2\, \varepsilon\, y^3 \sin(k\, z-\omega\, t)\, \omega^2\, \mu-2\, \varepsilon\, y \sin(k\, z-\omega\, t)\, \omega^2\, \mu\, x^2$$

$$-2\, \varepsilon\, y \sin(k\, z-\omega\, t)\, \omega^2\, \mu\, z^2-\varepsilon\, y \sin(k\, z-\omega\, t)\, \omega^2\, \mu\, x^4-2\, \varepsilon\, y^3 \sin(k\, z-\omega\, t)\, \omega^2\, \mu\, x^2$$

$$-2\, \varepsilon\, y^3 \sin(k\, z-\omega\, t)\, \omega^2\, \mu\, z^2-\varepsilon\, y \sin(k\, z-\omega\, t)\, \omega^2\, \mu\, z^4)\big)\big),$$

$$-\frac{1}{(1+x^2+y^2+z^2)^3\, \mu}\big(\cos(k\, z-\omega\, t)\, \big(2\, k^2\, x^3 \sin(k\, z-\omega\, t)-2 \cos(k\, z-\omega\, t)\, y\, z^2$$

$$-2\, y \cos(k\, z-\omega\, t)+x\, k^2 \sin(k\, z-\omega\, t)+2\, k\, x \cos(k\, z-\omega\, t)\, z+2\, k^2\, x \sin(k\, z-\omega\, t)\, y^2$$

$$+2\, k^2\, x \sin(k\, z-\omega\, t)\, z^2-2\, \varepsilon\, x \sin(k\, z-\omega\, t)\, \omega^2\, \mu\, y^2\, z^2+k^2\, x^5 \sin(k\, z-\omega\, t)$$

$$+2\, k\, x \cos(k\, z-\omega\, t)\, z\, y^2+2\, k^2\, x \sin(k\, z-\omega\, t)\, y^2\, z^2-\varepsilon\, x^5 \sin(k\, z-\omega\, t)\, \omega^2\, \mu$$

$$-\varepsilon\, x \sin(k\, z-\omega\, t)\, \omega^2\, \mu-2\, \varepsilon\, x^3 \sin(k\, z-\omega\, t)\, \omega^2\, \mu-2\, \varepsilon\, x \sin(k\, z-\omega\, t)\, \omega^2\, \mu\, y^2$$

$$-2\, \varepsilon\, x \sin(k\, z-\omega\, t)\, \omega^2\, \mu\, z^2-2\, \varepsilon\, x^3 \sin(k\, z-\omega\, t)\, \omega^2\, \mu\, y^2-2\, \varepsilon\, x^3 \sin(k\, z-\omega\, t)\, \omega^2\, \mu\, z^2$$

$$-\varepsilon\, x \sin(k\, z-\omega\, t)\, \omega^2\, \mu\, y^4-\varepsilon\, x \sin(k\, z-\omega\, t)\, \omega^2\, \mu\, z^4+2\, k^2\, x^3 \sin(k\, z-\omega\, t)\, y^2$$

$$+2\, k^2\, x^3 \sin(k\, z-\omega\, t)\, z^2+k^2\, x \sin(k\, z-\omega\, t)\, y^4+k^2\, x \sin(k\, z-\omega\, t)\, z^4+2\, k\, x^3 \cos(k\, z$$

$$-\omega\, t)\, z+2\, k\, x \cos(k\, z-\omega\, t)\, z^3\big)\big),\ -\frac{1}{(1+x^2+y^2+z^2)^3\, \mu}\big(\cos(k\, z-\omega\, t)\, \big(\sin(k\, z$$

$$-\omega\, t)\, k+\sin(k\, z-\omega\, t)\, k\, x^2+\sin(k\, z-\omega\, t)\, k\, y^2+\sin(k\, z-\omega\, t)\, k\, z^2+2 \cos(k\, z$$

$$-\omega t)\, z)\, (x^2+y^2)),\ -\frac{\cos(kz-\omega t)\,\varepsilon \sin(kz-\omega t)\,\omega\,(x^2+y^2)}{(1+x^2+y^2+z^2)^2}\Big]$$

Topological SPIN 3-form $= \frac{1}{(1+x^2+y^2+z^2)^3\,\mu}\Big(\cos(kz-\omega t)\,\big(2x\cos(kz-\omega t)$

$+2k^2y^3\sin(kz-\omega t)+yk^2\sin(kz-\omega t)+2k^2y\sin(kz-\omega t)\,x^2+2ky\cos(kz$

$-\omega t)\,z+2k^2y\sin(kz-\omega t)\,z^2+k^2y^5\sin(kz-\omega t)-2\varepsilon y\sin(kz-\omega t)\,\omega^2\mu x^2z^2$

$+k^2y\sin(kz-\omega t)\,x^4+2k^2y^3\sin(kz-\omega t)\,x^2+2k^2y^3\sin(kz-\omega t)\,z^2+k^2y\sin(kz$

$-\omega t)\,z^4+2ky^3\cos(kz-\omega t)\,z+2ky\cos(kz-\omega t)\,z^3+2\cos(kz-\omega t)\,xz^2$

$+2ky\cos(kz-\omega t)\,zx^2+2k^2y\sin(kz-\omega t)\,x^2z^2-\varepsilon y^5\sin(kz-\omega t)\,\omega^2\mu$

$-\varepsilon y\sin(kz-\omega t)\,\omega^2\mu-2\varepsilon y^3\sin(kz-\omega t)\,\omega^2\mu-2\varepsilon y\sin(kz-\omega t)\,\omega^2\mu x^2$

$-2\varepsilon y\sin(kz-\omega t)\,\omega^2\mu z^2-\varepsilon y\sin(kz-\omega t)\,\omega^2\mu x^4-2\varepsilon y^3\sin(kz-\omega t)\,\omega^2\mu x^2$

$-2\varepsilon y^3\sin(kz-\omega t)\,\omega^2\mu z^2-\varepsilon y\sin(kz-\omega t)\,\omega^2\mu z^4\big)$ &^(d(y), d(z), d(t)) $\Big)$

$+\frac{1}{(1+x^2+y^2+z^2)^3\,\mu}\Big(\cos(kz-\omega t)\,\big(2k^2x^3\sin(kz-\omega t)-2\cos(kz-\omega t)\,yz^2$

$-2y\cos(kz-\omega t)+xk^2\sin(kz-\omega t)+2kx\cos(kz-\omega t)\,z+2k^2x\sin(kz-\omega t)\,y^2$

$+2k^2x\sin(kz-\omega t)\,z^2-2\varepsilon x\sin(kz-\omega t)\,\omega^2\mu y^2z^2+k^2x^5\sin(kz-\omega t)$

$+2kx\cos(kz-\omega t)\,zy^2+2k^2x\sin(kz-\omega t)\,y^2z^2-\varepsilon x^5\sin(kz-\omega t)\,\omega^2\mu$

$-\varepsilon x\sin(kz-\omega t)\,\omega^2\mu-2\varepsilon x^3\sin(kz-\omega t)\,\omega^2\mu-2\varepsilon x\sin(kz-\omega t)\,\omega^2\mu y^2$

$-2\varepsilon x\sin(kz-\omega t)\,\omega^2\mu z^2-2\varepsilon x^3\sin(kz-\omega t)\,\omega^2\mu y^2-2\varepsilon x^3\sin(kz-\omega t)\,\omega^2\mu z^2$

$-\varepsilon x\sin(kz-\omega t)\,\omega^2\mu y^4-\varepsilon x\sin(kz-\omega t)\,\omega^2\mu z^4+2k^2x^3\sin(kz-\omega t)\,y^2$

$+2k^2x^3\sin(kz-\omega t)\,z^2+k^2x\sin(kz-\omega t)\,y^4+k^2x\sin(kz-\omega t)\,z^4+2kx^3\cos(kz$

$-\omega t)\,z+2kx\cos(kz-\omega t)\,z^3\big)$ &^(d(x), d(z), d(t)) $\Big)$

$-\frac{1}{(1+x^2+y^2+z^2)^3\,\mu}\Big(\cos(kz-\omega t)\,\big(\sin(kz-\omega t)\,k+\sin(kz-\omega t)\,kx^2+\sin(kz$

$-\omega t)\,ky^2+\sin(kz-\omega t)\,kz^2+2\cos(kz-\omega t)\,z\big)\,(x^2+y^2)$ &^(d(x), d(y), d(t)) $\Big)$

$+\frac{\cos(kz-\omega t)\,\varepsilon\sin(kz-\omega t)\,\omega\,(x^2+y^2)\ \text{\&\^{}}(d(x),d(y),d(z))}{(1+x^2+y^2+z^2)^2}$

Spin density rho_spin $= -\frac{\cos(kz-\omega t)\,\varepsilon\sin(kz-\omega t)\,\omega\,(x^2+y^2)}{(1+x^2+y^2+z^2)^2}$

$$
\begin{aligned}
&\textit{LaGrange field energy density (B.H-D.E)} = \frac{1}{\left(1+x^2+y^2+z^2\right)^4 \mu} \big(k^2 \sin(kz-\omega t)^2 x^2 \\
&+ k^2 \sin(kz-\omega t)^2 y^2 + 4\cos(kz-\omega t)^2 + 8\cos(kz-\omega t)^2 z^2 + 8x^2 \sin(kz-\omega t)\, k y^2 \cos(kz-\omega t)\, z \\
&+ 4\sin(kz-\omega t)\, k x^2 \cos(kz-\omega t)\, z + 4\sin(kz-\omega t)\, k y^2 \cos(kz-\omega t)\, z + 4x^4 \sin(kz-\omega t)\, k \cos(kz-\omega t)\, z \\
&+ 4x^2 \sin(kz-\omega t)^2 k^2 y^2 z^2 + 4x^2 \sin(kz-\omega t)\, k z^3 \cos(kz-\omega t) + 4y^4 \sin(kz-\omega t)\, k \cos(kz-\omega t)\, z \\
&+ 4y^2 \sin(kz-\omega t)\, k z^3 \cos(kz-\omega t) + 2\sin(kz-\omega t)^2 k^2 x^4 + 2\sin(kz-\omega t)^2 k^2 y^4 \\
&+ 4x^2 \cos(kz-\omega t)^2 z^2 + x^6 \sin(kz-\omega t)^2 k^2 + 4y^2 \cos(kz-\omega t)^2 z^2 \\
&+ y^6 \sin(kz-\omega t)^2 k^2 + 4\sin(kz-\omega t)^2 k^2 x^2 y^2 + 2\sin(kz-\omega t)^2 k^2 x^2 z^2 \\
&+ 2\sin(kz-\omega t)^2 k^2 y^2 z^2 + 3x^2 \sin(kz-\omega t)^2 k^2 y^4 + x^2 \sin(kz-\omega t)^2 k^2 z^4 \\
&+ 3x^4 \sin(kz-\omega t)^2 k^2 y^2 + 2x^4 \sin(kz-\omega t)^2 k^2 z^2 + y^2 \sin(kz-\omega t)^2 k^2 z^4 \\
&+ 2y^4 \sin(kz-\omega t)^2 k^2 z^2 + 4\cos(kz-\omega t)^2 z^4 - 4\varepsilon \sin(kz-\omega t)^2 \omega^2 \mu x^2 y^2 \\
&- 2\varepsilon \sin(kz-\omega t)^2 \omega^2 \mu x^2 z^2 - 2\varepsilon \sin(kz-\omega t)^2 \omega^2 \mu y^2 z^2 - 3\varepsilon \sin(kz-\omega t)^2 \omega^2 \mu x^2 y^4 \\
&- \varepsilon \sin(kz-\omega t)^2 \omega^2 \mu z^4 x^2 - 3\varepsilon \sin(kz-\omega t)^2 \omega^2 \mu x^4 y^2 \\
&- 2\varepsilon \sin(kz-\omega t)^2 \omega^2 \mu x^4 z^2 - \varepsilon \sin(kz-\omega t)^2 \omega^2 \mu z^4 y^2 - 2\varepsilon \sin(kz-\omega t)^2 \omega^2 \mu z^2 y^4 \\
&- 4\varepsilon \sin(kz-\omega t)^2 \omega^2 \mu x^2 z^2 y^2 - 2\varepsilon \sin(kz-\omega t)^2 \omega^2 \mu x^4 - 2\varepsilon \sin(kz-\omega t)^2 \omega^2 \mu y^4 \\
&- \varepsilon \sin(kz-\omega t)^2 \omega^2 \mu x^6 - \varepsilon \sin(kz-\omega t)^2 \omega^2 \mu y^2 - \varepsilon \sin(kz-\omega t)^2 \omega^2 \mu y^6 \\
&- \varepsilon \sin(kz-\omega t)^2 \omega^2 \mu x^2\big)
\end{aligned}
$$

$$
\begin{aligned}
&\textit{B.H} = \frac{1}{\left(1+x^2+y^2+z^2\right)^4 \mu} \big(k^2 \sin(kz-\omega t)^2 x^2 + k^2 \sin(kz-\omega t)^2 y^2 + 4\cos(kz-\omega t)^2 \\
&+ 8\cos(kz-\omega t)^2 z^2 + 8x^2 \sin(kz-\omega t)\, k y^2 \cos(kz-\omega t)\, z + 4\sin(kz-\omega t)\, k x^2 \cos(kz-\omega t)\, z \\
&+ 4\sin(kz-\omega t)\, k y^2 \cos(kz-\omega t)\, z + 4x^4 \sin(kz-\omega t)\, k \cos(kz-\omega t)\, z + 4x^2 \sin(kz-\omega t)^2 k^2 y^2 z^2 \\
&+ 4x^2 \sin(kz-\omega t)\, k z^3 \cos(kz-\omega t) + 4y^4 \sin(kz-\omega t)\, k \cos(kz-\omega t)\, z + 4y^2 \sin(kz-\omega t)\, k z^3 \cos(kz-\omega t) \\
&+ 2\sin(kz-\omega t)^2 k^2 x^4 + 2\sin(kz-\omega t)^2 k^2 y^4 + 4x^2 \cos(kz-\omega t)^2 z^2 + x^6 \sin(kz-\omega t)^2 k^2 \\
&+ 4y^2 \cos(kz-\omega t)^2 z^2 + y^6 \sin(kz-\omega t)^2 k^2 + 4\sin(kz-\omega t)^2 k^2 x^2 y^2 \\
&+ 2\sin(kz-\omega t)^2 k^2 x^2 z^2 + 2\sin(kz-\omega t)^2 k^2 y^2 z^2 + 3x^2 \sin(kz-\omega t)^2 k^2 y^4 \\
&+ x^2 \sin(kz-\omega t)^2 k^2 z^4 + 3x^4 \sin(kz-\omega t)^2 k^2 y^2 + 2x^4 \sin(kz-\omega t)^2 k^2 z^2
\end{aligned}
$$

$$+ y^2 \sin(kz-\omega t)^2 k^2 z^4 + 2 y^4 \sin(kz-\omega t)^2 k^2 z^2 + 4\cos(kz-\omega t)^2 z^4)$$

$$D.E = \frac{\varepsilon \sin(kz-\omega t)^2 \omega^2 (x^2+y^2)}{(1+x^2+y^2+z^2)^2}$$

$$A.J = -\frac{1}{(1+x^2+y^2+z^2)^4 \mu} \big(\cos(kz-\omega t) \,(-k^2\cos(kz-\omega t) - 10\cos(kz-\omega t)$$
$$- 2k^2\cos(kz-\omega t)\, x^2 - 2k^2\cos(kz-\omega t)\, y^2 + 4\sin(kz-\omega t)\, kz - 2k^2\cos(kz$$
$$-\omega t)\, z^2 - 2y^2\cos(kz-\omega t) - 2x^2\cos(kz-\omega t) - 2\cos(kz-\omega t)\, z^2 - \cos(kz$$
$$-\omega t)\, k^2 x^4 - \cos(kz-\omega t)\, k^2 y^4 - \cos(kz-\omega t)\, k^2 z^4 + 4\sin(kz-\omega t)\, kz^3$$
$$+ 4x^2\sin(kz-\omega t)\, kz + 4y^2\sin(kz-\omega t)\, kz - 2\cos(kz-\omega t)\, k^2x^2y^2 - 2\cos(kz$$
$$-\omega t)\, k^2x^2z^2 - 2\cos(kz-\omega t)\, k^2y^2z^2 + \varepsilon\omega^2\cos(kz-\omega t)\,\mu + 2\varepsilon\omega^2\cos(kz$$
$$-\omega t)\,\mu x^2 + 2\varepsilon\omega^2\cos(kz-\omega t)\,\mu y^2 + 2\varepsilon\omega^2\cos(kz-\omega t)\,\mu z^2 + \varepsilon\omega^2\cos(kz$$
$$-\omega t)\,\mu x^4 + \varepsilon\omega^2\cos(kz-\omega t)\,\mu y^4 + \varepsilon\omega^2\cos(kz-\omega t)\,\mu z^4 + 2\varepsilon\omega^2\cos(kz$$
$$-\omega t)\,\mu x^2y^2 + 2\varepsilon\omega^2\cos(kz-\omega t)\,\mu x^2z^2 + 2\varepsilon\omega^2\cos(kz-\omega t)\,\mu y^2z^2)\,(x^2+y^2)\big)$$

$$-rho.phi = 0$$

Poincare I $\quad (B.H - D.E)\text{-}(A.J - rho.phi) = \dfrac{1}{(1+x^2+y^2+z^2)^4\mu}\big(k^2\sin(kz-\omega t)^2 x^2$

$$+ k^2\sin(kz-\omega t)^2 y^2 - k^2\cos(kz-\omega t)^2 x^2 + 4\cos(kz-\omega t)^2 - 10\cos(kz-\omega t)^2 x^2$$
$$- 10\cos(kz-\omega t)^2 y^2 + 8\cos(kz-\omega t)^2 z^2 + 16x^2\sin(kz-\omega t)\, ky^2\cos(kz-\omega t)\, z$$
$$+ 8\sin(kz-\omega t)\, kx^2\cos(kz-\omega t)\, z + 8\sin(kz-\omega t)\, ky^2\cos(kz-\omega t)\, z$$
$$+ 8x^4\sin(kz-\omega t)\, k\cos(kz-\omega t)\, z + 4x^2\sin(kz-\omega t)^2 k^2y^2z^2 + 8x^2\sin(kz$$
$$-\omega t)\, kz^3\cos(kz-\omega t) + 8y^4\sin(kz-\omega t)\, k\cos(kz-\omega t)\, z + 8y^2\sin(kz$$
$$-\omega t)\, kz^3\cos(kz-\omega t) + 2\sin(kz-\omega t)^2 k^2x^4 + 2\sin(kz-\omega t)^2 k^2y^4$$
$$+ 2x^2\cos(kz-\omega t)^2 z^2 + x^6\sin(kz-\omega t)^2 k^2 + 2y^2\cos(kz-\omega t)^2 z^2 + y^6\sin(kz$$
$$-\omega t)^2 k^2 + 4\sin(kz-\omega t)^2 k^2x^2y^2 + 2\sin(kz-\omega t)^2 k^2x^2z^2 + 2\sin(kz-\omega t)^2 k^2y^2z^2$$
$$+ 3x^2\sin(kz-\omega t)^2 k^2y^4 + x^2\sin(kz-\omega t)^2 k^2z^4 + 3x^4\sin(kz-\omega t)^2 k^2y^2$$
$$+ 2x^4\sin(kz-\omega t)^2 k^2z^2 + y^2\sin(kz-\omega t)^2 k^2z^4 + 2y^4\sin(kz-\omega t)^2 k^2z^2$$
$$+ 4\cos(kz-\omega t)^2 z^4 - 4\varepsilon\sin(kz-\omega t)^2\omega^2\mu x^2y^2 - 2\varepsilon\sin(kz-\omega t)^2\omega^2\mu x^2z^2$$
$$- 2\varepsilon\sin(kz-\omega t)^2\omega^2\mu y^2z^2 - 3\varepsilon\sin(kz-\omega t)^2\omega^2\mu x^2y^4 - \varepsilon\sin(kz-\omega t)^2\omega^2\mu z^4x^2$$

$$- 3\,\varepsilon \sin(k z - \omega t)^2 \omega^2 \mu x^4 y^2 - 2\,\varepsilon \sin(k z - \omega t)^2 \omega^2 \mu x^4 z^2 - \varepsilon \sin(k z - \omega t)^2 \omega^2 \mu z^4 y^2$$
$$- 2\,\varepsilon \sin(k z - \omega t)^2 \omega^2 \mu z^2 y^4 + 4\,\varepsilon \omega^2 \cos(k z - \omega t)^2 \mu x^2 z^2 y^2 - 4\,\varepsilon \sin(k z$$
$$- \omega t)^2 \omega^2 \mu x^2 z^2 y^2 - 4\,y^2 \cos(k z - \omega t)^2 x^2 - 2\,\varepsilon \sin(k z - \omega t)^2 \omega^2 \mu x^4 - 2\,\varepsilon \sin(k z$$
$$- \omega t)^2 \omega^2 \mu y^4 - \varepsilon \sin(k z - \omega t)^2 \omega^2 \mu x^6 - \varepsilon \sin(k z - \omega t)^2 \omega^2 \mu y^2 - \varepsilon \sin(k z$$
$$- \omega t)^2 \omega^2 \mu y^6 - \varepsilon \sin(k z - \omega t)^2 \omega^2 \mu x^2 + \varepsilon \omega^2 \cos(k z - \omega t)^2 \mu z^4 y^2 + \varepsilon \omega^2 \cos(k z$$
$$- \omega t)^2 \mu z^4 x^2 + 2\,\varepsilon \omega^2 \cos(k z - \omega t)^2 \mu x^4 z^2 + 2\,\varepsilon \omega^2 \cos(k z - \omega t)^2 \mu y^4 z^2$$
$$+ 2\,\varepsilon \omega^2 \cos(k z - \omega t)^2 \mu y^2 z^2 + 3\,\varepsilon \omega^2 \cos(k z - \omega t)^2 \mu x^4 y^2 + 3\,\varepsilon \omega^2 \cos(k z$$
$$- \omega t)^2 \mu x^2 y^4 + 4\,\varepsilon \omega^2 \cos(k z - \omega t)^2 \mu x^2 y^2 + 2\,\varepsilon \omega^2 \cos(k z - \omega t)^2 \mu x^2 z^2$$
$$- 4\cos(k z - \omega t)^2 k^2 x^2 y^2 - 2\cos(k z - \omega t)^2 k^2 x^2 z^2 - 2\cos(k z - \omega t)^2 k^2 y^2 z^2$$
$$- 3\cos(k z - \omega t)^2 k^2 x^4 y^2 - 2\cos(k z - \omega t)^2 k^2 x^4 z^2 - 3\cos(k z - \omega t)^2 k^2 x^2 y^4$$
$$- \cos(k z - \omega t)^2 k^2 x^2 z^4 - 2\cos(k z - \omega t)^2 k^2 y^4 z^2 - \cos(k z - \omega t)^2 k^2 y^2 z^4$$
$$- 4\cos(k z - \omega t)^2 k^2 x^2 y^2 z^2 + \varepsilon \omega^2 \cos(k z - \omega t)^2 \mu x^2 + \varepsilon \omega^2 \cos(k z - \omega t)^2 \mu y^2$$
$$+ 2\,\varepsilon \omega^2 \cos(k z - \omega t)^2 \mu x^4 + 2\,\varepsilon \omega^2 \cos(k z - \omega t)^2 \mu y^4 + \varepsilon \omega^2 \cos(k z - \omega t)^2 \mu x^6$$
$$+ \varepsilon \omega^2 \cos(k z - \omega t)^2 \mu y^6 - 2\cos(k z - \omega t)^2 x^4 - 2\cos(k z - \omega t)^2 y^4 - k^2 \cos(k z$$
$$- \omega t)^2 y^2 - 2\cos(k z - \omega t)^2 k^2 x^4 - 2\cos(k z - \omega t)^2 k^2 y^4 - \cos(k z - \omega t)^2 k^2 x^6$$
$$- \cos(k z - \omega t)^2 k^2 y^6)$$

London Coefficient $LC = 0$

$$\textit{PROCA coefficient curlcurlB} = \left[-\frac{1}{(1 + x^2 + y^2 + z^2)^4} \left(x \left(20 \sin(k z - \omega t)\, k y^2 + 14 \sin(k z \right.\right.\right.$$
$$- \omega t)\, k + 20 \sin(k z - \omega t)\, k x^2 + 4 \sin(k z - \omega t)\, k z^2 + 56 \cos(k z - \omega t)\, z + k^3 \sin(k z$$
$$- \omega t) + 3\,k^3 \sin(k z - \omega t)\, x^2 + 3\,k^3 \sin(k z - \omega t)\, y^2 + 3\,k^3 \sin(k z - \omega t)\, z^2$$
$$+ 3\,k^3 \sin(k z - \omega t)\, x^4 + 3\,k^3 \sin(k z - \omega t)\, y^4 + 3\,k^3 \sin(k z - \omega t)\, z^4 + k^3 \sin(k z$$
$$- \omega t)\, x^6 + k^3 \sin(k z - \omega t)\, y^6 + k^3 \sin(k z - \omega t)\, z^6 + 12\,k^2 \cos(k z - \omega t)\, z^3$$
$$+ 6\,k^2 \cos(k z - \omega t)\, z^5 + 6\,k \sin(k z - \omega t)\, x^4 + 6\,k \sin(k z - \omega t)\, y^4 - 10\,k \sin(k z$$

$$- \omega t)\ z^4 + 8\cos(kz - \omega t)\ z x^2 + 8\cos(kz - \omega t)\ z y^2 + 6 k^2 \cos(kz - \omega t)\ z$$

$$+ 12 k \sin(kz - \omega t)\ x^2 y^2 - 4 k \sin(kz - \omega t)\ x^2 z^2 - 4 k \sin(kz - \omega t)\ y^2 z^2 + 6 k^3 \sin(kz$$

$$- \omega t)\ x^2 y^2 + 6 k^3 \sin(kz - \omega t)\ x^2 z^2 + 6 k^3 \sin(kz - \omega t)\ y^2 z^2 + 3 k^3 \sin(kz - \omega t)\ x^4 y^2$$

$$+ 3 k^3 \sin(kz - \omega t)\ x^4 z^2 + 3 k^3 \sin(kz - \omega t)\ x^2 y^4 + 3 k^3 \sin(kz - \omega t)\ x^2 z^4$$

$$+ 3 k^3 \sin(kz - \omega t)\ y^4 z^2 + 3 k^3 \sin(kz - \omega t)\ y^2 z^4 + 12 k^2 \cos(kz - \omega t)\ z x^2$$

$$+ 12 k^2 \cos(kz - \omega t)\ z y^2 + 6 k^2 \cos(kz - \omega t)\ z x^4 + 12 k^2 \cos(kz - \omega t)\ z^3 x^2$$

$$+ 6 k^2 \cos(kz - \omega t)\ z y^4 + 12 k^2 \cos(kz - \omega t)\ z^3 y^2 + 8\cos(kz - \omega t)\ z^3 + 6 k^3 \sin(kz$$

$$- \omega t)\ x^2 y^2 z^2 + 12 k^2 \cos(kz - \omega t)\ z x^2 y^2)),\ -\frac{1}{(1 + x^2 + y^2 + z^2)^4} (y\ (20 \sin(kz$$

$$- \omega t)\ k y^2 + 14 \sin(kz - \omega t)\ k + 20 \sin(kz - \omega t)\ k x^2 + 4 \sin(kz - \omega t)\ k z^2$$

$$+ 56 \cos(kz - \omega t)\ z + k^3 \sin(kz - \omega t) + 3 k^3 \sin(kz - \omega t)\ x^2 + 3 k^3 \sin(kz - \omega t)\ y^2$$

$$+ 3 k^3 \sin(kz - \omega t)\ z^2 + 3 k^3 \sin(kz - \omega t)\ x^4 + 3 k^3 \sin(kz - \omega t)\ y^4 + 3 k^3 \sin(kz$$

$$- \omega t)\ z^4 + k^3 \sin(kz - \omega t)\ x^6 + k^3 \sin(kz - \omega t)\ y^6 + k^3 \sin(kz - \omega t)\ z^6 + 12 k^2 \cos(kz$$

$$- \omega t)\ z^3 + 6 k^2 \cos(kz - \omega t)\ z^5 + 6 k \sin(kz - \omega t)\ x^4 + 6 k \sin(kz - \omega t)\ y^4$$

$$- 10 k \sin(kz - \omega t)\ z^4 + 8 \cos(kz - \omega t)\ z x^2 + 8 \cos(kz - \omega t)\ z y^2 + 6 k^2 \cos(kz$$

$$- \omega t)\ z + 12 k \sin(kz - \omega t)\ x^2 y^2 - 4 k \sin(kz - \omega t)\ x^2 z^2 - 4 k \sin(kz - \omega t)\ y^2 z^2$$

$+6\,k^3 \sin(k\,z-\omega\,t)\;x^2\,y^2+6\,k^3 \sin(k\,z-\omega\,t)\;x^2\,z^2+6\,k^3 \sin(k\,z-\omega\,t)\;y^2\,z^2$

$+3\,k^3 \sin(k\,z-\omega\,t)\;x^4\,y^2+3\,k^3 \sin(k\,z-\omega\,t)\;x^4\,z^2+3\,k^3 \sin(k\,z-\omega\,t)\;x^2\,y^4$

$+3\,k^3 \sin(k\,z-\omega\,t)\;x^2\,z^4+3\,k^3 \sin(k\,z-\omega\,t)\;y^4\,z^2+3\,k^3 \sin(k\,z-\omega\,t)\;y^2\,z^4$

$+12\,k^2 \cos(k\,z-\omega\,t)\;z\,x^2+12\,k^2 \cos(k\,z-\omega\,t)\;z\,y^2+6\,k^2 \cos(k\,z-\omega\,t)\;z\,x^4$

$+12\,k^2 \cos(k\,z-\omega\,t)\;z^3\,x^2+6\,k^2 \cos(k\,z-\omega\,t)\;z\,y^4+12\,k^2 \cos(k\,z-\omega\,t)\;z^3\,y^2$

$+8\cos(k\,z-\omega\,t)\;z^3+6\,k^3 \sin(k\,z-\omega\,t)\;x^2\,y^2\,z^2+12\,k^2 \cos(k\,z-\omega\,t)\;z\,x^2\,y^2)\,)$,

$-\frac{1}{(1+x^2+y^2+z^2)^4}\,(2\,(k^2 \cos(k\,z-\omega\,t)\;+10\cos(k\,z-\omega\,t)\;+2\,k^2 \cos(k\,z-\omega\,t)\;x^2$
$+2\,k^2 \cos(k\,z-\omega\,t)\;y^2-4\sin(k\,z-\omega\,t)\;k\,z+3\,k^2 \cos(k\,z-\omega\,t)\;z^2-16\,y^2 \cos(k\,z$
$-\omega\,t)\;-16\,x^2 \cos(k\,z-\omega\,t)\;-2\cos(k\,z-\omega\,t)\;x^4+12\cos(k\,z-\omega\,t)\;z^2-2\cos(k\,z$
$-\omega\,t)\;y^4+2\cos(k\,z-\omega\,t)\;z^4-4\cos(k\,z-\omega\,t)\;x^2\,y^2+\cos(k\,z-\omega\,t)\;k^2\,x^4+\cos(k\,z$
$-\omega\,t)\;k^2\,y^4+3\cos(k\,z-\omega\,t)\;k^2\,z^4+\cos(k\,z-\omega\,t)\;k^2\,z^6-8\sin(k\,z-\omega\,t)\;k\,z^3$
$-4\,k\sin(k\,z-\omega\,t)\;z^5+2\cos(k\,z-\omega\,t)\;k^2\,x^2\,y^2+4\cos(k\,z-\omega\,t)\;k^2\,x^2\,z^2+4\cos(k\,z$
$-\omega\,t)\;k^2\,y^2\,z^2+\cos(k\,z-\omega\,t)\;k^2\,x^4\,z^2+2\cos(k\,z-\omega\,t)\;k^2\,x^2\,z^4+\cos(k\,z-\omega\,t)\;k^2\,y^4\,z^2$
$+2\cos(k\,z-\omega\,t)\;k^2\,y^2\,z^4+4\,z\,k\sin(k\,z-\omega\,t)\;x^4+4\,z\,k\sin(k\,z-\omega\,t)\;y^4+2\cos(k\,z$
$-\omega\,t)\;k^2\,x^2\,y^2\,z^2+8\,z\,k\sin(k\,z-\omega\,t)\;x^2\,y^2)\,)\,]$

Amperian Current 4Vector *curlH-dD/dt=J4* $=\left[-\frac{1}{(1+x^2+y^2+z^2)^3\,\mu}\,(y\,(-k^2 \cos(k\,z\right.$

$-\omega\,t)\;-10\cos(k\,z-\omega\,t)\;-2\,k^2 \cos(k\,z-\omega\,t)\;x^2-2\,k^2 \cos(k\,z-\omega\,t)\;y^2+4\sin(k\,z$

$-\omega\,t)\;k\,z-2\,k^2 \cos(k\,z-\omega\,t)\;z^2-2\,y^2 \cos(k\,z-\omega\,t)\;-2\,x^2 \cos(k\,z-\omega\,t)\;-2\cos(k\,z$

$$-\omega t)\ z^2 - \cos(kz-\omega t)\ k^2 x^4 - \cos(kz-\omega t)\ k^2 y^4 - \cos(kz-\omega t)\ k^2 z^4 + 4\sin(kz$$

$$-\omega t)\ k z^3 + 4 x^2 \sin(kz-\omega t)\ k z + 4 y^2 \sin(kz-\omega t)\ k z - 2\cos(kz-\omega t)\ k^2 x^2 y^2$$

$$-2\cos(kz-\omega t)\ k^2 x^2 z^2 - 2\cos(kz-\omega t)\ k^2 y^2 z^2 + \varepsilon\,\omega^2 \cos(kz-\omega t)\ \mu$$

$$+2\,\varepsilon\,\omega^2\cos(kz-\omega t)\ \mu x^2 + 2\,\varepsilon\,\omega^2\cos(kz-\omega t)\ \mu y^2 + 2\,\varepsilon\,\omega^2\cos(kz-\omega t)\ \mu z^2$$

$$+\varepsilon\,\omega^2\cos(kz-\omega t)\ \mu x^4 + \varepsilon\,\omega^2\cos(kz-\omega t)\ \mu y^4 + \varepsilon\,\omega^2\cos(kz-\omega t)\ \mu z^4$$

$$+2\,\varepsilon\,\omega^2\cos(kz-\omega t)\ \mu x^2 y^2 + 2\,\varepsilon\,\omega^2\cos(kz-\omega t)\ \mu x^2 z^2 + 2\,\varepsilon\,\omega^2\cos(kz$$

$$-\omega t)\ \mu y^2 z^2)\,)\,,\ \frac{1}{(1+x^2+y^2+z^2)^3\,\mu}\big(x\,\big(-k^2\cos(kz-\omega t) - 10\cos(kz-\omega t)$$

$$-2k^2\cos(kz-\omega t)\ x^2 - 2k^2\cos(kz-\omega t)\ y^2 + 4\sin(kz-\omega t)\ kz - 2k^2\cos(kz$$

$$-\omega t)\ z^2 - 2y^2\cos(kz-\omega t) - 2x^2\cos(kz-\omega t) - 2\cos(kz-\omega t)\ z^2 - \cos(kz$$

$$-\omega t)\ k^2 x^4 - \cos(kz-\omega t)\ k^2 y^4 - \cos(kz-\omega t)\ k^2 z^4 + 4\sin(kz-\omega t)\ k z^3$$

$$+4x^2\sin(kz-\omega t)\ kz + 4y^2\sin(kz-\omega t)\ kz - 2\cos(kz-\omega t)\ k^2x^2y^2 - 2\cos(kz$$

$$-\omega t)\ k^2x^2z^2 - 2\cos(kz-\omega t)\ k^2y^2z^2 + \varepsilon\,\omega^2\cos(kz-\omega t)\ \mu + 2\,\varepsilon\,\omega^2\cos(kz$$

$$-\omega t)\ \mu x^2 + 2\,\varepsilon\,\omega^2\cos(kz-\omega t)\ \mu y^2 + 2\,\varepsilon\,\omega^2\cos(kz-\omega t)\ \mu z^2 + \varepsilon\,\omega^2\cos(kz$$

$$-\omega t)\ \mu x^4 + \varepsilon\,\omega^2\cos(kz-\omega t)\ \mu y^4 + \varepsilon\,\omega^2\cos(kz-\omega t)\ \mu z^4 + 2\,\varepsilon\,\omega^2\cos(kz$$

$$-\omega t)\ \mu x^2y^2 + 2\,\varepsilon\,\omega^2\cos(kz-\omega t)\ \mu x^2z^2 + 2\,\varepsilon\,\omega^2\cos(kz-\omega t)\ \mu y^2z^2)\,)\,,0,0\big]$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B)

$$=\Big[\frac{1}{(1+x^2+y^2+z^2)^5\,\mu}\big(2x\,\big(-k^2\cos(kz-\omega t) - 10\cos(kz-\omega t) - 2k^2\cos(kz$$

$$-\omega t)\ x^2 - 2k^2\cos(kz-\omega t)\ y^2 + 4\sin(kz-\omega t)\ kz - 2k^2\cos(kz-\omega t)\ z^2$$

$$-2y^2\cos(kz-\omega t) - 2x^2\cos(kz-\omega t) - 2\cos(kz-\omega t)\ z^2 - \cos(kz-\omega t)\ k^2x^4$$

$$- \cos(k z - \omega t)\ k^2 y^4 - \cos(k z - \omega t)\ k^2 z^4 + 4 \sin(k z - \omega t)\ k z^3 + 4 x^2 \sin(k z - \omega t)\ k z$$

$$+ 4 y^2 \sin(k z - \omega t)\ k z - 2 \cos(k z - \omega t)\ k^2 x^2 y^2 - 2 \cos(k z - \omega t)\ k^2 x^2 z^2 - 2 \cos(k z$$

$$- \omega t)\ k^2 y^2 z^2 + \varepsilon \omega^2 \cos(k z - \omega t)\ \mu + 2 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^2 + 2 \varepsilon \omega^2 \cos(k z$$

$$- \omega t)\ \mu y^2 + 2 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu z^2 + \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^4 + \varepsilon \omega^2 \cos(k z$$

$$- \omega t)\ \mu y^4 + \varepsilon \omega^2 \cos(k z - \omega t)\ \mu z^4 + 2 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^2 y^2 + 2 \varepsilon \omega^2 \cos(k z$$

$$- \omega t)\ \mu x^2 z^2 + 2 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu y^2 z^2)\ \cos(k z - \omega t)\ (1 + z^2)\,)\,,$$

$$\frac{1}{(1 + x^2 + y^2 + z^2)^5\ \mu} \big(2 y\ (-k^2 \cos(k z - \omega t) - 10 \cos(k z - \omega t) - 2 k^2 \cos(k z - \omega t)\ x^2$$

$$- 2 k^2 \cos(k z - \omega t)\ y^2 + 4 \sin(k z - \omega t)\ k z - 2 k^2 \cos(k z - \omega t)\ z^2 - 2 y^2 \cos(k z - \omega t)$$

$$- 2 x^2 \cos(k z - \omega t) - 2 \cos(k z - \omega t)\ z^2 - \cos(k z - \omega t)\ k^2 x^4 - \cos(k z - \omega t)\ k^2 y^4$$

$$- \cos(k z - \omega t)\ k^2 z^4 + 4 \sin(k z - \omega t)\ k z^3 + 4 x^2 \sin(k z - \omega t)\ k z + 4 y^2 \sin(k z$$

$$- \omega t)\ k z - 2 \cos(k z - \omega t)\ k^2 x^2 y^2 - 2 \cos(k z - \omega t)\ k^2 x^2 z^2 - 2 \cos(k z - \omega t)\ k^2 y^2 z^2$$

$$+ \varepsilon \omega^2 \cos(k z - \omega t)\ \mu + 2 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^2 + 2 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu y^2$$

$$+ 2 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu z^2 + \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^4 + \varepsilon \omega^2 \cos(k z - \omega t)\ \mu y^4$$

$$+ \varepsilon \omega^2 \cos(k z - \omega t)\ \mu z^4 + 2 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^2 y^2 + 2 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^2 z^2$$

$$+ 2\,\varepsilon\,\omega^2 \cos(k\,z - \omega\,t)\,\mu\,y^2\,z^2)\,\cos(k\,z - \omega\,t)\,(1 + z^2)\,),\ -\frac{1}{(1 + x^2 + y^2 + z^2)^5\,\mu}\,((\,$$

$$-k^2 \cos(k\,z - \omega\,t) - 10\cos(k\,z - \omega\,t) - 2\,k^2 \cos(k\,z - \omega\,t)\,x^2 - 2\,k^2 \cos(k\,z - \omega\,t)\,y^2$$

$$+ 4\sin(k\,z - \omega\,t)\,k\,z - 2\,k^2 \cos(k\,z - \omega\,t)\,z^2 - 2\,y^2 \cos(k\,z - \omega\,t) - 2\,x^2 \cos(k\,z - \omega\,t)$$

$$- 2\cos(k\,z - \omega\,t)\,z^2 - \cos(k\,z - \omega\,t)\,k^2\,x^4 - \cos(k\,z - \omega\,t)\,k^2\,y^4 - \cos(k\,z - \omega\,t)\,k^2\,z^4$$

$$+ 4\sin(k\,z - \omega\,t)\,k\,z^3 + 4\,x^2 \sin(k\,z - \omega\,t)\,k\,z + 4\,y^2 \sin(k\,z - \omega\,t)\,k\,z - 2\cos(k\,z$$

$$- \omega\,t)\,k^2\,x^2\,y^2 - 2\cos(k\,z - \omega\,t)\,k^2\,x^2\,z^2 - 2\cos(k\,z - \omega\,t)\,k^2\,y^2\,z^2 + \varepsilon\,\omega^2 \cos(k\,z - \omega\,t)\,\mu$$

$$+ 2\,\varepsilon\,\omega^2 \cos(k\,z - \omega\,t)\,\mu\,x^2 + 2\,\varepsilon\,\omega^2 \cos(k\,z - \omega\,t)\,\mu\,y^2 + 2\,\varepsilon\,\omega^2 \cos(k\,z - \omega\,t)\,\mu\,z^2$$

$$+ \varepsilon\,\omega^2 \cos(k\,z - \omega\,t)\,\mu\,x^4 + \varepsilon\,\omega^2 \cos(k\,z - \omega\,t)\,\mu\,y^4 + \varepsilon\,\omega^2 \cos(k\,z - \omega\,t)\,\mu\,z^4$$

$$+ 2\,\varepsilon\,\omega^2 \cos(k\,z - \omega\,t)\,\mu\,x^2\,y^2 + 2\,\varepsilon\,\omega^2 \cos(k\,z - \omega\,t)\,\mu\,x^2\,z^2 + 2\,\varepsilon\,\omega^2 \cos(k\,z$$

$$- \omega\,t)\,\mu\,y^2\,z^2)\,(\sin(k\,z - \omega\,t)\,k + \sin(k\,z - \omega\,t)\,k\,x^2 + \sin(k\,z - \omega\,t)\,k\,y^2 + \sin(k\,z$$

$$- \omega\,t)\,k\,z^2 + 2\cos(k\,z - \omega\,t)\,z)\,(x^2 + y^2)\,)\,]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] $= \left[\frac{2\cos(k\,z - \omega\,t)^2\,\omega\,x\,z}{(1 + x^2 + y^2 + z^2)^2},\right.$

$$\left.\frac{2\cos(k\,z - \omega\,t)^2\,\omega\,y\,z}{(1 + x^2 + y^2 + z^2)^2},\ \frac{2\cos(k\,z - \omega\,t)^2\,\omega\,(1 + z^2)}{(1 + x^2 + y^2 + z^2)^2},\ \frac{2\cos(k\,z - \omega\,t)^2\,k\,(1 + z^2)}{(1 + x^2 + y^2 + z^2)^2}\right]$$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{1}{(1 + x^2 + y^2 + z^2)^3\,\mu}\,(\cos(k\,z - \omega\,t)\,(2\,x\cos(k\,z - \omega\,t) + 2\,k^2\,y^3 \sin(k\,z - \omega\,t)\right.$$

$$+ y\,k^2 \sin(k\,z - \omega\,t) + 2\,k^2\,y \sin(k\,z - \omega\,t)\,x^2 + 2\,k\,y\cos(k\,z - \omega\,t)\,z + 2\,k^2\,y\sin(k\,z$$

$$- \omega\,t)\,z^2 + k^2\,y^5 \sin(k\,z - \omega\,t) - 2\,\varepsilon\,y\sin(k\,z - \omega\,t)\,\omega^2\,\mu\,x^2\,z^2 + k^2\,y\sin(k\,z - \omega\,t)\,x^4$$

$$+ 2 k^2 y^3 \sin(k z - \omega t)\ x^2 + 2 k^2 y^3 \sin(k z - \omega t)\ z^2 + k^2 y \sin(k z - \omega t)\ z^4 + 2 k y^3 \cos(k z$$

$$- \omega t)\ z + 2 k y \cos(k z - \omega t)\ z^3 + 2 \cos(k z - \omega t)\ x z^2 + 2 k y \cos(k z - \omega t)\ z x^2$$

$$+ 2 k^2 y \sin(k z - \omega t)\ x^2 z^2 - \varepsilon y^5 \sin(k z - \omega t)\ \omega^2 \mu - \varepsilon y \sin(k z - \omega t)\ \omega^2 \mu$$

$$- 2 \varepsilon y^3 \sin(k z - \omega t)\ \omega^2 \mu - 2 \varepsilon y \sin(k z - \omega t)\ \omega^2 \mu x^2 - 2 \varepsilon y \sin(k z - \omega t)\ \omega^2 \mu z^2$$

$$- \varepsilon y \sin(k z - \omega t)\ \omega^2 \mu x^4 - 2 \varepsilon y^3 \sin(k z - \omega t)\ \omega^2 \mu x^2 - 2 \varepsilon y^3 \sin(k z - \omega t)\ \omega^2 \mu z^2$$

$$- \varepsilon y \sin(k z - \omega t)\ \omega^2 \mu z^4)),\ - \frac{1}{(1 + x^2 + y^2 + z^2)^3 \mu} \left(\cos(k z - \omega t)\ \left(2 k^2 x^3 \sin(k z\right.\right.$$

$$- \omega t) - 2 \cos(k z - \omega t)\ y z^2 - 2 y \cos(k z - \omega t) + x k^2 \sin(k z - \omega t) + 2 k x \cos(k z$$

$$- \omega t)\ z + 2 k^2 x \sin(k z - \omega t)\ y^2 + 2 k^2 x \sin(k z - \omega t)\ z^2 - 2 \varepsilon x \sin(k z - \omega t)\ \omega^2 \mu y^2 z^2$$

$$+ k^2 x^5 \sin(k z - \omega t) + 2 k x \cos(k z - \omega t)\ z y^2 + 2 k^2 x \sin(k z - \omega t)\ y^2 z^2 - \varepsilon x^5 \sin(k z$$

$$- \omega t)\ \omega^2 \mu - \varepsilon x \sin(k z - \omega t)\ \omega^2 \mu - 2 \varepsilon x^3 \sin(k z - \omega t)\ \omega^2 \mu - 2 \varepsilon x \sin(k z$$

$$- \omega t)\ \omega^2 \mu y^2 - 2 \varepsilon x \sin(k z - \omega t)\ \omega^2 \mu z^2 - 2 \varepsilon x^3 \sin(k z - \omega t)\ \omega^2 \mu y^2 - 2 \varepsilon x^3 \sin(k z$$

$$- \omega t)\ \omega^2 \mu z^2 - \varepsilon x \sin(k z - \omega t)\ \omega^2 \mu y^4 - \varepsilon x \sin(k z - \omega t)\ \omega^2 \mu z^4 + 2 k^2 x^3 \sin(k z$$

$$- \omega t)\ y^2 + 2 k^2 x^3 \sin(k z - \omega t)\ z^2 + k^2 x \sin(k z - \omega t)\ y^4 + k^2 x \sin(k z - \omega t)\ z^4$$

$$+ 2 k x^3 \cos(k z - \omega t)\ z + 2 k x \cos(k z - \omega t)\ z^3)),\ - \frac{1}{(1 + x^2 + y^2 + z^2)^3 \mu} \left(\cos(k z\right.$$

$$- \omega t)\ \left(\sin(k z - \omega t)\ k + \sin(k z - \omega t)\ k x^2 + \sin(k z - \omega t)\ k y^2 + \sin(k z - \omega t)\ k z^2\right.$$

$$+ 2\cos(k z-\omega t)\, z)\, (x^2+y^2)\,), -\frac{\cos(k z-\omega t)\,\varepsilon \sin(k z-\omega t)\,\omega\,(x^2+y^2)}{(1+x^2+y^2+z^2)^2}\Bigg]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B)

$$= \Bigg[\frac{1}{(1+x^2+y^2+z^2)^5\,\mu}\Big(\cos(k z-\omega t)\,\big(4\cos(k z-\omega t)^2 z^2 y^3 - 6\sin(k z$$

$$-\omega t)\cos(k z-\omega t)\, k^2 x z^2 - 6\sin(k z-\omega t)\cos(k z-\omega t)\, k^2 x z^4 - 4\sin(k z$$

$$-\omega t)\cos(k z-\omega t)\, k^2 x y^2 + 4\sin(k z-\omega t)^2 k^2 x^2 z^2 y^3 + 4\sin(k z-\omega t)\, k z^3 \cos(k z$$

$$-\omega t)\, y^3 + 4\sin(k z-\omega t)\, k y^5 \cos(k z-\omega t)\, z + 2 y \sin(k z-\omega t)^2 k^2 x^2 z^2$$

$$+ 2 y \sin(k z-\omega t)^2 k^2 x^4 z^2 + y \sin(k z-\omega t)^2 k^2 z^4 x^2 + 4\sin(k z-\omega t)\, k y^3 \cos(k z$$

$$-\omega t)\, z - 2\cos(k z-\omega t)\, k^2 x^5 \sin(k z-\omega t)\, z^2 - 2\cos(k z-\omega t)\, k^2 x \sin(k z-\omega t)\, z^6$$

$$-4\cos(k z-\omega t)\, k^2 x^3 \sin(k z-\omega t)\, z^4 - 4 k x \cos(k z-\omega t)^2 z^3 y^2 - 4 k x \cos(k z$$

$$-\omega t)^2 z y^2 - 2\cos(k z-\omega t)\, k^2 x \sin(k z-\omega t)\, y^4 - 4\cos(k z-\omega t)\, k^2 x^3 \sin(k z$$

$$-\omega t)\, y^2 - 8\cos(k z-\omega t)\, k^2 x^3 \sin(k z-\omega t)\, z^2 + 4\cos(k z-\omega t)\,\varepsilon x^3 \sin(k z$$

$$-\omega t)\,\omega^2 \mu y^2 z^2 - 2\varepsilon \sin(k z-\omega t)^2 \omega^2 y^5 \mu - \varepsilon \sin(k z-\omega t)^2 \omega^2 y^7 \mu - \varepsilon \sin(k z$$

$$-\omega t)^2 \omega^2 y^3 \mu - 4\varepsilon \sin(k z-\omega t)^2 \omega^2 y^3 \mu x^2 - \varepsilon \sin(k z-\omega t)^2 \omega^2 y \mu x^6$$

$$-3\varepsilon \sin(k z-\omega t)^2 \omega^2 y^3 \mu x^4 - 3\varepsilon \sin(k z-\omega t)^2 \omega^2 y^5 \mu x^2 - 2\varepsilon \sin(k z$$

$$-\omega t)^2 \omega^2 y^3 \mu z^2 - 2\varepsilon \sin(k z-\omega t)^2 \omega^2 y^5 \mu z^2 - \varepsilon \sin(k z-\omega t)^2 \omega^2 y^3 \mu z^4 + 4\sin(k z$$

$-\omega t)\ \cos(k z-\omega t)\ \mu x y^2 \varepsilon \omega^2+4 \cos(k z-\omega t)\ \varepsilon x^3 \sin(k z-\omega t)\ \omega^2 \mu y^2+8 \cos(k z$

$-\omega t)\ \varepsilon x^3 \sin(k z-\omega t)\ \omega^2 \mu z^2+2 \cos(k z-\omega t)\ \varepsilon x \sin(k z-\omega t)\ \omega^2 \mu y^4+4 \cos(k z$

$-\omega t)\ \varepsilon x^3 \sin(k z-\omega t)\ \omega^2 \mu z^4+2 \cos(k z-\omega t)\ \varepsilon x \sin(k z-\omega t)\ \omega^2 \mu z^6+2 \cos(k z$

$-\omega t)\ \varepsilon x^5 \sin(k z-\omega t)\ \omega^2 \mu z^2+6 \sin(k z-\omega t)\ \cos(k z-\omega t)\ \mu x z^2 \varepsilon \omega^2+6 \sin(k z$

$-\omega t)\ \cos(k z-\omega t)\ \mu x z^4 \varepsilon \omega^2+4 \sin(k z-\omega t)\ \cos(k z-\omega t)\ \mu x^3 \varepsilon \omega^2+2 \sin(k z$

$-\omega t)\ \cos(k z-\omega t)\ \mu x \varepsilon \omega^2-4 \cos(k z-\omega t)\ k^2 x \sin(k z-\omega t)\ y^2 z^4-2 \cos(k z$

$-\omega t)\ k^2 x \sin(k z-\omega t)\ y^4 z^2-4 \cos(k z-\omega t)\ k^2 x^3 \sin(k z-\omega t)\ y^2 z^2-8 \cos(k z$

$-\omega t)\ k^2 x \sin(k z-\omega t)\ y^2 z^2+2 \cos(k z-\omega t)\ \varepsilon x^5 \sin(k z-\omega t)\ \omega^2 \mu+4 y \sin(k z$

$-\omega t)\ k z^3 \cos(k z-\omega t)\ x^2+4 y \sin(k z-\omega t)\ k x^4 \cos(k z-\omega t)\ z+8 \sin(k z$

$-\omega t)\ k x^2 \cos(k z-\omega t)\ z y^3+4 y \sin(k z-\omega t)\ k x^2 \cos(k z-\omega t)\ z-4 \sin(k z$

$-\omega t)\ \cos(k z-\omega t)\ k^2 x^3-2 \sin(k z-\omega t)\ \cos(k z-\omega t)\ k^2 x-4 \cos(k z-\omega t)^2 k x z$

$-8 \cos(k z-\omega t)^2 k x z^3-4 k x^3 \cos(k z-\omega t)^2 z^3-4 k x \cos(k z-\omega t)^2 z^5-2 \cos(k z$

$-\omega t)\ k^2 x^5 \sin(k z-\omega t)\ -4 k x^3 \cos(k z-\omega t)^2 z+2 \sin(k z-\omega t)^2 k^2 y^3 z^2$

$+3 \sin(k z-\omega t)^2 k^2 x^4 y^3+3 \sin(k z-\omega t)^2 k^2 x^2 y^5+2 \sin(k z-\omega t)^2 k^2 y^5 z^2$

$$+\sin(kz-\omega t)^2 k^2 z^4 y^3+y\sin(kz-\omega t)^2 k^2 x^2+2y\sin(kz-\omega t)^2 k^2 x^4$$

$$+4y\cos(kz-\omega t)^2 z^2 x^2+y\sin(kz-\omega t)^2 k^2 x^6+4\sin(kz-\omega t)^2 k^2 x^2 y^3$$

$$-2\varepsilon\sin(kz-\omega t)^2 \omega^2 y\mu x^2 z^2-2\varepsilon\sin(kz-\omega t)^2 \omega^2 y\mu x^4 z^2-4\varepsilon\sin(kz$$

$$-\omega t)^2 \omega^2 y^3 \mu x^2 z^2-\varepsilon\sin(kz-\omega t)^2 \omega^2 y\mu x^2 z^4-\varepsilon\sin(kz-\omega t)^2 \omega^2 y\mu x^2$$

$$-2\varepsilon\sin(kz-\omega t)^2 \omega^2 y\mu x^4+8\cos(kz-\omega t)\,\varepsilon x\sin(kz-\omega t)\,\omega^2 \mu y^2 z^2+4\cos(kz$$

$$-\omega t)\,\varepsilon x\sin(kz-\omega t)\,\omega^2 \mu y^2 z^4+2\cos(kz-\omega t)\,\varepsilon x\sin(kz-\omega t)\,\omega^2 \mu y^4 z^2$$

$$+8\cos(kz-\omega t)^2 y z^2+4y\cos(kz-\omega t)^2+4\cos(kz-\omega t)^2 y z^4+\sin(kz$$

$$-\omega t)^2 k^2 y^3+2\sin(kz-\omega t)^2 k^2 y^5+\sin(kz-\omega t)^2 k^2 y^7)),$$

$$-\frac{1}{(1+x^2+y^2+z^2)^5 \mu}\Big(\cos(kz-\omega t)\,\big(4\sin(kz-\omega t)^2 k^2 x^3 z^2 y^2+4\sin(kz$$

$$-\omega t)\,k z^3\cos(kz-\omega t)\,x^3+4\sin(kz-\omega t)\,k x^5\cos(kz-\omega t)\,z+2x\sin(kz$$

$$-\omega t)^2 k^2 y^2 z^2+2x\sin(kz-\omega t)^2 k^2 y^4 z^2+x\sin(kz-\omega t)^2 k^2 z^4 y^2+4\sin(kz$$

$$-\omega t)\,k x^3\cos(kz-\omega t)\,z+4ky\cos(kz-\omega t)^2 z^3 x^2+2\cos(kz-\omega t)\,k^2 y^5\sin(kz$$

$$-\omega t)\,z^2+2\cos(kz-\omega t)\,k^2 y\sin(kz-\omega t)\,z^6+4\cos(kz-\omega t)\,k^2 y^3\sin(kz-\omega t)\,z^4$$

$$+4ky\cos(kz-\omega t)^2 z x^2+2\cos(kz-\omega t)\,k^2 y\sin(kz-\omega t)\,x^4+4\cos(kz$$

$$- \omega t)\ k^2 y^3 \sin(k z - \omega t)\ x^2 + 8 \cos(k z - \omega t)\ k^2 y^3 \sin(k z - \omega t)\ z^2 + 6 \sin(k z$$

$$- \omega t)\ \cos(k z - \omega t)\ k^2 y z^2 + 6 \sin(k z - \omega t)\ \cos(k z - \omega t)\ k^2 y z^4 + 4 \sin(k z$$

$$- \omega t)\ \cos(k z - \omega t)\ k^2 x^2 y + 8 \cos(k z - \omega t)^2 x z^2 + 4 \cos(k z - \omega t)^2 x z^4 - 2 \cos(k z$$

$$- \omega t)\ \varepsilon y \sin(k z - \omega t)\ \omega^2 \mu x^4 z^2 - 2 \sin(k z - \omega t)\ \cos(k z - \omega t)\ \mu y \varepsilon \omega^2 - 4 \sin(k z$$

$$- \omega t)\ \cos(k z - \omega t)\ \mu y^3 \varepsilon \omega^2 + 4 x \sin(k z - \omega t)\ k z^3 \cos(k z - \omega t)\ y^2 + 4 x \sin(k z$$

$$- \omega t)\ k y^4 \cos(k z - \omega t)\ z + 8 \sin(k z - \omega t)\ k x^3 \cos(k z - \omega t)\ z y^2 + 4 x \sin(k z$$

$$- \omega t)\ k y^2 \cos(k z - \omega t)\ z + 4 \cos(k z - \omega t)\ k^2 y \sin(k z - \omega t)\ x^2 z^4 + 2 \cos(k z$$

$$- \omega t)\ k^2 y \sin(k z - \omega t)\ x^4 z^2 + 4 \cos(k z - \omega t)\ k^2 y^3 \sin(k z - \omega t)\ x^2 z^2 + 8 \cos(k z$$

$$- \omega t)\ k^2 y \sin(k z - \omega t)\ x^2 z^2 - 2 \cos(k z - \omega t)\ \varepsilon y^5 \sin(k z - \omega t)\ \omega^2 \mu + \sin(k z$$

$$- \omega t)^2 k^2 x^3 + 2 \sin(k z - \omega t)^2 k^2 x^5 + 4 \cos(k z - \omega t)^2 z^2 x^3 - 4 \varepsilon \sin(k z$$

$$- \omega t)^2 \omega^2 x^3 \mu y^2 z^2 - 2 \varepsilon \sin(k z - \omega t)^2 \omega^2 x \mu y^2 z^2 - 2 \varepsilon \sin(k z - \omega t)^2 \omega^2 x \mu y^4 z^2$$

$$- \varepsilon \sin(k z - \omega t)^2 \omega^2 x \mu y^2 z^4 + \sin(k z - \omega t)^2 k^2 x^7 - 4 \cos(k z - \omega t)\ \varepsilon y \sin(k z$$

$$- \omega t)\ \omega^2 \mu x^2 z^4 - 4 \cos(k z - \omega t)\ \varepsilon y^3 \sin(k z - \omega t)\ \omega^2 \mu x^2 z^2 - 8 \cos(k z$$

$$- \omega t)\ \varepsilon y \sin(k z - \omega t)\ \omega^2 \mu x^2 z^2 + 4 \cos(k z - \omega t)^2 k y z + 8 \cos(k z - \omega t)^2 k y z^3$$

$$+2\sin(kz-\omega t)\cos(kz-\omega t)k^2y+4\sin(kz-\omega t)\cos(kz-\omega t)k^2y^3$$

$$+4\sin(kz-\omega t)^2k^2x^3y^2+2\sin(kz-\omega t)^2k^2x^3z^2+3\sin(kz-\omega t)^2k^2x^5y^2$$

$$+3\sin(kz-\omega t)^2k^2x^3y^4+2\sin(kz-\omega t)^2k^2x^5z^2+\sin(kz-\omega t)^2k^2z^4x^3$$

$$+x\sin(kz-\omega t)^2k^2y^2+2x\sin(kz-\omega t)^2k^2y^4+4x\cos(kz-\omega t)^2z^2y^2$$

$$+x\sin(kz-\omega t)^2k^2y^6+2\cos(kz-\omega t)k^2y^5\sin(kz-\omega t)+4ky^3\cos(kz-\omega t)^2z^3$$

$$+4ky\cos(kz-\omega t)^2z^5+4ky^3\cos(kz-\omega t)^2z+4\cos(kz-\omega t)^2x-2\cos(kz$$

$$-\omega t)\varepsilon y^5\sin(kz-\omega t)\omega^2\mu z^2-4\cos(kz-\omega t)\varepsilon y^3\sin(kz-\omega t)\omega^2\mu z^4-2\cos(kz$$

$$-\omega t)\varepsilon y\sin(kz-\omega t)\omega^2\mu z^6-2\cos(kz-\omega t)\varepsilon y\sin(kz-\omega t)\omega^2\mu x^4-4\cos(kz$$

$$-\omega t)\varepsilon y^3\sin(kz-\omega t)\omega^2\mu x^2-8\cos(kz-\omega t)\varepsilon y^3\sin(kz-\omega t)\omega^2\mu z^2-6\sin(kz$$

$$-\omega t)\cos(kz-\omega t)\mu yz^2\varepsilon\omega^2-6\sin(kz-\omega t)\cos(kz-\omega t)\mu yz^4\varepsilon\omega^2-4\sin(kz$$

$$-\omega t)\cos(kz-\omega t)\mu x^2y\varepsilon\omega^2-4\varepsilon\sin(kz-\omega t)^2\omega^2x^3\mu y^2-2\varepsilon\sin(kz$$

$$-\omega t)^2\omega^2x^3\mu z^2-3\varepsilon\sin(kz-\omega t)^2\omega^2x^5\mu y^2-2\varepsilon\sin(kz-\omega t)^2\omega^2x^5\mu z^2$$

$$-3\varepsilon\sin(kz-\omega t)^2\omega^2x^3\mu y^4-\varepsilon\sin(kz-\omega t)^2\omega^2x^3\mu z^4-\varepsilon\sin(kz-\omega t)^2\omega^2x\mu y^2$$

$$-2\varepsilon\sin(kz-\omega t)^2\omega^2x\mu y^4-\varepsilon\sin(kz-\omega t)^2\omega^2x\mu y^6-\varepsilon\sin(kz-\omega t)^2\omega^2x^3\mu$$

$$-2\,\varepsilon\sin(kz-\omega t)^2\,\omega^2\,x^5\,\mu-\varepsilon\sin(kz-\omega t)^2\,\omega^2\,x^7\,\mu)),\ \frac{1}{\mu\,(1+x^2+y^2+z^2)^4}\,((x^2$$

$$+y^2)\,(k^2\sin(kz-\omega t)\,x^2-\varepsilon\sin(kz-\omega t)\,\omega^2\,\mu\,x^2-\varepsilon\sin(kz-\omega t)\,\omega^2\,\mu+k^2\sin(kz$$

$$-\omega t)+k^2\sin(kz-\omega t)\,y^2-\varepsilon\sin(kz-\omega t)\,\omega^2\,\mu\,z^2+k^2\sin(kz-\omega t)\,z^2+2\,k\cos(kz$$

$$-\omega t)\,z-\varepsilon\sin(kz-\omega t)\,\omega^2\,\mu\,y^2)\,\cos(kz-\omega t)\,(\sin(kz-\omega t)\,k+\sin(kz-\omega t)\,k\,x^2$$

$$+\sin(kz-\omega t)\,k\,y^2+\sin(kz-\omega t)\,k\,z^2+2\cos(kz-\omega t)\,z))\,]$$

Spin Dissipation J_spin dot E $= -\frac{1}{(1+x^2+y^2+z^2)^3\,\mu}\,((x^2+y^2)\,(k^2\sin(kz-\omega t)\,x^2$

$$-\varepsilon\sin(kz-\omega t)\,\omega^2\,\mu\,x^2-\varepsilon\sin(kz-\omega t)\,\omega^2\,\mu+k^2\sin(kz-\omega t)+k^2\sin(kz-\omega t)\,y^2$$

$$-\varepsilon\sin(kz-\omega t)\,\omega^2\,\mu\,z^2+k^2\sin(kz-\omega t)\,z^2+2\,k\cos(kz-\omega t)\,z-\varepsilon\sin(kz$$

$$-\omega t)\,\omega^2\,\mu\,y^2)\,\cos(kz-\omega t)\,\sin(kz-\omega t)\,\omega)$$

Disspative Force 3 vector $= \Big[\frac{1}{(1+x^2+y^2+z^2)^5\,\mu}\,(\cos(kz-\omega t)\,(-2\,\varepsilon\sin(kz$

$$-\omega t)^2\,\omega^2\,y^5\,\mu^2-\varepsilon\sin(kz-\omega t)^2\,\omega^2\,y^3\,\mu^2+8\cos(kz-\omega t)\,\varepsilon\,x\sin(kz-\omega t)\,\omega^2\,\mu^2\,y^2\,z^2$$

$$+\mu\,y\sin(kz-\omega t)^2\,k^2\,x^6+4\,\mu\sin(kz-\omega t)^2\,k^2\,x^2\,y^3-\varepsilon\sin(kz-\omega t)^2\,\omega^2\,y^7\,\mu^2$$

$$-4\,x\cos(kz-\omega t)\,k^2\,y^2\,z^4+8\,x\,y^2\sin(kz-\omega t)\,k\,z^3-2\,x\cos(kz-\omega t)\,k^2\,y^4\,z^2$$

$$-4\cos(kz-\omega t)\,k^2\,x^3\,y^2\,z^2+2\,\varepsilon\,\omega^2\cos(kz-\omega t)\,\mu\,x^5+8\,x\,y^2\sin(kz-\omega t)\,k\,z$$

$$-8\,x\cos(kz-\omega t)\,k^2\,y^2\,z^2-4\,\mu\sin(kz-\omega t)\cos(kz-\omega t)\,k^2\,x^3-2\,\mu\sin(kz$$

$$-\omega t)\cos(kz-\omega t)\,k^2\,x-4\,\mu\cos(kz-\omega t)^2\,k\,x\,z-8\,\mu\cos(kz-\omega t)^2\,k\,x\,z^3$$

$$-4\,\mu\,k\,x^3\cos(kz-\omega t)^2\,z^3-4\,\mu\,k\,x\cos(kz-\omega t)^2\,z^5-2\,\mu\cos(kz-\omega t)\,k^2\,x^5\sin(kz$$

$$-\omega t)-4\,\mu\,k\,x^3\cos(kz-\omega t)^2\,z+2\,\mu\sin(kz-\omega t)^2\,k^2\,y^3\,z^2+3\,\mu\sin(kz$$

$$-\omega t)^2 k^2 x^4 y^3 + 3 \mu \sin(k z - \omega t)^2 k^2 x^2 y^5 + 2 \mu \sin(k z - \omega t)^2 k^2 y^5 z^2 + \mu \sin(k z$$

$$-\omega t)^2 k^2 z^4 y^3 + \mu y \sin(k z - \omega t)^2 k^2 x^2 + 8 x \varepsilon \omega^2 \cos(k z - \omega t) \mu y^2 z^2 + 4 \varepsilon \omega^2 \cos(k z$$

$$-\omega t) \mu x^3 y^2 z^2 + 2 x \varepsilon \omega^2 \cos(k z - \omega t) \mu y^4 z^2 + 4 x \varepsilon \omega^2 \cos(k z - \omega t) \mu y^2 z^4 + 4 \sin(k z$$

$$-\omega t) \cos(k z - \omega t) \mu^2 x y^2 \varepsilon \omega^2 - 2 \varepsilon \sin(k z - \omega t)^2 \omega^2 y \mu^2 x^2 z^2 - 2 \varepsilon \sin(k z$$

$$-\omega t)^2 \omega^2 y \mu^2 x^4 z^2 - 4 \varepsilon \sin(k z - \omega t)^2 \omega^2 y^3 \mu^2 x^2 z^2 - \varepsilon \sin(k z - \omega t)^2 \omega^2 y \mu^2 x^2 z^4$$

$$+ 4 \cos(k z - \omega t) \varepsilon x^3 \sin(k z - \omega t) \omega^2 \mu^2 y^2 + 8 \cos(k z - \omega t) \varepsilon x^3 \sin(k z - \omega t) \omega^2 \mu^2 z^2$$

$$+ 2 \cos(k z - \omega t) \varepsilon x \sin(k z - \omega t) \omega^2 \mu^2 y^4 + 4 \cos(k z - \omega t) \varepsilon x^3 \sin(k z - \omega t) \omega^2 \mu^2 z^4$$

$$+ 2 \cos(k z - \omega t) \varepsilon x \sin(k z - \omega t) \omega^2 \mu^2 z^6 + 2 \cos(k z - \omega t) \varepsilon x^5 \sin(k z - \omega t) \omega^2 \mu^2 z^2$$

$$- 4 \mu \cos(k z - \omega t) k^2 x \sin(k z - \omega t) y^2 z^4 - 2 \mu \cos(k z - \omega t) k^2 x \sin(k z - \omega t) y^4 z^2$$

$$- 4 \mu \cos(k z - \omega t) k^2 x^3 \sin(k z - \omega t) y^2 z^2 - 8 \mu \cos(k z - \omega t) k^2 x \sin(k z - \omega t) y^2 z^2$$

$$+ 4 \mu y \sin(k z - \omega t) k z^3 \cos(k z - \omega t) x^2 + 4 \mu y \sin(k z - \omega t) k x^4 \cos(k z - \omega t) z$$

$$+ 8 \mu \sin(k z - \omega t) k x^2 \cos(k z - \omega t) z y^3 + 4 \mu y \sin(k z - \omega t) k x^2 \cos(k z - \omega t) z$$

$$+ 6 \sin(k z - \omega t) \cos(k z - \omega t) \mu^2 x z^2 \varepsilon \omega^2 + 6 \sin(k z - \omega t) \cos(k z - \omega t) \mu^2 x z^4 \varepsilon \omega^2$$

$$+ 2 \cos(k z - \omega t) \varepsilon x \sin(k z - \omega t) \omega^2 \mu^2 y^4 z^2 + \mu \sin(k z - \omega t)^2 k^2 y^3 + 2 \mu \sin(k z$$

$$- \omega t)^2 k^2 y^5 + 4 \mu \cos(k z - \omega t)^2 z^2 y^3 + \mu \sin(k z - \omega t)^2 k^2 y^7 - 6 \cos(k z - \omega t) k^2 x z^4$$

$$+ 8 x \sin(k z - \omega t) k z^5 - 4 x y^2 \cos(k z - \omega t) z^2 - 2 x \cos(k z - \omega t) k^2 z^6 - 4 \cos(k z$$

$$- \omega t) k^2 x^3 z^4 + 8 x^3 \sin(k z - \omega t) k z^3 - 2 \cos(k z - \omega t) k^2 x^5 z^2 - 2 x \cos(k z - \omega t) k^2 y^4$$

$$+ 16 x \sin(k z - \omega t) k z^3 + 8 x^3 \sin(k z - \omega t) k z - 4 \cos(k z - \omega t) k^2 x^3 y^2 - 8 \cos(k z$$

$$- \omega t) k^2 x^3 z^2 + 8 \mu \cos(k z - \omega t)^2 y z^2 + 4 \mu \cos(k z - \omega t)^2 y z^4 + 4 \varepsilon \omega^2 \cos(k z$$

$$- \omega t) \mu x^3 y^2 + 8 \varepsilon \omega^2 \cos(k z - \omega t) \mu x^3 z^2 + 2 x \varepsilon \omega^2 \cos(k z - \omega t) \mu z^6$$

$$+ 2 x \varepsilon \omega^2 \cos(k z - \omega t) \mu y^4 + 6 x \varepsilon \omega^2 \cos(k z - \omega t) \mu z^4 + 2 \varepsilon \omega^2 \cos(k z - \omega t) \mu x^5 z^2$$

$$+ 4 \varepsilon \omega^2 \cos(k z - \omega t) \mu x^3 z^4 - 6 \mu \sin(k z - \omega t) \cos(k z - \omega t) k^2 x z^2 - 6 \mu \sin(k z$$

$$- \omega t) \cos(k z - \omega t) k^2 x z^4 - 4 \mu \sin(k z - \omega t) \cos(k z - \omega t) k^2 x y^2 + 4 \sin(k z$$

$$- \omega t) \cos(k z - \omega t) \mu^2 x^3 \varepsilon \omega^2 + 2 \sin(k z - \omega t) \cos(k z - \omega t) \mu^2 x \varepsilon \omega^2$$

$$+ 4 \mu \sin(k z - \omega t)^2 k^2 x^2 z^2 y^3 + 4 \mu \sin(k z - \omega t) k z^3 \cos(k z - \omega t) y^3 + 4 \mu \sin(k z$$

$$- \omega t) k y^5 \cos(k z - \omega t) z + 2 \mu y \sin(k z - \omega t)^2 k^2 x^2 z^2 + 2 \mu y \sin(k z - \omega t)^2 k^2 x^4 z^2$$

$$+ \mu y \sin(k z - \omega t)^2 k^2 z^4 x^2 + 4 \mu \sin(k z - \omega t) k y^3 \cos(k z - \omega t) z + 2 \cos(k z$$

$$- \omega t) \varepsilon x^5 \sin(k z - \omega t) \omega^2 \mu^2 - 2 \mu \cos(k z - \omega t) k^2 x^5 \sin(k z - \omega t) z^2 - 2 \mu \cos(k z$$

$$-\omega t)\, k^2 x \sin(k z-\omega t)\, z^6-4\,\mu \cos(k z-\omega t)\, k^2 x^3 \sin(k z-\omega t)\, z^4$$

$$-4\,\mu\, k\, x \cos(k z-\omega t)^2 z^3 y^2-4\,\mu\, k\, x \cos(k z-\omega t)^2 z\, y^2-2\,\mu \cos(k z-\omega t)\, k^2 x \sin(k z$$

$$-\omega t)\, y^4-4\,\mu \cos(k z-\omega t)\, k^2 x^3 \sin(k z-\omega t)\, y^2-8\,\mu \cos(k z-\omega t)\, k^2 x^3 \sin(k z$$

$$-\omega t)\, z^2-\varepsilon \sin(k z-\omega t)^2\, \omega^2 y\, \mu^2 x^2-2\,\varepsilon \sin(k z-\omega t)^2\, \omega^2 y\, \mu^2 x^4-4\,\varepsilon \sin(k z$$

$$-\omega t)^2\, \omega^2 y^3 \mu^2 x^2-\varepsilon \sin(k z-\omega t)^2\, \omega^2 y\, \mu^2 x^6-3\,\varepsilon \sin(k z-\omega t)^2\, \omega^2 y^3 \mu^2 x^4$$

$$-3\,\varepsilon \sin(k z-\omega t)^2\, \omega^2 y^5 \mu^2 x^2-2\,\varepsilon \sin(k z-\omega t)^2\, \omega^2 y^3 \mu^2 z^2-2\,\varepsilon \sin(k z$$

$$-\omega t)^2\, \omega^2 y^5 \mu^2 z^2-\varepsilon \sin(k z-\omega t)^2\, \omega^2 y^3 \mu^2 z^4+2\,\mu\, y \sin(k z-\omega t)^2 k^2 x^4$$

$$+4\,\mu\, y \cos(k z-\omega t)^2 z^2 x^2-2\,k^2 \cos(k z-\omega t)\, x-4\,x^3 \cos(k z-\omega t)\, z^2-4\,x\, y^2 \cos(k z$$

$$-\omega t)+4\,\mu\, y \cos(k z-\omega t)^2-4 \cos(k z-\omega t)\, x\, z^4-20\,x \cos(k z-\omega t)+4 \cos(k z$$

$$-\omega t)\, \varepsilon\, x^3 \sin(k z-\omega t)\, \omega^2 \mu^2 y^2 z^2-2 \cos(k z-\omega t)\, k^2 x^5+8\,x \sin(k z-\omega t)\, k z$$

$$-4 \cos(k z-\omega t)\, k^2 x\, y^2-6 \cos(k z-\omega t)\, k^2 x\, z^2-4 \cos(k z-\omega t)\, k^2 x^3+4\,\varepsilon\, \omega^2 \cos(k z$$

$$-\omega t)\, \mu\, x\, y^2+6\,\varepsilon\, \omega^2 \cos(k z-\omega t)\, \mu\, x\, z^2+2\,\varepsilon\, \omega^2 \cos(k z-\omega t)\, \mu\, x+4\,\varepsilon\, \omega^2 \cos(k z$$

$$-\omega t)\, \mu\, x^3-24 \cos(k z-\omega t)\, x\, z^2-4\,x^3 \cos(k z-\omega t)+4 \cos(k z-\omega t)\, \varepsilon\, x \sin(k z$$

$$-\omega t)\, \omega^2 \mu^2 y^2 z^4)\Big),\ -\frac{1}{(1+x^2+y^2+z^2)^5\, \mu}\Big(\cos(k z-\omega t)\, \big(-8\, y \sin(k z-\omega t)\, k z^5$$

$+ 4\, y\, x^2 \cos(k\, z - \omega\, t)\; z^2 + 2\, y \cos(k\, z - \omega\, t)\; k^2 z^6 + 4 \cos(k\, z - \omega\, t)\; k^2 y^3 z^4 - 8\, y^3 \sin(k\, z$

$- \omega\, t)\; k\, z^3 + 2 \cos(k\, z - \omega\, t)\; k^2 y^5 z^2 + 2\, y \cos(k\, z - \omega\, t)\; k^2 x^4 - 16\, y \sin(k\, z - \omega\, t)\; k\, z^3$

$- 8\, y^3 \sin(k\, z - \omega\, t)\; k\, z + 4 \cos(k\, z - \omega\, t)\; k^2 x^2 y^3 + 8 \cos(k\, z - \omega\, t)\; k^2 y^3 z^2$

$+ \mu \sin(k\, z - \omega\, t)^2 k^2 x^3 + 2\, \mu \sin(k\, z - \omega\, t)^2 k^2 x^5 + 4\, \mu \cos(k\, z - \omega\, t)^2 z^2 x^3$

$+ \mu \sin(k\, z - \omega\, t)^2 k^2 x^7 + 8\, \mu \cos(k\, z - \omega\, t)^2 x\, z^2 + 4\, \mu \cos(k\, z - \omega\, t)^2 x\, z^4 + 6 \cos(k\, z$

$- \omega\, t)\; k^2 y\, z^4 - 4\, y\, \varepsilon\, \omega^2 \cos(k\, z - \omega\, t)\; \mu\, x^2 z^4 + 4\, \mu\, x \sin(k\, z - \omega\, t)\; k\, z^3 \cos(k\, z - \omega\, t)\; y^2$

$+ 4\, \mu\, x \sin(k\, z - \omega\, t)\; k\, y^4 \cos(k\, z - \omega\, t)\; z + 8\, \mu \sin(k\, z - \omega\, t)\; k\, x^3 \cos(k\, z - \omega\, t)\; z\, y^2$

$+ 4\, \mu\, x \sin(k\, z - \omega\, t)\; k\, y^2 \cos(k\, z - \omega\, t)\; z + 4\, \mu \cos(k\, z - \omega\, t)\; k^2 y \sin(k\, z - \omega\, t)\; x^2 z^4$

$+ 2\, \mu \cos(k\, z - \omega\, t)\; k^2 y \sin(k\, z - \omega\, t)\; x^4 z^2 + 4\, \mu \cos(k\, z - \omega\, t)\; k^2 y^3 \sin(k\, z - \omega\, t)\; x^2 z^2$

$+ 8\, \mu \cos(k\, z - \omega\, t)\; k^2 y \sin(k\, z - \omega\, t)\; x^2 z^2 - 2 \cos(k\, z - \omega\, t)\; \varepsilon\, y^5 \sin(k\, z - \omega\, t)\; \omega^2 \mu^2 z^2$

$- 4 \cos(k\, z - \omega\, t)\; \varepsilon\, y^3 \sin(k\, z - \omega\, t)\; \omega^2 \mu^2 z^4 - 2 \cos(k\, z - \omega\, t)\; \varepsilon\, y \sin(k\, z - \omega\, t)\; \omega^2 \mu^2 z^6$

$- 2 \cos(k\, z - \omega\, t)\; \varepsilon\, y \sin(k\, z - \omega\, t)\; \omega^2 \mu^2 x^4 - 4 \cos(k\, z - \omega\, t)\; \varepsilon\, y \sin(k\, z$

$- \omega\, t)\; \omega^2 \mu^2 x^2 z^4 - 2 \cos(k\, z - \omega\, t)\; \varepsilon\, y \sin(k\, z - \omega\, t)\; \omega^2 \mu^2 x^4 z^2 - 4 \cos(k\, z$

$- \omega\, t)\; \varepsilon\, y^3 \sin(k\, z - \omega\, t)\; \omega^2 \mu^2 x^2 z^2 - 8 \cos(k\, z - \omega\, t)\; \varepsilon\, y \sin(k\, z - \omega\, t)\; \omega^2 \mu^2 x^2 z^2$

$$-4\cos(kz-\omega t)\,\varepsilon y^3\sin(kz-\omega t)\,\omega^2\mu^2x^2-8\cos(kz-\omega t)\,\varepsilon y^3\sin(kz-\omega t)\,\omega^2\mu^2z^2$$

$$-6\sin(kz-\omega t)\cos(kz-\omega t)\,\mu^2yz^2\varepsilon\omega^2-6\sin(kz-\omega t)\cos(kz-\omega t)\,\mu^2yz^4\varepsilon\omega^2$$

$$-4\sin(kz-\omega t)\cos(kz-\omega t)\,\mu^2x^2y\varepsilon\omega^2-4\varepsilon\sin(kz-\omega t)^2\omega^2x^3\mu^2y^2z^2$$

$$-2\varepsilon\sin(kz-\omega t)^2\omega^2x\mu^2y^2z^2-2\varepsilon\sin(kz-\omega t)^2\omega^2x\mu^2y^4z^2-\varepsilon\sin(kz$$

$$-\omega t)^2\omega^2x\mu^2y^2z^4+4y^3\cos(kz-\omega t)+24\cos(kz-\omega t)\,yz^2+2k^2\cos(kz-\omega t)\,y$$

$$-8y\varepsilon\omega^2\cos(kz-\omega t)\,\mu x^2z^2-4\varepsilon\omega^2\cos(kz-\omega t)\,\mu x^2y^3z^2-2y\varepsilon\omega^2\cos(kz$$

$$-\omega t)\,\mu x^4z^2+20y\cos(kz-\omega t)+4y\cos(kz-\omega t)\,k^2x^2z^4-8y\sin(kz-\omega t)\,kz$$

$$+4\cos(kz-\omega t)\,k^2x^2y+6\cos(kz-\omega t)\,k^2yz^2+4\cos(kz-\omega t)\,k^2y^3-4\varepsilon\omega^2\cos(kz$$

$$-\omega t)\,\mu x^2y-6\varepsilon\omega^2\cos(kz-\omega t)\,\mu yz^2-2\varepsilon\omega^2\cos(kz-\omega t)\,\mu y-4\varepsilon\omega^2\cos(kz$$

$$-\omega t)\,\mu y^3+4\cos(kz-\omega t)^2\mu x+4y^3\cos(kz-\omega t)\,z^2+2\cos(kz-\omega t)\,k^2y^5$$

$$+4yx^2\cos(kz-\omega t)+4\cos(kz-\omega t)\,yz^4-8yx^2\sin(kz-\omega t)\,kz^3+2y\cos(kz$$

$$-\omega t)\,k^2x^4z^2+4\cos(kz-\omega t)\,k^2x^2y^3z^2-2\varepsilon\omega^2\cos(kz-\omega t)\,\mu y^5-8yx^2\sin(kz$$

$$-\omega t)\,kz+8y\cos(kz-\omega t)\,k^2x^2z^2+4\mu\cos(kz-\omega t)^2kyz+8\mu\cos(kz$$

$$-\omega t)^2kyz^3+2\mu\sin(kz-\omega t)\cos(kz-\omega t)\,k^2y+4\mu\sin(kz-\omega t)\cos(kz$$

$$-\omega t)\ k^2 y^3 - \varepsilon \sin(k z - \omega t)^2 \omega^2 x^3 \mu^2 + 4 \mu \sin(k z - \omega t)^2 k^2 x^3 y^2 + 2 \mu \sin(k z$$

$$-\omega t)^2 k^2 x^3 z^2 + 3 \mu \sin(k z - \omega t)^2 k^2 x^5 y^2 + 3 \mu \sin(k z - \omega t)^2 k^2 x^3 y^4 + 2 \mu \sin(k z$$

$$-\omega t)^2 k^2 x^5 z^2 + \mu \sin(k z - \omega t)^2 k^2 z^4 x^3 + \mu x \sin(k z - \omega t)^2 k^2 y^2 + 2 \mu x \sin(k z$$

$$-\omega t)^2 k^2 y^4 + 4 \mu x \cos(k z - \omega t)^2 z^2 y^2 + \mu x \sin(k z - \omega t)^2 k^2 y^6 + 2 \mu \cos(k z$$

$$-\omega t)\ k^2 y^5 \sin(k z - \omega t)\ + 4 \mu k y^3 \cos(k z - \omega t)^2 z^3 + 4 \mu k y \cos(k z - \omega t)^2 z^5$$

$$+ 4 \mu k y^3 \cos(k z - \omega t)^2 z - 2 \varepsilon \sin(k z - \omega t)^2 \omega^2 x^5 \mu^2 - \varepsilon \sin(k z - \omega t)^2 \omega^2 x^7 \mu^2$$

$$-4 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^2 y^3 - 8 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu y^3 z^2 - 2 y \varepsilon \omega^2 \cos(k z - \omega t)\ \mu z^6$$

$$-2 y \varepsilon \omega^2 \cos(k z - \omega t)\ \mu x^4 - 6 y \varepsilon \omega^2 \cos(k z - \omega t)\ \mu z^4 - 2 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu y^5 z^2$$

$$-4 \varepsilon \omega^2 \cos(k z - \omega t)\ \mu y^3 z^4 + 4 \mu \sin(k z - \omega t)^2 k^2 x^3 z^2 y^2 + 4 \mu \sin(k z$$

$$-\omega t)\ k z^3 \cos(k z - \omega t)\ x^3 + 4 \mu \sin(k z - \omega t)\ k x^5 \cos(k z - \omega t)\ z + 2 \mu x \sin(k z$$

$$-\omega t)^2 k^2 y^2 z^2 + 2 \mu x \sin(k z - \omega t)^2 k^2 y^4 z^2 + \mu x \sin(k z - \omega t)^2 k^2 z^4 y^2 + 4 \mu \sin(k z$$

$$-\omega t)\ k x^3 \cos(k z - \omega t)\ z + 4 \mu k y \cos(k z - \omega t)^2 z^3 x^2 + 2 \mu \cos(k z - \omega t)\ k^2 y^5 \sin(k z$$

$$-\omega t)\ z^2 + 2 \mu \cos(k z - \omega t)\ k^2 y \sin(k z - \omega t)\ z^6 + 4 \mu \cos(k z - \omega t)\ k^2 y^3 \sin(k z$$

$$-\omega t)\ z^4 + 4 \mu k y \cos(k z - \omega t)^2 z x^2 + 2 \mu \cos(k z - \omega t)\ k^2 y \sin(k z - \omega t)\ x^4$$

$+ 4\,\mu \cos(k\,z - \omega\,t)\;k^2\,y^3 \sin(k\,z - \omega\,t)\;x^2 + 8\,\mu \cos(k\,z - \omega\,t)\;k^2\,y^3 \sin(k\,z - \omega\,t)\;z^2$

$+ 6\,\mu \sin(k\,z - \omega\,t)\;\cos(k\,z - \omega\,t)\;k^2\,y\,z^2 + 6\,\mu \sin(k\,z - \omega\,t)\;\cos(k\,z - \omega\,t)\;k^2\,y\,z^4$

$+ 4\,\mu \sin(k\,z - \omega\,t)\;\cos(k\,z - \omega\,t)\;k^2\,x^2\,y - 2 \sin(k\,z - \omega\,t)\;\cos(k\,z - \omega\,t)\;\mu^2\,y\,\varepsilon\,\omega^2$

$- 4 \sin(k\,z - \omega\,t)\;\cos(k\,z - \omega\,t)\;\mu^2\,y^3\,\varepsilon\,\omega^2 - 2 \cos(k\,z - \omega\,t)\;\varepsilon\,y^5 \sin(k\,z - \omega\,t)\;\omega^2\,\mu^2$

$- 4\,\varepsilon \sin(k\,z - \omega\,t)^2\,\omega^2\,x^3\,\mu^2\,y^2 - 2\,\varepsilon \sin(k\,z - \omega\,t)^2\,\omega^2\,x^3\,\mu^2\,z^2 - 3\,\varepsilon \sin(k\,z$

$- \omega\,t)^2\,\omega^2\,x^5\,\mu^2\,y^2 - 2\,\varepsilon \sin(k\,z - \omega\,t)^2\,\omega^2\,x^5\,\mu^2\,z^2 - 3\,\varepsilon \sin(k\,z - \omega\,t)^2\,\omega^2\,x^3\,\mu^2\,y^4$

$- \varepsilon \sin(k\,z - \omega\,t)^2\,\omega^2\,x^3\,\mu^2\,z^4 - \varepsilon \sin(k\,z - \omega\,t)^2\,\omega^2\,x\,\mu^2\,y^2 - 2\,\varepsilon \sin(k\,z - \omega\,t)^2\,\omega^2\,x\,\mu^2\,y^4$

$- \varepsilon \sin(k\,z - \omega\,t)^2\,\omega^2\,x\,\mu^2\,y^6)\,),\; \dfrac{1}{(1 + x^2 + y^2 + z^2)^5\,\mu}\,(\,(\sin(k\,z - \omega\,t)\;k + \sin(k\,z$

$- \omega\,t)\;k\,x^2 + \sin(k\,z - \omega\,t)\;k\,y^2 + \sin(k\,z - \omega\,t)\;k\,z^2 + 2 \cos(k\,z - \omega\,t)\;z)\;(x^2$

$+ y^2)\;(k^2 \cos(k\,z - \omega\,t)\; + 10 \cos(k\,z - \omega\,t)\; + 2\,k^2 \cos(k\,z - \omega\,t)\;x^2 + 2\,k^2 \cos(k\,z$

$- \omega\,t)\;y^2 - 4 \sin(k\,z - \omega\,t)\;k\,z + 2\,k^2 \cos(k\,z - \omega\,t)\;z^2 + 2\,y^2 \cos(k\,z - \omega\,t)\; + 2\,x^2 \cos(k\,z$

$- \omega\,t)\; + 2 \cos(k\,z - \omega\,t)\;z^2 + \cos(k\,z - \omega\,t)\;k^2\,x^4 + \cos(k\,z - \omega\,t)\;k^2\,y^4 + \cos(k\,z$

$- \omega\,t)\;k^2\,z^4 - 4 \sin(k\,z - \omega\,t)\;k\,z^3 - 4\,x^2 \sin(k\,z - \omega\,t)\;k\,z - 4\,y^2 \sin(k\,z - \omega\,t)\;k\,z$

$+ 2 \cos(k\,z - \omega\,t)\;k^2\,x^2\,y^2 + 2 \cos(k\,z - \omega\,t)\;k^2\,x^2\,z^2 + 2 \cos(k\,z - \omega\,t)\;k^2\,y^2\,z^2$

$+ 2\,\mu \cos(k\,z - \omega\,t)\;k^2 \sin(k\,z - \omega\,t)\;z^2 + \mu \cos(k\,z - \omega\,t)\;k^2 \sin(k\,z - \omega\,t)\;z^4 + \mu \cos(k\,z$

$- \omega\,t)\;k^2\,x^4 \sin(k\,z - \omega\,t)\; + 2\,\mu \cos(k\,z - \omega\,t)\;k^2\,x^2 \sin(k\,z - \omega\,t)\; + 2\,\mu \cos(k\,z$

$- \omega\,t)^2\,k\,z\,y^2 - \mu^2 \cos(k\,z - \omega\,t)\;\varepsilon \sin(k\,z - \omega\,t)\;\omega^2 + \mu \cos(k\,z - \omega\,t)\;k^2 \sin(k\,z - \omega\,t)\;y^4$

$+ 2\,\mu \cos(k\,z - \omega\,t)\;k^2 \sin(k\,z - \omega\,t)\;y^2 + 2\,\mu \cos(k\,z - \omega\,t)^2\,k\,x^2\,z + 2\,\mu \cos(k\,z$

$- \omega\,t)^2\,k\,z + 2\,\mu \cos(k\,z - \omega\,t)^2\,k\,z^3 + \mu \cos(k\,z - \omega\,t)\;k^2 \sin(k\,z - \omega\,t)\; - \varepsilon\,\omega^2 \cos(k\,z$

$- \omega\,t)\;\mu - 2\,\mu^2 \cos(k\,z - \omega\,t)\;\varepsilon\,x^2 \sin(k\,z - \omega\,t)\;\omega^2\,y^2 - 2\,\mu^2 \cos(k\,z - \omega\,t)\;\varepsilon\,x^2 \sin(k\,z$

$$- \omega t)\,\omega^2 z^2 - 2\,\mu^2 \cos(k z - \omega t)\,\varepsilon \sin(k z - \omega t)\,\omega^2 y^2 z^2 - 2\,\varepsilon\,\omega^2 \cos(k z - \omega t)\,\mu\,x^2$$

$$- 2\,\varepsilon\,\omega^2 \cos(k z - \omega t)\,\mu\,y^2 - 2\,\varepsilon\,\omega^2 \cos(k z - \omega t)\,\mu\,z^2 - \varepsilon\,\omega^2 \cos(k z - \omega t)\,\mu\,x^4$$

$$- \varepsilon\,\omega^2 \cos(k z - \omega t)\,\mu\,y^4 - \varepsilon\,\omega^2 \cos(k z - \omega t)\,\mu\,z^4 - 2\,\varepsilon\,\omega^2 \cos(k z - \omega t)\,\mu\,x^2 y^2$$

$$- 2\,\varepsilon\,\omega^2 \cos(k z - \omega t)\,\mu\,x^2 z^2 - 2\,\varepsilon\,\omega^2 \cos(k z - \omega t)\,\mu\,y^2 z^2 - 2\,\mu^2 \cos(k z - \omega t)\,\varepsilon \sin(k z$$

$$- \omega t)\,\omega^2 y^2 - \mu^2 \cos(k z - \omega t)\,\varepsilon \sin(k z - \omega t)\,\omega^2 y^4 + 2\,\mu \cos(k z - \omega t)\,k^2 \sin(k z$$

$$- \omega t)\,y^2 z^2 - \mu^2 \cos(k z - \omega t)\,\varepsilon\,x^4 \sin(k z - \omega t)\,\omega^2 - 2\,\mu^2 \cos(k z - \omega t)\,\varepsilon\,x^2 \sin(k z$$

$$- \omega t)\,\omega^2 - 2\,\mu^2 \cos(k z - \omega t)\,\varepsilon \sin(k z - \omega t)\,\omega^2 z^2 - \mu^2 \cos(k z - \omega t)\,\varepsilon \sin(k z$$

$$- \omega t)\,\omega^2 z^4 + 2\,\mu \cos(k z - \omega t)\,k^2 x^2 \sin(k z - \omega t)\,y^2 + 2\,\mu \cos(k z - \omega t)\,k^2 x^2 \sin(k z$$

$$- \omega t)\,z^2))\,]$$

$$Dissipation = -\frac{\cos(k z - \omega t)\,\omega\,(\mu\,\varepsilon \sin(k z - \omega t)\,x^2 + \mu\,\varepsilon \sin(k z - \omega t)\,y^2 - 2 \cos(k z - \omega t)\,z\,x)}{(1 + x^2 + y^2 + z^2)^2}$$

****************************** *END PROCEDURE* ****************************** **(7)**

Enter the name of the problem, and the components of the 4 potential
p-2, n=4

```
> NAME:=`Example 7a = Index 1 Irreversible solution EdotB < 0 (kinematic out)
  Type 1`;
```

```
> Ax:=y*z;Ay:=-x*z;Az:=C*t*1;phi:=+C*z*z*1;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

NAME := *Example 7a = Index 1 Irreversible solution EdotB* <0 *(kinematic out) Type 1*

$$Ax := y\,z$$

$$Ay := -x\,z$$

$$Az := C\,t$$

$$\phi := C\,z^2$$

Example 7a = Index 1 Irreversible solution EdotB <0 *(kinematic out) Type 1*

********************************* *Differential Form Format* ********************
****

$$Action\ 1\text{-}form = y\,z\,d(x) - x\,z\,d(y) + C\,t\,d(z) - C\,z^2\,d(t)$$

$$Intensity\ 2\text{-}form\ F{=}dA\ = -2\,z\,(d(x))\,\&\hat{}\,(d(y)) - y\,(d(x))\,\&\hat{}\,(d(z)) + x\,(d(y))\,\&\hat{}\,(d(z)) + (-C - 2\,C\,z)\,(d(z))\,\&\hat{}\,(d(t))$$

$$Topological\ Torsion\ 3\text{-}form\quad A\hat{}F = 2\,z^3\,C\,\&\hat{}(d(x), d(y), d(t)) - 2\,C\,t\,z\,\&\hat{}(d(x), d(y), d(z)) + \left(-y\,z\,C\,(1 + 2\,z) + C\,z^2\,y\right)\,\&\hat{}(d(x), d(z), d(t)) + \left(x\,z\,C\,(1 + 2\,z) - C\,z^2\,x\right)\,\&\hat{}(d(y), d(z), d(t))$$

$$Topological\ Parity\ 4\text{-}form\quad F\hat{}F = 4\,C\,(1 + 2\,z)\,z\,\&\hat{}(d(x), d(y), d(z), d(t))$$

********************************* *Using EM format* *****************************
****

$$E\ field = [0, 0, -C\,(1 + 2\,z)]$$

$$B\ field = [x, y, -2\,z]$$

$$Topological\ TORSION\ 4\ vector\quad T4 = \text{-}[ExA + Bphi,AdotB] = \left[z\,x\,C\,(1 + z), z\,y\,C\,(1 + z), 2\,z^3\,C, 2\,C\,t\,z\right]$$

$$Helicity\ AdotB = -2\,C\,t\,z$$

$$Poincare\ II\ {=}2(E.B) = 4\,C\,(1 + 2\,z)\,z$$

$$coefficient\ of\ Topological\ Parity\ 4\text{-}form\ = 4\,C\,(1 + 2\,z)\,z$$

$$Pfaff\ Topological\ Dimension\quad PTD = 4$$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******

****

Xm *or linear (Mean) curvature* $= 0$

Yg *or quadratic (GAUSS) curvature* $= z\left(z + 2\,C^2\right)$

Za *or Cubic (Interaction internal energy) curvature* $= 0$

Tk *or quartic (4D expansion) curvature* $= 2\,z^3\,C^2$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor $CH = 0$

$$D\ field = \left[0,\ 0,\ -\varepsilon\,C\,(1 + 2\,z)\right]$$

$$H\ field = \left[\frac{x}{\mu},\ \frac{y}{\mu},\ -\frac{2\,z}{\mu}\right]$$

Poynting vector $ExH = EXH$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \left[0,\ 0,\ 0,\ -2\,\varepsilon\,C\right]$

Amerian charge density *divD = rho* $= -2\,\varepsilon\,C$

divergence Lorentz Current 4Vector, *4div(J4)* $= 0$

$$\text{Topological SPIN 4 vector S4} = \left[-\frac{-2\,x\,z^2 + C\,t\,y}{\mu},\ \frac{C\,t\,x + 2\,y\,z^2}{\mu},\ -\frac{z\left(-y^2 - x^2 + \varepsilon\,C^2\,\mu\,z + 2\,\varepsilon\,C^2\,\mu\,z^2\right)}{\mu},\ -C^2\,t\,\varepsilon\,(1 + 2\,z)\right]$$

$$\text{Topological SPIN 3-form} = -\frac{\left(-2\,x\,z^2 + C\,t\,y\right)\ \&\text{^}(d(y),\ d(z),\ d(t))}{\mu} - \frac{\left(C\,t\,x + 2\,y\,z^2\right)\ \&\text{^}(d(x),\ d(z),\ d(t))}{\mu} - \frac{z\left(-y^2 - x^2 + \varepsilon\,C^2\,\mu\,z + 2\,\varepsilon\,C^2\,\mu\,z^2\right)\ \&\text{^}(d(x),\ d(y),\ d(t))}{\mu} + C^2\,t\,\varepsilon\,(1 + 2\,z)\ \&\text{^}(d(x),\ d(y),\ d(z))$$

Spin density rho_spin $= -C^2\,t\,\varepsilon\,(1 + 2\,z)$

$$\text{LaGrange field energy density (B.H-D.E)} = -\frac{-x^2 - y^2 - 4\,z^2 + \varepsilon\,C^2\,\mu + 4\,\varepsilon\,C^2\,\mu\,z + 4\,\varepsilon\,C^2\,\mu\,z^2}{\mu}$$

$$B.H = \frac{x^2 + y^2 + 4\,z^2}{\mu}$$

$$D.E = \varepsilon\,C^2\,(1 + 2\,z)^2$$

$$A.J = 0$$

$$\text{-rho.phi} = -2\,C^2\,z^2\,\varepsilon$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* =

$$-\frac{-x^2 - y^2 - 4\,z^2 + \varepsilon\,C^2\,\mu + 4\,\varepsilon\,C^2\,\mu\,z + 6\,\varepsilon\,C^2\,\mu\,z^2}{\mu}$$

London Coefficient $LC = 0$

PROCA coefficient curlcurlB $= [0, 0, 0]$

Amperian Current 4Vector *curlH-dD/dt=J4* $= [0, 0, 0, -2\,\varepsilon\,C]$

Lorentz Force 3 vector due to Ampere current *FL = -(rho_ampere E + J_ampere x B)* $= [0, 0, -2\,C^2\,\varepsilon\,(1+2\,z)]$

Amperian Dissipation *Jampere dot E* $= 0$

Lorentz Force Spin factor $LFSPIN = 0$

Topological Torsion current 4 vector *T4 = -[ExA + B.phi,AdotB]* $= [z\,x\,C\,(1+z), z\,y\,C\,(1+z), 2\,z^3\,C, 2\,C\,t\,z]$

Lorentz Force 3 vector due to Torsion current *TF = -(rho_torsion E + J_torsion x B)* $= [-2\,C\,z^2\,y\,(1+2\,z), 2\,C\,z^2\,x\,(1+2\,z), -2\,C^2\,t\,z\,(1+2\,z)]$

Torsion Dissipation *Jtorsion dot E* $= 2\,z^3\,C^2\,(1+2\,z)$

Topological Spin current 4 vector *TS4 = -[A x H + D.phi,AdotD]* $= \left[-\frac{-2\,x\,z^2 + C\,t\,y}{\mu}, \frac{C\,t\,x + 2\,y\,z^2}{\mu}, -\frac{z\,(-y^2 - x^2 + \varepsilon\,C^2\,\mu\,z + 2\,\varepsilon\,C^2\,\mu\,z^2)}{\mu}, -C^2\,t\,\varepsilon\,(1+2\,z)\right]$

Lorentz Force 3 vector due to Spin current *SF = --(rho_spin E + J_spin x B)* =

$$\left[-\frac{z\,(-2\,C\,t\,x - 4\,y\,z^2 - y^3 - y\,x^2 + y\,\varepsilon\,C^2\,\mu\,z + 2\,y\,\varepsilon\,C^2\,\mu\,z^2)}{\mu},\right.$$

$$\frac{z\,(-x\,y^2 - x^3 + x\,\varepsilon\,C^2\,\mu\,z + 2\,x\,\varepsilon\,C^2\,\mu\,z^2 - 4\,x\,z^2 + 2\,C\,t\,y)}{\mu},$$

$$\left.-\frac{C\,t\,(\varepsilon\,C^2\,\mu + 4\,\varepsilon\,C^2\,\mu\,z + 4\,\varepsilon\,C^2\,\mu\,z^2 - x^2 - y^2)}{\mu}\right]$$

Spin Dissipation *J_spin dot E* $= \frac{z\,(-y^2 - x^2 + \varepsilon\,C^2\,\mu\,z + 2\,\varepsilon\,C^2\,\mu\,z^2)\,C\,(1+2\,z)}{\mu}$

$$\mathit{Disspative\ Force\ 3\ vector} = \left[-z \left(-4\, y\, z^2 + 2\, y\, \varepsilon\, C^2\, \mu\, z^2 + 4\, C\, z^2\, y + y\, \varepsilon\, C^2\, \mu\, z + 2\, z\, y\, C - 2\, C\, t\, x - y^3 - y\, x^2 \right), z \left(2\, x\, \varepsilon\, C^2\, \mu\, z^2 - 4\, x\, z^2 + 4\, C\, z^2\, x + x\, \varepsilon\, C^2\, \mu\, z + 2\, z\, x\, C - x\, y^2 - x^3 + 2\, C\, t\, y \right), -C \left(2\, \varepsilon\, C + 4\, C\, \varepsilon\, z + C^2\, t\, \varepsilon\, \mu + 4\, C^2\, t\, \varepsilon\, \mu\, z + 4\, C^2\, t\, \varepsilon\, \mu\, z^2 - t\, x^2 - t\, y^2 + 2\, C\, t\, z + 4\, C\, t\, z^2 \right) \right]$$

$$\mathit{Dissipation} \ = -C \left(2\, \varepsilon + C\, t\, \varepsilon\, \mu + 2\, C\, t\, \varepsilon\, \mu\, z - x\, z - x\, z^2 \right)$$

*****************************   *END PROCEDURE*   *****************************   **(8)**

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

p.155 vol4

```
> NAME:=`Example EVANS B# p155 vol4`;
```

```
> Ax:=y*cos(-kappa*z+omega*t)/(0+x^2+y^2+1*z^2);Ay:=-x*cos(-kappa*z+omega*t)/(0+x^2+y^2+1*z^2);Az:=0*kappa*cos(-kappa*z+omega*t);phi:=0*omega*cos(-kappa*z+omega*t);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0);
```

****

$$\mathit{NAME} := \mathit{Example\ EVANS\ B\#\ p155\ vol4}$$

$$\mathit{Ax} := \frac{y \cos(-\kappa\, z + \omega\, t)}{x^2 + y^2 + z^2}$$

$$\mathit{Ay} := -\frac{x \cos(-\kappa\, z + \omega\, t)}{x^2 + y^2 + z^2}$$

$$\mathit{Az} := 0$$

$$\phi := 0$$

Example EVANS B# p155 vol4

********************************   *Differential Form Format*   ********************************

$$\mathit{Action\ 1\text{-}form} = \frac{y \cos(-\kappa\, z + \omega\, t)\, d(x)}{x^2 + y^2 + z^2} - \frac{x \cos(-\kappa\, z + \omega\, t)\, d(y)}{x^2 + y^2 + z^2}$$

$$\mathit{Intensity\ 2\text{-}form\ F{=}dA} \ = \frac{y \sin(-\kappa\, z + \omega\, t)\, \omega\, (d(x))\ \&\wedge\ (d(t))}{x^2 + y^2 + z^2} + \left(-\frac{\cos(-\kappa\, z + \omega\, t)\, (x^2 - y^2 + z^2)}{(x^2 + y^2 + z^2)^2} + \frac{\cos(-\kappa\, z + \omega\, t)\, (x^2 - y^2 - z^2)}{(x^2 + y^2 + z^2)^2} \right) (d(x))\ \&\wedge\ (d(y)) - \frac{1}{(x^2 + y^2 + z^2)^2} \left(x \left(-\sin(-\kappa\, z + \omega\, t)\, \kappa\, x^2 - \sin(-\kappa\, z + \omega\, t)\, \kappa\, y^2 - \sin(-\kappa\, z \right.\right.$$

$+\omega t)\ \kappa z^2 + 2\cos(-\kappa z+\omega t)\ z)\ (d(y))\ \&\wedge\ (d(z))\big) + \frac{1}{(x^2+y^2+z^2)^2}\big(y\ (-\sin($

$-\kappa z+\omega t)\ \kappa x^2 - \sin(-\kappa z+\omega t)\ \kappa y^2 - \sin(-\kappa z+\omega t)\ \kappa z^2 + 2\cos(-\kappa z+\omega t)\ z)$

$(d(x))\ \&\wedge\ (d(z))\big) - \frac{x\sin(-\kappa z+\omega t)\ \omega\ (d(y))\ \&\wedge\ (d(t))}{x^2+y^2+z^2}$

Topological Torsion 3-form $A\wedge F = 0$

Topological Parity 4-form $F\wedge F = 0$

******************************** *Using EM format* ********************************

$$E\ field = \left[\frac{y\sin(-\kappa z+\omega t)\ \omega}{x^2+y^2+z^2}, -\frac{x\sin(-\kappa z+\omega t)\ \omega}{x^2+y^2+z^2}, 0\right]$$

$$B\ field = \left[-\frac{1}{(x^2+y^2+z^2)^2}\big(x\ (-\sin(-\kappa z+\omega t)\ \kappa x^2 - \sin(-\kappa z+\omega t)\ \kappa y^2 - \sin(-\kappa z+\omega t)\ \kappa z^2 + 2\cos(-\kappa z+\omega t)\ z)\big), -\frac{1}{(x^2+y^2+z^2)^2}\big(y\ (-\sin(-\kappa z+\omega t)\ \kappa x^2 - \sin(-\kappa z+\omega t)\ \kappa y^2 - \sin(-\kappa z+\omega t)\ \kappa z^2 + 2\cos(-\kappa z+\omega t)\ z)\big), -\frac{2\cos(-\kappa z+\omega t)\ z^2}{(x^2+y^2+z^2)^2}\right]$$

Topological TORSION 4 vector *T4 = -[ExA + Bphi,AdotB]* $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension *PTD* $= 2$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= -\frac{\cos(-\kappa z+\omega t)^2\ (x^2+y^2-z^2)}{(x^2+y^2+z^2)^3}$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $= 0$

$$D\ field = \left[\frac{\varepsilon\, y \sin(-\kappa z + \omega t)\, \omega}{x^2 + y^2 + z^2},\ -\frac{\varepsilon\, x \sin(-\kappa z + \omega t)\, \omega}{x^2 + y^2 + z^2},\ 0 \right]$$

$$H\ field = \left[-\frac{1}{(x^2 + y^2 + z^2)^2\, \mu} \left(x \left(-\sin(-\kappa z + \omega t)\, \kappa x^2 - \sin(-\kappa z + \omega t)\, \kappa y^2 - \sin(-\kappa z + \omega t)\, \kappa z^2 + 2 \cos(-\kappa z + \omega t)\, z \right) \right),\ -\frac{1}{(x^2 + y^2 + z^2)^2\, \mu} \left(y \left(-\sin(-\kappa z + \omega t)\, \kappa x^2 - \sin(-\kappa z + \omega t)\, \kappa y^2 - \sin(-\kappa z + \omega t)\, \kappa z^2 + 2 \cos(-\kappa z + \omega t)\, z \right) \right),\ -\frac{2 \cos(-\kappa z + \omega t)\, z^2}{(x^2 + y^2 + z^2)^2\, \mu} \right]$$

$$Poynting\ vector\ ExH = \left[\frac{2\, x \sin(-\kappa z + \omega t)\, \omega \cos(-\kappa z + \omega t)\, z^2}{(x^2 + y^2 + z^2)^3\, \mu},\ \frac{2\, y \sin(-\kappa z + \omega t)\, \omega \cos(-\kappa z + \omega t)\, z^2}{(x^2 + y^2 + z^2)^3\, \mu},\ -\frac{1}{\mu\, (x^2 + y^2 + z^2)^3} \left(\sin(-\kappa z + \omega t)\, \omega \left(-\sin(-\kappa z + \omega t)\, \kappa x^2 - \sin(-\kappa z + \omega t)\, \kappa y^2 - \sin(-\kappa z + \omega t)\, \kappa z^2 + 2 \cos(-\kappa z + \omega t)\, z \right) (x^2 + y^2) \right) \right]$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt=J4 \quad = \left[-\frac{1}{(x^2 + y^2 + z^2)^2\, \mu} \left(y \left(-\cos(-\kappa z + \omega t)\, \kappa^2 x^2 - 4 \sin(-\kappa z + \omega t)\, \kappa z - \cos(-\kappa z + \omega t)\, \kappa^2 z^2 - \cos(-\kappa z + \omega t)\, \kappa^2 y^2 - 2 \cos(-\kappa z + \omega t) + \varepsilon \cos(-\kappa z + \omega t)\, \omega^2 \mu x^2 + \varepsilon \cos(-\kappa z + \omega t)\, \omega^2 \mu y^2 + \varepsilon \cos(-\kappa z + \omega t)\, \omega^2 \mu z^2 \right) \right),\ \frac{1}{(x^2 + y^2 + z^2)^2\, \mu} \left(x \left(-\cos(-\kappa z + \omega t)\, \kappa^2 x^2 - 4 \sin(-\kappa z + \omega t)\, \kappa z - \cos(-\kappa z + \omega t)\, \kappa^2 z^2 - \cos(-\kappa z + \omega t)\, \kappa^2 y^2 - 2 \cos(-\kappa z + \omega t) + \varepsilon \cos(-\kappa z + \omega t)\, \omega^2 \mu x^2 + \varepsilon \cos(-\kappa z + \omega t)\, \omega^2 \mu y^2 + \varepsilon \cos(-\kappa z + \omega t)\, \omega^2 \mu z^2 \right) \right),\ 0,\ 0 \right]$$

Amerian charge density $\quad divD = rho = 0$

divergence Lorentz Current 4Vector, $\quad 4div(J4) = 0$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[\frac{2\, x \cos(-\kappa z + \omega t)^2\, z^2}{(x^2 + y^2 + z^2)^3\, \mu},\ \frac{2\, y \cos(-\kappa z + \omega t)^2\, z^2}{(x^2 + y^2 + z^2)^3\, \mu}, \right.$$

$$-\frac{1}{\mu\,(x^2+y^2+z^2)^3}\Big(\cos(-\kappa z+\omega t)\,\big(-\sin(-\kappa z+\omega t)\,\kappa x^2-\sin(-\kappa z+\omega t)\,\kappa y^2$$

$$-\sin(-\kappa z+\omega t)\,\kappa z^2+2\cos(-\kappa z+\omega t)\,z\big)\,(x^2+y^2)\Big),$$

$$\left.\frac{\cos(-\kappa z+\omega t)\,\varepsilon\sin(-\kappa z+\omega t)\,\omega\,(x^2+y^2)}{(x^2+y^2+z^2)^2}\right]$$

$$\textit{Topological SPIN 3-form} = \frac{2\,x\cos(-\kappa z+\omega t)^2\,z^2\ \&\wedge(d(y),d(z),d(t))}{(x^2+y^2+z^2)^3\,\mu}$$

$$-\frac{2\,y\cos(-\kappa z+\omega t)^2\,z^2\ \&\wedge(d(x),d(z),d(t))}{(x^2+y^2+z^2)^3\,\mu}-\frac{1}{\mu\,(x^2+y^2+z^2)^3}\Big(\cos(-\kappa z$$

$$+\omega t)\,\big(-\sin(-\kappa z+\omega t)\,\kappa x^2-\sin(-\kappa z+\omega t)\,\kappa y^2-\sin(-\kappa z+\omega t)\,\kappa z^2+2\cos(-\kappa z$$

$$+\omega t)\,z\big)\,(x^2+y^2)\ \&\wedge(d(x),d(y),d(t))\Big)$$

$$-\frac{\cos(-\kappa z+\omega t)\,\varepsilon\sin(-\kappa z+\omega t)\,\omega\,(x^2+y^2)\ \&\wedge(d(x),d(y),d(z))}{(x^2+y^2+z^2)^2}$$

$$\textit{Spin density rho\_spin} = \frac{\cos(-\kappa z+\omega t)\,\varepsilon\sin(-\kappa z+\omega t)\,\omega\,(x^2+y^2)}{(x^2+y^2+z^2)^2}$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{1}{\mu\,(x^2+y^2+z^2)^3}\Big(\sin(-\kappa z+\omega t)^2\,\kappa^2\,x^4+2\sin($$

$$-\kappa z+\omega t)^2\,\kappa^2\,x^2\,y^2+\sin(-\kappa z+\omega t)^2\,\kappa^2\,x^2\,z^2-4\sin(-\kappa z+\omega t)\,\kappa x^2\cos(-\kappa z+\omega t)\,z$$

$$+\sin(-\kappa z+\omega t)^2\,\kappa^2\,y^2\,z^2+\sin(-\kappa z+\omega t)^2\,\kappa^2\,y^4+4\cos(-\kappa z+\omega t)^2\,z^2-4\sin(-\kappa z$$

$$+\omega t)\,\kappa y^2\cos(-\kappa z+\omega t)\,z-\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\,\mu\,x^4-2\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\,\mu\,x^2\,y^2$$

$$-\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\,\mu\,x^2\,z^2-\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\,\mu\,y^4-\varepsilon\sin(-\kappa z$$

$$+\omega t)^2\,\omega^2\,\mu\,y^2\,z^2\Big)$$

$$B.H = \frac{1}{\mu\,(x^2+y^2+z^2)^3}\Big(\sin(-\kappa z+\omega t)^2\,\kappa^2\,x^4+2\sin(-\kappa z+\omega t)^2\,\kappa^2\,x^2\,y^2+\sin(-\kappa z$$

$$+\omega t)^2\,\kappa^2\,x^2\,z^2-4\sin(-\kappa z+\omega t)\,\kappa x^2\cos(-\kappa z+\omega t)\,z+\sin(-\kappa z+\omega t)^2\,\kappa^2\,y^2\,z^2$$

$$+\sin(-\kappa z+\omega t)^2\,\kappa^2\,y^4+4\cos(-\kappa z+\omega t)^2\,z^2-4\sin(-\kappa z+\omega t)\,\kappa y^2\cos(-\kappa z$$

$$+\omega t)\,z\Big)$$

$$D.E = \frac{\varepsilon \sin(-\kappa z + \omega t)^2 \omega^2 (x^2 + y^2)}{(x^2 + y^2 + z^2)^2}$$

$$A.J = -\frac{1}{\mu (x^2 + y^2 + z^2)^3} \left(\cos(-\kappa z + \omega t) \left(-\cos(-\kappa z + \omega t) \kappa^2 x^2 - 4 \sin(-\kappa z + \omega t) \kappa z - \cos(-\kappa z + \omega t) \kappa^2 z^2 - \cos(-\kappa z + \omega t) \kappa^2 y^2 - 2 \cos(-\kappa z + \omega t) + \varepsilon \cos(-\kappa z + \omega t) \omega^2 \mu x^2 + \varepsilon \cos(-\kappa z + \omega t) \omega^2 \mu y^2 + \varepsilon \cos(-\kappa z + \omega t) \omega^2 \mu z^2\right) (x^2 + y^2)\right)$$

$$\text{-}rho.phi = 0$$

Poincare I $\quad (B.H - D.E)\text{-}(A.J - rho.phi) = \frac{1}{\mu (x^2 + y^2 + z^2)^3} \big(\sin(-\kappa z + \omega t)^2 \kappa^2 x^4 + \sin(-\kappa z + \omega t)^2 \kappa^2 y^4 - 2 x^2 \cos(-\kappa z + \omega t)^2 - 2 y^2 \cos(-\kappa z + \omega t)^2 - \varepsilon \sin(-\kappa z + \omega t)^2 \omega^2 \mu x^4 - \varepsilon \sin(-\kappa z + \omega t)^2 \omega^2 \mu y^4 + \varepsilon \cos(-\kappa z + \omega t)^2 \omega^2 \mu x^4 + \varepsilon \cos(-\kappa z + \omega t)^2 \omega^2 \mu y^4 - \cos(-\kappa z + \omega t)^2 x^4 \kappa^2 - \cos(-\kappa z + \omega t)^2 y^4 \kappa^2 - 8 \sin(-\kappa z + \omega t) \kappa x^2 \cos(-\kappa z + \omega t) z - 8 \sin(-\kappa z + \omega t) \kappa y^2 \cos(-\kappa z + \omega t) z - 2 \varepsilon \sin(-\kappa z + \omega t)^2 \omega^2 \mu x^2 y^2 - \varepsilon \sin(-\kappa z + \omega t)^2 \omega^2 \mu x^2 z^2 - \varepsilon \sin(-\kappa z + \omega t)^2 \omega^2 \mu y^2 z^2 + 2 \varepsilon \cos(-\kappa z + \omega t)^2 \omega^2 \mu x^2 y^2 + \varepsilon \cos(-\kappa z + \omega t)^2 \omega^2 \mu z^2 x^2 + \varepsilon \cos(-\kappa z + \omega t)^2 \omega^2 \mu z^2 y^2 + 2 \sin(-\kappa z + \omega t)^2 \kappa^2 x^2 y^2 + \sin(-\kappa z + \omega t)^2 \kappa^2 x^2 z^2 + \sin(-\kappa z + \omega t)^2 \kappa^2 y^2 z^2 - 2 \cos(-\kappa z + \omega t)^2 x^2 \kappa^2 y^2 - x^2 \cos(-\kappa z + \omega t)^2 \kappa^2 z^2 - \cos(-\kappa z + \omega t)^2 \kappa^2 y^2 z^2 + 4 \cos(-\kappa z + \omega t)^2 z^2\big)$

London Coefficient $\quad LC = -\frac{1}{(x^2 + y^2 + z^2) \mu \cos(-\kappa z + \omega t)} \big(-\cos(-\kappa z + \omega t) \kappa^2 x^2 - 4 \sin(-\kappa z + \omega t) \kappa z - \cos(-\kappa z + \omega t) \kappa^2 z^2 - \cos(-\kappa z + \omega t) \kappa^2 y^2 - 2 \cos(-\kappa z + \omega t) + \varepsilon \cos(-\kappa z + \omega t) \omega^2 \mu x^2 + \varepsilon \cos(-\kappa z + \omega t) \omega^2 \mu y^2 + \varepsilon \cos(-\kappa z + \omega t) \omega^2 \mu z^2\big)$

PROCA coefficient curlcurlB $= \Big[-\frac{1}{(x^2 + y^2 + z^2)^3} \big(x \big(-\sin(-\kappa z + \omega t) \kappa^3 x^4 + 6 x^2 \cos(-\kappa z + \omega t) \kappa^2 z - 2 x^2 \sin(-\kappa z + \omega t) \kappa^3 z^2 - 2 x^2 \sin(-\kappa z + \omega t) \kappa^3 y^2 - 6 \sin(-\kappa z$

$+ \omega t) \kappa x^2 - 2 \sin(-\kappa z + \omega t) \kappa^3 y^2 z^2 + 10 \sin(-\kappa z + \omega t) \kappa z^2 + 6 \cos(-\kappa z$

$+\omega t)\ \kappa^2 z y^2 + 8\cos(-\kappa z + \omega t)\ z - \sin(-\kappa z + \omega t)\ \kappa^3 y^4 - \sin(-\kappa z + \omega t)\ \kappa^3 z^4$

$-6\sin(-\kappa z + \omega t)\ \kappa y^2 + 6\cos(-\kappa z + \omega t)\ \kappa^2 z^3)),\ -\frac{1}{(x^2+y^2+z^2)^3}\left(y\left(-\sin(\right.\right.$

$-\kappa z + \omega t)\ \kappa^3 x^4 + 6x^2\cos(-\kappa z + \omega t)\ \kappa^2 z - 2x^2\sin(-\kappa z + \omega t)\ \kappa^3 z^2 - 2x^2\sin(-\kappa z$

$+\omega t)\ \kappa^3 y^2 - 6\sin(-\kappa z + \omega t)\ \kappa x^2 - 2\sin(-\kappa z + \omega t)\ \kappa^3 y^2 z^2 + 10\sin(-\kappa z + \omega t)\ \kappa z^2$

$+6\cos(-\kappa z + \omega t)\ \kappa^2 z y^2 + 8\cos(-\kappa z + \omega t)\ z - \sin(-\kappa z + \omega t)\ \kappa^3 y^4 - \sin(-\kappa z$

$+\omega t)\ \kappa^3 z^4 - 6\sin(-\kappa z + \omega t)\ \kappa y^2 + 6\cos(-\kappa z + \omega t)\ \kappa^2 z^3)),$

$-\frac{1}{(x^2+y^2+z^2)^3}\left(2\left(x^2\cos(-\kappa z + \omega t)\ \kappa^2 z^2 - 4x^2\sin(-\kappa z + \omega t)\ \kappa z - 2x^2\cos(-\kappa z\right.\right.$

$+\omega t) - 2y^2\cos(-\kappa z + \omega t) + \cos(-\kappa z + \omega t)\ \kappa^2 z^4 + \cos(-\kappa z + \omega t)\ \kappa^2 y^2 z^2$

$-4y^2\sin(-\kappa z + \omega t)\ \kappa z + 4z^3\kappa\sin(-\kappa z + \omega t) + 2\cos(-\kappa z + \omega t)\ z^2))]$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \left[-\frac{1}{(x^2+y^2+z^2)^2\mu}\left(y\left(-\cos(-\kappa z\right.\right.\right.$

$+\omega t)\ \kappa^2 x^2 - 4\sin(-\kappa z + \omega t)\ \kappa z - \cos(-\kappa z + \omega t)\ \kappa^2 z^2 - \cos(-\kappa z + \omega t)\ \kappa^2 y^2$

$-2\cos(-\kappa z + \omega t) + \varepsilon\cos(-\kappa z + \omega t)\ \omega^2\mu x^2 + \varepsilon\cos(-\kappa z + \omega t)\ \omega^2\mu y^2 + \varepsilon\cos(-\kappa z$

$+\omega t)\ \omega^2\mu z^2)),\ \frac{1}{(x^2+y^2+z^2)^2\mu}\left(x\left(-\cos(-\kappa z + \omega t)\ \kappa^2 x^2 - 4\sin(-\kappa z\right.\right.$

$+\omega t)\ \kappa z - \cos(-\kappa z + \omega t)\ \kappa^2 z^2 - \cos(-\kappa z + \omega t)\ \kappa^2 y^2 - 2\cos(-\kappa z + \omega t) + \varepsilon\cos($

$-\kappa z + \omega t)\ \omega^2\mu x^2 + \varepsilon\cos(-\kappa z + \omega t)\ \omega^2\mu y^2 + \varepsilon\cos(-\kappa z + \omega t)\ \omega^2\mu z^2)),\ 0,\ 0]$

Lorentz Force 3 vector due to Ampere current *FL = -(rho_ampere E + J_ampere x B)*

$= \left[\frac{1}{(x^2+y^2+z^2)^4\mu}\left(2x\left(-\cos(-\kappa z + \omega t)\ \kappa^2 x^2 - 4\sin(-\kappa z + \omega t)\ \kappa z - \cos(-\kappa z\right.\right.\right.$

$$+ \omega t)\ \kappa^2 z^2 - \cos(-\kappa z + \omega t)\ \kappa^2 y^2 - 2\cos(-\kappa z + \omega t) + \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu x^2$$

$$+ \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu y^2 + \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu z^2)\ \cos(-\kappa z + \omega t)\ z^2),$$

$$\frac{1}{(x^2+y^2+z^2)^4 \mu} (2\, y\, (-\cos(-\kappa z + \omega t)\ \kappa^2 x^2 - 4\sin(-\kappa z + \omega t)\ \kappa z - \cos(-\kappa z$$

$$+ \omega t)\ \kappa^2 z^2 - \cos(-\kappa z + \omega t)\ \kappa^2 y^2 - 2\cos(-\kappa z + \omega t) + \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu x^2$$

$$+ \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu y^2 + \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu z^2)\ \cos(-\kappa z + \omega t)\ z^2),$$

$$-\frac{1}{(x^2+y^2+z^2)^4 \mu} ((-\cos(-\kappa z + \omega t)\ \kappa^2 x^2 - 4\sin(-\kappa z + \omega t)\ \kappa z - \cos(-\kappa z$$

$$+ \omega t)\ \kappa^2 z^2 - \cos(-\kappa z + \omega t)\ \kappa^2 y^2 - 2\cos(-\kappa z + \omega t) + \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu x^2$$

$$+ \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu y^2 + \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu z^2)\ (-\sin(-\kappa z + \omega t)\ \kappa x^2 - \sin($$

$$-\kappa z + \omega t)\ \kappa y^2 - \sin(-\kappa z + \omega t)\ \kappa z^2 + 2\cos(-\kappa z + \omega t)\ z)\ (x^2 + y^2)))]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= (\cos(-\kappa z + \omega t)\ (x^2+y^2+z^2)) \Big/ (-\cos(-\kappa z$
$+ \omega t)\ \kappa^2 x^2 - 4\sin(-\kappa z + \omega t)\ \kappa z - \cos(-\kappa z + \omega t)\ \kappa^2 z^2 - \cos(-\kappa z + \omega t)\ \kappa^2 y^2$
$- 2\cos(-\kappa z + \omega t) + \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu x^2 + \varepsilon \cos(-\kappa z + \omega t)\ \omega^2 \mu y^2 + \varepsilon \cos(-\kappa z$
$+ \omega t)\ \omega^2 \mu z^2)$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] $= \Bigg[\frac{2\, x \cos(-\kappa z + \omega t)^2 z^2}{(x^2+y^2+z^2)^3 \mu},$

$$\frac{2\, y \cos(-\kappa z + \omega t)^2 z^2}{(x^2+y^2+z^2)^3 \mu}, -\frac{1}{\mu\,(x^2+y^2+z^2)^3} (\cos(-\kappa z + \omega t)\ (-\sin(-\kappa z + \omega t)\ \kappa x^2$$

$$- \sin(-\kappa z+\omega t)\ \kappa y^2 - \sin(-\kappa z+\omega t)\ \kappa z^2 + 2\cos(-\kappa z+\omega t)\ z)\ (x^2+y^2)),$$

$$\left.\frac{\cos(-\kappa z+\omega t)\ \varepsilon \sin(-\kappa z+\omega t)\ \omega\ (x^2+y^2)}{(x^2+y^2+z^2)^2}\right]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B)

$$= \left[\frac{1}{(x^2+y^2+z^2)^4\,\mu}\left(\cos(-\kappa z+\omega t)\ y\ (\sin(-\kappa z+\omega t)^2\,\kappa^2 x^4 + 2\sin(-\kappa z\right.\right.$$

$$+\omega t)^2\,\kappa^2 x^2 y^2 + \sin(-\kappa z+\omega t)^2\,\kappa^2 x^2 z^2 - 4\sin(-\kappa z+\omega t)\ \kappa x^2 \cos(-\kappa z+\omega t)\ z$$

$$+\sin(-\kappa z+\omega t)^2\,\kappa^2 y^2 z^2 + \sin(-\kappa z+\omega t)^2\,\kappa^2 y^4 + 4\cos(-\kappa z+\omega t)^2 z^2 - 4\sin(-\kappa z$$

$$+\omega t)\ \kappa y^2\cos(-\kappa z+\omega t)\ z - \varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\mu x^4 - 2\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\mu x^2 y^2$$

$$-\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\mu x^2 z^2 - \varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\mu y^4 - \varepsilon\sin(-\kappa z$$

$$+\omega t)^2\,\omega^2\mu y^2 z^2)),\ -\frac{1}{(x^2+y^2+z^2)^4\,\mu}\left(x\cos(-\kappa z+\omega t)\ (\sin(-\kappa z+\omega t)^2\,\kappa^2 x^4\right.$$

$$+2\sin(-\kappa z+\omega t)^2\,\kappa^2 x^2 y^2 + \sin(-\kappa z+\omega t)^2\,\kappa^2 x^2 z^2 - 4\sin(-\kappa z+\omega t)\ \kappa x^2\cos(-\kappa z$$

$$+\omega t)\ z + \sin(-\kappa z+\omega t)^2\,\kappa^2 y^2 z^2 + \sin(-\kappa z+\omega t)^2\,\kappa^2 y^4 + 4\cos(-\kappa z+\omega t)^2 z^2$$

$$-4\sin(-\kappa z+\omega t)\ \kappa y^2\cos(-\kappa z+\omega t)\ z - \varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\mu x^4 - 2\,\varepsilon\sin(-\kappa z$$

$$+\omega t)^2\,\omega^2\mu x^2 y^2 - \varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\mu x^2 z^2 - \varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\mu y^4 - \varepsilon\sin($$

$$\left.-\kappa z+\omega t)^2\,\omega^2\mu y^2 z^2)),\ 0\right]$$

Spin Dissipation J_spin dot E $=0$

Disspative Force 3 vector $= \left[\frac{1}{(x^2+y^2+z^2)^4\,\mu}\left(\cos(-\kappa z+\omega t)\ (-2\,x^3 z^2\cos(-\kappa z+\omega t)\ \kappa^2\right.\right.$

$$-8\,x z^3\,\kappa\sin(-\kappa z+\omega t) - 2\,x z^4\cos(-\kappa z+\omega t)\ \kappa^2 - 2\,x z^2\cos(-\kappa z+\omega t)\ \kappa^2 y^2$$

$$-4\cos(-\kappa z+\omega t)\ z^2 x + 2\,x^3 z^2\,\varepsilon\cos(-\kappa z+\omega t)\ \omega^2\mu + 2\,x z^2\,\varepsilon\cos(-\kappa z+\omega t)\ \omega^2\mu y^2$$

$$+2\,x\,z^4\,\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\,\mu+\mu\,y\sin(-\kappa z+\omega t)^2\,\kappa^2\,x^4+2\,\mu\,y^3\sin(-\kappa z+\omega t)^2\,\kappa^2\,x^2$$

$$+\mu\,y\sin(-\kappa z+\omega t)^2\,\kappa^2\,x^2\,z^2-4\,\mu\,y\sin(-\kappa z+\omega t)\,\kappa\,x^2\cos(-\kappa z+\omega t)\,z$$

$$+\mu\,y^3\sin(-\kappa z+\omega t)^2\,\kappa^2\,z^2+\mu\,y^5\sin(-\kappa z+\omega t)^2\,\kappa^2+4\,\mu\,y\cos(-\kappa z+\omega t)^2\,z^2$$

$$-4\,\mu\,y^3\sin(-\kappa z+\omega t)\,\kappa\cos(-\kappa z+\omega t)\,z-\mu^2\,y\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\,x^4$$

$$-2\,\mu^2\,y^3\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\,x^2-\mu^2\,y\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\,x^2\,z^2-\mu^2\,y^5\,\varepsilon\sin(-\kappa z$$

$$+\omega t)^2\,\omega^2-\mu^2\,y^3\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\,z^2)),\ -\frac{1}{(x^2+y^2+z^2)^4\,\mu}\Big(\cos(-\kappa z$$

$$+\omega t)\,\big(2\,y\,z^2\cos(-\kappa z+\omega t)\,\kappa^2\,x^2+8\,y\,z^3\,\kappa\sin(-\kappa z+\omega t)+2\,y\,z^4\cos(-\kappa z+\omega t)\,\kappa^2$$

$$+2\,y^3\,z^2\cos(-\kappa z+\omega t)\,\kappa^2+4\cos(-\kappa z+\omega t)\,z^2\,y-2\,y\,z^2\,\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\,\mu\,x^2$$

$$-2\,y^3\,z^2\,\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\,\mu-2\,y\,z^4\,\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\,\mu+\mu\,x^5\sin(-\kappa z+\omega t)^2\,\kappa^2$$

$$+2\,\mu\,x^3\sin(-\kappa z+\omega t)^2\,\kappa^2\,y^2+\mu\,x^3\sin(-\kappa z+\omega t)^2\,\kappa^2\,z^2-4\,\mu\,x^3\sin(-\kappa z$$

$$+\omega t)\,\kappa\cos(-\kappa z+\omega t)\,z+\mu\,x\sin(-\kappa z+\omega t)^2\,\kappa^2\,y^2\,z^2+\mu\,x\sin(-\kappa z+\omega t)^2\,\kappa^2\,y^4$$

$$+4\,\mu\,x\cos(-\kappa z+\omega t)^2\,z^2-4\,\mu\,x\sin(-\kappa z+\omega t)\,\kappa\,y^2\cos(-\kappa z+\omega t)\,z$$

$$-\mu^2\,x^5\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2-2\,\mu^2\,x^3\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\,y^2-\mu^2\,x^3\,\varepsilon\sin(-\kappa z$$

$$+\omega t)^2\,\omega^2\,z^2-\mu^2\,x\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\,y^4-\mu^2\,x\,\varepsilon\sin(-\kappa z+\omega t)^2\,\omega^2\,y^2\,z^2)),$$

$$-\frac{1}{(x^2+y^2+z^2)^4\mu}\Big(\big(-\cos(-\kappa z+\omega t)\,\kappa^2 x^2-4\sin(-\kappa z+\omega t)\,\kappa z-\cos(-\kappa z+\omega t)\,\kappa^2 z^2-\cos(-\kappa z+\omega t)\,\kappa^2 y^2-2\cos(-\kappa z+\omega t)+\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\mu x^2+\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\mu y^2+\varepsilon\cos(-\kappa z+\omega t)\,\omega^2\mu z^2\big)\big(-\sin(-\kappa z+\omega t)\,\kappa x^2-\sin(-\kappa z+\omega t)\,\kappa y^2-\sin(-\kappa z+\omega t)\,\kappa z^2+2\cos(-\kappa z+\omega t)\,z\big)\,(x^2+y^2)\Big)\Big]$$

$$Dissipation\ =\frac{\mu\cos(-\kappa z+\omega t)\,\varepsilon\sin(-\kappa z+\omega t)\,\omega\,(x^2+y^2)}{(x^2+y^2+z^2)^2}$$

***************************** *END PROCEDURE* ***************************** **(9)**

```
> EH:=crossprod(E,HF):EH[1];EH[2];factor(EH[3]);
```

$$\frac{2x\sin(-\kappa z+\omega t)\,\omega\cos(-\kappa z+\omega t)\,z^2}{(x^2+y^2+z^2)^3\mu}$$

$$\frac{2y\sin(-\kappa z+\omega t)\,\omega\cos(-\kappa z+\omega t)\,z^2}{(x^2+y^2+z^2)^3\mu}$$

$$-\frac{1}{\mu(x^2+y^2+z^2)^3}\Big(\sin(-\kappa z+\omega t)\,\omega\,\big(-\sin(-\kappa z+\omega t)\,\kappa x^2-\sin(-\kappa z+\omega t)\,\kappa y^2-\sin(-\kappa z+\omega t)\,\kappa z^2+2\cos(-\kappa z+\omega t)\,z\big)\,(x^2+y^2)\Big) \quad \textbf{(10)}$$

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.
p=2, n=2, euclidean signature index 0.   p160,vol4

```
> NAME:=`Example A-- Hopf signature index 0 or 1. The 1-form is divided by the
  Holder norm any p, n=2  EdotB =0 `;
```

```
> Ax:=y*1*Omega;Ay:=-x*Omega;Az:=C*t;phi:=z*C;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,-1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME :=

Example A-- Hopf signature index 0 or 1. The 1-form is divided by the Holder norm any p, n=2 EdotB =0

$$Ax := y\,\Omega$$

$$Ay := -x\,\Omega$$

$$Az := C\,t$$

$$\phi := C\,z$$

Example A-- Hopf signature index 0 or 1. The 1-form is divided by the Holder norm any p, n=2 EdotB =0

*********************************** *Differential Form Format* *********************************

$$\textit{Action 1-form} = y\,\Omega\,d(x) - x\,\Omega\,d(y) + C\,t\,d(z) - C\,z\,d(t)$$

$$\textit{Intensity 2-form F=dA} \;= -2\,\Omega\,(d(x))\;\&\hat{}\;(d(y)) - 2\,C\,(d(z))\;\&\hat{}\;(d(t))$$

$$\textit{Topological Torsion 3-form} \quad A\hat{}F = -2\,C\,t\,\Omega\,\&\hat{}(d(x), d(y), d(z)) + 2\,C\,z\,\Omega\,\&\hat{}(d(x), d(y), d(t)) - 2\,y\,\Omega\,C\,\&\hat{}(d(x), d(z), d(t)) + 2\,x\,\Omega\,C\,\&\hat{}(d(y), d(z), d(t))$$

$$\textit{Topological Parity 4-form} \quad F\hat{}F = 8\,\Omega\,C\,\&\hat{}(d(x), d(y), d(z), d(t))$$

*********************************** *Using EM format* ********************************

$$\textit{E field} = [0, 0, -2\,C]$$

$$\textit{B field} = [0, 0, -2\,\Omega]$$

$$\textit{Topological TORSION 4 vector} \quad T4 = -[ExA + Bphi, AdotB] = [2\,x\,\Omega\,C, 2\,y\,\Omega\,C, 2\,C\,z\,\Omega, 2\,C\,t\,\Omega]$$

$$\textit{Helicity AdotB} = -2\,C\,t\,\Omega$$

$$\textit{Poincare II} = 2(E.B) = 8\,\Omega\,C$$

$$\textit{coefficient of Topological Parity 4-form} = 8\,\Omega\,C$$

$$\textit{Pfaff Topological Dimension} \quad PTD = 4$$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

$$\textit{Xm or linear (Mean) curvature} = 0$$

$$\textit{Yg or quadratic (GAUSS) curvature} = \Omega^2 + C^2$$

$$\textit{Za or Cubic (Interaction internal energy) curvature} = 0$$

$$\textit{Tk or quartic (4D expansion) curvature} = \Omega^2\,C^2$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\textit{Chirality factor CH} = 0$$

$$\textit{D field} = [0, 0, -2\,\varepsilon\,C]$$

$$\textit{H field} = \left[0, 0, -\frac{2\,\Omega}{\mu}\right]$$

Poynting vector ExH = *EXH*

Amperian Current 4Vector curlH-dD/dt=J4 = [0, 0, 0, 0]

Amerian charge density divD = rho = 0

divergence Lorentz Current 4Vector, 4div(J4) = 0

$$\text{Topological SPIN 4 vector S4} = \left[\frac{2\, x\, \Omega^2}{\mu}, \frac{2\, y\, \Omega^2}{\mu}, -2\, \varepsilon\, C^2 z, -2\, C^2\, t\, \varepsilon \right]$$

$$\text{Topological SPIN 3-form} = \frac{2\, x\, \Omega^2\, \&\wedge(d(y), d(z), d(t))}{\mu} - \frac{2\, y\, \Omega^2\, \&\wedge(d(x), d(z), d(t))}{\mu}$$
$$- 2\, \varepsilon\, C^2\, z\, \&\wedge(d(x), d(y), d(t)) + 2\, C^2\, t\, \varepsilon\, \&\wedge(d(x), d(y), d(z))$$

$$\text{Spin density rho\_spin} = -2\, C^2\, t\, \varepsilon$$

$$\text{LaGrange field energy density (B.H-D.E)} = -\frac{4\left(-\Omega^2 + \varepsilon\, C^2\, \mu\right)}{\mu}$$

$$B.H = \frac{4\, \Omega^2}{\mu}$$

$$D.E = 4\, \varepsilon\, C^2$$

A.J = 0

-rho.phi = 0

$$\text{Poincare I} \quad \text{(B.H - D.E)-(A.J - rho.phi)} = -\frac{4\left(-\Omega^2 + \varepsilon\, C^2\, \mu\right)}{\mu}$$

London Coefficient LC = 0

PROCA coefficient curlcurlB = [0, 0, 0]

Amperian Current 4Vector curlH-dD/dt=J4 = [0, 0, 0, 0]

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) = [0, 0, 0]

Amperian Dissipation Jampere dot E = 0

Lorentz Force Spin factor LFSPIN = 0

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] = $[2\, x\, \Omega\, C, 2\, y\, \Omega\, C, 2\, C\, z\, \Omega, 2\, C\, t\, \Omega]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) = $[-4\, y\, \Omega^2\, C, 4\, x\, \Omega^2\, C, -4\, C^2\, t\, \Omega]$

Torsion Dissipation Jtorsion dot E = $4\, C^2\, z\, \Omega$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] $= \left[\frac{2\,x\,\Omega^2}{\mu}, \frac{2\,y\,\Omega^2}{\mu}, -2\,\varepsilon\,C^2\,z, -2\,C^2\,t\,\varepsilon\right]$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $= \left[\frac{4\,y\,\Omega^3}{\mu}, -\frac{4\,x\,\Omega^3}{\mu}, -4\,C^3\,t\,\varepsilon\right]$

Spin Dissipation J_spin dot E $= 4\,\varepsilon\,C^3\,z$

Disspative Force 3 vector $= \left[-4\,y\,\Omega^2\,(-\Omega + C), 4\,x\,\Omega^2\,(-\Omega + C), -4\,C^2\,t\,(C\,\varepsilon\,\mu + \Omega)\right]$

Dissipation $= -2\,C\,(C\,t\,\varepsilon\,\mu - x\,\Omega)$

*****************************   *END PROCEDURE*   ***************************** **(11)**

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.
p=2, n=4, euclidean signature index 0   p 160 vol4

```
> NAME:=`Example A-- Hopf signature index 0. The 1-form is divided by the Holder
  norm any p=2, n=4 `;
```

```
> Ax:=y*Omega;Ay:=-x*Omega;Az:=C*t*1;phi:=z*C*1;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,4,0*alpha*(g+I*gamma),0):
```

****

NAME :=

Example A-- Hopf signature index 0. The 1-form is divided by the Holder norm any p=2, n=4

$$Ax := y\,\Omega$$

$$Ay := -x\,\Omega$$

$$Az := C\,t$$

$$\phi := C\,z$$

Example A-- Hopf signature index 0. The 1-form is divided by the Holder norm any p=2, n=4

*********************************   *Differential Form Format*   *********************************

Action 1-form $= y\,\Omega\,d(x) - x\,\Omega\,d(y) + C\,t\,d(z) - C\,z\,d(t)$

Intensity 2-form F=dA $= -2\,\Omega\,(d(x))\,\&\wedge\,(d(y)) - 2\,C\,(d(z))\,\&\wedge\,(d(t))$

Topological Torsion 3-form $A\wedge F = -2\,C\,t\,\Omega\,\&\wedge(d(x), d(y), d(z)) + 2\,C\,z\,\Omega\,\&\wedge(d(x), d(y), d(t)) - 2\,y\,\Omega\,C\,\&\wedge(d(x), d(z), d(t)) + 2\,x\,\Omega\,C\,\&\wedge(d(y), d(z), d(t))$

Topological Parity 4-form $F\wedge F = 8\,\Omega\,C\,\&\wedge(d(x), d(y), d(z), d(t))$

*********************************** *Using EM format* ***********************************

$E\ field = [0, 0, -2\,C]$

$B\ field = [0, 0, -2\,\Omega]$

Topological TORSION 4 vector $T4 = -[ExA + Bphi, AdotB] = [2\,x\,\Omega\,C, 2\,y\,\Omega\,C, 2\,C\,z\,\Omega, 2\,C\,t\,\Omega]$

Helicity AdotB $= -2\,C\,t\,\Omega$

Poincare II =2(E.B) $= 8\,\Omega\,C$

coefficient of Topological Parity 4-form $= 8\,\Omega\,C$

Pfaff Topological Dimension $PTD = 4$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************************

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= \Omega^2 + C^2$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= \Omega^2\,C^2$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor $CH = 0$

$D\ field = [0, 0, -2\,\varepsilon\,C]$

$$H\ field = \left[0, 0, -\frac{2\,\Omega}{\mu}\right]$$

Poynting vector ExH $= EXH$

Amperian Current 4Vector $curlH\text{-}dD/dt = J4 = [0, 0, 0, 0]$

Amerian charge density $divD = rho = 0$

divergence Lorentz Current 4Vector, $4div(J4) = 0$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[\frac{2\,x\,\Omega^2}{\mu}, \frac{2\,y\,\Omega^2}{\mu}, -2\,\varepsilon\,C^2\,z, -2\,C^2\,t\,\varepsilon\right]$$

$$Topological\ SPIN\ 3\text{-}form = \frac{2\,x\,\Omega^2\,\&\wedge(d(y), d(z), d(t))}{\mu} - \frac{2\,y\,\Omega^2\,\&\wedge(d(x), d(z), d(t))}{\mu}$$

$-2\,\varepsilon\,C^2\,z\,\&\wedge(d(x),d(y),d(t)) + 2\,C^2\,t\,\varepsilon\,\&\wedge(d(x),d(y),d(z))$

Spin density rho_spin $= -2\,C^2\,t\,\varepsilon$

LaGrange field energy density (B.H-D.E) $= -\dfrac{4\,(-\Omega^2+\varepsilon\,C^2\,\mu)}{\mu}$

$B.H = \dfrac{4\,\Omega^2}{\mu}$

$D.E = 4\,\varepsilon\,C^2$

$A.J = 0$

$-rho.phi = 0$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* $= -\dfrac{4\,(-\Omega^2+\varepsilon\,C^2\,\mu)}{\mu}$

London Coefficient *LC* $= 0$

PROCA coefficient curlcurlB $= [0, 0, 0]$

Amperian Current 4Vector *curlH-dD/dt=J4* $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Ampere current *FL = -(rho_ampere E + J_ampere x B)* $= [0, 0, 0]$

Amperian Dissipation *Jampere dot E* $= 0$

Lorentz Force Spin factor *LFSPIN* $= 0$

Topological Torsion current 4 vector *T4 = -[ExA + B.phi,AdotB]* $= [2\,x\,\Omega\,C, 2\,y\,\Omega\,C, 2\,C\,z\,\Omega, 2\,C\,t\,\Omega]$

Lorentz Force 3 vector due to Torsion current *TF = -(rho_torsion E + J_torsion x B)* $= [-4\,y\,\Omega^2\,C, 4\,x\,\Omega^2\,C, -4\,C^2\,t\,\Omega]$

Torsion Dissipation *Jtorsion dot E* $= 4\,C^2\,z\,\Omega$

Topological Spin current 4 vector *TS4 = -[A x H + D.phi,AdotD]* $= \left[\dfrac{2\,x\,\Omega^2}{\mu}, \dfrac{2\,y\,\Omega^2}{\mu}, -2\,\varepsilon\,C^2\,z, -2\,C^2\,t\,\varepsilon\right]$

Lorentz Force 3 vector due to Spin current *SF = --(rho_spin E + J_spin x B)* $= \Big[\dfrac{4\,y\,\Omega^3}{\mu},$

$$-\frac{4 x \Omega^3}{\mu}, -4 C^3 t \varepsilon \Big]$$

$$\text{Spin Dissipation J\_spin dot E} \quad = 4 \varepsilon C^3 z$$

$$\text{Disspative Force 3 vector} = \left[-4 y \Omega^2 (-\Omega + C), 4 x \Omega^2 (-\Omega + C), -4 C^2 t (C \varepsilon \mu + \Omega)\right]$$

$$\text{Dissipation} \quad = -2 C (C t \varepsilon \mu - x \Omega)$$

********************************* END PROCEDURE ******************************* **(12)**

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.
p=2, n=4, Minkowski signature index 1 negative ambiguity p 161 vol4

```
> NAME:=`Example B-- Hopf signature index 1. The 1-form is divided by the Holder
  norm p=2, n=4 `;

> Ax:=-y;Ay:=+x;Az:=-C*t;phi:=-z*C;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,-1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME :=

Example B-- Hopf signature index 1. The 1-form is divided by the Holder norm p=2, n=4

$$Ax := -y$$

$$Ay := x$$

$$Az := -C t$$

$$\phi := -C z$$

Example B-- Hopf signature index 1. The 1-form is divided by the Holder norm p=2, n=4

************************************* Differential Form Format ************************
****

$$\text{Action 1-form} = -y\, d(x) + x\, d(y) - C t\, d(z) + C z\, d(t)$$

$$\text{Intensity 2-form F=dA} \quad = 2\, (d(x)) \,\&\wedge\, (d(y)) + 2 C\, (d(z)) \,\&\wedge\, (d(t))$$

$$\text{Topological Torsion 3-form} \quad A\wedge F = -2 C t \,\&\wedge(d(x), d(y), d(z)) + 2 C z \,\&\wedge(d(x), d(y), d(t)) - 2 y C \,\&\wedge(d(x), d(z), d(t)) + 2 x C \,\&\wedge(d(y), d(z), d(t))$$

$$\text{Topological Parity 4-form} \quad F\wedge F = 8 C \,\&\wedge(d(x), d(y), d(z), d(t))$$

************************************* Using EM format *********************************
******

$$\text{E field} = [0, 0, 2 C]$$

$$\text{B field} = [0, 0, 2]$$

$$\text{Topological TORSION 4 vector} \quad T4 = -[ExA + Bphi,AdotB] = [2 x C, 2 y C, 2 C z, 2 C t]$$

$$\text{Helicity AdotB} = -2 C t$$

$$\text{Poincare II} = 2(E.B) = 8 C$$

$$\text{coefficient of Topological Parity 4-form} = 8 C$$

Pfaff Topological Dimension $PTD = 4$

******************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= C^2 + 1$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= C^2$

**********   *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor $CH = 0$

$$D\ field = [0, 0, 2\,\varepsilon\, C]$$

$$H\ field = \left[0, 0, \frac{2}{\mu}\right]$$

Poynting vector ExH $= EXH$

Amperian Current 4Vector   *curlH-dD/dt=J4* $= [0, 0, 0, 0]$

Amerian charge density   *divD = rho* $= 0$

divergence Lorentz Current 4Vector,   *4div(J4)* $= 0$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[\frac{2\,x}{\mu}, \frac{2\,y}{\mu}, -2\,\varepsilon\,C^2\,z, -2\,C^2\,t\,\varepsilon\right]$$

$$Topological\ SPIN\ 3\text{-}form = \frac{2\,x\ \&\hat{}(d(y), d(z), d(t))}{\mu} - \frac{2\,y\ \&\hat{}(d(x), d(z), d(t))}{\mu} - 2\,\varepsilon\,C^2\,z\ \&\hat{}(d(x), d(y), d(t)) + 2\,C^2\,t\,\varepsilon\ \&\hat{}(d(x), d(y), d(z))$$

Spin density rho_spin $= -2\,C^2\,t\,\varepsilon$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{4\,(-1 + \varepsilon\,C^2\,\mu)}{\mu}$$

$$B.H = \frac{4}{\mu}$$

$$D.E = 4\,\varepsilon\,C^2$$

$$A.J = 0$$

$$\text{-}rho.phi = 0$$

$$Poincare\ I\quad (B.H - D.E)\text{-}(A.J - rho.phi) = -\frac{4\,(-1 + \varepsilon\,C^2\,\mu)}{\mu}$$

London Coefficient $LC = 0$

PROCA coefficient curlcurlB $= [0, 0, 0]$

$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt=J4 \quad = [0, 0, 0, 0]$

$Lorentz\ Force\ 3\ vector\ due\ to\ Ampere\ current \quad FL = \text{-}(rho\_ampere\ E + J\_ampere\ x\ B) \quad = [0, 0, 0]$

$Amperian\ Dissipation \ \ Jampere\ dot\ E \quad = 0$

$Lorentz\ Force\ Spin\ factor \ \ LFSPIN = 0$

$Topological\ Torsion\ current\ 4\ vector \quad T4 = \text{-}[ExA + B.phi,AdotB] = [2\,x\,C, 2\,y\,C, 2\,C\,z, 2\,C\,t]$

$Lorentz\ Force\ 3\ vector\ due\ to\ Torsion\ current \quad TF = \text{-}(rho\_torsion\ E + J\_torsion\ x\ B)$
$= [4\,y\,C, -4\,x\,C, 4\,C^2\,t]$

$Torsion\ Dissipation \ \ Jtorsion\ dot\ E \quad = -4\,C^2\,z$

$Topological\ Spin\ current\ 4\ vector \quad TS4 = \text{-}[A\ x\ H + D.phi,AdotD] = \left[\frac{2\,x}{\mu}, \frac{2\,y}{\mu}, -2\,\varepsilon\,C^2\,z, -2\,C^2\,t\,\varepsilon\right]$

$Lorentz\ Force\ 3\ vector\ due\ to\ Spin\ current \quad SF = \text{--}(rho\_spin\ E + J\_spin\ x\ B) \quad = \left[-\frac{4\,y}{\mu}, \frac{4\,x}{\mu}, 4\,C^3\,t\,\varepsilon\right]$

$Spin\ Dissipation \ \ J\_spin\ dot\ E \quad = -4\,\varepsilon\,C^3\,z$

$Disspative\ Force\ 3\ vector = [4\,y\,(C-1), -4\,x\,(C-1), 4\,C^2\,t\,(C\,\varepsilon\,\mu+1)]$

$Dissipation \quad = -2\,C\,(C\,t\,\varepsilon\,\mu - x)$

***************************** *END PROCEDURE* ***************************** **(13)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 1a-- Real Linear Polarization A^G<>0, A^F = 0  INBOUND `;
> theta:=(k*z+omega*t);
> Ax:=cos(theta);Ay:=cos(theta);Az:=0;phi:=0;
```

****

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0);
```

$NAME := Example\ 1a\text{--}\ Real\ Linear\ Polarization\ A\text{^}G <> 0, A\text{^}F = 0\ \ INBOUND$

$$\theta := k\,z + \omega\,t$$

$$Ax := \cos(k\,z + \omega\,t)$$

$$Ay := \cos(k\,z + \omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

Example 1a-- Real Linear Polarization A^G <>0, A^F = 0 INBOUND

******************************** *Differential Form Format* ********************
****

$$\textit{Action 1-form} = \cos(k\,z + \omega\,t)\; d(x) + \cos(k\,z + \omega\,t)\; d(y)$$

$$\textit{Intensity 2-form F=dA} = \sin(k\,z + \omega\,t)\;\omega\;(d(x))\;\&\wedge\;(d(t)) + \sin(k\,z + \omega\,t)\;k\;(d(x))\;\&\wedge\;(d(z)) + \sin(k\,z + \omega\,t)\;\omega\;(d(y))\;\&\wedge\;(d(t)) + \sin(k\,z + \omega\,t)\;k\;(d(y))\;\&\wedge\;(d(z))$$

Topological Torsion 3-form $A\wedge F = 0$

Topological Parity 4-form $F\wedge F = 0$

******************************** *Using EM format* ****************************
****

$$\textit{E field} = [\sin(k\,z + \omega\,t)\;\omega,\; \sin(k\,z + \omega\,t)\;\omega,\; 0]$$

$$\textit{B field} = [\sin(k\,z + \omega\,t)\;k,\; -\sin(k\,z + \omega\,t)\;k,\; 0]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension *PTD* $= 2$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= 0$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $= 0$

$$\textit{D field} = [\varepsilon\,\sin(k\,z + \omega\,t)\;\omega,\; \varepsilon\,\sin(k\,z + \omega\,t)\;\omega,\; 0]$$

$$\textit{H field} = \left[\frac{\sin(k\,z + \omega\,t)\;k}{\mu},\; -\frac{\sin(k\,z + \omega\,t)\;k}{\mu},\; 0\right]$$

$$\textit{Poynting vector ExH} = \left[\begin{array}{ccc} 0 & 0 & \frac{2\,\omega\,k\left(-1+\cos(k\,z+\omega\,t)^2\right)}{\mu} \end{array}\right]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} \quad = \left[\frac{\cos(k\,z+\omega\,t)\,\left(k^2-\mu\,\omega^2\,\varepsilon\right)}{\mu},\right.$$

$$\left.\frac{\cos(k\,z+\omega\,t)\,\left(k^2-\mu\,\omega^2\,\varepsilon\right)}{\mu}, 0, 0\right]$$

$$\textit{Amerian charge density} \quad \textit{divD = rho} = 0$$

$$\textit{divergence Lorentz Current 4Vector,} \quad \textit{4div(J4)} = 0$$

$$\textit{Topological SPIN 4 vector S4} = \left[0, 0, -\frac{2\cos(k\,z+\omega\,t)\,\sin(k\,z+\omega\,t)\,k}{\mu}, 2\cos(k\,z\right.$$

$$\left.+\,\omega\,t)\,\varepsilon\sin(k\,z+\omega\,t)\,\omega\right]$$

$$\textit{Topological SPIN 3-form} = -\frac{2\cos(k\,z+\omega\,t)\,\sin(k\,z+\omega\,t)\,k\,\&\hat{}\,(d(x), d(y), d(t))}{\mu}$$

$$-\,2\cos(k\,z+\omega\,t)\,\varepsilon\sin(k\,z+\omega\,t)\,\omega\,\&\hat{}\,(d(x), d(y), d(z))$$

$$\textit{Spin density rho\_spin} = 2\cos(k\,z+\omega\,t)\,\varepsilon\sin(k\,z+\omega\,t)\,\omega$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{2\sin(k\,z+\omega\,t)^2\,\left(k^2-\mu\,\omega^2\,\varepsilon\right)}{\mu}$$

$$\textit{B.H} = \frac{2\sin(k\,z+\omega\,t)^2\,k^2}{\mu}$$

$$\textit{D.E} = 2\,\varepsilon\sin(k\,z+\omega\,t)^2\,\omega^2$$

$$\textit{A.J} = \frac{2\cos(k\,z+\omega\,t)^2\,\left(k^2-\mu\,\omega^2\,\varepsilon\right)}{\mu}$$

$$\textit{-rho.phi} = 0$$

$$\textit{Poincare I} \quad \textit{(B.H - D.E)-(A.J - rho.phi)} =$$

$$-\frac{2\left(k^2-\mu\,\omega^2\,\varepsilon\right)\left(\cos(k\,z+\omega\,t)-\sin(k\,z+\omega\,t)\right)\left(\cos(k\,z+\omega\,t)+\sin(k\,z+\omega\,t)\right)}{\mu}$$

$$\textit{London Coefficient} \quad \textit{LC} = \frac{k^2-\mu\,\omega^2\,\varepsilon}{\mu}$$

$$\textit{PROCA coefficient curlcurlB} = \left[\sin(k\,z+\omega\,t)\,k^3, -\sin(k\,z+\omega\,t)\,k^3, 0\right]$$

Amperian Current 4Vector curlH-dD/dt=J4 $= \left[\frac{\cos(kz+\omega t)\,(k^2-\mu\omega^2\varepsilon)}{\mu}, \frac{\cos(kz+\omega t)\,(k^2-\mu\omega^2\varepsilon)}{\mu}, 0, 0 \right]$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= \left[0, 0, \frac{2\cos(kz+\omega t)\,(k^2-\mu\omega^2\varepsilon)\sin(kz+\omega t)\,k}{\mu} \right]$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \left[0, 0, -\frac{2\cos(kz+\omega t)\sin(kz+\omega t)\,k}{\mu}, 2\cos(kz+\omega t)\,\varepsilon\sin(kz+\omega t)\,\omega \right]$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B)

$$= \left[\frac{2\cos(kz+\omega t)\sin(kz+\omega t)^2\,(k^2-\mu\omega^2\varepsilon)}{\mu}, \frac{2\cos(kz+\omega t)\sin(kz+\omega t)^2\,(k^2-\mu\omega^2\varepsilon)}{\mu}, 0 \right]$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \left[2\cos(kz+\omega t)\sin(kz+\omega t)^2\,(k^2-\mu\omega^2\varepsilon), 2\cos(kz+\omega t)\sin(kz+\omega t)^2\,(k^2-\mu\omega^2\varepsilon), \frac{2\cos(kz+\omega t)\,(k^2-\mu\omega^2\varepsilon)\sin(kz+\omega t)\,k}{\mu} \right]$

Dissipation $= 2\,\mu\cos(kz+\omega t)\,\varepsilon\sin(kz+\omega t)\,\omega$

********************************** END PROCEDURE ******************************** **(14)**

```
> factor(B[3]);
```

$$0 \quad (15)$$

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 1b-- Real Linear Polarization A^G<>0, A^F = 0  OUTBOUND `;
> theta:=(-k*z+omega*t);
> Ax:=cos(theta);Ay:=cos(theta);Az:=0;phi:=0;
```

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,g+I*gamma,0);
```

****

NAME := *Example 1b-- Real Linear Polarization A^G <>0, A^F = 0 OUTBOUND*

$$\theta := -k\,z + \omega\,t$$

$$Ax := \cos(k\,z - \omega\,t)$$

$$Ay := \cos(k\,z - \omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

Example 1b-- Real Linear Polarization A^G <>0, A^F = 0 OUTBOUND

********************************* *Differential Form Format* **********************************

$$\textit{Action 1-form} = d(x)\,\cos(k\,z - \omega\,t) + d(y)\,\cos(k\,z - \omega\,t)$$

$$\textit{Intensity 2-form F=dA} = -\sin(k\,z - \omega\,t)\,\omega\,(d(x))\ \&\wedge\ (d(t)) + \sin(k\,z - \omega\,t)\,k\,(d(x))\ \&\wedge\ (d(z)) - \sin(k\,z - \omega\,t)\,\omega\,(d(y))\ \&\wedge\ (d(t)) + \sin(k\,z - \omega\,t)\,k\,(d(y))\ \&\wedge\ (d(z))$$

$$\textit{Topological Torsion 3-form} \quad A\wedge F = 0$$

$$\textit{Topological Parity 4-form} \quad F\wedge F = 0$$

********************************* *Using EM format* ********************************

$$\textit{E field} = [-\sin(k\,z - \omega\,t)\,\omega,\ -\sin(k\,z - \omega\,t)\,\omega,\ 0]$$

$$\textit{B field} = [\sin(k\,z - \omega\,t)\,k,\ -\sin(k\,z - \omega\,t)\,k,\ 0]$$

$$\textit{Topological TORSION 4 vector} \quad T4 = -[ExA + Bphi, AdotB] = [0, 0, 0, 0]$$

$$\textit{Helicity AdotB} = 0$$

$$\textit{Poincare II} = 2(E.B) = 0$$

$$\textit{coefficient of Topological Parity 4-form} = 0$$

$$\textit{Pfaff Topological Dimension} \quad PTD = 2$$

******************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ****** ********************

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= 0$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\textit{Chirality factor CH} = (g + I\gamma)\sqrt{\frac{\mu}{\varepsilon}}$$

$$\textit{D field} = \left[I\sin(kz - \omega t)\left(I\varepsilon\omega - I\sqrt{\frac{\mu}{\varepsilon}}\,k\,g + \sqrt{\frac{\mu}{\varepsilon}}\,k\,\gamma \right), -I\sin(kz - \omega t)\left(-I\varepsilon\omega - I\sqrt{\frac{\mu}{\varepsilon}}\,k\,g + \sqrt{\frac{\mu}{\varepsilon}}\,k\,\gamma \right), 0 \right]$$

$$\textit{H field} = \left[\frac{I\sin(kz - \omega t)\left(-Ik - I\sqrt{\frac{\mu}{\varepsilon}}\,\omega\mu g + \sqrt{\frac{\mu}{\varepsilon}}\,\omega\mu\gamma \right)}{\mu}, \frac{I\sin(kz - \omega t)\left(Ik - I\sqrt{\frac{\mu}{\varepsilon}}\,\omega\mu g + \sqrt{\frac{\mu}{\varepsilon}}\,\omega\mu\gamma \right)}{\mu}, 0 \right]$$

$$\textit{Poynting vector ExH} = \left[\begin{array}{ccc} 0 & 0 & -\dfrac{2\omega k\left(-1 + \cos(kz - \omega t)^2\right)}{\mu} \end{array} \right]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} \; = \left[\frac{\cos(kz - \omega t)\left(k^2 - \mu\omega^2\varepsilon\right)}{\mu}, \frac{\cos(kz - \omega t)\left(k^2 - \mu\omega^2\varepsilon\right)}{\mu}, 0, 0 \right]$$

Amerian charge density *divD = rho* $= 0$

divergence Lorentz Current 4Vector, *4div(J4)* $= 0$

$$\textit{Topological SPIN 4 vector S4} = \left[0, 0, -\frac{2\cos(kz - \omega t)\sin(kz - \omega t)\,k}{\mu}, -2\cos(kz - \omega t)\sin(kz - \omega t)\,\varepsilon\omega \right]$$

$$\textit{Topological SPIN 3-form} = -\frac{2\cos(kz - \omega t)\sin(kz - \omega t)\,k\,\&\text{^}(d(x), d(y), d(t))}{\mu} + 2\cos(kz - \omega t)\sin(kz - \omega t)\,\varepsilon\omega\,\&\text{^}(d(x), d(y), d(z))$$

$$\textit{Spin density rho\_spin} = -2\cos(kz-\omega t)\sin(kz-\omega t)\,\varepsilon\,\omega$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{2\sin(kz-\omega t)^2\,(k^2-\mu\,\omega^2\,\varepsilon)}{\mu}$$

$$B.H = \frac{2\sin(kz-\omega t)^2\,k^2}{\mu}$$

$$D.E = 2\sin(kz-\omega t)^2\,\omega^2\,\varepsilon$$

$$A.J = \frac{2\cos(kz-\omega t)^2\,(k^2-\mu\,\omega^2\,\varepsilon)}{\mu}$$

$$\textit{-rho.phi} = 0$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)*

$$= \frac{2\,(k^2-\mu\,\omega^2\,\varepsilon)\,(\sin(kz-\omega t)-\cos(kz-\omega t))\,(\sin(kz-\omega t)+\cos(kz-\omega t))}{\mu}$$

$$\textit{London Coefficient} \quad LC = \frac{k^2-\mu\,\omega^2\,\varepsilon}{\mu}$$

$$\textit{PROCA coefficient curlcurlB} = [k^3\sin(kz-\omega t),\ -k^3\sin(kz-\omega t),\ 0]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} = \left[\frac{\cos(kz-\omega t)\,(k^2-\mu\,\omega^2\,\varepsilon)}{\mu},\ \frac{\cos(kz-\omega t)\,(k^2-\mu\,\omega^2\,\varepsilon)}{\mu},\ 0,\ 0\right]$$

$$\textit{Lorentz Force 3 vector due to Ampere current} \quad FL = \textit{-(rho\_ampere E} + \textit{J\_ampere x B)} = \left[0,\ 0,\ \frac{2\cos(kz-\omega t)\,(k^2-\mu\,\omega^2\,\varepsilon)\sin(kz-\omega t)\,k}{\mu}\right]$$

$$\textit{Amperian Dissipation Jampere dot E} = 0$$

$$\textit{Lorentz Force Spin factor LFSPIN} = 0$$

$$\textit{Topological Torsion current 4 vector} \quad T4 = \textit{-[ExA} + \textit{B.phi,AdotB]} = [0, 0, 0, 0]$$

$$\textit{Lorentz Force 3 vector due to Torsion current} \quad TF = \textit{-(rho\_torsion E} + \textit{J\_torsion x B)} = [0, 0, 0]$$

$$\textit{Torsion Dissipation Jtorsion dot E} = 0$$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] $= \left[0, 0, -\frac{2\cos(kz-\omega t)\sin(kz-\omega t)\,k}{\mu}, -2\cos(kz-\omega t)\sin(kz-\omega t)\,\varepsilon\,\omega\right]$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B)

$$= \left[\frac{2\cos(kz-\omega t)\sin(kz-\omega t)^2\left(k^2-\mu\omega^2\varepsilon\right)}{\mu}, \frac{2\cos(kz-\omega t)\sin(kz-\omega t)^2\left(k^2-\mu\omega^2\varepsilon\right)}{\mu}, 0\right]$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \left[2\cos(kz-\omega t)\sin(kz-\omega t)^2\left(k^2-\mu\omega^2\varepsilon\right), 2\cos(kz-\omega t)\sin(kz-\omega t)^2\left(k^2-\mu\omega^2\varepsilon\right), \frac{2\cos(kz-\omega t)\left(k^2-\mu\omega^2\varepsilon\right)\sin(kz-\omega t)\,k}{\mu}\right]$

Dissipation $= -2\,\mu\cos(kz-\omega t)\sin(kz-\omega t)\,\varepsilon\,\omega$

************************************ *END PROCEDURE* ************************************ **(16)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 2a-- Real Circular Polarization A^G=0, A^F <> 0 INBOUND`;
> theta:=(k*z+omega*t);
> Ax:=cos(theta);Ay:=sin(theta);Az:=0;phi:=0;
```

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,g+I*gamma,0):
```

****

NAME := *Example 2a-- Real Circular Polarization A^G=0, A^F <> 0 INBOUND*

$$\theta := k\,z + \omega\,t$$

$$Ax := \cos(k\,z + \omega\,t)$$

$$Ay := \sin(k\,z + \omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

Example 2a-- Real Circular Polarization A^G=0, A^F <> 0 INBOUND

******************************** *Differential Form Format* ********************
****

$$Action\ 1\text{-}form = \cos(k\,z + \omega\,t)\ d(x) + \sin(k\,z + \omega\,t)\ d(y)$$

$$Intensity\ 2\text{-}form\ F{=}dA\ = \sin(k\,z + \omega\,t)\ \omega\ (d(x))\ \&\text{^}\ (d(t)) + \sin(k\,z + \omega\,t)\ k\ (d(x))\ \&\text{^}\ (d(z)) - \cos(k\,z + \omega\,t)\ \omega\ (d(y))\ \&\text{^}\ (d(t)) - \cos(k\,z + \omega\,t)\ k\ (d(y))\ \&\text{^}\ (d(z))$$

$$Topological\ Torsion\ 3\text{-}form\quad A\text{^}F = \left(-\sin(k\,z + \omega\,t)^2\,\omega - \cos(k\,z + \omega\,t)^2\,\omega\right)\ \&\text{^}(d(x), d(y), d(t)) + \left(-\sin(k\,z + \omega\,t)^2\,k - \cos(k\,z + \omega\,t)^2\,k\right)\ \&\text{^}(d(x), d(y), d(z))$$

$$Topological\ Parity\ 4\text{-}form\quad F\text{^}F = 0$$

******************************** *Using EM format* ****************************
****

$$E\ field = [\sin(k\,z + \omega\,t)\ \omega,\ -\cos(k\,z + \omega\,t)\ \omega,\ 0]$$

$$B\ field = [-\cos(k\,z + \omega\,t)\ k,\ -\sin(k\,z + \omega\,t)\ k,\ 0]$$

$$Topological\ TORSION\ 4\ vector\quad T4 = -[ExA + Bphi, AdotB] = \left[0, 0, -\omega\left(\cos(k\,z + \omega\,t)^2 + \sin(k\,z + \omega\,t)^2\right), k\left(\cos(k\,z + \omega\,t)^2 + \sin(k\,z + \omega\,t)^2\right)\right]$$

$$Helicity\ AdotB = -k\left(\cos(k\,z + \omega\,t)^2 + \sin(k\,z + \omega\,t)^2\right)$$

$$Poincare\ II\ = 2(E.B) = 0$$

$$coefficient\ of\ Topological\ Parity\ 4\text{-}form\ = 0$$

Pfaff Topological Dimension $PTD = 3$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ****** *********************

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= 0$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\textit{Chirality factor CH} = (g + \mathrm{I}\,\gamma)\sqrt{\frac{\mu}{\varepsilon}}$$

$$\textit{D field} = \left[-\mathrm{I}\left(\mathrm{I}\,\varepsilon\sin(k z+\omega t)\,\omega - \mathrm{I}\sqrt{\frac{\mu}{\varepsilon}}\cos(k z+\omega t)\,k g + \sqrt{\frac{\mu}{\varepsilon}}\cos(k z+\omega t)\,k\gamma\right),\right.$$
$$\left.-\varepsilon\cos(k z+\omega t)\,\omega - \sqrt{\frac{\mu}{\varepsilon}}\sin(k z+\omega t)\,k g - \mathrm{I}\sqrt{\frac{\mu}{\varepsilon}}\sin(k z+\omega t)\,k\gamma, 0\right]$$

$$\textit{H field} = \left[-\frac{\cos(k z+\omega t)\,k + \sqrt{\frac{\mu}{\varepsilon}}\sin(k z+\omega t)\,\omega\,\mu\,g + \mathrm{I}\sqrt{\frac{\mu}{\varepsilon}}\sin(k z+\omega t)\,\omega\,\mu\,\gamma}{\mu},\right.$$
$$\left.\frac{\mathrm{I}\left(\mathrm{I}\sin(k z+\omega t)\,k - \mathrm{I}\sqrt{\frac{\mu}{\varepsilon}}\cos(k z+\omega t)\,\omega\,\mu\,g + \sqrt{\frac{\mu}{\varepsilon}}\cos(k z+\omega t)\,\omega\,\mu\,\gamma\right)}{\mu}, 0\right]$$

$$\textit{Poynting vector ExH} = \begin{bmatrix} 0 & 0 & -\frac{\omega k}{\mu} \end{bmatrix}$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} \quad = \left[\frac{\cos(k z+\omega t)\,(k^2-\mu\,\omega^2\,\varepsilon)}{\mu},\right.$$
$$\left.\frac{\sin(k z+\omega t)\,(k^2-\mu\,\omega^2\,\varepsilon)}{\mu}, 0, 0\right]$$

Amerian charge density *divD = rho* $= 0$

divergence Lorentz Current 4Vector, *4div(J4)* $= 0$

$$\textit{Topological SPIN 4 vector S4} = \left[0, 0, \mathrm{I}\,(\cos(k z+\omega t) + \mathrm{I}\sin(k z+\omega t))\,(\cos(k z+\omega t)\right.$$
$$- \mathrm{I}\sin(k z+\omega t))\,(-\mathrm{I}\,g+\gamma)\sqrt{\frac{\mu}{\varepsilon}}\,\omega, -\mathrm{I}\,(\cos(k z+\omega t) + \mathrm{I}\sin(k z+\omega t))\,(\cos(k z$$

$$+\omega t) - I\sin(kz+\omega t))\,(-Ig+\gamma)\sqrt{\frac{\mu}{\varepsilon}}\,k\Big]$$

Topological SPIN 3-form $= I\,(\cos(kz+\omega t) + I\sin(kz+\omega t))\,(\cos(kz+\omega t) - I\sin(kz$
$+\omega t))\,(-Ig+\gamma)\sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\&\wedge(d(x), d(y), d(t)) + I\,(\cos(kz+\omega t) + I\sin(kz$
$+\omega t))\,(\cos(kz+\omega t) - I\sin(kz+\omega t))\,(-Ig+\gamma)\sqrt{\frac{\mu}{\varepsilon}}\,k\,\&\wedge(d(x), d(y), d(z))$

Spin density rho_spin $= -I\,(\cos(kz+\omega t) + I\sin(kz+\omega t))\,(\cos(kz+\omega t) - I\sin(kz$
$+\omega t))\,(-Ig+\gamma)\sqrt{\frac{\mu}{\varepsilon}}\,k$

LaGrange field energy density (B.H-D.E) $= \dfrac{(k^2-\mu\omega^2\varepsilon)\,(\cos(kz+\omega t)^2+\sin(kz+\omega t)^2)}{\mu}$

$$B.H = \frac{k^2\,(\cos(kz+\omega t)^2+\sin(kz+\omega t)^2)}{\mu}$$

$$D.E = \omega^2\varepsilon\,(\cos(kz+\omega t)^2+\sin(kz+\omega t)^2)$$

$$A.J = \frac{(k^2-\mu\omega^2\varepsilon)\,(\cos(kz+\omega t)^2+\sin(kz+\omega t)^2)}{\mu}$$

-rho.phi $= 0$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* $= 0$

London Coefficient $LC = \dfrac{k^2-\mu\omega^2\varepsilon}{\mu}$

PROCA coefficient curlcurlB $= [-\cos(kz+\omega t)\,k^3, -\sin(kz+\omega t)\,k^3, 0]$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \left[\dfrac{\cos(kz+\omega t)\,(k^2-\mu\omega^2\varepsilon)}{\mu},\right.$
$\left.\dfrac{\sin(kz+\omega t)\,(k^2-\mu\omega^2\varepsilon)}{\mu}, 0, 0\right]$

Lorentz Force 3 vector due to Ampere current *FL = -(rho_ampere E + J_ampere x B)* = [0,
0, 0]

Amperian Dissipation Jampere dot E = 0

Lorentz Force Spin factor LFSPIN = 0

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] $= \left[0, 0, -\omega \left(\cos(k z + \omega t)^2 + \sin(k z + \omega t)^2\right), k \left(\cos(k z + \omega t)^2 + \sin(k z + \omega t)^2\right)\right]$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] $= \left[0, 0, \mathrm{I} \left(\cos(k z + \omega t) + \mathrm{I} \sin(k z + \omega t)\right) \left(\cos(k z + \omega t) - \mathrm{I} \sin(k z + \omega t)\right) (-\mathrm{I} g + \gamma) \sqrt{\frac{\mu}{\varepsilon}}\, \omega, -\mathrm{I} \left(\cos(k z + \omega t) + \mathrm{I} \sin(k z + \omega t)\right) \left(\cos(k z + \omega t) - \mathrm{I} \sin(k z + \omega t)\right) (-\mathrm{I} g + \gamma) \sqrt{\frac{\mu}{\varepsilon}}\, k\right]$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $= [0, 0, 0]$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= [0, 0, 0]$

Dissipation $= -\mathrm{I} \mu \left(\cos(k z + \omega t) + \mathrm{I} \sin(k z + \omega t)\right) \left(\cos(k z + \omega t) - \mathrm{I} \sin(k z + \omega t)\right) (-\mathrm{I} g + \gamma) \sqrt{\frac{\mu}{\varepsilon}}\, k$

****************************** *END PROCEDURE* ****************************** **(17)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 2b-- Real Circular Polarization A^G<>0, A^F = 0  OUTBOUND `;
> theta:=(-k*z+omega*t);
> Ax:=cos(theta);Ay:=-sin(theta);Az:=0;phi:=0;
```

****

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,alpha,0):
```

NAME := Example 2b-- Real Circular Polarization A^G <>0, A^F = 0 OUTBOUND

$$\theta := -k z + \omega t$$

$$Ax := \cos(k z - \omega t)$$

$$Ay := \sin(k z - \omega t)$$

$$Az := 0$$

$$\phi := 0$$

Example 2b-- Real Circular Polarization A^G <>0, A^F = 0 OUTBOUND

********************************* *Differential Form Format* *******************************

$$Action\ 1\text{-}form = d(x)\cos(kz-\omega t)+\sin(kz-\omega t)\, d(y)$$

$$Intensity\ 2\text{-}form\ F=dA = -\sin(kz-\omega t)\,\omega\,(d(x))\ \&\wedge\ (d(t)) + \sin(kz-\omega t)\,k\,(d(x))\ \&\wedge\ (d(z)) + \cos(kz-\omega t)\,\omega\,(d(y))\ \&\wedge\ (d(t)) - \cos(kz-\omega t)\,k\,(d(y))\ \&\wedge\ (d(z))$$

$$Topological\ Torsion\ 3\text{-}form\quad A\wedge F = \left(\sin(kz-\omega t)^2\omega+\cos(kz-\omega t)^2\omega\right)\ \&\wedge(d(x),d(y),d(t)) + \left(-\sin(kz-\omega t)^2 k-\cos(kz-\omega t)^2 k\right)\ \&\wedge(d(x),d(y),d(z))$$

$$Topological\ Parity\ 4\text{-}form\quad F\wedge F = 0$$

********************************* *Using EM format* ********************************

$$E\ field = [-\sin(kz-\omega t)\,\omega,\ \cos(kz-\omega t)\,\omega,\ 0]$$

$$B\ field = [-\cos(kz-\omega t)\,k,\ -\sin(kz-\omega t)\,k,\ 0]$$

$$Topological\ TORSION\ 4\ vector\quad T4 = -[ExA + Bphi, AdotB] = \left[0, 0, \omega\left(\cos(kz-\omega t)^2+\sin(kz-\omega t)^2\right), k\left(\cos(kz-\omega t)^2+\sin(kz-\omega t)^2\right)\right]$$

$$Helicity\ AdotB = -k\left(\cos(kz-\omega t)^2+\sin(kz-\omega t)^2\right)$$

$$Poincare\ II = 2(E.B) = 0$$

$$coefficient\ of\ Topological\ Parity\ 4\text{-}form = 0$$

$$Pfaff\ Topological\ Dimension\quad PTD = 3$$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************************

$$Xm\ or\ linear\ (Mean)\ curvature = 0$$

$$Yg\ or\ quadratic\ (GAUSS)\ curvature = 0$$

$$Za\ or\ Cubic\ (Interaction\ internal\ energy)\ curvature = 0$$

$$Tk\ or\ quartic\ (4D\ expansion)\ curvature = 0$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$Chirality\ factor\ CH = \alpha\sqrt{\frac{\mu}{\varepsilon}}$$

$$D\ field = \left[-\varepsilon\sin(kz-\omega t)\,\omega - \alpha\sqrt{\frac{\mu}{\varepsilon}}\cos(kz-\omega t)\,k,\ \varepsilon\cos(kz-\omega t)\,\omega\right.$$

$$-\alpha\sqrt{\frac{\mu}{\varepsilon}}\sin(kz-\omega t)\,k,\,0\Bigg]$$

$$H\ field=\left[\frac{-\cos(kz-\omega t)\,k+\alpha\sqrt{\frac{\mu}{\varepsilon}}\sin(kz-\omega t)\,\omega\,\mu}{\mu},\right.$$

$$\left.-\frac{\sin(kz-\omega t)\,k+\alpha\sqrt{\frac{\mu}{\varepsilon}}\cos(kz-\omega t)\,\omega\,\mu}{\mu},\,0\right]$$

$$Poynting\ vector\ ExH=\left[\begin{array}{ccc}0 & 0 & \frac{\omega k}{\mu}\end{array}\right]$$

$$Amperian\ Current\ 4Vector\quad curlH\text{-}dD/dt=J4\ =\left[\frac{\cos(kz-\omega t)\,(k^2-\mu\,\omega^2\,\varepsilon)}{\mu},\right.$$

$$\left.\frac{\sin(kz-\omega t)\,(k^2-\mu\,\omega^2\,\varepsilon)}{\mu},\,0,\,0\right]$$

$$Amerian\ charge\ density\qquad divD\ =\ rho=0$$

$$divergence\ Lorentz\ Current\ 4Vector,\quad 4div(J4)\ =0$$

$$Topological\ SPIN\ 4\ vector\ S4=\left[0,\,0,\,-\alpha\sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\left(\cos(kz-\omega t)^2+\sin(kz-\omega t)^2\right),\right.$$

$$\left.-\alpha\sqrt{\frac{\mu}{\varepsilon}}\,k\,\left(\cos(kz-\omega t)^2+\sin(kz-\omega t)^2\right)\right]$$

$$Topological\ SPIN\ 3\text{-}form=-\alpha\sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\left(\cos(kz-\omega t)^2+\sin(kz-\omega t)^2\right)\ \&\text{^}(d(x),d(y),$$

$$d(t))+\alpha\sqrt{\frac{\mu}{\varepsilon}}\,k\,\left(\cos(kz-\omega t)^2+\sin(kz-\omega t)^2\right)\ \&\text{^}(d(x),d(y),d(z))$$

$$Spin\ density\ rho\_spin=-\alpha\sqrt{\frac{\mu}{\varepsilon}}\,k\,\left(\cos(kz-\omega t)^2+\sin(kz-\omega t)^2\right)$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E)=\frac{(k^2-\mu\,\omega^2\,\varepsilon)\,\left(\cos(kz-\omega t)^2+\sin(kz-\omega t)^2\right)}{\mu}$$

$$B.H=\frac{k^2\left(\cos(kz-\omega t)^2+\sin(kz-\omega t)^2\right)}{\mu}$$

$$D.E=\omega^2\,\varepsilon\,\left(\cos(kz-\omega t)^2+\sin(kz-\omega t)^2\right)$$

$$A.J = \frac{(k^2 - \mu\,\omega^2\,\varepsilon)\,(\cos(k\,z - \omega\,t)^2 + \sin(k\,z - \omega\,t)^2)}{\mu}$$

$$\textit{-rho.phi} = 0$$

Poincare I $\quad$ *(B.H - D.E)-(A.J - rho.phi)* = 0

London Coefficient $\quad LC = \dfrac{k^2 - \mu\,\omega^2\,\varepsilon}{\mu}$

PROCA coefficient curlcurlB = $[-k^3\cos(k\,z - \omega\,t),\ -k^3\sin(k\,z - \omega\,t),\ 0]$

Amperian Current 4Vector $\quad$ *curlH-dD/dt=J4* $= \left[\dfrac{\cos(k\,z - \omega\,t)\,(k^2 - \mu\,\omega^2\,\varepsilon)}{\mu},\ \dfrac{\sin(k\,z - \omega\,t)\,(k^2 - \mu\,\omega^2\,\varepsilon)}{\mu},\ 0,\ 0\right]$

Lorentz Force 3 vector due to Ampere current $\quad$ *FL = -(rho_ampere E + J_ampere x B)* = [0, 0, 0]

Amperian Dissipation Jampere dot E = 0

Lorentz Force Spin factor LFSPIN = 0

Topological Torsion current 4 vector $\quad$ *T4 = -[ExA + B.phi,AdotB]* $= [0,\ 0,\ \omega\,(\cos(k\,z - \omega\,t)^2 + \sin(k\,z - \omega\,t)^2),\ k\,(\cos(k\,z - \omega\,t)^2 + \sin(k\,z - \omega\,t)^2)]$

Lorentz Force 3 vector due to Torsion current $\quad$ *TF = -(rho_torsion E + J_torsion x B)* = [0, 0, 0]

Torsion Dissipation Jtorsion dot E = 0

Topological Spin current 4 vector $\quad$ *TS4 = -[A x H + D.phi,AdotD]* $= \left[0,\ 0,\ -\alpha\sqrt{\dfrac{\mu}{\varepsilon}}\ \omega\,(\cos(k\,z - \omega\,t)^2 + \sin(k\,z - \omega\,t)^2),\ -\alpha\sqrt{\dfrac{\mu}{\varepsilon}}\ k\,(\cos(k\,z - \omega\,t)^2 + \sin(k\,z - \omega\,t)^2)\right]$

Lorentz Force 3 vector due to Spin current $\quad$ *SF = --(rho_spin E + J_spin x B)* = [0, 0, 0]

Spin Dissipation J_spin dot E = 0

Disspative Force 3 vector = [0, 0, 0]

$$Dissipation \;= -\mu\,\alpha\,\sqrt{\frac{\mu}{\varepsilon}}\;k\,\left(\cos(k\,z-\omega\,t)^2+\sin(k\,z-\omega\,t)^2\right)$$

***************************** *END PROCEDURE* ***************************** **(18)**

Enter the name of the procedure and then the components of the 4 potential

```
> NAME:=`Example 3a-- Complex Linear Polarization A^G<>0, A^F = 0  INBOUND `;
> theta:=(k*z+omega*t);
> Ax:=cos(theta);Ay:=I*cos(theta);Az:=0;phi:=0;
```

****

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,gamma,0):
```

NAME := *Example 3a-- Complex Linear Polarization A^G <>0, A^F = 0 INBOUND*

$$\theta := k\,z+\omega\,t$$

$$Ax := \cos(k\,z+\omega\,t)$$

$$Ay := I\cos(k\,z+\omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

Example 3a-- Complex Linear Polarization A^G <>0, A^F = 0 INBOUND

********************************* *Differential Form Format* ********************
****

$$Action\ 1\text{-}form = \cos(k\,z+\omega\,t)\;d(x)\;+I\cos(k\,z+\omega\,t)\;d(y)$$

$$Intensity\ 2\text{-}form\ F{=}dA\;\; = \sin(k\,z+\omega\,t)\;\omega\;(d(x))\;\&\wedge\;(d(t))\;+\sin(k\,z+\omega\,t)\;k\;(d(x))\;\&\wedge\;(d(z))\;+I\sin(k\,z+\omega\,t)\;\omega\;(d(y))\;\&\wedge\;(d(t))\;+I\sin(k\,z+\omega\,t)\;k\;(d(y))\;\&\wedge\;(d(z))$$

$$Topological\ Torsion\ 3\text{-}form\quad A\wedge F = 0$$

$$Topological\ Parity\ 4\text{-}form\quad F\wedge F = 0$$

********************************* *Using EM format* *****************************
****

$$E\ field = [\sin(k\,z+\omega\,t)\;\omega,\ I\sin(k\,z+\omega\,t)\;\omega,\ 0]$$

$$B\ field = [I\sin(k\,z+\omega\,t)\;k,\ -\sin(k\,z+\omega\,t)\;k,\ 0]$$

Topological TORSION 4 vector   T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension *PTD* = 2

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ****** ********************

Xm or linear (Mean) curvature = 0

Yg or quadratic (GAUSS) curvature = 0

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\text{Chirality factor } CH = \gamma\sqrt{\frac{\mu}{\varepsilon}}$$

$$D\ field = \left[I\sin(kz+\omega t)\left(-I\varepsilon\omega + \sqrt{\frac{\mu}{\varepsilon}}\,k\gamma\right), -\sin(kz+\omega t)\left(-I\varepsilon\omega + \sqrt{\frac{\mu}{\varepsilon}}\,k\gamma\right), 0\right]$$

$$H\ field = \left[-\frac{\sin(kz+\omega t)\left(-Ik + \sqrt{\frac{\mu}{\varepsilon}}\,\omega\mu\gamma\right)}{\mu}, -\frac{I\sin(kz+\omega t)\left(-Ik + \sqrt{\frac{\mu}{\varepsilon}}\,\omega\mu\gamma\right)}{\mu}, 0\right]$$

Poynting vector ExH = *EXH*

$$\text{Amperian Current 4Vector} \quad curlH\text{-}dD/dt=J4 = \left[\frac{\cos(kz+\omega t)\left(k^2-\mu\omega^2\varepsilon\right)}{\mu}, \frac{I\cos(kz+\omega t)\left(k^2-\mu\omega^2\varepsilon\right)}{\mu}, 0, 0\right]$$

Amerian charge density *divD = rho* = 0

divergence Lorentz Current 4Vector, *4div(J4)* = 0

Topological SPIN 4 vector S4 = [0, 0, 0, 0]

Topological SPIN 3-form = 0

Spin density rho_spin = 0

LaGrange field energy density (B.H-D.E) = 0

B.H = 0

D.E = 0

A.J = 0

-rho.phi = 0

Poincare I *(B.H - D.E)-(A.J - rho.phi)* $= 0$

London Coefficient $LC = \frac{k^2 - \mu\,\omega^2\,\varepsilon}{\mu}$

PROCA coefficient curlcurlB $= [\mathrm{I}\sin(k\,z + \omega\,t)\,k^3, -\sin(k\,z + \omega\,t)\,k^3, 0]$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \left[\frac{\cos(k\,z + \omega\,t)\,(k^2 - \mu\,\omega^2\,\varepsilon)}{\mu}, \frac{\mathrm{I}\cos(k\,z + \omega\,t)\,(k^2 - \mu\,\omega^2\,\varepsilon)}{\mu}, 0, 0\right]$

Lorentz Force 3 vector due to Ampere current *FL = -(rho_ampere E + J_ampere x B)* $= [0, 0, 0]$

Amperian Dissipation *Jampere dot E* $= 0$

Lorentz Force Spin factor $LFSPIN = 0$

Topological Torsion current 4 vector *T4 = -[ExA + B.phi,AdotB]* $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current *TF = -(rho_torsion E + J_torsion x B)* $= [0, 0, 0]$

Torsion Dissipation *Jtorsion dot E* $= 0$

Topological Spin current 4 vector *TS4 = -[A x H + D.phi,AdotD]* $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Spin current *SF = --(rho_spin E + J_spin x B)* $= [0, 0, 0]$

Spin Dissipation *J_spin dot E* $= 0$

Disspative Force 3 vector $= [0, 0, 0]$

Dissipation $= 0$

****************************** *END PROCEDURE* ****************************** **(19)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 3b-- Complex Linear Polarization A^G = 0, A^F = 0   OUTBOUND`;
> theta:=(-k*z+omega*t);
> Ax:=cos(theta);Ay:=I*cos(theta);Az:=0;phi:=0;
```

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,gamma,0):
```

****

NAME := *Example 3b-- Complex Linear Polarization A^G = 0, A^F = 0   OUTBOUND*

$$\theta := -k\,z + \omega\,t$$

$$Ax := \cos(k\,z - \omega\,t)$$

$$Ay := I\cos(k\,z - \omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

Example 3b-- Complex Linear Polarization A^G = 0, A^F = 0   OUTBOUND

******************************** *Differential Form Format* ********************
****

$$Action\ 1\text{-}form = d(x)\cos(k\,z - \omega\,t) + I\cos(k\,z - \omega\,t)\,d(y)$$

$$Intensity\ 2\text{-}form\ F=dA\ = -\sin(k\,z - \omega\,t)\,\omega\,(d(x))\ \&\wedge\ (d(t)) + \sin(k\,z - \omega\,t)\,k\,(d(x))\ \&\wedge\ (d(z)) - I\sin(k\,z - \omega\,t)\,\omega\,(d(y))\ \&\wedge\ (d(t)) + I\sin(k\,z - \omega\,t)\,k\,(d(y))\ \&\wedge\ (d(z))$$

$$Topological\ Torsion\ 3\text{-}form\quad A\wedge F = 0$$

$$Topological\ Parity\ 4\text{-}form\quad F\wedge F = 0$$

******************************** *Using EM format* ****************************
****

$$E\ field = [-\sin(k\,z - \omega\,t)\,\omega,\ -I\sin(k\,z - \omega\,t)\,\omega,\ 0]$$

$$B\ field = [I\sin(k\,z - \omega\,t)\,k,\ -\sin(k\,z - \omega\,t)\,k,\ 0]$$

$$Topological\ TORSION\ 4\ vector\quad T4 = -[ExA + Bphi, AdotB] = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ II = 2(E.B) = 0$$

$$coefficient\ of\ Topological\ Parity\ 4\text{-}form = 0$$

$$Pfaff\ Topological\ Dimension\quad PTD = 2$$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

$$Xm\ or\ linear\ (Mean)\ curvature = 0$$

$$Yg\ or\ quadratic\ (GAUSS)\ curvature = 0$$

$$Za\ or\ Cubic\ (Interaction\ internal\ energy)\ curvature = 0$$

$$Tk\ or\ quartic\ (4D\ expansion)\ curvature = 0$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$Chirality\ factor\ CH = \gamma\sqrt{\frac{\mu}{\varepsilon}}$$

$$D\ field = \left[I\sin(k\,z - \omega\,t)\left(I\,\varepsilon\,\omega + \sqrt{\frac{\mu}{\varepsilon}}\,k\,\gamma\right),\ -\sin(k\,z - \omega\,t)\left(I\,\varepsilon\,\omega + \sqrt{\frac{\mu}{\varepsilon}}\,k\,\gamma\right),\ 0\right]$$

$$H\ field = \left[\frac{\sin(kz - \omega t)\left(\mathrm{I}\,k + \sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\mu\,\gamma\right)}{\mu}, \frac{\mathrm{I}\sin(kz - \omega t)\left(\mathrm{I}\,k + \sqrt{\frac{\mu}{\varepsilon}}\,\omega\,\mu\,\gamma\right)}{\mu}, 0 \right]$$

Poynting vector ExH = EXH

Amperian Current 4Vector curlH-dD/dt=J4 $= \left[\frac{\cos(kz - \omega t)\,(k^2 - \mu\,\omega^2\,\varepsilon)}{\mu}, \frac{\mathrm{I}\cos(kz - \omega t)\,(k^2 - \mu\,\omega^2\,\varepsilon)}{\mu}, 0, 0 \right]$

Amerian charge density divD = rho $= 0$

divergence Lorentz Current 4Vector, 4div(J4) $= 0$

Topological SPIN 4 vector S4 $= [0, 0, 0, 0]$

Topological SPIN 3-form $= 0$

Spin density rho_spin $= 0$

LaGrange field energy density (B.H-D.E) $= 0$

B.H $= 0$

D.E $= 0$

A.J $= 0$

-rho.phi $= 0$

Poincare I (B.H - D.E)-(A.J - rho.phi) $= 0$

London Coefficient LC $= \frac{k^2 - \mu\,\omega^2\,\varepsilon}{\mu}$

PROCA coefficient curlcurlB $= [\mathrm{I}\sin(kz - \omega t)\,k^3, -k^3\sin(kz - \omega t), 0]$

Amperian Current 4Vector curlH-dD/dt=J4 $= \left[\frac{\cos(kz - \omega t)\,(k^2 - \mu\,\omega^2\,\varepsilon)}{\mu}, \frac{\mathrm{I}\cos(kz - \omega t)\,(k^2 - \mu\,\omega^2\,\varepsilon)}{\mu}, 0, 0 \right]$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= [0, 0, 0]$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0,$

0, 0]

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $= [0, 0, 0]$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= [0, 0, 0]$

Dissipation $= 0$

*******************************   *END PROCEDURE*   *****************************   **(20)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 4a-- Complex Circular Polarization A^G<>0, A^F <> 0   INBOUND`;
> theta:=(k*z+omega*t);
> Ax:=cos(theta);Ay:=I*sin(theta);Az:=0;phi:=0;
```

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,gamma,0):
```

****

NAME := Example 4a-- Complex Circular Polarization A^G <>0, A^F <> 0   INBOUND

$$\theta := k\,z + \omega\,t$$

$$Ax := \cos(k\,z + \omega\,t)$$

$$Ay := \mathrm{I}\sin(k\,z + \omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

Example 4a-- Complex Circular Polarization A^G <>0, A^F <> 0   INBOUND

**********************************   *Differential Form Format*   ********************
****

$$\textit{Action 1-form} = \cos(k\,z + \omega\,t)\ d(x) + \mathrm{I}\sin(k\,z + \omega\,t)\ d(y)$$

$$\textit{Intensity 2-form F=dA} = \sin(k\,z + \omega\,t)\ \omega\ (d(x))\ \&\wedge\ (d(t)) + \sin(k\,z + \omega\,t)\ k\ (d(x))\ \&\wedge\ (d(z)) - \mathrm{I}\cos(k\,z + \omega\,t)\ \omega\ (d(y))\ \&\wedge\ (d(t)) - \mathrm{I}\cos(k\,z + \omega\,t)\ k\ (d(y))\ \&\wedge\ (d(z))$$

$$\textit{Topological Torsion 3-form}\quad A\wedge F = \left(-\mathrm{I}\sin(k\,z + \omega\,t)^2\,\omega - \mathrm{I}\cos(k\,z + \omega\,t)^2\,\omega\right)\ \&\wedge(d(x), d(y), d(t)) + \left(-\mathrm{I}\sin(k\,z + \omega\,t)^2\,k - \mathrm{I}\cos(k\,z + \omega\,t)^2\,k\right)\ \&\wedge(d(x), d(y), d(z))$$

$$\textit{Topological Parity 4-form}\quad F\wedge F = 0$$

**********************************   *Using EM format*   *****************************

****

$$E\ field = [\sin(kz+\omega t)\,\omega,\ -I\cos(kz+\omega t)\,\omega,\ 0]$$

$$B\ field = [-I\cos(kz+\omega t)\,k,\ -\sin(kz+\omega t)\,k,\ 0]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, -I\,(\cos(kz+\omega t) + I\sin(kz+\omega t))\,(\cos(kz+\omega t) - I\sin(kz+\omega t))\,\omega, I\,(\cos(kz+\omega t) + I\sin(kz+\omega t))\,(\cos(kz+\omega t) - I\sin(kz+\omega t))\,k]$

Helicity AdotB $= -I\,(\cos(kz+\omega t) + I\sin(kz+\omega t))\,(\cos(kz+\omega t) - I\sin(kz+\omega t))\,k$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension   PTD $= 3$

******************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* **************************

Xm or linear (Mean) curvature $= 0$

Yg or quadratic (GAUSS) curvature $= 0$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$Chirality\ factor\ CH = \gamma\sqrt{\frac{\mu}{\varepsilon}}$$

$$D\ field = \left[-I\left(I\,\varepsilon\sin(kz+\omega t)\,\omega + \sqrt{\frac{\mu}{\varepsilon}}\cos(kz+\omega t)\,k\,\gamma\right), -I\left(\varepsilon\cos(kz+\omega t)\,\omega - I\sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)\,k\,\gamma\right), 0\right]$$

$$H\ field = \left[-\frac{I\left(\cos(kz+\omega t)\,k - I\sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)\,\omega\,\mu\,\gamma\right)}{\mu}, \frac{I\left(I\sin(kz+\omega t)\,k + \sqrt{\frac{\mu}{\varepsilon}}\cos(kz+\omega t)\,\omega\,\mu\,\gamma\right)}{\mu}, 0\right]$$

$$Poynting\ vector\ ExH = \left[\begin{array}{ccc} 0 & 0 & \frac{\omega k\,(-1 + 2\cos(kz+\omega t)^2)}{\mu}\end{array}\right]$$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \left[\frac{\cos(kz+\omega t)\,(k^2-\mu\omega^2\varepsilon)}{\mu}, \frac{I\sin(kz+\omega t)\,(k^2-\mu\omega^2\varepsilon)}{\mu}, 0, 0 \right]$

Amerian charge density *divD = rho* $= 0$

divergence Lorentz Current 4Vector, *4div(J4)* $= 0$

Topological SPIN 4 vector S4 $= \left[0, 0, \frac{1}{\mu}\left(I\left(2 I\sin(kz+\omega t)\cos(kz+\omega t)\,k + \cos(kz+\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,\omega\mu\gamma + \sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)^2\omega\mu\gamma\right)\right), -I\left(2 I\sin(kz+\omega t)\,\varepsilon\cos(kz+\omega t)\,\omega + \cos(kz+\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,k\gamma + \sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)^2 k\gamma\right)\right]$

Topological SPIN 3-form $= \frac{1}{\mu}\left(I\left(2 I\sin(kz+\omega t)\cos(kz+\omega t)\,k + \cos(kz+\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,\omega\mu\gamma + \sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)^2\omega\mu\gamma\right)\,\&\wedge(d(x), d(y), d(t))\right) + I\left(2 I\sin(kz+\omega t)\,\varepsilon\cos(kz+\omega t)\,\omega + \cos(kz+\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,k\gamma + \sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)^2 k\gamma\right)\,\&\wedge(d(x), d(y), d(z))$

Spin density rho_spin $= -I\left(2 I\sin(kz+\omega t)\,\varepsilon\cos(kz+\omega t)\,\omega + \cos(kz+\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,k\gamma + \sqrt{\frac{\mu}{\varepsilon}}\sin(kz+\omega t)^2 k\gamma\right)$

LaGrange field energy density (B.H-D.E) =

$$-\frac{(k^2-\mu\omega^2\varepsilon)\,(\cos(kz+\omega t)-\sin(kz+\omega t))\,(\cos(kz+\omega t)+\sin(kz+\omega t))}{\mu}$$

$$B.H = -\frac{k^2\,(\cos(kz+\omega t)-\sin(kz+\omega t))\,(\cos(kz+\omega t)+\sin(kz+\omega t))}{\mu}$$

$$D.E = -\omega^2\varepsilon\,(\cos(kz+\omega t)-\sin(kz+\omega t))\,(\cos(kz+\omega t)+\sin(kz+\omega t))$$

$$A.J = \frac{(k^2 - \mu\,\omega^2\,\varepsilon)\,(\cos(k\,z + \omega\,t) - \sin(k\,z + \omega\,t))\,(\cos(k\,z + \omega\,t) + \sin(k\,z + \omega\,t))}{\mu}$$

$$\textit{-rho.phi} = 0$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* =

$$-\frac{2\,(k^2 - \mu\,\omega^2\,\varepsilon)\,(\cos(k\,z + \omega\,t) - \sin(k\,z + \omega\,t))\,(\cos(k\,z + \omega\,t) + \sin(k\,z + \omega\,t))}{\mu}$$

London Coefficient $LC = \dfrac{k^2 - \mu\,\omega^2\,\varepsilon}{\mu}$

PROCA coefficient curlcurlB $= [-I\cos(k\,z + \omega\,t)\,k^3,\ -\sin(k\,z + \omega\,t)\,k^3,\ 0]$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \left[\dfrac{\cos(k\,z + \omega\,t)\,(k^2 - \mu\,\omega^2\,\varepsilon)}{\mu},\right.$
$\left.\dfrac{I\sin(k\,z + \omega\,t)\,(k^2 - \mu\,\omega^2\,\varepsilon)}{\mu},\ 0,\ 0\right]$

Lorentz Force 3 vector due to Ampere current *FL = -(rho_ampere E + J_ampere x B)* $= \left[0,\right.$
$\left.0,\ \dfrac{2\cos(k\,z + \omega\,t)\,(k^2 - \mu\,\omega^2\,\varepsilon)\,\sin(k\,z + \omega\,t)\,k}{\mu}\right]$

Amperian Dissipation *Jampere dot E* $= 0$

Lorentz Force Spin factor *LFSPIN* $= 0$

Topological Torsion current 4 vector *T4 = -[ExA + B.phi,AdotB]* $= [0,\ 0,\ -I\,(\cos(k\,z + \omega\,t)$
$+ I\sin(k\,z + \omega\,t))\,(\cos(k\,z + \omega\,t) - I\sin(k\,z + \omega\,t))\,\omega,\ I\,(\cos(k\,z + \omega\,t) + I\sin(k\,z$
$+ \omega\,t))\,(\cos(k\,z + \omega\,t) - I\sin(k\,z + \omega\,t))\,k]$

Lorentz Force 3 vector due to Torsion current *TF = -(rho_torsion E + J_torsion x B)* $= [0,$
$0,\ 0]$

Torsion Dissipation *Jtorsion dot E* $= 0$

Topological Spin current 4 vector *TS4 = -[A x H + D.phi,AdotD]* $= \left[0,\ 0,\ \dfrac{1}{\mu}\left(I\left(2\,I\sin(k\,z\right.\right.\right.$
$\left.\left.+ \omega\,t)\cos(k\,z + \omega\,t)\,k + \cos(k\,z + \omega\,t)^2\sqrt{\dfrac{\mu}{\varepsilon}}\,\omega\,\mu\,\gamma + \sqrt{\dfrac{\mu}{\varepsilon}}\sin(k\,z + \omega\,t)^2\,\omega\,\mu\,\gamma\right)\right),$

$$-I\left(2\,I\sin(k\,z+\omega\,t)\,\varepsilon\cos(k\,z+\omega\,t)\,\omega+\cos(k\,z+\omega\,t)^2\sqrt{\frac{\mu}{\varepsilon}}\,k\,\gamma+\sqrt{\frac{\mu}{\varepsilon}}\,\sin(k\,z+\omega\,t)^2\,k\,\gamma\right)\Bigg]$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B)

$$=\left[\frac{2\cos(k\,z+\omega\,t)\,\sin(k\,z+\omega\,t)^2\,(k^2-\mu\,\omega^2\,\varepsilon)}{\mu},\;-\frac{2\,I\cos(k\,z+\omega\,t)^2\,\sin(k\,z+\omega\,t)\,(k^2-\mu\,\omega^2\,\varepsilon)}{\mu},\,0\right]$$

Spin Dissipation J_spin dot E $=0$

Disspative Force 3 vector $=\Big[2\cos(k\,z+\omega\,t)\,\sin(k\,z+\omega\,t)^2\,(k^2-\mu\,\omega^2\,\varepsilon),\,-2\,I\cos(k\,z+\omega\,t)^2\sin(k\,z+\omega\,t)\,(k^2-\mu\,\omega^2\,\varepsilon),\,\frac{2\cos(k\,z+\omega\,t)\,(k^2-\mu\,\omega^2\,\varepsilon)\,\sin(k\,z+\omega\,t)\,k}{\mu}\Big]$

Dissipation $=-I\,\mu\left(2\,I\sin(k\,z+\omega\,t)\,\varepsilon\cos(k\,z+\omega\,t)\,\omega+\cos(k\,z+\omega\,t)^2\sqrt{\frac{\mu}{\varepsilon}}\,k\,\gamma+\sqrt{\frac{\mu}{\varepsilon}}\,\sin(k\,z+\omega\,t)^2\,k\,\gamma\right)$

*****************************   *END PROCEDURE*   ***************************** **(21)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 4b-- Complex Circular Polarization A^G<>0, A^F <> 0
  OUTBOUND`;
> theta:=(-k*z+omega*t);
> Ax:=cos(theta);Ay:=-I*sin(theta);Az:=0;phi:=0;
```

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,gamma,0):
```

****

NAME := *Example 4b-- Complex Circular Polarization A^G <>0, A^F <> 0   OUTBOUND*

$$\theta := -k\,z+\omega\,t$$

$$Ax := \cos(k\,z-\omega\,t)$$

$$Ay := I\sin(k\,z-\omega\,t)$$

$$Az := 0$$

$\phi := 0$

Example 4b-- Complex Circular Polarization A^G <>0, A^F <> 0 OUTBOUND

********************************* *Differential Form Format* ********************
****

$$Action\ 1\text{-}form = d(x)\cos(kz-\omega t) + I\sin(kz-\omega t)\,d(y)$$

$$Intensity\ 2\text{-}form\ F{=}dA = -\sin(kz-\omega t)\,\omega\,(d(x))\ \&\wedge\ (d(t)) + \sin(kz-\omega t)\,k\,(d(x))\ \&\wedge\ (d(z)) + I\cos(kz-\omega t)\,\omega\,(d(y))\ \&\wedge\ (d(t)) - I\cos(kz-\omega t)\,k\,(d(y))\ \&\wedge\ (d(z))$$

$$Topological\ Torsion\ 3\text{-}form\quad A{\wedge}F = \left(I\sin(kz-\omega t)^2\,\omega + I\cos(kz-\omega t)^2\,\omega\right)\ \&\wedge(d(x), d(y), d(t)) + \left(-I\sin(kz-\omega t)^2\,k - I\cos(kz-\omega t)^2\,k\right)\ \&\wedge(d(x), d(y), d(z))$$

$$Topological\ Parity\ 4\text{-}form\quad F{\wedge}F = 0$$

******************************** *Using EM format* ****************************
****

$$E\ field = [-\sin(kz-\omega t)\,\omega,\ I\cos(kz-\omega t)\,\omega,\ 0]$$

$$B\ field = [-I\cos(kz-\omega t)\,k,\ -\sin(kz-\omega t)\,k,\ 0]$$

$$Topological\ TORSION\ 4\ vector\quad T4 = -[ExA + Bphi, AdotB] = [0, 0, I\,(\sin(kz-\omega t) - I\cos(kz-\omega t))\,(\sin(kz-\omega t) + I\cos(kz-\omega t))\,\omega,\ I\,(\sin(kz-\omega t) - I\cos(kz-\omega t))\,(\sin(kz-\omega t) + I\cos(kz-\omega t))\,k]$$

$$Helicity\ AdotB = -I\,(\sin(kz-\omega t) - I\cos(kz-\omega t))\,(\sin(kz-\omega t) + I\cos(kz-\omega t))\,k$$

$$Poincare\ II = 2(E.B) = 0$$

$$coefficient\ of\ Topological\ Parity\ 4\text{-}form = 0$$

$$Pfaff\ Topological\ Dimension\quad PTD = 3$$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

$$Xm\ or\ linear\ (Mean)\ curvature = 0$$

$$Yg\ or\ quadratic\ (GAUSS)\ curvature = 0$$

$$Za\ or\ Cubic\ (Interaction\ internal\ energy)\ curvature = 0$$

$$Tk\ or\ quartic\ (4D\ expansion)\ curvature = 0$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$Chirality\ factor\ CH = \gamma\sqrt{\frac{\mu}{\varepsilon}}$$

$$D\ field = \left[-\varepsilon \sin(kz - \omega t)\ \omega - I\gamma \sqrt{\frac{\mu}{\varepsilon}} \cos(kz - \omega t)\ k, I\varepsilon \cos(kz - \omega t)\ \omega - \sqrt{\frac{\mu}{\varepsilon}} \sin(kz - \omega t)\ k\gamma, 0 \right]$$

$$H\ field = \left[\frac{-I\cos(kz - \omega t)\ k + \gamma \sqrt{\frac{\mu}{\varepsilon}} \sin(kz - \omega t)\ \omega\mu}{\mu}, -\frac{\sin(kz - \omega t)\ k + I\gamma \sqrt{\frac{\mu}{\varepsilon}} \cos(kz - \omega t)\ \omega\mu}{\mu}, 0 \right]$$

$$Poynting\ vector\ ExH = \left[\begin{matrix} 0 & 0 & -\frac{\omega k \left(2\cos(kz - \omega t)^2 - 1\right)}{\mu} \end{matrix} \right]$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt\text{=}J4 \; = \left[\frac{\cos(kz - \omega t)\ (k^2 - \mu\omega^2\varepsilon)}{\mu}, \frac{I\sin(kz - \omega t)\ (k^2 - \mu\omega^2\varepsilon)}{\mu}, 0, 0 \right]$$

$$Amerian\ charge\ density \quad divD\ = rho = 0$$

$$divergence\ Lorentz\ Current\ 4Vector, \quad 4div(J4)\ = 0$$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[0, 0, -\frac{1}{\mu} \left(I \left(-2I\cos(kz - \omega t)\sin(kz - \omega t)\ k + \gamma\sqrt{\frac{\mu}{\varepsilon}} \cos(kz - \omega t)^2 \omega\mu + \sin(kz - \omega t)^2 \gamma \sqrt{\frac{\mu}{\varepsilon}}\ \omega\mu \right) \right), -I \left(-2I\cos(kz - \omega t)\sin(kz - \omega t)\ \varepsilon\omega + \gamma\sqrt{\frac{\mu}{\varepsilon}} \cos(kz - \omega t)^2 k + \sin(kz - \omega t)^2 \sqrt{\frac{\mu}{\varepsilon}}\ k\gamma \right) \right]$$

$$Topological\ SPIN\ 3\text{-}form = -\frac{1}{\mu} \left(I \left(-2I\cos(kz - \omega t)\sin(kz - \omega t)\ k + \gamma\sqrt{\frac{\mu}{\varepsilon}} \cos(kz - \omega t)^2 \omega\mu + \sin(kz - \omega t)^2 \gamma\sqrt{\frac{\mu}{\varepsilon}}\ \omega\mu \right) \&\wedge(d(x), d(y), d(t)) \right) + I \left(-2I\cos(kz - \omega t)\sin(kz - \omega t)\ \varepsilon\omega + \gamma\sqrt{\frac{\mu}{\varepsilon}} \cos(kz - \omega t)^2 k + \sin(kz - \omega t)^2 \sqrt{\frac{\mu}{\varepsilon}}\ k\gamma \right) \&\wedge(d(x), d(y), d(z))$$

Spin density rho_spin $= -I\left(-2\,I\cos(kz-\omega t)\,\sin(kz-\omega t)\,\varepsilon\,\omega+\gamma\sqrt{\frac{\mu}{\varepsilon}}\,\cos(kz-\omega t)^2\,k\right.$

$$\left.+\sin(kz-\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,k\,\gamma\right)$$

LaGrange field energy density (B.H-D.E)

$$= \frac{(k^2-\mu\,\omega^2\,\varepsilon)\,(\sin(kz-\omega t)-\cos(kz-\omega t))\,(\sin(kz-\omega t)+\cos(kz-\omega t))}{\mu}$$

$$B.H = \frac{k^2\,(\sin(kz-\omega t)-\cos(kz-\omega t))\,(\sin(kz-\omega t)+\cos(kz-\omega t))}{\mu}$$

$$D.E = \omega^2\,\varepsilon\,(\sin(kz-\omega t)-\cos(kz-\omega t))\,(\sin(kz-\omega t)+\cos(kz-\omega t))$$

$$A.J = -\frac{(k^2-\mu\,\omega^2\,\varepsilon)\,(\sin(kz-\omega t)-\cos(kz-\omega t))\,(\sin(kz-\omega t)+\cos(kz-\omega t))}{\mu}$$

$$\textit{-rho.phi} = 0$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)*

$$= \frac{2\,(k^2-\mu\,\omega^2\,\varepsilon)\,(\sin(kz-\omega t)-\cos(kz-\omega t))\,(\sin(kz-\omega t)+\cos(kz-\omega t))}{\mu}$$

London Coefficient $LC = \dfrac{k^2-\mu\,\omega^2\,\varepsilon}{\mu}$

PROCA coefficient curlcurlB $= [-I\cos(kz-\omega t)\,k^3,\ -k^3\sin(kz-\omega t),\ 0]$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \left[\frac{\cos(kz-\omega t)\,(k^2-\mu\,\omega^2\,\varepsilon)}{\mu},\right.$

$$\left.\frac{I\sin(kz-\omega t)\,(k^2-\mu\,\omega^2\,\varepsilon)}{\mu},\ 0,\ 0\right]$$

Lorentz Force 3 vector due to Ampere current *FL = -(rho_ampere E + J_ampere x B)* $= \left[0,\right.$

$$\left.0,\ \frac{2\cos(kz-\omega t)\,(k^2-\mu\,\omega^2\,\varepsilon)\,\sin(kz-\omega t)\,k}{\mu}\right]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector *T4 = -[ExA + B.phi,AdotB]* $= [0, 0, I\,(\sin(kz-\omega t)$

$$-\mathrm{I}\cos(kz-\omega t))\,(\sin(kz-\omega t)+\mathrm{I}\cos(kz-\omega t))\,\omega,\ \mathrm{I}\,(\sin(kz-\omega t)-\mathrm{I}\cos(kz-\omega t))\,(\sin(kz-\omega t)+\mathrm{I}\cos(kz-\omega t))\,k]$$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \left[0, 0, -\frac{1}{\mu}\left(\mathrm{I}\left(-2\,\mathrm{I}\cos(kz-\omega t)\sin(kz-\omega t)\,k+\gamma\sqrt{\frac{\mu}{\varepsilon}}\cos(kz-\omega t)^2\omega\mu+\sin(kz-\omega t)^2\gamma\sqrt{\frac{\mu}{\varepsilon}}\,\omega\mu\right)\right),\right.$
$\left.-\mathrm{I}\left(-2\,\mathrm{I}\cos(kz-\omega t)\sin(kz-\omega t)\,\varepsilon\omega+\gamma\sqrt{\frac{\mu}{\varepsilon}}\cos(kz-\omega t)^2k+\sin(kz-\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,k\gamma\right)\right]$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B)

$$= \left[\frac{2\cos(kz-\omega t)\sin(kz-\omega t)^2(k^2-\mu\omega^2\varepsilon)}{\mu}, -\frac{2\,\mathrm{I}\cos(kz-\omega t)^2\sin(kz-\omega t)\,(k^2-\mu\omega^2\varepsilon)}{\mu}, 0\right]$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \left[2\cos(kz-\omega t)\sin(kz-\omega t)^2(k^2-\mu\omega^2\varepsilon), -2\,\mathrm{I}\cos(kz-\omega t)^2\sin(kz-\omega t)\,(k^2-\mu\omega^2\varepsilon), \frac{2\cos(kz-\omega t)\,(k^2-\mu\omega^2\varepsilon)\sin(kz-\omega t)\,k}{\mu}\right]$

Dissipation $= -\mathrm{I}\,\mu\left(-2\,\mathrm{I}\cos(kz-\omega t)\sin(kz-\omega t)\,\varepsilon\omega+\gamma\sqrt{\frac{\mu}{\varepsilon}}\cos(kz-\omega t)^2k+\sin(kz-\omega t)^2\sqrt{\frac{\mu}{\varepsilon}}\,k\gamma\right)$

***************************** *END PROCEDURE* *************************** **(22)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 5a-- waveguide TM mode  (group kinematic in, wave in)`;
> theta:=(k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=-vg*f(x,y)*cos(theta);
```
Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)
```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,gamma,0):
```
****

NAME := *Example 5a-- waveguide TM mode (group kinematic in, wave in)*

$$\theta := k\,z + \omega\,t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z+\omega\,t)$$

$$\phi := -vg\,\left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z+\omega\,t)$$

Example 5a-- waveguide TM mode (group kinematic in, wave in)

********************************** *Differential Form Format* *********************************

$$\textit{Action 1-form} = \left(vg\cos(k\,z+\omega\,t)\,x(x,y)^2 + vg\cos(k\,z+\omega\,t)\,y(x,y)^2\right)d(t) + \left(\cos(k\,z+\omega\,t)\,x(x,y)^2 + \cos(k\,z+\omega\,t)\,y(x,y)^2\right)d(z)$$

$$\textit{Intensity 2-form F=dA} = \left(2\,vg\cos(k\,z+\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) + 2\,vg\cos(k\,z+\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)\right)(d(x))\ \&\wedge\ (d(t)) + \left(2\cos(k\,z+\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) + 2\cos(k\,z+\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)\right)(d(x))\ \&\wedge\ (d(z)) + \left(2\,vg\cos(k\,z+\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) + 2\,vg\cos(k\,z+\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)\right)(d(y))\ \&\wedge\ (d(t)) + \left(2\cos(k\,z+\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) + 2\cos(k\,z+\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)\right)(d(y))\ \&\wedge\ (d(z)) + \left(\sin(k\,z+\omega\,t)\,\omega\,x(x,y)^2 + \sin(k\,z+\omega\,t)\,\omega\,y(x,y)^2 - vg\sin(k\,z+\omega\,t)\,k\,x(x,y)^2 - vg\sin(k\,z+\omega\,t)\,k\,y(x,y)^2\right)(d(z))\ \&\wedge\ (d(t))$$

Topological Torsion 3-form $A\wedge F = 0$

Topological Parity 4-form $F\wedge F = 0$

********************************* *Using EM format* ******************************

$$\textit{E field} = \left[2\,vg\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z+\omega\,t),\ 2\,vg\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z+\omega\,t),\ -\sin(k\,z+\omega\,t)\left(x(x,y)^2\right.\right.$$

$+ y(x,y)^2)\ (-\omega + vg\, k)\Big]$

$$B\ field = \left[2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z+\omega\,t),\ -2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z+\omega\,t),\ 0\right]$$

Topological TORSION 4 vector   T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension   PTD = 2

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

$$Xm\ or\ linear\ (Mean)\ curvature\ = -\sin(k\,z+\omega\,t)\ (x(x,y)^2 + y(x,y)^2)\ (k + \omega\,vg)$$

Yg or quadratic (GAUSS) curvature = 0

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$Chirality\ factor\ CH = \gamma\sqrt{\frac{\mu}{\varepsilon}}$$

$$D\ field = \left[2\cos(k\,z+\omega\,t)\left(\varepsilon\,vg\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + \left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\,vg\,\varepsilon + \gamma\sqrt{\frac{\mu}{\varepsilon}}\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + \gamma\sqrt{\frac{\mu}{\varepsilon}}\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right),\ 2\cos(k\,z+\omega\,t)\left(\varepsilon\,vg\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + \varepsilon\,vg\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) - \gamma\sqrt{\frac{\mu}{\varepsilon}}\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) - \left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\sqrt{\frac{\mu}{\varepsilon}}\,\gamma\right),\ -\varepsilon\sin(k\,z+\omega\,t)\ (x(x,y)^2 + y(x,y)^2)\ (-\omega + vg\,k)\right]$$

$$H\ field = \left[\frac{1}{\mu} \left(2\cos(k z + \omega t) \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) - \gamma \sqrt{\frac{\mu}{\varepsilon}}\, vg\, \mu\, x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) - \gamma \sqrt{\frac{\mu}{\varepsilon}}\, vg\, \mu\, y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \right), -\frac{1}{\mu} \left(2\cos(k z + \omega t) \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) + \gamma \sqrt{\frac{\mu}{\varepsilon}}\, vg\, \mu\, x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + \gamma \sqrt{\frac{\mu}{\varepsilon}}\, vg\, \mu\, y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \right), \gamma \sqrt{\frac{\mu}{\varepsilon}} \sin(k z + \omega t) \left(x(x,y)^2 + y(x,y)^2 \right) (-\omega + vg\, k) \right]$$

$$Poynting\ vector\ ExH = \left[-\frac{1}{\mu} \left(2\cos(k z + \omega t) \sin(k z + \omega t) \left(x(x,y)^2 + y(x,y)^2 \right) (-\omega + vg\, k) \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \right), -\frac{1}{\mu} \left(2\cos(k z + \omega t) \sin(k z + \omega t) \left(x(x,y)^2 + y(x,y)^2 \right) (-\omega + vg\, k) \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \right), -\frac{1}{\mu} \left(4\cos(k z + \omega t)^2 vg \left(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 \right) \right) \right]$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt\text{=}J4 \; = \left[-\frac{2\sin(k z + \omega t)\,(k - \omega \mu \varepsilon\, vg) \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right)}{\mu}, -\frac{2\sin(k z + \omega t)\,(k - \omega \mu \varepsilon\, vg) \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right)}{\mu}, \right.$$

$$-\frac{1}{\mu}\left(\cos(kz+\omega t)\left(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\right.\right.$$

$$+2y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$

$$+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\,\omega^2\,\mu\,x(x,y)^2-\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2$$

$$\left.\left.+\varepsilon\,\omega^2\,\mu\,y(x,y)^2-\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\right)\right),\ \varepsilon\cos(kz+\omega t)\left(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2\right.$$

$$+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 vg$$

$$+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg\,y(x,$$

$$\left.\left.y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+x(x,y)^2\,\omega\,k-k^2\,vg\,x(x,y)^2+k\,\omega\,y(x,y)^2-k^2\,vg\,y(x,y)^2\right)\right]$$

Amerian charge density $\quad divD = rho = \varepsilon\cos(kz+\omega t)\left(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,\right.$

$$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 vg$$

$$+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg\,y(x,$$

$$\left.y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+x(x,y)^2\,\omega\,k-k^2\,vg\,x(x,y)^2+k\,\omega\,y(x,y)^2-k^2\,vg\,y(x,y)^2\right)$$

divergence Lorentz Current 4Vector, $\quad$ *4div(J4)* $=0$

Topological SPIN 4 vector S4 $=\left[-\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\left(-1\right.\right.\right.$

$$\left.\left.+vg^2\,\varepsilon\,\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\right),\ -\frac{1}{\mu}\left(2\left(x(x,y)^2\right.\right.$$

$$\left.+y(x,y)^2\right)\cos(kz+\omega t)^2\left(-1+vg^2\,\varepsilon\,\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,\right.\right.$$

$$\left.\left.\left.y)\right)\right)\right),\ \varepsilon\sin(kz+\omega t)\left(x(x,y)^2+y(x,y)^2\right)^2\left(-\omega+vg\,k\right)vg\cos(kz+\omega t),$$

$$\left.-\left(x(x,y)^2+y(x,y)^2\right)^2\cos(kz+\omega t)\,\varepsilon\sin(kz+\omega t)\left(-\omega+vg\,k\right)\right]$$

Topological SPIN 3-form $=-\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\left(-1+vg^2\,\varepsilon\,\mu\right)\left(x(x,\right.\right.$

$$y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\&\text{^}(d(y),d(z),d(t))\Big)+\frac{1}{\mu}\Big(2\,(x(x,y)^2+y(x,y)^2)\cos(k z+\omega t)^2(-1+vg^2\varepsilon\mu)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\&\text{^}(d(x),d(z),d(t))\Big)+\varepsilon\sin(k z+\omega t)\,(x(x,y)^2+y(x,y)^2)^2(-\omega+vg\,k)\,vg\cos(k z+\omega t)\,\&\text{^}(d(x),d(y),d(t))+(x(x,y)^2+y(x,y)^2)^2\cos(k z+\omega t)\,\varepsilon\sin(k z+\omega t)\,(-\omega+vg\,k)\,\&\text{^}(d(x),d(y),d(z))$$

Spin density rho_spin $= -(x(x,y)^2+y(x,y)^2)^2\cos(k z+\omega t)\,\varepsilon\sin(k z+\omega t)\,(-\omega+vg\,k)$

LaGrange field energy density (B.H-D.E) $= -\frac{1}{\mu}\Big(-4\left(\frac{\partial}{\partial y}x(x,y)\right)^2x(x,y)^2\cos(k z+\omega t)^2-8\left(\frac{\partial}{\partial y}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)x(x,y)\cos(k z+\omega t)^2-4\left(\frac{\partial}{\partial y}y(x,y)\right)^2y(x,y)^2\cos(k z+\omega t)^2-4\left(\frac{\partial}{\partial x}x(x,y)\right)^2x(x,y)^2\cos(k z+\omega t)^2-8\left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)x(x,y)\cos(k z+\omega t)^2-4\,y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\cos(k z+\omega t)^2+4\,\mu\,\varepsilon\cos(k z+\omega t)^2vg^2x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+8\,\mu\,\varepsilon\cos(k z+\omega t)^2vg^2x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+4\,\mu\,\varepsilon\cos(k z+\omega t)^2vg^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2y(x,y)^2+4\,\mu\,\varepsilon\cos(k z+\omega t)^2vg^2x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+8\,\mu\,\varepsilon\cos(k z+\omega t)^2vg^2x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+4\,\mu\,\varepsilon\cos(k z+\omega t)^2vg^2y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+\mu\,\varepsilon\sin(k z+\omega t)^2x(x,y)^4\omega^2-2\,\mu\,\varepsilon\sin(k z+\omega t)^2\omega\,vg\,k\,x(x,y)^4+\mu\,\varepsilon\sin(k z+\omega t)^2x(x,y)^4vg^2k^2+2\,\mu\,\varepsilon\sin(k z+\omega t)^2x(x,y)^2y(x,y)^2\omega^2-4\,\mu\,\varepsilon\sin(k z+\omega t)^2\omega\,vg\,k\,x(x,y)^2y(x,y)^2+2\,\mu\,\varepsilon\sin(k z+\omega t)^2x(x,y)^2y(x,y)^2vg^2k^2+\mu\,\varepsilon\sin(k z+\omega t)^2y(x,y)^4\omega^2-2\,\mu\,\varepsilon\sin(k z+\omega t)^2\omega\,vg\,k\,y(x,y)^4+\mu\,\varepsilon\sin(k z+\omega t)^2y(x,y)^4vg^2k^2\Big)$

B.H $=\frac{1}{\mu}\Big(4\cos(k z+\omega t)^2\Big(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,$

$$y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Bigg)\Bigg)$$

$$D.E=\varepsilon\Bigg(4\cos(kz+\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+8\cos(kz+\omega t)^2 vg^2 x(x,$$
$$y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+4\cos(kz+\omega t)^2 vg^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 y(x,y)^2$$
$$+4\cos(kz+\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+8\cos(kz+\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial y}x(x,$$
$$y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+4\cos(kz+\omega t)^2 vg^2 y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+\sin(kz$$
$$+\omega t)^2 x(x,y)^4\omega^2-2\sin(kz+\omega t)^2\omega\, vg\, k\, x(x,y)^4+\sin(kz+\omega t)^2 x(x,y)^4 vg^2 k^2$$
$$+2\sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2\omega^2-4\sin(kz+\omega t)^2\omega\, vg\, k\, x(x,y)^2 y(x,y)^2$$
$$+2\sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2+\sin(kz+\omega t)^2 y(x,y)^4\omega^2-2\sin(kz$$
$$+\omega t)^2\omega\, vg\, k\, y(x,y)^4+\sin(kz+\omega t)^2 y(x,y)^4 vg^2 k^2\Bigg)$$

$$A.J=-\frac{1}{\mu}\Bigg(\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\Bigg(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,$$
$$y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2x(x,$$
$$y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\omega^2\mu\, x(x,y)^2$$
$$-\varepsilon\omega\mu\, vg\, k\, x(x,y)^2+\varepsilon\omega^2\mu\, y(x,y)^2-\varepsilon\omega\mu\, vg\, k\, y(x,y)^2\Bigg)\Bigg)$$

$$\text{-}rho.phi=-vg\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\varepsilon\Bigg(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,$$
$$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 vg$$
$$+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg\,y(x,$$
$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+x(x,y)^2\omega k-k^2 vg\,x(x,y)^2+k\omega\,y(x,y)^2-k^2 vg\,y(x,y)^2\Bigg)$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* $=-\frac{1}{\mu}\Bigg(-6\,y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\cos(kz+\omega t)^2$

$$-6\left(\frac{\partial}{\partial y}x(x,y)\right)^2 x(x,y)^2\cos(kz+\omega t)^2-6\left(\frac{\partial}{\partial y}y(x,y)\right)^2 y(x,y)^2\cos(kz+\omega t)^2$$

$$-6\left(\frac{\partial}{\partial x}x(x,y)\right)^2 x(x,y)^2\cos(kz+\omega t)^2-2\cos(kz+\omega t)^2 x(x,y)^3\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$

$$-2\cos(kz+\omega t)^2 y(x,y)^3\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)-2\cos(kz+\omega t)^2 y(x,y)^3\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)$$

$$-2\cos(kz+\omega t)^2 x(x,y)^3\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\cos(kz+\omega t)^2 x(x,y)^2 vg^2\varepsilon\mu\, y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+2\cos(kz+\omega t)^2 x(x,y)^2 vg^2\varepsilon\mu\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)$$

$$+2\cos(kz+\omega t)^2 y(x,y)^2 vg^2\varepsilon\mu\, x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\cos(kz+\omega t)^2 y(x,$$

$$y)^2 vg^2\varepsilon\mu\, x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-2\cos(kz+\omega t)^2 x(x,y)^2\varepsilon\,\omega^2\mu\, y(x,y)^2+2\cos(kz$$

$$+\omega t)^2 x(x,y)^2 vg^2\varepsilon\mu\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\cos(kz+\omega t)^2 x(x,y)^2 vg^2\varepsilon\mu\left(\frac{\partial}{\partial y}y(x,y)\right)^2$$

$$-\cos(kz+\omega t)^2 x(x,y)^4 vg^2\varepsilon\mu k^2+2\cos(kz+\omega t)^2 y(x,y)^2 vg^2\varepsilon\mu\left(\frac{\partial}{\partial x}x(x,y)\right)^2$$

$$+2\cos(kz+\omega t)^2 y(x,y)^2 vg^2\varepsilon\mu\left(\frac{\partial}{\partial y}x(x,y)\right)^2-\cos(kz+\omega t)^2 y(x,y)^4 vg^2\varepsilon\mu k^2$$

$$-2\cos(kz+\omega t)^2 x(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2-2\cos(kz+\omega t)^2 x(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2$$

$$-2\cos(kz+\omega t)^2 y(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2-2\cos(kz+\omega t)^2 y(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$-2\cos(kz+\omega t)^2 x(x,y)^2 y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-2\cos(kz+\omega t)^2 y(x,y)^2\left(\frac{\partial^2}{\partial y^2}x(x,$$

$$y)\right)x(x,y)-2\cos(kz+\omega t)^2 y(x,y)^2\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)x(x,y)+6\mu\varepsilon\cos(kz$$

$$+\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+6\mu\varepsilon\cos(kz+\omega t)^2 vg^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 y(x,y)^2$$

$$+6\mu\varepsilon\cos(kz+\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+6\mu\varepsilon\cos(kz+\omega t)^2 vg^2 y(x,$$

$$y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+\mu\varepsilon\sin(kz+\omega t)^2 x(x,y)^4 vg^2 k^2+2\mu\varepsilon\sin(kz+\omega t)^2 x(x,$$

$$y)^2 y(x,y)^2 \omega^2 + \mu\,\varepsilon \sin(k z+\omega t)^2 y(x,y)^4 vg^2 k^2 - \cos(k z+\omega t)^2 x(x,y)^4 \varepsilon\,\omega^2 \mu$$

$$- \cos(k z+\omega t)^2 y(x,y)^4 \varepsilon\,\omega^2 \mu + 8\,\mu\,\varepsilon \cos(k z+\omega t)^2 vg^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 8\,\mu\,\varepsilon \cos(k z+\omega t)^2 vg^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)$$

$$- 4\,\mu\,\varepsilon \sin(k z+\omega t)^2 \omega\, vg\, k\, x(x,y)^2 y(x,y)^2 - 2\,\mu\,\varepsilon \sin(k z+\omega t)^2 \omega\, vg\, k\, y(x,y)^4$$

$$+ 2\,\mu\,\varepsilon \sin(k z+\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 - 2\,\mu\,\varepsilon \sin(k z+\omega t)^2 \omega\, vg\, k\, x(x,y)^4$$

$$+ 4 \cos(k z+\omega t)^2 x(x,y)^2 \varepsilon\,\omega\,\mu\, vg\, k\, y(x,y)^2 + 2 \cos(k z+\omega t)^2 x(x,y)^4 \varepsilon\,\omega\,\mu\, vg\, k$$

$$- 2 \cos(k z+\omega t)^2 x(x,y)^2 vg^2 \varepsilon\,\mu\, k^2 y(x,y)^2 + 2 \cos(k z+\omega t)^2 y(x,y)^4 \varepsilon\,\omega\,\mu\, vg\, k$$

$$- 8 \left(\frac{\partial}{\partial y} y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) x(x,y) \cos(k z+\omega t)^2 - 8 \left(\frac{\partial}{\partial x} y(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial x} x(x,y)\right) x(x,y) \cos(k z+\omega t)^2 + \mu\,\varepsilon \sin(k z+\omega t)^2 x(x,y)^4 \omega^2 + \mu\,\varepsilon \sin(k z$$

$$+ \omega t)^2 y(x,y)^4 \omega^2 + 2 \cos(k z+\omega t)^2 x(x,y)^3 vg^2 \varepsilon\,\mu \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2 \cos(k z$$

$$+ \omega t)^2 x(x,y)^3 vg^2 \varepsilon\,\mu \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2 \cos(k z+\omega t)^2 y(x,y)^3 vg^2 \varepsilon\,\mu \left(\frac{\partial^2}{\partial x^2} y(x,y)\right)$$

$$+ 2 \cos(k z+\omega t)^2 y(x,y)^3 vg^2 \varepsilon\,\mu \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - 2 \cos(k z+\omega t)^2 x(x,y)^2 y(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right)\Bigg)$$

$$\textit{London Coefficient} \qquad LC = \frac{k^2 - \mu\,\omega^2 \varepsilon}{\mu}$$

$$\textit{PROCA coefficient curlcurlB} = \Bigg[2 \cos(k z+\omega t) \Bigg(-2 \left(\frac{\partial}{\partial x} x(x,y)\right) \left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right)$$

$$- \left(\frac{\partial}{\partial y} x(x,y)\right) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - x(x,y) \left(\frac{\partial^3}{\partial y\,\partial x^2} x(x,y)\right) - 2 \left(\frac{\partial}{\partial x} y(x,$$

$$y)\right) \left(\frac{\partial^2}{\partial y\,\partial x} y(x,y)\right) - \left(\frac{\partial}{\partial y} y(x,y)\right) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) - y(x,y) \left(\frac{\partial^3}{\partial y\,\partial x^2} y(x,y)\right)$$

$$-3\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-x(x,y)\left(\frac{\partial^3}{\partial y^3}x(x,y)\right)-3\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-y(x,y)\left(\frac{\partial^3}{\partial y^3}y(x,y)\right)+k^2x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+k^2y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Bigg),$$

$$2\cos(kz+\omega t)\Bigg(-k^2x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)-k^2y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+3\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+x(x,y)\left(\frac{\partial^3}{\partial x^3}x(x,y)\right)+3\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+y(x,y)\left(\frac{\partial^3}{\partial x^3}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}x(x,y)\right)+\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+x(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}y(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+y(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}y(x,y)\right)\Bigg),0\Bigg]$$

Amperian Current 4Vector *curlH-dD/dt=J4* $=\Bigg[$

$$-\frac{2\sin(kz+\omega t)\,(k-\omega\mu\varepsilon vg)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)}{\mu},$$

$$-\frac{2\sin(kz+\omega t)\,(k-\omega\mu\varepsilon vg)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)}{\mu},$$

$$-\frac{1}{\mu}\Bigg(\cos(kz+\omega t)\Bigg(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\omega^2\mu\,x(x,y)^2-\varepsilon\omega\mu\,vg\,k\,x(x,y)^2+\varepsilon\omega^2\mu\,y(x,y)^2-\varepsilon\omega\mu\,vg\,k\,y(x,y)^2\Bigg)\Bigg),\ \varepsilon\cos(kz+\omega t)\Bigg(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2vg$$

$$+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+x(x,y)^2\,\omega\,k-k^2\,vg\,x(x,y)^2+k\,\omega\,y(x,y)^2-k^2\,vg\,y(x,y)^2\Big)\Big]$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B)   = [

$$-\frac{1}{\mu}\Big(2\cos(k\,z+\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\Big($$

$$-2\left(\frac{\partial}{\partial x}x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-2\left(\frac{\partial}{\partial x}y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,$$

$$y)\right)-2\left(\frac{\partial}{\partial y}x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-2\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-\varepsilon\,\omega^2\,\mu\,x(x,y)^2+2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2-\varepsilon\,\omega^2\,\mu\,y(x,y)^2$$

$$+2\,\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-vg^2\,\varepsilon\,\mu\,k^2\,x(x,y)^2-vg^2\,\varepsilon\,\mu\,k^2\,y(x,y)^2\Big)\Big),\ -\frac{1}{\mu}\Big(2\cos(k\,z$$

$$+\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\Big(-2\left(\frac{\partial}{\partial x}x(x,y)\right)^2-2\,x(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-2\left(\frac{\partial}{\partial x}y(x,y)\right)^2-2y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)-2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$-2x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-2y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-\varepsilon\,\omega^2\mu\,x(x,y)^2$$

$$+2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2-\varepsilon\,\omega^2\mu\,y(x,y)^2+2\,\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2+2\,vg^2\varepsilon\,\mu\left(\frac{\partial}{\partial x}x(x,y)\right)^2$$

$$+2\,vg^2\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\,vg^2\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)$$

$$+2\,vg^2\varepsilon\,\mu\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,vg^2\varepsilon\,\mu\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg^2\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$

$$+2\,vg^2\varepsilon\,\mu\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg^2\varepsilon\,\mu\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-vg^2\varepsilon\,\mu\,k^2x(x,y)^2$$

$$-vg^2\varepsilon\,\mu\,k^2y(x,y)^2\Big)\Big)\Big],\ \frac{1}{\mu}\Big(\sin(k\,z+\omega\,t)\cos(k\,z+\omega\,t)\Big(-2\,\mu\,\varepsilon\,vg\,x(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)y(x,y)^2\omega-2\,\mu\,\varepsilon\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)y(x,y)^2\omega$$

$$+2\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg^2x(x,y)^2k-2\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg\,x(x,y)^2\omega$$

$$+2\,\mu\,\varepsilon\,vg^2x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)y(x,y)^2k+2\,\mu\,\varepsilon\,vg^2y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)x(x,y)^2k$$

$$-2\,\mu\,\varepsilon\,vg\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)x(x,y)^2\omega+2\,\mu\,\varepsilon\,vg^2x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)y(x,y)^2k$$

$$-\mu\,\varepsilon\,\omega^2k\,x(x,y)^4-\mu\,\varepsilon\,k^3vg^2x(x,y)^4-\mu\,\varepsilon\,\omega^2k\,y(x,y)^4-\mu\,\varepsilon\,k^3vg^2y(x,y)^4-8\,k\,x(x,$$

$$y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)-8\,k\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)$$

$$-2\,\mu\,\varepsilon\left(\frac{\partial}{\partial x}y(x,y)\right)^2vg\,x(x,y)^2\omega+2\,\mu\,\varepsilon\left(\frac{\partial}{\partial x}y(x,y)\right)^2vg^2x(x,y)^2k$$

$$+2\,\mu\,\varepsilon\,vg^{y(x,y)^2}\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2\omega+2\,\mu\,\varepsilon\,vg^2\,y(x,y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 k$$

$$+2\,\mu\,\varepsilon\,vg^{x(x,y)^2}\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2\omega+2\,\mu\,\varepsilon\,vg^2\,x(x,y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 k$$

$$-2\,\mu\,\varepsilon\,vg\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 y(x,y)^2\,\omega+2\,\mu\,\varepsilon\,vg^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 y(x,y)^2\,k$$

$$-2\,\mu\,\varepsilon\,vg\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 x(x,y)^2\,\omega+2\,\mu\,\varepsilon\,vg^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 x(x,y)^2\,k$$

$$+2\,\mu\,\varepsilon\,vg^{y(x,y)^2}\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2\omega+2\,\mu\,\varepsilon\,vg^2\,y(x,y)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 k$$

$$+2\,\mu\,\varepsilon\,\omega\,k^2\,x(x,y)^4\,vg-2\,\mu\,\varepsilon\,\omega^2\,k\,x(x,y)^2\,y(x,y)^2-2\,\mu\,\varepsilon\,k^3\,vg^2\,x(x,y)^2\,y(x,y)^2$$

$$+2\,\mu\,\varepsilon\,\omega\,k^2\,y(x,y)^4\,vg+2\,\mu\,\varepsilon\,vg\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2\omega+2\,\mu\,\varepsilon\,vg^2\,x(x,$$

$$y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 k-2\,\mu\,\varepsilon\,vg\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 y(x,y)^2\,\omega+2\,\mu\,\varepsilon\,vg^2\left(\frac{\partial}{\partial x}\,x(x,\right.$$

$$\left.y)\right)^2 y(x,y)^2\,k+4\,\mu\,\varepsilon\,\omega\,k^2\,x(x,y)^2\,y(x,y)^2\,vg+2\,\mu\,\varepsilon\,vg^2\,x(x,y)^3\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)k$$

$$+2\,\mu\,\varepsilon\,vg^2\,y(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)k-2\,\mu\,\varepsilon\,vg\,x(x,y)^3\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)\omega$$

$$+2\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)^3\,vg^2\,k-2\,\mu\,\varepsilon\,vg\,x(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)\omega$$

$$+2\,\mu\,\varepsilon\,vg^2\,x(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)k-2\,\mu\,\varepsilon\,vg\,y(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)\omega$$

$$-2\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)^3\,vg\,\omega+8\,\omega\,\mu\,\varepsilon\,vg\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,y(x,\right.$$

$$\left.y)\right)+8\,\omega\,\mu\,\varepsilon\,vg\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)-4\,k\,x(x,y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2$$

$$-4\,k\,y(x,y)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2-4\,k\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2-4\,k\,y(x,y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2\Bigg)$$

$$\Bigg)\Bigg]$$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] = [0, 0, 0, 0]

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) = [0, 0, 0]

Torsion Dissipation Jtorsion dot E = 0

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] =

$$\left[-\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\left(-1+vg^2\varepsilon\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\right), -\frac{1}{\mu}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\left(-1+vg^2\varepsilon\mu\right)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\right), \varepsilon\sin(kz+\omega t)\left(x(x,y)^2+y(x,y)^2\right)^2(-\omega+vg\,k)\,vg\cos(kz+\omega t), -\left(x(x,y)^2+y(x,y)^2\right)^2\cos(kz+\omega t)\,\varepsilon\sin(kz+\omega t)\,(-\omega+vg\,k)\right]$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) =

$$\left[0, 0, -\frac{1}{\mu}\left(\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)\left(-4\left(\frac{\partial}{\partial y}x(x,y)\right)^2 x(x,y)^2\cos(kz+\omega t)^2 - 8\left(\frac{\partial}{\partial y}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)x(x,y)\cos(kz+\omega t)^2 - 4\left(\frac{\partial}{\partial y}y(x,y)\right)^2 y(x,y)^2\cos(kz+\omega t)^2 - 4\left(\frac{\partial}{\partial x}x(x,y)\right)^2 x(x,y)^2\cos(kz+\omega t)^2 - 8\left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)x(x,y)\cos(kz+\omega t)^2 - 4\,y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\cos(kz+\omega t)^2 + 4\mu\varepsilon\cos(kz+\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2 + 8\mu\varepsilon\cos(kz+\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) + 4\mu\varepsilon\cos(kz+\omega t)^2 vg^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 y(x,y)^2 + 4\mu\varepsilon\cos(kz+\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2 + 8\mu\varepsilon\cos(kz+\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) + 4\mu\varepsilon\cos(kz+\omega t)^2 vg^2 y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2\right.$$

$$+\mu\varepsilon\sin(kz+\omega t)^2 x(x,y)^4\omega^2 - 2\mu\varepsilon\sin(kz+\omega t)^2\omega\, vg\, k\, x(x,y)^4 + \mu\varepsilon\sin(kz+\omega t)^2 x(x,y)^4 vg^2 k^2 + 2\mu\varepsilon\sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2\omega^2 - 4\mu\varepsilon\sin(kz+\omega t)^2\omega\, vg\, k\, x(x,y)^2 y(x,y)^2 + 2\mu\varepsilon\sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 + \mu\varepsilon\sin(kz+\omega t)^2 y(x,y)^4\omega^2 - 2\mu\varepsilon\sin(kz+\omega t)^2\omega\, vg\, k\, y(x,y)^4 + \mu\varepsilon\sin(kz+\omega t)^2 y(x,y)^4 vg^2 k^2)\,)\,]$$

Spin Dissipation J_spin dot E $= -\frac{1}{\mu}\Big((x(x,y)^2+y(x,y)^2)\cos(kz+\omega t)\, vg\Big(-4\left(\frac{\partial}{\partial y}x(x,y)\right)^2 x(x,y)^2\cos(kz+\omega t)^2 - 8\left(\frac{\partial}{\partial y}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)x(x,y)\cos(kz+\omega t)^2 - 4\left(\frac{\partial}{\partial y}y(x,y)\right)^2 y(x,y)^2\cos(kz+\omega t)^2 - 4\left(\frac{\partial}{\partial x}x(x,y)\right)^2 x(x,y)^2\cos(kz+\omega t)^2 - 8\left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)x(x,y)\cos(kz+\omega t)^2 - 4y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\cos(kz+\omega t)^2 + 4\mu\varepsilon\cos(kz+\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2 + 8\mu\varepsilon\cos(kz+\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) + 4\mu\varepsilon\cos(kz+\omega t)^2 vg^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 y(x,y)^2 + 4\mu\varepsilon\cos(kz+\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2 + 8\mu\varepsilon\cos(kz+\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) + 4\mu\varepsilon\cos(kz+\omega t)^2 vg^2 y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 + \mu\varepsilon\sin(kz+\omega t)^2 x(x,y)^4\omega^2 - 2\mu\varepsilon\sin(kz+\omega t)^2\omega\, vg\, k\, x(x,y)^4 + \mu\varepsilon\sin(kz+\omega t)^2 x(x,y)^4 vg^2 k^2 + 2\mu\varepsilon\sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2\omega^2 - 4\mu\varepsilon\sin(kz+\omega t)^2\omega\, vg\, k\, x(x,y)^2 y(x,y)^2 + 2\mu\varepsilon\sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 + \mu\varepsilon\sin(kz+\omega t)^2 y(x,y)^4\omega^2 - 2\mu\varepsilon\sin(kz+\omega t)^2\omega\, vg\, k\, y(x,y)^4 + \mu\varepsilon\sin(kz+\omega t)^2 y(x,y)^4 vg^2 k^2\Big)\Big)$

Disspative Force 3 vector $= \Big[-\frac{1}{\mu}\Big(2\cos(kz+\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\Big(-2\left(\frac{\partial}{\partial x}x(x,y)\right)^2 - 2x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right) - 2\left(\frac{\partial}{\partial x}y(x,y)\right)^2$

$$- 2\, y(x, y) \left(\frac{\partial^2}{\partial x^2} y(x, y)\right) - 2 \left(\frac{\partial}{\partial y} x(x, y)\right)^2 - 2\, x(x, y) \left(\frac{\partial^2}{\partial y^2} x(x, y)\right)$$

$$- 2 \left(\frac{\partial}{\partial y} y(x, y)\right)^2 - 2\, y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y)\right) - \varepsilon\, \omega^2 \mu\, x(x, y)^2 + 2\, \varepsilon\, \omega\, \mu\, vg\, k\, x(x, y)^2$$

$$- \varepsilon\, \omega^2 \mu\, y(x, y)^2 + 2\, \varepsilon\, \omega\, \mu\, vg\, k\, y(x, y)^2 + 2\, vg^2\, \varepsilon\, \mu \left(\frac{\partial}{\partial x} x(x, y)\right)^2 + 2\, vg^2\, \varepsilon\, \mu\, x(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2} x(x, y)\right) + 2\, vg^2\, \varepsilon\, \mu \left(\frac{\partial^2}{\partial x^2} y(x, y)\right) y(x, y) + 2\, vg^2\, \varepsilon\, \mu \left(\frac{\partial}{\partial x} y(x, y)\right)^2$$

$$+ 2\, vg^2\, \varepsilon\, \mu \left(\frac{\partial}{\partial y} x(x, y)\right)^2 + 2\, vg^2\, \varepsilon\, \mu\, x(x, y) \left(\frac{\partial^2}{\partial y^2} x(x, y)\right) + 2\, vg^2\, \varepsilon\, \mu \left(\frac{\partial}{\partial y} y(x, y)\right)^2$$

$$+ 2\, vg^2\, \varepsilon\, \mu\, y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y)\right) - vg^2\, \varepsilon\, \mu\, k^2\, x(x, y)^2 - vg^2\, \varepsilon\, \mu\, k^2\, y(x, y)^2\Big)\Big),$$

$$- \frac{1}{\mu} \Bigg(2 \cos(k\, z + \omega\, t)^2 \left(x(x, y) \left(\frac{\partial}{\partial y} x(x, y)\right) + y(x, y) \left(\frac{\partial}{\partial y} y(x, y)\right)\right) \Bigg($$

$$-2 \left(\frac{\partial}{\partial x} x(x, y)\right)^2 - 2\, x(x, y) \left(\frac{\partial^2}{\partial x^2} x(x, y)\right) - 2 \left(\frac{\partial}{\partial x} y(x, y)\right)^2 - 2\, y(x, y) \left(\frac{\partial^2}{\partial x^2} y(x,$$

$$y)\right) - 2 \left(\frac{\partial}{\partial y} x(x, y)\right)^2 - 2\, x(x, y) \left(\frac{\partial^2}{\partial y^2} x(x, y)\right) - 2 \left(\frac{\partial}{\partial y} y(x, y)\right)^2 - 2\, y(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} y(x, y)\right) - \varepsilon\, \omega^2 \mu\, x(x, y)^2 + 2\, \varepsilon\, \omega\, \mu\, vg\, k\, x(x, y)^2 - \varepsilon\, \omega^2 \mu\, y(x, y)^2$$

$$+ 2\, \varepsilon\, \omega\, \mu\, vg\, k\, y(x, y)^2 + 2\, vg^2\, \varepsilon\, \mu \left(\frac{\partial}{\partial x} x(x, y)\right)^2 + 2\, vg^2\, \varepsilon\, \mu\, x(x, y) \left(\frac{\partial^2}{\partial x^2} x(x, y)\right)$$

$$+ 2\, vg^2\, \varepsilon\, \mu \left(\frac{\partial^2}{\partial x^2}\, y(x, y) \right) y(x, y) + 2\, vg^2\, \varepsilon\, \mu \left(\frac{\partial}{\partial x}\, y(x, y) \right)^2 + 2\, vg^2\, \varepsilon\, \mu \left(\frac{\partial}{\partial y}\, x(x, y) \right)^2$$

$$+ 2\, vg^2\, \varepsilon\, \mu\, x(x, y) \left(\frac{\partial^2}{\partial y^2}\, x(x, y) \right) + 2\, vg^2\, \varepsilon\, \mu \left(\frac{\partial}{\partial y}\, y(x, y) \right)^2 + 2\, vg^2\, \varepsilon\, \mu\, y(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2}\, y(x, y) \right) - vg^2\, \varepsilon\, \mu\, k^2\, x(x, y)^2 - vg^2\, \varepsilon\, \mu\, k^2\, y(x, y)^2 \Bigg) \Bigg), -\frac{1}{\mu} \Bigg(\cos(k\, z$$

$$+ \omega\, t) \Bigg(8\, y(x, y)^3 \cos(k\, z + \omega\, t)^2\, vg^2\, \mu^2\, \varepsilon\, x(x, y) \left(\frac{\partial}{\partial x}\, x(x, y) \right) \left(\frac{\partial}{\partial x}\, y(x, y) \right)$$

$$+ 8\, y(x, y)^3 \cos(k\, z + \omega\, t)^2\, vg^2\, \mu^2\, \varepsilon\, x(x, y) \left(\frac{\partial}{\partial y}\, x(x, y) \right) \left(\frac{\partial}{\partial y}\, y(x, y) \right) + 8\, x(x,$$

$$y)^3 \cos(k\, z + \omega\, t)^2\, vg^2\, \mu^2\, \varepsilon \left(\frac{\partial}{\partial x}\, x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial x}\, y(x, y) \right) - 8\, \mu\, y(x, y)^3 \cos(k\, z$$

$$+ \omega\, t)^2\, x(x, y) \left(\frac{\partial}{\partial y}\, x(x, y) \right) \left(\frac{\partial}{\partial y}\, y(x, y) \right) - 6\, x(x, y)^2\, \varepsilon \sin(k\, z + \omega\, t)^2\, \mu^2\, y(x,$$

$$y)^4\, \omega\, vg\, k + 8\, x(x, y)^3 \cos(k\, z + \omega\, t)^2\, vg^2\, \mu^2\, \varepsilon \left(\frac{\partial}{\partial y}\, x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial y}\, y(x, y) \right)$$

$$- 6\, x(x, y)^4\, \varepsilon \sin(k\, z + \omega\, t)^2\, \mu^2\, y(x, y)^2\, \omega\, vg\, k - 8\, \mu\, x(x, y)^3 \cos(k\, z + \omega\, t)^2 \left(\frac{\partial}{\partial x}\, x(x,$$

$$y) \right) y(x, y) \left(\frac{\partial}{\partial x}\, y(x, y) \right) + 4 \sin(k\, z + \omega\, t)\, k\, x(x, y)^2 \left(\frac{\partial}{\partial x}\, x(x, y) \right)^2 + 4 \sin(k\, z$$

$$+ \omega\, t)\, k\, y(x, y)^2 \left(\frac{\partial}{\partial x}\, y(x, y) \right)^2 + 4 \sin(k\, z + \omega\, t)\, k\, x(x, y)^2 \left(\frac{\partial}{\partial y}\, x(x, y) \right)^2 + 4 \sin(k\, z$$

$$+ \omega\, t)\, k\, y(x, y)^2 \left(\frac{\partial}{\partial y}\, y(x, y) \right)^2 - 4\, \mu\, y(x, y)^2 \cos(k\, z + \omega\, t)^2\, x(x, y)^2 \left(\frac{\partial}{\partial x}\, x(x, y) \right)^2$$

$$- 4\, \mu\, x(x, y)^2 \cos(k\, z + \omega\, t)^2\, y(x, y)^2 \left(\frac{\partial}{\partial x}\, y(x, y) \right)^2 - 4\, \mu\, x(x, y)^2 \cos(k\, z$$

$$+ \omega\, t)^2\, y(x, y)^2 \left(\frac{\partial}{\partial y}\, y(x, y) \right)^2 - 4\, \mu\, y(x, y)^2 \cos(k\, z + \omega\, t)^2\, x(x, y)^2 \left(\frac{\partial}{\partial y}\, x(x, y) \right)^2$$

$$+ x(x, y)^6\, \varepsilon \sin(k\, z + \omega\, t)^2\, \mu^2\, \omega^2 + y(x, y)^6\, \varepsilon \sin(k\, z + \omega\, t)^2\, \mu^2\, \omega^2 + 3\, x(x,$$

$$y)^2\, \varepsilon \sin(k\, z + \omega\, t)^2\, \mu^2\, y(x, y)^4\, vg^2\, k^2 - 2\, y(x, y)^6\, \varepsilon \sin(k\, z + \omega\, t)^2\, \mu^2\, \omega\, vg\, k + 4\, x(x,$$

$$y)^2 \cos(k z + \omega t)^2 vg^2 \mu^2 \varepsilon y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y)\right)^2 + 4 x(x, y)^2 \cos(k z$$

$$+ \omega t)^2 vg^2 \mu^2 \left(\frac{\partial}{\partial x} y(x, y)\right)^2 y(x, y)^2 \varepsilon + 4 y(x, y)^2 \cos(k z + \omega t)^2 vg^2 \mu^2 \varepsilon x(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} x(x, y)\right)^2 + 4 y(x, y)^2 \cos(k z + \omega t)^2 vg^2 \mu^2 \varepsilon x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y)\right)^2$$

$$- 2 x(x, y)^6 \varepsilon \sin(k z + \omega t)^2 \mu^2 \omega vg k + 3 x(x, y)^4 \varepsilon \sin(k z + \omega t)^2 \mu^2 y(x, y)^2 vg^2 k^2$$

$$- 8 \sin(k z + \omega t) \omega \mu \varepsilon vg x(x, y) \left(\frac{\partial}{\partial y} x(x, y)\right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y)\right) - 8 \sin(k z$$

$$+ \omega t) \omega \mu \varepsilon vg x(x, y) \left(\frac{\partial}{\partial x} x(x, y)\right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y)\right) - 4 \sin(k z$$

$$+ \omega t) \mu \varepsilon \omega k^2 x(x, y)^2 y(x, y)^2 vg + 2 \sin(k z + \omega t) \mu \varepsilon \left(\frac{\partial^2}{\partial x^2} y(x, y)\right) y(x,$$

$$y) vg x(x, y)^2 \omega + 2 \sin(k z + \omega t) \mu \varepsilon vg x(x, y) \left(\frac{\partial^2}{\partial x^2} x(x, y)\right) y(x, y)^2 \omega - 2 \sin(k z$$

$$+ \omega t) \mu \varepsilon vg^2 x(x, y) \left(\frac{\partial^2}{\partial x^2} x(x, y)\right) y(x, y)^2 k - 2 \sin(k z + \omega t) \mu \varepsilon vg^2 y(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} y(x, y)\right) x(x, y)^2 k - 2 \sin(k z + \omega t) \mu \varepsilon \left(\frac{\partial^2}{\partial x^2} y(x, y)\right) y(x, y) vg^2 x(x, y)^2 k$$

$$+ 2 \sin(k z + \omega t) \mu \varepsilon vg x(x, y) \left(\frac{\partial^2}{\partial y^2} x(x, y)\right) y(x, y)^2 \omega - 2 \sin(k z + \omega t) \mu \varepsilon vg^2 x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x, y)\right) y(x, y)^2 k + 2 \sin(k z + \omega t) \mu \varepsilon vg y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y)\right) x(x, y)^2 \omega$$

$$- 2 \sin(k z + \omega t) \mu \varepsilon vg^2 y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y)\right)^2 k - 2 \sin(k z + \omega t) \mu \varepsilon vg x(x,$$

$$y)^2 \left(\frac{\partial}{\partial y} x(x, y)\right)^2 \omega - 2 \sin(k z + \omega t) \mu \varepsilon vg^2 x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y)\right)^2 k + 2 \sin(k z$$

$$+ \omega t) \mu \varepsilon vg \left(\frac{\partial}{\partial y} x(x, y)\right)^2 y(x, y)^2 \omega - 2 \sin(k z + \omega t) \mu \varepsilon vg^2 \left(\frac{\partial}{\partial y} x(x, y)\right)^2 y(x, y)^2 k$$

$$+ 2 \sin(k z + \omega t) \mu \varepsilon vg \left(\frac{\partial}{\partial y} y(x, y)\right)^2 x(x, y)^2 \omega - 2 \sin(k z$$

$$+ \omega t) \mu \varepsilon vg^2 \left(\frac{\partial}{\partial y} y(x, y)\right)^2 x(x, y)^2 k - 2 \sin(k z + \omega t) \mu \varepsilon vg y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y)\right)^2 \omega$$

$$-2\sin(kz+\omega t)\,\mu\,\varepsilon\,vg^2\,y(x,y)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 k-2\sin(kz+\omega t)\,\mu\,\varepsilon\,\omega\,k^2\,x(x,y)^4\,vg$$

$$+2\sin(kz+\omega t)\,\mu\,\varepsilon\,\omega^2\,k\,x(x,y)^2\,y(x,y)^2+2\sin(kz+\omega t)\,\mu\,\varepsilon\,k^3\,vg^2\,x(x,y)^2\,y(x,y)^2$$

$$-2\sin(kz+\omega t)\,\mu\,\varepsilon\,\omega\,k^2\,y(x,y)^4\,vg-2\sin(kz+\omega t)\,\mu\,\varepsilon\,vg\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2\omega$$

$$-2\sin(kz+\omega t)\,\mu\,\varepsilon\,vg^2\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 k+2\sin(kz$$

$$+\omega t)\,\mu\,\varepsilon\,vg\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 y(x,y)^2\,\omega-2\sin(kz+\omega t)\,\mu\,\varepsilon\,vg^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 y(x,y)^2\,k$$

$$+2\sin(kz+\omega t)\,\mu\,\varepsilon\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 vg\,x(x,y)^2\,\omega-2\sin(kz+\omega t)\,\mu\,\varepsilon\left(\frac{\partial}{\partial x}\,y(x,$$

$$y)\right)^2 vg^2\,x(x,y)^2\,k-2\sin(kz+\omega t)\,\mu\,\varepsilon\,vg\,y(x,y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2\omega-2\sin(kz$$

$$+\omega t)\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)^3\,vg^2\,k+2\sin(kz+\omega t)\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)^3\,vg\,\omega$$

$$-2\sin(kz+\omega t)\,\mu\,\varepsilon\,vg^2\,x(x,y)^3\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)k+2\sin(kz+\omega t)\,\mu\,\varepsilon\,vg\,x(x,$$

$$y)^3\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)\omega-2\sin(kz+\omega t)\,\mu\,\varepsilon\,vg^2\,y(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)k+2\sin(kz$$

$$+\omega t)\,\mu\,\varepsilon\,vg\,y(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)\omega-2\sin(kz+\omega t)\,\mu\,\varepsilon\,vg^2\,x(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)k$$

$$+2\sin(kz+\omega t)\,\mu\,\varepsilon\,vg\,x(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)\omega+x(x,y)^6\,\varepsilon\sin(kz+\omega t)^2\,\mu^2\,vg^2\,k^2$$

$$+3\,x(x,y)^4\,\varepsilon\sin(kz+\omega t)^2\,\mu^2\,y(x,y)^2\,\omega^2+4\,y(x,y)^4\cos(kz$$

$$+\omega t)^2\,vg^2\,\mu^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2\varepsilon-8\,\mu\,y(x,y)^3\cos(kz+\omega t)^2\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,$$

$$y)\right)\left(\frac{\partial}{\partial x}\,y(x,y)\right)+4\,x(x,y)^4\cos(kz+\omega t)^2\,vg^2\,\mu^2\,\varepsilon\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2+y(x,$$

$$y)^6\,\varepsilon\sin(kz+\omega t)^2\,\mu^2\,vg^2\,k^2+4\,y(x,y)^4\cos(kz+\omega t)^2\,vg^2\,\mu^2\,\varepsilon\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2$$

$$-8\,\mu\,x(x,y)^3\cos(kz+\omega t)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)+3\,x(x,$$

$$y)^2 \varepsilon \sin(kz+\omega t)^2 \mu^2 y(x,y)^4 \omega^2 + 4\, x(x,y)^4 \cos(kz+\omega t)^2 vg^2 \mu^2 \varepsilon \left(\frac{\partial}{\partial x} x(x,y)\right)^2$$

$$-4\,\mu\, y(x,y)^4 \cos(kz+\omega t)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - 4\,\mu\, y(x,y)^4 \cos(kz+\omega t)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2$$

$$-4\,\mu\, x(x,y)^4 \cos(kz+\omega t)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 4\,\mu\, x(x,y)^4 \cos(kz+\omega t)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2$$

$$+\sin(kz+\omega t)\, \mu\, \varepsilon\, \omega^2 k\, x(x,y)^4 + \sin(kz+\omega t)\, \mu\, \varepsilon\, k^3 vg^2 x(x,y)^4 + \sin(kz$$

$$+\omega t)\, \mu\, \varepsilon\, \omega^2 k\, y(x,y)^4 + \sin(kz+\omega t)\, \mu\, \varepsilon\, k^3 vg^2 y(x,y)^4 + 8 \sin(kz+\omega t)\, k\, x(x,$$

$$y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) + 8 \sin(kz+\omega t)\, k\, x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial x} y(x,y)\right)\Big)\Big)\Big]$$

$$Dissipation\ = \cos(kz+\omega t)\, \varepsilon \left(2\, vg \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right)\right.$$

$$+2 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y)\, vg + 2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 vg + 2\, vg \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\, vg\, x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\, vg \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 2\, vg\, y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + x(x,y)^2 \omega k$$

$$-k^2 vg\, x(x,y)^2 + k\, \omega\, y(x,y)^2 - k^2 vg\, y(x,y)^2 + \mu \sin(kz+\omega t)\, x(x,y)^4 \omega - \mu \sin(kz$$

$$+\omega t)\, x(x,y)^4 vg\, k + 2\, \mu \sin(kz+\omega t)\, x(x,y)^2 y(x,y)^2 \omega - 2\, \mu \sin(kz+\omega t)\, x(x,$$

$$y)^2 y(x,y)^2 vg\, k + \mu \sin(kz+\omega t)\, y(x,y)^4 \omega - \mu \sin(kz+\omega t)\, y(x,y)^4 vg\, k\Big)$$

******************************** *END PROCEDURE* ****************************** **(23)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 5b-- waveguide TM mode  (group kinematic in, wave out)`;
> theta:=(-k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=-vg*f(x,y)*cos(theta);
```

Then call the procedure JCM(Ax,Ay,Az,phi,a,b,c,e,p,N,H)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,gamma,0):
```

****

NAME := *Example 5b-- waveguide TM mode (group kinematic in, wave out)*

$$\theta := -k\, z + \omega\, t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right)\cos(kz - \omega t)$$

$$\phi := -vg\left(x(x,y)^2 + y(x,y)^2\right)\cos(kz - \omega t)$$

Example 5b-- waveguide TM mode (group kinematic in, wave out)

******************************** *Differential Form Format* ********************
****

$$\text{Action 1-form} = \left(vg\cos(kz-\omega t)\,x(x,y)^2 + vg\cos(kz-\omega t)\,y(x,y)^2\right)d(t) + \left(\cos(kz-\omega t)\,x(x,y)^2 + \cos(kz-\omega t)\,y(x,y)^2\right)d(z)$$

$$\text{Intensity 2-form } F{=}dA = \left(2\,vg\cos(kz-\omega t)\,y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) + 2\,vg\cos(kz-\omega t)\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)\right)(d(x))\ \&\wedge\ (d(t)) + \left(2\cos(kz-\omega t)\,y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) + 2\cos(kz-\omega t)\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)\right)(d(x))\ \&\wedge\ (d(z)) + \left(2\,vg\cos(kz-\omega t)\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) + 2\,vg\cos(kz-\omega t)\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)\right)(d(y))\ \&\wedge\ (d(t)) + \left(2\cos(kz-\omega t)\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) + 2\cos(kz-\omega t)\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)\right)(d(y))\ \&\wedge\ (d(z)) + \left(-\sin(kz-\omega t)\,\omega\,x(x,y)^2 - \sin(kz-\omega t)\,\omega\,y(x,y)^2 - vg\sin(kz-\omega t)\,k\,x(x,y)^2 - vg\sin(kz-\omega t)\,k\,y(x,y)^2\right)(d(z))\ \&\wedge\ (d(t))$$

Topological Torsion 3-form $A\wedge F = 0$

Topological Parity 4-form $F\wedge F = 0$

******************************** *Using EM format* ****************************
****

$$E\ field = \left[2\,vg\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz-\omega t),\ 2\,vg\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz-\omega t),\ -\sin(kz-\omega t)\left(x(x,y)^2 + y(x,y)^2\right)(\omega + vg\,k)\right]$$

$$B\ field = \left[2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz-\omega t),\ -2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz-\omega t),\ 0\right]$$

Topological TORSION 4 vector $T4 = -[ExA + Bphi, AdotB] = [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension *PTD* $= 2$

*********************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ************************

Xm or linear (Mean) curvature $= -\sin(k\,z - \omega\,t)\,\left(x(x,y)^2 + y(x,y)^2\right)\,(k - \omega\,vg)$

Yg or quadratic (GAUSS) curvature $= 0$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\textit{Chirality factor CH} = \gamma\sqrt{\frac{\mu}{\varepsilon}}$$

$$\textit{D field} = \left[2\cos(k\,z-\omega\,t)\left(\varepsilon\,vg\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + \left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\,vg\,\varepsilon + \gamma\sqrt{\frac{\mu}{\varepsilon}}\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + \gamma\sqrt{\frac{\mu}{\varepsilon}}\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right), 2\cos(k\,z-\omega\,t)\left(\varepsilon\,vg\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + \varepsilon\,vg\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) - \gamma\sqrt{\frac{\mu}{\varepsilon}}\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) - \left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\sqrt{\frac{\mu}{\varepsilon}}\,\gamma\right), -\varepsilon\sin(k\,z-\omega\,t)\left(x(x,y)^2+y(x,y)^2\right)(\omega+vg\,k)\right]$$

$$\textit{H field} = \left[\frac{1}{\mu}\left(2\cos(k\,z-\omega\,t)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) - \gamma\sqrt{\frac{\mu}{\varepsilon}}\,vg\,\mu\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) - \gamma\sqrt{\frac{\mu}{\varepsilon}}\,vg\,\mu\,y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\right), -\frac{1}{\mu}\left(2\cos(k\,z-\omega\,t)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) + \gamma\sqrt{\frac{\mu}{\varepsilon}}\,vg\,\mu\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + \gamma\sqrt{\frac{\mu}{\varepsilon}}\,vg\,\mu\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\right), \gamma\sqrt{\frac{\mu}{\varepsilon}}\sin(k\,z-\omega\,t)\left(x(x,y)^2\right.$$

$$+ y(x,y)^2\big)\,(\omega + vg\,k)\Big]$$

Poynting vector ExH $= \Bigg[-\frac{1}{\mu}\Big(2\cos(k z-\omega t)\sin(k z-\omega t)\,\big(x(x,y)^2+y(x,y)^2\big)\,(\omega + vg\,k)\Big(x(x,y)\Big(\frac{\partial}{\partial x}x(x,y)\Big)+y(x,y)\Big(\frac{\partial}{\partial x}y(x,y)\Big)\Big)\Big), -\frac{1}{\mu}\Big(2\cos(k z-\omega t)\sin(k z-\omega t)\,\big(x(x,y)^2+y(x,y)^2\big)\,(\omega + vg\,k)\Big(x(x,y)\Big(\frac{\partial}{\partial y}x(x,y)\Big)+y(x,y)\Big(\frac{\partial}{\partial y}y(x,y)\Big)\Big)\Big), -\frac{1}{\mu}\Big(4\cos(k z-\omega t)^2 vg\Big(x(x,y)^2\Big(\frac{\partial}{\partial y}x(x,y)\Big)^2+2\,x(x,y)\Big(\frac{\partial}{\partial y}x(x,y)\Big)y(x,y)\Big(\frac{\partial}{\partial y}y(x,y)\Big)+y(x,y)^2\Big(\frac{\partial}{\partial y}y(x,y)\Big)^2+x(x,y)^2\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2+2\,x(x,y)\Big(\frac{\partial}{\partial x}x(x,y)\Big)y(x,y)\Big(\frac{\partial}{\partial x}y(x,y)\Big)+y(x,y)^2\Big(\frac{\partial}{\partial x}y(x,y)\Big)^2\Big)\Big)\Bigg]$

Amperian Current 4Vector curlH-dD/dt=J4 $= \Bigg[$

$$-\frac{2\sin(k z-\omega t)\,(k+\omega\,\mu\,\varepsilon\,vg)\Big(x(x,y)\Big(\frac{\partial}{\partial x}x(x,y)\Big)+y(x,y)\Big(\frac{\partial}{\partial x}y(x,y)\Big)\Big)}{\mu},$$

$$-\frac{2\sin(k z-\omega t)\,(k+\omega\,\mu\,\varepsilon\,vg)\Big(x(x,y)\Big(\frac{\partial}{\partial y}x(x,y)\Big)+y(x,y)\Big(\frac{\partial}{\partial y}y(x,y)\Big)\Big)}{\mu},$$

$$-\frac{1}{\mu}\Bigg(\cos(k z-\omega t)\Bigg(2\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2+2\,x(x,y)\Big(\frac{\partial^2}{\partial x^2}x(x,y)\Big)+2\Big(\frac{\partial}{\partial x}y(x,y)\Big)^2 +2\,y(x,y)\Big(\frac{\partial^2}{\partial x^2}y(x,y)\Big)+2\Big(\frac{\partial}{\partial y}x(x,y)\Big)^2+2\,x(x,y)\Big(\frac{\partial^2}{\partial y^2}x(x,y)\Big) +2\Big(\frac{\partial}{\partial y}y(x,y)\Big)^2+2\,y(x,y)\Big(\frac{\partial^2}{\partial y^2}y(x,y)\Big)+\varepsilon\,\omega^2\mu\,x(x,y)^2+\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2 +\varepsilon\,\omega^2\mu\,y(x,y)^2+\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\Bigg)\Bigg), \varepsilon\cos(k z-\omega t)\Bigg(2\,vg\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2$$

$$+2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y)\, vg + 2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 vg$$

$$+2\, vg \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\, vg \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 2\, vg\, y(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - x(x,y)^2\, \omega\, k - k^2\, vg\, x(x,y)^2 - k\, \omega\, y(x,y)^2 - k^2\, vg\, y(x,y)^2 \Big) \Big]$$

Amerian charge density $\quad divD = rho = \varepsilon \cos(k z - \omega t) \left(2\, vg \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\, vg\, x(x, \right.$

$$y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y)\, vg + 2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 vg$$

$$+2\, vg \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\, vg \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 2\, vg\, y(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - x(x,y)^2\, \omega\, k - k^2\, vg\, x(x,y)^2 - k\, \omega\, y(x,y)^2 - k^2\, vg\, y(x,y)^2 \Big)$$

divergence Lorentz Current 4Vector, $\quad$ *4div(J4)* $= 0$

Topological SPIN 4 vector S4 $= \Big[-\frac{1}{\mu} \Big(2 \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z - \omega t)^2 \,(-1$

$$+ vg^2\, \varepsilon\, \mu) \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right)\right)\Big), -\frac{1}{\mu} \Big(2 \left(x(x,y)^2\right.$$

$$\left.+ y(x,y)^2\right) \cos(k z - \omega t)^2 \left(-1 + vg^2\, \varepsilon\, \mu\right) \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,\right.\right.$$

$$y)\Big)\Big)\Big), \varepsilon \sin(k z - \omega t) \left(x(x,y)^2 + y(x,y)^2\right)^2 (\omega + vg\, k)\, vg \cos(k z - \omega t),$$

$$-\left(x(x,y)^2 + y(x,y)^2\right)^2 \cos(k z - \omega t)\, \varepsilon \sin(k z - \omega t)\, (\omega + vg\, k) \Big]$$

Topological SPIN 3-form $= -\frac{1}{\mu} \Big(2 \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z - \omega t)^2 \left(-1 + vg^2\, \varepsilon\, \mu\right) \Big(x(x,$

$$y) \left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right)\Big) \&\wedge(d(y), d(z), d(t)) \Big) + \frac{1}{\mu} \Big(2 \left(x(x,y)^2\right.$$

$$\left.+ y(x,y)^2\right) \cos(k z - \omega t)^2 \left(-1 + vg^2\, \varepsilon\, \mu\right) \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,\right.\right.$$

$$y)\Big)\Big) \&\wedge(d(x), d(z), d(t)) \Big) + \varepsilon \sin(k z - \omega t) \left(x(x,y)^2 + y(x,y)^2\right)^2 (\omega$$

$$+ vg\, k)\, vg \cos(k z - \omega t) \&\wedge(d(x), d(y), d(t)) + \left(x(x,y)^2 + y(x,y)^2\right)^2 \cos(k z$$

$$- \omega t)\, \varepsilon \sin(k z - \omega t)\, (\omega + vg\, k) \&\wedge(d(x), d(y), d(z))$$

Spin density rho_spin = $-\left(x(x,y)^2+y(x,y)^2\right)^2\cos(kz-\omega t)\,\varepsilon\,\sin(kz-\omega t)\,(\omega+vg\,k)$

LaGrange field energy density (B.H-D.E) = $-\frac{1}{\mu}\Big(-4\left(\frac{\partial}{\partial x}x(x,y)\right)^2 x(x,y)^2\cos(kz-\omega t)^2$

$-8\left(\frac{\partial}{\partial x}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)x(x,y)\cos(kz-\omega t)^2-4\,y(x,$

$y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\cos(kz-\omega t)^2-8\left(\frac{\partial}{\partial y}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)x(x,$

$y)\cos(kz-\omega t)^2-4\left(\frac{\partial}{\partial y}y(x,y)\right)^2 y(x,y)^2\cos(kz-\omega t)^2-4\left(\frac{\partial}{\partial y}x(x,y)\right)^2 x(x,$

$y)^2\cos(kz-\omega t)^2+4\,\varepsilon\,\mu\cos(kz-\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+8\,\varepsilon\,\mu\cos(kz$

$-\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+4\,\varepsilon\,\mu\cos(kz$

$-\omega t)^2 vg^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 y(x,y)^2+4\,\varepsilon\,\mu\cos(kz-\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$

$+8\,\varepsilon\,\mu\cos(kz-\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+4\,\varepsilon\,\mu\cos(kz$

$-\omega t)^2 vg^2 y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+\varepsilon\,\mu\sin(kz-\omega t)^2 x(x,y)^4\omega^2+2\,\varepsilon\,\mu\sin(kz$

$-\omega t)^2\omega\,vg\,k\,x(x,y)^4+\varepsilon\,\mu\sin(kz-\omega t)^2 x(x,y)^4 vg^2 k^2+2\,\varepsilon\,\mu\sin(kz-\omega t)^2 x(x,$

$y)^2 y(x,y)^2\omega^2+4\,\varepsilon\,\mu\sin(kz-\omega t)^2\omega\,vg\,k\,x(x,y)^2 y(x,y)^2+2\,\varepsilon\,\mu\sin(kz$

$-\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2+\varepsilon\,\mu\sin(kz-\omega t)^2 y(x,y)^4\omega^2+2\,\varepsilon\,\mu\sin(kz$

$-\omega t)^2\omega\,vg\,k\,y(x,y)^4+\varepsilon\,\mu\sin(kz-\omega t)^2 y(x,y)^4 vg^2 k^2\Big)$

B.H = $\frac{1}{\mu}\Big(4\cos(kz-\omega t)^2\Big(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,$

$y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,$

$y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Big)\Big)$

D.E = $\varepsilon\Big(4\cos(kz-\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+8\cos(kz-\omega t)^2 vg^2 x(x,$

$y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+4\cos(kz-\omega t)^2 vg^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 y(x,y)^2$

$+4\cos(kz-\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+8\cos(kz-\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial y}x(x,$

$$y)\Big)\, y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right) + 4\cos(k z - \omega t)^2 vg^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + \sin(k z$$

$$- \omega t)^2 x(x,y)^4 \omega^2 + 2\sin(k z - \omega t)^2 \omega\, vg\, k\, x(x,y)^4 + \sin(k z - \omega t)^2 x(x,y)^4 vg^2 k^2$$

$$+ 2\sin(k z - \omega t)^2 x(x,y)^2 y(x,y)^2 \omega^2 + 4\sin(k z - \omega t)^2 \omega\, vg\, k\, x(x,y)^2 y(x,y)^2$$

$$+ 2\sin(k z - \omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 + \sin(k z - \omega t)^2 y(x,y)^4 \omega^2 + 2\sin(k z$$

$$- \omega t)^2 \omega\, vg\, k\, y(x,y)^4 + \sin(k z - \omega t)^2 y(x,y)^4 vg^2 k^2\Big)$$

$$A.J = -\frac{1}{\mu}\Bigg(\left(x(x,y)^2 + y(x,y)^2\right)\cos(k z - \omega t)^2 \Bigg(2\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\, x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,$$

$$y)\right) + 2\left(\frac{\partial}{\partial x} y(x,y)\right)^2 + 2\, y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + 2\left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\, x(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 2\, y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + \varepsilon\,\omega^2 \mu\, x(x,y)^2$$

$$+ \varepsilon\,\omega\,\mu\, vg\, k\, x(x,y)^2 + \varepsilon\,\omega^2 \mu\, y(x,y)^2 + \varepsilon\,\omega\,\mu\, vg\, k\, y(x,y)^2\Bigg)\Bigg)$$

$$\text{-rho.phi} = -vg\left(x(x,y)^2 + y(x,y)^2\right)\cos(k z - \omega t)^2 \varepsilon \Bigg(2\, vg\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\, vg\, x(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y)\, vg + 2\left(\frac{\partial}{\partial x} y(x,y)\right)^2 vg$$

$$+ 2\, vg\left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\, vg\, x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\, vg\left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 2\, vg\, y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - x(x,y)^2 \omega\, k - k^2 vg\, x(x,y)^2 - k\,\omega\, y(x,y)^2 - k^2 vg\, y(x,y)^2\Bigg)$$

Poincare I $\quad$ $$(B.H - D.E)\text{-}(A.J - \text{rho.phi}) = -\frac{1}{\mu}\Bigg(6\,\varepsilon\,\mu\cos(k z - \omega t)^2 vg^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,$$

$$y)\right)^2 + 6\,\varepsilon\,\mu\cos(k z - \omega t)^2 vg^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 y(x,y)^2 + 6\,\varepsilon\,\mu\cos(k z$$

$$- \omega t)^2 vg^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 6\,\varepsilon\,\mu\cos(k z - \omega t)^2 vg^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2$$

$$+ \varepsilon\,\mu\sin(k z - \omega t)^2 x(x,y)^4 vg^2 k^2 + 2\,\varepsilon\,\mu\sin(k z - \omega t)^2 x(x,y)^2 y(x,y)^2 \omega^2$$

$$+ \varepsilon\,\mu\sin(k z - \omega t)^2 y(x,y)^4 vg^2 k^2 - 2\cos(k z - \omega t)^2 x(x,y)^4 \varepsilon\,\omega\,\mu\, vg\, k - 2\cos(k z$$

$$-\omega t)^2 x(x,y)^2 vg^2 \varepsilon \mu k^2 y(x,y)^2 - 2\cos(kz-\omega t)^2 y(x,y)^4 \varepsilon \omega \mu vg k - 8 \left(\frac{\partial}{\partial x} y(x,\right.$$

$$\left.y)\right) y(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) x(x,y) \cos(kz-\omega t)^2 - 8 \left(\frac{\partial}{\partial y} y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} x(x,\right.$$

$$\left.y)\right) x(x,y) \cos(kz-\omega t)^2 + \varepsilon \mu \sin(kz-\omega t)^2 x(x,y)^4 \omega^2 + \varepsilon \mu \sin(kz-\omega t)^2 y(x,$$

$$y)^4 \omega^2 - 6 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 x(x,y)^2 \cos(kz-\omega t)^2 - 6 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 \cos(kz$$

$$-\omega t)^2 - 6 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 y(x,y)^2 \cos(kz-\omega t)^2 - 6 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 x(x,y)^2 \cos(kz$$

$$-\omega t)^2 + 2\cos(kz-\omega t)^2 y(x,y)^2 vg^2 \varepsilon \mu x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\cos(kz$$

$$-\omega t)^2 x(x,y)^2 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y) + 2\cos(kz-\omega t)^2 x(x,y)^2 vg^2 \varepsilon \mu y(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + 2\cos(kz-\omega t)^2 y(x,y)^2 vg^2 \varepsilon \mu x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right)$$

$$+ 2\cos(kz-\omega t)^2 x(x,y)^3 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2\cos(kz-\omega t)^2 x(x,$$

$$y)^3 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\cos(kz-\omega t)^2 y(x,y)^3 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial x^2} y(x,y)\right)$$

$$+ 2\cos(kz-\omega t)^2 y(x,y)^3 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + 8 \varepsilon \mu \cos(kz-\omega t)^2 vg^2 x(x,$$

$$y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 8 \varepsilon \mu \cos(kz-\omega t)^2 vg^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,\right.$$

$$\left.y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) + 4 \varepsilon \mu \sin(kz-\omega t)^2 \omega vg k x(x,y)^2 y(x,y)^2 - 2\cos(kz$$

$$-\omega t)^2 x(x,y)^2 y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) - 2\cos(kz-\omega t)^2 x(x,y)^2 y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right)$$

$$-2\cos(kz-\omega t)^2 y(x,y)^2 x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - 2\cos(kz-\omega t)^2 y(x,y)^2 x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) - 2\cos(kz-\omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) - 2\cos(kz-\omega t)^2 x(x,$$

$$y)^3 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) - 2\cos(kz-\omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - 2\cos(kz-\omega t)^2 x(x,$$

$$y)^3 \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - 2\cos(kz-\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - 2\cos(kz-\omega t)^2 x(x,$$

$$y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - 2\cos(kz-\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 2\cos(kz-\omega t)^2 y(x,$$

$$y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 2\cos(kz-\omega t)^2 x(x,y)^2 \varepsilon\,\omega^2 \mu\, y(x,y)^2 + 2\cos(kz-\omega t)^2 x(x,$$

$$y)^2 vg^2 \varepsilon\,\mu \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + 2\cos(kz-\omega t)^2 x(x,y)^2 vg^2 \varepsilon\,\mu \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - \cos(kz$$

$$-\omega t)^2 x(x,y)^4 vg^2 \varepsilon\,\mu\, k^2 + 2\cos(kz-\omega t)^2 y(x,y)^2 vg^2 \varepsilon\,\mu \left(\frac{\partial}{\partial x} x(x,y)\right)^2$$

$$+2\cos(kz-\omega t)^2 y(x,y)^2 vg^2 \varepsilon\,\mu \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - \cos(kz-\omega t)^2 y(x,y)^4 vg^2 \varepsilon\,\mu\, k^2$$

$$-4\cos(kz-\omega t)^2 x(x,y)^2 \varepsilon\,\omega\,\mu\, vg\, k\, y(x,y)^2 + 2\,\varepsilon\,\mu \sin(kz-\omega t)^2 \omega\, vg\, k\, x(x,y)^4$$

$$+2\,\varepsilon\,\mu \sin(kz-\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 + 2\,\varepsilon\,\mu \sin(kz-\omega t)^2 \omega\, vg\, k\, y(x,y)^4$$

$$-\cos(kz-\omega t)^2 x(x,y)^4 \varepsilon\,\omega^2 \mu - \cos(kz-\omega t)^2 y(x,y)^4 \varepsilon\,\omega^2 \mu\Big)$$

$$\textit{London Coefficient} \qquad LC = \frac{k^2 - \mu\,\omega^2 \varepsilon}{\mu}$$

$$\textit{PROCA coefficient curlcurlB} = \Bigg[2\cos(kz-\omega t)\left(-2\left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right)\right.$$

$$-\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - x(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2} x(x,y)\right) - 2\left(\frac{\partial}{\partial x} y(x,$$

$$y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} y(x,y)\right) - \left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) - y(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2} y(x,y)\right)$$

$$-3\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) - x(x,y)\left(\frac{\partial^3}{\partial y^3} x(x,y)\right) - 3\left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial y^2} y(x,$$

$$\left. y)\right) - y(x,y)\left(\frac{\partial^3}{\partial y^3} y(x,y)\right) + k^2 x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + k^2 y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right),$$

$$2\cos(kz-\omega t)\left(-k^2 x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) - k^2 y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right) + 3\left(\frac{\partial}{\partial x} x(x,\right.$$

$$y)\right)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial x^3} x(x,y)\right) + 3\left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + y(x,$$

$$y)\left(\frac{\partial^3}{\partial x^3} y(x,y)\right) + 2\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right) + \left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$+x(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}y(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+y(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}y(x,y)\right)\Bigg),0\Bigg]$$

Amperian Current 4Vector   curlH-dD/dt=J4 $=\Bigg[$

$$-\frac{2\sin(kz-\omega t)\,(k+\omega\,\mu\,\varepsilon\,vg)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)}{\mu},$$

$$-\frac{2\sin(kz-\omega t)\,(k+\omega\,\mu\,\varepsilon\,vg)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)}{\mu},$$

$$-\frac{1}{\mu}\Bigg(\cos(kz-\omega t)\Bigg(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2$$
$$+2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$
$$+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\,\omega^2\,\mu\,x(x,y)^2+\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2$$
$$+\varepsilon\,\omega^2\,\mu\,y(x,y)^2+\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\Bigg)\Bigg),\ \varepsilon\cos(kz-\omega t)\Bigg(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2$$
$$+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2vg$$
$$+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-x(x,y)^2\,\omega\,k-k^2\,vg\,x(x,y)^2-k\,\omega\,y(x,y)^2-k^2\,vg\,y(x,y)^2\Bigg)\Bigg]$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B) $=\Bigg[$

$$-\frac{1}{\mu}\Bigg(2\cos(kz-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\Bigg($$

$$-2\left(\frac{\partial}{\partial x}x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-2\left(\frac{\partial}{\partial x}y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,$$

$$y)\right)-2\left(\frac{\partial}{\partial y}x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-2\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-\varepsilon\,\omega^2\,\mu\,x(x,y)^2-2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2-\varepsilon\,\omega^2\,\mu\,y(x,y)^2$$

$$-2\,\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-vg^2\,\varepsilon\,\mu\,k^2\,x(x,y)^2-vg^2\,\varepsilon\,\mu\,k^2\,y(x,y)^2\Big)\Big),\ -\frac{1}{\mu}\Big(2\cos(k\,z$$

$$-\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\Big(-2\left(\frac{\partial}{\partial x}x(x,y)\right)^2-2\,x(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-2\left(\frac{\partial}{\partial x}y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)-2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$-2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-\varepsilon\,\omega^2\,\mu\,x(x,y)^2$$

$$-2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2-\varepsilon\,\omega^2\,\mu\,y(x,y)^2-2\,\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}x(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,y(x,y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)-vg^2\,\varepsilon\,\mu\,k^2\,x(x,y)^2$$

$$-vg^2\,\varepsilon\,\mu\,k^2\,y(x,y)^2\Big)\Big)\Big],\ \frac{1}{\mu}\Big(\sin(k\,z-\omega\,t)\,\cos(k\,z-\omega\,t)\,\Big(2\,\mu\,\varepsilon\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}\,x(x,\right.$$

$$\left.y)\right)y(x,y)^2\,\omega+2\,\mu\,\varepsilon\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)y(x,y)^2\,\omega+2\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,$$

$$y)\,vg^2\,x(x,y)^2\,k+2\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)\,vg\,x(x,y)^2\,\omega+2\,\mu\,\varepsilon\,vg^2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}\,x(x,\right.$$

$$\left.y)\right)y(x,y)^2\,k+2\,\mu\,\varepsilon\,vg^2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)x(x,y)^2\,k+2\,\mu\,\varepsilon\,vg\,y(x,y)\left(\frac{\partial^2}{\partial y^2}\,y(x,\right.$$

$$\left.y)\right)x(x,y)^2\,\omega+2\,\mu\,\varepsilon\,vg^2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)y(x,y)^2\,k-\mu\,\varepsilon\,\omega^2\,k\,x(x,y)^4$$

$$-\mu\,\varepsilon\,k^3\,vg^2\,x(x,y)^4-\mu\,\varepsilon\,\omega^2\,k\,y(x,y)^4-\mu\,\varepsilon\,k^3\,vg^2\,y(x,y)^4-8\,k\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,\right.$$

$$\left.y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)-8\,k\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)$$

$$+2\,\mu\,\varepsilon\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 vg\,x(x,y)^2\,\omega+2\,\mu\,\varepsilon\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 vg^2\,x(x,y)^2\,k$$

$$-2\,\mu\,\varepsilon\,vg^{y(x,y)^2}\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2\omega+2\,\mu\,\varepsilon\,vg^2\,y(x,y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 k$$

$$-2\,\mu\,\varepsilon\,vg^{x(x,y)^2}\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2\omega+2\,\mu\,\varepsilon\,vg^2\,x(x,y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 k$$

$$+2\,\mu\,\varepsilon\,vg\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 y(x,y)^2\,\omega+2\,\mu\,\varepsilon\,vg^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 y(x,y)^2\,k$$

$$+2\,\mu\,\varepsilon\,vg\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 x(x,y)^2\,\omega+2\,\mu\,\varepsilon\,vg^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 x(x,y)^2\,k$$

$$- 2\,\mu\,\varepsilon\,vg^{y(x,y)^2} \left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 \omega + 2\,\mu\,\varepsilon\,vg^2\,y(x,y)^2 \left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 k$$

$$- 2\,\mu\,\varepsilon\,\omega\,k^2\,x(x,y)^4\,vg - 2\,\mu\,\varepsilon\,\omega^2\,k\,x(x,y)^2\,y(x,y)^2 - 2\,\mu\,\varepsilon\,k^3\,vg^2\,x(x,y)^2\,y(x,y)^2$$

$$- 2\,\mu\,\varepsilon\,\omega\,k^2\,y(x,y)^4\,vg - 2\,\mu\,\varepsilon\,vg\,x(x,y)^2 \left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 \omega + 2\,\mu\,\varepsilon\,vg^2\,x(x,$$

$$y)^2 \left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 k + 2\,\mu\,\varepsilon\,vg \left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 y(x,y)^2\,\omega + 2\,\mu\,\varepsilon\,vg^2 \left(\frac{\partial}{\partial x}\,x(x,$$

$$y)\right)^2 y(x,y)^2\,k - 4\,\mu\,\varepsilon\,\omega\,k^2\,x(x,y)^2\,y(x,y)^2\,vg + 2\,\mu\,\varepsilon\,vg^2\,x(x,y)^3 \left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right) k$$

$$+ 2\,\mu\,\varepsilon\,vg^2\,y(x,y)^3 \left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right) k + 2\,\mu\,\varepsilon\,vg\,x(x,y)^3 \left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right) \omega$$

$$+ 2\,\mu\,\varepsilon \left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right) y(x,y)^3\,vg^2\,k + 2\,\mu\,\varepsilon\,vg\,x(x,y)^3 \left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right) \omega$$

$$+ 2\,\mu\,\varepsilon\,vg^{2\,x(x,y)^3} \left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right) k + 2\,\mu\,\varepsilon\,vg\,y(x,y)^3 \left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right) \omega$$

$$+ 2\,\mu\,\varepsilon \left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right) y(x,y)^3\,vg\,\omega - 8\,\omega\,\mu\,\varepsilon\,vg\,x(x,y) \left(\frac{\partial}{\partial y}\,x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y}\,y(x,$$

$$y)\right) - 8\,\omega\,\mu\,\varepsilon\,vg\,x(x,y) \left(\frac{\partial}{\partial x}\,x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x}\,y(x,y)\right) - 4\,k\,x(x,y)^2 \left(\frac{\partial}{\partial y}\,x(x,y)\right)^2$$

$$- 4\,k\,y(x,y)^2 \left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 - 4\,k\,x(x,y)^2 \left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 - 4\,k\,y(x,y)^2 \left(\frac{\partial}{\partial x}\,y(x,y)\right)^2\Bigg)$$

$$\Bigg)\Bigg]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \left[-\frac{1}{\mu}\left(2\,\left(x(x,y)^2 + y(x,\right.\right.\right.$

$$y)^2)\cos(kz-\omega t)^2\,(-1+vg^2\,\varepsilon\,\mu)\,\Big(x(x,y)\,\Big(\frac{\partial}{\partial x}x(x,y)\Big)+y(x,y)\,\Big(\frac{\partial}{\partial x}y(x,$$

$$y)\Big)\Big)\Big),\ -\frac{1}{\mu}\Big(2\,(x(x,y)^2+y(x,y)^2)\cos(kz-\omega t)^2\,(-1+vg^2\,\varepsilon\,\mu)\,\Big(x(x,y)\,\Big(\frac{\partial}{\partial y}x(x,$$

$$y)\Big)+y(x,y)\,\Big(\frac{\partial}{\partial y}y(x,y)\Big)\Big)\Big),\ \varepsilon\sin(kz-\omega t)\,(x(x,y)^2+y(x,y)^2)^2\,(\omega$$

$$+vg\,k)\,vg\cos(kz-\omega t),\ -(x(x,y)^2+y(x,y)^2)^2\cos(kz-\omega t)\,\varepsilon\sin(kz-\omega t)\,(\omega$$

$$+vg\,k)\Big]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $=\Big[0,\ 0,$

$$-\frac{1}{\mu}\Big(\cos(kz-\omega t)\,(x(x,y)^2+y(x,y)^2)\,\Big(-4\,\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2 x(x,y)^2\cos(kz-\omega t)^2$$

$$-8\,\Big(\frac{\partial}{\partial x}y(x,y)\Big)\,y(x,y)\,\Big(\frac{\partial}{\partial x}x(x,y)\Big)\,x(x,y)\cos(kz-\omega t)^2-4\,y(x,$$

$$y)^2\,\Big(\frac{\partial}{\partial x}y(x,y)\Big)^2\cos(kz-\omega t)^2-8\,\Big(\frac{\partial}{\partial y}y(x,y)\Big)\,y(x,y)\,\Big(\frac{\partial}{\partial y}x(x,y)\Big)\,x(x,$$

$$y)\cos(kz-\omega t)^2-4\,\Big(\frac{\partial}{\partial y}y(x,y)\Big)^2 y(x,y)^2\cos(kz-\omega t)^2-4\,\Big(\frac{\partial}{\partial y}x(x,y)\Big)^2 x(x,$$

$$y)^2\cos(kz-\omega t)^2+4\,\varepsilon\,\mu\cos(kz-\omega t)^2\,vg^2\,x(x,y)^2\,\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2+8\,\varepsilon\,\mu\cos(kz$$

$$-\omega t)^2\,vg^2\,x(x,y)\,\Big(\frac{\partial}{\partial x}x(x,y)\Big)\,y(x,y)\,\Big(\frac{\partial}{\partial x}y(x,y)\Big)+4\,\varepsilon\,\mu\cos(kz$$

$$-\omega t)^2\,vg^2\,\Big(\frac{\partial}{\partial x}y(x,y)\Big)^2 y(x,y)^2+4\,\varepsilon\,\mu\cos(kz-\omega t)^2\,vg^2\,x(x,y)^2\,\Big(\frac{\partial}{\partial y}x(x,y)\Big)^2$$

$$+8\,\varepsilon\,\mu\cos(kz-\omega t)^2\,vg^2\,x(x,y)\,\Big(\frac{\partial}{\partial y}x(x,y)\Big)\,y(x,y)\,\Big(\frac{\partial}{\partial y}y(x,y)\Big)+4\,\varepsilon\,\mu\cos(kz$$

$$-\omega t)^2\,vg^2\,y(x,y)^2\,\Big(\frac{\partial}{\partial y}y(x,y)\Big)^2+\varepsilon\,\mu\sin(kz-\omega t)^2\,x(x,y)^4\,\omega^2+2\,\varepsilon\,\mu\sin(kz$$

$$-\omega t)^2\,\omega\,vg\,k\,x(x,y)^4+\varepsilon\,\mu\sin(kz-\omega t)^2\,x(x,y)^4\,vg^2\,k^2+2\,\varepsilon\,\mu\sin(kz-\omega t)^2\,x(x,$$

$$y)^2\,y(x,y)^2\,\omega^2+4\,\varepsilon\,\mu\sin(kz-\omega t)^2\,\omega\,vg\,k\,x(x,y)^2\,y(x,y)^2+2\,\varepsilon\,\mu\sin(kz$$

$$-\omega t)^2\,x(x,y)^2\,y(x,y)^2\,vg^2\,k^2+\varepsilon\,\mu\sin(kz-\omega t)^2\,y(x,y)^4\,\omega^2+2\,\varepsilon\,\mu\sin(kz$$

$$-\omega t)^2\,\omega\,vg\,k\,y(x,y)^4+\varepsilon\,\mu\sin(kz-\omega t)^2\,y(x,y)^4\,vg^2\,k^2)\Big)\Big]$$

Spin Dissipation J_spin dot E $=-\frac{1}{\mu}\Big(vg\cos(kz-\omega t)\,(x(x,y)^2+y(x,y)^2)\,\Big(-4\,\Big(\frac{\partial}{\partial x}x(x,$

$$y)\Big)^2 x(x,y)^2 \cos(k z-\omega t)^2 - 8 \left(\frac{\partial}{\partial x} y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) x(x,y) \cos(k z$$

$$-\omega t)^2 - 4 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 \cos(k z-\omega t)^2 - 8 \left(\frac{\partial}{\partial y} y(x,y)\right) y(x,y) \Big(\frac{\partial}{\partial y} x(x,$$

$$y)\Big) x(x,y) \cos(k z-\omega t)^2 - 4 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 y(x,y)^2 \cos(k z-\omega t)^2 - 4 \Big(\frac{\partial}{\partial y} x(x,$$

$$y)\Big)^2 x(x,y)^2 \cos(k z-\omega t)^2 + 4 \varepsilon \mu \cos(k z-\omega t)^2 vg^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2$$

$$+ 8 \varepsilon \mu \cos(k z-\omega t)^2 vg^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 4 \varepsilon \mu \cos(k z$$

$$-\omega t)^2 vg^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 y(x,y)^2 + 4 \varepsilon \mu \cos(k z-\omega t)^2 vg^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2$$

$$+ 8 \varepsilon \mu \cos(k z-\omega t)^2 vg^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) + 4 \varepsilon \mu \cos(k z$$

$$-\omega t)^2 vg^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + \varepsilon \mu \sin(k z-\omega t)^2 x(x,y)^4 \omega^2 + 2 \varepsilon \mu \sin(k z$$

$$-\omega t)^2 \omega\, vg\, k\, x(x,y)^4 + \varepsilon \mu \sin(k z-\omega t)^2 x(x,y)^4 vg^2 k^2 + 2 \varepsilon \mu \sin(k z-\omega t)^2 x(x,$$

$$y)^2 y(x,y)^2 \omega^2 + 4 \varepsilon \mu \sin(k z-\omega t)^2 \omega\, vg\, k\, x(x,y)^2 y(x,y)^2 + 2 \varepsilon \mu \sin(k z$$

$$-\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 + \varepsilon \mu \sin(k z-\omega t)^2 y(x,y)^4 \omega^2 + 2 \varepsilon \mu \sin(k z$$

$$-\omega t)^2 \omega\, vg\, k\, y(x,y)^4 + \varepsilon \mu \sin(k z-\omega t)^2 y(x,y)^4 vg^2 k^2\Big)\Big)$$

$$\textit{Disspative Force 3 vector} = \Bigg[-\frac{1}{\mu} \Big(2 \cos(k z-\omega t)^2 \Big(x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,$$

$$y) \left(\frac{\partial}{\partial x} y(x,y)\right)\Big)\Big) \Big(-2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 2 x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - 2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2$$

$$- 2 y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) - 2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 2 x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$- 2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - 2 y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - \varepsilon \omega^2 \mu\, x(x,y)^2 - 2 \varepsilon \omega \mu\, vg\, k\, x(x,y)^2$$

$$- \varepsilon \omega^2 \mu\, y(x,y)^2 - 2 \varepsilon \omega \mu\, vg\, k\, y(x,y)^2 + 2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2 vg^2 \varepsilon \mu\, x(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\,y(x,y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)-vg^2\,\varepsilon\,\mu\,k^2\,x(x,y)^2-vg^2\,\varepsilon\,\mu\,k^2\,y(x,y)^2\Big)\Big),$$

$$-\frac{1}{\mu}\Big(2\cos(k\,z-\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)\right)\Big($$

$$-2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)-2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}\,y(x,$$

$$y)\Big)-2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)-2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2-2\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)-\varepsilon\,\omega^2\,\mu\,x(x,y)^2-2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2-\varepsilon\,\omega^2\,\mu\,y(x,y)^2$$

$$-2\,\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)-vg^2\,\varepsilon\,\mu\,k^2\,x(x,y)^2-vg^2\,\varepsilon\,\mu\,k^2\,y(x,y)^2\Big)\Big),\ -\frac{1}{\mu}\Big(\cos(k\,z$$

$$-\omega\,t)\Big(8\sin(k\,z-\omega\,t)\,k\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)+\sin(k\,z$$

$$-\omega t)\ \mu\,\varepsilon\,k^{3}\,vg^{2}\,x(x,y)^{4}+\sin(k\,z-\omega\,t)\ \mu\,\varepsilon\,\omega^{2}\,k\,x(x,y)^{4}+\sin(k\,z-\omega\,t)\ \mu\,\varepsilon\,\omega^{2}\,k\,y(x,y)^{4}$$

$$+\sin(k\,z-\omega\,t)\ \mu\,\varepsilon\,k^{3}\,vg^{2}\,y(x,y)^{4}+8\sin(k\,z-\omega\,t)\ k\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,$$

$$y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)+y(x,y)^{6}\,\varepsilon\,\mu^{2}\sin(k\,z-\omega\,t)^{2}\,\omega^{2}-4\,\mu\,x(x,y)^{2}\,y(x,y)^{2}\left(\frac{\partial}{\partial x}\,y(x,\right.$$

$$\left.y)\right)^{2}\cos(k\,z-\omega\,t)^{2}-4\,\mu\,x(x,y)^{2}\left(\frac{\partial}{\partial y}\,y(x,y)\right)^{2}y(x,y)^{2}\cos(k\,z-\omega\,t)^{2}-4\,\mu\,y(x,$$

$$y)^{2}\left(\frac{\partial}{\partial x}\,x(x,y)\right)^{2}x(x,y)^{2}\cos(k\,z-\omega\,t)^{2}-4\,\mu\,y(x,y)^{2}\left(\frac{\partial}{\partial y}\,x(x,y)\right)^{2}x(x,$$

$$y)^{2}\cos(k\,z-\omega\,t)^{2}+x(x,y)^{6}\,\varepsilon\,\mu^{2}\sin(k\,z-\omega\,t)^{2}\,\omega^{2}+4\sin(k\,z-\omega\,t)\ k\,x(x,$$

$$y)^{2}\left(\frac{\partial}{\partial x}\,x(x,y)\right)^{2}+4\sin(k\,z-\omega\,t)\ k\,y(x,y)^{2}\left(\frac{\partial}{\partial x}\,y(x,y)\right)^{2}+4\sin(k\,z$$

$$-\omega\,t)\ k\,x(x,y)^{2}\left(\frac{\partial}{\partial y}\,x(x,y)\right)^{2}+4\sin(k\,z-\omega\,t)\ k\,y(x,y)^{2}\left(\frac{\partial}{\partial y}\,y(x,y)\right)^{2}+4\sin(k\,z$$

$$-\omega\,t)\ \mu\,\varepsilon\,\omega\,k^{2}\,x(x,y)^{2}\,y(x,y)^{2}\,vg+8\sin(k\,z-\omega\,t)\ \omega\,\mu\,\varepsilon\,vg\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,$$

$$y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)+8\sin(k\,z-\omega\,t)\ \omega\,\mu\,\varepsilon\,vg\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)$$

$$+2\sin(k\,z-\omega\,t)\ \mu\,\varepsilon\,\omega\,k^{2}\,x(x,y)^{4}\,vg-2\sin(k\,z-\omega\,t)\ \mu\,\varepsilon\,vg^{2}\left(\frac{\partial}{\partial y}\,y(x,y)\right)^{2}x(x,y)^{2}\,k$$

$$+2\sin(k\,z-\omega\,t)\ \mu\,\varepsilon\,vg\,y(x,y)^{2}\left(\frac{\partial}{\partial y}\,y(x,y)\right)^{2}\omega-2\sin(k\,z-\omega\,t)\ \mu\,\varepsilon\,vg\left(\frac{\partial}{\partial y}\,y(x,\right.$$

$$\left.y)\right)^{2}x(x,y)^{2}\,\omega-2\sin(k\,z-\omega\,t)\ \mu\,\varepsilon\,vg^{2}\,x(x,y)^{2}\left(\frac{\partial}{\partial y}\,x(x,y)\right)^{2}k-2\sin(k\,z$$

$$-\omega\,t)\ \mu\,\varepsilon\,vg\left(\frac{\partial}{\partial y}\,x(x,y)\right)^{2}y(x,y)^{2}\,\omega-2\sin(k\,z-\omega\,t)\ \mu\,\varepsilon\,vg^{2}\left(\frac{\partial}{\partial y}\,x(x,y)\right)^{2}y(x,y)^{2}\,k$$

$$-2\sin(k\,z-\omega\,t)\ \mu\,\varepsilon\left(\frac{\partial}{\partial x}\,y(x,y)\right)^{2}vg\,x(x,y)^{2}\,\omega-2\sin(k\,z-\omega\,t)\ \mu\,\varepsilon\left(\frac{\partial}{\partial x}\,y(x,\right.$$

$$\left.y)\right)^{2}vg^{2}\,x(x,y)^{2}\,k+2\sin(k\,z-\omega\,t)\ \mu\,\varepsilon\,vg\,y(x,y)^{2}\left(\frac{\partial}{\partial x}\,y(x,y)\right)^{2}\omega-2\sin(k\,z$$

$$-\omega\,t)\ \mu\,\varepsilon\,vg^{2}\,y(x,y)^{2}\left(\frac{\partial}{\partial x}\,y(x,y)\right)^{2}k+2\sin(k\,z-\omega\,t)\ \mu\,\varepsilon\,vg\,x(x,y)^{2}\left(\frac{\partial}{\partial y}\,x(x,y)\right)^{2}\omega$$

$$-2\sin(kz-\omega t)\,\mu\,\varepsilon\,vg^2\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 k-2\sin(kz$$

$$-\omega t)\,\mu\,\varepsilon\,vg\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 y(x,y)^2\,\omega-2\sin(kz-\omega t)\,\mu\,\varepsilon\,vg^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 y(x,y)^2\,k$$

$$+2\sin(kz-\omega t)\,\mu\,\varepsilon\,\omega^2\,k\,x(x,y)^2\,y(x,y)^2+2\sin(kz-\omega t)\,\mu\,\varepsilon\,k^3\,vg^2\,x(x,y)^2\,y(x,y)^2$$

$$+2\sin(kz-\omega t)\,\mu\,\varepsilon\,\omega\,k^2\,y(x,y)^4\,vg+2\sin(kz-\omega t)\,\mu\,\varepsilon\,vg\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2\omega$$

$$-2\sin(kz-\omega t)\,\mu\,\varepsilon\,vg^2\,y(x,y)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 k-4\,\mu\,x(x,y)^4\left(\frac{\partial}{\partial x}\,x(x,$$

$$y)\right)^2\cos(kz-\omega t)^2-4\,\mu\,x(x,y)^4\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2\cos(kz-\omega t)^2-4\,\mu\,y(x,$$

$$y)^4\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2\cos(kz-\omega t)^2-4\,\mu\,y(x,y)^4\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2\cos(kz-\omega t)^2$$

$$+8\,x(x,y)^3\,\varepsilon\,\mu^2\cos(kz-\omega t)^2\,vg^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)+8\,x(x,$$

$$y)^3\,\varepsilon\,\mu^2\cos(kz-\omega t)^2\,vg^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)+6\,x(x,y)^4\,\varepsilon\,\mu^2\sin(kz$$

$$-\omega t)^2\,\omega\,vg\,k\,y(x,y)^2+6\,x(x,y)^2\,\varepsilon\,\mu^2\sin(kz-\omega t)^2\,\omega\,vg\,k\,y(x,y)^4+8\,y(x,$$

$$y)^3\,\varepsilon\,\mu^2\cos(kz-\omega t)^2\,vg^2\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)\left(\frac{\partial}{\partial x}\,y(x,y)\right)+8\,y(x,$$

$$y)^3\,\varepsilon\,\mu^2\cos(kz-\omega t)^2\,vg^2\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)\left(\frac{\partial}{\partial y}\,y(x,y)\right)-2\sin(kz$$

$$-\omega t)\,\mu\,\varepsilon\,vg^2\,x(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)k-2\sin(kz-\omega t)\,\mu\,\varepsilon\,vg\,x(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)\omega$$

$$-2\sin(kz-\omega t)\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)^3\,vg^2\,k-2\sin(kz-\omega t)\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}\,y(x,$$

$$y)\right)y(x,y)^3\,vg\,\omega-2\sin(kz-\omega t)\,\mu\,\varepsilon\,vg\,x(x,y)^3\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)\omega-2\sin(kz$$

$$-\omega t)\,\mu\,\varepsilon\,vg^2\,x(x,y)^3\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)k-2\sin(kz-\omega t)\,\mu\,\varepsilon\,vg\,y(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)\omega$$

$$-2\sin(kz-\omega t)\,\mu\,\varepsilon\,vg^2\,y(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)k-2\sin(kz-\omega t)\,\mu\,\varepsilon\,vg\,y(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) x(x, y)^2 \omega - 2 \sin(k z - \omega t) \mu \varepsilon vg^2 y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) x(x, y)^2 k$$

$$- 2 \sin(k z - \omega t) \mu \varepsilon \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) y(x, y)\, vg\, x(x, y)^2 \omega - 2 \sin(k z - \omega t) \mu \varepsilon vg^2 x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) y(x, y)^2 k - 2 \sin(k z - \omega t) \mu \varepsilon vg^2 x(x, y) \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) y(x, y)^2 k$$

$$- 2 \sin(k z - \omega t) \mu \varepsilon vg\, x(x, y) \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) y(x, y)^2 \omega - 2 \sin(k z - \omega t) \mu \varepsilon vg\, x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) y(x, y)^2 \omega - 2 \sin(k z - \omega t) \mu \varepsilon \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) y(x, y)\, vg^2 x(x, y)^2 k$$

$$+ 4 x(x, y)^2 \varepsilon \mu^2 \cos(k z - \omega t)^2 vg^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 y(x, y)^2 + 4 x(x, y)^2 \varepsilon \mu^2 \cos(k z$$

$$- \omega t)^2 vg^2 y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 + 2 x(x, y)^6 \varepsilon \mu^2 \sin(k z - \omega t)^2 \omega\, vg\, k + 3 x(x,$$

$$y)^4 \varepsilon \mu^2 \sin(k z - \omega t)^2 y(x, y)^2 vg^2 k^2 + 3 x(x, y)^2 \varepsilon \mu^2 \sin(k z - \omega t)^2 y(x, y)^4 vg^2 k^2$$

$$+ 4 y(x, y)^2 \varepsilon \mu^2 \cos(k z - \omega t)^2 vg^2 x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 + 4 y(x, y)^2 \varepsilon \mu^2 \cos(k z$$

$$- \omega t)^2 vg^2 x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 + 2 y(x, y)^6 \varepsilon \mu^2 \sin(k z - \omega t)^2 \omega\, vg\, k - 8 \mu\, x(x,$$

$$y)^3 \left(\frac{\partial}{\partial x} y(x, y) \right) y(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) \cos(k z - \omega t)^2 - 8 \mu\, x(x, y)^3 \left(\frac{\partial}{\partial y} y(x, y) \right) y(x,$$

$$y) \left(\frac{\partial}{\partial y} x(x, y) \right) \cos(k z - \omega t)^2 - 8 \mu\, y(x, y)^3 \left(\frac{\partial}{\partial x} y(x, y) \right) \left(\frac{\partial}{\partial x} x(x, y) \right) x(x,$$

$$y) \cos(k z - \omega t)^2 - 8 \mu\, y(x, y)^3 \left(\frac{\partial}{\partial y} y(x, y) \right) \left(\frac{\partial}{\partial y} x(x, y) \right) x(x, y) \cos(k z - \omega t)^2$$

$$+ 4 x(x, y)^4 \varepsilon \mu^2 \cos(k z - \omega t)^2 vg^2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 + 4 x(x, y)^4 \varepsilon \mu^2 \cos(k z$$

$$- \omega t)^2 vg^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 + x(x, y)^6 \varepsilon \mu^2 \sin(k z - \omega t)^2 vg^2 k^2 + 3 x(x,$$

$$y)^4 \varepsilon \mu^2 \sin(k z - \omega t)^2 y(x, y)^2 \omega^2 + 3 x(x, y)^2 \varepsilon \mu^2 \sin(k z - \omega t)^2 y(x, y)^4 \omega^2 + 4 y(x,$$

$$y)^4 \varepsilon \mu^2 \cos(k z - \omega t)^2 vg^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 + 4 y(x, y)^4 \varepsilon \mu^2 \cos(k z$$

$$- \omega t)^2 vg^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 + y(x, y)^6 \varepsilon \mu^2 \sin(k z - \omega t)^2 vg^2 k^2))]$$

$$Dissipation \;= \varepsilon \cos(k z - \omega t) \Big(2\, vg \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y)\, vg + 2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 vg + 2\, vg \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\, vg \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 2\, vg\, y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - x(x,y)^2 \omega k - k^2 vg\, x(x,y)^2 - k \omega\, y(x,y)^2 - k^2 vg\, y(x,y)^2 - \mu \sin(k z - \omega t)\, x(x,y)^4 \omega - \mu \sin(k z - \omega t)\, x(x,y)^4 vg\, k - 2 \mu \sin(k z - \omega t)\, x(x,y)^2 y(x,y)^2 \omega - 2 \mu \sin(k z - \omega t)\, x(x,y)^2 y(x,y)^2 vg\, k - \mu \sin(k z - \omega t)\, y(x,y)^4 \omega - \mu \sin(k z - \omega t)\, y(x,y)^4 vg\, k \Big)$$

******************************* *END PROCEDURE* ***************************** **(24)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 5c-- waveguide TM mode phi (group kinematic out, wave in)`;
> theta:=(k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=+vg*f(x,y)*cos(theta);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 5c-- waveguide TM mode phi (group kinematic out, wave in)

$$\theta := k z + \omega t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z + \omega t)$$

$$\phi := vg \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z + \omega t)$$

Example 5c-- waveguide TM mode phi (group kinematic out, wave in)

********************************** *Differential Form Format* *********************************

$$Action\ 1\text{-}form = \left(-vg \cos(k z + \omega t)\, x(x,y)^2 - vg \cos(k z + \omega t)\, y(x,y)^2\right) d(t) + \left(\cos(k z + \omega t)\, x(x,y)^2 + \cos(k z + \omega t)\, y(x,y)^2\right) d(z)$$

$$Intensity\ 2\text{-}form\ F{=}dA \;= \left(-2\, vg \cos(k z + \omega t)\, y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) - 2\, vg \cos(k z + \omega t)\, x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right)\right) (d(x))\ \&\wedge\ (d(t)) + \left(2 \cos(k z + \omega t)\, y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 2 \cos(k z + \omega t)\, x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right)\right) (d(x))\ \&\wedge\ (d(z)) + \Big(-2\, vg \cos(k z$$

$+\omega t)\, y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) - 2\, vg \cos(k z + \omega t)\, x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right)\Big)$

$(d(y))$ &^ $(d(t)) + \Big(2 \cos(k z + \omega t)\, y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) + 2 \cos(k z + \omega t)\, x(x,$

$y) \left(\frac{\partial}{\partial y} x(x,y)\right)\Big)$ $(d(y))$ &^ $(d(z)) + \big(\sin(k z + \omega t)\, \omega\, x(x,y)^2 + \sin(k z$

$+ \omega t)\, \omega\, y(x,y)^2 + vg \sin(k z + \omega t)\, k\, x(x,y)^2 + vg \sin(k z + \omega t)\, k\, y(x,y)^2\big)$

$(d(z))$ &^ $(d(t))$

Topological Torsion 3-form *A^F* = 0

Topological Parity 4-form *F^F* = 0

******************************** *Using EM format* ******************************
****

E field $= \Big[-2\, vg \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right)\right) \cos(k z + \omega t),\ -2\, vg \Big(x(x,$

$y) \left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)\Big) \cos(k z + \omega t),\ \sin(k z + \omega t)\, \big(x(x,y)^2$

$+ y(x,y)^2\big)\, (\omega + vg\, k)\Big]$

B field $= \Big[2 \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)\right) \cos(k z + \omega t),\ -2 \Big(x(x,$

$y) \left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right)\Big) \cos(k z + \omega t),\ 0\Big]$

Topological TORSION 4 vector *T4 = -[ExA + Bphi,AdotB]* = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension *PTD* = 2

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

Xm or linear (Mean) curvature $= -\sin(k z + \omega t)\, \big(x(x,y)^2 + y(x,y)^2\big)\, (k$

$- \omega\, vg)$

Yg or quadratic (GAUSS) curvature = 0

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

$$D\ field = \Big[-2\,\varepsilon\, vg \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \cos(k z + \omega t), -2\,\varepsilon\, vg \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \cos(k z + \omega t), \varepsilon \sin(k z + \omega t) \left(x(x,y)^2 + y(x,y)^2 \right) (\omega + vg\, k) \Big]$$

$$H\ field = \left[\frac{2 \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \cos(k z + \omega t)}{\mu}, - \frac{2 \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \cos(k z + \omega t)}{\mu}, 0 \right]$$

$$Poynting\ vector\ ExH = \Big[\frac{1}{\mu} \Big(2 \sin(k z + \omega t) \left(x(x,y)^2 + y(x,y)^2 \right) (\omega + vg\, k) \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \cos(k z + \omega t) \Big), \frac{1}{\mu} \Big(2 \sin(k z + \omega t) \left(x(x,y)^2 + y(x,y)^2 \right) (\omega + vg\, k) \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \cos(k z + \omega t) \Big), \frac{1}{\mu} \Big(4\, vg \cos(k z + \omega t)^2 \Big(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2\, x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2\, x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 \Big) \Big) \Big]$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt=J4 = \Big[- \frac{2 \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \sin(k z + \omega t) (k + \omega\, \mu\, \varepsilon\, vg)}{\mu}, - \frac{2 \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \sin(k z + \omega t) (k + \omega\, \mu\, \varepsilon\, vg)}{\mu},$$

$$-\frac{1}{\mu}\left(\cos(k z+\omega t)\left(2\left(\frac{\partial}{\partial x} x(x, y)\right)^2+2 x(x, y)\left(\frac{\partial^2}{\partial x^2} x(x, y)\right)+2\left(\frac{\partial}{\partial x} y(x, y)\right)^2\right.\right.$$

$$+2 y(x, y)\left(\frac{\partial^2}{\partial x^2} y(x, y)\right)+2\left(\frac{\partial}{\partial y} x(x, y)\right)^2+2 x(x, y)\left(\frac{\partial^2}{\partial y^2} x(x, y)\right)$$

$$+2\left(\frac{\partial}{\partial y} y(x, y)\right)^2+2 y(x, y)\left(\frac{\partial^2}{\partial y^2} y(x, y)\right)+\varepsilon \omega^2 \mu x(x, y)^2+\varepsilon \omega \mu vg\, k x(x, y)^2$$

$$\left.\left.+\varepsilon \omega^2 \mu y(x, y)^2+\varepsilon \omega \mu vg\, k y(x, y)^2\right)\right), -\cos(k z+\omega t)\, \varepsilon\left(2 vg\left(\frac{\partial}{\partial x} x(x, y)\right)^2\right.$$

$$+2 vg\, x(x, y)\left(\frac{\partial^2}{\partial x^2} x(x, y)\right)+2\left(\frac{\partial^2}{\partial x^2} y(x, y)\right) y(x, y)\, vg+2\left(\frac{\partial}{\partial x} y(x, y)\right)^2 vg$$

$$+2 vg\left(\frac{\partial}{\partial y} x(x, y)\right)^2+2 vg\, x(x, y)\left(\frac{\partial^2}{\partial y^2} x(x, y)\right)+2 vg\left(\frac{\partial}{\partial y} y(x, y)\right)^2+2 vg\, y(x,$$

$$\left.\left. y)\left(\frac{\partial^2}{\partial y^2} y(x, y)\right)-x(x, y)^2 \omega k-k^2 vg\, x(x, y)^2-k \omega y(x, y)^2-k^2 vg\, y(x, y)^2\right)\right]$$

Amerian charge density $\quad divD = rho = -\cos(k z+\omega t)\, \varepsilon\left(2 vg\left(\frac{\partial}{\partial x} x(x, y)\right)^2+2 vg\, x(x,\right.$

$$y)\left(\frac{\partial^2}{\partial x^2} x(x, y)\right)+2\left(\frac{\partial^2}{\partial x^2} y(x, y)\right) y(x, y)\, vg+2\left(\frac{\partial}{\partial x} y(x, y)\right)^2 vg$$

$$+2 vg\left(\frac{\partial}{\partial y} x(x, y)\right)^2+2 vg\, x(x, y)\left(\frac{\partial^2}{\partial y^2} x(x, y)\right)+2 vg\left(\frac{\partial}{\partial y} y(x, y)\right)^2+2 vg\, y(x,$$

$$\left. y)\left(\frac{\partial^2}{\partial y^2} y(x, y)\right)-x(x, y)^2 \omega k-k^2 vg\, x(x, y)^2-k \omega y(x, y)^2-k^2 vg\, y(x, y)^2\right)$$

divergence Lorentz Current 4Vector, $\quad$ *4div(J4)* $=0$

Topological SPIN 4 vector S4 $=\left[-\frac{1}{\mu}\left(2\left(x(x, y)^2+y(x, y)^2\right) \cos(k z+\omega t)^2\left(x(x,\right.\right.\right.$

$$\left.\left. y)\left(\frac{\partial}{\partial x} x(x, y)\right)+y(x, y)\left(\frac{\partial}{\partial x} y(x, y)\right)\right)\left(-1+vg^2 \varepsilon \mu\right)\right), -\frac{1}{\mu}\left(2\left(x(x, y)^2\right.\right.$$

$$\left.+y(x, y)^2\right) \cos(k z+\omega t)^2\left(x(x, y)\left(\frac{\partial}{\partial y} x(x, y)\right)+y(x, y)\left(\frac{\partial}{\partial y} y(x, y)\right)\right)(-1$$

$$\left.\left.+vg^2 \varepsilon \mu\right)\right), \varepsilon \sin(k z+\omega t)\left(x(x, y)^2+y(x, y)^2\right)^2(\omega+vg\, k)\, vg \cos(k z+\omega t),$$

$$\left.\left(x(x, y)^2+y(x, y)^2\right)^2 \cos(k z+\omega t)\, \varepsilon \sin(k z+\omega t)\,(\omega+vg\, k)\right]$$

Topological SPIN 3-form $=-\frac{1}{\mu}\left(2\left(x(x, y)^2+y(x, y)^2\right) \cos(k z+\omega t)^2\left(-1+vg^2 \varepsilon \mu\right)\left(x(x,\right.\right.$

$y) \left(\frac{\partial}{\partial x} x(x, y)\right) + y(x, y) \left(\frac{\partial}{\partial x} y(x, y)\right)\Big) \&\text{^}(d(y), d(z), d(t))\Big) + \frac{1}{\mu}\Big(2 \big(x(x, y)^2$

$+ y(x, y)^2\big) \cos(k z + \omega t)^2 \left(-1 + vg^2 \varepsilon \mu\right) \Big(x(x, y) \left(\frac{\partial}{\partial y} x(x, y)\right) + y(x, y) \left(\frac{\partial}{\partial y} y(x,$

$y)\right)\Big) \&\text{^}(d(x), d(z), d(t))\Big) + \varepsilon \sin(k z + \omega t) \left(x(x, y)^2 + y(x, y)^2\right)^2 (\omega$

$+ vg\, k)\, vg \cos(k z + \omega t) \&\text{^}(d(x), d(y), d(t)) - \left(x(x, y)^2 + y(x, y)^2\right)^2 \cos(k z$

$+ \omega t)\, \varepsilon \sin(k z + \omega t)\, (\omega + vg\, k) \&\text{^}(d(x), d(y), d(z))$

Spin density rho_spin $= \left(x(x, y)^2 + y(x, y)^2\right)^2 \cos(k z + \omega t)\, \varepsilon \sin(k z + \omega t)\, (\omega + vg\, k)$

LaGrange field energy density (B.H-D.E) $= -\frac{1}{\mu}\Big(-4 \left(\frac{\partial}{\partial y} x(x, y)\right)^2 x(x, y)^2 \cos(k z + \omega t)^2$

$- 8 \left(\frac{\partial}{\partial y} y(x, y)\right) y(x, y) \left(\frac{\partial}{\partial y} x(x, y)\right) x(x, y) \cos(k z + \omega t)^2 - 4 \left(\frac{\partial}{\partial y} y(x,$

$y)\right)^2 y(x, y)^2 \cos(k z + \omega t)^2 - 4 \left(\frac{\partial}{\partial x} x(x, y)\right)^2 x(x, y)^2 \cos(k z + \omega t)^2 - 8 \left(\frac{\partial}{\partial x} y(x,$

$y)\right) y(x, y) \left(\frac{\partial}{\partial x} x(x, y)\right) x(x, y) \cos(k z + \omega t)^2 - 4 \left(\frac{\partial}{\partial x} y(x, y)\right)^2 y(x, y)^2 \cos(k z$

$+ \omega t)^2 + 4 \varepsilon \mu vg^2 \cos(k z + \omega t)^2 x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y)\right)^2 + 8 \varepsilon \mu vg^2 \cos(k z + \omega t)^2 x(x,$

$y) \left(\frac{\partial}{\partial x} x(x, y)\right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y)\right) + 4 \varepsilon \mu vg^2 \cos(k z + \omega t)^2 y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y)\right)^2$

$+ 4 \varepsilon \mu vg^2 \cos(k z + \omega t)^2 x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y)\right)^2 + 8 \varepsilon \mu vg^2 \cos(k z + \omega t)^2 x(x,$

$y) \left(\frac{\partial}{\partial y} x(x, y)\right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y)\right) + 4 \varepsilon \mu vg^2 \cos(k z + \omega t)^2 y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y)\right)^2$

$+ \varepsilon \mu \sin(k z + \omega t)^2 x(x, y)^4 \omega^2 + 2 \varepsilon \mu \sin(k z + \omega t)^2 x(x, y)^4 \omega\, vg\, k + \varepsilon \mu \sin(k z$

$+ \omega t)^2 x(x, y)^4 vg^2 k^2 + 2 \varepsilon \mu \sin(k z + \omega t)^2 x(x, y)^2 y(x, y)^2 \omega^2 + 4 \varepsilon \mu \sin(k z$

$+ \omega t)^2 x(x, y)^2 y(x, y)^2 \omega\, vg\, k + 2 \varepsilon \mu \sin(k z + \omega t)^2 x(x, y)^2 y(x, y)^2 vg^2 k^2$

$+ \varepsilon \mu \sin(k z + \omega t)^2 y(x, y)^4 \omega^2 + 2 \varepsilon \mu \sin(k z + \omega t)^2 y(x, y)^4 \omega\, vg\, k + \varepsilon \mu \sin(k z$

$+ \omega t)^2 y(x, y)^4 vg^2 k^2\Big)$

B.H $= \frac{1}{\mu}\Big(4 \cos(k z + \omega t)^2 \Big(x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y)\right)^2 + 2\, x(x, y) \left(\frac{\partial}{\partial y} x(x, y)\right) y(x,$

$y) \left(\frac{\partial}{\partial y} y(x, y)\right) + y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y)\right)^2 + x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y)\right)^2 + 2\, x(x,$

$$y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Bigg)\Bigg)$$

$$D.E=\varepsilon\Bigg(4\cos(kz+\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+8\cos(kz+\omega t)^2 vg^2 x(x,$$
$$y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+4\cos(kz+\omega t)^2 vg^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 y(x,y)^2$$
$$+4\cos(kz+\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+8\cos(kz+\omega t)^2 vg^2 x(x,y)\left(\frac{\partial}{\partial y}x(x,$$
$$y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+4\cos(kz+\omega t)^2 vg^2 y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+\sin(kz$$
$$+\omega t)^2 x(x,y)^4\omega^2+2\sin(kz+\omega t)^2\omega\, vg\, k\, x(x,y)^4+\sin(kz+\omega t)^2 x(x,y)^4 vg^2 k^2$$
$$+2\sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2\omega^2+4\sin(kz+\omega t)^2\omega\, vg\, k\, x(x,y)^2 y(x,y)^2$$
$$+2\sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2+\sin(kz+\omega t)^2 y(x,y)^4\omega^2+2\sin(kz$$
$$+\omega t)^2\omega\, vg\, k\, y(x,y)^4+\sin(kz+\omega t)^2 y(x,y)^4 vg^2 k^2\Bigg)$$

$$A.J=-\frac{1}{\mu}\Bigg((x(x,y)^2+y(x,y)^2)\cos(kz+\omega t)^2\Bigg(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,$$
$$y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,$$
$$y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\,\omega^2\mu\, x(x,y)^2$$
$$+\varepsilon\,\omega\,\mu\, vg\, k\, x(x,y)^2+\varepsilon\,\omega^2\mu\, y(x,y)^2+\varepsilon\,\omega\,\mu\, vg\, k\, y(x,y)^2\Bigg)\Bigg)$$

$$-rho.phi=-vg\,(x(x,y)^2+y(x,y)^2)\cos(kz+\omega t)^2\varepsilon\Bigg(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,$$
$$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 vg$$
$$+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg\,y(x,$$
$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-x(x,y)^2\omega k-k^2 vg\,x(x,y)^2-k\,\omega\,y(x,y)^2-k^2 vg\,y(x,y)^2\Bigg)$$

Poincare I $(B.H - D.E)-(A.J - rho.phi) = -\frac{1}{\mu}\Bigg(2\cos(kz+\omega t)^2 x(x,y)^2 vg^2\varepsilon\mu\left(\frac{\partial}{\partial y}y(x,\right.$

$$y)\Big)^2 + 2\cos(kz+\omega t)^2 y(x,y)^2 vg^2 \varepsilon\mu \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\cos(kz+\omega t)^2 y(x,$$

$$y)^2 vg^2 \varepsilon\mu \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - \cos(kz+\omega t)^2 x(x,y)^4 vg^2 \varepsilon\mu k^2 - \cos(kz+\omega t)^2 y(x,$$

$$y)^4 vg^2 \varepsilon\mu k^2 - 2\cos(kz+\omega t)^2 x(x,y)^2 \varepsilon\omega^2 \mu y(x,y)^2 + 2\cos(kz+\omega t)^2 x(x,$$

$$y)^2 vg^2 \varepsilon\mu \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - 2\cos(kz+\omega t)^2 y(x,y)^4 \varepsilon\omega\mu\, vg\, k + 2\varepsilon\mu \sin(kz$$

$$+\omega t)^2 y(x,y)^4 \omega\, vg\, k - 4\cos(kz+\omega t)^2 x(x,y)^2 \varepsilon\omega\mu\, vg\, k\, y(x,y)^2 + 2\cos(kz$$

$$+\omega t)^2 y(x,y)^2 vg^2 \varepsilon\mu\, x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\cos(kz+\omega t)^2 y(x,y)^2 vg^2 \varepsilon\mu\, x(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2\cos(kz+\omega t)^2 x(x,y)^2 vg^2 \varepsilon\mu\, y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right)$$

$$+2\cos(kz+\omega t)^2 x(x,y)^2 vg^2 \varepsilon\mu \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y) - 2\cos(kz+\omega t)^2 x(x,$$

$$y)^2 vg^2 \varepsilon\mu k^2 y(x,y)^2 - 2\cos(kz+\omega t)^2 x(x,y)^4 \varepsilon\omega\mu\, vg\, k - 6\left(\frac{\partial}{\partial x} y(x,y)\right)^2 y(x,$$

$$y)^2 \cos(kz+\omega t)^2 - 6\left(\frac{\partial}{\partial y} y(x,y)\right)^2 y(x,y)^2 \cos(kz+\omega t)^2 - 6\left(\frac{\partial}{\partial x} x(x,y)\right)^2 x(x,$$

$$y)^2 \cos(kz+\omega t)^2 - 6\left(\frac{\partial}{\partial y} x(x,y)\right)^2 x(x,y)^2 \cos(kz+\omega t)^2 - 2\cos(kz+\omega t)^2 x(x,$$

$$y)^3 \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - 2\cos(kz+\omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - 2\cos(kz+\omega t)^2 y(x,$$

$$y)^3 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) - 2\cos(kz+\omega t)^2 x(x,y)^3 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + \varepsilon\mu \sin(kz$$

$$+\omega t)^2 y(x,y)^4 vg^2 k^2 + 6\varepsilon\mu\, vg^2 \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + \varepsilon\mu \sin(kz$$

$$+\omega t)^2 x(x,y)^4 vg^2 k^2 + 2\varepsilon\mu \sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2 \omega^2 + 6\varepsilon\mu\, vg^2 \cos(kz$$

$$+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 6\varepsilon\mu\, vg^2 \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2$$

$$+6\varepsilon\mu\, vg^2 \cos(kz+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 2\cos(kz+\omega t)^2 y(x,$$

$$y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 - 2\cos(kz+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 - 2\cos(kz+\omega t)^2 y(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 - 2\cos(kz+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 + 8\,\varepsilon\,\mu\, vg^2 \cos(kz$$

$$+\omega t)^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) + \varepsilon\,\mu \sin(kz+\omega t)^2 x(x,y)^4 \omega^2$$

$$+8\,\varepsilon\,\mu\, vg^2 \cos(kz+\omega t)^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) + 4\,\varepsilon\,\mu \sin(kz$$

$$+\omega t)^2 x(x,y)^2 y(x,y)^2 \omega\, vg\, k + \varepsilon\,\mu \sin(kz+\omega t)^2 y(x,y)^4 \omega^2 - 8 \left(\frac{\partial}{\partial x} y(x,y) \right) y(x,$$

$$y) \left(\frac{\partial}{\partial x} x(x,y) \right) x(x,y) \cos(kz+\omega t)^2 - 8 \left(\frac{\partial}{\partial y} y(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) x(x,$$

$$y) \cos(kz+\omega t)^2 - \cos(kz+\omega t)^2 x(x,y)^4 \varepsilon\, \omega^2 \mu - \cos(kz+\omega t)^2 y(x,y)^4 \varepsilon\, \omega^2 \mu$$

$$+2\cos(kz+\omega t)^2 x(x,y)^3 vg^2 \varepsilon\,\mu \left(\frac{\partial^2}{\partial y^2} x(x,y) \right) + 2\cos(kz+\omega t)^2 y(x,$$

$$y)^3 vg^2 \varepsilon\,\mu \left(\frac{\partial^2}{\partial x^2} y(x,y) \right) + 2\cos(kz+\omega t)^2 y(x,y)^3 vg^2 \varepsilon\,\mu \left(\frac{\partial^2}{\partial y^2} y(x,y) \right)$$

$$+2\cos(kz+\omega t)^2 x(x,y)^3 vg^2 \varepsilon\,\mu \left(\frac{\partial^2}{\partial x^2} x(x,y) \right) - 2\cos(kz+\omega t)^2 y(x,y)^2 x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x,y) \right) - 2\cos(kz+\omega t)^2 y(x,y)^2 x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y) \right) - 2\cos(kz$$

$$+\omega t)^2 x(x,y)^2 y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y) \right) - 2\cos(kz+\omega t)^2 x(x,y)^2 y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y) \right)$$

$$+2\,\varepsilon\,\mu \sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 + 2\,\varepsilon\,\mu \sin(kz+\omega t)^2 x(x,y)^4 \omega\, vg\, k \Big)$$

$$\textit{London Coefficient} \qquad LC = \frac{k^2 - \mu\,\omega^2 \varepsilon}{\mu}$$

$$\textit{PROCA coefficient curlcurlB} = \Bigg[2\cos(kz+\omega t) \left(-2 \left(\frac{\partial}{\partial x} x(x,y) \right) \left(\frac{\partial^2}{\partial y\,\partial x} x(x,y) \right) \right.$$

$$- \left(\frac{\partial}{\partial y} x(x,y) \right) \left(\frac{\partial^2}{\partial x^2} x(x,y) \right) - x(x,y) \left(\frac{\partial^3}{\partial y\,\partial x^2} x(x,y) \right) - 2 \left(\frac{\partial}{\partial x} y(x,$$

$$y) \right) \left(\frac{\partial^2}{\partial y\,\partial x} y(x,y) \right) - \left(\frac{\partial}{\partial y} y(x,y) \right) \left(\frac{\partial^2}{\partial x^2} y(x,y) \right) - y(x,y) \left(\frac{\partial^3}{\partial y\,\partial x^2} y(x,y) \right)$$

$$-3\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-x(x,y)\left(\frac{\partial^3}{\partial y^3}x(x,y)\right)-3\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-y(x,y)\left(\frac{\partial^3}{\partial y^3}y(x,y)\right)+k^2x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+k^2y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Bigg),$$

$$2\cos(kz+\omega t)\Bigg(-k^2x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)-k^2y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+3\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+x(x,y)\left(\frac{\partial^3}{\partial x^3}x(x,y)\right)+3\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+y(x,y)\left(\frac{\partial^3}{\partial x^3}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}x(x,y)\right)+\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+x(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}y(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+y(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}y(x,y)\right)\Bigg),0\Bigg]$$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \Bigg[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz+\omega t)\,(k+\omega\,\mu\,\varepsilon\,vg)}{\mu},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\sin(kz+\omega t)\,(k+\omega\,\mu\,\varepsilon\,vg)}{\mu},$$

$$-\frac{1}{\mu}\Bigg(\cos(kz+\omega t)\Bigg(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\,\omega^2\mu\,x(x,y)^2+\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2+\varepsilon\,\omega^2\mu\,y(x,y)^2+\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\Bigg)\Bigg),\ -\cos(kz+\omega t)\,\varepsilon\Bigg(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2$$

$$+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2vg$$

$$+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-x(x,y)^2\,\omega\,k-k^2\,vg\,x(x,y)^2-k\,\omega\,y(x,y)^2-k^2\,vg\,y(x,y)^2\Big)\Big]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) = $\Big[$

$$-\frac{1}{\mu}\Big(2\cos(k\,z+\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\Big($$

$$-2\left(\frac{\partial}{\partial x}x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-2\left(\frac{\partial}{\partial x}y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,$$

$$y)\right)-2\left(\frac{\partial}{\partial y}x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-2\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-\varepsilon\,\omega^2\,\mu\,x(x,y)^2-2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2-\varepsilon\,\omega^2\,\mu\,y(x,y)^2$$

$$-2\,\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-vg^2\,\varepsilon\,\mu\,k^2\,x(x,y)^2-vg^2\,\varepsilon\,\mu\,k^2\,y(x,y)^2\Big)\Big),\ -\frac{1}{\mu}\Big(2\cos(k\,z$$

$$+\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\Big(-2\left(\frac{\partial}{\partial x}x(x,y)\right)^2-2\,x(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-2\left(\frac{\partial}{\partial x}y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)-2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$-2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-\varepsilon\,\omega^2\,\mu\,x(x,y)^2$$

$$-2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2-\varepsilon\,\omega^2\,\mu\,y(x,y)^2-2\,\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}x(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-vg^2\,\varepsilon\,\mu\,k^2\,x(x,y)^2$$

$$-vg^2\,\varepsilon\,\mu\,k^2\,y(x,y)^2\Big)\Big)\Big],\ \frac{1}{\mu}\Big(\sin(k\,z+\omega\,t)\,\cos(k\,z+\omega\,t)\,\Big(-\mu\,\varepsilon\,\omega^2\,k\,x(x,y)^4$$

$$-\mu\,\varepsilon\,k^3\,vg^2\,x(x,y)^4-\mu\,\varepsilon\,\omega^2\,k\,y(x,y)^4-\mu\,\varepsilon\,k^3\,vg^2\,y(x,y)^4-8\,k\,x(x,y)\left(\frac{\partial}{\partial y}x(x,\right.$$

$$\left.y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)-8\,k\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)$$

$$+2\,\mu\,\varepsilon\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)y(x,y)^2\,\omega+2\,\mu\,\varepsilon\,vg^2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)y(x,y)^2\,k$$

$$+2\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg\,x(x,y)^2\,\omega+2\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg^2\,x(x,y)^2\,k$$

$$+2\,\mu\,\varepsilon\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)y(x,y)^2\,\omega+2\,\mu\,\varepsilon\,vg^2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)y(x,y)^2\,k$$

$$+2\,\mu\,\varepsilon\,vg\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)x(x,y)^2\,\omega+2\,\mu\,\varepsilon\,vg^2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)x(x,y)^2\,k$$

$$+2\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)^3\,vg\,\omega+2\,\mu\,\varepsilon\left(\frac{\partial}{\partial x}y(x,y)\right)^2vg\,x(x,y)^2\,\omega$$

$$+2\mu\varepsilon\left(\frac{\partial}{\partial x}y(x,y)\right)^2 vg^2 x(x,y)^2 k-2\mu\varepsilon vg\, y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\omega$$

$$+2\mu\varepsilon vg^2 y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 k-2\mu\varepsilon vg\, x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2\omega$$

$$+2\mu\varepsilon vg^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2 k+2\mu\varepsilon vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2 y(x,y)^2\omega$$

$$+2\mu\varepsilon vg^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2 y(x,y)^2 k+2\mu\varepsilon vg\left(\frac{\partial}{\partial y}y(x,y)\right)^2 x(x,y)^2\omega$$

$$+2\mu\varepsilon vg^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 x(x,y)^2 k-2\mu\varepsilon vg\, y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2\omega$$

$$+2\mu\varepsilon vg^2 y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 k-2\mu\varepsilon\omega k^2 x(x,y)^4 vg-2\mu\varepsilon\omega^2 k\, x(x,y)^2 y(x,y)^2$$

$$-2\mu\varepsilon k^3 vg^2 x(x,y)^2 y(x,y)^2-2\mu\varepsilon\omega k^2 y(x,y)^4 vg-2\mu\varepsilon vg\, x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2\omega$$

$$+2\mu\varepsilon vg^2 x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2 k+2\mu\varepsilon vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2 y(x,y)^2\omega$$

$$+2\mu\varepsilon vg^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2 y(x,y)^2 k-4\mu\varepsilon\omega k^2 x(x,y)^2 y(x,y)^2 vg-8\omega\mu\varepsilon vg\, x(x,$$

$$y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)-8\omega\mu\varepsilon vg\, x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,$$

$$y)\left(\frac{\partial}{\partial x}y(x,y)\right)+2\mu\varepsilon\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)^3 vg^2 k+2\mu\varepsilon vg\, x(x,y)^3\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)\omega$$

$$+2\mu\varepsilon vg^2 x(x,y)^3\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)k+2\mu\varepsilon vg\, y(x,y)^3\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\omega$$

$$+2\mu\varepsilon vg^2 y(x,y)^3\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)k+2\mu\varepsilon vg\, x(x,y)^3\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)\omega$$

$$+2\mu\varepsilon vg^2 x(x,y)^3\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)k-4k\,x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2-4k\,y(x,$$

$$y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-4k\,x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2-4k\,y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Bigg)\Bigg)\Bigg]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \Big[-\frac{1}{\mu}\Big(2\,(x(x,y)^2 + y(x,y)^2)\cos(kz+\omega t)^2 \Big(x(x,y)\Big(\frac{\partial}{\partial x}x(x,y)\Big) + y(x,y)\Big(\frac{\partial}{\partial x}y(x,y)\Big)\Big)(-1 + vg^2\,\varepsilon\,\mu)\Big), -\frac{1}{\mu}\Big(2\,(x(x,y)^2 + y(x,y)^2)\cos(kz+\omega t)^2\Big(x(x,y)\Big(\frac{\partial}{\partial y}x(x,y)\Big) + y(x,y)\Big(\frac{\partial}{\partial y}y(x,y)\Big)\Big)(-1 + vg^2\,\varepsilon\,\mu)\Big), \varepsilon\sin(kz+\omega t)\,(x(x,y)^2 + y(x,y)^2)^2\,(\omega + vg\,k)\,vg\cos(kz+\omega t), (x(x,y)^2 + y(x,y)^2)^2\cos(kz+\omega t)\,\varepsilon\sin(kz+\omega t)\,(\omega + vg\,k)\Big]$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= \Big[0, 0, -\frac{1}{\mu}\Big((x(x,y)^2 + y(x,y)^2)\cos(kz+\omega t)\Big(-4\Big(\frac{\partial}{\partial y}x(x,y)\Big)^2 x(x,y)^2\cos(kz+\omega t)^2 - 8\Big(\frac{\partial}{\partial y}y(x,y)\Big)y(x,y)\Big(\frac{\partial}{\partial y}x(x,y)\Big)x(x,y)\cos(kz+\omega t)^2 - 4\Big(\frac{\partial}{\partial y}y(x,y)\Big)^2 y(x,y)^2\cos(kz+\omega t)^2 - 4\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2 x(x,y)^2\cos(kz+\omega t)^2 - 8\Big(\frac{\partial}{\partial x}y(x,y)\Big)y(x,y)\Big(\frac{\partial}{\partial x}x(x,y)\Big)x(x,y)\cos(kz+\omega t)^2 - 4\Big(\frac{\partial}{\partial x}y(x,y)\Big)^2 y(x,y)^2\cos(kz+\omega t)^2 + 4\,\varepsilon\,\mu\,vg^2\cos(kz+\omega t)^2 x(x,y)^2\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2 + 8\,\varepsilon\,\mu\,vg^2\cos(kz+\omega t)^2 x(x,y)\Big(\frac{\partial}{\partial x}x(x,y)\Big)y(x,y)\Big(\frac{\partial}{\partial x}y(x,y)\Big) + 4\,\varepsilon\,\mu\,vg^2\cos(kz+\omega t)^2 y(x,y)^2\Big(\frac{\partial}{\partial x}y(x,y)\Big)^2 + 4\,\varepsilon\,\mu\,vg^2\cos(kz+\omega t)^2 x(x,y)^2\Big(\frac{\partial}{\partial y}x(x,y)\Big)^2 + 8\,\varepsilon\,\mu\,vg^2\cos(kz+\omega t)^2 x(x,y)\Big(\frac{\partial}{\partial y}x(x,y)\Big)y(x,y)\Big(\frac{\partial}{\partial y}y(x,y)\Big) + 4\,\varepsilon\,\mu\,vg^2\cos(kz+\omega t)^2 y(x,y)^2\Big(\frac{\partial}{\partial y}y(x,y)\Big)^2 + \varepsilon\,\mu\sin(kz+\omega t)^2 x(x,y)^4\omega^2 + 2\,\varepsilon\,\mu\sin(kz+\omega t)^2 x(x,y)^4\,\omega\,vg\,k + \varepsilon\,\mu\sin(kz+\omega t)^2 x(x,y)^4 vg^2 k^2 + 2\,\varepsilon\,\mu\sin(kz+\omega t)^2 x(x,y)^2 y(x,y)^2\omega^2 + 4\,\varepsilon\,\mu\sin(kz$

$$+ \omega t)^2 x(x,y)^2 y(x,y)^2 \omega\, vg\, k + 2\, \varepsilon\, \mu \sin(k z + \omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2$$
$$+ \varepsilon\, \mu \sin(k z + \omega t)^2 y(x,y)^4 \omega^2 + 2\, \varepsilon\, \mu \sin(k z + \omega t)^2 y(x,y)^4 \omega\, vg\, k + \varepsilon\, \mu \sin(k z$$
$$+ \omega t)^2 y(x,y)^4 vg^2 k^2)\,)\,]$$

Spin Dissipation J_spin dot E $= \frac{1}{\mu}\Big((x(x,y)^2 + y(x,y)^2) \cos(k z + \omega t)\, vg \Big(-4 \left(\frac{\partial}{\partial y} x(x,$

$$y)\right)^2 x(x,y)^2 \cos(k z + \omega t)^2 - 8 \left(\frac{\partial}{\partial y} y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) x(x,y) \cos(k z$$
$$+ \omega t)^2 - 4 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 y(x,y)^2 \cos(k z + \omega t)^2 - 4 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 x(x,y)^2 \cos(k z$$
$$+ \omega t)^2 - 8 \left(\frac{\partial}{\partial x} y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) x(x,y) \cos(k z + \omega t)^2$$
$$- 4 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 y(x,y)^2 \cos(k z + \omega t)^2 + 4\, \varepsilon\, \mu\, vg^2 \cos(k z + \omega t)^2 x(x,$$
$$y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 8\, \varepsilon\, \mu\, vg^2 \cos(k z + \omega t)^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right)$$
$$+ 4\, \varepsilon\, \mu\, vg^2 \cos(k z + \omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + 4\, \varepsilon\, \mu\, vg^2 \cos(k z + \omega t)^2 x(x,$$
$$y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 8\, \varepsilon\, \mu\, vg^2 \cos(k z + \omega t)^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)$$
$$+ 4\, \varepsilon\, \mu\, vg^2 \cos(k z + \omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + \varepsilon\, \mu \sin(k z + \omega t)^2 x(x,y)^4 \omega^2$$
$$+ 2\, \varepsilon\, \mu \sin(k z + \omega t)^2 x(x,y)^4 \omega\, vg\, k + \varepsilon\, \mu \sin(k z + \omega t)^2 x(x,y)^4 vg^2 k^2$$
$$+ 2\, \varepsilon\, \mu \sin(k z + \omega t)^2 x(x,y)^2 y(x,y)^2 \omega^2 + 4\, \varepsilon\, \mu \sin(k z + \omega t)^2 x(x,y)^2 y(x,y)^2 \omega\, vg\, k$$
$$+ 2\, \varepsilon\, \mu \sin(k z + \omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 + \varepsilon\, \mu \sin(k z + \omega t)^2 y(x,y)^4 \omega^2$$
$$+ 2\, \varepsilon\, \mu \sin(k z + \omega t)^2 y(x,y)^4 \omega\, vg\, k + \varepsilon\, \mu \sin(k z + \omega t)^2 y(x,y)^4 vg^2 k^2 \Big)\Big)$$

Disspative Force 3 vector $= \Big[-\frac{1}{\mu}\Big(2 \cos(k z + \omega t)^2 \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,\right.$

$$\left. y) \left(\frac{\partial}{\partial x} y(x,y)\right)\right) \left(-2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 2\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - 2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 \right.$$

$$-2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)-2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)$$

$$-2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)-\varepsilon\,\omega^2\,\mu\,x(x,y)^2-2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2$$

$$-\varepsilon\,\omega^2\,\mu\,y(x,y)^2-2\,\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\,y(x,y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)-vg^2\,\varepsilon\,\mu\,k^2\,x(x,y)^2-vg^2\,\varepsilon\,\mu\,k^2\,y(x,y)^2\Big)\Big),$$

$$-\frac{1}{\mu}\Bigg(2\cos(k\,z+\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)\right)\Bigg($$

$$-2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)-2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}\,y(x,$$

$$y)\right)-2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)-2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2-2\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)-\varepsilon\,\omega^2\,\mu\,x(x,y)^2-2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2-\varepsilon\,\omega^2\,\mu\,y(x,y)^2$$

$$-2\,\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)-vg^2\,\varepsilon\,\mu\,k^2\,x(x,y)^2-vg^2\,\varepsilon\,\mu\,k^2\,y(x,y)^2\Big)\Big)\Big],\ -\frac{1}{\mu}\Big(\cos(k\,z$$

$$+\omega\,t)\Big(4\sin(k\,z+\omega\,t)\,k\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2+4\sin(k\,z+\omega\,t)\,k\,y(x,$$

$$y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2+4\sin(k\,z+\omega\,t)\,k\,x(x,y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2+4\sin(k\,z$$

$$+\omega\,t)\,k\,y(x,y)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2+4\,x(x,y)^2\,\varepsilon\,\mu^2\,vg^2\cos(k\,z+\omega\,t)^2\,y(x,$$

$$y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2+4\,x(x,y)^2\,\varepsilon\,\mu^2\,vg^2\cos(k\,z+\omega\,t)^2\,y(x,y)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2$$

$$+2\,x(x,y)^6\,\varepsilon\,\mu^2\sin(k\,z+\omega\,t)^2\,\omega\,vg\,k+3\,x(x,y)^4\,\varepsilon\,\mu^2\sin(k\,z+\omega\,t)^2\,y(x,y)^2\,vg^2\,k^2$$

$$+3\,x(x,y)^2\,\varepsilon\,\mu^2\sin(k\,z+\omega\,t)^2\,y(x,y)^4\,vg^2\,k^2+4\,y(x,y)^2\,\varepsilon\,\mu^2\,vg^2\cos(k\,z+\omega\,t)^2\,x(x,$$

$$y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2+4\,y(x,y)^2\,\varepsilon\,\mu^2\,vg^2\cos(k\,z+\omega\,t)^2\,x(x,y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2$$

$$+2\,y(x,y)^6\,\varepsilon\,\mu^2\sin(k\,z+\omega\,t)^2\,\omega\,vg\,k+8\,x(x,y)^3\,\varepsilon\,\mu^2\,vg^2\cos(k\,z+\omega\,t)^2\left(\frac{\partial}{\partial x}\,x(x,\right.$$

$$\left.y)\right)y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)+8\,x(x,y)^3\,\varepsilon\,\mu^2\,vg^2\cos(k\,z+\omega\,t)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,$$

$$y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)+6\,x(x,y)^4\,\varepsilon\,\mu^2\sin(k\,z+\omega\,t)^2\,y(x,y)^2\,\omega\,vg\,k+6\,x(x,$$

$$y)^2\,\varepsilon\,\mu^2\sin(k\,z+\omega\,t)^2\,y(x,y)^4\,\omega\,vg\,k+8\,y(x,y)^3\,\varepsilon\,\mu^2\,vg^2\cos(k\,z+\omega\,t)^2\,x(x,$$

$$y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)\left(\frac{\partial}{\partial x}\,y(x,y)\right)+8\,y(x,y)^3\,\varepsilon\,\mu^2\,vg^2\cos(k\,z+\omega\,t)^2\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,\right.$$

$$\left.y)\right)\left(\frac{\partial}{\partial y}\,y(x,y)\right)-2\sin(k\,z+\omega\,t)\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)\,vg\,x(x,y)^2\,\omega-2\sin(k\,z$$

$$+ \omega t)\, \mu\, \varepsilon\, vg\, x(x, y) \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) y(x, y)^2\, \omega - 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg^2\, x(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) y(x, y)^2\, k - 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg^2\, y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) x(x, y)^2\, k$$

$$- 2 \sin(k z + \omega t)\, \mu\, \varepsilon \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) y(x, y)\, vg^2\, x(x, y)^2\, k - 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg\, x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) y(x, y)^2\, \omega - 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg^2\, x(x, y) \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) y(x, y)^2\, k$$

$$- 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg\, y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) x(x, y)^2\, \omega - 2 \sin(k z$$

$$+ \omega t)\, \mu\, \varepsilon\, vg\, x(x, y)^3 \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) \omega - 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg^2\, x(x, y)^3 \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) k$$

$$- 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg\, y(x, y)^3 \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) \omega - 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg^2\, y(x,$$

$$y)^3 \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) k - 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg\, x(x, y)^3 \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) \omega - 2 \sin(k z$$

$$+ \omega t)\, \mu\, \varepsilon\, vg^2\, x(x, y)^3 \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) k - 2 \sin(k z + \omega t)\, \mu\, \varepsilon \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) y(x, y)^3\, vg\, \omega$$

$$- 2 \sin(k z + \omega t)\, \mu\, \varepsilon \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) y(x, y)^3\, vg^2\, k + y(x, y)^6\, \varepsilon\, \mu^2 \sin(k z + \omega t)^2\, \omega^2$$

$$- 4\, \mu\, x(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 y(x, y)^2 \cos(k z + \omega t)^2 - 4\, \mu\, x(x, y)^2 \left(\frac{\partial}{\partial x} y(x,$$

$$y) \right)^2 y(x, y)^2 \cos(k z + \omega t)^2 - 4\, \mu\, y(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 x(x, y)^2 \cos(k z + \omega t)^2$$

$$- 4\, \mu\, y(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 x(x, y)^2 \cos(k z + \omega t)^2 + x(x, y)^6\, \varepsilon\, \mu^2 \sin(k z + \omega t)^2\, \omega^2$$

$$+ 8 \sin(k z + \omega t)\, \omega\, \mu\, \varepsilon\, vg\, x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) + 8 \sin(k z$$

$$+ \omega t)\, \omega\, \mu\, \varepsilon\, vg\, x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) + 4 \sin(k z$$

$$+ \omega t)\, \mu\, \varepsilon\, \omega\, k^2\, x(x, y)^2\, y(x, y)^2\, vg - 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg^2\, y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 k$$

$$+ 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg\, x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 \omega - 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg^2\, x(x,$$

$$y)^2 \left(\frac{\partial}{\partial y}\, x(x,y)\right)^2 k - 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg \left(\frac{\partial}{\partial y}\, x(x,y)\right)^2 y(x,y)^2\, \omega - 2 \sin(k z$$

$$+ \omega t)\, \mu\, \varepsilon\, vg^2 \left(\frac{\partial}{\partial y}\, x(x,y)\right)^2 y(x,y)^2\, k - 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg \left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 x(x,y)^2\, \omega$$

$$- 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg^2 \left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 x(x,y)^2\, k + 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg\, y(x,$$

$$y)^2 \left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 \omega - 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg^2\, y(x,y)^2 \left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 k + 2 \sin(k z$$

$$+ \omega t)\, \mu\, \varepsilon\, \omega\, k^2\, x(x,y)^4\, vg + 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, \omega^2\, k\, x(x,y)^2\, y(x,y)^2 + 2 \sin(k z$$

$$+ \omega t)\, \mu\, \varepsilon\, k^3\, vg^2\, x(x,y)^2\, y(x,y)^2 + 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, \omega\, k^2\, y(x,y)^4\, vg + 2 \sin(k z$$

$$+ \omega t)\, \mu\, \varepsilon\, vg\, x(x,y)^2 \left(\frac{\partial}{\partial x}\, x(x,y)\right)^2 \omega - 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg^2\, x(x,y)^2 \left(\frac{\partial}{\partial x}\, x(x,y)\right)^2 k$$

$$- 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg \left(\frac{\partial}{\partial x}\, x(x,y)\right)^2 y(x,y)^2\, \omega - 2 \sin(k z$$

$$+ \omega t)\, \mu\, \varepsilon\, vg^2 \left(\frac{\partial}{\partial x}\, x(x,y)\right)^2 y(x,y)^2\, k - 2 \sin(k z + \omega t)\, \mu\, \varepsilon \left(\frac{\partial}{\partial x}\, y(x,y)\right)^2 vg\, x(x,y)^2\, \omega$$

$$- 2 \sin(k z + \omega t)\, \mu\, \varepsilon \left(\frac{\partial}{\partial x}\, y(x,y)\right)^2 vg^2\, x(x,y)^2\, k + 2 \sin(k z + \omega t)\, \mu\, \varepsilon\, vg\, y(x,$$

$$y)^2 \left(\frac{\partial}{\partial x}\, y(x,y)\right)^2 \omega - 8\, \mu\, x(x,y)^3 \left(\frac{\partial}{\partial y}\, y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y}\, x(x,y)\right) \cos(k z + \omega t)^2$$

$$- 8\, \mu\, x(x,y)^3 \left(\frac{\partial}{\partial x}\, y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x}\, x(x,y)\right) \cos(k z + \omega t)^2 - 8\, \mu\, y(x,$$

$$y)^3 \left(\frac{\partial}{\partial y}\, y(x,y)\right) \left(\frac{\partial}{\partial y}\, x(x,y)\right) x(x,y) \cos(k z + \omega t)^2 - 8\, \mu\, y(x,y)^3 \left(\frac{\partial}{\partial x}\, y(x,$$

$$y)\right) \left(\frac{\partial}{\partial x}\, x(x,y)\right) x(x,y) \cos(k z + \omega t)^2 + 4\, x(x,y)^4\, \varepsilon\, \mu^2\, vg^2 \cos(k z$$

$$+ \omega t)^2 \left(\frac{\partial}{\partial x}\, x(x,y)\right)^2 + 4\, x(x,y)^4\, \varepsilon\, \mu^2\, vg^2 \cos(k z + \omega t)^2 \left(\frac{\partial}{\partial y}\, x(x,y)\right)^2 + x(x,$$

$$y)^6\, \varepsilon\, \mu^2 \sin(k z + \omega t)^2\, vg^2\, k^2 + 3\, x(x,y)^4\, \varepsilon\, \mu^2 \sin(k z + \omega t)^2\, y(x,y)^2\, \omega^2 + 3\, x(x,$$

$$y)^2\, \varepsilon\, \mu^2 \sin(k z + \omega t)^2\, y(x,y)^4\, \omega^2 + 4\, y(x,y)^4\, \varepsilon\, \mu^2\, vg^2 \cos(k z + \omega t)^2 \left(\frac{\partial}{\partial x}\, y(x,y)\right)^2$$

$$+ 4\, y(x,y)^4\, \varepsilon\, \mu^2\, vg^2 \cos(k z + \omega t)^2 \left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 + y(x,y)^6\, \varepsilon\, \mu^2 \sin(k z + \omega t)^2\, vg^2\, k^2$$

$$-4\,\mu\,x(x,y)^4\left(\frac{\partial}{\partial y}x(x,y)\right)^2\cos(k\,z+\omega\,t)^2-4\,\mu\,x(x,y)^4\left(\frac{\partial}{\partial x}x(x,y)\right)^2\cos(k\,z+\omega\,t)^2$$

$$-4\,\mu\,y(x,y)^4\left(\frac{\partial}{\partial y}y(x,y)\right)^2\cos(k\,z+\omega\,t)^2-4\,\mu\,y(x,y)^4\left(\frac{\partial}{\partial x}y(x,y)\right)^2\cos(k\,z+\omega\,t)^2$$

$$+\sin(k\,z+\omega\,t)\,\mu\,\varepsilon\,\omega^2\,k\,x(x,y)^4+\sin(k\,z+\omega\,t)\,\mu\,\varepsilon\,k^3\,vg^2\,x(x,y)^4+\sin(k\,z$$

$$+\omega\,t)\,\mu\,\varepsilon\,\omega^2\,k\,y(x,y)^4+\sin(k\,z+\omega\,t)\,\mu\,\varepsilon\,k^3\,vg^2\,y(x,y)^4+8\sin(k\,z+\omega\,t)\,k\,x(x,$$

$$y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+8\sin(k\,z+\omega\,t)\,k\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,$$

$$y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Bigg)\Bigg)\Bigg)\Bigg]$$

$$Dissipation\ = -\cos(k\,z+\omega\,t)\,\varepsilon\left(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)\right.$$

$$+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2vg+2\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-x(x,y)^2\,\omega\,k$$

$$-k^2\,vg\,x(x,y)^2-k\,\omega\,y(x,y)^2-k^2\,vg\,y(x,y)^2-\mu\sin(k\,z+\omega\,t)\,x(x,y)^4\,\omega-\mu\sin(k\,z$$

$$+\omega\,t)\,x(x,y)^4\,vg\,k-2\,\mu\sin(k\,z+\omega\,t)\,x(x,y)^2\,y(x,y)^2\,\omega-2\,\mu\sin(k\,z+\omega\,t)\,x(x,$$

$$\left.y)^2\,y(x,y)^2\,vg\,k-\mu\sin(k\,z+\omega\,t)\,y(x,y)^4\,\omega-\mu\sin(k\,z+\omega\,t)\,y(x,y)^4\,vg\,k\right)$$

********************************* END PROCEDURE ****************************** **(25)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 5d -- waveguide TM mode (group kinematic out, wave out)`;
> theta:=(k*z-omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=+vg*f(x,y)*cos(theta);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 5d -- waveguide TM mode (group kinematic out, wave out)

$$\theta := k\,z - \omega\,t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z - \omega\,t)$$

$$\phi := vg\left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z - \omega\,t)$$

Example 5d -- waveguide TM mode (group kinematic out, wave out)

********************************* *Differential Form Format* ********************
****

$$\textit{Action 1-form} = \left(-vg\cos(k\,z-\omega\,t)\,x(x,y)^2 - vg\cos(k\,z-\omega\,t)\,y(x,y)^2\right)d(t) + \left(\cos(k\,z-\omega\,t)\,x(x,y)^2 + \cos(k\,z-\omega\,t)\,y(x,y)^2\right)d(z)$$

$$\textit{Intensity 2-form F=dA} = \left(-2\,vg\cos(k\,z-\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) - 2\,vg\cos(k\,z-\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)\right)(d(x))\ \&\wedge\ (d(t)) + \left(2\cos(k\,z-\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right) + 2\cos(k\,z-\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)\right)(d(x))\ \&\wedge\ (d(z)) + \left(-2\,vg\cos(k\,z-\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) - 2\,vg\cos(k\,z-\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)\right)(d(y))\ \&\wedge\ (d(t)) + \left(2\cos(k\,z-\omega\,t)\,y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right) + 2\cos(k\,z-\omega\,t)\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)\right)(d(y))\ \&\wedge\ (d(z)) + \left(-\sin(k\,z-\omega\,t)\,\omega\,x(x,y)^2 - \sin(k\,z-\omega\,t)\,\omega\,y(x,y)^2 + vg\sin(k\,z-\omega\,t)\,k\,x(x,y)^2 + vg\sin(k\,z-\omega\,t)\,k\,y(x,y)^2\right)(d(z))\ \&\wedge\ (d(t))$$

$$\textit{Topological Torsion 3-form} \quad A\wedge F = 0$$

$$\textit{Topological Parity 4-form} \quad F\wedge F = 0$$

********************************* *Using EM format* ******************************
****

$$E\ field = \left[-2\, vg \left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\cos(k z - \omega t),\ -2\, vg \left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right)\cos(k z - \omega t),\ \sin(k z - \omega t)\left(x(x,y)^2 + y(x,y)^2\right)(-\omega + vg\, k)\right]$$

$$B\ field = \left[2\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right)\cos(k z - \omega t),\ -2\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\cos(k z - \omega t),\ 0\right]$$

Topological TORSION 4 vector   T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension   PTD = 2

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ****** *********************

Xm or linear (Mean) curvature $= -\sin(k z - \omega t)\left(x(x,y)^2 + y(x,y)^2\right)(k + \omega\, vg)$

Yg or quadratic (GAUSS) curvature = 0

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

$$D\ field = \left[-2\,\varepsilon\, vg \left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\cos(k z - \omega t),\ -2\,\varepsilon\, vg \left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right)\cos(k z - \omega t),\ \varepsilon \sin(k z - \omega t)\left(x(x,y)^2 + y(x,y)^2\right)(-\omega + vg\, k)\right]$$

$$H\ field = \left[\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right)\cos(k z - \omega t)}{\mu},\ -\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\cos(k z - \omega t)}{\mu},\ 0\right]$$

$$\textit{Poynting vector ExH} = \Bigg[\frac{1}{\mu} \Big(2 \sin(k z - \omega t) \left(x(x, y)^2 + y(x, y)^2 \right) (-\omega + vg\, k) \Big(x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) + y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) \Big) \cos(k z - \omega t) \Big), \frac{1}{\mu} \Big(2 \sin(k z - \omega t) \left(x(x, y)^2 + y(x, y)^2 \right) (-\omega + vg\, k) \Big(x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) + y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) \Big) \cos(k z - \omega t) \Big), \frac{1}{\mu} \Big(4\, vg \cos(k z - \omega t)^2 \Big(x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 + 2\, x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) + y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 + x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 + 2\, x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) + y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 \Big) \Big) \Bigg]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} \quad = \Bigg[-\frac{2 \Big(x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) + y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) \Big) \sin(k z - \omega t) \left(k - \omega \mu \varepsilon\, vg \right)}{\mu}, -\frac{2 \Big(x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) + y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) \Big) \sin(k z - \omega t) \left(k - \omega \mu \varepsilon\, vg \right)}{\mu}, -\frac{1}{\mu} \Big(\cos(k z - \omega t) \Big(2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 + 2\, x(x, y) \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) + 2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 + 2\, y(x, y) \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) + 2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 + 2\, x(x, y) \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) + 2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 + 2\, y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) + \varepsilon \omega^2 \mu\, x(x, y)^2 + \varepsilon \omega^2 \mu\, y(x, y)^2 - \varepsilon \omega \mu\, vg\, k\, x(x, y)^2 - \varepsilon \omega \mu\, vg\, k\, y(x, y)^2 \Big) \Big), -\varepsilon \cos(k z - \omega t) \Big(2\, vg \left(\frac{\partial}{\partial x} x(x, y) \right)^2 + 2\, vg\, x(x, y) \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) + 2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 vg + 2 \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) y(x, y)\, vg + 2\, vg \left(\frac{\partial}{\partial y} x(x, y) \right)^2 + 2\, vg\, x(x, y) \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) + 2\, vg \left(\frac{\partial}{\partial y} y(x, y) \right)^2 + 2\, vg\, y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+x(x,y)^2\omega k+k\omega y(x,y)^2-k^2 vg\, x(x,y)^2-k^2 vg\, y(x,y)^2\Big)\Big]$$

Amerian charge density $\quad$ *divD = rho* $= -\varepsilon\cos(kz-\omega t)\Big(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg\,x(x,$

$$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 vg+2\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg$$

$$+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+x(x,y)^2\omega k+k\omega y(x,y)^2-k^2 vg\, x(x,y)^2-k^2 vg\, y(x,y)^2\Big)$$

divergence Lorentz Current 4Vector, $\quad$ *4div(J4)* $=0$

Topological SPIN 4 vector S4 $=\Big[-\frac{1}{\mu}\Big(2\,(x(x,y)^2+y(x,y)^2)\cos(kz-\omega t)^2(-1$

$$+vg^2\varepsilon\mu)\Big(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)\Big),-\frac{1}{\mu}\Big(2\,(x(x,y)^2$$

$$+y(x,y)^2)\cos(kz-\omega t)^2(-1+vg^2\varepsilon\mu)\Big(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,$$

$$y)\right)\Big)\Big),\varepsilon\sin(kz-\omega t)\,(x(x,y)^2+y(x,y)^2)^2(-\omega+vg\,k)\,vg\cos(kz-\omega t),$$

$$(x(x,y)^2+y(x,y)^2)^2\cos(kz-\omega t)\,\varepsilon\sin(kz-\omega t)\,(-\omega+vg\,k)\Big]$$

Topological SPIN 3-form $=-\frac{1}{\mu}\Big(2\,(x(x,y)^2+y(x,y)^2)\cos(kz-\omega t)^2(-1+vg^2\varepsilon\mu)\Big(x(x,$

$$y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)\ \&\wedge(d(y),d(z),d(t))\Big)+\frac{1}{\mu}\Big(2\,(x(x,y)^2$$

$$+y(x,y)^2)\cos(kz-\omega t)^2(-1+vg^2\varepsilon\mu)\Big(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,$$

$$y)\right)\Big)\ \&\wedge(d(x),d(z),d(t))\Big)+\varepsilon\sin(kz-\omega t)\,(x(x,y)^2+y(x,y)^2)^2(-\omega$$

$$+vg\,k)\,vg\cos(kz-\omega t)\ \&\wedge(d(x),d(y),d(t))-(x(x,y)^2+y(x,y)^2)^2\cos(kz$$

$$-\omega t)\,\varepsilon\sin(kz-\omega t)\,(-\omega+vg\,k)\ \&\wedge(d(x),d(y),d(z))$$

Spin density rho_spin $=(x(x,y)^2+y(x,y)^2)^2\cos(kz-\omega t)\,\varepsilon\sin(kz-\omega t)\,(-\omega+vg\,k)$

LaGrange field energy density (B.H-D.E) $=-\frac{1}{\mu}\Big(-4\left(\frac{\partial}{\partial y}x(x,y)\right)^2 x(x,y)^2\cos(kz-\omega t)^2$

$$- 8 \left(\frac{\partial}{\partial y} y(x, y) \right) y(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) x(x, y) \cos(k z - \omega t)^2 - 4 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 y(x, y)^2 \cos(k z - \omega t)^2 - 4 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 x(x, y)^2 \cos(k z - \omega t)^2 - 8 \left(\frac{\partial}{\partial x} y(x, y) \right) y(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) x(x, y) \cos(k z - \omega t)^2 - 4 y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 \cos(k z - \omega t)^2 + 4 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 + 8 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) + 4 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 y(x, y)^2 + 4 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 + 8 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) + 4 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 + \varepsilon \mu \sin(k z - \omega t)^2 x(x, y)^4 \omega^2 - 2 \varepsilon \mu \sin(k z - \omega t)^2 \omega\, vg\, k\, x(x, y)^4 + \varepsilon \mu \sin(k z - \omega t)^2 x(x, y)^4 vg^2 k^2 + 2 \varepsilon \mu \sin(k z - \omega t)^2 x(x, y)^2 y(x, y)^2 \omega^2 - 4 \varepsilon \mu \sin(k z - \omega t)^2 \omega\, vg\, k\, x(x, y)^2 y(x, y)^2 + 2 \varepsilon \mu \sin(k z - \omega t)^2 x(x, y)^2 y(x, y)^2 vg^2 k^2 + \varepsilon \mu \sin(k z - \omega t)^2 y(x, y)^4 \omega^2 - 2 \varepsilon \mu \sin(k z - \omega t)^2 \omega\, vg\, k\, y(x, y)^4 + \varepsilon \mu \sin(k z - \omega t)^2 y(x, y)^4 vg^2 k^2 \Big)$$

$$B.H = \frac{1}{\mu} \left(4 \cos(k z - \omega t)^2 \left(x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 + 2 x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) + y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 + x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 + 2 x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) + y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 \right) \right)$$

$$D.E = \varepsilon \Big(4 vg^2 \cos(k z - \omega t)^2 x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 + 8 vg^2 \cos(k z - \omega t)^2 x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) + 4 vg^2 \cos(k z - \omega t)^2 y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 + 4 vg^2 \cos(k z - \omega t)^2 x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 + 8 vg^2 \cos(k z - \omega t)^2 x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) + 4 vg^2 \cos(k z - \omega t)^2 y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 + \sin(k z - \omega t)^2 x(x, y)^4 \omega^2 - 2 \sin(k z - \omega t)^2 \omega\, vg\, k\, x(x, y)^4 + \sin(k z - \omega t)^2 x(x, y)^4 vg^2 k^2$$

$$+2\sin(kz-\omega t)^2 x(x,y)^2 y(x,y)^2 \omega^2 - 4\sin(kz-\omega t)^2 \omega\, vg\, k\, x(x,y)^2 y(x,y)^2$$

$$+2\sin(kz-\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 + \sin(kz-\omega t)^2 y(x,y)^4 \omega^2 - 2\sin(kz$$
$$-\omega t)^2 \omega\, vg\, k\, y(x,y)^4 + \sin(kz-\omega t)^2 y(x,y)^4 vg^2 k^2\Big)$$

$$A.J = -\frac{1}{\mu}\Bigg((x(x,y)^2+y(x,y)^2)\cos(kz-\omega t)^2\Bigg(2\left(\frac{\partial}{\partial x}x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,$$
$$y)\right) + 2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 + 2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right) + 2\left(\frac{\partial}{\partial y}x(x,y)\right)^2 + 2\,x(x,$$
$$y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right) + 2\left(\frac{\partial}{\partial y}y(x,y)\right)^2 + 2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right) + \varepsilon\,\omega^2\mu\,x(x,y)^2$$
$$+\varepsilon\,\omega^2\mu\,y(x,y)^2 - \varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2 - \varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\Bigg)\Bigg)$$

$$-rho.phi = -vg\,(x(x,y)^2+y(x,y)^2)\cos(kz-\omega t)^2\,\varepsilon\Bigg(2\,vg\left(\frac{\partial}{\partial x}x(x,y)\right)^2 + 2\,vg\,x(x,$$
$$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right) + 2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 vg + 2\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)\,vg$$
$$+2\,vg\left(\frac{\partial}{\partial y}x(x,y)\right)^2 + 2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right) + 2\,vg\left(\frac{\partial}{\partial y}y(x,y)\right)^2 + 2\,vg\,y(x,$$
$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right) + x(x,y)^2\omega\,k + k\,\omega\,y(x,y)^2 - k^2 vg\,x(x,y)^2 - k^2 vg\,y(x,y)^2\Bigg)$$

Poincare I $\quad$ $(B.H - D.E)-(A.J - rho.phi) = -\frac{1}{\mu}\Bigg(6\,\varepsilon\,\mu\cos(kz-\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial x}x(x,$

$$y)\right)^2 + 6\,\varepsilon\,\mu\cos(kz-\omega t)^2 vg^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2 y(x,y)^2 + 6\,\varepsilon\,\mu\cos(kz$$
$$-\omega t)^2 vg^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2 + 6\,\varepsilon\,\mu\cos(kz-\omega t)^2 vg^2 y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2$$
$$+\varepsilon\,\mu\sin(kz-\omega t)^2 x(x,y)^4 vg^2 k^2 + 2\,\varepsilon\,\mu\sin(kz-\omega t)^2 x(x,y)^2 y(x,y)^2\omega^2$$
$$+\varepsilon\,\mu\sin(kz-\omega t)^2 y(x,y)^4 vg^2 k^2 + 2\cos(kz-\omega t)^2 x(x,y)^4\varepsilon\,\omega\,\mu\,vg\,k - 2\cos(kz$$
$$-\omega t)^2 x(x,y)^2 vg^2\varepsilon\,\mu\,k^2 y(x,y)^2 + 2\cos(kz-\omega t)^2 y(x,y)^4\varepsilon\,\omega\,\mu\,vg\,k - 8\left(\frac{\partial}{\partial x}y(x,$$
$$y)\right)y(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)x(x,y)\cos(kz-\omega t)^2 - 8\left(\frac{\partial}{\partial y}y(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}x(x,$$

$$y)\Big)\, x(x, y) \cos(k z - \omega t)^2 + \varepsilon \mu \sin(k z - \omega t)^2 x(x, y)^4 \omega^2 + \varepsilon \mu \sin(k z - \omega t)^2 y(x,$$

$$y)^4 \omega^2 - 6 \left(\frac{\partial}{\partial x} x(x, y)\right)^2 x(x, y)^2 \cos(k z - \omega t)^2 - 6\, y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y)\right)^2 \cos(k z$$

$$- \omega t)^2 - 6 \left(\frac{\partial}{\partial y} y(x, y)\right)^2 y(x, y)^2 \cos(k z - \omega t)^2 - 6 \left(\frac{\partial}{\partial y} x(x, y)\right)^2 x(x, y)^2 \cos(k z$$

$$- \omega t)^2 - 2 \cos(k z - \omega t)^2 x(x, y)^2 y(x, y) \left(\frac{\partial^2}{\partial x^2} y(x, y)\right) - 2 \cos(k z - \omega t)^2 x(x,$$

$$y)^2 y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y)\right) - 2 \cos(k z - \omega t)^2 y(x, y)^2 x(x, y) \left(\frac{\partial^2}{\partial x^2} x(x, y)\right)$$

$$- 2 \cos(k z - \omega t)^2 y(x, y)^2 x(x, y) \left(\frac{\partial^2}{\partial y^2} x(x, y)\right) + 2 \cos(k z - \omega t)^2 x(x,$$

$$y)^3 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial x^2} x(x, y)\right) + 2 \cos(k z - \omega t)^2 x(x, y)^3 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial y^2} x(x, y)\right)$$

$$+ 2 \cos(k z - \omega t)^2 y(x, y)^3 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial x^2} y(x, y)\right) + 2 \cos(k z - \omega t)^2 y(x,$$

$$y)^3 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial y^2} y(x, y)\right) + 8 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x, y) \left(\frac{\partial}{\partial x} x(x, y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial x} y(x, y)\right) + 8 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x, y) \left(\frac{\partial}{\partial y} x(x, y)\right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y)\right)$$

$$- 4 \varepsilon \mu \sin(k z - \omega t)^2 \omega\, vg\, k\, x(x, y)^2 y(x, y)^2 + 2 \cos(k z - \omega t)^2 y(x, y)^2 vg^2 \varepsilon \mu\, x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x, y)\right) + 2 \cos(k z - \omega t)^2 x(x, y)^2 vg^2 \varepsilon \mu \left(\frac{\partial^2}{\partial x^2} y(x, y)\right) y(x, y)$$

$$+ 2 \cos(k z - \omega t)^2 x(x, y)^2 vg^2 \varepsilon \mu\, y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y)\right) + 2 \cos(k z - \omega t)^2 y(x,$$

$$y)^2 vg^2 \varepsilon \mu\, x(x, y) \left(\frac{\partial^2}{\partial x^2} x(x, y)\right) - 2 \cos(k z - \omega t)^2 x(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y)\right)^2$$

$$- 2 \cos(k z - \omega t)^2 x(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y)\right)^2 - 2 \cos(k z - \omega t)^2 y(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y)\right)^2$$

$$- 2 \cos(k z - \omega t)^2 y(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y)\right)^2 - 2 \cos(k z - \omega t)^2 x(x, y)^2 \varepsilon \omega^2 \mu\, y(x, y)^2$$

$$+ 2 \cos(k z - \omega t)^2 x(x, y)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial x} y(x, y)\right)^2 + 2 \cos(k z - \omega t)^2 x(x,$$

$$y)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - \cos(kz-\omega t)^2 x(x,y)^4 vg^2 \varepsilon \mu k^2 + 2\cos(kz-\omega t)^2 y(x,$$

$$y)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\cos(kz-\omega t)^2 y(x,y)^2 vg^2 \varepsilon \mu \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - \cos(kz$$

$$-\omega t)^2 y(x,y)^4 vg^2 \varepsilon \mu k^2 - 2\cos(kz-\omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - 2\cos(kz$$

$$-\omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) - 2\cos(kz-\omega t)^2 x(x,y)^3 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 4\cos(kz$$

$$-\omega t)^2 x(x,y)^2 \varepsilon \omega \mu \, vg\, k\, y(x,y)^2 - 2\varepsilon\mu \sin(kz-\omega t)^2 \omega\, vg\, k\, x(x,y)^4 + 2\varepsilon\mu \sin(kz$$

$$-\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 - 2\varepsilon\mu \sin(kz-\omega t)^2 \omega\, vg\, k\, y(x,y)^4 - \cos(kz$$

$$-\omega t)^2 x(x,y)^4 \varepsilon \omega^2 \mu - \cos(kz-\omega t)^2 y(x,y)^4 \varepsilon \omega^2 \mu - 2\cos(kz-\omega t)^2 x(x,$$

$$y)^3 \left(\frac{\partial^2}{\partial x^2} x(x,y)\right)\Bigg)$$

$$\textit{London Coefficient} \qquad LC = \frac{k^2 - \mu \omega^2 \varepsilon}{\mu}$$

$$\textit{PROCA coefficient curlcurlB} = \Bigg[2\cos(kz-\omega t)\left(-2\left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right)\right.$$

$$-\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - x(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2} x(x,y)\right) - 2\left(\frac{\partial}{\partial x} y(x,$$

$$y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} y(x,y)\right) - \left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) - y(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2} y(x,y)\right)$$

$$-3\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) - x(x,y)\left(\frac{\partial^3}{\partial y^3} x(x,y)\right) - 3\left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial y^2} y(x,$$

$$\left.y)\right) - y(x,y)\left(\frac{\partial^3}{\partial y^3} y(x,y)\right) + k^2 x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + k^2 y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right),$$

$$2\cos(kz-\omega t)\left(-k^2 x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) - k^2 y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right) + 3\left(\frac{\partial}{\partial x} x(x,\right.$$

$$y)\right)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial x^3} x(x,y)\right) + 3\left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + y(x,$$

$$y)\left(\frac{\partial^3}{\partial x^3} y(x,y)\right) + 2\left(\frac{\partial}{\partial y} x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right) + \left(\frac{\partial}{\partial x} x(x,y)\right)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$+ x(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x} x(x,y)\right) + 2\left(\frac{\partial}{\partial y} y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} y(x,y)\right) + \left(\frac{\partial}{\partial x} y(x,y)\right)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x} y(x,y)\right)\Bigg), 0\Bigg]$$

Amperian Current 4Vector   curlH-dD/dt=J4 $=\Bigg[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\sin(k\,z-\omega\,t)\,(k-\omega\,\mu\,\varepsilon\,vg)}{\mu},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right)\sin(k\,z-\omega\,t)\,(k-\omega\,\mu\,\varepsilon\,vg)}{\mu},$$

$$-\frac{1}{\mu}\Bigg(\cos(k\,z-\omega\,t)\Bigg(2\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2\left(\frac{\partial}{\partial x} y(x,y)\right)^2 + 2\,y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + 2\left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 2\,y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + \varepsilon\,\omega^2\,\mu\,x(x,y)^2 + \varepsilon\,\omega^2\,\mu\,y(x,y)^2 - \varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2 - \varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2\Bigg)\Bigg), -\varepsilon\cos(k\,z-\omega\,t)\Bigg(2\,vg\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2\left(\frac{\partial}{\partial x} y(x,y)\right)^2 vg + 2\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y)\,vg + 2\,vg\left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\,vg\,x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\,vg\left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 2\,vg\,y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + x(x,y)^2\,\omega\,k + k\,\omega\,y(x,y)^2 - k^2\,vg\,x(x,y)^2 - k^2\,vg\,y(x,y)^2\Bigg)\Bigg]$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B) $=\Bigg[$

$$-\frac{1}{\mu}\Bigg(2\cos(k\,z-\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\Bigg($$

$$-2\left(\frac{\partial}{\partial x}x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-2\left(\frac{\partial}{\partial x}y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,$$

$$y)\right)-2\left(\frac{\partial}{\partial y}x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-2\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-\varepsilon\,\omega^2\,\mu\,x(x,y)^2-\varepsilon\,\omega^2\,\mu\,y(x,y)^2+2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2$$

$$+2\,\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-vg^2\,\varepsilon\,\mu\,k^2\,x(x,y)^2-vg^2\,\varepsilon\,\mu\,k^2\,y(x,y)^2\Big)\Big),\ -\frac{1}{\mu}\Big(2\cos(k\,z$$

$$-\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\Big(-2\left(\frac{\partial}{\partial x}x(x,y)\right)^2-2\,x(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-2\left(\frac{\partial}{\partial x}y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)-2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$-2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-\varepsilon\,\omega^2\,\mu\,x(x,y)^2$$

$$-\varepsilon\,\omega^2\,\mu\,y(x,y)^2+2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2+2\,\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}x(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,y(x,y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)-vg^2\,\varepsilon\,\mu\,k^2\,x(x,y)^2$$

$$-vg^2\,\varepsilon\,\mu\,k^2\,y(x,y)^2\Big)\Big)\Big],\ \frac{1}{\mu}\Big(\sin(k\,z-\omega\,t)\,\cos(k\,z-\omega\,t)\,\Big(-\mu\,\varepsilon\,\omega^2\,k\,x(x,y)^4$$

$$-\mu\,\varepsilon\,k^3\,vg^2\,x(x,y)^4-\mu\,\varepsilon\,\omega^2\,k\,y(x,y)^4-\mu\,\varepsilon\,k^3\,vg^2\,y(x,y)^4-8\,k\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)-8\,k\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)$$

$$-2\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)^3\,vg\,\omega+2\,\mu\,\varepsilon\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)y(x,y)^3\,vg^2\,k$$

$$-2\,\mu\,\varepsilon\,vg^{x(x,y)^3}\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)\omega+2\,\mu\,\varepsilon\,vg^2\,x(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)k$$

$$-2\,\mu\,\varepsilon\,vg^{y(x,y)^3}\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)\omega+2\,\mu\,\varepsilon\,vg^2\,y(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)k$$

$$-2\,\mu\,\varepsilon\,vg^{x(x,y)^3}\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)\omega+2\,\mu\,\varepsilon\,vg^2\,x(x,y)^3\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)k$$

$$-2\,\mu\,\varepsilon\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 vg\,x(x,y)^2\,\omega+2\,\mu\,\varepsilon\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 vg^2\,x(x,y)^2\,k$$

$$+2\,\mu\,\varepsilon\,vg^{y(x,y)^2}\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2\omega+2\,\mu\,\varepsilon\,vg^2\,y(x,y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 k$$

$$+2\,\mu\,\varepsilon\,vg^{x(x,y)^2}\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2\omega+2\,\mu\,\varepsilon\,vg^2\,x(x,y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 k$$

$$-2\,\mu\,\varepsilon\,vg\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 y(x,y)^2\,\omega+2\,\mu\,\varepsilon\,vg^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 y(x,y)^2\,k$$

$$-2\,\mu\,\varepsilon\,vg\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 x(x,y)^2\,\omega+2\,\mu\,\varepsilon\,vg^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 x(x,y)^2\,k$$

$$+ 2\,\mu\,\varepsilon\, vg^{y(x,y)^2} \left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 \omega + 2\,\mu\,\varepsilon\, vg^2\, y(x,y)^2 \left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 k$$

$$+ 2\,\mu\,\varepsilon\,\omega\, k^2\, x(x,y)^4\, vg - 2\,\mu\,\varepsilon\,\omega^2\, k\, x(x,y)^2\, y(x,y)^2 - 2\,\mu\,\varepsilon\, k^3\, vg^2\, x(x,y)^2\, y(x,y)^2$$

$$+ 2\,\mu\,\varepsilon\,\omega\, k^2\, y(x,y)^4\, vg + 2\,\mu\,\varepsilon\, vg\, x(x,y)^2 \left(\frac{\partial}{\partial x}\, x(x,y)\right)^2 \omega + 2\,\mu\,\varepsilon\, vg^2\, x(x,$$

$$y)^2 \left(\frac{\partial}{\partial x}\, x(x,y)\right)^2 k - 2\,\mu\,\varepsilon\, vg \left(\frac{\partial}{\partial x}\, x(x,y)\right)^2 y(x,y)^2\, \omega + 2\,\mu\,\varepsilon\, vg^2 \left(\frac{\partial}{\partial x}\, x(x,$$

$$y)\right)^2 y(x,y)^2\, k + 4\,\mu\,\varepsilon\,\omega\, k^2\, x(x,y)^2\, y(x,y)^2\, vg + 8\,\omega\,\mu\,\varepsilon\, vg\, x(x,y) \left(\frac{\partial}{\partial y}\, x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial y}\, y(x,y)\right) + 8\,\omega\,\mu\,\varepsilon\, vg\, x(x,y) \left(\frac{\partial}{\partial x}\, x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x}\, y(x,y)\right) - 4\, k\, x(x,$$

$$y)^2 \left(\frac{\partial}{\partial y}\, x(x,y)\right)^2 - 4\, k\, y(x,y)^2 \left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 - 4\, k\, x(x,y)^2 \left(\frac{\partial}{\partial x}\, x(x,y)\right)^2$$

$$- 4\, k^{y(x,y)^2} \left(\frac{\partial}{\partial x}\, y(x,y)\right)^2 - 2\,\mu\,\varepsilon\, vg\, x(x,y) \left(\frac{\partial^2}{\partial x^2}\, x(x,y)\right) y(x,y)^2\, \omega + 2\,\mu\,\varepsilon\, vg^2\, x(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2}\, x(x,y)\right) y(x,y)^2\, k - 2\,\mu\,\varepsilon \left(\frac{\partial^2}{\partial x^2}\, y(x,y)\right) y(x,y)\; vg\, x(x,y)^2\, \omega + 2\,\mu\,\varepsilon \left(\frac{\partial^2}{\partial x^2}\, y(x,$$

$$y)\right) y(x,y)\; vg^2\, x(x,y)^2\, k - 2\,\mu\,\varepsilon\, vg\, x(x,y) \left(\frac{\partial^2}{\partial y^2}\, x(x,y)\right) y(x,y)^2\, \omega + 2\,\mu\,\varepsilon\, vg^2\, x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2}\, x(x,y)\right) y(x,y)^2\, k - 2\,\mu\,\varepsilon\, vg\, y(x,y) \left(\frac{\partial^2}{\partial y^2}\, y(x,y)\right) x(x,y)^2\, \omega + 2\,\mu\,\varepsilon\, vg^2\, y(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2}\, y(x,y)\right) x(x,y)^2\, k \Big) \Big) \Big]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] $= \Big[-\frac{1}{\mu} \Big(2\, \big(x(x,y)^2 + y(x,$

$y)^2) \cos(k z - \omega t)^2 (-1 + vg^2 \varepsilon \mu) \left(x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) + y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) \right) \right), -\frac{1}{\mu} \left(2 (x(x, y)^2 + y(x, y)^2) \cos(k z - \omega t)^2 (-1 + vg^2 \varepsilon \mu) \left(x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) + y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) \right) \right), \varepsilon \sin(k z - \omega t) (x(x, y)^2 + y(x, y)^2)^2 (-\omega + vg\, k)\, vg \cos(k z - \omega t), (x(x, y)^2 + y(x, y)^2)^2 \cos(k z - \omega t)\, \varepsilon \sin(k z - \omega t) (-\omega + vg\, k) \Big]$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $= \Big[0, 0,$

$-\frac{1}{\mu} \Big((x(x, y)^2 + y(x, y)^2) \cos(k z - \omega t) \Big(-4 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 x(x, y)^2 \cos(k z - \omega t)^2 - 8 \left(\frac{\partial}{\partial y} y(x, y) \right) y(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) x(x, y) \cos(k z - \omega t)^2 - 4 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 y(x, y)^2 \cos(k z - \omega t)^2 - 4 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 x(x, y)^2 \cos(k z - \omega t)^2 - 8 \left(\frac{\partial}{\partial x} y(x, y) \right) y(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) x(x, y) \cos(k z - \omega t)^2 - 4 y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 \cos(k z - \omega t)^2 + 4 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 + 8 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) + 4 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 y(x, y)^2 + 4 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 + 8 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) + 4 \varepsilon \mu \cos(k z - \omega t)^2 vg^2 y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 + \varepsilon \mu \sin(k z - \omega t)^2 x(x, y)^4 \omega^2 - 2 \varepsilon \mu \sin(k z - \omega t)^2 \omega\, vg\, k\, x(x, y)^4 + \varepsilon \mu \sin(k z - \omega t)^2 x(x, y)^4 vg^2 k^2 + 2 \varepsilon \mu \sin(k z - \omega t)^2 x(x, y)^2 y(x, y)^2 \omega^2 - 4 \varepsilon \mu \sin(k z - \omega t)^2 \omega\, vg\, k\, x(x, y)^2 y(x, y)^2 + 2 \varepsilon \mu \sin(k z - \omega t)^2 x(x, y)^2 y(x, y)^2 vg^2 k^2 + \varepsilon \mu \sin(k z - \omega t)^2 y(x, y)^4 \omega^2 - 2 \varepsilon \mu \sin(k z - \omega t)^2 \omega\, vg\, k\, y(x, y)^4 + \varepsilon \mu \sin(k z - \omega t)^2 y(x, y)^4 vg^2 k^2 \Big) \Big) \Big]$

Spin Dissipation J_spin dot E $= \frac{1}{\mu} \Big((x(x, y)^2 + y(x, y)^2) \cos(k z - \omega t)\, vg \Big(-4 \Big(\frac{\partial}{\partial y} x(x,$

$$y)\Big)^2 x(x,y)^2 \cos(kz-\omega t)^2 - 8\left(\frac{\partial}{\partial y} y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) x(x,y) \cos(kz$$

$$-\omega t)^2 - 4\left(\frac{\partial}{\partial y} y(x,y)\right)^2 y(x,y)^2 \cos(kz-\omega t)^2 - 4\left(\frac{\partial}{\partial x} x(x,y)\right)^2 x(x,y)^2 \cos(kz$$

$$-\omega t)^2 - 8\left(\frac{\partial}{\partial x} y(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) x(x,y) \cos(kz-\omega t)^2 - 4\, y(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 \cos(kz-\omega t)^2 + 4\,\varepsilon\,\mu \cos(kz-\omega t)^2 vg^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2$$

$$+ 8\,\varepsilon\,\mu \cos(kz-\omega t)^2 vg^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 4\,\varepsilon\,\mu \cos(kz$$

$$-\omega t)^2 vg^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 y(x,y)^2 + 4\,\varepsilon\,\mu \cos(kz-\omega t)^2 vg^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2$$

$$+ 8\,\varepsilon\,\mu \cos(kz-\omega t)^2 vg^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) + 4\,\varepsilon\,\mu \cos(kz$$

$$-\omega t)^2 vg^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + \varepsilon\,\mu \sin(kz-\omega t)^2 x(x,y)^4 \omega^2 - 2\,\varepsilon\,\mu \sin(kz$$

$$-\omega t)^2 \omega\, vg\, k\, x(x,y)^4 + \varepsilon\,\mu \sin(kz-\omega t)^2 x(x,y)^4 vg^2 k^2 + 2\,\varepsilon\,\mu \sin(kz-\omega t)^2 x(x,$$

$$y)^2 y(x,y)^2 \omega^2 - 4\,\varepsilon\,\mu \sin(kz-\omega t)^2 \omega\, vg\, k\, x(x,y)^2 y(x,y)^2 + 2\,\varepsilon\,\mu \sin(kz$$

$$-\omega t)^2 x(x,y)^2 y(x,y)^2 vg^2 k^2 + \varepsilon\,\mu \sin(kz-\omega t)^2 y(x,y)^4 \omega^2 - 2\,\varepsilon\,\mu \sin(kz$$

$$-\omega t)^2 \omega\, vg\, k\, y(x,y)^4 + \varepsilon\,\mu \sin(kz-\omega t)^2 y(x,y)^4 vg^2 k^2)\Big)$$

$$\textit{Disspative Force 3 vector} = \Bigg[-\frac{1}{\mu}\Bigg(2\cos(kz-\omega t)^2 \left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,\right.$$

$$\left. y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\Bigg) \left(-2\left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 2\,x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - 2\left(\frac{\partial}{\partial x} y(x,y)\right)^2\right.$$

$$-2\,y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) - 2\left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 2\,x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$-2\left(\frac{\partial}{\partial y} y(x,y)\right)^2 - 2\,y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - \varepsilon\,\omega^2 \mu\, x(x,y)^2 - \varepsilon\,\omega^2 \mu\, y(x,y)^2$$

$$+ 2\,\varepsilon\,\omega\,\mu\, vg\, k\, x(x,y)^2 + 2\,\varepsilon\,\omega\,\mu\, vg\, k\, y(x,y)^2 + 2\, vg^2 \varepsilon\,\mu \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\, vg^2 \varepsilon\,\mu\, x(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}y(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\,y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-vg^2\,\varepsilon\,\mu\,k^2\,x(x,y)^2-vg^2\,\varepsilon\,\mu\,k^2\,y(x,y)^2\Big)\Big),$$

$$-\frac{1}{\mu}\Big(2\cos(k\,z-\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\Big($$

$$-2\left(\frac{\partial}{\partial x}x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)-2\left(\frac{\partial}{\partial x}y(x,y)\right)^2-2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,$$

$$y)\right)-2\left(\frac{\partial}{\partial y}x(x,y)\right)^2-2\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)-2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-2\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-\varepsilon\,\omega^2\,\mu\,x(x,y)^2-\varepsilon\,\omega^2\,\mu\,y(x,y)^2+2\,\varepsilon\,\omega\,\mu\,vg\,k\,x(x,y)^2$$

$$+2\,\varepsilon\,\omega\,\mu\,vg\,k\,y(x,y)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$$

$$+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial x}y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)y(x,y)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$+2\,vg^2\,\varepsilon\,\mu\,x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+2\,vg^2\,\varepsilon\,\mu\left(\frac{\partial}{\partial y}y(x,y)\right)^2+2\,vg^2\,\varepsilon\,\mu\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-vg^2\,\varepsilon\,\mu\,k^2\,x(x,y)^2-vg^2\,\varepsilon\,\mu\,k^2\,y(x,y)^2\Big)\Big),\ -\frac{1}{\mu}\Big(\cos(k\,z$$

$$-\omega\,t)\Big(8\sin(k\,z-\omega\,t)\,k\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+\sin(k\,z$$

$$- \omega t) \mu \varepsilon k^3 vg^2 x(x,y)^4 + \sin(kz - \omega t) \mu \varepsilon \omega^2 k x(x,y)^4 + \sin(kz - \omega t) \mu \varepsilon \omega^2 k y(x,y)^4$$

$$+ \sin(kz - \omega t) \mu \varepsilon k^3 vg^2 y(x,y)^4 + 8 \sin(kz - \omega t) k x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 4 \sin(kz - \omega t) k x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 4 \sin(kz - \omega t) k y(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + 4 \sin(kz - \omega t) k x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 4 \sin(kz$$

$$- \omega t) k y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - 4 \sin(kz - \omega t) \mu \varepsilon \omega k^2 x(x,y)^2 y(x,y)^2 vg - 8 \sin(kz$$

$$- \omega t) \omega \mu \varepsilon vg\, x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) - 8 \sin(kz$$

$$- \omega t) \omega \mu \varepsilon vg\, x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 2 \sin(kz - \omega t) \mu \varepsilon vg\, y(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) x(x,y)^2 \omega - 2 \sin(kz - \omega t) \mu \varepsilon vg^2 y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) x(x,y)^2 k$$

$$+ 2 \sin(kz - \omega t) \mu \varepsilon \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y)\, vg\, x(x,y)^2 \omega - 2 \sin(kz - \omega t) \mu \varepsilon vg^2 x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) y(x,y)^2 k - 2 \sin(kz - \omega t) \mu \varepsilon vg^2 x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) y(x,y)^2 k$$

$$+ 2 \sin(kz - \omega t) \mu \varepsilon vg\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) y(x,y)^2 \omega + 2 \sin(kz - \omega t) \mu \varepsilon vg\, x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) y(x,y)^2 \omega - 2 \sin(kz - \omega t) \mu \varepsilon \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y)\, vg^2 x(x,y)^2 k$$

$$+ y(x,y)^6 \varepsilon \sin(kz - \omega t)^2 \mu^2 \omega^2 - 4 \mu x(x,y)^2 \cos(kz - \omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2$$

$$- 4 \mu x(x,y)^2 \cos(kz - \omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - 4 \mu y(x,y)^2 \cos(kz$$

$$- \omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 4 \mu y(x,y)^2 \cos(kz - \omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2$$

$$+ x(x,y)^6 \varepsilon \sin(kz - \omega t)^2 \mu^2 \omega^2 - 6 x(x,y)^2 \varepsilon \sin(kz - \omega t)^2 \mu^2 y(x,y)^4 \omega\, vg\, k$$

$$+ 8^{x(x,y)^3} \cos(kz - \omega t)^2 vg^2 \varepsilon \mu^2 \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 8 x(x,$$

$$y)^{3} \cos(k z - \omega t)^{2} vg^{2} \varepsilon \mu^{2} \left(\frac{\partial}{\partial y} x(x, y)\right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y)\right) + 8 y(x, y)^{3} \cos(k z$$

$$- \omega t)^{2} vg^{2} \varepsilon \mu^{2} x(x, y) \left(\frac{\partial}{\partial x} x(x, y)\right) \left(\frac{\partial}{\partial x} y(x, y)\right) + 8 y(x, y)^{3} \cos(k z$$

$$- \omega t)^{2} vg^{2} \varepsilon \mu^{2} x(x, y) \left(\frac{\partial}{\partial y} x(x, y)\right) \left(\frac{\partial}{\partial y} y(x, y)\right) - 6 x(x, y)^{4} \varepsilon \sin(k z$$

$$- \omega t)^{2} \mu^{2} y(x, y)^{2} \omega vg k - 4 \mu x(x, y)^{4} \cos(k z - \omega t)^{2} \left(\frac{\partial}{\partial x} x(x, y)\right)^{2} - 4 \mu x(x,$$

$$y)^{4} \cos(k z - \omega t)^{2} \left(\frac{\partial}{\partial y} x(x, y)\right)^{2} - 4 \mu y(x, y)^{4} \cos(k z - \omega t)^{2} \left(\frac{\partial}{\partial x} y(x, y)\right)^{2}$$

$$- 4 \mu^{y(x, y)^{4}} \cos(k z - \omega t)^{2} \left(\frac{\partial}{\partial y} y(x, y)\right)^{2} - 2 \sin(k z - \omega t) \mu \varepsilon \omega k^{2} x(x, y)^{4} vg$$

$$- 2 \sin(k z - \omega t) \mu \varepsilon vg^{2} \left(\frac{\partial}{\partial y} y(x, y)\right)^{2} x(x, y)^{2} k - 2 \sin(k z - \omega t) \mu \varepsilon vg y(x,$$

$$y)^{2} \left(\frac{\partial}{\partial y} y(x, y)\right)^{2} \omega + 2 \sin(k z - \omega t) \mu \varepsilon vg \left(\frac{\partial}{\partial y} y(x, y)\right)^{2} x(x, y)^{2} \omega - 2 \sin(k z$$

$$- \omega t) \mu \varepsilon vg^{2} x(x, y)^{2} \left(\frac{\partial}{\partial y} x(x, y)\right)^{2} k + 2 \sin(k z - \omega t) \mu \varepsilon vg \left(\frac{\partial}{\partial y} x(x, y)\right)^{2} y(x, y)^{2} \omega$$

$$- 2 \sin(k z - \omega t) \mu \varepsilon vg^{2} \left(\frac{\partial}{\partial y} x(x, y)\right)^{2} y(x, y)^{2} k + 2 \sin(k z - \omega t) \mu \varepsilon \left(\frac{\partial}{\partial x} y(x,\right.$$

$$\left.y)\right)^{2} vg x(x, y)^{2} \omega - 2 \sin(k z - \omega t) \mu \varepsilon \left(\frac{\partial}{\partial x} y(x, y)\right)^{2} vg^{2} x(x, y)^{2} k - 2 \sin(k z$$

$$- \omega t) \mu \varepsilon vg y(x, y)^{2} \left(\frac{\partial}{\partial x} y(x, y)\right)^{2} \omega - 2 \sin(k z - \omega t) \mu \varepsilon vg^{2} y(x, y)^{2} \left(\frac{\partial}{\partial x} y(x, y)\right)^{2} k$$

$$- 2 \sin(k z - \omega t) \mu \varepsilon vg x(x, y)^{2} \left(\frac{\partial}{\partial y} x(x, y)\right)^{2} \omega - 2 \sin(k z - \omega t) \mu \varepsilon vg^{2} x(x,$$

$$y)^{2} \left(\frac{\partial}{\partial x} x(x, y)\right)^{2} k + 2 \sin(k z - \omega t) \mu \varepsilon vg \left(\frac{\partial}{\partial x} x(x, y)\right)^{2} y(x, y)^{2} \omega - 2 \sin(k z$$

$$- \omega t) \mu \varepsilon vg^{2} \left(\frac{\partial}{\partial x} x(x, y)\right)^{2} y(x, y)^{2} k + 2 \sin(k z - \omega t) \mu \varepsilon \omega^{2} k x(x, y)^{2} y(x, y)^{2}$$

$$+ 2 \sin(k z - \omega t) \mu \varepsilon k^{3} vg^{2} x(x, y)^{2} y(x, y)^{2} - 2 \sin(k z - \omega t) \mu \varepsilon \omega k^{2} y(x, y)^{4} vg$$

$$- 2 \sin(k z - \omega t) \mu \varepsilon vg x(x, y)^{2} \left(\frac{\partial}{\partial x} x(x, y)\right)^{2} \omega - 2 \sin(k z - \omega t) \mu \varepsilon vg^{2} y(x,$$

$$y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 k - 2 \sin(k z - \omega t) \mu \varepsilon vg^2 x(x,y)^3 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) k + 2 \sin(k z$$

$$- \omega t) \mu \varepsilon vg\, x(x,y)^3 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) \omega - 2 \sin(k z - \omega t) \mu \varepsilon \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y)^3 vg^2 k$$

$$+ 2 \sin(k z - \omega t) \mu \varepsilon \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y)^3 vg\, \omega + 2 \sin(k z - \omega t) \mu \varepsilon vg\, x(x,$$

$$y)^3 \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) \omega - 2 \sin(k z - \omega t) \mu \varepsilon vg^2 x(x,y)^3 \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) k + 2 \sin(k z$$

$$- \omega t) \mu \varepsilon vg\, y(x,y)^3 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) \omega - 2 \sin(k z - \omega t) \mu \varepsilon vg^2 y(x,y)^3 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) k$$

$$- 2 x(x,y)^6 \varepsilon \sin(k z - \omega t)^2 \mu^2 \omega vg\, k + 3 x(x,y)^4 \varepsilon \sin(k z - \omega t)^2 \mu^2 y(x,y)^2 vg^2 k^2$$

$$+ 3 x(x,y)^2 \varepsilon \sin(k z - \omega t)^2 \mu^2 y(x,y)^4 vg^2 k^2 + 4 x(x,y)^2 \cos(k z$$

$$- \omega t)^2 \varepsilon vg^2 \mu^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 y(x,y)^2 + 4 x(x,y)^2 \cos(k z - \omega t)^2 \varepsilon vg^2 \mu^2 \left(\frac{\partial}{\partial y} y(x,\right.$$

$$\left. y)\right)^2 y(x,y)^2 - 2 y(x,y)^6 \varepsilon \sin(k z - \omega t)^2 \mu^2 \omega vg\, k + 4 y(x,y)^2 \cos(k z$$

$$- \omega t)^2 \varepsilon vg^2 \mu^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 x(x,y)^2 + 4 y(x,y)^2 \cos(k z - \omega t)^2 \varepsilon vg^2 \mu^2 \left(\frac{\partial}{\partial y} x(x,\right.$$

$$\left. y)\right)^2 x(x,y)^2 - 8 \mu\, x(x,y)^3 \cos(k z - \omega t)^2 \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right)$$

$$- 8 \mu\, x(x,y)^3 \cos(k z - \omega t)^2 \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) - 8 \mu\, y(x,$$

$$y)^3 \cos(k z - \omega t)^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) \left(\frac{\partial}{\partial x} y(x,y)\right) - 8 \mu\, y(x,y)^3 \cos(k z - \omega t)^2 x(x,$$

$$y) \left(\frac{\partial}{\partial y} x(x,y)\right) \left(\frac{\partial}{\partial y} y(x,y)\right) + x(x,y)^6 \varepsilon \sin(k z - \omega t)^2 \mu^2 vg^2 k^2 + 3 x(x,$$

$$y)^4 \varepsilon \sin(k z - \omega t)^2 \mu^2 y(x,y)^2 \omega^2 + 3 x(x,y)^2 \varepsilon \sin(k z - \omega t)^2 \mu^2 y(x,y)^4 \omega^2 + 4 x(x,$$

$$y)^4 \cos(k z - \omega t)^2 \varepsilon vg^2 \mu^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 4 x(x,y)^4 \cos(k z$$

$$- \omega t)^2 \varepsilon vg^2 \mu^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + y(x,y)^6 \varepsilon \sin(k z - \omega t)^2 \mu^2 vg^2 k^2 + 4 y(x,$$

$$y)^4 \cos(k z - \omega t)^2 \varepsilon vg^2 \mu^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + 4 y(x,y)^4 \cos(k z$$

$$-\omega t)^2 \varepsilon vg^2 \mu^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2\Big)\Big)\Big]$$

$$Dissipation = -\varepsilon \cos(k z - \omega t) \left(2\, vg \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + 2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 vg + 2 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) y(x,y)\, vg + 2\, vg \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\, vg\, x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + 2\, vg \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + 2\, vg\, y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + x(x,y)^2 \omega k + k \omega y(x,y)^2 - k^2 vg\, x(x,y)^2 - k^2 vg\, y(x,y)^2 + \mu \sin(k z - \omega t)\, x(x,y)^4 \omega - \mu \sin(k z - \omega t)\, x(x,y)^4 vg\, k + 2 \mu \sin(k z - \omega t)\, x(x,y)^2 y(x,y)^2 \omega - 2 \mu \sin(k z - \omega t)\, x(x,y)^2 y(x,y)^2 vg\, k + \mu \sin(k z - \omega t)\, y(x,y)^4 \omega - \mu \sin(k z - \omega t)\, y(x,y)^4 vg\, k\right)$$

*****************************   *END PROCEDURE*   ***************************** **(26)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 6a-- Wave guide TTM (kinematic in, wave in)`;
> theta:=(k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=-(omega/k)*f(x,y)*cos(theta);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 6a-- Wave guide TTM (kinematic in, wave in)

$$\theta := k z + \omega t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z + \omega t)$$

$$\phi := -\frac{\omega \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z + \omega t)}{k}$$

Example 6a-- Wave guide TTM (kinematic in, wave in)

********************************   *Differential Form Format*   ********************************

$$Action\ 1\text{-}form = \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z + \omega t)\, d(z) + \frac{\omega \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z + \omega t)\, d(t)}{k}$$

Intensity 2-form F=dA $= 2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz + \omega t)\,(d(x))\,\&\wedge\,(d(z)) + 2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz + \omega t)\,(d(y))\,\&\wedge\,(d(z)) + \frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz+\omega t)\,(d(x))\,\&\wedge\,(d(t))}{k} + \frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz+\omega t)\,(d(y))\,\&\wedge\,(d(t))}{k}$

Topological Torsion 3-form A^F $= 0$

Topological Parity 4-form F^F $= 0$

********************************* *Using EM format* ********************************

E field $= \left[\frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz+\omega t)}{k}, \frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz+\omega t)}{k}, 0\right]$

B field $= \left[2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(kz+\omega t), -2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(kz+\omega t), 0\right]$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension PTD $= 2$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ************************

Xm or linear (Mean) curvature $=$

$$-\frac{\left(x(x,y)^2 + y(x,y)^2\right)\sin(kz+\omega t)\left(k^2+\omega^2\right)}{k}$$

Yg or quadratic (GAUSS) curvature $= 0$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

***********   Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH ************

Chirality factor $CH = 0$

$$D\,field = \left[\frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z+\omega\,t)}{k}, \frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z+\omega\,t)}{k}, 0\right]$$

$$H\,field = \left[\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z+\omega\,t)}{\mu}, -\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z+\omega\,t)}{\mu}, 0\right]$$

$$Poynting\ vector\ ExH = \left[0, 0, -\frac{1}{\mu\,k}\left(4\,\omega\cos(k\,z+\omega\,t)^2\left(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\right)\right)\right]$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt\text{=}J4 \;= \Bigg[$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(k\,z+\omega\,t)\left(k^2-\mu\,\omega^2\,\varepsilon\right)}{\mu\,k},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\sin(k\,z+\omega\,t)\left(k^2-\mu\,\omega^2\,\varepsilon\right)}{\mu\,k},$$

$$-\frac{1}{\mu}\left(2\cos(k\,z+\omega\,t)\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,\right.\right.$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Big)\Big)\Big],\ \frac{1}{k}\Big(2\,\varepsilon\cos(k\,z+\omega\,t)\,\omega\Big(\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Big)\Big)\Big]$$

Amerian charge density *divD = rho* $= \frac{1}{k}\Big(2\,\varepsilon\cos(k\,z+\omega\,t)\,\omega\Big(\Big(\frac{\partial}{\partial x}x(x,y)\Big)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Big)\Big)$

divergence Lorentz Current 4Vector, *4div(J4)* $=0$

Topological SPIN 4 vector S4 $=\Big[\frac{1}{\mu k^2}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\Big(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Big),\ \frac{1}{\mu k^2}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\Big(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Big),0,0\Big]$

Topological SPIN 3-form $=\frac{1}{\mu k^2}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\Big(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\ \&\wedge(d(y),d(z),d(t))\Big)-\frac{1}{\mu k^2}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\Big(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\ \&\wedge(d(x),d(z),d(t))\Big)$

Spin density rho_spin $=0$

LaGrange field energy density (B.H-D.E) $=\frac{1}{\mu k^2}\Big(4\cos(k\,z+\omega\,t)^2\Big(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,$

$$y)^2 \left(\frac{\partial}{\partial x} x(x, y)\right)^2 + 2\, x(x, y) \left(\frac{\partial}{\partial x} x(x, y)\right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y)\right) + y(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} y(x, y)\right)^2\Big) \left(k^2 - \mu\, \omega^2\, \varepsilon\right)\Big)$$

$$B.H = \frac{1}{\mu} \Big(4 \cos(k\, z + \omega\, t)^2 \Big(x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y)\right)^2 + 2\, x(x, y) \left(\frac{\partial}{\partial y} x(x, y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial y} y(x, y)\right) + y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y)\right)^2 + x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y)\right)^2 + 2\, x(x,$$

$$y) \left(\frac{\partial}{\partial x} x(x, y)\right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y)\right) + y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y)\right)^2\Big)\Big)$$

$$D.E = \frac{1}{k^2} \Big(4\, \varepsilon\, \omega^2 \cos(k\, z + \omega\, t)^2 \Big(x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y)\right)^2 + 2\, x(x, y) \left(\frac{\partial}{\partial y} x(x, y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial y} y(x, y)\right) + y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y)\right)^2 + x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y)\right)^2 + 2\, x(x,$$

$$y) \left(\frac{\partial}{\partial x} x(x, y)\right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y)\right) + y(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y)\right)^2\Big)\Big)$$

$$A.J = -\frac{1}{\mu} \Big(2 \left(x(x, y)^2 + y(x, y)^2\right) \cos(k\, z + \omega\, t)^2 \Big(\left(\frac{\partial}{\partial x} x(x, y)\right)^2 + x(x, y) \left(\frac{\partial^2}{\partial x^2} x(x, y)\right)$$

$$+ \left(\frac{\partial}{\partial x} y(x, y)\right)^2 + y(x, y) \left(\frac{\partial^2}{\partial x^2} y(x, y)\right) + \left(\frac{\partial}{\partial y} x(x, y)\right)^2 + x(x, y) \left(\frac{\partial^2}{\partial y^2} x(x, y)\right)$$

$$+ \left(\frac{\partial}{\partial y} y(x, y)\right)^2 + y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y)\right)\Big)\Big)$$

$$\text{-rho.phi} = -\frac{1}{k^2} \Big(2\, \omega^2 \left(x(x, y)^2 + y(x, y)^2\right) \cos(k\, z + \omega\, t)^2\, \varepsilon \Big(\left(\frac{\partial}{\partial x} x(x, y)\right)^2 + x(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2} x(x, y)\right) + \left(\frac{\partial}{\partial x} y(x, y)\right)^2 + y(x, y) \left(\frac{\partial^2}{\partial x^2} y(x, y)\right) + \left(\frac{\partial}{\partial y} x(x, y)\right)^2 + x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x, y)\right) + \left(\frac{\partial}{\partial y} y(x, y)\right)^2 + y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y)\right)\Big)\Big)$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* $= \dfrac{1}{\mu\, k^2} \Big(2 \cos(k\, z + \omega\, t)^2 \left(k^2 - \mu\, \omega^2\, \varepsilon\right) \Big(3\, x(x,$

$$y)^2 \left(\frac{\partial}{\partial x} x(x, y)\right)^2 + 4\, x(x, y) \left(\frac{\partial}{\partial x} x(x, y)\right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y)\right) + 3\, y(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} y(x, y)\right)^2 + 3\, x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y)\right)^2 + 4\, x(x, y) \left(\frac{\partial}{\partial y} x(x, y)\right) y(x, y) \left(\frac{\partial}{\partial y} y(x,\right.$$

$$y)\Big) + 3\,y(x,y)^2 \left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 + \left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right) x(x,y)^3 + \left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 x(x,y)^2$$

$$+ y(x,y)\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right) x(x,y)^2 + \left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right) x(x,y)^3 + \left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 x(x,y)^2$$

$$+ y(x,y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right) x(x,y)^2 + \left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 y(x,y)^2 + \left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right) x(x,$$

$$y)\,y(x,y)^2 + y(x,y)^3\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right) + \left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 y(x,y)^2 + \left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right) x(x,$$

$$y)\,y(x,y)^2 + y(x,y)^3\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)\Big)\Big)$$

$$\textit{London Coefficient} \qquad LC = \frac{k^2 - \mu\,\omega^2\,\varepsilon}{\mu}$$

$$\textit{PROCA coefficient curlcurlB} = \Big[2\cos(k\,z + \omega\,t)\,\Big(-2\left(\frac{\partial}{\partial x}\,x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\,x(x,y)\right)$$

$$-\left(\frac{\partial}{\partial y}\,x(x,y)\right)\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right) - x(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2}\,x(x,y)\right) - 2\left(\frac{\partial}{\partial x}\,y(x,$$

$$y)\Big)\left(\frac{\partial^2}{\partial y\,\partial x}\,y(x,y)\right) - \left(\frac{\partial}{\partial y}\,y(x,y)\right)\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right) - y(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2}\,y(x,y)\right)$$

$$-3\left(\frac{\partial}{\partial y}\,x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right) - x(x,y)\left(\frac{\partial^3}{\partial y^3}\,x(x,y)\right) - 3\left(\frac{\partial}{\partial y}\,y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}\,y(x,$$

$$y)\Big) - y(x,y)\left(\frac{\partial^3}{\partial y^3}\,y(x,y)\right) + k^2\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right) + k^2\,y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)\Big),$$

$$2\cos(k\,z + \omega\,t)\,\Big(-k^2\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right) - k^2\,y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right) + 3\left(\frac{\partial}{\partial x}\,x(x,$$

$$y)\Big)\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right) + x(x,y)\left(\frac{\partial^3}{\partial x^3}\,x(x,y)\right) + 3\left(\frac{\partial}{\partial x}\,y(x,y)\right)\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right) + y(x,$$

$$y)\left(\frac{\partial^3}{\partial x^3}\,y(x,y)\right) + 2\left(\frac{\partial}{\partial y}\,x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\,x(x,y)\right) + \left(\frac{\partial}{\partial x}\,x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)$$

$$+ x(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}\,x(x,y)\right) + 2\left(\frac{\partial}{\partial y}\,y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\,y(x,y)\right) + \left(\frac{\partial}{\partial x}\,y(x,$$

$$y)\Big)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right) + y(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}\,y(x,y)\right)\Big), 0\Big]$$

Amperian Current 4Vector   curlH-dD/dt=J4 $= \Bigg[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz+\omega t)\left(k^2-\mu\,\omega^2\,\varepsilon\right)}{\mu\,k},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\sin(kz+\omega t)\left(k^2-\mu\,\omega^2\,\varepsilon\right)}{\mu\,k},$$

$$-\frac{1}{\mu}\Bigg(2\cos(kz+\omega t)\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg),$$

$$\frac{1}{k}\Bigg(2\,\varepsilon\cos(kz+\omega t)\,\omega\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg)\Bigg]$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B)

$$=\Bigg[\frac{1}{\mu\,k^2}\Bigg(4\cos(kz+\omega t)^2\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Bigg),$$

$$\frac{1}{\mu\,k^2}\Bigg(4\cos(kz+\omega t)^2\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial x} y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial y} y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right)\Bigg)\Bigg)\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,\right.\right.$$

$$\left.\left. y)\right)\right)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Bigg), -\frac{1}{\mu\,k}\Bigg(4\sin(k\,z+\omega\,t)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\cos(k\,z+\omega\,t)\Bigg(x(x,$$

$$y)^2\left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right) + y(x,$$

$$y)^2\left(\frac{\partial}{\partial y} y(x,y)\right)^2 + x(x,y)^2\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\,x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial x} y(x,\right.$$

$$\left. y)\right) + y(x,y)^2\left(\frac{\partial}{\partial x} y(x,y)\right)^2\Bigg)\Bigg)\Bigg]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \Bigg[\frac{1}{\mu\,k^2}\Big(2\left(x(x,y)^2 + y(x,$

$$y)^2\right)\cos(k\,z+\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Big),$$

$$\frac{1}{\mu\,k^2}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z+\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,\right.\right.$$

$$\left.\left. y)\right)\right)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Big), 0, 0\Bigg]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= \Big[0, 0,$

$$\frac{1}{\mu\,k^2}\Big(4\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z+\omega\,t)^3\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Big(x(x,y)^2\left(\frac{\partial}{\partial y} x(x,y)\right)^2$$

$$+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Big)\Big)\Big]$$

Spin Dissipation J_spin dot E $$=\frac{1}{\mu k^3}\Big(4\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^3\left(k^2-\mu\omega^2\varepsilon\right)\omega\Big(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Big)\Big)$$

Disspative Force 3 vector $$=\Bigg[\frac{1}{\mu k^2}\Bigg(4\cos(kz+\omega t)^2\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\left(k^2-\mu\omega^2\varepsilon\right)\Bigg),\frac{1}{\mu k^2}\Bigg(4\cos(kz+\omega t)^2\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\left(k^2-\mu\omega^2\varepsilon\right)\Bigg),\frac{1}{\mu k^2}\Bigg(4\left(k^2-\mu\omega^2\varepsilon\right)\cos(kz+\omega t)\Big(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,$$

$$y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,$$

$$y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Big)\left(-\sin(k\,z+\omega\,t)\,k+\cos(k\,z+\omega\,t)^2\,\mu\,x(x,y)^2+\cos(k\,z\right.$$

$$\left.+\,\omega\,t)^2\,\mu\,y(x,y)^2)\Big)\right]$$

$$Dissipation\ =\frac{1}{k}\Bigg(2\,\varepsilon\cos(k\,z+\omega\,t)\,\omega\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg)$$

***************************** *END PROCEDURE* *************************** **(27)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 6b-- Wave guide TTE (kinematic in, wave out)`;\
> theta:=(k*z-omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=-(omega/k)*f(x,y)*cos(theta);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 6b-- Wave guide TTE (kinematic in, wave out)

$$\theta := k\,z-\omega\,t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)$$

$$\phi := -\frac{\omega\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)}{k}$$

Example 6b-- Wave guide TTE (kinematic in, wave out)

********************************* *Differential Form Format* ********************

****

$$Action\ 1\text{-}form=\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)\,d(z)$$

$$+\frac{\omega\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)\,d(t)}{k}$$

Intensity 2-form F=dA $= -2\,\left(x(x,y)^2+y(x,y)^2\right)\sin(k\,z-\omega\,t)\,\omega\,(d(z))\ \&\wedge\ (d(t))$

$$+2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z-\omega\,t)\,(d(x))\ \&\wedge\ (d(z))$$

$$+2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z-\omega\,t)\,(d(y))\ \&\wedge\ (d(z))$$

$$+\frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z-\omega\,t)\,(d(x))\ \&\wedge\ (d(t))}{k}$$

$$+\frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z-\omega\,t)\,(d(y))\ \&\wedge\ (d(t))}{k}$$

Topological Torsion 3-form *A^F* = 0

Topological Parity 4-form *F^F* = 0

******************************** *Using EM format* ******************************

$$E\ field=\left[\frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{k},\ \frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{k},\ -2\left(x(x,y)^2+y(x,y)^2\right)\sin(k\,z-\omega\,t)\,\omega\right]$$

$$B\ field=\left[2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z-\omega\,t),\ -2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z-\omega\,t),\ 0\right]$$

Topological TORSION 4 vector *T4 = -[ExA + Bphi,AdotB]* = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension *PTD* = 2

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ***************************

Xm or linear (Mean) curvature =

$$-\frac{\left(x(x,y)^2+y(x,y)^2\right)\sin(k\,z-\omega\,t)\,(k-\omega)\,(k+\omega)}{k}$$

Yg or quadratic (GAUSS) curvature = 0

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $= 0$

$$D\ field = \left[\frac{2\,\varepsilon\,\omega \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \cos(k\,z - \omega\,t)}{k}, \frac{2\,\varepsilon\,\omega \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \cos(k\,z - \omega\,t)}{k}, -2\,\varepsilon \left(x(x,y)^2 + y(x,y)^2 \right) \sin(k\,z - \omega\,t)\,\omega \right]$$

$$H\ field = \left[\frac{2 \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \cos(k\,z - \omega\,t)}{\mu}, -\frac{2 \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \cos(k\,z - \omega\,t)}{\mu}, 0 \right]$$

$$Poynting\ vector\ ExH = \left[-\frac{1}{\mu} \left(4 \left(x(x,y)^2 + y(x,y)^2 \right) \sin(k\,z - \omega\,t)\,\omega \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \cos(k\,z - \omega\,t) \right), -\frac{1}{\mu} \left(4 \left(x(x,y)^2 + y(x,y)^2 \right) \sin(k\,z - \omega\,t)\,\omega \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \cos(k\,z - \omega\,t) \right), -\frac{1}{\mu\,k} \left(4\,\omega \cos(k\,z - \omega\,t)^2 \left(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2\,x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2\,x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 \right) \right) \right]$$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \Big[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz-\omega t)\left(k^2+\mu\omega^2\varepsilon\right)}{\mu k},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\sin(kz-\omega t)\left(k^2+\mu\omega^2\varepsilon\right)}{\mu k},$$

$$-\frac{1}{\mu}\left(2\cos(kz-\omega t)\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\omega^2\mu\,x(x,y)^2+\varepsilon\omega^2\mu\,y(x,y)^2\right)\right),\ \frac{1}{k}\left(2\varepsilon\cos(kz-\omega t)\,\omega\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-x(x,y)^2k^2-k^2y(x,y)^2\right)\right)\Bigg]$$

Amerian charge density *divD = rho* $= \frac{1}{k}\left(2\varepsilon\cos(kz-\omega t)\,\omega\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-x(x,y)^2k^2-k^2y(x,y)^2\right)\right)$

divergence Lorentz Current 4Vector, *4div(J4)* $=0$

Topological SPIN 4 vector S4 $=\left[\frac{1}{\mu k^2}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\left(k^2-\mu\omega^2\varepsilon\right)\right),\ \frac{1}{\mu k^2}\left(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\left(k^2\right.\right.\right.$

$$-\mu\,\omega^2\,\varepsilon\big)\Big), \frac{2\,\varepsilon\,\left(x(x,y)^2+y(x,y)^2\right)^2 \sin(k\,z-\omega\,t)\,\omega^2\cos(k\,z-\omega\,t)}{k}, -2\,\big(x(x,y)^2$$

$$+y(x,y)^2\big)^2\cos(k\,z-\omega\,t)\,\varepsilon\sin(k\,z-\omega\,t)\,\omega\Big]$$

$$\textit{Topological SPIN 3-form} = \frac{1}{\mu\,k^2}\Big(2\,\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)^2\Big(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\ \&\text{^}(d(y),d(z),d(t))\Big)$$

$$-\frac{1}{\mu\,k^2}\Big(2\,\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)^2\Big(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\ \&\text{^}(d(x),d(z),d(t))\Big)$$

$$+\frac{2\,\varepsilon\,\left(x(x,y)^2+y(x,y)^2\right)^2\sin(k\,z-\omega\,t)\,\omega^2\cos(k\,z-\omega\,t)\ \&\text{^}(d(x),d(y),d(t))}{k}$$

$$+2\,\left(x(x,y)^2+y(x,y)^2\right)^2\cos(k\,z-\omega\,t)\,\varepsilon\sin(k\,z-\omega\,t)\,\omega\ \&\text{^}(d(x),d(y),d(z))$$

$$\textit{Spin density rho\_spin} = -2\,\left(x(x,y)^2+y(x,y)^2\right)^2\cos(k\,z-\omega\,t)\,\varepsilon\sin(k\,z-\omega\,t)\,\omega$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{1}{\mu\,k^2}\Big(4\,\Big(\cos(k\,z-\omega\,t)^2 k^2 x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2$$

$$+2\cos(k\,z-\omega\,t)^2 k^2 x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+\cos(k\,z$$

$$-\omega\,t)^2 k^2 y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+\cos(k\,z-\omega\,t)^2 k^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$+2\cos(k\,z-\omega\,t)^2 k^2 x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+\cos(k\,z$$

$$-\omega\,t)^2 k^2 y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-\mu\,\omega^2\,\varepsilon\cos(k\,z-\omega\,t)^2 x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2$$

$$-2\,\mu\,\omega^2\,\varepsilon\cos(k\,z-\omega\,t)^2 x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)-\mu\,\omega^2\,\varepsilon\cos(k\,z$$

$$-\omega\,t)^2 y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2-\mu\,\omega^2\,\varepsilon\cos(k\,z-\omega\,t)^2 x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$-2\,\mu\,\omega^2\,\varepsilon\cos(k\,z-\omega\,t)^2 x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)-\mu\,\omega^2\,\varepsilon\cos(k\,z$$

$$-\omega\,t)^2 y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-\mu\,\omega^2\,\varepsilon\sin(k\,z-\omega\,t)^2 k^2 x(x,y)^4-2\,\mu\,\omega^2\,\varepsilon\sin(k\,z$$

$$-\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 - \mu \omega^2 \varepsilon \sin(kz-\omega t)^2 k^2 y(x,y)^4 \Big) \Big)$$

$$B.H = \frac{1}{\mu} \Big(4 \cos(kz-\omega t)^2 \Big(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2\, x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,$$

$$y) \left(\frac{\partial}{\partial y} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2\, x(x,$$

$$y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 \Big) \Big)$$

$$D.E = \frac{1}{k^2} \Big(4 \varepsilon \omega^2 \Big(\left(\frac{\partial}{\partial x} x(x,y) \right)^2 x(x,y)^2 \cos(kz-\omega t)^2 + 2 \left(\frac{\partial}{\partial x} y(x,y) \right) y(x,$$

$$y) \left(\frac{\partial}{\partial x} x(x,y) \right) x(x,y) \cos(kz-\omega t)^2 + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 \cos(kz-\omega t)^2$$

$$+ \left(\frac{\partial}{\partial y} x(x,y) \right)^2 x(x,y)^2 \cos(kz-\omega t)^2 + 2 \left(\frac{\partial}{\partial y} y(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) x(x,$$

$$y) \cos(kz-\omega t)^2 + \left(\frac{\partial}{\partial y} y(x,y) \right)^2 y(x,y)^2 \cos(kz-\omega t)^2 + \sin(kz-\omega t)^2 k^2 x(x,y)^4$$

$$+ 2 \sin(kz-\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 + \sin(kz-\omega t)^2 k^2 y(x,y)^4 \Big) \Big)$$

$$A.J = -\frac{1}{\mu} \Big(2 \left(x(x,y)^2 + y(x,y)^2 \right) \cos(kz-\omega t)^2 \Big(\left(\frac{\partial}{\partial x} x(x,y) \right)^2 + x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y) \right)$$

$$+ \left(\frac{\partial}{\partial x} y(x,y) \right)^2 + y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y) \right) + \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y) \right)$$

$$+ \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y) \right) + \varepsilon \omega^2 \mu\, x(x,y)^2 + \varepsilon \omega^2 \mu\, y(x,y)^2 \Big) \Big)$$

$$-rho.phi = -\frac{1}{k^2} \Big(2 \omega^2 \left(x(x,y)^2 + y(x,y)^2 \right) \cos(kz-\omega t)^2 \varepsilon \Big(\left(\frac{\partial}{\partial x} x(x,y) \right)^2 + x(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2} x(x,y) \right) + \left(\frac{\partial}{\partial x} y(x,y) \right)^2 + y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y) \right) + \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + x(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} x(x,y) \right) + \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y) \right) - x(x,y)^2 k^2 - k^2 y(x,y)^2 \Big) \Big)$$

Poincare I $\quad (B.H - D.E)\text{-}(A.J - rho.phi) = \frac{1}{\mu k^2} \Big(2 \Big(-4 \mu \omega^2 \varepsilon \cos(kz-\omega t)^2 x(x,$

$$y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) - 4 \mu \omega^2 \varepsilon \cos(kz-\omega t)^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,$$

$$y)\Big)\, y(x,y)\left(\frac{\partial}{\partial y}\, y(x,y)\right) + 3\cos(k\,z-\omega\,t)^2\,k^2\,x(x,y)^2\left(\frac{\partial}{\partial x}\, x(x,y)\right)^2 + 3\cos(k\,z$$

$$-\omega\,t)^2\,k^2\,y(x,y)^2\left(\frac{\partial}{\partial x}\, y(x,y)\right)^2 + 3\cos(k\,z-\omega\,t)^2\,k^2\,x(x,y)^2\left(\frac{\partial}{\partial y}\, x(x,y)\right)^2$$

$$+3\cos(k\,z-\omega\,t)^2\,k^2\,y(x,y)^2\left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 + \cos(k\,z-\omega\,t)^2\,x(x,y)^2\left(\frac{\partial}{\partial x}\, y(x,y)\right)^2 k^2$$

$$+\cos(k\,z-\omega\,t)^2\,x(x,y)^2\left(\frac{\partial}{\partial y}\, y(x,y)\right)^2 k^2 + \cos(k\,z-\omega\,t)^2\,y(x,y)^2\left(\frac{\partial}{\partial x}\, x(x,y)\right)^2 k^2$$

$$+\cos(k\,z-\omega\,t)^2\,y(x,y)^2\left(\frac{\partial}{\partial y}\, x(x,y)\right)^2 k^2 - \cos(k\,z-\omega\,t)^2\,x(x,y)^3\left(\frac{\partial^2}{\partial x^2}\, x(x,\right.$$

$$\left.y)\right)\mu\,\omega^2\,\varepsilon - \cos(k\,z-\omega\,t)^2\,x(x,y)^3\left(\frac{\partial^2}{\partial y^2}\, x(x,y)\right)\mu\,\omega^2\,\varepsilon - \cos(k\,z-\omega\,t)^2\,y(x,$$

$$y)^3\left(\frac{\partial^2}{\partial x^2}\, y(x,y)\right)\mu\,\omega^2\,\varepsilon - \cos(k\,z-\omega\,t)^2\,y(x,y)^3\left(\frac{\partial^2}{\partial y^2}\, y(x,y)\right)\mu\,\omega^2\,\varepsilon + \cos(k\,z$$

$$-\omega\,t)^2\,x(x,y)^2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}\, y(x,y)\right)k^2 + \cos(k\,z-\omega\,t)^2\,x(x,y)^2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}\, y(x,\right.$$

$$\left.y)\right)k^2 + \cos(k\,z-\omega\,t)^2\,y(x,y)^2\left(\frac{\partial^2}{\partial x^2}\, x(x,y)\right)x(x,y)\,k^2 + \cos(k\,z-\omega\,t)^2\,y(x,$$

$$y)^2\left(\frac{\partial^2}{\partial y^2}\, x(x,y)\right)x(x,y)\,k^2 - \cos(k\,z-\omega\,t)^2\,x(x,y)^2\,y(x,y)\left(\frac{\partial^2}{\partial x^2}\, y(x,y)\right)\mu\,\omega^2\,\varepsilon$$

$$-\cos(k\,z-\omega\,t)^2\,x(x,y)^2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}\, y(x,y)\right)\mu\,\omega^2\,\varepsilon - \cos(k\,z-\omega\,t)^2\,y(x,$$

$$y)^2\left(\frac{\partial^2}{\partial x^2}\, x(x,y)\right)x(x,y)\,\mu\,\omega^2\,\varepsilon - \cos(k\,z-\omega\,t)^2\,y(x,y)^2\left(\frac{\partial^2}{\partial y^2}\, x(x,y)\right)x(x,y)\,\mu\,\omega^2\,\varepsilon$$

$$-4\,\mu\,\omega^2\,\varepsilon\sin(k\,z-\omega\,t)^2\,k^2\,x(x,y)^2\,y(x,y)^2 + 4\cos(k\,z-\omega\,t)^2\,x(x,y)^2\,k^2\,\varepsilon\,\omega^2\,\mu\,y(x,y)^2$$

$$+2\cos(k\,z-\omega\,t)^2\,x(x,y)^4\,k^2\,\varepsilon\,\omega^2\,\mu - \cos(k\,z-\omega\,t)^2\,x(x,y)^2\left(\frac{\partial}{\partial x}\, y(x,y)\right)^2\mu\,\omega^2\,\varepsilon$$

$$-\cos(k\,z-\omega\,t)^2\,x(x,y)^2\left(\frac{\partial}{\partial y}\, y(x,y)\right)^2\mu\,\omega^2\,\varepsilon + 2\cos(k\,z-\omega\,t)^2\,y(x,y)^4\,k^2\,\varepsilon\,\omega^2\,\mu$$

$$-\cos(k\,z-\omega\,t)^2\,y(x,y)^2\left(\frac{\partial}{\partial x}\, x(x,y)\right)^2\mu\,\omega^2\,\varepsilon - \cos(k\,z-\omega\,t)^2\,y(x,y)^2\left(\frac{\partial}{\partial y}\, x(x,\right.$$

$$\left.y)\right)^2\mu\,\omega^2\,\varepsilon + \cos(k\,z-\omega\,t)^2\,x(x,y)^3\left(\frac{\partial^2}{\partial x^2}\, x(x,y)\right)k^2 + \cos(k\,z-\omega\,t)^2\,x(x,$$

$$y)^3 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) k^2 + \cos(k z - \omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) k^2 + \cos(k z$$

$$- \omega t)^2 y(x,y)^3 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) k^2 + 4 \cos(k z - \omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 4 \cos(k z - \omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)$$

$$- 3 \mu \omega^2 \varepsilon \cos(k z - \omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 3 \mu \omega^2 \varepsilon \cos(k z - \omega t)^2 y(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - 3 \mu \omega^2 \varepsilon \cos(k z - \omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 3 \mu \omega^2 \varepsilon \cos(k z$$

$$- \omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - 2 \mu \omega^2 \varepsilon \sin(k z - \omega t)^2 k^2 x(x,y)^4 - 2 \mu \omega^2 \varepsilon \sin(k z$$

$$- \omega t)^2 k^2 y(x,y)^4 \Big) \Big)$$

$$\textit{London Coefficient} \quad LC = \frac{k^2 - \mu \omega^2 \varepsilon}{\mu}$$

$$\textit{PROCA coefficient curlcurlB} = \Bigg[2 \cos(k z - \omega t) \left(-2 \left(\frac{\partial}{\partial x} x(x,y)\right) \left(\frac{\partial^2}{\partial y \, \partial x} x(x,y)\right) \right.$$

$$- \left(\frac{\partial}{\partial y} x(x,y)\right) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) - x(x,y) \left(\frac{\partial^3}{\partial y \, \partial x^2} x(x,y)\right) - 2 \left(\frac{\partial}{\partial x} y(x,$$

$$y)\right) \left(\frac{\partial^2}{\partial y \, \partial x} y(x,y)\right) - \left(\frac{\partial}{\partial y} y(x,y)\right) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) - y(x,y) \left(\frac{\partial^3}{\partial y \, \partial x^2} y(x,y)\right)$$

$$- 3 \left(\frac{\partial}{\partial y} x(x,y)\right) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) - x(x,y) \left(\frac{\partial^3}{\partial y^3} x(x,y)\right) - 3 \left(\frac{\partial}{\partial y} y(x,y)\right) \left(\frac{\partial^2}{\partial y^2} y(x,$$

$$\left. y)\right) - y(x,y) \left(\frac{\partial^3}{\partial y^3} y(x,y)\right) + k^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) + k^2 y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) \right),$$

$$2 \cos(k z - \omega t) \left(-k^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) - k^2 y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 3 \left(\frac{\partial}{\partial x} x(x,\right.\right.$$

$$y)\Big) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + x(x,y) \left(\frac{\partial^3}{\partial x^3} x(x,y)\right) + 3 \left(\frac{\partial}{\partial x} y(x,y)\right) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + y(x,$$

$$y) \left(\frac{\partial^3}{\partial x^3} y(x,y)\right) + 2 \left(\frac{\partial}{\partial y} x(x,y)\right) \left(\frac{\partial^2}{\partial y \, \partial x} x(x,y)\right) + \left(\frac{\partial}{\partial x} x(x,y)\right) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$+ x(x,y) \left(\frac{\partial^3}{\partial y^2 \, \partial x} x(x,y)\right) + 2 \left(\frac{\partial}{\partial y} y(x,y)\right) \left(\frac{\partial^2}{\partial y \, \partial x} y(x,y)\right) + \left(\frac{\partial}{\partial x} y(x,\right.$$

$$y)\Big)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)+y(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}\,y(x,y)\right)\Big),0\Big]$$

Amperian Current 4Vector curlH-dD/dt=J4 $= \Big[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)\right)\sin(k\,z-\omega\,t)\,(k^2+\mu\,\omega^2\,\varepsilon)}{\mu\,k},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)\right)\sin(k\,z-\omega\,t)\,(k^2+\mu\,\omega^2\,\varepsilon)}{\mu\,k},$$

$$-\frac{1}{\mu}\Bigg(2\cos(k\,z-\omega\,t)\Bigg(\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)+\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2+y(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)+\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)+\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2+y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)+\varepsilon\,\omega^2\,\mu\,x(x,y)^2+\varepsilon\,\omega^2\,\mu\,y(x,y)^2\Bigg)\Bigg),\ \frac{1}{k}\Bigg(2\,\varepsilon\cos(k\,z$$

$$-\omega\,t)\,\omega\Bigg(\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)+\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2+y(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)+\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)+\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2+y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)-x(x,y)^2\,k^2-k^2\,y(x,y)^2\Bigg)\Bigg)\Big]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B)

$$=\Bigg[\frac{1}{\mu\,k^2}\Bigg(4\cos(k\,z-\omega\,t)^2\left(x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}\,y(x,\right.\right.$$

$$y)\Big)\Big)\Bigg(\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2k^2+\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)x(x,y)\,k^2+\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2k^2+y(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)k^2+\left(\frac{\partial}{\partial y}x(x,y)\right)^2k^2+\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)x(x,y)\,k^2+\left(\frac{\partial}{\partial y}y(x,y)\right)^2k^2$$

$$+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)k^2+2\,k^2\,\varepsilon\,\omega^2\,\mu\,x(x,y)^2+2\,k^2\,\varepsilon\,\omega^2\,\mu\,y(x,y)^2-\left(\frac{\partial}{\partial x}x(x,\right.$$

$$\left.y)\right)^2\mu\,\omega^2\,\varepsilon-\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)x(x,y)\,\mu\,\omega^2\,\varepsilon-\left(\frac{\partial}{\partial x}y(x,y)\right)^2\mu\,\omega^2\,\varepsilon-y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,\right.$$

$$\left.y)\right)\mu\,\omega^2\,\varepsilon-\left(\frac{\partial}{\partial y}x(x,y)\right)^2\mu\,\omega^2\,\varepsilon-\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)x(x,y)\,\mu\,\omega^2\,\varepsilon-\left(\frac{\partial}{\partial y}y(x,y)\right)^2\mu\,\omega^2\,\varepsilon$$

$$-y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\mu\,\omega^2\,\varepsilon\Bigg)\Bigg),\ \frac{1}{\mu\,k^2}\Bigg(4\cos(k\,z-\omega\,t)^2\Bigg(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)$$

$$+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Bigg)\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2k^2+\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)x(x,y)\,k^2+\left(\frac{\partial}{\partial x}y(x,\right.$$

$$\left.y)\right)^2k^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)k^2+\left(\frac{\partial}{\partial y}x(x,y)\right)^2k^2+\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)x(x,y)\,k^2$$

$$+\left(\frac{\partial}{\partial y}y(x,y)\right)^2k^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)k^2+2\,k^2\,\varepsilon\,\omega^2\,\mu\,x(x,y)^2+2\,k^2\,\varepsilon\,\omega^2\,\mu\,y(x,y)^2$$

$$-\left(\frac{\partial}{\partial x}x(x,y)\right)^2\mu\,\omega^2\,\varepsilon-\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)x(x,y)\,\mu\,\omega^2\,\varepsilon-\left(\frac{\partial}{\partial x}y(x,y)\right)^2\mu\,\omega^2\,\varepsilon-y(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)\mu\,\omega^2\,\varepsilon-\left(\frac{\partial}{\partial y}x(x,y)\right)^2\mu\,\omega^2\,\varepsilon-\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)x(x,y)\,\mu\,\omega^2\,\varepsilon$$

$$-\left(\frac{\partial}{\partial y}y(x,y)\right)^2\mu\,\omega^2\,\varepsilon-y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\mu\,\omega^2\,\varepsilon\Bigg)\Bigg),\ \frac{1}{\mu\,k}\Bigg(4\cos(k\,z$$

$$-\omega\,t)\,\sin(k\,z-\omega\,t)\Bigg(-x(x,y)^4\,k^2\,\varepsilon\,\omega^2\,\mu+x(x,y)^3\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)\mu\,\omega^2\,\varepsilon+x(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 \mu\,\omega^2\,\varepsilon + x(x,y)^3 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) \mu\,\omega^2\,\varepsilon + x(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 \mu\,\omega^2\,\varepsilon$$

$$-y(x,y)^4 k^2\,\varepsilon\,\omega^2\,\mu + y(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 \mu\,\omega^2\,\varepsilon + y(x,y)^3 \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) \mu\,\omega^2\,\varepsilon$$

$$+y(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 \mu\,\omega^2\,\varepsilon + y(x,y)^3 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) \mu\,\omega^2\,\varepsilon - 2\,x(x,y) \left(\frac{\partial}{\partial x} x(x,$$

$$y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) \mu\,\omega^2\,\varepsilon - 2\,x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) \mu\,\omega^2\,\varepsilon$$

$$-2\,x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) k^2 - 2\,x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial y} y(x,y)\right) k^2 - 2\,x(x,y)^2 k^2\,\varepsilon\,\omega^2\,\mu\,y(x,y)^2 + x(x,y)^2 y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) \mu\,\omega^2\,\varepsilon$$

$$+x(x,y)^2 y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) \mu\,\omega^2\,\varepsilon + y(x,y)^2 \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) x(x,y)\,\mu\,\omega^2\,\varepsilon$$

$$+y(x,y)^2 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) x(x,y)\,\mu\,\omega^2\,\varepsilon - x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 k^2 - y(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 k^2 - x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 k^2 - y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 k^2\Bigg)\Bigg)\Bigg]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0,$
$0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \Bigg[\frac{1}{\mu k^2} \Bigg(2\,\left(x(x,y)^2 + y(x,$

$$y)^2\right) \cos(k\,z - \omega\,t)^2 \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right)\right) \left(k^2 - \mu\,\omega^2\,\varepsilon\right)\Bigg),$$

$$\frac{1}{\mu k^2} \Bigg(2\,\left(x(x,y)^2 + y(x,y)^2\right) \cos(k\,z - \omega\,t)^2 \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,$$

$y)\Big)\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Big),\ \dfrac{2\,\varepsilon\,\left(x(x,y)^2+y(x,y)^2\right)^2\sin(k\,z-\omega\,t)\,\omega^2\cos(k\,z-\omega\,t)}{k},$

$-2\,\left(x(x,y)^2+y(x,y)^2\right)^2\cos(k\,z-\omega\,t)\,\varepsilon\sin(k\,z-\omega\,t)\,\omega\Big]$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $=\Big[0,0,$

$\dfrac{1}{\mu\,k^2}\Big(4\,\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)\,\Big(\cos(k\,z-\omega\,t)^2\,k^2\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2$

$+2\cos(k\,z-\omega\,t)^2\,k^2\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)+\cos(k\,z$

$-\omega\,t)^2\,k^2\,y(x,y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2+\cos(k\,z-\omega\,t)^2\,k^2\,x(x,y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2$

$+2\cos(k\,z-\omega\,t)^2\,k^2\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)+\cos(k\,z$

$-\omega\,t)^2\,k^2\,y(x,y)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2-\mu\,\omega^2\,\varepsilon\cos(k\,z-\omega\,t)^2\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2$

$-2\,\mu\,\omega^2\,\varepsilon\cos(k\,z-\omega\,t)^2\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)-\mu\,\omega^2\,\varepsilon\cos(k\,z$

$-\omega\,t)^2\,y(x,y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2-\mu\,\omega^2\,\varepsilon\cos(k\,z-\omega\,t)^2\,x(x,y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2$

$-2\,\mu\,\omega^2\,\varepsilon\cos(k\,z-\omega\,t)^2\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)-\mu\,\omega^2\,\varepsilon\cos(k\,z$

$-\omega\,t)^2\,y(x,y)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2-\mu\,\omega^2\,\varepsilon\sin(k\,z-\omega\,t)^2\,k^2\,x(x,y)^4-2\,\mu\,\omega^2\,\varepsilon\sin(k\,z$

$-\omega\,t)^2\,k^2\,x(x,y)^2\,y(x,y)^2-\mu\,\omega^2\,\varepsilon\sin(k\,z-\omega\,t)^2\,k^2\,y(x,y)^4\Big)\Big)\Big]$

Spin Dissipation J_spin dot E $=\dfrac{1}{\mu\,k^3}\Big(4\,\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)\,\omega\,\Big(\cos(k\,z$

$-\omega\,t)^2\,k^2\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2+2\cos(k\,z-\omega\,t)^2\,k^2\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,$

$y)\left(\frac{\partial}{\partial x}\,y(x,y)\right)+\cos(k\,z-\omega\,t)^2\,k^2\,y(x,y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2+\cos(k\,z-\omega\,t)^2\,k^2\,x(x,$

$y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2+2\cos(k\,z-\omega\,t)^2\,k^2\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)$

$$+\cos(kz-\omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - \mu\,\omega^2\,\varepsilon\cos(kz-\omega t)^2 x(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 2\,\mu\,\omega^2\,\varepsilon\cos(kz-\omega t)^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right)$$

$$-\mu\,\omega^2\,\varepsilon\cos(kz-\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - \mu\,\omega^2\,\varepsilon\cos(kz-\omega t)^2 x(x,$$

$$y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 2\,\mu\,\omega^2\,\varepsilon\cos(kz-\omega t)^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)$$

$$-\mu\,\omega^2\,\varepsilon\cos(kz-\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - \mu\,\omega^2\,\varepsilon\sin(kz-\omega t)^2 k^2 x(x,y)^4$$

$$-2\,\mu\,\omega^2\,\varepsilon\sin(kz-\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 - \mu\,\omega^2\,\varepsilon\sin(kz-\omega t)^2 k^2 y(x,y)^4\Big)\Big)$$

$$\textit{Disspative Force 3 vector} = \left[\frac{1}{\mu k^2}\Big(4\cos(kz-\omega t)^2 \Big(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,\right.$$

$$y)\left(\frac{\partial}{\partial x} y(x,y)\right)\Big)\Big(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 k^2 + \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) x(x,y)\, k^2 + \left(\frac{\partial}{\partial x} y(x,y)\right)^2 k^2$$

$$+y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) k^2 + \left(\frac{\partial}{\partial y} x(x,y)\right)^2 k^2 + \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) x(x,y)\, k^2$$

$$+\left(\frac{\partial}{\partial y} y(x,y)\right)^2 k^2 + y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) k^2 + 2\,k^2\,\varepsilon\,\omega^2\,\mu\, x(x,y)^2 + 2\,k^2\,\varepsilon\,\omega^2\,\mu\, y(x,y)^2$$

$$-\left(\frac{\partial}{\partial x} x(x,y)\right)^2 \mu\,\omega^2\,\varepsilon - \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) x(x,y)\,\mu\,\omega^2\,\varepsilon - \left(\frac{\partial}{\partial x} y(x,y)\right)^2 \mu\,\omega^2\,\varepsilon - y(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right)\mu\,\omega^2\,\varepsilon - \left(\frac{\partial}{\partial y} x(x,y)\right)^2 \mu\,\omega^2\,\varepsilon - \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) x(x,y)\,\mu\,\omega^2\,\varepsilon$$

$$-\left(\frac{\partial}{\partial y} y(x,y)\right)^2 \mu\,\omega^2\,\varepsilon - y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right)\mu\,\omega^2\,\varepsilon\Big)\Big), \frac{1}{\mu k^2}\Big(4\cos(kz$$

$$- \omega t)^2 \left(x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) + y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) \right) \left(\left(\frac{\partial}{\partial x} x(x, y) \right)^2 k^2 \right.$$

$$+ \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) x(x, y)\, k^2 + \left(\frac{\partial}{\partial x} y(x, y) \right)^2 k^2 + y(x, y) \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) k^2$$

$$+ \left(\frac{\partial}{\partial y} x(x, y) \right)^2 k^2 + \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) x(x, y)\, k^2 + \left(\frac{\partial}{\partial y} y(x, y) \right)^2 k^2 + y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x,\right.$$

$$\left. y) \right) k^2 + 2\, k^2\, \varepsilon\, \omega^2\, \mu\, x(x, y)^2 + 2\, k^2\, \varepsilon\, \omega^2\, \mu\, y(x, y)^2 - \left(\frac{\partial}{\partial x} x(x, y) \right)^2 \mu\, \omega^2\, \varepsilon - \left(\frac{\partial^2}{\partial x^2} x(x,\right.$$

$$\left. y) \right) x(x, y)\, \mu\, \omega^2\, \varepsilon - \left(\frac{\partial}{\partial x} y(x, y) \right)^2 \mu\, \omega^2\, \varepsilon - y(x, y) \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) \mu\, \omega^2\, \varepsilon$$

$$- \left(\frac{\partial}{\partial y} x(x, y) \right)^2 \mu\, \omega^2\, \varepsilon - \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) x(x, y)\, \mu\, \omega^2\, \varepsilon - \left(\frac{\partial}{\partial y} y(x, y) \right)^2 \mu\, \omega^2\, \varepsilon - y(x,$$

$$\left. y) \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) \mu\, \omega^2\, \varepsilon \right) \Bigg), \frac{1}{\mu\, k^2} \Bigg(4 \cos(k\, z - \omega\, t) \Big(\sin(k\, z - \omega\, t)\, k\, x(x, y)^2\, y(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) \mu\, \omega^2\, \varepsilon + \sin(k\, z - \omega\, t)\, k\, x(x, y)^2\, y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) \mu\, \omega^2\, \varepsilon + \sin(k\, z$$

$$- \omega\, t)\, k\, y(x, y)^2 \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) x(x, y)\, \mu\, \omega^2\, \varepsilon + \sin(k\, z - \omega\, t)\, k\, y(x, y)^2 \left(\frac{\partial^2}{\partial y^2} x(x,\right.$$

$$\left. y) \right) x(x, y)\, \mu\, \omega^2\, \varepsilon + \sin(k\, z - \omega\, t)\, k\, x(x, y)^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 \mu\, \omega^2\, \varepsilon + \sin(k\, z$$

$$- \omega\, t)\, k\, x(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 \mu\, \omega^2\, \varepsilon + \sin(k\, z - \omega\, t)\, k\, y(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 \mu\, \omega^2\, \varepsilon$$

$$+ \sin(k\, z - \omega\, t)\, k\, y(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 \mu\, \omega^2\, \varepsilon - 2 \sin(k\, z - \omega\, t)\, k^3\, x(x,$$

$$y)^2\, \varepsilon\, \omega^2\, \mu\, y(x, y)^2 + \sin(k\, z - \omega\, t)\, k\, x(x, y)^3 \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) \mu\, \omega^2\, \varepsilon + \sin(k\, z$$

$$- \omega\, t)\, k\, x(x, y)^3 \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) \mu\, \omega^2\, \varepsilon + \sin(k\, z - \omega\, t)\, k\, y(x, y)^3 \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) \mu\, \omega^2\, \varepsilon$$

$$+\sin(kz-\omega t)\, k\, y(x,y)^3 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) \mu\,\omega^2\,\varepsilon - 2\sin(kz-\omega t)\, k\, x(x,y) \left(\frac{\partial}{\partial x} x(x,$$

$$y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) \mu\,\omega^2\,\varepsilon - 2\sin(kz-\omega t)\, k\, x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial y} y(x,y)\right) \mu\,\omega^2\,\varepsilon - 3\, x(x,y)^4 k^2 \varepsilon\,\omega^2 \mu^2 y(x,y)^2 \sin(kz-\omega t)^2 - 3\, x(x,$$

$$y)^2 k^2 y(x,y)^4 \varepsilon\,\omega^2 \mu^2 \sin(kz-\omega t)^2 - x(x,y)^2 \cos(kz-\omega t)^2 \mu^2 \omega^2 \varepsilon\, y(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - x(x,y)^2 \cos(kz-\omega t)^2 \mu^2 \omega^2 \varepsilon\, y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - y(x,$$

$$y)^2 \cos(kz-\omega t)^2 \mu^2 \omega^2 \varepsilon\, x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - y(x,y)^2 \cos(kz$$

$$-\omega t)^2 \mu^2 \omega^2 \varepsilon\, x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - x(x,y)^6 k^2 \varepsilon\,\omega^2 \mu^2 \sin(kz-\omega t)^2 - x(x,$$

$$y)^4 \cos(kz-\omega t)^2 \mu^2 \omega^2 \varepsilon \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - x(x,y)^4 \cos(kz-\omega t)^2 \mu^2 \omega^2 \varepsilon \left(\frac{\partial}{\partial y} x(x,y)\right)^2$$

$$-y(x,y)^6 k^2 \varepsilon\,\omega^2 \mu^2 \sin(kz-\omega t)^2 - y(x,y)^4 \cos(kz-\omega t)^2 \mu^2 \omega^2 \varepsilon \left(\frac{\partial}{\partial x} y(x,y)\right)^2$$

$$-y(x,y)^4 \cos(kz-\omega t)^2 \mu^2 \omega^2 \varepsilon \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + \mu\, x(x,y)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,$$

$$y)\right)^2 \cos(kz-\omega t)^2 + \mu\, x(x,y)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 \cos(kz-\omega t)^2 + \mu\, y(x,$$

$$y)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 \cos(kz-\omega t)^2 + \mu\, y(x,y)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,$$

$$y)\right)^2 \cos(kz-\omega t)^2 + 2\,\mu\, x(x,y)^3 k^2 \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) \cos(kz-\omega t)^2$$

$$+2\,\mu\, x(x,y)^3 k^2 \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) \cos(kz-\omega t)^2 + 2\,\mu\, y(x,y)^3 k^2 x(x,$$

$$y) \left(\frac{\partial}{\partial x} x(x,y)\right) \left(\frac{\partial}{\partial x} y(x,y)\right) \cos(kz-\omega t)^2 + 2\,\mu\, y(x,y)^3 k^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,$$

$$y)\right) \left(\frac{\partial}{\partial y} y(x,y)\right) \cos(kz-\omega t)^2 + \mu\, x(x,y)^4 k^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 \cos(kz-\omega t)^2$$

$$+\mu^{x(x,y)^4} k^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 \cos(kz-\omega t)^2 + \mu\, y(x,y)^4 k^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 \cos(kz$$

$$-\omega t)^2 + \mu\, y(x,y)^4 k^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 \cos(kz-\omega t)^2 - 2\, x(x,y)^3 \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial x} y(x,y)\right) \mu^2 \omega^2 \varepsilon \cos(kz-\omega t)^2 - 2\, x(x,y)^3 \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,\right.$$

$$\left. y)\right) \mu^2 \omega^2 \varepsilon \cos(kz-\omega t)^2 - 2\, y(x,y)^3 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) \left(\frac{\partial}{\partial x} y(x,\right.$$

$$\left. y)\right) \mu^2 \omega^2 \varepsilon \cos(kz-\omega t)^2 - 2\, y(x,y)^3 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) \left(\frac{\partial}{\partial y} y(x,\right.$$

$$\left. y)\right) \mu^2 \omega^2 \varepsilon \cos(kz-\omega t)^2 - \sin(kz-\omega t)\, k^3 x(x,y)^4 \varepsilon \omega^2 \mu - \sin(kz-\omega t)\, k^3 y(x,$$

$$y)^4 \varepsilon \omega^2 \mu - 2 \sin(kz-\omega t)\, k^3 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) - 2 \sin(kz$$

$$- \omega t)\, k^3 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) - \sin(kz-\omega t)\, k^3 x(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - \sin(kz-\omega t)\, k^3 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - \sin(kz-\omega t)\, k^3 x(x,$$

$$y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - \sin(kz-\omega t)\, k^3 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 \Bigg)\Bigg)\Bigg]$$

$$Dissipation = \frac{1}{k} \Bigg(2\varepsilon \cos(kz-\omega t)\, \omega \Bigg(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right)$$

$$+ \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$+ \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - x(x,y)^2 k^2 - k^2 y(x,y)^2 - \mu \sin(kz$$

$$- \omega t)\, k\, x(x,y)^4 - 2\mu \sin(kz-\omega t)\, k\, x(x,y)^2 y(x,y)^2 - \mu \sin(kz-\omega t)\, k\, y(x,y)^4 \Bigg)\Bigg)$$

****************************** *END PROCEDURE* ***************************** **(28)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 6c-- Wave guide TTM (kinematic out, wave in)`;
> theta:=(k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=+(omega/k)*f(x,y)*cos(theta);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 6c-- Wave guide TTM (kinematic out, wave in)

$$\theta := kz + \omega t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z + \omega t)$$

$$\phi := \frac{\omega \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z + \omega t)}{k}$$

Example 6c-- Wave guide TTM (kinematic out, wave in)

************************************ *Differential Form Format* **********************************

$$\textit{Action 1-form} = \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z + \omega t)\, d(z) - \frac{\omega \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z + \omega t)\, d(t)}{k}$$

$$\textit{Intensity 2-form F=dA} = 2\left(x(x,y)^2 + y(x,y)^2\right) \sin(k z + \omega t)\, \omega\, (d(z)) \,\&\wedge\, (d(t)) + 2\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right) \cos(k z + \omega t)\, (d(x)) \,\&\wedge\, (d(z)) + 2\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right) \cos(k z + \omega t)\, (d(y)) \,\&\wedge\, (d(z)) - \frac{2\omega\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right) \cos(k z + \omega t)\, (d(x)) \,\&\wedge\, (d(t))}{k} - \frac{2\omega\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right) \cos(k z + \omega t)\, (d(y)) \,\&\wedge\, (d(t))}{k}$$

Topological Torsion 3-form $A\wedge F = 0$

Topological Parity 4-form $F\wedge F = 0$

************************************ *Using EM format* ************************************

$$\textit{E field} = \left[-\frac{2\omega\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right) \cos(k z + \omega t)}{k}, -\frac{2\omega\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right) \cos(k z + \omega t)}{k}, 2\left(x(x,y)^2 + y(x,y)^2\right) \sin(k z + \omega t)\, \omega \right]$$

$$\textit{B field} = \left[2\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right)\right) \cos(k z + \omega t), -2\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right) \cos(k z + \omega t), 0 \right]$$

Topological TORSION 4 vector   T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension   PTD = 2

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

Xm or linear (Mean) curvature =

$$-\frac{\left(x(x,y)^2+y(x,y)^2\right)\sin(k\,z+\omega\,t)\,(k-\omega)\,(k+\omega)}{k}$$

Yg or quadratic (GAUSS) curvature = 0

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

**********   *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

$$D\ field=\left[-\frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z+\omega\,t)}{k},\right.$$
$$-\frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z+\omega\,t)}{k},2\,\varepsilon\left(x(x,y)^2\right.$$
$$\left.\left.+y(x,y)^2\right)\sin(k\,z+\omega\,t)\,\omega\right]$$

$$H\ field=\left[\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z+\omega\,t)}{\mu},\right.$$
$$\left.-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z+\omega\,t)}{\mu},0\right]$$

$$Poynting\ vector\ ExH=\left[\frac{1}{\mu}\left(4\left(x(x,y)^2+y(x,y)^2\right)\sin(k\,z+\omega\,t)\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)\right.\right.\right.$$
$$\left.\left.+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z+\omega\,t)\right),\frac{1}{\mu}\left(4\left(x(x,y)^2+y(x,y)^2\right)\sin(k\,z\right.$$

$$+ \omega t) \, \omega \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \cos(k z + \omega t) \Big),$$

$$\frac{1}{k \mu} \Big(4 \omega \cos(k z + \omega t)^2 \Big(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2 \, x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,$$

$$y) \left(\frac{\partial}{\partial y} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2 \, x(x,$$

$$y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 \Big) \Big) \Big]$$

Amperian Current 4Vector *curlH-dD/dt=J4* $= \Big[$

$$-\frac{2 \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \sin(k z + \omega t) \left(k^2 + \mu \omega^2 \varepsilon \right)}{k \mu},$$

$$-\frac{2 \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \sin(k z + \omega t) \left(k^2 + \mu \omega^2 \varepsilon \right)}{k \mu},$$

$$-\frac{1}{\mu} \Big(2 \cos(k z + \omega t) \Big(\left(\frac{\partial}{\partial x} x(x,y) \right)^2 + x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y) \right) + \left(\frac{\partial}{\partial x} y(x,y) \right)^2 + y(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2} y(x,y) \right) + \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y) \right) + \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + y(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} y(x,y) \right) + \varepsilon \omega^2 \mu \, x(x,y)^2 + \varepsilon \omega^2 \mu \, y(x,y)^2 \Big) \Big), \; -\frac{1}{k} \Big(2 \, \varepsilon \cos(k z$$

$$+ \omega t) \, \omega \Big(\left(\frac{\partial}{\partial x} x(x,y) \right)^2 + x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y) \right) + \left(\frac{\partial}{\partial x} y(x,y) \right)^2 + y(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2} y(x,y) \right) + \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y) \right) + \left(\frac{\partial}{\partial y} y(x,y) \right)^2 + y(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} y(x,y) \right) - x(x,y)^2 k^2 - k^2 y(x,y)^2 \Big) \Big) \Big]$$

Amerian charge density *divD = rho* $= -\frac{1}{k} \Big(2 \, \varepsilon \cos(k z + \omega t) \, \omega \Big(\left(\frac{\partial}{\partial x} x(x,y) \right)^2 + x(x,$

$$y) \left(\frac{\partial^2}{\partial x^2} x(x,y) \right) + \left(\frac{\partial}{\partial x} y(x,y) \right)^2 + y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y) \right) + \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + x(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-x(x,y)^2k^2-k^2y(x,y)^2\Big)\Big)$$

divergence Lorentz Current 4Vector, *4div(J4)* $=0$

Topological SPIN 4 vector S4 $=\Bigg[\frac{1}{\mu k^2}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\Big(x(x,$

$y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Big), \frac{1}{\mu k^2}\Big(2\,(x(x,y)^2$

$+y(x,y)^2)\cos(kz+\omega t)^2\Big(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Big)\,(k^2$

$-\mu\,\omega^2\,\varepsilon)\Big), \frac{2\,\varepsilon\left(x(x,y)^2+y(x,y)^2\right)^2\sin(kz+\omega t)\,\omega^2\cos(kz+\omega t)}{k}, 2\,(x(x,y)^2$

$+y(x,y)^2)^2\cos(kz+\omega t)\,\varepsilon\sin(kz+\omega t)\,\omega\Bigg]$

Topological SPIN 3-form $=\frac{1}{\mu k^2}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\Big(x(x,y)\Big(\frac{\partial}{\partial x}x(x,$

$y)\Big)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\,\&\text{^}(d(y),d(z),d(t))\Big)$

$-\frac{1}{\mu k^2}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\Big(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,$

$y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\,\&\text{^}(d(x),d(z),d(t))\Big)$

$+\frac{2\,\varepsilon\left(x(x,y)^2+y(x,y)^2\right)^2\sin(kz+\omega t)\,\omega^2\cos(kz+\omega t)\,\&\text{^}(d(x),d(y),d(t))}{k}$

$-2\left(x(x,y)^2+y(x,y)^2\right)^2\cos(kz+\omega t)\,\varepsilon\sin(kz+\omega t)\,\omega\,\&\text{^}(d(x),d(y),d(z))$

Spin density rho_spin $=2\left(x(x,y)^2+y(x,y)^2\right)^2\cos(kz+\omega t)\,\varepsilon\sin(kz+\omega t)\,\omega$

LaGrange field energy density (B.H-D.E) $=\frac{1}{\mu k^2}\Big(4\Big(\cos(kz+\omega t)^2k^2x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2$

$+2\cos(kz+\omega t)^2k^2x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+\cos(kz$

$+\omega t)^2k^2y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+\cos(kz+\omega t)^2k^2x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$

$+2\cos(kz+\omega t)^2k^2x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+\cos(kz$

$$+\omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - \mu\,\omega^2\,\varepsilon\cos(kz+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2$$

$$-2\,\mu\,\omega^2\,\varepsilon\cos(kz+\omega t)^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) - \mu\,\omega^2\,\varepsilon\cos(kz$$

$$+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - \mu\,\omega^2\,\varepsilon\cos(kz+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2$$

$$-2\,\mu\,\omega^2\,\varepsilon\cos(kz+\omega t)^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) - \mu\,\omega^2\,\varepsilon\cos(kz$$

$$+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - \mu\,\omega^2\,\varepsilon\sin(kz+\omega t)^2 k^2 x(x,y)^4 - 2\,\mu\,\omega^2\,\varepsilon\sin(kz$$

$$+\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 - \mu\,\omega^2\,\varepsilon\sin(kz+\omega t)^2 k^2 y(x,y)^4\Big)\Big)$$

$$B.H = \frac{1}{\mu}\Bigg(4\cos(kz+\omega t)^2 \Bigg(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\,x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial y} y(x,y)\right) + y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\,x(x,$$

$$y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2\Bigg)\Bigg)$$

$$D.E = \frac{1}{k^2}\Bigg(4\,\omega^2\,\varepsilon\Bigg(\cos(kz+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\cos(kz+\omega t)^2 x(x,$$

$$y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2$$

$$+\cos(kz+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\cos(kz+\omega t)^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial y} y(x,y)\right) + \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + \sin(kz+\omega t)^2 k^2 x(x,y)^4$$

$$+2\sin(kz+\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 + \sin(kz+\omega t)^2 k^2 y(x,y)^4\Bigg)\Bigg)$$

$$A.J = -\frac{1}{\mu}\Bigg(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2 \Bigg(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial x} y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial y} y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right) + \varepsilon\,\omega^2\,\mu\,x(x,y)^2 + \varepsilon\,\omega^2\,\mu\,y(x,y)^2\Bigg)\Bigg)$$

$$-rho.phi = -\frac{1}{k^2}\Bigg(2\,\omega^2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz+\omega t)^2\,\varepsilon\Bigg(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + x(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-x(x,y)^2k^2-k^2y(x,y)^2\Big)\Big)$$

Poincare I (B.H - D.E)-(A.J - rho.phi) $= \frac{1}{\mu k^2}\Big(2\Big(\cos(kz+\omega t)^2k^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2y(x,$

$$y)^2+\cos(kz+\omega t)^2k^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2x(x,y)^2+\cos(kz+\omega t)^2k^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2y(x,y)^2$$

$$+\cos(kz+\omega t)^2k^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2x(x,y)^2+\cos(kz+\omega t)^2k^2x(x,y)^3\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$$

$$+\cos(kz+\omega t)^2k^2y(x,y)^3\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\cos(kz+\omega t)^2k^2x(x,y)^3\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$

$$+\cos(kz+\omega t)^2k^2y(x,y)^3\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+3\cos(kz+\omega t)^2k^2x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2$$

$$+3\cos(kz+\omega t)^2k^2y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+3\cos(kz+\omega t)^2k^2x(x,$$

$$y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+3\cos(kz+\omega t)^2k^2y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2-\cos(kz$$

$$+\omega t)^2x(x,y)^3\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)\mu\omega^2\varepsilon-\cos(kz+\omega t)^2y(x,y)^3\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\mu\omega^2\varepsilon$$

$$-\cos(kz+\omega t)^2y(x,y)^3\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)\mu\omega^2\varepsilon-\cos(kz+\omega t)^2x(x,y)^3\left(\frac{\partial^2}{\partial y^2}x(x,$$

$$y)\right)\mu\omega^2\varepsilon+\cos(kz+\omega t)^2k^2x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)y(x,y)^2+\cos(kz+\omega t)^2k^2y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)x(x,y)^2-\cos(kz+\omega t)^2y(x,y)^2\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)x(x,y)\,\mu\omega^2\varepsilon$$

$$-\cos(kz+\omega t)^2y(x,y)^2\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)x(x,y)\,\mu\omega^2\varepsilon-\cos(kz+\omega t)^2x(x,y)^2y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\mu\omega^2\varepsilon-\cos(kz+\omega t)^2x(x,y)^2y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)\mu\omega^2\varepsilon$$

$$-4\mu\omega^2\varepsilon\sin(kz+\omega t)^2k^2x(x,y)^2y(x,y)^2+4\cos(kz+\omega t)^2k^2x(x,y)^2\varepsilon\omega^2\mu y(x,y)^2$$

$$+2\cos(kz+\omega t)^2k^2x(x,y)^4\varepsilon\omega^2\mu+2\cos(kz+\omega t)^2k^2y(x,y)^4\varepsilon\omega^2\mu-\cos(kz$$

$$+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 \mu\,\omega^2\,\varepsilon - \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 \mu\,\omega^2\,\varepsilon$$

$$-\cos(kz+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 \mu\,\omega^2\,\varepsilon - \cos(kz+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} y(x,$$

$$y)\right)^2 \mu\,\omega^2\,\varepsilon + \cos(kz+\omega t)^2 k^2 x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) y(x,y)^2 + \cos(kz+\omega t)^2 k^2 y(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) x(x,y)^2 - 4\,\mu\,\omega^2\,\varepsilon \cos(kz+\omega t)^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial x} y(x,y)\right) - 4\,\mu\,\omega^2\,\varepsilon \cos(kz+\omega t)^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)$$

$$+4\cos(kz+\omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 4\cos(kz$$

$$+\omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) - 3\,\mu\,\omega^2\,\varepsilon \cos(kz+\omega t)^2 x(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 3\,\mu\,\omega^2\,\varepsilon \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - 3\,\mu\,\omega^2\,\varepsilon \cos(kz$$

$$+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 3\,\mu\,\omega^2\,\varepsilon \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2$$

$$-2\,\mu\,\omega^2\,\varepsilon \sin(kz+\omega t)^2 k^2 x(x,y)^4 - 2\,\mu\,\omega^2\,\varepsilon \sin(kz+\omega t)^2 k^2 y(x,y)^4\Big)\Big)$$

$$\textit{London Coefficient} \qquad LC = \frac{k^2 - \mu\,\omega^2\,\varepsilon}{\mu}$$

$$\textit{PROCA coefficient curlcurlB} = \Bigg[-2\cos(kz+\omega t) \left(2 \left(\frac{\partial}{\partial x} x(x,y)\right) \left(\frac{\partial^2}{\partial y\,\partial x} x(x,y)\right)\right.$$

$$+\left(\frac{\partial}{\partial y} x(x,y)\right) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + x(x,y) \left(\frac{\partial^3}{\partial y\,\partial x^2} x(x,y)\right) + 2 \left(\frac{\partial}{\partial x} y(x,$$

$$y)\right) \left(\frac{\partial^2}{\partial y\,\partial x} y(x,y)\right) + \left(\frac{\partial}{\partial y} y(x,y)\right) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + y(x,y) \left(\frac{\partial^3}{\partial y\,\partial x^2} y(x,y)\right)$$

$$+3 \left(\frac{\partial}{\partial y} x(x,y)\right) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + x(x,y) \left(\frac{\partial^3}{\partial y^3} x(x,y)\right) + 3 \left(\frac{\partial}{\partial y} y(x,y)\right) \left(\frac{\partial^2}{\partial y^2} y(x,$$

$$\left. y)\right) + y(x,y) \left(\frac{\partial^3}{\partial y^3} y(x,y)\right) - k^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) - k^2 y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)\right),$$

$$2\cos(kz+\omega t) \left(-k^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) - k^2 y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + 3 \left(\frac{\partial}{\partial x} x(x,\right.\right.$$

$$y)\Big)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+x(x,y)\left(\frac{\partial^3}{\partial x^3}x(x,y)\right)+3\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+y(x,$$

$$y)\left(\frac{\partial^3}{\partial x^3}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}x(x,y)\right)+\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$

$$+x(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}y(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,$$

$$y)\Big)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+y(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}y(x,y)\right)\Big),0\Big]$$

Amperian Current 4Vector curlH-dD/dt=J4 $= \Big[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz+\omega t)\left(k^2+\mu\,\omega^2\,\varepsilon\right)}{k\,\mu},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\sin(kz+\omega t)\left(k^2+\mu\,\omega^2\,\varepsilon\right)}{k\,\mu},$$

$$-\frac{1}{\mu}\Bigg(2\cos(kz+\omega t)\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+\varepsilon\,\omega^2\,\mu\,x(x,y)^2+\varepsilon\,\omega^2\,\mu\,y(x,y)^2\Bigg)\Bigg),-\frac{1}{k}\Bigg(2\,\varepsilon\cos(kz$$

$$+\omega t)\,\omega\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,$$

$$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)-x(x,y)^2k^2-k^2y(x,y)^2\Bigg)\Bigg)\Bigg]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B)

$$
= \left[\frac{1}{\mu k^2} \left(4 \cos(k z + \omega t)^2 \left(x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) + y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) \right) \left(\left(\frac{\partial}{\partial x} x(x, y) \right)^2 k^2 + \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) x(x, y) k^2 + \left(\frac{\partial}{\partial x} y(x, y) \right)^2 k^2 + y(x, y) \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) k^2 + \left(\frac{\partial}{\partial y} x(x, y) \right)^2 k^2 + \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) x(x, y) k^2 + \left(\frac{\partial}{\partial y} y(x, y) \right)^2 k^2 + y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) k^2 + 2 k^2 \varepsilon \omega^2 \mu x(x, y)^2 + 2 k^2 \varepsilon \omega^2 \mu y(x, y)^2 - \left(\frac{\partial}{\partial x} x(x, y) \right)^2 \mu \omega^2 \varepsilon - \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) x(x, y) \mu \omega^2 \varepsilon - \left(\frac{\partial}{\partial x} y(x, y) \right)^2 \mu \omega^2 \varepsilon - y(x, y) \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) \mu \omega^2 \varepsilon - \left(\frac{\partial}{\partial y} x(x, y) \right)^2 \mu \omega^2 \varepsilon - \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) x(x, y) \mu \omega^2 \varepsilon - \left(\frac{\partial}{\partial y} y(x, y) \right)^2 \mu \omega^2 \varepsilon - y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) \mu \omega^2 \varepsilon \right) \right), \frac{1}{\mu k^2} \left(4 \cos(k z + \omega t)^2 \left(x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) + y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) \right) \left(\left(\frac{\partial}{\partial x} x(x, y) \right)^2 k^2 + \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) x(x, y) k^2 + \left(\frac{\partial}{\partial x} y(x, y) \right)^2 k^2 + y(x, y) \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) k^2 + \left(\frac{\partial}{\partial y} x(x, y) \right)^2 k^2 + \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) x(x, y) k^2 + \left(\frac{\partial}{\partial y} y(x, y) \right)^2 k^2 + y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) k^2 + 2 k^2 \varepsilon \omega^2 \mu x(x, y)^2 + 2 k^2 \varepsilon \omega^2 \mu y(x, y)^2 - \left(\frac{\partial}{\partial x} x(x, y) \right)^2 \mu \omega^2 \varepsilon - \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) x(x, y) \mu \omega^2 \varepsilon - \left(\frac{\partial}{\partial x} y(x, y) \right)^2 \mu \omega^2 \varepsilon - y(x,
$$

$$y) \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) \mu \omega^2 \varepsilon - \left(\frac{\partial}{\partial y} x(x, y) \right)^2 \mu \omega^2 \varepsilon - \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) x(x, y) \mu \omega^2 \varepsilon$$

$$- \left(\frac{\partial}{\partial y} y(x, y) \right)^2 \mu \omega^2 \varepsilon - y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) \mu \omega^2 \varepsilon \Big) \Big), \frac{1}{k \mu} \Big(4 \sin(k z$$

$$+ \omega t) \cos(k z + \omega t) \Big(-x(x, y)^4 k^2 \varepsilon \omega^2 \mu + x(x, y)^3 \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) \mu \omega^2 \varepsilon + x(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 \mu \omega^2 \varepsilon + x(x, y)^3 \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) \mu \omega^2 \varepsilon + x(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 \mu \omega^2 \varepsilon$$

$$- y(x, y)^4 k^2 \varepsilon \omega^2 \mu + y(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 \mu \omega^2 \varepsilon + y(x, y)^3 \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) \mu \omega^2 \varepsilon$$

$$+ y(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 \mu \omega^2 \varepsilon + y(x, y)^3 \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) \mu \omega^2 \varepsilon - 2 x(x, y) \Big(\frac{\partial}{\partial x} x(x,$$

$$y) \Big) y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) \mu \omega^2 \varepsilon - 2 x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) \mu \omega^2 \varepsilon$$

$$- 2 x(x, y) \left(\frac{\partial}{\partial x} x(x, y) \right) y(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) k^2 - 2 x(x, y) \left(\frac{\partial}{\partial y} x(x, y) \right) y(x,$$

$$y) \left(\frac{\partial}{\partial y} y(x, y) \right) k^2 - 2 x(x, y)^2 k^2 \varepsilon \omega^2 \mu y(x, y)^2 + x(x, y)^2 y(x, y) \left(\frac{\partial^2}{\partial x^2} y(x, y) \right) \mu \omega^2 \varepsilon$$

$$+ x(x, y)^2 y(x, y) \left(\frac{\partial^2}{\partial y^2} y(x, y) \right) \mu \omega^2 \varepsilon + y(x, y)^2 \left(\frac{\partial^2}{\partial x^2} x(x, y) \right) x(x, y) \mu \omega^2 \varepsilon$$

$$+ y(x, y)^2 \left(\frac{\partial^2}{\partial y^2} x(x, y) \right) x(x, y) \mu \omega^2 \varepsilon - x(x, y)^2 \left(\frac{\partial}{\partial x} x(x, y) \right)^2 k^2 - y(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} y(x, y) \right)^2 k^2 - x(x, y)^2 \left(\frac{\partial}{\partial y} x(x, y) \right)^2 k^2 - y(x, y)^2 \left(\frac{\partial}{\partial y} y(x, y) \right)^2 k^2 \Big) \Big) \Big]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] = [0, 0, 0, 0]

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) = [0, 0, 0]

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] $= \left[\frac{1}{\mu k^2} \left(2 \left(x(x,y)^2 + y(x,y)^2 \right) \cos(kz+\omega t)^2 \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) \right) \left(k^2 - \mu \omega^2 \varepsilon \right) \right), \frac{1}{\mu k^2} \left(2 \left(x(x,y)^2 + y(x,y)^2 \right) \cos(kz+\omega t)^2 \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) \right) \left(k^2 - \mu \omega^2 \varepsilon \right) \right), \frac{2 \varepsilon \left(x(x,y)^2 + y(x,y)^2 \right)^2 \sin(kz+\omega t)\, \omega^2 \cos(kz+\omega t)}{k}, 2 \left(x(x,y)^2 + y(x,y)^2 \right)^2 \cos(kz+\omega t)\, \varepsilon \sin(kz+\omega t)\, \omega \right]$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $= \left[0, 0, \frac{1}{\mu k^2} \left(4 \left(x(x,y)^2 + y(x,y)^2 \right) \cos(kz+\omega t) \left(\cos(kz+\omega t)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 + 2 \cos(kz+\omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) + \cos(kz+\omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 + \cos(kz+\omega t)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 + 2 \cos(kz+\omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) + \cos(kz+\omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 - \mu \omega^2 \varepsilon \cos(kz+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y) \right)^2 - 2 \mu \omega^2 \varepsilon \cos(kz+\omega t)^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y) \right) - \mu \omega^2 \varepsilon \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y) \right)^2 - \mu \omega^2 \varepsilon \cos(kz+\omega t)^2 x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y) \right)^2 - 2 \mu \omega^2 \varepsilon \cos(kz+\omega t)^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y) \right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y) \right) - \mu \omega^2 \varepsilon \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y) \right)^2 - \mu \omega^2 \varepsilon \sin(kz+\omega t)^2 k^2 x(x,y)^4 - 2 \mu \omega^2 \varepsilon \sin(kz+\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 - \mu \omega^2 \varepsilon \sin(kz+\omega t)^2 k^2 y(x,y)^4 \right) \right) \right]$

Spin Dissipation J_spin dot E $= - \frac{1}{\mu k^3} \left(4 \left(x(x,y)^2 + y(x,y)^2 \right) \cos(kz+\omega t)\, \omega \left(\cos(kz \right. \right.$

$$+\omega t)^2 k^2 x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\cos(kz+\omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial x} y(x,y)\right) + \cos(kz+\omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + \cos(kz+\omega t)^2 k^2 x(x,$$

$$y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\cos(kz+\omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)$$

$$+\cos(kz+\omega t)^2 k^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - \mu\,\omega^2\,\varepsilon \cos(kz+\omega t)^2 x(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - 2\,\mu\,\omega^2\,\varepsilon \cos(kz+\omega t)^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right)$$

$$-\mu\,\omega^2\,\varepsilon \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - \mu\,\omega^2\,\varepsilon \cos(kz+\omega t)^2 x(x,$$

$$y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - 2\,\mu\,\omega^2\,\varepsilon \cos(kz+\omega t)^2 x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)$$

$$-\mu\,\omega^2\,\varepsilon \cos(kz+\omega t)^2 y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 - \mu\,\omega^2\,\varepsilon \sin(kz+\omega t)^2 k^2 x(x,y)^4$$

$$-2\,\mu\,\omega^2\,\varepsilon \sin(kz+\omega t)^2 k^2 x(x,y)^2 y(x,y)^2 - \mu\,\omega^2\,\varepsilon \sin(kz+\omega t)^2 k^2 y(x,y)^4 \Big) \Big)$$

$$\mathit{Disspative\ Force\ 3\ vector} = \left[\frac{1}{\mu k^2} \Big(4\cos(kz+\omega t)^2 \left(x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) + y(x, \right.\right.$$

$$y) \left(\frac{\partial}{\partial x} y(x,y)\right)\Big) \Big(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 k^2 + \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) x(x,y)\, k^2 + \left(\frac{\partial}{\partial x} y(x,y)\right)^2 k^2$$

$$+ y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) k^2 + \left(\frac{\partial}{\partial y} x(x,y)\right)^2 k^2 + \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) x(x,y)\, k^2$$

$$+ \left(\frac{\partial}{\partial y} y(x,y)\right)^2 k^2 + y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) k^2 + 2\,k^2\,\varepsilon\,\omega^2\,\mu\, x(x,y)^2 + 2\,k^2\,\varepsilon\,\omega^2\,\mu\, y(x,y)^2$$

$$- \left(\frac{\partial}{\partial x} x(x,y)\right)^2 \mu\,\omega^2\,\varepsilon - \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) x(x,y)\,\mu\,\omega^2\,\varepsilon - \left(\frac{\partial}{\partial x} y(x,y)\right)^2 \mu\,\omega^2\,\varepsilon - y(x,$$

$$y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) \mu\,\omega^2\,\varepsilon - \left(\frac{\partial}{\partial y} x(x,y)\right)^2 \mu\,\omega^2\,\varepsilon - \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) x(x,y)\,\mu\,\omega^2\,\varepsilon$$

$$- \left(\frac{\partial}{\partial y} y(x,y)\right)^2 \mu \omega^2 \varepsilon - y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) \mu \omega^2 \varepsilon \Bigg) \Bigg), \frac{1}{\mu k^2} \Bigg(4 \cos(k z$$

$$+ \omega t)^2 \left(x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)\right) \Bigg(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 k^2$$

$$+ \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) x(x,y) k^2 + \left(\frac{\partial}{\partial x} y(x,y)\right)^2 k^2 + y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) k^2$$

$$+ \left(\frac{\partial}{\partial y} x(x,y)\right)^2 k^2 + \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) x(x,y) k^2 + \left(\frac{\partial}{\partial y} y(x,y)\right)^2 k^2 + y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,$$

$$y)\right) k^2 + 2 k^2 \varepsilon \omega^2 \mu x(x,y)^2 + 2 k^2 \varepsilon \omega^2 \mu y(x,y)^2 - \left(\frac{\partial}{\partial x} x(x,y)\right)^2 \mu \omega^2 \varepsilon - \left(\frac{\partial^2}{\partial x^2} x(x,$$

$$y)\right) x(x,y) \mu \omega^2 \varepsilon - \left(\frac{\partial}{\partial x} y(x,y)\right)^2 \mu \omega^2 \varepsilon - y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) \mu \omega^2 \varepsilon$$

$$- \left(\frac{\partial}{\partial y} x(x,y)\right)^2 \mu \omega^2 \varepsilon - \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) x(x,y) \mu \omega^2 \varepsilon - \left(\frac{\partial}{\partial y} y(x,y)\right)^2 \mu \omega^2 \varepsilon - y(x,$$

$$y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) \mu \omega^2 \varepsilon \Bigg) \Bigg), \frac{1}{\mu k^2} \Bigg(4 \cos(k z + \omega t) \Big(\sin(k z + \omega t) k x(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 \mu \omega^2 \varepsilon + \sin(k z + \omega t) k x(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 \mu \omega^2 \varepsilon + \sin(k z$$

$$+ \omega t) k y(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 \mu \omega^2 \varepsilon + \sin(k z + \omega t) k y(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 \mu \omega^2 \varepsilon$$

$$- 2 \sin(k z + \omega t) k^3 x(x,y)^2 \varepsilon \omega^2 \mu y(x,y)^2 - 2 \sin(k z + \omega t) k x(x,y) \left(\frac{\partial}{\partial x} x(x,$$

$$y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) \mu \omega^2 \varepsilon - 2 \sin(k z + \omega t) k x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,$$

$$y) \left(\frac{\partial}{\partial y} y(x,y)\right) \mu \omega^2 \varepsilon - 2 x(x,y)^3 \cos(k z + \omega t)^2 \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,$$

$$y)\Big)\,\mu^2\,\omega^2\,\varepsilon - 2\,x(x,y)^3\cos(k\,z+\omega\,t)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}\,y(x,y)\right)\mu^2\,\omega^2\,\varepsilon$$

$$-2\,y(x,y)^3\cos(k\,z+\omega\,t)^2\,x(x,y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)\left(\frac{\partial}{\partial x}\,y(x,y)\right)\mu^2\,\omega^2\,\varepsilon - 2\,y(x,$$

$$y)^3\cos(k\,z+\omega\,t)^2\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)\left(\frac{\partial}{\partial y}\,y(x,y)\right)\mu^2\,\omega^2\,\varepsilon - \sin(k\,z$$

$$+\,\omega\,t)\,k^3\,x(x,y)^4\,\varepsilon\,\omega^2\,\mu - \sin(k\,z+\omega\,t)\,k^3\,y(x,y)^4\,\varepsilon\,\omega^2\,\mu - 2\sin(k\,z+\omega\,t)\,k^3\,x(x,$$

$$y)\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right) - 2\sin(k\,z+\omega\,t)\,k^3\,x(x,y)\left(\frac{\partial}{\partial y}\,x(x,y)\right)y(x,$$

$$y)\left(\frac{\partial}{\partial y}\,y(x,y)\right) - x(x,y)^6\,\varepsilon\sin(k\,z+\omega\,t)^2\,\omega^2\,\mu^2\,k^2 - x(x,y)^4\cos(k\,z$$

$$+\,\omega\,t)^2\,\mu^2\,\omega^2\,\varepsilon\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 - x(x,y)^4\cos(k\,z+\omega\,t)^2\,\mu^2\,\omega^2\,\varepsilon\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2$$

$$-\,y(x,y)^6\,\varepsilon\sin(k\,z+\omega\,t)^2\,\omega^2\,\mu^2\,k^2 - y(x,y)^4\cos(k\,z+\omega\,t)^2\,\mu^2\,\omega^2\,\varepsilon\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2$$

$$-\,y(x,y)^4\cos(k\,z+\omega\,t)^2\,\mu^2\,\omega^2\,\varepsilon\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 + \mu\,y(x,y)^2\cos(k\,z+\omega\,t)^2\,k^2\,x(x,$$

$$y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 + \mu\,y(x,y)^2\cos(k\,z+\omega\,t)^2\,k^2\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 + \mu\,x(x,$$

$$y)^2\cos(k\,z+\omega\,t)^2\,k^2\,y(x,y)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 + \mu\,x(x,y)^2\cos(k\,z+\omega\,t)^2\,k^2\,y(x,$$

$$y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 - \sin(k\,z+\omega\,t)\,k^3\,x(x,y)^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)^2 - \sin(k\,z+\omega\,t)\,k^3\,y(x,$$

$$y)^2\left(\frac{\partial}{\partial x}\,y(x,y)\right)^2 - \sin(k\,z+\omega\,t)\,k^3\,x(x,y)^2\left(\frac{\partial}{\partial y}\,x(x,y)\right)^2 - \sin(k\,z+\omega\,t)\,k^3\,y(x,$$

$$y)^2\left(\frac{\partial}{\partial y}\,y(x,y)\right)^2 + \sin(k\,z+\omega\,t)\,k\,y(x,y)^2\left(\frac{\partial^2}{\partial x^2}\,x(x,y)\right)x(x,y)\,\mu\,\omega^2\,\varepsilon + \sin(k\,z$$

$$+\,\omega\,t)\,k\,y(x,y)^2\left(\frac{\partial^2}{\partial y^2}\,x(x,y)\right)x(x,y)\,\mu\,\omega^2\,\varepsilon + \sin(k\,z+\omega\,t)\,k\,x(x,y)^2\,y(x,$$

$$y)\left(\frac{\partial^2}{\partial x^2}\,y(x,y)\right)\mu\,\omega^2\,\varepsilon + \sin(k\,z+\omega\,t)\,k\,x(x,y)^2\,y(x,y)\left(\frac{\partial^2}{\partial y^2}\,y(x,y)\right)\mu\,\omega^2\,\varepsilon$$

$$+\,2\,\mu\,x(x,y)^3\cos(k\,z+\omega\,t)^2\,k^2\left(\frac{\partial}{\partial x}\,x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}\,y(x,y)\right) + 2\,\mu\,x(x,$$

$$y)^3 \cos(k z+\omega t)^2 k^2 \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) + 2 \mu y(x,y)^3 \cos(k z$$

$$+ \omega t)^2 k^2 x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) \left(\frac{\partial}{\partial x} y(x,y)\right) + 2 \mu y(x,y)^3 \cos(k z+\omega t)^2 k^2 x(x,$$

$$y) \left(\frac{\partial}{\partial y} x(x,y)\right) \left(\frac{\partial}{\partial y} y(x,y)\right) - 3 x(x,y)^4 \varepsilon \sin(k z+\omega t)^2 \omega^2 \mu^2 k^2 y(x,y)^2 - 3 x(x,$$

$$y)^2 \varepsilon \sin(k z+\omega t)^2 \omega^2 \mu^2 k^2 y(x,y)^4 - x(x,y)^2 \cos(k z+\omega t)^2 \mu^2 \omega^2 \varepsilon y(x,$$

$$y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 - y(x,y)^2 \cos(k z+\omega t)^2 \mu^2 \omega^2 \varepsilon x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 - y(x,$$

$$y)^2 \cos(k z+\omega t)^2 \mu^2 \omega^2 \varepsilon x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 - x(x,y)^2 \cos(k z$$

$$+ \omega t)^2 \mu^2 \omega^2 \varepsilon y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + \sin(k z+\omega t)\, k\, x(x,y)^3 \left(\frac{\partial^2}{\partial y^2} x(x,y)\right) \mu \omega^2 \varepsilon$$

$$+ \sin(k z+\omega t)\, k\, x(x,y)^3 \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) \mu \omega^2 \varepsilon + \sin(k z+\omega t)\, k\, y(x,y)^3 \left(\frac{\partial^2}{\partial x^2} y(x,$$

$$y)\right) \mu \omega^2 \varepsilon + \sin(k z+\omega t)\, k\, y(x,y)^3 \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) \mu \omega^2 \varepsilon + \mu x(x,y)^4 \cos(k z$$

$$+ \omega t)^2 k^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + \mu y(x,y)^4 \cos(k z+\omega t)^2 k^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + \mu x(x,$$

$$y)^4 \cos(k z+\omega t)^2 k^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + \mu y(x,y)^4 \cos(k z+\omega t)^2 k^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 \Big)\Big)\Big]$$

$$Dissipation = -\frac{1}{k}\Bigg(2 \varepsilon \cos(k z+\omega t)\, \omega \Bigg(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right)$$

$$+ \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$+ \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + y(x,y) \left(\frac{\partial^2}{\partial y^2} y(x,y)\right) - x(x,y)^2 k^2 - k^2 y(x,y)^2 - \mu \sin(k z$$

$$+ \omega t)\, k\, x(x,y)^4 - 2 \mu \sin(k z+\omega t)\, k\, x(x,y)^2 y(x,y)^2 - \mu \sin(k z+\omega t)\, k\, y(x,y)^4\Bigg)\Bigg)$$

****************************** *END PROCEDURE* **************************** **(29)**

Enter the name of the problem, and the components of the 4 potential.

```
> NAME:=`Example 6d-- Wave guide TTM (kinematic out, wave out)`;
> theta:=(k*z-omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=+(omega/k)*f(x,y)*cos(theta);
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 6d-- Wave guide TTM (kinematic out, wave out)

$$\theta := k\,z - \omega\,t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z - \omega\,t)$$

$$\phi := \frac{\omega\left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z - \omega\,t)}{k}$$

Example 6d-- Wave guide TTM (kinematic out, wave out)

********************************** *Differential Form Format* **********************************

$$\text{Action 1-form} = \left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z - \omega\,t)\;d(z) - \frac{\omega\left(x(x,y)^2 + y(x,y)^2\right)\cos(k\,z - \omega\,t)\;d(t)}{k}$$

$$\text{Intensity 2-form } F{=}dA \;= 2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z - \omega\,t)\;(d(x))\;\&\wedge\;(d(z)) + 2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z - \omega\,t)\;(d(y))\;\&\wedge\;(d(z)) - \frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z - \omega\,t)\;(d(x))\;\&\wedge\;(d(t))}{k} - \frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z - \omega\,t)\;(d(y))\;\&\wedge\;(d(t))}{k}$$

Topological Torsion 3-form $A\wedge F = 0$

Topological Parity 4-form $F\wedge F = 0$

********************************** *Using EM format* *********************************

$$E\ field = \left[-\frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z - \omega\,t)}{k}, \right.$$

$$-\frac{2\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{k},0\right]$$

$$B\,field=\left[2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z-\omega\,t),\ -2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z-\omega\,t),0\right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension PTD = 2

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ****** ********************

Xm or linear (Mean) curvature =

$$-\frac{\left(x(x,y)^2+y(x,y)^2\right)\sin(k\,z-\omega\,t)\left(k^2+\omega^2\right)}{k}$$

Yg or quadratic (GAUSS) curvature = 0

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

$$D\,field=\left[-\frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{k},\ -\frac{2\,\varepsilon\,\omega\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{k},0\right]$$

$$H\,field=\left[\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{\mu},\ -\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\cos(k\,z-\omega\,t)}{\mu},0\right]$$

$$Poynting\ vector\ ExH=\left[0,0,\frac{1}{k\,\mu}\left(4\,\omega\cos(k\,z-\omega\,t)^2\left(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,\right.\right.\right.$$

$y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2$

$+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Big)\Big)\Big]$

Amperian Current 4Vector   curlH-dD/dt=J4 $=\Big[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz-\omega t)\left(k^2-\mu\,\omega^2\,\varepsilon\right)}{k\,\mu},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\sin(kz-\omega t)\left(k^2-\mu\,\omega^2\,\varepsilon\right)}{k\,\mu},$$

$-\frac{1}{\mu}\Bigg(2\cos(kz-\omega t)\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,$

$y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,$

$y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg),\ -\frac{1}{k}\Bigg(2\,\varepsilon\cos(kz-\omega t)\,\omega\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,$

$y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$

$+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg)\Big]$

Amerian charge density   divD = rho $=-\frac{1}{k}\Bigg(2\,\varepsilon\cos(kz-\omega t)\,\omega\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,$

$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,$

$y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg)$

divergence Lorentz Current 4Vector,   4div(J4) $=0$

Topological SPIN 4 vector S4 $=\Big[\frac{1}{\mu\,k^2}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(kz-\omega t)^2\Big(x(x,$

$$y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Big),\ \frac{1}{\mu\,k^2}\Big(2\left(x(x,y)^2\right.$$

$$\left.+y(x,y)^2\right)\cos(k\,z-\omega\,t)^2\Big(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Big)\Big(k^2$$

$$-\mu\,\omega^2\,\varepsilon\Big)\Big),0,0\Big]$$

$$\textit{Topological SPIN 3-form}=\frac{1}{\mu\,k^2}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)^2\Big(x(x,y)\left(\frac{\partial}{\partial x}x(x,\right.$$

$$\left.y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\ \&\wedge(d(y),d(z),d(t))\Big)$$

$$-\frac{1}{\mu\,k^2}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)^2\Big(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,$$

$$y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\ \&\wedge(d(x),d(z),d(t))\Big)$$

$$\textit{Spin density rho\_spin}=0$$

$$\textit{LaGrange field energy density (B.H-D.E)}=\frac{1}{\mu\,k^2}\Big(4\cos(k\,z-\omega\,t)^2\Big(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,$$

$$y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,$$

$$y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Big)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Big)$$

$$B.H=\frac{1}{\mu}\Big(4\cos(k\,z-\omega\,t)^2\Big(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,$$

$$y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,$$

$$y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Big)\Big)$$

$$D.E=\frac{1}{k^2}\Big(4\,\varepsilon\,\omega^2\cos(k\,z-\omega\,t)^2\Big(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,$$

$$y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,$$

$$y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Big)\Big)$$

$$A.J=-\frac{1}{\mu}\Big(2\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)^2\Big(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$
$$+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg)$$

-rho.phi $= -\frac{1}{k^2}\Bigg(2\,\omega^2\left(x(x,y)^2+y(x,y)^2\right)\cos(k\,z-\omega\,t)^2\,\varepsilon\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,$
$y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,$
$y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg)$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* $=\frac{1}{\mu\,k^2}\Bigg(2\cos(k\,z-\omega\,t)^2\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Bigg(3\,x(x,$
$y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+4\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+3\,y(x,$
$y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2+3\,x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+4\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,\right.$
$\left.y)\right)+3\,y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,y)^3\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+x(x,y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2$
$+x(x,y)^2y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+x(x,y)^3\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+x(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2$
$+x(x,y)^2y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+y(x,y)^2x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,\right.$
$\left.y)\right)+y(x,y)^3\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2+y(x,y)^2x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$
$+y(x,y)^3\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg)$

London Coefficient $LC=\frac{k^2-\mu\,\omega^2\,\varepsilon}{\mu}$

PROCA coefficient curlcurlB $=\Bigg[-2\cos(k\,z-\omega\,t)\Bigg(2\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}x(x,y)\right)$
$+\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+x(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2}x(x,y)\right)+2\left(\frac{\partial}{\partial x}y(x,\right.$
$\left.y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}y(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+y(x,y)\left(\frac{\partial^3}{\partial y\,\partial x^2}y(x,y)\right)$

$$+3\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+x(x,y)\left(\frac{\partial^3}{\partial y^3}x(x,y)\right)+3\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+y(x,y)\left(\frac{\partial^3}{\partial y^3}y(x,y)\right)-k^2x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)-k^2y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\Bigg),$$

$$2\cos(kz-\omega t)\Bigg(-k^2x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)-k^2y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+3\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+x(x,y)\left(\frac{\partial^3}{\partial x^3}x(x,y)\right)+3\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+y(x,y)\left(\frac{\partial^3}{\partial x^3}y(x,y)\right)+2\left(\frac{\partial}{\partial y}x(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}x(x,y)\right)+\left(\frac{\partial}{\partial x}x(x,y)\right)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$

$$+x(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}x(x,y)\right)+2\left(\frac{\partial}{\partial y}y(x,y)\right)\left(\frac{\partial^2}{\partial y\,\partial x}y(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)+y(x,y)\left(\frac{\partial^3}{\partial y^2\,\partial x}y(x,y)\right)\Bigg),0\Bigg]$$

Amperian Current 4Vector *curlH-dD/dt=J4* $=\Bigg[$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)\right)\sin(kz-\omega t)\left(k^2-\mu\,\omega^2\varepsilon\right)}{k\,\mu},$$

$$-\frac{2\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)\right)\sin(kz-\omega t)\left(k^2-\mu\,\omega^2\varepsilon\right)}{k\,\mu},$$

$$-\frac{1}{\mu}\Bigg(2\cos(kz-\omega t)\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg),$$

$$-\frac{1}{k}\Bigg(2\,\varepsilon\cos(kz-\omega t)\,\omega\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial y} y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right)\Bigg)\Bigg)\Bigg]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B)

$$= \Bigg[\frac{1}{\mu k^2}\Bigg(4\cos(kz-\omega t)^2\Bigg(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + \left(\frac{\partial}{\partial x} y(x,y)\right)^2$$

$$+ y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right) + \left(\frac{\partial}{\partial y} y(x,y)\right)^2$$

$$+ y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right)\Bigg)\left(x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial x} y(x,y)\right)\right)(k^2$$

$$-\mu\omega^2\varepsilon)\Bigg), \frac{1}{\mu k^2}\Bigg(4\cos(kz-\omega t)^2\Bigg(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial x^2} x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial x} y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + x(x,y)\left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial y} y(x,y)\right)^2 + y(x,y)\left(\frac{\partial^2}{\partial y^2} y(x,y)\right)\Bigg)\left(x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y)\left(\frac{\partial}{\partial y} y(x,\right.\right.$$

$$\left.\left.y)\right)\right)(k^2-\mu\omega^2\varepsilon)\Bigg), -\frac{1}{k\mu}\Bigg(4\sin(kz-\omega t)(k^2-\mu\omega^2\varepsilon)\cos(kz-\omega t)\Bigg(x(x,$$

$$y)^2\left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2x(x,y)\left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial y} y(x,y)\right) + y(x,$$

$$y)^2\left(\frac{\partial}{\partial y} y(x,y)\right)^2 + x(x,y)^2\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2x(x,y)\left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y)\left(\frac{\partial}{\partial x} y(x,\right.$$

$$\left.y)\right) + y(x,y)^2\left(\frac{\partial}{\partial x} y(x,y)\right)^2\Bigg)\Bigg)\Bigg]$$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $=0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \Big[\frac{1}{\mu k^2} \Big(2 \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z - \omega t)^2 \Big(x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right)\Big) \left(k^2 - \mu \omega^2 \varepsilon\right)\Big),$

$\frac{1}{\mu k^2} \Big(2 \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z - \omega t)^2 \Big(x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) + y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)\Big) \left(k^2 - \mu \omega^2 \varepsilon\right)\Big), 0, 0\Big]$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= \Big[0, 0,$

$\frac{1}{\mu k^2} \Big(4 \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z - \omega t)^3 \left(k^2 - \mu \omega^2 \varepsilon\right) \Big(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2$

$+ 2\, x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right) + y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\, x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 \Big)\Big)\Big]$

Spin Dissipation J_spin dot E $= - \frac{1}{\mu k^3} \Big(4 \left(x(x,y)^2 + y(x,y)^2\right) \cos(k z - \omega t)^3 \left(k^2 - \mu \omega^2 \varepsilon\right) \omega \Big(x(x,y)^2 \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + 2\, x(x,y) \left(\frac{\partial}{\partial y} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial y} y(x,y)\right)$

$+ y(x,y)^2 \left(\frac{\partial}{\partial y} y(x,y)\right)^2 + x(x,y)^2 \left(\frac{\partial}{\partial x} x(x,y)\right)^2 + 2\, x(x,y) \left(\frac{\partial}{\partial x} x(x,y)\right) y(x,y) \left(\frac{\partial}{\partial x} y(x,y)\right) + y(x,y)^2 \left(\frac{\partial}{\partial x} y(x,y)\right)^2 \Big)\Big)$

Disspative Force 3 vector $= \Big[\frac{1}{\mu k^2} \Big(4 \cos(k z - \omega t)^2 \Big(\left(\frac{\partial}{\partial x} x(x,y)\right)^2 + x(x,y) \left(\frac{\partial^2}{\partial x^2} x(x,y)\right) + \left(\frac{\partial}{\partial x} y(x,y)\right)^2 + y(x,y) \left(\frac{\partial^2}{\partial x^2} y(x,y)\right) + \left(\frac{\partial}{\partial y} x(x,y)\right)^2 + x(x,y) \left(\frac{\partial^2}{\partial y^2} x(x,y)\right)$

$$+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg)\left(x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial x}y(x,\right.\right.$$

$$\left.\left.y)\right)\right)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Bigg),\ \frac{1}{\mu\,k^2}\Bigg(4\cos(k\,z-\omega\,t)^2\Bigg(\left(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,\right.\right.$$

$$\left.\left.y)\right)+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg)\left(x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)+y(x,y)\left(\frac{\partial}{\partial y}y(x,\right.\right.$$

$$\left.\left.y)\right)\right)\left(k^2-\mu\,\omega^2\,\varepsilon\right)\Bigg),\ \frac{1}{\mu\,k^2}\Bigg(4\left(k^2-\mu\,\omega^2\,\varepsilon\right)\cos(k\,z-\omega\,t)\Bigg(x(x,y)^2\left(\frac{\partial}{\partial y}x(x,y)\right)^2$$

$$+2\,x(x,y)\left(\frac{\partial}{\partial y}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial y}y(x,y)\right)+y(x,y)^2\left(\frac{\partial}{\partial y}y(x,y)\right)^2+x(x,$$

$$y)^2\left(\frac{\partial}{\partial x}x(x,y)\right)^2+2\,x(x,y)\left(\frac{\partial}{\partial x}x(x,y)\right)y(x,y)\left(\frac{\partial}{\partial x}y(x,y)\right)+y(x,$$

$$y)^2\left(\frac{\partial}{\partial x}y(x,y)\right)^2\Bigg)\left(-\sin(k\,z-\omega\,t)\,k+\cos(k\,z-\omega\,t)^2\,\mu\,x(x,y)^2+\cos(k\,z\right.$$

$$\left.-\,\omega\,t)^2\,\mu\,y(x,y)^2\right)\Bigg)\Bigg]$$

$$Dissipation\ =-\frac{1}{k}\Bigg(2\,\varepsilon\cos(k\,z-\omega\,t)\,\omega\Bigg(\left(\frac{\partial}{\partial x}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial x^2}x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial x}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial x^2}y(x,y)\right)+\left(\frac{\partial}{\partial y}x(x,y)\right)^2+x(x,y)\left(\frac{\partial^2}{\partial y^2}x(x,y)\right)$$

$$+\left(\frac{\partial}{\partial y}y(x,y)\right)^2+y(x,y)\left(\frac{\partial^2}{\partial y^2}y(x,y)\right)\Bigg)\Bigg)$$

***************************** *END PROCEDURE* ***************************** **(30)**

****

Enter the name of the problem, and the components of the 4 potential
p-2, n=4

```
> NAME:=`Example 7a = Index 1 Irreversible solution EdotB < 0 (kinematic out)
  Type 1`;
> Holder:=(x^2+y^2+z^2-c^2*t^2)^(4/2);
```

```
> Ax:=y/Holder;Ay:=-x/Holder;Az:=c*t/Holder;phi:=+c*z/Holder;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

NAME := Example 7a = Index 1 Irreversible solution EdotB < 0 (kinematic out) Type 1

$$Holder := (x^2+y^2+z^2-c^2t^2)^2$$

$$Ax := \frac{y}{(x^2+y^2+z^2-c^2t^2)^2}$$

$$Ay := -\frac{x}{(x^2+y^2+z^2-c^2t^2)^2}$$

$$Az := \frac{c\,t}{(x^2+y^2+z^2-c^2t^2)^2}$$

$$\phi := \frac{c\,z}{(x^2+y^2+z^2-c^2t^2)^2}$$

Example 7a = Index 1 Irreversible solution EdotB < 0 (kinematic out) Type 1

************************************ *Differential Form Format* *********************************

$$\text{Action 1-form} = -\frac{c\,z\,d(t)}{(-x^2-y^2-z^2+c^2t^2)^2}+\frac{y\,d(x)}{(-x^2-y^2-z^2+c^2t^2)^2}-\frac{x\,d(y)}{(-x^2-y^2-z^2+c^2t^2)^2}+\frac{c\,t\,d(z)}{(-x^2-y^2-z^2+c^2t^2)^2}$$

$$\text{Intensity 2-form } F=dA = \left(-\frac{4\,c\,z\,x}{(-x^2-y^2-z^2+c^2t^2)^3}+\frac{4\,y\,c^2\,t}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(x))\ \&\wedge\ (d(t)) + \left(-\frac{3\,x^2-y^2-z^2+c^2t^2}{(-x^2-y^2-z^2+c^2t^2)^3}-\frac{-x^2+3\,y^2-z^2+c^2t^2}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(x))\ \&\wedge\ (d(y)) + \left(-\frac{4\,y\,z}{(-x^2-y^2-z^2+c^2t^2)^3}+\frac{4\,c\,t\,x}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(x))\ \&\wedge\ (d(z)) + \left(-\frac{4\,c\,z\,y}{(-x^2-y^2-z^2+c^2t^2)^3}-\frac{4\,x\,c^2\,t}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(y))\ \&\wedge\ (d(t)) + \left(\frac{4\,x\,z}{(-x^2-y^2-z^2+c^2t^2)^3}+\frac{4\,c\,t\,y}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(y))\ \&\wedge\ (d(z)) + \left(-\frac{c\,(-x^2-y^2+3\,z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3}\right.$$

$$+ \frac{c\,(x^2+y^2+z^2+3\,c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^3}\Bigg)\,(d(z))\ \&\hat{}\ (d(t))$$

Topological Torsion 3-form $A\hat{}F = \Bigg(\frac{2\,c\,z\,(x^2+y^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5} + \frac{4\,x\,c\,(c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2\,t^2)^5}$

$$- \frac{4\,y\,c\,(c\,t\,x+y\,z)}{(-x^2-y^2-z^2+c^2\,t^2)^5}\Bigg)\,\&\hat{}(d(x),d(y),d(t)) + \Bigg(-\frac{2\,c\,t\,(x^2+y^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5}$$

$$+ \frac{4\,x\,(-y\,z+c\,t\,x)}{(-x^2-y^2-z^2+c^2\,t^2)^5} + \frac{4\,y\,(c\,t\,y+x\,z)}{(-x^2-y^2-z^2+c^2\,t^2)^5}\Bigg)\,\&\hat{}(d(x),d(y),d(z))$$

$$+ \Bigg(\frac{2\,y\,c\,(x^2+y^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5} - \frac{4\,c\,z\,(-y\,z+c\,t\,x)}{(-x^2-y^2-z^2+c^2\,t^2)^5}$$

$$- \frac{4\,c^2\,t\,(c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2\,t^2)^5}\Bigg)\,\&\hat{}(d(x),d(z),d(t)) + \Bigg(\frac{4\,c^2\,t\,(c\,t\,x+y\,z)}{(-x^2-y^2-z^2+c^2\,t^2)^5}$$

$$- \frac{2\,x\,c\,(x^2+y^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5} - \frac{4\,c\,z\,(c\,t\,y+x\,z)}{(-x^2-y^2-z^2+c^2\,t^2)^5}\Bigg)\,\&\hat{}(d(y),d(z),d(t))$$

Topological Parity 4-form $F\hat{}F = -\frac{8\,c\,\&\hat{}(d(x),d(y),d(z),d(t))}{(-x^2-y^2-z^2+c^2\,t^2)^4}$

******************************** *Using EM format* ****************************
****

E field $= \left[\frac{4\,c\,(c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2\,t^2)^3},\ -\frac{4\,c\,(c\,t\,x+y\,z)}{(-x^2-y^2-z^2+c^2\,t^2)^3},\ \frac{2\,c\,(x^2+y^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^3}\right]$

B field $= \left[\frac{4\,(c\,t\,y+x\,z)}{(-x^2-y^2-z^2+c^2\,t^2)^3},\ -\frac{4\,(-y\,z+c\,t\,x)}{(-x^2-y^2-z^2+c^2\,t^2)^3},\ -\frac{2\,(x^2+y^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^3}\right]$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= \Bigg[\frac{2\,x\,c}{(-x^2-y^2-z^2+c^2\,t^2)^4},$

$$\frac{2\,y\,c}{(-x^2-y^2-z^2+c^2\,t^2)^4},\ \frac{2\,c\,z}{(-x^2-y^2-z^2+c^2\,t^2)^4},\ \frac{2\,c\,t}{(-x^2-y^2-z^2+c^2\,t^2)^4}\Bigg]$$

Helicity AdotB $= -\frac{2\,c\,t}{(-x^2-y^2-z^2+c^2\,t^2)^4}$

Poincare II =2(E.B) $= -\frac{8\,c}{(-x^2-y^2-z^2+c^2\,t^2)^4}$

coefficient of Topological Parity 4-form $= -\frac{8\,c}{(-x^2-y^2-z^2+c^2\,t^2)^4}$

Pfaff Topological Dimension PTD = 4

******************** Correlation Similarity Invariants of Jacobian of (Ak/lambda_N) ****** ***********************

$$\textit{Xm or linear (Mean) curvature} = \frac{4\,c\,t\,z\,(1+c^2)}{(-x^2-y^2-z^2+c^2t^2)^3}$$

$$\textit{Yg or quadratic (GAUSS) curvature} = -\frac{3c^4t^2-c^2t^2-3c^2z^2+c^2x^2+c^2y^2-3x^2+z^2-3y^2}{(-x^2-y^2-z^2+c^2t^2)^5}$$

$$\textit{Za or Cubic (Interaction internal energy) curvature} = \frac{4\,c\,t\,z\,(1+c^2)}{(-x^2-y^2-z^2+c^2t^2)^7}$$

$$\textit{Tk or quartic (4D expansion) curvature} = -\frac{3c^2}{(-x^2-y^2-z^2+c^2t^2)^8}$$

*********** Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH *************

$$\textit{Chirality factor CH} = 0$$

$$\textit{D field} = \left[\frac{4\,\varepsilon\,c\,(c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2t^2)^3}, -\frac{4\,\varepsilon\,c\,(c\,t\,x+y\,z)}{(-x^2-y^2-z^2+c^2t^2)^3}, \frac{2\,\varepsilon\,c\,(x^2+y^2-z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3}\right]$$

$$\textit{H field} = \left[\frac{4\,(c\,t\,y+x\,z)}{(-x^2-y^2-z^2+c^2t^2)^3\,\mu}, -\frac{4\,(-y\,z+c\,t\,x)}{(-x^2-y^2-z^2+c^2t^2)^3\,\mu}, -\frac{2\,(x^2+y^2-z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3\,\mu}\right]$$

Poynting vector ExH

$$= \left[\frac{16\,c^2\,(x^2+y^2-z^2+c^2t^2)\,t\,x}{(-x^2-y^2-z^2+c^2t^2)^6\,\mu}, \frac{16\,c^2\,(x^2+y^2-z^2+c^2t^2)\,t\,y}{(-x^2-y^2-z^2+c^2t^2)^6\,\mu}, \frac{32\,c^2\,t\,z\,(x^2+y^2)}{(-x^2-y^2-z^2+c^2t^2)^6\,\mu}\right]$$

Amperian Current 4Vector curlH-dD/dt=J4

$$= \left[\frac{4\,(y\,x^2+y^3+y\,z^2+5\,y\,c^2t^2-6\,c\,t\,x\,z)\,(\varepsilon\,c^2\mu-1)}{(-x^2-y^2-z^2+c^2t^2)^4\,\mu}, -\frac{4\,(x^3+x\,y^2+x\,z^2+5\,c^2t^2x+6\,c\,t\,y\,z)\,(\varepsilon\,c^2\mu-1)}{(-x^2-y^2-z^2+c^2t^2)^4\,\mu}, \frac{8\,c\,t\,(2x^2+2y^2-z^2+c^2t^2)\,(\varepsilon\,c^2\mu-1)}{(-x^2-y^2-z^2+c^2t^2)^4\,\mu}, 0\right]$$

Amerian charge density $\quad$ *divD = rho* = 0

divergence Lorentz Current 4Vector, $\quad$ *4div(J4)* = 0

Topological SPIN 4 vector S4

$$= \left[\frac{2\,(x^3+x\,y^2-x\,z^2+3\,c^2\,t^2\,x-2\,c\,t\,y\,z+2\,\varepsilon\,c^3\,\mu\,t\,y\,z-2\,\varepsilon\,c^2\,\mu\,x\,z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu}, \right.$$

$$-\frac{2\,(-3\,y\,c^2\,t^2-2\,c\,t\,x\,z-y\,x^2-y^3+y\,z^2+2\,\varepsilon\,c^3\,\mu\,t\,x\,z+2\,\varepsilon\,c^2\,\mu\,y\,z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu},$$

$$\frac{2\,z\,(2\,y^2+2\,x^2+\varepsilon\,c^2\,\mu\,x^2+\varepsilon\,c^2\,\mu\,y^2-\varepsilon\,c^2\,z^2\,\mu+\varepsilon\,c^4\,\mu\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu},$$

$$\left. \frac{2\,\varepsilon\,c^2\,t\,(3\,y^2+3\,x^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5} \right]$$

Topological SPIN 3-form

$$= \frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu} \big(2\,(x^3+x\,y^2-x\,z^2+3\,c^2\,t^2\,x-2\,c\,t\,y\,z+2\,\varepsilon\,c^3\,\mu\,t\,y\,z$$

$$-2\,\varepsilon\,c^2\,\mu\,x\,z^2)\ \&\wedge(d(y),d(z),d(t))\big)$$

$$+\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu} \big(2\,(-3\,y\,c^2\,t^2-2\,c\,t\,x\,z-y\,x^2-y^3+y\,z^2+2\,\varepsilon\,c^3\,\mu\,t\,x\,z$$

$$+2\,\varepsilon\,c^2\,\mu\,y\,z^2)\ \&\wedge(d(x),d(z),d(t))\big)$$

$$+\frac{2\,z\,(2\,y^2+2\,x^2+\varepsilon\,c^2\,\mu\,x^2+\varepsilon\,c^2\,\mu\,y^2-\varepsilon\,c^2\,z^2\,\mu+\varepsilon\,c^4\,\mu\,t^2)\ \&\wedge(d(x),d(y),d(t))}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu}$$

$$-\frac{2\,\varepsilon\,c^2\,t\,(3\,y^2+3\,x^2-z^2+c^2\,t^2)\ \&\wedge(d(x),d(y),d(z))}{(-x^2-y^2-z^2+c^2\,t^2)^5}$$

$$\textit{Spin density rho\_spin} = \frac{2\,\varepsilon\,c^2\,t\,(3\,y^2+3\,x^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5}$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu} \big(4\,(6\,y^2\,c^2\,t^2+2\,x^2\,z^2$$

$$+2\,y^2\,z^2+6\,x^2\,c^2\,t^2+x^4+2\,x^2\,y^2+y^4+z^4-2\,c^2\,t^2\,z^2+c^4\,t^4)\,(\varepsilon\,c^2\,\mu-1)\big)$$

$$B.H = \frac{4\,(6\,y^2\,c^2\,t^2+2\,x^2\,z^2+2\,y^2\,z^2+6\,x^2\,c^2\,t^2+x^4+2\,x^2\,y^2+y^4+z^4-2\,c^2\,t^2\,z^2+c^4\,t^4)}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}$$

D.E

$$= \frac{4\,\varepsilon\,c^2\,(6\,y^2\,c^2\,t^2+2\,x^2\,z^2+2\,y^2\,z^2+6\,x^2\,c^2\,t^2+x^4+2\,x^2\,y^2+y^4+z^4-2\,c^2\,t^2\,z^2+c^4\,t^4)}{(-x^2-y^2-z^2+c^2\,t^2)^6}$$

A.J

$$= \frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^6 \mu} \left(4 \left(\varepsilon c^2 \mu - 1\right) \left(2 x^2 y^2 + y^4 + y^2 z^2 + 9 y^2 c^2 t^2 + x^4 + x^2 z^2 + 9 x^2 c^2 t^2 - 2 c^2 t^2 z^2 + 2 c^4 t^4\right)\right)$$

-rho.phi $= 0$

Poincare I *(B.H - D.E)-(A.J - rho.phi)*

$$= -\frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^6 \mu} \left(4 \left(\varepsilon c^2 \mu - 1\right) \left(15 y^2 c^2 t^2 + 3 x^2 z^2 + 3 y^2 z^2 + 15 x^2 c^2 t^2 + 2 x^4 + 4 x^2 y^2 + 2 y^4 + z^4 - 4 c^2 t^2 z^2 + 3 c^4 t^4\right)\right)$$

London Coefficient *LC* $= 0$

PROCA coefficient curlcurlB =

$$\left[-\frac{24 \left(3 c t y x^2 + 3 c t y z^2 + 7 x z c^2 t^2 + 3 c t y^3 + 5 c^3 t^3 y + x^3 z + x z^3 + y^2 z x\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5}, \frac{24 \left(-7 y z c^2 t^2 + 3 z^2 c t x + 3 c t x y^2 - y z x^2 + 3 c t x^3 + 5 c^3 t^3 x - y^3 z - y z^3\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5}, \frac{8 \left(2 x^4 + 4 x^2 y^2 + 2 y^4 - 4 c^2 t^2 z^2 + 17 x^2 c^2 t^2 + 17 y^2 c^2 t^2 - z^4 + x^2 z^2 + y^2 z^2 + 5 c^4 t^4\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5} \right]$$

Amperian Current 4Vector *curlH-dD/dt=J4*

$$= \left[\frac{4 \left(y x^2 + y^3 + y z^2 + 5 y c^2 t^2 - 6 c t x z\right) \left(\varepsilon c^2 \mu - 1\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^4 \mu}, -\frac{4 \left(x^3 + x y^2 + x z^2 + 5 c^2 t^2 x + 6 c t y z\right) \left(\varepsilon c^2 \mu - 1\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^4 \mu}, \frac{8 c t \left(2 x^2 + 2 y^2 - z^2 + c^2 t^2\right) \left(\varepsilon c^2 \mu - 1\right)}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^4 \mu}, 0 \right]$$

Lorentz Force 3 vector due to Ampere current *FL = -(rho_ampere E + J_ampere x B)* =

$$\left[-\frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^7 \mu} \left(8 \left(\varepsilon c^2 \mu - 1\right) \left(x^5 + 2 x^3 y^2 + 14 x^3 c^2 t^2 + x y^4 + 14 x y^2 c^2 t^2 - x z^4 - 8 x z^2 c^2 t^2 + 9 c^4 t^4 x - 2 c t y z x^2 - 2 c t y^3 z - 2 c t y z^3 + 2 c^3 t^3 y z\right)\right), \right.$$

$$-\frac{1}{(-x^2-y^2-z^2+c^2t^2)^7\mu}\Big(8(\varepsilon c^2\mu-1)(14c^2t^2yx^2+2ctx^3z+14c^2t^2y^3$$

$$+2cty^2zx-8c^2t^2yz^2+2ctxz^3+9c^4t^4y-2c^3t^3xz+yx^4+2y^3x^2+y^5-yz^4)\Big),$$

$$-\frac{16(\varepsilon c^2\mu-1)z(x^2+y^2)(x^2+y^2+11c^2t^2+z^2)}{(-x^2-y^2-z^2+c^2t^2)^7\mu}\Bigg]$$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB]

$$=\left[\frac{2xc}{(-x^2-y^2-z^2+c^2t^2)^4},\frac{2yc}{(-x^2-y^2-z^2+c^2t^2)^4},\frac{2cz}{(-x^2-y^2-z^2+c^2t^2)^4},\right.$$

$$\left.\frac{2ct}{(-x^2-y^2-z^2+c^2t^2)^4}\right]$$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B)

$$=\left[\frac{4yc}{(-x^2-y^2-z^2+c^2t^2)^6},-\frac{4xc}{(-x^2-y^2-z^2+c^2t^2)^6},\frac{4c^2t}{(-x^2-y^2-z^2+c^2t^2)^6}\right]$$

Torsion Dissipation Jtorsion dot E $=-\frac{4c^2z}{(-x^2-y^2-z^2+c^2t^2)^6}$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD]

$$=\left[\frac{2(x^3+xy^2-xz^2+3c^2t^2x-2ctyz+2\varepsilon c^3\mu tyz-2\varepsilon c^2\mu xz^2)}{(-x^2-y^2-z^2+c^2t^2)^5\mu},\right.$$

$$-\frac{2(-3yc^2t^2-2ctxz-yx^2-y^3+yz^2+2\varepsilon c^3\mu txz+2\varepsilon c^2\mu yz^2)}{(-x^2-y^2-z^2+c^2t^2)^5\mu},$$

$$\frac{2z(2y^2+2x^2+\varepsilon c^2\mu x^2+\varepsilon c^2\mu y^2-\varepsilon c^2z^2\mu+\varepsilon c^4\mu t^2)}{(-x^2-y^2-z^2+c^2t^2)^5\mu},$$

$$\left.\frac{2\varepsilon c^2t(3y^2+3x^2-z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5}\right]$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) $=\Big[$

$$- \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^8 \mu} (4 (-y^5 - 2 \varepsilon c^3 \mu t x^3 z - 2 \varepsilon c^3 \mu t x z^3 + 2 \varepsilon c^5 \mu t^3 x z$$

$$- 2 \varepsilon c^4 \mu y z^2 t^2 + 6 \varepsilon c^4 \mu t^2 y x^2 - 2 \varepsilon c^3 \mu t x z y^2 - 4 c^2 t^2 y^3 - 3 c^4 t^4 y - 2 y x^2 z^2$$

$$+ 2 c t y^2 z x + 6 \varepsilon c^4 \mu t^2 y^3 + 2 \varepsilon c^6 \mu t^4 y - y x^4 - 2 y^3 x^2 - 2 y^3 z^2 - y z^4 - 4 c^2 t^2 y x^2$$

$$+ 2 c t x^3 z + 4 c^2 t^2 y z^2 + 2 c t x z^3 - 2 c^3 t^3 x z)), \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^8 \mu} (4 ($$

$$-4 x y^2 c^2 t^2 + 4 x z^2 c^2 t^2 - 2 c t y^3 z - 2 c t y z^3 + 2 c^3 t^3 y z - x^5 + 6 \varepsilon c^4 \mu t^2 x^3 + 2 \varepsilon c^6 \mu t^4 x$$

$$+ 2 z \varepsilon c^3 \mu x^2 t y - 2 x^3 y^2 - 2 x^3 z^2 - x y^4 - x z^4 - 4 x^3 c^2 t^2 - 2 x y^2 z^2 - 3 c^4 t^4 x$$

$$+ 2 z \varepsilon c^3 \mu y^3 t + 2 \varepsilon c^3 z^3 \mu t y - 2 z \varepsilon c^5 \mu t^3 y - 2 \varepsilon c^4 \mu t^2 x z^2 - 2 c t y z x^2$$

$$+ 6 \varepsilon c^4 \mu t^2 x y^2)), - \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^8 \mu} (4 c t (6 \varepsilon c^2 \mu x^2 y^2 + 3 \varepsilon c^2 \mu y^4$$

$$+ 4 \varepsilon c^2 \mu y^2 z^2 + 4 \varepsilon c^4 \mu y^2 t^2 + 3 \varepsilon c^2 \mu x^4 + 4 \varepsilon c^2 \mu x^2 z^2 + 4 \varepsilon c^4 \mu x^2 t^2 + \varepsilon c^2 \mu z^4$$

$$- 2 \varepsilon c^4 \mu t^2 z^2 + \varepsilon c^6 \mu t^4 - 4 x^2 y^2 - 2 x^2 z^2 - 2 y^2 z^2 - 2 x^4 - 6 x^2 c^2 t^2 - 6 y^2 c^2 t^2 - 2 y^4))]$$

Spin Dissipation J_spin dot E $= \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^8 \mu} (4 c^3 z (2 \varepsilon \mu x^2 y^2 + 2 \varepsilon \mu x^2 z^2$

$$+ 2 \varepsilon \mu y^2 z^2 + 6 \varepsilon c^2 \mu t^2 y^2 + 6 \varepsilon c^2 \mu t^2 x^2 + \varepsilon \mu z^4 + \varepsilon \mu y^4 - 2 \varepsilon c^2 \mu t^2 z^2 + \varepsilon c^4 \mu t^4 - 8 y^2 t^2$$

$$- 8 x^2 t^2 + \varepsilon \mu x^4))$$

Disspative Force 3 vector $= \left[- \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^8 \mu} (4 (2 x^7 + 6 \varepsilon c^4 \mu^2 t^2 y^3 \right.$

$$+ 2 \varepsilon c^6 \mu^2 t^4 y - 4 \mu c^2 t^2 y x^2 + 2 \mu c t x^3 z + 4 \mu c^2 t^2 y z^2 + 2 \mu c t x z^3 - 2 \mu c^3 t^3 x z$$

$$- 2 y c \mu x^2 z^2 + 2 y c^3 \mu t^2 z^2 - 14 x z^4 c^2 t^2 + 34 x z^2 c^4 t^4 - 4 c t y z^5 + 8 c^3 t^3 y z^3 - 4 c^5 t^5 y z$$

$$- 4 \mu c^2 t^2 y^3 - 3 \mu c^4 t^4 y - 2 \mu y x^2 z^2 - 2 y^3 c \mu x^2 - y c \mu x^4 + 2 y^3 c^3 \mu t^2 - 2 y^3 c \mu z^2$$

$$- y\, c\, \mu\, z^4 - y\, c^5\, \mu\, t^4 + 6\, x^5\, y^2 + 6\, x^3\, y^4 + 2\, y^6\, x - \mu\, y\, z^4 - y^5\, c\, \mu - 18\, c^6\, t^6\, x - 2\, \mu\, y^3\, z^2$$

$$- \mu\, y\, x^4 - 2\, \mu\, y^3\, x^2 + 4\, c^3\, t\, y\, z^5\, \varepsilon\, \mu - 8\, c^5\, t^3\, y\, z^3\, \varepsilon\, \mu + 4\, c^7\, t^5\, y\, z\, \varepsilon\, \mu - 4\, x^3\, y^2\, \varepsilon\, c^2\, \mu\, z^2$$

$$+ 4\, y^5\, \varepsilon\, c^3\, \mu\, t\, z - 2\, y^4\, \varepsilon\, c^2\, \mu\, x\, z^2 + 8\, y^3\, z^3\, \varepsilon\, c^3\, \mu\, t + 2\, y^2\, z^4\, \varepsilon\, c^2\, \mu\, x - 8\, y^3\, c^5\, t^3\, \varepsilon\, \mu\, z$$

$$- 12\, x^3\, c^4\, t^2\, \varepsilon\, \mu\, z^2 - 52\, x^3\, y^2\, \varepsilon\, c^4\, \mu\, t^2 - 26\, x\, y^4\, \varepsilon\, c^4\, \mu\, t^2 + 10\, x\, y^2\, c^6\, t^4\, \varepsilon\, \mu + 14\, x\, z^4\, \varepsilon\, c^4\, \mu\, t^2$$

$$- 34\, x\, z^2\, c^6\, t^4\, \varepsilon\, \mu + 2\, y\, c^3\, \mu\, x^2\, t^2 - 2\, \varepsilon\, c^3\, \mu^2\, t\, x^3\, z - 2\, \varepsilon\, c^3\, \mu^2\, t\, x\, z^3 + 2\, \varepsilon\, c^5\, \mu^2\, t^3\, x\, z$$

$$- 2\, \varepsilon\, c^4\, \mu^2\, y\, z^2\, t^2 + 6\, \varepsilon\, c^4\, \mu^2\, t^2\, y\, x^2 + 2\, \mu\, c\, t\, y^2\, z\, x - 8\, x^2\, y^3\, c\, t\, z + 12\, y^2\, z^2\, c^2\, t^2\, x$$

$$+ 8\, x^2\, c^3\, t^3\, y\, z - 4\, x^4\, c\, t\, y\, z - 2\, x^5\, \varepsilon\, c^2\, \mu\, z^2 - 8\, x^2\, z^3\, c\, t\, y + 2\, x^3\, z^4\, \varepsilon\, c^2\, \mu - 6\, x^5\, \varepsilon\, c^2\, \mu\, y^2$$

$$- 26\, x^5\, \varepsilon\, c^4\, \mu\, t^2 - 6\, x^3\, y^4\, \varepsilon\, c^2\, \mu + 10\, x^3\, c^6\, t^4\, \varepsilon\, \mu - 2\, x\, y^6\, \varepsilon\, c^2\, \mu + 2\, x\, z^6\, \varepsilon\, c^2\, \mu + 18\, c^8\, t^6\, x\, \varepsilon\, \mu$$

$$+ 8\, x^2\, y^3\, \varepsilon\, c^3\, \mu\, t\, z - 12\, y^2\, c^4\, t^2\, \varepsilon\, \mu\, x\, z^2 - 8\, x^2\, c^5\, t^3\, \varepsilon\, \mu\, y\, z + 4\, x^4\, \varepsilon\, c^3\, \mu\, t\, y\, z + 8\, x^2\, z^3\, \varepsilon\, c^3\, \mu\, t\, y$$

$$- 2\, \varepsilon\, c^3\, \mu^2\, t\, x\, z\, y^2 + 12\, x^3\, c^2\, t^2\, z^2 - 2\, x^7\, \varepsilon\, c^2\, \mu + 52\, x^3\, y^2\, c^2\, t^2 + 26\, y^4\, c^2\, t^2\, x - 4\, y^5\, c\, t\, z$$

$$- 8\, y^3\, z^3\, c\, t - 10\, y^2\, c^4\, t^4\, x + 8\, y^3\, c^3\, t^3\, z + 4\, x^3\, y^2\, z^2 + 2\, y^4\, x\, z^2 - 2\, y^2\, z^4\, x + 26\, x^5\, c^2\, t^2$$

$$- 10\, x^3\, c^4\, t^4 - 2\, x\, z^6 - \mu\, y^5 + 2\, x^5\, z^2 - 2\, x^3\, z^4)) ,\ - \frac{1}{\left(-x^2 - y^2 - z^2 + c^2\, t^2 \right)^8\, \mu} \Big(4\, \Big($$

$$-18\, c^6\, t^6\, y + 2\, \mu\, x^3\, y^2 + 2\, \mu\, x^3\, z^2 + \mu\, x\, y^4 + \mu\, x\, z^4 + x^5\, c\, \mu - 2\, x\, c^3\, \mu\, y^2\, t^2 - 26\, c^4\, t^2\, y\, x^4\, \varepsilon\, \mu$$

$$- 52\, c^4\, t^2\, y^3\, x^2\, \varepsilon\, \mu + 10\, c^6\, t^4\, y\, x^2\, \varepsilon\, \mu - 4\, c^3\, t\, x^5\, z\, \varepsilon\, \mu - 8\, c^3\, t\, x^3\, z^3\, \varepsilon\, \mu + 8\, c^5\, t^3\, x^3\, z\, \varepsilon\, \mu$$

$$- 12\, c^4\, t^2\, y^3\, \varepsilon\, z^2\, \mu + 14\, c^4\, t^2\, y\, z^4\, \varepsilon\, \mu - 34\, c^6\, t^4\, y\, z^2\, \varepsilon\, \mu - 4\, c^3\, t\, x\, z^5\, \varepsilon\, \mu + 8\, c^5\, t^3\, x\, z^3\, \varepsilon\, \mu$$

$$- 4\, c^7\, t^5\, x\, z\, \varepsilon\, \mu - 2\, y\, x^4\, \varepsilon\, c^2\, z^2\, \mu - 4\, y^3\, x^2\, \varepsilon\, c^2\, z^2\, \mu + 2\, y\, z^4\, \varepsilon\, c^2\, \mu\, x^2 + 2\, y^7 + 4\, \mu\, x\, y^2\, c^2\, t^2$$

$$- 4\, \mu\, x\, z^2\, c^2\, t^2 + 2\, \mu\, c\, t\, y^3\, z + 2\, \mu\, c\, t\, y\, z^3 - 2\, \mu\, c^3\, t^3\, y\, z - 6\, \varepsilon\, c^4\, \mu^2\, t^2\, x^3 - 2\, \varepsilon\, c^6\, \mu^2\, t^4\, x$$

$$+ 2\, x\, c\, \mu\, y^2\, z^2 - 2\, x\, c^3\, \mu\, t^2\, z^2 - 14\, c^2\, t^2\, y\, z^4 + 34\, c^4\, t^4\, y\, z^2 + 4\, c\, t\, x\, z^5 - 8\, c^3\, t^3\, x\, z^3 + 4\, c^5\, t^5\, x\, z$$

$$+ 4\, \mu\, x^3\, c^2\, t^2 + 2\, \mu\, x\, y^2\, z^2 + 3\, \mu\, c^4\, t^4\, x + 2\, x^3\, c\, \mu\, y^2 - 2\, x^3\, c^3\, \mu\, t^2 + 2\, x^3\, c\, \mu\, z^2 + x\, c\, \mu\, y^4$$

$$+ x\, c\, \mu\, z^4 + x\, c^5\, \mu\, t^4 + 52\, c^2\, t^2\, y^3\, x^2 + 26\, c^2\, t^2\, y\, x^4 + 4\, c\, t\, x^5\, z + 12\, c^2\, t^2\, y^3\, z^2 + 8\, c\, t\, x^3\, z^3$$

$$- 10\, c^4\, t^4\, y\, x^2 - 8\, c^3\, t^3\, x^3\, z - 2\, y^7\, \varepsilon\, c^2\, \mu + 6\, y^3\, x^4 + 2\, y\, x^6 + 6\, y^5\, x^2 - 2\, y^3\, z^4 + 2\, y^5\, z^2$$

$$+ 26\, c^2\, t^2\, y^5 - 10\, c^4\, t^4\, y^3 - 2\, y\, z^4\, x^2 + 4\, x^2\, y^3\, z^2 + 2\, x^4\, y\, z^2 - 2\, z\, \varepsilon\, c^3\, \mu^2\, y^3\, t - 2\, \varepsilon\, c^3\, z^3\, \mu^2\, t\, y$$

$$+ 2\, z\, \varepsilon\, c^5\, \mu^2\, t^3\, y + 2\, \varepsilon\, c^4\, \mu^2\, t^2\, x\, z^2 + 2\, \mu\, c\, t\, y\, z\, x^2 - 6\, \varepsilon\, c^4\, \mu^2\, t^2\, x\, y^2 + 8\, c\, t\, x^3\, z\, y^2$$

$$- 26\, c^4\, t^2\, y^5\, \varepsilon\, \mu + 10\, c^6\, t^4\, y^3\, \varepsilon\, \mu + 4\, c\, t\, y^4\, z\, x + 12\, c^2\, t^2\, y\, z^2\, x^2 + 8\, c\, t\, x\, z^3\, y^2 + 18\, c^8\, t^6\, y\, \varepsilon\, \mu$$

$$- 8\, c^3\, t^3\, x\, z\, y^2 - 2\, y\, x^6\, \varepsilon\, c^2\, \mu - 6\, y^3\, x^4\, \varepsilon\, c^2\, \mu - 6\, y^5\, x^2\, \varepsilon\, c^2\, \mu - 2\, y^5\, \varepsilon\, c^2\, z^2\, \mu + 2\, y^3\, z^4\, \varepsilon\, c^2\, \mu$$

$$+ 2\, y\, z^6\, \varepsilon\, c^2\, \mu - 12\, c^4\, t^2\, y\, x^2\, \varepsilon\, z^2\, \mu - 8\, c^3\, t\, x^3\, z\, \varepsilon\, \mu\, y^2 - 4\, c^3\, t\, y^4\, z\, x\, \varepsilon\, \mu - 8\, c^3\, t\, y^2\, z^3\, x\, \varepsilon\, \mu$$

$$+ 8\, c^5\, t^3\, y^2\, z\, x\, \varepsilon\, \mu - 2\, z\, \varepsilon\, c^3\, \mu^2\, x^2\, t\, y - 2\, y\, z^6 + \mu\, x^5)\,)\,,$$

$$- \frac{1}{(-x^2 - y^2 - z^2 + c^2\, t^2)^8\, \mu} \left(4\, \left(4\, c^5\, t^3\, \mu^2\, \varepsilon\, x^2 + 4\, z\, y^6 - 8\, \varepsilon\, c^2\, \mu\, x^4\, z^3 - 4\, \varepsilon\, c^2\, \mu\, x^2\, z^5\right.\right.$$

$$- 8\, \varepsilon\, c^2\, \mu\, y^4\, z^3 - 4\, \varepsilon\, c^2\, \mu\, y^2\, z^5 + 80\, z\, x^2\, y^2\, c^2\, t^2 - 4\, z\, \varepsilon\, c^2\, \mu\, x^6 - 4\, z\, \varepsilon\, c^2\, \mu\, y^6 + 4\, c^5\, t^3\, \mu^2\, \varepsilon\, y^2$$

$$+3c^3t\mu^2\varepsilon x^4+c^3t\mu^2\varepsilon z^4-2c^5t^3\mu^2\varepsilon z^2-4ct\mu x^2y^2-2ct\mu x^2z^2-2ct\mu y^2z^2$$
$$-2c^2t\mu x^2z^2-2c^2t\mu y^2z^2-2c^2t\mu x^2y^2+4zx^6+8y^4z^3+4y^2z^5+8x^4z^3+4x^2z^5$$
$$-44x^2zc^4t^4-44y^2zc^4t^4+12zx^4y^2+12zx^2y^4+16x^2y^2z^3-c^6t^5\mu-12z\varepsilon c^2\mu x^4y^2$$
$$-40z\varepsilon c^4\mu x^4t^2-12z\varepsilon c^2\mu x^2y^4+44z\varepsilon c^6\mu x^2t^4-40z\varepsilon c^4\mu y^4t^2+44z\varepsilon c^6\mu y^2t^4$$
$$-16\varepsilon c^2\mu x^2y^2z^3-40\varepsilon c^4\mu x^2t^2z^3-40\varepsilon c^4\mu y^2t^2z^3+6c^3t\mu^2\varepsilon x^2y^2+4c^3t\mu^2\varepsilon y^2z^2$$
$$+4c^3t\mu^2\varepsilon x^2z^2+40y^2z^3c^2t^2+40x^2c^2t^2z^3+40zy^4c^2t^2+40zx^4c^2t^2+c^7t^5\mu^2\varepsilon$$
$$-2ct\mu x^4-6c^3t^3\mu x^2-6c^3t^3\mu y^2-2ct\mu y^4+2c^4t^3\mu x^2-c^2t\mu x^4+2c^4t^3\mu y^2$$
$$-c^2t\mu y^4+2c^4t^3\mu z^2-c^2t\mu z^4-80z\varepsilon c^4\mu x^2y^2t^2+3c^3t\mu^2\varepsilon y^4)\,)\,]$$

$$\mathit{Dissipation} = \frac{2c\,(3cty^2\varepsilon\mu+3x^2ct\varepsilon\mu-\mu\varepsilon ctz^2+\mu\varepsilon c^3t^3-x^3-xy^2-xz^2+c^2t^2x)}{(-x^2-y^2-z^2+c^2t^2)^5}$$

*****************************   *END PROCEDURE*   ***************************   **(31)**

```
> NAME:=`Example 7b = Index 1 Irreversible solution EdotB < 0 (kinematic out)
  Type 1`;
> Holder:=(x^2+y^2+z^2-c^2*t^2)^(4/2);

> Ax:=m*y/Holder;Ay:=-m*x/Holder;Az:=-c*t/Holder;phi:=z*c/Holder;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 7b = Index 1 Irreversible solution EdotB <0 (kinematic out) Type 1

$$\mathit{Holder} := (x^2+y^2+z^2-c^2t^2)^2$$

$$\mathit{Ax} := \frac{my}{(x^2+y^2+z^2-c^2t^2)^2}$$

$$\mathit{Ay} := -\frac{mx}{(x^2+y^2+z^2-c^2t^2)^2}$$

$$\mathit{Az} := -\frac{ct}{(x^2+y^2+z^2-c^2t^2)^2}$$

$$\phi := \frac{cz}{(x^2+y^2+z^2-c^2t^2)^2}$$

Example 7b = Index 1 Irreversible solution EdotB <0 (kinematic out) Type 1

*********************************   *Differential Form Format*   ********************

****

Action 1-form $= -\frac{c\,z\,d(t)}{(-x^2-y^2-z^2+c^2t^2)^2} + \frac{m\,y\,d(x)}{(-x^2-y^2-z^2+c^2t^2)^2} - \frac{m\,x\,d(y)}{(-x^2-y^2-z^2+c^2t^2)^2} - \frac{c\,t\,d(z)}{(-x^2-y^2-z^2+c^2t^2)^2}$

Intensity 2-form F=dA $= \left(-\frac{4\,c\,z\,x}{(-x^2-y^2-z^2+c^2t^2)^3} + \frac{4\,m\,y\,c^2\,t}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(x))\ \&\wedge\ (d(t)) + \left(-\frac{m\,(3x^2-y^2-z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3} - \frac{m\,(-x^2+3y^2-z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(x))\ \&\wedge\ (d(y)) + \left(-\frac{4\,m\,y\,z}{(-x^2-y^2-z^2+c^2t^2)^3} - \frac{4\,c\,t\,x}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(x))\ \&\wedge\ (d(z)) + \left(-\frac{4\,c\,z\,y}{(-x^2-y^2-z^2+c^2t^2)^3} - \frac{4\,m\,x\,c^2\,t}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(y))\ \&\wedge\ (d(t)) + \left(\frac{4\,m\,x\,z}{(-x^2-y^2-z^2+c^2t^2)^3} - \frac{4\,c\,t\,y}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(y))\ \&\wedge\ (d(z)) + \left(-\frac{c\,(-x^2-y^2+3z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3} - \frac{c\,(x^2+y^2+z^2+3c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(z))\ \&\wedge\ (d(t))$

Topological Torsion 3-form $A\wedge F = \left(\frac{2\,c\,z\,m\,(x^2+y^2-z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5} + \frac{4\,m\,x\,c\,(m\,c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5} - \frac{4\,m\,y\,c\,(m\,x\,c\,t+y\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}\right)\&\wedge(d(x),d(y),d(t)) + \left(\frac{2\,c\,t\,m\,(x^2+y^2-z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5} - \frac{4\,m\,x\,(m\,y\,z+c\,t\,x)}{(-x^2-y^2-z^2+c^2t^2)^5} - \frac{4\,m\,y\,(c\,t\,y-m\,x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}\right)\&\wedge(d(x),d(y),d(z)) + \left(-\frac{4\,m\,y\,c\,(c^2t^2+z^2)}{(-x^2-y^2-z^2+c^2t^2)^5} + \frac{4\,c\,z\,(m\,y\,z+c\,t\,x)}{(-x^2-y^2-z^2+c^2t^2)^5} + \frac{4\,c^2\,t\,(m\,c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}\right)\&\wedge(d(x),d(z),d(t)) + \left(-\frac{4\,c^2\,t\,(m\,x\,c\,t+y\,z)}{(-x^2-y^2-z^2+c^2t^2)^5} + \frac{4\,m\,x\,c\,(c^2t^2+z^2)}{(-x^2-y^2-z^2+c^2t^2)^5} + \frac{4\,c\,z\,(c\,t\,y-m\,x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}\right)\&\wedge(d(y),d(z),d(t))$

Topological Parity 4-form $F\wedge F = \dfrac{16\, m\, c\, (c^2 t^2 + z^2)\ \&\wedge(d(x), d(y), d(z), d(t))}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}$

******************************** *Using EM format* ********************************

$$E\ field = \left[\frac{4\,c\,(m\,c\,t\,y - x\,z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, -\frac{4\,c\,(m\,x\,c\,t + y\,z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, -\frac{4\,c\,(c^2 t^2 + z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}\right]$$

$$B\ field = \left[-\frac{4\,(c\,t\,y - m\,x\,z)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, \frac{4\,(m\,y\,z + c\,t\,x)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, -\frac{2\,m\,(x^2 + y^2 - z^2 + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}\right]$$

Topological TORSION 4 vector $T4$ = -[ExA + Bphi,AdotB] $= \left[0, 0, \dfrac{2\,c\,z\,m}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}, -\dfrac{2\,m\,c\,t}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}\right]$

$$Helicity\ AdotB = \frac{2\,m\,c\,t}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}$$

$$Poincare\ II = 2(E.B) = \frac{16\,m\,c\,(c^2 t^2 + z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}$$

$$coefficient\ of\ Topological\ Parity\ 4\text{-}form = \frac{16\,m\,c\,(c^2 t^2 + z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}$$

Pfaff Topological Dimension $PTD = 4$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

$$Xm\ or\ linear\ (Mean)\ curvature = \frac{4\,c\,t\,z\,(c-1)\,(1+c)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}$$

Yg or quadratic (GAUSS) curvature

$$= \frac{3\,c^4 t^2 + m^2 c^2 t^2 - 3\,c^2 z^2 + c^2 x^2 + c^2 y^2 + 3\,m^2 y^2 + 3\,m^2 x^2 - m^2 z^2}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}$$

$$Za\ or\ Cubic\ (Interaction\ internal\ energy)\ curvature = \frac{4\,m^2 c\,t\,z\,(c-1)\,(1+c)}{(-x^2 - y^2 - z^2 + c^2 t^2)^7}$$

$$Tk\ or\ quartic\ (4D\ expansion)\ curvature = \frac{3\,m^2 c^2}{(-x^2 - y^2 - z^2 + c^2 t^2)^8}$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor $CH = 0$

$$D\ field = \left[\frac{4\,\varepsilon\,c\,(m\,c\,t\,y - x\,z)}{(-x^2 - y^2 - z^2 + c^2\,t^2)^3}, -\frac{4\,\varepsilon\,c\,(m\,x\,c\,t + y\,z)}{(-x^2 - y^2 - z^2 + c^2\,t^2)^3}, -\frac{4\,\varepsilon\,c\,(c^2\,t^2 + z^2)}{(-x^2 - y^2 - z^2 + c^2\,t^2)^3} \right]$$

$$H\ field = \left[-\frac{4\,(c\,t\,y - m\,x\,z)}{(-x^2 - y^2 - z^2 + c^2\,t^2)^3\,\mu}, \frac{4\,(m\,y\,z + c\,t\,x)}{(-x^2 - y^2 - z^2 + c^2\,t^2)^3\,\mu}, -\frac{2\,m\,(x^2 + y^2 - z^2 + c^2\,t^2)}{(-x^2 - y^2 - z^2 + c^2\,t^2)^3\,\mu} \right]$$

$$Poynting\ vector\ ExH = \left[\frac{1}{(-x^2 - y^2 - z^2 + c^2\,t^2)^6\,\mu} (8\,c\,(m^2\,x^3\,c\,t + c\,t\,y^2\,m^2\,x - m^2\,x\,z^2\,c\,t + m^2\,x\,c^3\,t^3 + m\,y\,z\,x^2 + m\,y^3\,z + m\,y\,z^3 + 3\,m\,y\,z\,c^2\,t^2 + 2\,c^3\,t^3\,x + 2\,z^2\,c\,t\,x)), \frac{1}{(-x^2 - y^2 - z^2 + c^2\,t^2)^6\,\mu} (8\,c\,(2\,c^3\,t^3\,y - 3\,m\,x\,z\,c^2\,t^2 + 2\,c\,t\,y\,z^2 - m\,x\,z^3 + c\,t\,x^2\,m^2\,y + m^2\,c\,t\,y^3 - m^2\,y\,z^2\,c\,t + m^2\,c^3\,t^3\,y - m\,x^3\,z - m\,x\,z\,y^2)), \frac{16\,c^2\,t\,z\,(m^2\,y^2 - x^2 + m^2\,x^2 - y^2)}{(-x^2 - y^2 - z^2 + c^2\,t^2)^6\,\mu} \right]$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt{=}J4 = \left[\frac{1}{(-x^2 - y^2 - z^2 + c^2\,t^2)^4\,\mu} (4\,(-m\,y\,x^2 - m\,y^3 - m\,y\,z^2 - 5\,m\,y\,c^2\,t^2 - 6\,c\,t\,x\,z + \varepsilon\,c^2\,\mu\,m\,y\,x^2 + \varepsilon\,c^2\,\mu\,m\,y^3 + \varepsilon\,c^2\,\mu\,m\,y\,z^2 + 5\,\varepsilon\,c^4\,\mu\,m\,y\,t^2 - 6\,\varepsilon\,c^3\,\mu\,t\,x\,z)), -\frac{1}{(-x^2 - y^2 - z^2 + c^2\,t^2)^4\,\mu} (4\,(-m\,x^3 - m\,x\,y^2 - m\,x\,z^2 - 5\,c^2\,t^2\,m\,x + 6\,c\,t\,y\,z + \varepsilon\,c^2\,\mu\,m\,x^3 + \varepsilon\,c^2\,\mu\,m\,x\,y^2 + \varepsilon\,c^2\,\mu\,m\,x\,z^2 + 5\,\varepsilon\,c^4\,\mu\,t^2\,m\,x + 6\,\varepsilon\,c^3\,\mu\,t\,y\,z)), -\frac{8\,c\,t\,(-2\,x^2 - 2\,y^2 + z^2 - c^2\,t^2 + \varepsilon\,c^2\,\mu\,x^2 + \varepsilon\,c^2\,\mu\,y^2 + 4\,\varepsilon\,c^2\,z^2\,\mu + 2\,\varepsilon\,c^4\,\mu\,t^2)}{(-x^2 - y^2 - z^2 + c^2\,t^2)^4\,\mu}, -\frac{8\,\varepsilon\,c\,z\,(x^2 + y^2 + z^2 + 5\,c^2\,t^2)}{(-x^2 - y^2 - z^2 + c^2\,t^2)^4} \right]$$

Amerian charge density $\quad divD = rho = -\dfrac{8\,\varepsilon\,c\,z\,(x^2+y^2+z^2+5\,c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^4}$

divergence Lorentz Current 4Vector, $\quad 4div(J4) = 0$

Topological SPIN 4 vector S4

$$= \left[\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu}\left(2\,(m^2\,x^3+m^2\,x\,y^2-m^2\,x\,z^2+m^2\,x\,c^2\,t^2+2\,c\,t\,m\,y\,z+2\,c^2\,t^2\,x+2\,\varepsilon\,c^3\,z\,\mu\,m\,t\,y-2\,\varepsilon\,c^2\,\mu\,x\,z^2)\right), -\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu}\left(2\,(-2\,y\,c^2\,t^2+2\,c\,t\,m\,x\,z-m^2\,y\,x^2-m^2\,y^3+m^2\,y\,z^2-m^2\,y\,c^2\,t^2+2\,\varepsilon\,c^3\,z\,\mu\,m\,x\,t+2\,\varepsilon\,c^2\,\mu\,y\,z^2)\right), -\frac{4\,z\,(-m^2\,y^2-m^2\,x^2+\varepsilon\,c^4\,\mu\,t^2+\varepsilon\,c^2\,z^2\,\mu)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu}, \frac{4\,\varepsilon\,c^2\,t\,(m^2\,y^2+m^2\,x^2+c^2\,t^2+z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5} \right]$$

Topological SPIN 3-form $= \dfrac{1}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu}\left(2\,(m^2\,x^3+m^2\,x\,y^2-m^2\,x\,z^2+m^2\,x\,c^2\,t^2+2\,c\,t\,m\,y\,z+2\,c^2\,t^2\,x+2\,\varepsilon\,c^3\,z\,\mu\,m\,t\,y-2\,\varepsilon\,c^2\,\mu\,x\,z^2)\ \&\text{^}(d(y), d(z), d(t))\right) + \dfrac{1}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu}\left(2\,(-2\,y\,c^2\,t^2+2\,c\,t\,m\,x\,z-m^2\,y\,x^2-m^2\,y^3+m^2\,y\,z^2-m^2\,y\,c^2\,t^2+2\,\varepsilon\,c^3\,z\,\mu\,m\,x\,t+2\,\varepsilon\,c^2\,\mu\,y\,z^2)\ \&\text{^}(d(x), d(z), d(t))\right) - \dfrac{4\,z\,(-m^2\,y^2-m^2\,x^2+\varepsilon\,c^4\,\mu\,t^2+\varepsilon\,c^2\,z^2\,\mu)\ \&\text{^}(d(x), d(y), d(t))}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu} - \dfrac{4\,\varepsilon\,c^2\,t\,(m^2\,y^2+m^2\,x^2+c^2\,t^2+z^2)\ \&\text{^}(d(x), d(y), d(z))}{(-x^2-y^2-z^2+c^2\,t^2)^5}$

Spin density rho_spin $= \dfrac{4\,\varepsilon\,c^2\,t\,(m^2\,y^2+m^2\,x^2+c^2\,t^2+z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5}$

LaGrange field energy density (B.H-D.E) $= -\dfrac{1}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}\left(4\,(-4\,y^2\,c^2\,t^2-2\,m^2\,x^2\,z^2-2\,m^2\,y^2\,z^2-4\,x^2\,c^2\,t^2-m^2\,x^4-2\,m^2\,y^2\,x^2-2\,m^2\,x^2\,c^2\,t^2-m^2\,y^4-2\,m^2\,y^2\,c^2\,t^2-m^2\,z^4+2\,m^2\,c^2\,t^2\,z^2-m^2\,c^4\,t^4+4\,m^2\,y^2\,\varepsilon\,c^4\,\mu\,t^2+4\,\varepsilon\,c^2\,\mu\,x^2\,z^2+4\,m^2\,x^2\,\varepsilon\,c^4\,\mu\,t^2+4\,\varepsilon\,c^2\,\mu\,y^2\,z^2+4\,\varepsilon\,c^6\,\mu\,t^4+8\,\varepsilon\,c^4\,\mu\,t^2\,z^2+4\,\varepsilon\,c^2\,\mu\,z^4)\right)$

B.H $= \dfrac{1}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}\left(4\,(4\,y^2\,c^2\,t^2+2\,m^2\,x^2\,z^2+2\,m^2\,y^2\,z^2+4\,x^2\,c^2\,t^2+m^2\,x^4\right.$

$$+2m^2y^2x^2+2m^2x^2c^2t^2+m^2y^4+2m^2y^2c^2t^2+m^2z^4-2m^2c^2t^2z^2+m^2c^4t^4))$$

$$D.E=\frac{16\,\varepsilon\,c^2\,(m^2y^2c^2t^2+x^2z^2+m^2x^2c^2t^2+y^2z^2+c^4t^4+2c^2t^2z^2+z^4)}{(-x^2-y^2-z^2+c^2t^2)^6}$$

$$A.J=\frac{1}{(-x^2-y^2-z^2+c^2t^2)^6\,\mu}(4\,(2c^2t^2z^2-4x^2c^2t^2-4y^2c^2t^2-m^2x^4-m^2y^4-2c^4t^4$$
$$+m^2y^4\varepsilon c^2\mu+m^2x^4\varepsilon c^2\mu-m^2x^2z^2-m^2y^2z^2+2m^2y^2\varepsilon c^2\mu x^2+m^2y^2\varepsilon c^2\mu z^2$$
$$+5m^2y^2\varepsilon c^4\mu t^2+m^2x^2\varepsilon c^2\mu z^2+5m^2x^2\varepsilon c^4\mu t^2-2m^2y^2x^2+4\varepsilon c^6\mu t^4+2\varepsilon c^4\mu y^2t^2$$
$$+2\varepsilon c^4\mu x^2t^2+8\varepsilon c^4\mu t^2z^2-5m^2x^2c^2t^2-5m^2y^2c^2t^2))$$

$$\text{-rho.phi}=-\frac{8c^2z^2\varepsilon\,(x^2+y^2+z^2+5c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^6}$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* $=-\frac{1}{(-x^2-y^2-z^2+c^2t^2)^6\,\mu}(4\,(2c^2t^2z^2$
$$-8x^2c^2t^2-8y^2c^2t^2-2m^2x^4-2m^2y^4-m^2z^4-2c^4t^4+m^2y^4\varepsilon c^2\mu+m^2x^4\varepsilon c^2\mu$$
$$-3m^2x^2z^2-3m^2y^2z^2-m^2c^4t^4+2m^2y^2\varepsilon c^2\mu x^2+m^2y^2\varepsilon c^2\mu z^2+9m^2y^2\varepsilon c^4\mu t^2$$
$$+m^2x^2\varepsilon c^2\mu z^2+9m^2x^2\varepsilon c^4\mu t^2-4m^2y^2x^2+6\varepsilon c^2\mu z^4+8\varepsilon c^6\mu t^4+2\varepsilon c^4\mu y^2t^2$$
$$+6\varepsilon c^2\mu x^2z^2+6\varepsilon c^2\mu y^2z^2+2\varepsilon c^4\mu x^2t^2+26\varepsilon c^4\mu t^2z^2-7m^2x^2c^2t^2-7m^2y^2c^2t^2$$
$$+2m^2c^2t^2z^2))$$

London Coefficient $LC=0$

PROCA coefficient curlcurlB

$$=\left[\frac{1}{(-x^2-y^2-z^2+c^2t^2)^5}(24\,(-m\,x^3z-m\,x\,z^3+3\,c\,t\,y\,x^2+3\,c\,t\,y\,z^2+3\,c\,t\,y^3\right.$$
$$+5c^3t^3y-m\,x\,z\,y^2-7\,m\,x\,z\,c^2t^2)),$$
$$-\frac{24\,(m\,y^3z+m\,y\,z^3+3z^2c\,t\,x+3\,c\,t\,x\,y^2+3\,c\,t\,x^3+5c^3t^3x+m\,y\,z\,x^2+7\,m\,y\,z\,c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5},$$
$$\frac{1}{(-x^2-y^2-z^2+c^2t^2)^5}(8\,m\,(2x^4+4x^2y^2+2y^4-4c^2t^2z^2+17x^2c^2t^2+17y^2c^2t^2$$
$$\left.-z^4+x^2z^2+y^2z^2+5c^4t^4))\right]$$

Amperian Current 4Vector *curlH-dD/dt=J4* $=\left[\frac{1}{(-x^2-y^2-z^2+c^2t^2)^4\,\mu}(4\,(-m\,y\,x^2\right.$

$$- m y^3 - m y z^2 - 5 m y c^2 t^2 - 6 c t x z + \varepsilon c^2 \mu m y x^2 + \varepsilon c^2 \mu m y^3 + \varepsilon c^2 \mu m y z^2$$

$$+ 5 \varepsilon c^4 \mu m y t^2 - 6 \varepsilon c^3 \mu t x z)), - \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^4 \mu} (4 (-m x^3 - m x y^2$$

$$- m x z^2 - 5 c^2 t^2 m x + 6 c t y z + \varepsilon c^2 \mu m x^3 + \varepsilon c^2 \mu m x y^2 + \varepsilon c^2 \mu m x z^2 + 5 \varepsilon c^4 \mu t^2 m x$$

$$+ 6 \varepsilon c^3 \mu t y z)),$$

$$- \frac{8 c t (-2 x^2 - 2 y^2 + z^2 - c^2 t^2 + \varepsilon c^2 \mu x^2 + \varepsilon c^2 \mu y^2 + 4 \varepsilon c^2 z^2 \mu + 2 \varepsilon c^4 \mu t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4 \mu},$$

$$- \frac{8 \varepsilon c z (x^2 + y^2 + z^2 + 5 c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4} \Big]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B) $= \Big[$

$$- \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^7 \mu} (8 (-8 x y^2 c^2 t^2 + 4 x z^2 c^2 t^2 + 4 \varepsilon c^4 \mu t^2 x^3 + 8 \varepsilon c^6 \mu t^4 x$$

$$+ 6 \varepsilon c^4 \mu m^2 x^3 t^2 + \varepsilon c^2 \mu m^2 x y^4 - \varepsilon c^2 \mu m^2 x z^4 + 5 \varepsilon c^6 \mu t^4 m^2 x - 2 m c t y z x^2$$

$$+ 6 m z \varepsilon c^3 \mu x^2 t y + 2 \varepsilon c^2 \mu m^2 x^3 y^2 - 6 m^2 x^3 c^2 t^2 - 5 c^4 t^4 m^2 x - m^2 x^5$$

$$+ 6 \varepsilon c^4 \mu m^2 x y^2 t^2 - 4 \varepsilon c^4 \mu m^2 x z^2 t^2 + 6 m z \varepsilon c^3 \mu y^3 t + 6 m \varepsilon c^3 z^3 \mu t y - 6 m z \varepsilon c^5 \mu t^3 y$$

$$- 8 x^3 c^2 t^2 - 4 c^4 t^4 x + 4 \varepsilon c^2 \mu y^2 x z^2 + 36 \varepsilon c^4 \mu t^2 x z^2 + 4 \varepsilon c^4 \mu t^2 x y^2 + 4 \varepsilon c^2 \mu x^3 z^2$$

$$+ 4 \varepsilon c^2 z^4 \mu x - 2 m c t y^3 z - 2 m c t y z^3 + 2 m c^3 t^3 y z - 6 m^2 x y^2 c^2 t^2 + 4 m^2 x z^2 c^2 t^2$$

$$+ \varepsilon c^2 \mu m^2 x^5 + m^2 x z^4 - m^2 x y^4 - 2 m^2 x^3 y^2)),$$

$$- \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^7 \mu} (8 (2 c t m x z y^2 + \varepsilon c^2 \mu m^2 y x^4 + 2 \varepsilon c^2 \mu m^2 y^3 x^2$$

$$+6\,\varepsilon\,c^4\,\mu\,m^2\,y^3\,t^2-\varepsilon\,c^2\,\mu\,m^2\,y\,z^4+5\,\varepsilon\,c^6\,\mu\,m^2\,y\,t^4-6\,m^2\,y^3\,c^2\,t^2-5\,m^2\,y\,c^4\,t^4-m^2\,y^5$$

$$+4\,\varepsilon\,c^2\,\mu\,y\,z^2\,x^2+36\,\varepsilon\,c^4\,\mu\,y\,z^2\,t^2+4\,\varepsilon\,c^4\,\mu\,t^2\,y\,x^2-m^2\,y\,x^4-2\,m^2\,y^3\,x^2+m^2\,y\,z^4$$

$$-8\,c^2\,t^2\,y^3-4\,c^4\,t^4\,y+4\,\varepsilon\,c^2\,\mu\,y^3\,z^2+4\,\varepsilon\,c^2\,\mu\,y\,z^4+4\,\varepsilon\,c^4\,\mu\,t^2\,y^3+8\,\varepsilon\,c^6\,\mu\,t^4\,y$$

$$-6\,c^3\,t\,\varepsilon\,\mu\,y^2\,m\,x\,z+2\,c\,t\,m\,x^3\,z+2\,c\,t\,m\,x\,z^3-2\,c^3\,t^3\,m\,x\,z-6\,m^2\,y\,x^2\,c^2\,t^2+4\,m^2\,y\,z^2\,c^2\,t^2$$

$$+\varepsilon\,c^2\,\mu\,m^2\,y^5-8\,c^2\,t^2\,y\,x^2+4\,c^2\,t^2\,y\,z^2-6\,c^3\,t\,\varepsilon\,\mu\,x^3\,m\,z-6\,c^3\,t\,\varepsilon\,z^3\,\mu\,m\,x+6\,c^5\,t^3\,\varepsilon\,\mu\,m\,x\,z$$

$$+6\,\varepsilon\,c^4\,\mu\,m^2\,y\,x^2\,t^2-4\,\varepsilon\,c^4\,\mu\,m^2\,y\,z^2\,t^2)\,),\ -\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^7\,\mu}\,(16\,z\,($$

$$-6\,x^2\,c^2\,t^2-6\,y^2\,c^2\,t^2-m^2\,x^4-m^2\,y^4+m^2\,y^4\,\varepsilon\,c^2\,\mu+m^2\,x^4\,\varepsilon\,c^2\,\mu-m^2\,x^2\,z^2-m^2\,y^2\,z^2$$

$$+2\,m^2\,y^2\,\varepsilon\,c^2\,\mu\,x^2+m^2\,y^2\,\varepsilon\,c^2\,\mu\,z^2+5\,m^2\,y^2\,\varepsilon\,c^4\,\mu\,t^2+m^2\,x^2\,\varepsilon\,c^2\,\mu\,z^2+5\,m^2\,x^2\,\varepsilon\,c^4\,\mu\,t^2$$

$$-2\,m^2\,y^2\,x^2+2\,\varepsilon\,c^2\,\mu\,z^4+10\,\varepsilon\,c^6\,\mu\,t^4-4\,\varepsilon\,c^4\,\mu\,y^2\,t^2+2\,\varepsilon\,c^2\,\mu\,x^2\,z^2+2\,\varepsilon\,c^2\,\mu\,y^2\,z^2$$

$$-4\,\varepsilon\,c^4\,\mu\,x^2\,t^2+12\,\varepsilon\,c^4\,\mu\,t^2\,z^2-5\,m^2\,x^2\,c^2\,t^2-5\,m^2\,y^2\,c^2\,t^2)\,)\,]$$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $=\left[0,\ 0,\ \frac{2\,c\,z\,m}{(-x^2-y^2-z^2+c^2\,t^2)^4},\ -\frac{2\,m\,c\,t}{(-x^2-y^2-z^2+c^2\,t^2)^4}\right]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $=\left[-\frac{8\,c\,m^2\,(c^2\,t^2+z^2)\,y}{(-x^2-y^2-z^2+c^2\,t^2)^7},\ \frac{8\,c\,m^2\,(c^2\,t^2+z^2)\,x}{(-x^2-y^2-z^2+c^2\,t^2)^7},\ \frac{8\,m\,c^2\,t\,(c^2\,t^2+z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^7}\right]$

Torsion Dissipation Jtorsion dot E $=\frac{8\,c^2\,m\,z\,(c^2\,t^2+z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^7}$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^5 \mu} (2 (m^2 x^3 + m^2 x y^2 - m^2 x z^2 + m^2 x c^2 t^2 + 2 c t m y z + 2 c^2 t^2 x + 2 \varepsilon c^3 z \mu m t y - 2 \varepsilon c^2 \mu x z^2)), - \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^5 \mu} (2 (-2 y c^2 t^2 + 2 c t m x z - m^2 y x^2 - m^2 y^3 + m^2 y z^2 - m^2 y c^2 t^2 + 2 \varepsilon c^3 z \mu m x t + 2 \varepsilon c^2 \mu y z^2)), - \frac{4 z (-m^2 y^2 - m^2 x^2 + \varepsilon c^4 \mu t^2 + \varepsilon c^2 z^2 \mu)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5 \mu}, \frac{4 \varepsilon c^2 t (m^2 y^2 + m^2 x^2 + c^2 t^2 + z^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5} \right]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= \Bigg[$

$$- \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^8 \mu} (4 m (-2 m^2 y x^2 z^2 - 2 c t m x z y^2 + 4 \varepsilon c^4 \mu m^2 y^3 t^2 - 2 m^2 y^3 c^2 t^2 - m^2 y c^4 t^4 - m^2 y^5 + 2 \varepsilon c^2 \mu y z^2 x^2 + 10 \varepsilon c^4 \mu y z^2 t^2 - m^2 y x^4 - 2 m^2 y^3 x^2 - m^2 y z^4 - 2 m^2 y^3 z^2 - 2 c^2 t^2 y^3 - 2 c^4 t^4 y + 2 \varepsilon c^2 \mu y^3 z^2 + 2 \varepsilon c^2 \mu y z^4 + 4 \varepsilon c^6 \mu t^4 y - 2 c^3 t \varepsilon \mu y^2 m x z - 2 c t m x^3 z - 2 c t m x z^3 + 2 c^3 t^3 m x z - 2 m^2 y x^2 c^2 t^2 + 2 m^2 y z^2 c^2 t^2 - 2 c^2 t^2 y x^2 + 2 c^2 t^2 y z^2 - 2 c^3 t \varepsilon \mu x^3 m z - 2 c^3 t \varepsilon z^3 \mu m x + 2 c^5 t^3 \varepsilon \mu m x z + 4 \varepsilon c^4 \mu m^2 y x^2 t^2)), \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^8 \mu} (4 m (-2 m^2 x y^2 z^2 - 2 x y^2 c^2 t^2 + 2 x z^2 c^2 t^2 + 4 \varepsilon c^6 \mu t^4 x + 4 \varepsilon c^4 \mu m^2 x^3 t^2 + 2 m c t y z x^2 + 2 m z \varepsilon c^3 \mu x^2 t y - 2 m^2 x^3 c^2 t^2 - c^4 t^4 m^2 x - m^2 x^5 + 4 \varepsilon c^4 \mu m^2 x y^2 t^2 + 2 m z \varepsilon c^3 \mu y^3 t + 2 m \varepsilon c^3 z^3 \mu t y - 2 m z \varepsilon c^5 \mu t^3 y - 2 m^2 x^3 z^2 - 2 x^3 c^2 t^2 - 2 c^4 t^4 x + 2 \varepsilon c^2 \mu y^2 x z^2 + 10 \varepsilon c^4 \mu t^2 x z^2 + 2 \varepsilon c^2 \mu x^3 z^2 + 2 \varepsilon c^2 z^4 \mu x + 2 m c t y^3 z + 2 m c t y z^3 - 2 m c^3 t^3 y z - 2 m^2 x y^2 c^2 t^2$$

$+ 2\, m^2\, x\, z^2\, c^2\, t^2 - m^2\, x\, z^4 - m^2\, x\, y^4 - 2\, m^2\, x^3\, y^2)\,)\,,$

$\frac{1}{(-x^2 - y^2 - z^2 + c^2\, t^2)^8\, \mu} (8\, c\, t\, (2\, m^2\, y^2\, \varepsilon\, c^4\, \mu\, t^2 + 2\, m^2\, x^2\, \varepsilon\, c^4\, \mu\, t^2 + 2\, \varepsilon\, c^6\, \mu\, t^4$

$+ 4\, \varepsilon\, c^4\, \mu\, t^2\, z^2 + 2\, \varepsilon\, c^2\, \mu\, z^4 - 2\, m^2\, y^2\, x^2 - m^2\, x^2\, z^2 - m^2\, y^2\, z^2 - 2\, x^2\, c^2\, t^2 - 2\, y^2\, c^2\, t^2$

$- m^2\, y^2\, c^2\, t^2 - m^2\, x^4 - m^2\, x^2\, c^2\, t^2 - m^2\, y^4 + 2\, \varepsilon\, c^2\, \mu\, x^2\, z^2 + 2\, \varepsilon\, c^2\, \mu\, y^2\, z^2)\,)\,]$

Spin Dissipation J_spin dot E $= \frac{1}{(-x^2 - y^2 - z^2 + c^2\, t^2)^8\, \mu} (8\, c\, z\, (2\, m^2\, y^2\, \varepsilon\, c^4\, \mu\, t^2$

$+ 2\, m^2\, x^2\, \varepsilon\, c^4\, \mu\, t^2 + 2\, \varepsilon\, c^6\, \mu\, t^4 + 4\, \varepsilon\, c^4\, \mu\, t^2\, z^2 + 2\, \varepsilon\, c^2\, \mu\, z^4 - 2\, m^2\, y^2\, x^2 - m^2\, x^2\, z^2 - m^2\, y^2\, z^2$

$- 2\, x^2\, c^2\, t^2 - 2\, y^2\, c^2\, t^2 - m^2\, y^2\, c^2\, t^2 - m^2\, x^4 - m^2\, x^2\, c^2\, t^2 - m^2\, y^4 + 2\, \varepsilon\, c^2\, \mu\, x^2\, z^2$

$+ 2\, \varepsilon\, c^2\, \mu\, y^2\, z^2)\,)$

Disspative Force 3 vector $= \left[-\frac{1}{(-x^2 - y^2 - z^2 + c^2\, t^2)^8\, \mu} (4\, (18\, c^4\, t^4\, m^2\, x\, z^2 - 8\, \varepsilon\, c^2\, \mu\, x^5\, z^2 \right.$

$- 16\, \varepsilon\, c^2\, \mu\, x^3\, z^4 - 8\, \varepsilon\, c^2\, z^6\, \mu\, x + 4\, m\, c\, t\, y^5\, z + 8\, m\, c\, t\, y^3\, z^3 + 4\, m\, c\, t\, y\, z^5 - 8\, m\, c^3\, t^3\, y\, z^3$

$+ 4\, m\, c^5\, t^5\, y\, z - 8\, x^2\, c^3\, t^3\, m\, y\, z - 72\, x^3\, c^4\, t^2\, \varepsilon\, \mu\, z^2 + 4\, y^2\, c^2\, t^2\, m^2\, x\, z^2 - 2\, m^2\, x^5\, \varepsilon\, c^2\, \mu\, z^2$

$- 10\, m^2\, x^5\, \varepsilon\, c^4\, \mu\, t^2 + 2\, m^2\, x^3\, z^4\, \varepsilon\, c^2\, \mu + 2\, m^2\, x^3\, \varepsilon\, c^6\, \mu\, t^4 + 2\, \varepsilon\, c^2\, \mu\, z^6\, m^2\, x - 64\, \varepsilon\, c^4\, \mu\, z^4\, t^2\, x$

$+ 10\, \varepsilon\, c^8\, \mu\, t^6\, m^2\, x - 8\, \varepsilon\, c^6\, \mu\, y^2\, t^4\, x + 56\, \varepsilon\, c^6\, \mu\, t^4\, z^2\, x + 2\, m^2\, x^7 + 10\, x^5\, c^2\, t^2\, m^2 - 2\, x^3\, c^4\, t^4\, m^2$

$- 8\, y^2\, c^4\, t^4\, x - 16\, \varepsilon\, c^4\, \mu\, t^2\, x^3\, y^2 - 6\, \varepsilon\, c^2\, \mu\, m^2\, x^3\, y^4 - 2\, \varepsilon\, c^2\, \mu\, m^2\, x\, y^6 - 4\, m^2\, x^3\, \varepsilon\, c^4\, \mu\, z^2\, t^2$

$- 4\, m^2\, y^2\, x^3\, \varepsilon\, c^2\, \mu\, z^2 + 6\, \varepsilon\, c^4\, \mu\, z^4\, m^2\, x\, t^2 - 12\, \varepsilon\, c^3\, \mu\, z^5\, t\, m\, y - 18\, \varepsilon\, c^6\, \mu\, t^4\, m^2\, x\, z^2$

$- 12\, \varepsilon\, c^7\, \mu\, t^5\, m\, y\, z - 24\, \varepsilon\, c^3\, \mu\, y^3\, z^3\, t\, m + 24\, \varepsilon\, c^5\, \mu\, t^3\, z^3\, m\, y + 2\, m^2\, x\, y^6 - 2\, m^2\, x^3\, z^4$

$- 2\, m^2\, x\, z^6 + 24\, y^3\, c^5\, t^3\, \varepsilon\, z\, \mu\, m - 72\, y^2\, c^4\, t^2\, \varepsilon\, \mu\, x\, z^2 - 2\, m^2\, y^4\, \varepsilon\, c^2\, \mu\, x\, z^2 - 10\, m^2\, y^4\, \varepsilon\, c^4\, \mu\, t^2\, x$

$$+ 2\, m^2\, y^2\, z^4\, \varepsilon\, c^2\, \mu\, x - 20\, m^2\, y^2\, \varepsilon\, c^4\, \mu\, x^3\, t^2 + 2\, m^2\, y^2\, \varepsilon\, c^6\, \mu\, t^4\, x - m^3\, \mu\, y^5 + 6\, m^2\, x^3\, y^4$$

$$- 8\, c^6\, t^6\, x + 16\, x^5\, c^2\, t^2 - 6\, m^2\, x\, z^4\, c^2\, t^2 - 2\, \varepsilon\, c^2\, \mu\, m^2\, x^7 - 2\, m^3\, \mu\, y\, x^2\, z^2 - 2\, m^3\, \mu\, y^3\, c^2\, t^2$$

$$- m^3\, \mu\, y\, c^4\, t^4 - 2\, m\, \mu\, c^2\, t^2\, y^3 - 2\, m\, \mu\, c^4\, t^4\, y + 2\, c^5\, m^2\, y\, \mu\, t^4 - 2\, c\, m^2\, y^3\, \mu\, z^2 - 2\, c\, m^2\, y\, \mu\, z^4$$

$$+ 20\, x^3\, c^2\, t^2\, m^2\, y^2 + 4\, x^3\, c^2\, t^2\, m^2\, z^2 + 10\, y^4\, c^2\, t^2\, m^2\, x - 2\, y^2\, c^4\, t^4\, m^2\, x - 8\, y^3\, c^3\, t^3\, m\, z$$

$$+ 16\, \varepsilon\, c^8\, \mu\, t^6\, x - 8\, \varepsilon\, c^6\, \mu\, x^3\, t^4 - 12\, m\, z\, \varepsilon\, c^3\, \mu\, x^4\, t\, y - 24\, m\, z\, \varepsilon\, c^3\, \mu\, x^2\, t\, y^3$$

$$- 2\, m^2\, \mu^2\, c^3\, t\, \varepsilon\, y^2\, x\, z - 2\, c^3\, m^2\, y^3\, \mu\, t^2 + 32\, x^3\, y^2\, c^2\, t^2 + 16\, x\, y^4\, c^2\, t^2 + 8\, x^3\, z^2\, c^2\, t^2 - 8\, x\, z^4\, c^2\, t^2$$

$$+ 16\, x\, z^2\, c^4\, t^4 - 10\, c^6\, t^6\, m^2\, x - 2\, m^2\, x\, z^4\, y^2 + 2\, m^2\, x\, y^4\, z^2 + 4\, m^2\, x^3\, y^2\, z^2 - m^3\, \mu\, y\, x^4$$

$$- 2\, m^3\, \mu\, y^3\, x^2 - m^3\, \mu\, y\, z^4 - 2\, m^3\, \mu\, y^3\, z^2 - 2\, m^2\, \mu\, c\, t\, x\, z\, y^2 + 2\, m\, \mu^2\, \varepsilon\, c^2\, y\, z^2\, x^2$$

$$+ 10\, m\, \mu^2\, \varepsilon\, c^4\, y\, z^2\, t^2 - 2\, m^2\, \mu^2\, c^3\, t\, \varepsilon\, x^3\, z - 2\, m^2\, \mu^2\, c^3\, t\, \varepsilon\, z^3\, x + 2\, m^2\, \mu^2\, c^5\, t^3\, \varepsilon\, x\, z$$

$$+ 4\, m^3\, \mu^2\, \varepsilon\, c^4\, y\, x^2\, t^2 - 12\, m\, z\, \varepsilon\, c^3\, \mu\, y^5\, t + 4\, m\, c\, t\, y\, z\, x^4 + 8\, m\, c\, t\, y^3\, z\, x^2 + 8\, m\, c\, t\, y\, z^3\, x^2$$

$$- 6\, \varepsilon\, c^2\, \mu\, m^2\, x^5\, y^2 - 16\, \varepsilon\, c^2\, \mu\, y^2\, x^3\, z^2 - 8\, \varepsilon\, c^2\, \mu\, y^4\, x\, z^2 - 16\, \varepsilon\, c^2\, \mu\, y^2\, x\, z^4 - 8\, \varepsilon\, c^4\, \mu\, t^2\, x\, y^4$$

$$+ 4\, m^3\, \mu^2\, \varepsilon\, c^4\, y^3\, t^2 + 2\, m\, \mu^2\, \varepsilon\, c^2\, y^3\, z^2 + 2\, m\, \mu^2\, \varepsilon\, c^2\, y\, z^4 + 4\, m\, \mu^2\, \varepsilon\, c^6\, t^4\, y - 2\, m^2\, \mu\, c\, t\, x^3\, z$$

$$- 2\, m^2\, \mu\, c\, t\, x\, z^3 + 2\, m^2\, \mu\, c^3\, t^3\, x\, z - 2\, m^3\, \mu\, y\, x^2\, c^2\, t^2 + 2\, m^3\, \mu\, y\, z^2\, c^2\, t^2 - 2\, m\, \mu\, c^2\, t^2\, y\, x^2$$

$$+ 2\, m\, \mu\, c^2\, t^2\, y\, z^2 - 2\, c^3\, m^2\, y\, \mu\, x^2\, t^2 - 2\, c\, m^2\, y\, \mu\, x^2\, z^2 + 8\, x\, y^2\, c^2\, t^2\, z^2 - 8\, \varepsilon\, c^4\, \mu\, t^2\, x^5$$

$$-24\,\varepsilon\,c^3\,\mu\,x^2\,z^3\,t\,m\,y+2\,m^2\,x^5\,z^2-8\,x^3\,c^4\,t^4+24\,x^2\,c^5\,t^3\,\varepsilon\,z\,\mu\,m\,y-4\,m^2\,y^2\,\varepsilon\,c^4\,\mu\,z^2\,t^2\,x$$

$$+6\,m^2\,x^5\,y^2)\,)\,,\ -\frac{1}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^8\,\mu}\left(4\left(-4\,c^5\,t^5\,m\,x\,z-2\,\varepsilon\,c^2\,\mu\,m^2\,y^7\right.\right.$$

$$+8\,c^2\,t^2\,y\,x^2\,z^2+2\,m^3\,\mu\,x\,y^2\,z^2+2\,m^3\,\mu\,x^3\,c^2\,t^2+m^3\,\mu\,c^4\,t^4\,x+2\,m\,\mu\,x^3\,c^2\,t^2+2\,m\,\mu\,c^4\,t^4\,x$$

$$+2\,c^3\,m^2\,x^3\,\mu\,t^2-2\,c^5\,m^2\,x\,\mu\,t^4+2\,c\,m^2\,x^3\,\mu\,z^2+2\,c\,m^2\,x\,\mu\,z^4-2\,m^2\,\mu^2\,z\,\varepsilon\,c^3\,x^2\,t\,y$$

$$+2\,\varepsilon\,c^6\,\mu\,t^4\,y\,m^2\,x^2-24\,c^5\,t^3\,m\,x^3\,z\,\varepsilon\,\mu+24\,c^3\,t\,m\,x^3\,z^3\,\varepsilon\,\mu-24\,c^5\,t^3\,m\,x\,z^3\,\varepsilon\,\mu$$

$$+12\,c^3\,t\,m\,x\,z^5\,\varepsilon\,\mu+12\,c^7\,t^5\,m\,x\,z\,\varepsilon\,\mu-72\,c^4\,t^2\,y\,x^2\,\varepsilon\,z^2\,\mu-4\,m^2\,y^3\,c^4\,t^2\,\varepsilon\,z^2\,\mu$$

$$-18\,m^2\,y\,c^6\,t^4\,\varepsilon\,z^2\,\mu-4\,\varepsilon\,c^2\,\mu\,y^3\,z^2\,x^2\,m^2-2\,\varepsilon\,c^2\,\mu\,y\,z^2\,x^4\,m^2-20\,\varepsilon\,c^4\,\mu\,t^2\,y^3\,x^2\,m^2$$

$$-10\,\varepsilon\,c^4\,\mu\,t^2\,y\,x^4\,m^2+6\,m^2\,y\,z^4\,\varepsilon\,c^4\,\mu\,t^2+2\,\varepsilon\,c^2\,\mu\,y\,z^4\,m^2\,x^2+18\,m^2\,y\,c^4\,t^4\,z^2-6\,m^2\,y\,z^4\,c^2\,t^2$$

$$-8\,\varepsilon\,c^2\,\mu\,y^5\,z^2-16\,\varepsilon\,c^2\,\mu\,y^3\,z^4-8\,\varepsilon\,c^2\,\mu\,y\,z^6-8\,\varepsilon\,c^4\,\mu\,t^2\,y^5-4\,c\,t\,m\,x^5\,z-8\,c\,t\,m\,x^3\,z^3$$

$$-4\,c\,t\,m\,x\,z^5+8\,c^3\,t^3\,m\,x\,z^3-8\,\varepsilon\,c^2\,\mu\,y\,z^2\,x^4-16\,\varepsilon\,c^2\,\mu\,y^3\,z^2\,x^2-16\,\varepsilon\,c^2\,\mu\,y\,z^4\,x^2$$

$$-8\,\varepsilon\,c^4\,\mu\,t^2\,y\,x^4-16\,\varepsilon\,c^4\,\mu\,t^2\,y^3\,x^2+2\,m\,\mu\,x\,y^2\,c^2\,t^2-2\,m\,\mu\,x\,z^2\,c^2\,t^2-4\,m\,\mu^2\,\varepsilon\,c^6\,t^4\,x$$

$$-4\,m^3\,\mu^2\,\varepsilon\,c^4\,x^3\,t^2-2\,m\,\mu^2\,\varepsilon\,c^2\,x^3\,z^2-2\,m\,\mu^2\,\varepsilon\,c^2\,z^4\,x-2\,m^2\,\mu\,c\,t\,y^3\,z-2\,m^2\,\mu\,c\,t\,y\,z^3$$

$$+2\,m^2\,\mu\,c^3\,t^3\,y\,z+2\,m^3\,\mu\,x\,y^2\,c^2\,t^2-2\,m^3\,\mu\,x\,z^2\,c^2\,t^2+2\,c^3\,m^2\,x\,\mu\,y^2\,t^2+2\,c\,m^2\,x\,\mu\,y^2\,z^2$$

$$+6\,m^2\,y^5\,x^2+2\,m^2\,y^5\,z^2+2\,m^2\,y\,x^6+6\,m^2\,y^3\,x^4-2\,m^2\,y^3\,z^4-2\,m^2\,y\,z^6+16\,c^2\,t^2\,y^5$$

$-8\,c^6\,t^6\,y+m^3\,\mu\,x^5+10\,c^2\,t^2\,y^5\,m^2-2\,c^4\,t^4\,y^3\,m^2-8\,x^2\,c^4\,t^4\,y+12\,c^3\,t\,\varepsilon\,\mu\,x^5\,m\,z$

$-2\,m^2\,\mu\,c\,t\,y\,z\,x^2-4\,m^3\,\mu^2\,\varepsilon\,c^4\,x\,y^2\,t^2-2\,m^2\,\mu^2\,z\,\varepsilon\,c^3\,y^3\,t-2\,m^2\,\mu^2\,\varepsilon\,c^3\,z^3\,t\,y$

$+2\,m^2\,\mu^2\,z\,\varepsilon\,c^5\,t^3\,y-2\,m\,\mu^2\,\varepsilon\,c^2\,y^2\,x\,z^2-10\,m\,\mu^2\,\varepsilon\,c^4\,t^2\,x\,z^2+2\,m^2\,y^7-8\,c^2\,t^2\,y\,z^4$

$+2\,m^3\,\mu\,x^3\,z^2+m^3\,\mu\,x\,z^4+m^3\,\mu\,x\,y^4+2\,m^3\,\mu\,x^3\,y^2-8\,c\,t\,m\,x^3\,z\,y^2-4\,c\,t\,m\,x\,z\,y^4$

$-8\,c\,t\,m\,x\,z^3\,y^2-2\,\varepsilon\,c^2\,\mu\,m^2\,y\,x^6-6\,\varepsilon\,c^2\,\mu\,m^2\,y^3\,x^4-6\,\varepsilon\,c^2\,\mu\,m^2\,y^5\,x^2-10\,m^2\,y\,c^6\,t^6$

$+2\,m^2\,y\,x^4\,z^2+4\,m^2\,y^3\,x^2\,z^2-2\,m^2\,y\,z^4\,x^2+32\,c^2\,t^2\,y^3\,x^2+8\,c^2\,t^2\,y^3\,z^2+16\,c^4\,t^4\,y\,z^2$

$+16\,c^2\,t^2\,y\,x^4+12\,c^3\,t\,\varepsilon\,\mu\,y^4\,m\,x\,z+20\,c^2\,t^2\,y^3\,m^2\,x^2-8\,c^6\,t^4\,y^3\,\varepsilon\,\mu-2\,c^4\,t^4\,y\,m^2\,x^2$

$+16\,c^8\,t^6\,y\,\varepsilon\,\mu+10\,c^2\,t^2\,y\,x^4\,m^2+4\,c^2\,t^2\,y^3\,z^2\,m^2+8\,x^3\,c^3\,t^3\,m\,z-8\,c^6\,t^4\,y\,x^2\,\varepsilon\,\mu$

$+4\,c^2\,t^2\,y\,z^2\,m^2\,x^2-64\,c^4\,t^2\,y\,z^4\,\varepsilon\,\mu+8\,y^2\,c^3\,t^3\,m\,x\,z+24\,c^3\,t\,\varepsilon\,\mu\,y^2\,m\,x^3\,z+2\,m^2\,y^3\,c^6\,t^4\,\varepsilon\,\mu$

$+10\,m^2\,y\,c^8\,t^6\,\varepsilon\,\mu-10\,m^2\,y^5\,\varepsilon\,c^4\,\mu\,t^2-2\,m^2\,y^5\,\varepsilon\,c^2\,z^2\,\mu+2\,m^2\,y\,z^6\,\varepsilon\,c^2\,\mu-72\,c^4\,t^2\,y^3\,\varepsilon\,z^2\,\mu$

$+56\,c^6\,t^4\,y\,\varepsilon\,z^2\,\mu+2\,\varepsilon\,c^2\,\mu\,y^3\,z^4\,m^2-24\,c^5\,t^3\,m\,x\,z\,y^2\,\varepsilon\,\mu+24\,c^3\,t\,m\,x\,z^3\,y^2\,\varepsilon\,\mu$

$-4\,\varepsilon\,c^4\,\mu\,y\,z^2\,t^2\,m^2\,x^2-8\,y^3\,c^4\,t^4)),\ -\dfrac{1}{(-x^2-y^2-z^2+c^2\,t^2)^8\,\mu}(8\,(8\,z\,\varepsilon\,c^4\,\mu\,x^4\,t^2$

$-28\,z\,\varepsilon\,c^6\,\mu\,x^2\,t^4+8\,z\,\varepsilon\,c^4\,\mu\,y^4\,t^2-28\,z\,\varepsilon\,c^6\,\mu\,y^2\,t^4-2\,z\,m^2\,x^6\,\varepsilon\,c^2\,\mu-2\,z\,m^2\,y^6\,\varepsilon\,c^2\,\mu$

$+16\,z\,m^2\,y^2\,x^2\,c^2\,t^2-8\,\varepsilon\,c^2\,\mu\,x^2\,y^2\,z^3-12\,\varepsilon\,c^4\,\mu\,x^2\,t^2\,z^3-12\,\varepsilon\,c^4\,\mu\,y^2\,t^2\,z^3-4\,m^2\,x^4\,\varepsilon\,c^2\,z^3\,\mu$

$-4\,m^2\,y^4\,\varepsilon\,c^2\,z^3\,\mu-16\,z\,m^2\,y^2\,\varepsilon\,c^4\,\mu\,x^2\,t^2-12\,z\,x^2\,c^4\,t^4-12\,z\,c^4\,t^4\,y^2+12\,y^2\,z^3\,c^2\,t^2$

$+12\,x^2\,c^2\,t^2\,z^3+6\,z\,m^2\,x^4\,y^2+6\,z\,m^2\,y^4\,x^2+8\,m^2\,x^2\,z^3\,y^2-4\,\varepsilon\,c^2\,\mu\,z^7+12\,z\,y^4\,c^2\,t^2$

$+12\,z\,x^4\,c^2\,t^2-2\,c^7\,t^5\,\mu^2\,\varepsilon+2\,c^3\,t^3\,\mu\,x^2+2\,c^3\,t^3\,\mu\,y^2-m\,c^6\,t^5\,\mu+2\,z\,m^2\,x^6+2\,z\,m^2\,y^6$

$$+4\,m^2 x^4 z^3 + 4\,m^2 y^4 z^3 + 2\,m^2 x^2 z^5 + 2\,m^2 y^2 z^5 - 2\,m^2 x^2 z^5 \varepsilon c^2 \mu - 2\,m^2 y^2 z^5 \varepsilon c^2 \mu$$
$$-2\,c^5 t^3 \mu^2 m^2 y^2 \varepsilon - 2\,c^5 t^3 \mu^2 m^2 x^2 \varepsilon + 2\,c\,t\,\mu\,m^2 y^2 x^2 + c\,t\,\mu\,m^2 x^2 z^2 + c\,t\,\mu\,m^2 y^2 z^2$$
$$-2\,c^3 t\,\mu^2 \varepsilon x^2 z^2 - 2\,c^3 t\,\mu^2 \varepsilon y^2 z^2 + m\,c^2 t\,\mu\,x^2 z^2 + m\,c^2 t\,\mu\,y^2 z^2 + 16\,z\,\varepsilon c^4 \mu\,x^2 y^2 t^2$$
$$-8\,m^2 x^2 z^3 \varepsilon c^2 \mu\,y^2 - 8\,m^2 x^2 z^3 \varepsilon c^4 \mu\,t^2 - 8\,m^2 y^2 z^3 \varepsilon c^4 \mu\,t^2 - 6\,z\,m^2 x^4 \varepsilon c^2 \mu\,y^2$$
$$-8\,z\,m^2 x^4 \varepsilon c^4 \mu\,t^2 - 6\,z\,m^2 y^4 \varepsilon c^2 \mu\,x^2 - 8\,z\,m^2 y^4 \varepsilon c^4 \mu\,t^2 + 10\,z\,m^2 y^2 \varepsilon c^6 \mu\,t^4$$
$$+10\,z\,m^2 x^2 \varepsilon c^6 \mu\,t^4 - 4\,\varepsilon c^2 \mu\,x^4 z^3 - 8\,\varepsilon c^2 \mu\,x^2 z^5 - 4\,\varepsilon c^2 \mu\,y^4 z^3 - 8\,\varepsilon c^2 \mu\,y^2 z^5$$
$$+8\,m^2 x^2 z^3 c^2 t^2 + 8\,m^2 y^2 z^3 c^2 t^2 - 20\,\varepsilon c^4 \mu\,z^5 t^2 + 4\,\varepsilon c^6 \mu\,t^4 z^3 + 24\,z\,x^2 y^2 c^2 t^2$$
$$+8\,z\,m^2 x^4 c^2 t^2 + 8\,z\,m^2 y^4 c^2 t^2 + 20\,z\,\varepsilon c^8 \mu\,t^6 - 10\,z\,m^2 x^2 c^4 t^4 - 10\,z\,m^2 y^2 c^4 t^4$$
$$-4\,c^5 t^3 \mu^2 \varepsilon z^2 - 2\,c^3 t\,\mu^2 \varepsilon z^4 + c^3 t^3 \mu\,m^2 y^2 + c\,t\,\mu\,m^2 x^4 + c^3 t^3 \mu\,m^2 x^2 + c\,t\,\mu\,m^2 y^4$$
$$+m\,c^4 t^3 \mu\,x^2 + m\,c^4 t^3 \mu\,y^2 + m\,c^2 t\,\mu\,z^4)\,)\,]$$

$$Dissipation = -\frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}\,(4\,\varepsilon c\,(-2\,x^4 z - 4\,y^2 x^2 z - 4\,x^2 z^3 - 8\,c^2 t^2 x^2 z$$
$$-2\,y^4 z - 4\,y^2 z^3 - 8\,y^2 c^2 t^2 z - 2\,z^5 - 8\,c^2 t^2 z^3 + 10\,z\,c^4 t^4 - c\,t\,\mu\,m^2 y^2 - c\,t\,\mu\,m^2 x^2$$
$$-c^3 t^3 \mu - c\,t\,\mu\,z^2)\,)$$

***************************** *END PROCEDURE* ***************************** **(32)**

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.
p=2, n=4

```
> NAME:=`Example 8a Index 1 Irreversible solution EdotB >0 (kinematic out) Type 2`;

> Holder:=(x^2+y^2+z^2-(c*t)^2)^(4/2);
> Ax:=c*t*1/Holder;Ay:=-z*1/Holder;Az:=y*1/Holder;phi:=+x*c*1/Holder;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := *Example 8a Index 1 Irreversible solution EdotB >0 (kinematic out) Type 2*

$$Holder := (x^2 + y^2 + z^2 - c^2 t^2)^2$$

$$Ax := \frac{c\,t}{(x^2 + y^2 + z^2 - c^2 t^2)^2}$$

$$Ay := -\frac{z}{(x^2 + y^2 + z^2 - c^2 t^2)^2}$$

$$Az := \frac{y}{(x^2+y^2+z^2-c^2 t^2)^2}$$

$$\phi := \frac{x c}{(x^2+y^2+z^2-c^2 t^2)^2}$$

Example 8a Index 1 Irreversible solution EdotB >0 (kinematic out) Type 2

*********************************** *Differential Form Format* ************************************

$$\text{Action 1-form} = -\frac{x c\, d(t)}{(-x^2-y^2-z^2+c^2 t^2)^2} + \frac{c t\, d(x)}{(-x^2-y^2-z^2+c^2 t^2)^2} - \frac{z\, d(y)}{(-x^2-y^2-z^2+c^2 t^2)^2} + \frac{y\, d(z)}{(-x^2-y^2-z^2+c^2 t^2)^2}$$

$$\text{Intensity 2-form } F=dA = \left(-\frac{c\,(3x^2-y^2-z^2+c^2 t^2)}{(-x^2-y^2-z^2+c^2 t^2)^3} + \frac{c\,(x^2+y^2+z^2+3c^2 t^2)}{(-x^2-y^2-z^2+c^2 t^2)^3} \right) (d(x))\ \&\wedge\ (d(t)) + \left(-\frac{4xz}{(-x^2-y^2-z^2+c^2 t^2)^3} - \frac{4cty}{(-x^2-y^2-z^2+c^2 t^2)^3} \right) (d(x))\ \&\wedge\ (d(y)) + \left(-\frac{4ctz}{(-x^2-y^2-z^2+c^2 t^2)^3} + \frac{4yx}{(-x^2-y^2-z^2+c^2 t^2)^3} \right) (d(x))\ \&\wedge\ (d(z)) + \left(-\frac{4xcy}{(-x^2-y^2-z^2+c^2 t^2)^3} - \frac{4zc^2 t}{(-x^2-y^2-z^2+c^2 t^2)^3} \right) (d(y))\ \&\wedge\ (d(t)) + \left(\frac{-x^2-y^2+3z^2+c^2 t^2}{(-x^2-y^2-z^2+c^2 t^2)^3} + \frac{-x^2+3y^2-z^2+c^2 t^2}{(-x^2-y^2-z^2+c^2 t^2)^3} \right) (d(y))\ \&\wedge\ (d(z)) + \left(-\frac{4czx}{(-x^2-y^2-z^2+c^2 t^2)^3} + \frac{4yc^2 t}{(-x^2-y^2-z^2+c^2 t^2)^3} \right) (d(z))\ \&\wedge\ (d(t))$$

$$\text{Topological Torsion 3-form } A\wedge F = \left(\frac{4xc\,(cty+xz)}{(-x^2-y^2-z^2+c^2 t^2)^5} + \frac{2zc\,(-x^2+y^2+z^2+c^2 t^2)}{(-x^2-y^2-z^2+c^2 t^2)^5} - \frac{4c^2 t\,(ctz+yx)}{(-x^2-y^2-z^2+c^2 t^2)^5} \right) \&\wedge(d(x), d(y), d(t)) + \left(-\frac{4y\,(cty+xz)}{(-x^2-y^2-z^2+c^2 t^2)^5} - \frac{4z\,(ctz-yx)}{(-x^2-y^2-z^2+c^2 t^2)^5} + \frac{2ct\,(-x^2+y^2+z^2+c^2 t^2)}{(-x^2-y^2-z^2+c^2 t^2)^5} \right) \&\wedge(d(x), d(y), d(z)) + \left(\frac{4c^2 t\,(cty-xz)}{(-x^2-y^2-z^2+c^2 t^2)^5} + \frac{4xc\,(ctz-yx)}{(-x^2-y^2-z^2+c^2 t^2)^5} \right.$$

$$-\frac{2\,y\,c\,(-x^2+y^2+z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5}\Bigg)\&\text{^}(d(x),d(z),d(t))+\Bigg(\frac{4\,y\,c\,(c\,t\,z+y\,x)}{(-x^2-y^2-z^2+c^2t^2)^5}$$

$$-\frac{4\,z\,c\,(c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}-\frac{2\,x\,c\,(-x^2+y^2+z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5}\Bigg)\&\text{^}(d(y),d(z),d(t))$$

Topological Parity 4-form $\quad F\text{^}F=\dfrac{8\,c\,\&\text{^}(d(x),d(y),d(z),d(t))}{(-x^2-y^2-z^2+c^2t^2)^4}$

******************************** *Using EM format* ****************************
****

$$E\ field=\left[\frac{2\,c\,(-x^2+y^2+z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3},-\frac{4\,c\,(c\,t\,z+y\,x)}{(-x^2-y^2-z^2+c^2t^2)^3},\frac{4\,c\,(c\,t\,y-x\,z)}{(-x^2-y^2-z^2+c^2t^2)^3}\right]$$

$$B\ field=\left[\frac{2\,(-x^2+y^2+z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3},\frac{4\,(c\,t\,z-y\,x)}{(-x^2-y^2-z^2+c^2t^2)^3},-\frac{4\,(c\,t\,y+x\,z)}{(-x^2-y^2-z^2+c^2t^2)^3}\right]$$

Topological TORSION 4 vector $\quad T4$ = -[ExA + Bphi,AdotB] $=\Bigg[-\dfrac{2\,x\,c}{(-x^2-y^2-z^2+c^2t^2)^4},$

$$-\frac{2\,y\,c}{(-x^2-y^2-z^2+c^2t^2)^4},-\frac{2\,c\,z}{(-x^2-y^2-z^2+c^2t^2)^4},-\frac{2\,c\,t}{(-x^2-y^2-z^2+c^2t^2)^4}\Bigg]$$

Helicity AdotB $=\dfrac{2\,c\,t}{(-x^2-y^2-z^2+c^2t^2)^4}$

Poincare II =2(E.B) $=\dfrac{8\,c}{(-x^2-y^2-z^2+c^2t^2)^4}$

coefficient of Topological Parity 4-form $=\dfrac{8\,c}{(-x^2-y^2-z^2+c^2t^2)^4}$

Pfaff Topological Dimension $\quad PTD=4$

****************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

Xm or linear (Mean) curvature $=\dfrac{4\,c\,t\,x\,(1+c^2)}{(-x^2-y^2-z^2+c^2t^2)^3}$

Yg or quadratic (GAUSS) curvature =

$$-\frac{3\,c^4t^2-c^2t^2-3\,c^2x^2+c^2y^2+c^2z^2+x^2-3\,y^2-3\,z^2}{(-x^2-y^2-z^2+c^2t^2)^5}$$

Za or Cubic (Interaction internal energy) curvature $=\dfrac{4\,c\,t\,x\,(1+c^2)}{(-x^2-y^2-z^2+c^2t^2)^7}$

$$\textit{Tk or quartic (4D expansion) curvature} \;=-\frac{3\,c^2}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^8}$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\textit{Chirality factor } CH=0$$

$$\textit{D field}=\left[\frac{2\,\varepsilon\,c\,\left(-x^2+y^2+z^2+c^2\,t^2\right)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3},-\frac{4\,\varepsilon\,c\,(c\,t\,z+y\,x)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3},\frac{4\,\varepsilon\,c\,(c\,t\,y-x\,z)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3}\right]$$

$$\textit{H field}=\left[\frac{2\,\left(-x^2+y^2+z^2+c^2\,t^2\right)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3\,\mu},\frac{4\,(c\,t\,z-y\,x)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3\,\mu},-\frac{4\,(c\,t\,y+x\,z)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^3\,\mu}\right]$$

$$\textit{Poynting vector ExH}=\left[\frac{32\,c^2\,t\,x\,\left(z^2+y^2\right)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^6\,\mu},\frac{16\,c^2\,\left(-x^2+y^2+z^2+c^2\,t^2\right)\,t\,y}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^6\,\mu},\frac{16\,c^2\,\left(-x^2+y^2+z^2+c^2\,t^2\right)\,t\,z}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^6\,\mu}\right]$$

Amperian Current 4Vector curlH-dD/dt=J4

$$=\left[\frac{8\,c\,t\,\left(-x^2+2\,y^2+2\,z^2+c^2\,t^2\right)\,\left(\varepsilon\,c^2\,\mu-1\right)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^4\,\mu},-\frac{4\,\left(x^2\,z+y^2\,z+z^3+5\,z\,c^2\,t^2+6\,c\,t\,y\,x\right)\,\left(\varepsilon\,c^2\,\mu-1\right)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^4\,\mu},\frac{4\,\left(y\,x^2+y^3+y\,z^2+5\,y\,c^2\,t^2-6\,c\,t\,x\,z\right)\,\left(\varepsilon\,c^2\,\mu-1\right)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^4\,\mu},0\right]$$

$$\textit{Amerian charge density} \quad \textit{divD} = \textit{rho}=0$$

$$\textit{divergence Lorentz Current 4Vector,} \quad \textit{4div(J4)} \;=0$$

$$\textit{Topological SPIN 4 vector S4}=\left[\frac{2\,x\,\left(2\,z^2+2\,y^2-\varepsilon\,c^2\,\mu\,x^2+\varepsilon\,c^2\,\mu\,y^2+\varepsilon\,c^2\,z^2\,\mu+\varepsilon\,c^4\,\mu\,t^2\right)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^5\,\mu},-\frac{2\,\left(y\,x^2-y^3-y\,z^2-3\,y\,c^2\,t^2-2\,c\,t\,x\,z+2\,\varepsilon\,c^3\,\mu\,t\,x\,z+2\,\varepsilon\,c^2\,\mu\,y\,x^2\right)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^5\,\mu},\frac{2\,\left(3\,z\,c^2\,t^2-2\,c\,t\,y\,x-x^2\,z+y^2\,z+z^3+2\,\varepsilon\,c^3\,\mu\,t\,y\,x-2\,\varepsilon\,c^2\,\mu\,x^2\,z\right)}{\left(-x^2-y^2-z^2+c^2\,t^2\right)^5\,\mu},\right.$$

$$\left.\frac{2\,\varepsilon\,c^2\,t\,(c^2\,t^2 - x^2 + 3\,y^2 + 3\,z^2)}{(-x^2 - y^2 - z^2 + c^2\,t^2)^5}\right]$$

Topological SPIN 3-form

$$= \frac{2\,x\,(2\,z^2 + 2\,y^2 - \varepsilon\,c^2\,\mu\,x^2 + \varepsilon\,c^2\,\mu\,y^2 + \varepsilon\,c^2\,z^2\,\mu + \varepsilon\,c^4\,\mu\,t^2)\ \&\wedge(d(y), d(z), d(t))}{(-x^2 - y^2 - z^2 + c^2\,t^2)^5\,\mu}$$

$$+ \frac{1}{(-x^2 - y^2 - z^2 + c^2\,t^2)^5\,\mu}\,(2\,(y\,x^2 - y^3 - y\,z^2 - 3\,y\,c^2\,t^2 - 2\,c\,t\,x\,z + 2\,\varepsilon\,c^3\,\mu\,t\,x\,z$$

$$+ 2\,\varepsilon\,c^2\,\mu\,y\,x^2)\ \&\wedge(d(x), d(z), d(t)))$$

$$+ \frac{1}{(-x^2 - y^2 - z^2 + c^2\,t^2)^5\,\mu}\,(2\,(3\,z\,c^2\,t^2 - 2\,c\,t\,y\,x - x^2\,z + y^2\,z + z^3 + 2\,\varepsilon\,c^3\,\mu\,t\,y\,x$$

$$- 2\,\varepsilon\,c^2\,\mu\,x^2\,z)\ \&\wedge(d(x), d(y), d(t)))$$

$$- \frac{2\,\varepsilon\,c^2\,t\,(c^2\,t^2 - x^2 + 3\,y^2 + 3\,z^2)\ \&\wedge(d(x), d(y), d(z))}{(-x^2 - y^2 - z^2 + c^2\,t^2)^5}$$

$$\textit{Spin density rho\_spin} = \frac{2\,\varepsilon\,c^2\,t\,(c^2\,t^2 - x^2 + 3\,y^2 + 3\,z^2)}{(-x^2 - y^2 - z^2 + c^2\,t^2)^5}$$

LaGrange field energy density (B.H-D.E) $= -\frac{1}{(-x^2 - y^2 - z^2 + c^2\,t^2)^6\,\mu}\,(4\,(x^4 + 2\,x^2\,y^2$

$$+ 2\,x^2\,z^2 - 2\,x^2\,c^2\,t^2 + y^4 + 2\,y^2\,z^2 + 6\,y^2\,c^2\,t^2 + z^4 + 6\,c^2\,t^2\,z^2 + c^4\,t^4)\,(\varepsilon\,c^2\,\mu - 1))$$

$$B.H = \frac{4\,(x^4 + 2\,x^2\,y^2 + 2\,x^2\,z^2 - 2\,x^2\,c^2\,t^2 + y^4 + 2\,y^2\,z^2 + 6\,y^2\,c^2\,t^2 + z^4 + 6\,c^2\,t^2\,z^2 + c^4\,t^4)}{(-x^2 - y^2 - z^2 + c^2\,t^2)^6\,\mu}$$

D.E

$$= \frac{4\,\varepsilon\,c^2\,(x^4 + 2\,x^2\,y^2 + 2\,x^2\,z^2 - 2\,x^2\,c^2\,t^2 + y^4 + 2\,y^2\,z^2 + 6\,y^2\,c^2\,t^2 + z^4 + 6\,c^2\,t^2\,z^2 + c^4\,t^4)}{(-x^2 - y^2 - z^2 + c^2\,t^2)^6}$$

A.J

$$= \frac{1}{(-x^2 - y^2 - z^2 + c^2\,t^2)^6\,\mu}\,(4\,(\varepsilon\,c^2\,\mu - 1)\,(-2\,x^2\,c^2\,t^2 + 9\,y^2\,c^2\,t^2 + 9\,c^2\,t^2\,z^2$$

$$+ 2\,c^4\,t^4 + x^2\,z^2 + 2\,y^2\,z^2 + z^4 + x^2\,y^2 + y^4))$$

$$\textit{-rho.phi} = 0$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)* $= -\frac{1}{(-x^2 - y^2 - z^2 + c^2\,t^2)^6\,\mu}\,(4\,(\varepsilon\,c^2\,\mu - 1)\,(x^4$

$$+ 3\,x^2\,y^2 + 3\,x^2\,z^2 - 4\,x^2\,c^2\,t^2 + 2\,y^4 + 4\,y^2\,z^2 + 15\,y^2\,c^2\,t^2 + 2\,z^4 + 15\,c^2\,t^2\,z^2 + 3\,c^4\,t^4))$$

London Coefficient *LC* = 0

PROCA coefficient curlcurlB =

$$\left[-\frac{1}{(-x^2-y^2-z^2+c^2t^2)^5}\left(8\left(-x^4+x^2y^2+2y^4+17c^2t^2z^2-4x^2c^2t^2+17y^2c^2t^2+2z^4+x^2z^2+4y^2z^2+5c^4t^4\right)\right),\right.$$

$$-\frac{24\left(-yxz^2+3ctx^2z+3y^2zct-7yxc^2t^2-yx^3-y^3x+3ctz^3+5c^3t^3z\right)}{(-x^2-y^2-z^2+c^2t^2)^5},$$

$$\left.\frac{24\left(3ctyx^2+3ctyz^2+7xzc^2t^2+3cty^3+5c^3t^3y+x^3z+xz^3+y^2zx\right)}{(-x^2-y^2-z^2+c^2t^2)^5}\right]$$

Amperian Current 4Vector   curlH-dD/dt=J4

$$=\left[\frac{8ct\left(-x^2+2y^2+2z^2+c^2t^2\right)\left(\varepsilon c^2\mu-1\right)}{(-x^2-y^2-z^2+c^2t^2)^4\mu},\right.$$

$$-\frac{4\left(x^2z+y^2z+z^3+5zc^2t^2+6ctyx\right)\left(\varepsilon c^2\mu-1\right)}{(-x^2-y^2-z^2+c^2t^2)^4\mu},$$

$$\left.\frac{4\left(yx^2+y^3+yz^2+5yc^2t^2-6ctxz\right)\left(\varepsilon c^2\mu-1\right)}{(-x^2-y^2-z^2+c^2t^2)^4\mu},0\right]$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B)   =

$$\left[-\frac{16\left(\varepsilon c^2\mu-1\right)x\left(z^2+y^2\right)\left(x^2+y^2+11c^2t^2+z^2\right)}{(-x^2-y^2-z^2+c^2t^2)^7\mu},\right.$$

$$-\frac{1}{(-x^2-y^2-z^2+c^2t^2)^7\mu}\left(8\left(\varepsilon c^2\mu-1\right)\left(-yx^4-8c^2t^2yx^2+y^5+2y^3z^2+14c^2t^2y^3+yz^4+14c^2t^2yz^2+9c^4t^4y+2ctx^3z+2cty^2zx+2ctxz^3-2c^3t^3xz\right)\right),$$

$$\left.-\frac{1}{(-x^2-y^2-z^2+c^2t^2)^7\mu}\left(8\left(\varepsilon c^2\mu-1\right)\left(-8c^2t^2x^2z-2ctyx^3+14y^2c^2t^2z-2cty^3x+14c^2t^2z^3-2ctyxz^2+9zc^4t^4+2c^3t^3yx-x^4z+y^4z+2y^2z^3+z^5\right)\right)\right]$$

Amperian Dissipation Jampere dot E = 0

Lorentz Force Spin factor LFSPIN = 0

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] =

$$\left[-\frac{2\,x\,c}{(-x^2-y^2-z^2+c^2\,t^2)^4},\ -\frac{2\,y\,c}{(-x^2-y^2-z^2+c^2\,t^2)^4},\ -\frac{2\,c\,z}{(-x^2-y^2-z^2+c^2\,t^2)^4},\ -\frac{2\,c\,t}{(-x^2-y^2-z^2+c^2\,t^2)^4} \right]$$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) =

$$\left[-\frac{4\,c^2\,t}{(-x^2-y^2-z^2+c^2\,t^2)^6},\ \frac{4\,z\,c}{(-x^2-y^2-z^2+c^2\,t^2)^6},\ -\frac{4\,y\,c}{(-x^2-y^2-z^2+c^2\,t^2)^6} \right]$$

Torsion Dissipation Jtorsion dot E $= \dfrac{4\,c^2\,x}{(-x^2-y^2-z^2+c^2\,t^2)^6}$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{2\,x\,(2\,z^2+2\,y^2-\varepsilon\,c^2\,\mu\,x^2+\varepsilon\,c^2\,\mu\,y^2+\varepsilon\,c^2\,z^2\,\mu+\varepsilon\,c^4\,\mu\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu},\right.$$

$$-\frac{2\,\left(y\,x^2-y^3-y\,z^2-3\,y\,c^2\,t^2-2\,c\,t\,x\,z+2\,\varepsilon\,c^3\,\mu\,t\,x\,z+2\,\varepsilon\,c^2\,\mu\,y\,x^2\right)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu},$$

$$\frac{2\,\left(3\,z\,c^2\,t^2-2\,c\,t\,y\,x-x^2\,z+y^2\,z+z^3+2\,\varepsilon\,c^3\,\mu\,t\,y\,x-2\,\varepsilon\,c^2\,\mu\,x^2\,z\right)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu},$$

$$\left.\frac{2\,\varepsilon\,c^2\,t\,(c^2\,t^2-x^2+3\,y^2+3\,z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5}\right]$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) =

$$\left[-\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^8\,\mu}\,(4\,c\,t\,(-2\,\varepsilon\,c^4\,\mu\,x^2\,t^2+4\,\varepsilon\,c^4\,\mu\,y^2\,t^2+4\,\varepsilon\,c^4\,\mu\,t^2\,z^2+\varepsilon\,c^6\,\mu\,t^4\right.$$

$$+\varepsilon\,c^2\,\mu\,x^4+4\,\varepsilon\,c^2\,\mu\,x^2\,y^2+4\,\varepsilon\,c^2\,\mu\,x^2\,z^2+3\,\varepsilon\,c^2\,\mu\,y^4+6\,\varepsilon\,c^2\,\mu\,y^2\,z^2+3\,\varepsilon\,c^2\,\mu\,z^4-6\,y^2\,c^2\,t^2$$

$$-2\,x^2\,y^2-4\,y^2\,z^2-2\,x^2\,z^2-2\,y^4-6\,c^2\,t^2\,z^2-2\,z^4)),\ \frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^8\,\mu}\,(4\,($$

$$-3\,z\,c^4\,t^4-4\,c^2\,t^2\,z^3+2\,\varepsilon\,c^3\,\mu\,t\,y\,x\,z^2+6\,\varepsilon\,c^4\,\mu\,t^2\,y^2\,z-2\,\varepsilon\,c^4\,\mu\,t^2\,x^2\,z-2\,y^2\,x^2\,z-2\,c\,t\,y\,x^3$$

$$-2\,c\,t\,y^3\,x+2\,c^3\,t^3\,y\,x-z^5+6\,\varepsilon\,c^4\,\mu\,t^2\,z^3+2\,z\,\varepsilon\,c^6\,\mu\,t^4+2\,\varepsilon\,c^3\,\mu\,t\,y\,x^3+2\,\varepsilon\,c^3\,\mu\,t\,y^3\,x$$

$$-2\,\varepsilon\,c^5\,\mu\,t^3\,y\,x-4\,y^2\,c^2\,t^2\,z+4\,c^2\,t^2\,x^2\,z-2\,c\,t\,y\,x\,z^2-y^4\,z-2\,y^2\,z^3-x^4\,z-2\,x^2\,z^3))$$

$$,\ -\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^8\,\mu}(4\,(-y^5-2\,\varepsilon\,c^3\,\mu\,t\,x^3\,z-2\,\varepsilon\,c^3\,\mu\,t\,x\,z^3+2\,\varepsilon\,c^5\,\mu\,t^3\,x\,z$$
$$+6\,\varepsilon\,c^4\,\mu\,y\,z^2\,t^2-2\,\varepsilon\,c^4\,\mu\,t^2\,y\,x^2-2\,\varepsilon\,c^3\,\mu\,t\,x\,z\,y^2-4\,c^2\,t^2\,y^3-3\,c^4\,t^4\,y-2\,y\,x^2\,z^2$$
$$+2\,c\,t\,y^2\,z\,x+6\,\varepsilon\,c^4\,\mu\,t^2\,y^3+2\,\varepsilon\,c^6\,\mu\,t^4\,y-y\,x^4-2\,y^3\,x^2-2\,y^3\,z^2-y\,z^4+4\,c^2\,t^2\,y\,x^2$$
$$+2\,c\,t\,x^3\,z-4\,c^2\,t^2\,y\,z^2+2\,c\,t\,x\,z^3-2\,c^3\,t^3\,x\,z))\,]$$

Spin Dissipation J_spin dot E $=\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^8\,\mu}(4\,c^3\,x\,(-8\,y^2\,t^2-8\,t^2\,z^2$
$$-2\,\varepsilon\,c^2\,\mu\,t^2\,x^2+\varepsilon\,c^4\,\mu\,t^4+\varepsilon\,\mu\,x^4+2\,\varepsilon\,\mu\,y^2\,x^2+\varepsilon\,\mu\,y^4+2\,\varepsilon\,\mu\,y^2\,z^2+6\,\varepsilon\,c^2\,\mu\,t^2\,z^2$$
$$+6\,\varepsilon\,c^2\,\mu\,t^2\,y^2+2\,\varepsilon\,\mu\,x^2\,z^2+\varepsilon\,\mu\,z^4))$$

Disspative Force 3 vector $=\Big[-\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^8\,\mu}(4\,(-4\,\varepsilon\,c^2\,\mu\,x^5\,z^2-8\,\varepsilon\,c^2\,\mu\,x^3\,z^4$

$$-4\,\varepsilon\,c^2\,z^6\,\mu\,x-40\,x^3\,c^4\,t^2\,\varepsilon\,\mu\,z^2-40\,\varepsilon\,c^4\,\mu\,z^4\,t^2\,x+44\,\varepsilon\,c^6\,\mu\,y^2\,t^4\,x+44\,\varepsilon\,c^6\,\mu\,t^4\,z^2\,x$$

$$+16\,y^2\,x^3\,z^2+12\,y^4\,x\,z^2+12\,y^2\,x\,z^4+c^7\,t^5\,\mu^2\,\varepsilon-6\,c^3\,t^3\,\mu\,y^2-44\,y^2\,c^4\,t^4\,x$$

$$-40\,\varepsilon\,c^4\,\mu\,t^2\,x^3\,y^2-2\,c\,t\,\mu\,x^2\,z^2-4\,c\,t\,\mu\,y^2\,z^2+4\,x^5\,y^2+8\,x^3\,y^4+4\,x^5\,z^2+8\,x^3\,z^4+4\,z^6\,x$$

$$-80\,y^2\,c^4\,t^2\,\varepsilon\,\mu\,x\,z^2+4\,c^3\,t\,\mu^2\,\varepsilon\,x^2\,y^2+4\,x\,y^6-4\,\varepsilon\,c^2\,\mu\,x^5\,y^2-8\,\varepsilon\,c^2\,\mu\,x^3\,y^4-4\,x\,\varepsilon\,c^2\,\mu\,y^6$$

$$+4\,c^5\,t^3\,\mu^2\,\varepsilon\,y^2+c^3\,t\,\mu^2\,\varepsilon\,x^4+3\,c^3\,t\,\mu^2\,\varepsilon\,y^4-2\,c\,t\,\mu\,x^2\,y^2+2\,c^2\,t\,\mu\,x^2\,y^2+2\,c^2\,t\,\mu\,y^2\,z^2$$

$$-2\,c^5\,t^3\,\mu^2\,\varepsilon\,x^2+2\,c^2\,t\,\mu\,x^2\,z^2+40\,x^3\,y^2\,c^2\,t^2+40\,x\,y^4\,c^2\,t^2+40\,x^3\,z^2\,c^2\,t^2+40\,x\,z^4\,c^2\,t^2$$

$$- 44\, x\, z^2\, c^4\, t^4 + 4\, c^3\, t\, \mu^2\, \varepsilon\, x^2\, z^2 + 6\, c^3\, t\, \mu^2\, \varepsilon\, y^2\, z^2 - 16\, \varepsilon\, c^2\, \mu\, y^2\, x^3\, z^2 - 12\, \varepsilon\, c^2\, \mu\, y^4\, x\, z^2$$

$$- 12\, \varepsilon\, c^2\, \mu\, y^2\, x\, z^4 - 40\, \varepsilon\, c^4\, \mu\, t^2\, x\, y^4 + 80\, x\, y^2\, c^2\, t^2\, z^2 + 4\, c^5\, t^3\, \mu^2\, \varepsilon\, z^2 + 3\, c^3\, t\, \mu^2\, \varepsilon\, z^4 + c^6\, t^5\, \mu$$

$$- 6\, c^3\, t^3\, \mu\, z^2 - 2\, c\, t\, \mu\, z^4 - 2\, c\, t\, \mu\, y^4 - 2\, c^4\, t^3\, \mu\, x^2 + c^2\, t\, \mu\, x^4 - 2\, c^4\, t^3\, \mu\, y^2 + c^2\, t\, \mu\, y^4$$

$$- 2\, c^4\, t^3\, \mu\, z^2 + c^2\, t\, \mu\, z^4)\big)\big), -\frac{1}{(-x^2 - y^2 - z^2 + c^2\, t^2)^8\, \mu} \big(4\, \big(12\, c^2\, t^2\, y\, x^2\, z^2$$

$$- 8\, y^2\, z^3\, \varepsilon\, c^3\, \mu\, t\, x + 8\, y^2\, c^5\, t^3\, \varepsilon\, \mu\, x\, z - 8\, x^3\, y^2\, \varepsilon\, c^3\, \mu\, t\, z - 4\, y^4\, \varepsilon\, c^3\, \mu\, t\, x\, z - 12\, c^4\, t^2\, y\, x^2\, \varepsilon\, z^2\, \mu$$

$$- 6\, \varepsilon\, c^2\, \mu\, y^5\, z^2 - 6\, \varepsilon\, c^2\, \mu\, y^3\, z^4 - 2\, \varepsilon\, c^2\, \mu\, y\, z^6 - 26\, \varepsilon\, c^4\, \mu\, t^2\, y^5 + 6\, y^5\, z^2 + 6\, y^3\, z^4 + 2\, y\, z^6$$

$$+ 2\, y^7 + \mu\, y^4\, z + 2\, \mu\, y^2\, z^3 + \mu\, x^4\, z + 2\, \mu\, x^2\, z^3 - z^5\, c\, \mu - 2\, y\, x^6 - 4\, c^3\, t\, x^5\, z\, \varepsilon\, \mu$$

$$+ 8\, c^5\, t^3\, x^3\, z\, \varepsilon\, \mu - 4\, c^7\, t^5\, x\, z\, \varepsilon\, \mu + 2\, \varepsilon\, c^2\, \mu\, y\, z^2\, x^4 - 4\, \varepsilon\, c^2\, \mu\, y^3\, z^2\, x^2 - 2\, \varepsilon\, c^2\, \mu\, y\, z^4\, x^2$$

$$+ 14\, \varepsilon\, c^4\, \mu\, t^2\, y\, x^4 - 12\, \varepsilon\, c^4\, \mu\, t^2\, y^3\, x^2 - 6\, \varepsilon\, c^4\, \mu^2\, t^2\, y^2\, z + 2\, \varepsilon\, c^4\, \mu^2\, t^2\, x^2\, z - 2\, \varepsilon\, c^3\, \mu^2\, t\, y\, x^3$$

$$- 2\, \varepsilon\, c^3\, \mu^2\, t\, y^3\, x + 2\, \varepsilon\, c^5\, \mu^2\, t^3\, y\, x + 2\, \mu\, c\, t\, y\, x\, z^2 + 26\, c^2\, t^2\, y^5 - 18\, c^6\, t^6\, y + 34\, x^2\, c^4\, t^4\, y$$

$$- 8\, x^3\, z^3\, \varepsilon\, c^3\, \mu\, t - 4\, z^5\, \varepsilon\, c^3\, \mu\, t\, x + 8\, c^5\, t^3\, z^3\, \varepsilon\, \mu\, x + 26\, c^2\, t^2\, y\, z^4 + \mu\, z^5 + 12\, c^2\, t^2\, y^3\, x^2$$

$$+ 52\, c^2\, t^2\, y^3\, z^2 - 10\, c^4\, t^4\, y\, z^2 - 14\, c^2\, t^2\, y\, x^4 - 2\, x^4\, y^3 + 8\, y^2\, z^3\, c\, t\, x - 8\, y^2\, c^3\, t^3\, x\, z$$

$$+ 8\, x^3\, y^2\, c\, t\, z + 2\, x^4\, y^3\, \varepsilon\, c^2\, \mu + 4\, y^4\, c\, t\, x\, z - 2\, y^5\, \varepsilon\, c^2\, \mu\, x^2 + 2\, y\, x^6\, \varepsilon\, c^2\, \mu - 2\, \varepsilon\, c^3\, \mu^2\, t\, y\, x\, z^2$$

$$+ 2\, \mu\, c\, t\, y\, x^3 + 2\, \mu\, c\, t\, y^3\, x - 2\, \mu\, c^3\, t^3\, y\, x - 6\, \varepsilon\, c^4\, \mu^2\, t^2\, z^3 - 2\, z\, \varepsilon\, c^6\, \mu^2\, t^4 + 4\, \mu\, y^2\, c^2\, t^2\, z$$

$$
\begin{aligned}
& -4\,\mu\,c^{2}\,t^{2}\,x^{2}\,z-2\,z\,c\,\mu\,x^{2}\,y^{2}+2\,z\,c^{3}\,\mu\,y^{2}\,t^{2}+8\,x^{3}\,z^{3}\,c\,t+4\,z^{5}\,c\,t\,x-8\,c^{3}\,t^{3}\,z^{3}\,x-2\,y^{7}\,\varepsilon\,c^{2}\,\mu \\
& +2\,z\,c^{3}\,\mu\,x^{2}\,t^{2}+10\,c^{6}\,t^{4}\,y^{3}\,\varepsilon\,\mu+18\,c^{8}\,t^{6}\,y\,\varepsilon\,\mu+2\,x^{2}\,y^{5}-34\,c^{6}\,t^{4}\,y\,x^{2}\,\varepsilon\,\mu-26\,c^{4}\,t^{2}\,y\,z^{4}\,\varepsilon\,\mu \\
& +4\,c\,t\,x^{5}\,z-8\,c^{3}\,t^{3}\,x^{3}\,z+4\,c^{5}\,t^{5}\,x\,z+3\,\mu\,z\,c^{4}\,t^{4}+4\,\mu\,c^{2}\,t^{2}\,z^{3}+2\,\mu\,y^{2}\,x^{2}\,z-2\,z^{3}\,c\,\mu\,x^{2} \\
& -z\,c\,\mu\,x^{4}-2\,z^{3}\,c\,\mu\,y^{2}-z\,c\,\mu\,y^{4}+2\,z^{3}\,c^{3}\,\mu\,t^{2}-z\,c^{5}\,\mu\,t^{4}-52\,c^{4}\,t^{2}\,y^{3}\,\varepsilon\,z^{2}\,\mu+10\,c^{6}\,t^{4}\,y\,\varepsilon\,z^{2}\,\mu \\
& -10\,y^{3}\,c^{4}\,t^{4}-2\,y\,z^{2}\,x^{4}+4\,y^{3}\,z^{2}\,x^{2}+2\,y\,z^{4}\,x^{2}\big)\big)\big),
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{\left(-x^{2}-y^{2}-z^{2}+c^{2}\,t^{2}\right)^{8}\mu}\Big(4\,\big(14\,z\,\varepsilon\,c^{4}\,\mu\,x^{4}\,t^{2}-34\,z\,\varepsilon\,c^{6}\,\mu\,x^{2}\,t^{4}-26\,z\,\varepsilon\,c^{4}\,\mu\,y^{4}\,t^{2} \\
& +10\,z\,\varepsilon\,c^{6}\,\mu\,y^{2}\,t^{4}-4\,\varepsilon\,c^{2}\,\mu\,x^{2}\,y^{2}\,z^{3}-12\,\varepsilon\,c^{4}\,\mu\,x^{2}\,t^{2}\,z^{3}-52\,\varepsilon\,c^{4}\,\mu\,y^{2}\,t^{2}\,z^{3}+34\,z\,x^{2}\,c^{4}\,t^{4} \\
& -10\,z\,c^{4}\,t^{4}\,y^{2}+52\,y^{2}\,z^{3}\,c^{2}\,t^{2}+12\,x^{2}\,c^{2}\,t^{2}\,z^{3}-2\,\varepsilon\,c^{2}\,\mu\,z^{7}+26\,z\,y^{4}\,c^{2}\,t^{2}-14\,z\,x^{4}\,c^{2}\,t^{2} \\
& +4\,c^{3}\,t\,y\,x^{5}\,\varepsilon\,\mu+8\,c^{3}\,t\,y^{3}\,x^{3}\,\varepsilon\,\mu-8\,c^{5}\,t^{3}\,y\,x^{3}\,\varepsilon\,\mu+4\,c^{3}\,t\,y^{5}\,x\,\varepsilon\,\mu-8\,c^{5}\,t^{3}\,y^{3}\,x\,\varepsilon\,\mu \\
& +4\,c^{7}\,t^{5}\,y\,x\,\varepsilon\,\mu+2\,x^{4}\,z\,\varepsilon\,c^{2}\,\mu\,y^{2}-2\,y^{4}\,z\,\varepsilon\,c^{2}\,\mu\,x^{2}+4\,x^{2}\,y^{2}\,z^{3}+8\,c^{3}\,t\,y\,x^{3}\,\varepsilon\,z^{2}\,\mu \\
& +8\,c^{3}\,t\,y^{3}\,x\,\varepsilon\,z^{2}\,\mu+4\,c^{3}\,t\,y\,x\,z^{4}\,\varepsilon\,\mu-8\,c^{5}\,t^{3}\,y\,x\,z^{2}\,\varepsilon\,\mu+2\,y^{6}\,z-2\,\varepsilon\,c^{3}\,\mu^{2}\,t\,x\,z\,y^{2} \\
& +6\,\varepsilon\,c^{4}\,\mu^{2}\,t^{2}\,y^{3}+2\,\varepsilon\,c^{6}\,\mu^{2}\,t^{4}\,y+4\,\mu\,c^{2}\,t^{2}\,y\,x^{2}+2\,\mu\,c\,t\,x^{3}\,z-4\,\mu\,c^{2}\,t^{2}\,y\,z^{2}+2\,\mu\,c\,t\,x\,z^{3} \\
& -2\,\mu\,c^{3}\,t^{3}\,x\,z+2\,y\,c\,\mu\,x^{2}\,z^{2}-2\,y\,c^{3}\,\mu\,t^{2}\,z^{2}-8\,c\,t\,y^{3}\,x^{3}-4\,c\,t\,y^{5}\,x+8\,c^{3}\,t^{3}\,y^{3}\,x \\
& -2\,y\,c^{3}\,\mu\,x^{2}\,t^{2}-8\,c\,t\,y\,x^{3}\,z^{2}-8\,c\,t\,y^{3}\,x\,z^{2}-4\,c\,t\,y\,x\,z^{4}+8\,c^{3}\,t^{3}\,y\,x\,z^{2}+2\,x^{6}\,z\,\varepsilon\,c^{2}\,\mu \\
& -2\,y^{6}\,z\,\varepsilon\,c^{2}\,\mu+2\,z^{7}-4\,c\,t\,y\,x^{5}+8\,c^{3}\,t^{3}\,y\,x^{3}-4\,c^{5}\,t^{5}\,y\,x-4\,\mu\,c^{2}\,t^{2}\,y^{3}-3\,\mu\,c^{4}\,t^{4}\,y \\
& -2\,\mu\,y\,x^{2}\,z^{2}+2\,y^{3}\,c\,\mu\,x^{2}+y\,c\,\mu\,x^{4}-2\,y^{3}\,c^{3}\,\mu\,t^{2}+2\,y^{3}\,c\,\mu\,z^{2}+y\,c\,\mu\,z^{4}+y\,c^{5}\,\mu\,t^{4} \\
& -12\,z\,\varepsilon\,c^{4}\,\mu\,x^{2}\,y^{2}\,t^{2}-2\,x^{6}\,z-\mu\,y^{5}+2\,\varepsilon\,c^{2}\,\mu\,x^{4}\,z^{3}-2\,\varepsilon\,c^{2}\,\mu\,x^{2}\,z^{5}-6\,\varepsilon\,c^{2}\,\mu\,y^{4}\,z^{3} \\
& -6\,\varepsilon\,c^{2}\,\mu\,y^{2}\,z^{5}-26\,\varepsilon\,c^{4}\,\mu\,z^{5}\,t^{2}+10\,\varepsilon\,c^{6}\,\mu\,t^{4}\,z^{3}+12\,z\,x^{2}\,y^{2}\,c^{2}\,t^{2}+18\,z\,\varepsilon\,c^{8}\,\mu\,t^{6}-\mu\,y\,x^{4} \\
& -2\,\mu\,y^{3}\,x^{2}-2\,\mu\,y^{3}\,z^{2}-\mu\,y\,z^{4}+y^{5}\,c\,\mu-2\,\varepsilon\,c^{3}\,\mu^{2}\,t\,x^{3}\,z-2\,\varepsilon\,c^{3}\,\mu^{2}\,t\,x\,z^{3}+2\,\varepsilon\,c^{5}\,\mu^{2}\,t^{3}\,x\,z \\
& +6\,\varepsilon\,c^{4}\,\mu^{2}\,y\,z^{2}\,t^{2}-2\,\varepsilon\,c^{4}\,\mu^{2}\,t^{2}\,y\,x^{2}+2\,\mu\,c\,t\,y^{2}\,z\,x-2\,x^{4}\,z\,y^{2}+2\,x^{2}\,y^{4}\,z-2\,x^{4}\,z^{3}+2\,x^{2}\,z^{5} \\
& +6\,y^{4}\,z^{3}+6\,y^{2}\,z^{5}+26\,c^{2}\,z^{5}\,t^{2}-10\,c^{4}\,t^{4}\,z^{3}-18\,z\,c^{6}\,t^{6}\big)\Big)\Big]
\end{aligned}
$$

$$
\mathit{Dissipation} = \frac{2\,c\,\left(\mu\,\varepsilon\,c^{3}\,t^{3}-\varepsilon\,c\,\mu\,t\,x^{2}+3\,\varepsilon\,c\,\mu\,y^{2}\,t+3\,c\,t\,z^{2}\,\varepsilon\,\mu+x^{3}+x\,y^{2}+x\,z^{2}-c^{2}\,t^{2}\,x\right)}{\left(-x^{2}-y^{2}-z^{2}+c^{2}\,t^{2}\right)^{5}}
$$

$$\text{******************************}\quad \textit{END PROCEDURE} \quad \text{******************************} \tag{33}$$

Enter the name of the problem, and the components of the 4 potential.

p=2 n=4

```
> NAME:=`Example 9-- Index 1 Irreversible solution EdotB =0 Type 1 + Type 2`;

> Holder:=(x^2+y^2+z^2-(c*t)^2)^(4/2);
> Ax:=(c*t+y)*1/Holder;Ay:=(-z-x)*1/Holder;Az:=(c*t+y)*1/Holder;phi:=(+x+z)*c*
  1/Holder;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 9-- Index 1 Irreversible solution EdotB =0 Type 1 + Type 2

$$Holder := (x^2+y^2+z^2-c^2t^2)^2$$

$$Ax := \frac{c\,t+y}{(x^2+y^2+z^2-c^2t^2)^2}$$

$$Ay := \frac{-z-x}{(x^2+y^2+z^2-c^2t^2)^2}$$

$$Az := \frac{c\,t+y}{(x^2+y^2+z^2-c^2t^2)^2}$$

$$\phi := \frac{(x+z)\,c}{(x^2+y^2+z^2-c^2t^2)^2}$$

Example 9-- Index 1 Irreversible solution EdotB =0 Type 1 + Type 2

********************************* *Differential Form Format* ********************

****

$$\textit{Action 1-form} = \frac{(-x\,c-z\,c)\,d(t)}{(-x^2-y^2-z^2+c^2t^2)^2} + \frac{(c\,t+y)\,d(x)}{(-x^2-y^2-z^2+c^2t^2)^2} + \frac{(-z-x)\,d(y)}{(-x^2-y^2-z^2+c^2t^2)^2} + \frac{(c\,t+y)\,d(z)}{(-x^2-y^2-z^2+c^2t^2)^2}$$

$$\textit{Intensity 2-form F=dA} = \left(-\frac{c\,(3x^2-y^2-z^2+c^2t^2+4\,x\,z)}{(-x^2-y^2-z^2+c^2t^2)^3} + \frac{c\,(x^2+y^2+z^2+3\,c^2t^2+4\,c\,t\,y)}{(-x^2-y^2-z^2+c^2t^2)^3} \right)(d(x))\ \&\wedge\ (d(t)) + \Bigg(-\frac{3x^2-y^2-z^2+c^2t^2+4\,x\,z}{(-x^2-y^2-z^2+c^2t^2)^3} - \frac{-x^2+3y^2-z^2+c^2t^2+4\,c\,t\,y}{(-x^2-y^2-z^2+c^2t^2)^3} \Bigg)(d(x))\ \&\wedge\ (d(y)) + \Bigg($$

$$- \frac{4\,(c\,t+y)\,z}{(-x^2-y^2-z^2+c^2t^2)^3} + \frac{4\,(c\,t+y)\,x}{(-x^2-y^2-z^2+c^2t^2)^3}\Bigg)\,(d(x))\ \&\wedge\ (d(z)) + \Bigg($$

$$- \frac{4\,(x+z)\,c\,y}{(-x^2-y^2-z^2+c^2t^2)^3} - \frac{4\,(x+z)\,c^2\,t}{(-x^2-y^2-z^2+c^2t^2)^3}\Bigg)\,(d(y))\ \&\wedge\ (d(t))$$

$$+ \left(\frac{-x^2-y^2+3\,z^2+c^2t^2+4\,x\,z}{(-x^2-y^2-z^2+c^2t^2)^3} + \frac{-x^2+3\,y^2-z^2+c^2t^2+4\,c\,t\,y}{(-x^2-y^2-z^2+c^2t^2)^3}\right)(d(y))\ \&\wedge\ (d(z))$$

$$+ \Bigg(- \frac{c\,(-x^2-y^2+3\,z^2+c^2t^2+4\,x\,z)}{(-x^2-y^2-z^2+c^2t^2)^3}$$

$$+ \frac{c\,(x^2+y^2+z^2+3\,c^2t^2+4\,c\,t\,y)}{(-x^2-y^2-z^2+c^2t^2)^3}\Bigg)\,(d(z))\ \&\wedge\ (d(t))$$

Topological Torsion 3-form $A\wedge F = \Bigg(\frac{2\,(x+z)\,c\,(x^2+y^2-z^2+c^2t^2+2\,x\,z+2\,c\,t\,y)}{(-x^2-y^2-z^2+c^2t^2)^5}$

$$+ \frac{2\,(x+z)\,c\,(-x^2+y^2+z^2+c^2t^2+2\,c\,t\,y-2\,x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5} - \frac{4\,(c\,t+y)^2\,(x+z)\,c}{(-x^2-y^2-z^2+c^2t^2)^5}\Bigg)$$

$$\&\wedge(d(x), d(y), d(t)) + \Bigg(- \frac{2\,(c\,t+y)\,(x^2+y^2-z^2+c^2t^2+2\,x\,z+2\,c\,t\,y)}{(-x^2-y^2-z^2+c^2t^2)^5}$$

$$+ \frac{4\,(x+z)\,(c\,t+y)\,(-z+x)}{(-x^2-y^2-z^2+c^2t^2)^5} + \frac{2\,(c\,t+y)\,(-x^2+y^2+z^2+c^2t^2+2\,c\,t\,y+2\,x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}\Bigg)$$

$$\&\wedge(d(x), d(y), d(z)) + \Bigg(\frac{2\,(c\,t+y)\,c\,(x^2+y^2-z^2+c^2t^2+2\,c\,t\,y-2\,x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}$$

$$- \frac{4\,(x+z)\,c\,(c\,t+y)\,(-z+x)}{(-x^2-y^2-z^2+c^2t^2)^5}$$

$$- \frac{2\,(c\,t+y)\,c\,(-x^2+y^2+z^2+c^2t^2+2\,c\,t\,y-2\,x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}\Bigg)\ \&\wedge(d(x), d(z), d(t))$$

$$+ \Bigg(\frac{4\,(c\,t+y)^2\,(x+z)\,c}{(-x^2-y^2-z^2+c^2t^2)^5} - \frac{2\,(x+z)\,c\,(x^2+y^2-z^2+c^2t^2+2\,c\,t\,y-2\,x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}$$

$$- \frac{2\,(x+z)\,c\,(-x^2+y^2+z^2+c^2t^2+2\,c\,t\,y+2\,x\,z)}{(-x^2-y^2-z^2+c^2t^2)^5}\Bigg)\ \&\wedge(d(y), d(z), d(t))$$

Topological Parity 4-form $F\wedge F = 0$

******************************** *Using EM format* ****************************
****

$$E\ field = \left[\frac{2\,c\,(-x^2+y^2+z^2+c^2 t^2+2\,c\,t\,y-2\,x\,z)}{(-x^2-y^2-z^2+c^2 t^2)^3},\ -\frac{4\,(x+z)\,c\,(c\,t+y)}{(-x^2-y^2-z^2+c^2 t^2)^3},\right.$$
$$\left. \frac{2\,c\,(x^2+y^2-z^2+c^2 t^2+2\,c\,t\,y-2\,x\,z)}{(-x^2-y^2-z^2+c^2 t^2)^3} \right]$$

$$B\ field = \left[\frac{2\,(-x^2+y^2+z^2+c^2 t^2+2\,c\,t\,y+2\,x\,z)}{(-x^2-y^2-z^2+c^2 t^2)^3},\ -\frac{4\,(c\,t+y)\,(-z+x)}{(-x^2-y^2-z^2+c^2 t^2)^3},\right.$$
$$\left. -\frac{2\,(x^2+y^2-z^2+c^2 t^2+2\,x\,z+2\,c\,t\,y)}{(-x^2-y^2-z^2+c^2 t^2)^3} \right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension PTD = 2

******************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

Xm or linear (Mean) curvature $= \dfrac{4\,c\,t\,(1+c^2)\,(x+z)}{(-x^2-y^2-z^2+c^2 t^2)^3}$

Yg or quadratic (GAUSS) curvature =

$$-\frac{1}{(-x^2-y^2-z^2+c^2 t^2)^5}\,(2\,(3\,c^4 t^2+4\,y\,c^3\,t-c^2 t^2-c^2 z^2-c^2 x^2-4\,c^2 z\,x+c^2 y^2$$
$$-4\,c\,t\,y-4\,x\,z-3\,y^2-x^2-z^2))$$

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

$$D\ field = \left[\frac{2\,\varepsilon\,c\,(-x^2+y^2+z^2+c^2 t^2+2\,c\,t\,y-2\,x\,z)}{(-x^2-y^2-z^2+c^2 t^2)^3},\ -\frac{4\,\varepsilon\,(x+z)\,c\,(c\,t+y)}{(-x^2-y^2-z^2+c^2 t^2)^3},\right.$$
$$\left. \frac{2\,\varepsilon\,c\,(x^2+y^2-z^2+c^2 t^2+2\,c\,t\,y-2\,x\,z)}{(-x^2-y^2-z^2+c^2 t^2)^3} \right]$$

$$H\ field = \left[\frac{2\,(-x^2+y^2+z^2+c^2 t^2+2\,c\,t\,y+2\,x\,z)}{(-x^2-y^2-z^2+c^2 t^2)^3\,\mu},\ -\frac{4\,(c\,t+y)\,(-z+x)}{(-x^2-y^2-z^2+c^2 t^2)^3\,\mu},\right.$$

$$-\frac{2\,(x^2+y^2-z^2+c^2t^2+2\,x\,z+2\,c\,t\,y)}{(-x^2-y^2-z^2+c^2t^2)^3\,\mu}\Bigg]$$

Poynting vector ExH $=\Bigg[\dfrac{16\,(c\,t+y)\,c\,x\,(z^2+x^2+y^2+2\,c\,t\,y+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^6\,\mu},$

$$\frac{8\,c\,(-x^4+y^4+6\,y^2c^2t^2-z^4+4\,c\,t\,y^3+4\,c^3t^3y-2\,x^2z^2+c^4t^4)}{(-x^2-y^2-z^2+c^2t^2)^6\,\mu},$$

$$\frac{16\,(c\,t+y)\,c\,z\,(z^2+x^2+y^2+2\,c\,t\,y+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^6\,\mu}\Bigg]$$

Amperian Current 4Vector   curlH-dD/dt=J4

$$=\Bigg[\frac{1}{(-x^2-y^2-z^2+c^2t^2)^4\,\mu}(4\,(-2\,c\,t\,x^2+4\,c\,t\,y^2+4\,c\,t\,z^2+2\,c^3t^3+5\,y\,c^2t^2$$

$$+y\,x^2+y^3+y\,z^2-6\,c\,t\,x\,z)\,(\varepsilon\,c^2\mu-1)),$$

$$-\frac{4\,(x+z)\,(x^2+y^2+z^2+5\,c^2t^2+6\,c\,t\,y)\,(\varepsilon\,c^2\mu-1)}{(-x^2-y^2-z^2+c^2t^2)^4\,\mu},$$

$$\frac{1}{(-x^2-y^2-z^2+c^2t^2)^4\,\mu}(4\,(4\,c\,t\,x^2+4\,c\,t\,y^2-2\,c\,t\,z^2+2\,c^3t^3+5\,y\,c^2t^2+y\,x^2+y^3$$

$$+y\,z^2-6\,c\,t\,x\,z)\,(\varepsilon\,c^2\mu-1)),0\Bigg]$$

Amerian charge density   divD = rho $=0$

divergence Lorentz Current 4Vector,   4div(J4) $=0$

Topological SPIN 4 vector S4 $=\Bigg[\dfrac{1}{(-x^2-y^2-z^2+c^2t^2)^5\,\mu}(2\,(x^3+2\,\varepsilon\,c^3\mu\,t\,y\,z-z\,c^2t^2-z^3$

$$+\varepsilon\,c^2z^3\mu-\varepsilon\,c^2\mu\,x^3+\varepsilon\,c^2\mu\,x\,y^2+\varepsilon\,c^4\mu\,x\,t^2+3\,x^2z-y^2z+3\,x\,y^2+\varepsilon\,c^4z\,\mu\,t^2-2\,c\,t\,y\,z$$

$$-3\,\varepsilon\,c^2\mu\,x^2z+\varepsilon\,c^2\mu\,y^2z+x\,z^2+6\,c\,t\,y\,x+3\,c^2t^2x-\varepsilon\,c^2\mu\,x\,z^2+2\,\varepsilon\,c^3\mu\,t\,y\,x)),$$

$$-\frac{4\,(c\,t+y)\,(-y^2-c^2t^2-2\,c\,t\,y-2\,x\,z+\varepsilon\,c^2\mu\,x^2+2\,\varepsilon\,c^2\mu\,x\,z+\varepsilon\,c^2z^2\mu)}{(-x^2-y^2-z^2+c^2t^2)^5\,\mu},$$

$$\frac{1}{(-x^2-y^2-z^2+c^2t^2)^5\,\mu}(2\,(-x^3+2\,\varepsilon\,c^3\mu\,t\,y\,z+3\,z\,c^2t^2+z^3-\varepsilon\,c^2z^3\mu+\varepsilon\,c^2\mu\,x^3$$

$$+\varepsilon c^2 \mu x y^2 + \varepsilon c^4 \mu x t^2 + x^2 z + 3 y^2 z - x y^2 + \varepsilon c^4 z \mu t^2 + 6 c t y z - \varepsilon c^2 \mu x^2 z + \varepsilon c^2 \mu y^2 z$$

$$+ 3 x z^2 - 2 c t y x - c^2 t^2 x - 3 \varepsilon c^2 \mu x z^2 + 2 \varepsilon c^3 \mu t y x)),$$

$$\left.\frac{4 (c t + y) \varepsilon c (z^2 + x^2 + y^2 + 2 c t y + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}\right]$$

$$\textit{Topological SPIN 3-form} = \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^5 \mu} \Big(2 \big(x^3 + 2 \varepsilon c^3 \mu t y z - z c^2 t^2 - z^3$$
$$+ \varepsilon c^2 z^3 \mu - \varepsilon c^2 \mu x^3 + \varepsilon c^2 \mu x y^2 + \varepsilon c^4 \mu x t^2 + 3 x^2 z - y^2 z + 3 x y^2 + \varepsilon c^4 z \mu t^2 - 2 c t y z$$
$$- 3 \varepsilon c^2 \mu x^2 z + \varepsilon c^2 \mu y^2 z + x z^2 + 6 c t y x + 3 c^2 t^2 x - \varepsilon c^2 \mu x z^2 + 2 \varepsilon c^3 \mu t y x\big)$$
$$\&\wedge(d(y), d(z), d(t))\Big) + \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^5 \mu} \Big(4 (c t + y) \big(-y^2 - c^2 t^2 - 2 c t y$$
$$- 2 x z + \varepsilon c^2 \mu x^2 + 2 \varepsilon c^2 \mu x z + \varepsilon c^2 z^2 \mu\big) \&\wedge(d(x), d(z), d(t))\Big)$$
$$+ \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^5 \mu} \Big(2 \big(-x^3 + 2 \varepsilon c^3 \mu t y z + 3 z c^2 t^2 + z^3 - \varepsilon c^2 z^3 \mu + \varepsilon c^2 \mu x^3$$
$$+ \varepsilon c^2 \mu x y^2 + \varepsilon c^4 \mu x t^2 + x^2 z + 3 y^2 z - x y^2 + \varepsilon c^4 z \mu t^2 + 6 c t y z - \varepsilon c^2 \mu x^2 z + \varepsilon c^2 \mu y^2 z$$
$$+ 3 x z^2 - 2 c t y x - c^2 t^2 x - 3 \varepsilon c^2 \mu x z^2 + 2 \varepsilon c^3 \mu t y x\big) \&\wedge(d(x), d(y), d(t))\Big)$$
$$- \frac{4 (c t + y) \varepsilon c (z^2 + x^2 + y^2 + 2 c t y + c^2 t^2) \&\wedge(d(x), d(y), d(z))}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}$$

$$\textit{Spin density rho\_spin} = \frac{4 (c t + y) \varepsilon c (z^2 + x^2 + y^2 + 2 c t y + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{8 (z^2 + x^2 + y^2 + 2 c t y + c^2 t^2)^2 (\varepsilon c^2 \mu - 1)}{(-x^2 - y^2 - z^2 + c^2 t^2)^6 \mu}$$

$$\textit{B.H} = \frac{8 (z^2 + x^2 + y^2 + 2 c t y + c^2 t^2)^2}{(-x^2 - y^2 - z^2 + c^2 t^2)^6 \mu}$$

$$\textit{D.E} = \frac{8 \varepsilon c^2 (z^2 + x^2 + y^2 + 2 c t y + c^2 t^2)^2}{(-x^2 - y^2 - z^2 + c^2 t^2)^6}$$

$$\textit{A.J} = \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^6 \mu} \Big(4 (\varepsilon c^2 \mu - 1) \big(x^4 + 3 x^2 y^2 + 2 y^4 + 7 c^2 t^2 z^2 + 7 x^2 c^2 t^2$$
$$+ 18 y^2 c^2 t^2 + z^4 + 10 c t y x^2 + 10 c t y z^2 - 2 x z c^2 t^2 + 10 c t y^3 + 14 c^3 t^3 y + 2 x^2 z^2$$
$$+ 3 y^2 z^2 + 4 c^4 t^4 + 2 x^3 z + 2 x z^3 + 2 y^2 z x\big)\Big)$$

$$-rho.phi = 0$$

Poincare I $\quad$ *(B.H - D.E)-(A.J - rho.phi)* $= -\frac{1}{(-x^2-y^2-z^2+c^2t^2)^6\mu}\big(4(\varepsilon c^2\mu-1)(3x^4 + 7x^2y^2 + 4y^4 + 11c^2t^2z^2 + 11x^2c^2t^2 + 30y^2c^2t^2 + 3z^4 + 18ctyx^2 + 18ctyz^2 - 2xzc^2t^2 + 18cty^3 + 22c^3t^3y + 6x^2z^2 + 7y^2z^2 + 6c^4t^4 + 2x^3z + 2xz^3 + 2y^2zx)\big)$

London Coefficient $\quad LC = 0$

PROCA coefficient curlcurlB $= \Big[-\frac{1}{(-x^2-y^2-z^2+c^2t^2)^5}\big(8(-x^4 + x^2y^2 + 2y^4 + 17c^2t^2z^2 - 4x^2c^2t^2 + 17y^2c^2t^2 + 2z^4 + 9ctyx^2 + 9ctyz^2 + 21xzc^2t^2 + 9cty^3 + 15c^3t^3y + x^2z^2 + 4y^2z^2 + 5c^4t^4 + 3x^3z + 3xz^3 + 3y^2zx)\big),$

$\frac{24(-z+x)(3ctx^2 + yx^2 + yz^2 + 3ctz^2 + 3cty^2 + 5c^3t^3 + 7yc^2t^2 + y^3)}{(-x^2-y^2-z^2+c^2t^2)^5},$

$\frac{1}{(-x^2-y^2-z^2+c^2t^2)^5}\big(8(2x^4 + 4x^2y^2 + 2y^4 - 4c^2t^2z^2 + 17x^2c^2t^2 + 17y^2c^2t^2 - z^4 + 9ctyx^2 + 9ctyz^2 + 21xzc^2t^2 + 9cty^3 + 15c^3t^3y + x^2z^2 + y^2z^2 + 5c^4t^4 + 3x^3z + 3xz^3 + 3y^2zx)\big)\Big]$

Amperian Current 4Vector $\quad$ *curlH-dD/dt=J4*

$= \Big[\frac{1}{(-x^2-y^2-z^2+c^2t^2)^4\mu}\big(4(-2ctx^2 + 4cty^2 + 4ctz^2 + 2c^3t^3 + 5yc^2t^2 + yx^2 + y^3 + yz^2 - 6ctxz)(\varepsilon c^2\mu-1)\big),$

$-\frac{4(x+z)(x^2+y^2+z^2+5c^2t^2+6cty)(\varepsilon c^2\mu-1)}{(-x^2-y^2-z^2+c^2t^2)^4\mu},$

$\frac{1}{(-x^2-y^2-z^2+c^2t^2)^4\mu}\big(4(4ctx^2 + 4cty^2 - 2ctz^2 + 2c^3t^3 + 5yc^2t^2 + yx^2 + y^3 + yz^2 - 6ctxz)(\varepsilon c^2\mu-1)\big), 0\Big]$

Lorentz Force 3 vector due to Ampere current $\quad$ *FL = -(rho_ampere E + J_ampere x B)* $= \Big[$

$$- \frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^7 \mu} \left(8 \left(\varepsilon c^2 \mu - 1\right) \left(36 x y^2 c^2 t^2 + 14 x z^2 c^2 t^2 - 2 c t y^3 z\right.\right.$$

$$- 2 c t y z^3 + 2 c^3 t^3 y z + x^5 + z c^4 t^4 + 2 y^2 x^2 z + 18 c t y x^3 + 18 c t y^3 x + 30 c^3 t^3 y x - z^5$$

$$+ 4 x^3 y^2 + 2 x^3 z^2 + 3 x y^4 + x z^4 - 4 c^2 t^2 x^2 z + 18 c t y x z^2 + 14 x^3 c^2 t^2 + 4 x y^2 z^2$$

$$\left.\left. + 9 c^4 t^4 x - 2 c t y z x^2 - y^4 z - 2 y^2 z^3 + 3 x^4 z + 2 x^2 z^3\right)\right),$$

$$- \frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^7 \mu} \left(16 \left(\varepsilon c^2 \mu - 1\right) \left(2 c^5 t^5 + 2 x y z^3 + y^5 - 3 c t x^4 + c^3 t^3 x^2\right.\right.$$

$$+ 16 y^2 c^3 t^3 + 3 y^2 x^2 c t + 3 y^2 z^2 c t - 6 c t x^2 z^2 + 14 c^2 t^2 y^3 + 9 c^4 t^4 y + 2 c t y^2 z x + y^3 x^2$$

$$+ y^3 z^2 - 2 z x y c^2 t^2 + 3 c^2 t^2 y x^2 + 2 c t x^3 z + 3 c^2 t^2 y z^2 + 2 c t x z^3 - 2 c^3 t^3 x z + 2 y x^3 z$$

$$\left.\left. + 6 y^4 c t + 2 y^3 x z + z^2 c^3 t^3 - 3 z^4 c t\right)\right), - \frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^7 \mu} \left(8 \left(\varepsilon c^2 \mu\right.\right.$$

$$\left. - 1\right) \left(-4 x z^2 c^2 t^2 + 18 c t y^3 z + 18 c t y z^3 + 30 c^3 t^3 y z - x^5 + 9 z c^4 t^4 + 14 c^2 t^2 z^3\right.$$

$$+ 4 y^2 x^2 z - 2 c t y x^3 - 2 c t y^3 x + 2 c^3 t^3 y x + z^5 - 2 x^3 y^2 + 2 x^3 z^2 - x y^4 + 3 x z^4$$

$$+ 36 y^2 c^2 t^2 z + 14 c^2 t^2 x^2 z - 2 c t y x z^2 + 2 x y^2 z^2 + c^4 t^4 x + 18 c t y z x^2 + 3 y^4 z + 4 y^2 z^3$$

$$\left.\left.\left. + x^4 z + 2 x^2 z^3\right)\right)\right]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{1}{\left(-x^2 - y^2 - z^2 + c^2 t^2\right)^5 \mu} \left(2 \left(x^3 + 2 \varepsilon c^3 \mu t y z - z c^2 t^2 - z^3 + \varepsilon c^2 z^3 \mu - \varepsilon c^2 \mu x^3\right.\right.\right.$$

$$+\varepsilon c^2 \mu x y^2+\varepsilon c^4 \mu x t^2+3 x^2 z-y^2 z+3 x y^2+\varepsilon c^4 z \mu t^2-2 c t y z-3 \varepsilon c^2 \mu x^2 z$$

$$+\varepsilon c^2 \mu y^2 z+x z^2+6 c t y x+3 c^2 t^2 x-\varepsilon c^2 \mu x z^2+2 \varepsilon c^3 \mu t y x)),$$

$$-\frac{4 (c t+y) (-y^2-c^2 t^2-2 c t y-2 x z+\varepsilon c^2 \mu x^2+2 \varepsilon c^2 \mu x z+\varepsilon c^2 z^2 \mu)}{(-x^2-y^2-z^2+c^2 t^2)^5 \mu},$$

$$\frac{1}{(-x^2-y^2-z^2+c^2 t^2)^5 \mu} (2 (-x^3+2 \varepsilon c^3 \mu t y z+3 z c^2 t^2+z^3-\varepsilon c^2 z^3 \mu+\varepsilon c^2 \mu x^3$$

$$+\varepsilon c^2 \mu x y^2+\varepsilon c^4 \mu x t^2+x^2 z+3 y^2 z-x y^2+\varepsilon c^4 z \mu t^2+6 c t y z-\varepsilon c^2 \mu x^2 z+\varepsilon c^2 \mu y^2 z$$

$$+3 x z^2-2 c t y x-c^2 t^2 x-3 \varepsilon c^2 \mu x z^2+2 \varepsilon c^3 \mu t y x)),$$

$$\left.\frac{4 (c t+y) \varepsilon c (z^2+x^2+y^2+2 c t y+c^2 t^2)}{(-x^2-y^2-z^2+c^2 t^2)^5}\right]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= \Bigg[$

$$-\frac{8 (z^2+x^2+y^2+2 c t y+c^2 t^2)^2 (c t+y) (\varepsilon c^2 \mu-1)}{(-x^2-y^2-z^2+c^2 t^2)^8 \mu},$$

$$\frac{8 (x+z) (z^2+x^2+y^2+2 c t y+c^2 t^2)^2 (\varepsilon c^2 \mu-1)}{(-x^2-y^2-z^2+c^2 t^2)^8 \mu},$$

$$\left.-\frac{8 (z^2+x^2+y^2+2 c t y+c^2 t^2)^2 (c t+y) (\varepsilon c^2 \mu-1)}{(-x^2-y^2-z^2+c^2 t^2)^8 \mu}\right]$$

Spin Dissipation J_spin dot E $= \dfrac{8 c (x+z) (z^2+x^2+y^2+2 c t y+c^2 t^2)^2 (\varepsilon c^2 \mu-1)}{(-x^2-y^2-z^2+c^2 t^2)^8 \mu}$

Disspative Force 3 vector $= \left[-\dfrac{1}{(-x^2-y^2-z^2+c^2 t^2)^8 \mu} (8 (\varepsilon c^2 \mu-1) (-y^4 z c^2 t^2\right.$

$$-2 y^2 z^3 c^2 t^2+\mu c t x^4+2 \mu c^3 t^3 x^2+10 \mu y^2 c^3 t^3+10 \mu c^2 t^2 y^3+5 \mu c^4 t^4 y+2 \mu y x^2 z^2$$

$$+5 \mu y^4 c t+2 \mu z^2 c^3 t^3+\mu z^4 c t-4 c^3 t^3 y^3 z+2 c t y z^5-13 x z^4 c^2 t^2+5 x z^2 c^4 t^4$$

$$+ 2\, c\, t\, y^5 z + 4\, c\, t\, y^3 z^3 - 4\, c^3 t^3 y\, z^3 - 46\, x^3 y^2 c^2 t^2 - 33\, x\, y^4 c^2 t^2 + 27\, x\, y^2 c^4 t^4$$

$$- 26\, x^3 z^2 c^2 t^2 + 2\, c^5 t^5 y\, z - 5\, z\, c^4 t^4 x^2 - z\, c^4 t^4 y^2 - 18\, c\, t\, y\, x^5 - 36\, c\, t\, y^3 x^3 - 12\, c^3 t^3 y\, x^3$$

$$- 18\, c\, t\, y^5 x - 12\, c^3 t^3 y^3 x + 30\, c^5 t^5 y\, x + 7\, c^2 t^2 x^4 z + 6\, c^2 t^2 x^2 z^3 + 3\, z^5 y^2 - 7\, x^3 y^4$$

$$- 3\, x^3 z^4 - 3\, x\, y^6 - x\, z^6 + y^6 z + 3\, y^4 z^3 - 3\, x^6 z - 5\, x^4 z^3 + \mu\, y^5 - x^7 - 5\, x^5 y^2 - 3\, x^5 z^2$$

$$- z^5 x^2 + \mu\, c^5 t^5 + \mu\, y\, x^4 + 2\, \mu\, y^3 x^2 + 2\, \mu\, y^3 z^2 + \mu\, y\, z^4 - 2\, y^2 x^2 z^3 - y^4 x^2 z + 5\, x^3 c^4 t^4$$

$$- 5\, y^2 x^4 z - 7\, x\, y^4 z^2 - 5\, x\, z^4 y^2 - 10\, x^3 y^2 z^2 - z^3 c^4 t^4 + z\, c^6 t^6 - z^5 c^2 t^2 + 9\, c^6 t^6 x + z^7$$

$$+ 4\, c\, t\, y^3 z\, x^2 + 4\, c\, t\, y\, z^3 x^2 - 4\, c^3 t^3 y\, z\, x^2 + 6\, y^2 x^2 z\, c^2 t^2 - 36\, c\, t\, y\, x^3 z^2 - 36\, c\, t\, y^3 x\, z^2$$

$$- 12\, c^3 t^3 y\, x\, z^2 - 18\, c\, t\, y\, x\, z^4 + 2\, c\, t\, y\, z\, x^4 + 6\, \mu\, y^2 x^2 c\, t + 6\, \mu\, y^2 z^2 c\, t + 2\, \mu\, c\, t\, x^2 z^2$$

$$+ 6\, \mu\, c^2 t^2 y\, x^2 + 6\, \mu\, c^2 t^2 y\, z^2 - 46\, x\, y^2 c^2 t^2 z^2 - 13\, x^5 c^2 t^2)),$$

$$- \frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^8 \mu} \left(8\, (\varepsilon\, c^2 \mu - 1)\, (4\, c^7 t^7 - 4\, y^5 x^2 - 4\, y^5 z^2 - 2\, y^3 x^4 - 2\, y^3 z^4\right.$$

$$- \mu\, x^5 - \mu\, z^5 - 26\, y^5 c^2 t^2 + 6\, c\, t\, x^6 - 8\, c^3 t^3 x^4 - 20\, y^4 c^3 t^3 + 10\, c^4 t^4 y^3 + 18\, c^6 t^6 y$$

$$- 4\, y^3 x^2 z^2 - 4\, y\, x^5 z - 8\, y^3 x^3 z - 12\, y^6 c\, t - 4\, y^5 x\, z - 8\, z^4 c^3 t^3 + 6\, z^6 c\, t - 2\, \mu\, x^3 y^2$$

$$- 2\, \mu\, x^3 z^2 - \mu\, x\, y^4 - \mu\, x\, z^4 - \mu\, y^4 z - 2\, \mu\, y^2 z^3 - \mu\, x^4 z - 2\, \mu\, x^2 z^3 - 2\, c^5 t^5 x^2 + 28\, c^5 t^5 y^2$$

$$- 2\, c^5 t^5 z^2 - 8\, x^3 y\, z^3 - 8\, x\, y^3 z^3 - 4\, x\, y\, z^5 - 4\, c\, t\, x\, z^5 + 8\, c^3 t^3 x\, z^3 - 4\, c^5 t^5 x\, z - \mu\, z\, c^4 t^4$$

$$- 2\, \mu\, c^2 t^2 z^3 - 2\, \mu\, y^2 x^2 z - 2\, \mu\, x^3 c^2 t^2 - 2\, \mu\, x\, y^2 z^2 - \mu\, c^4 t^4 x - 2\, y^7 + 18\, c\, t\, x^4 z^2$$

$$- 28\, c^3 t^3 x^2 y^2 - 16\, c^3 t^3 x^2 z^2 - 28\, y^2 c^3 t^3 z^2 - 18\, y^4 x^2 c\, t - 18\, y^4 z^2 c\, t + 18\, c\, t\, x^2 z^4$$

$$- 32\, c^2 t^2 y^3 x^2 - 32\, c^2 t^2 y^3 z^2 - 12\, c^4 t^4 y\, x^2 - 12\, c^4 t^4 y\, z^2 - 6\, c^2 t^2 y\, x^4 - 4\, c\, t\, x^5 z$$

$$- 8\, c\, t\, x^3 z^3 + 8\, c^3 t^3 x^3 z - 6\, c^2 t^2 y\, z^4 + 8\, c^3 t^3 y^2 z\, x + 8\, z\, x^3 y\, c^2 t^2 + 8\, z\, x\, y^3 c^2 t^2$$

$$- 4\, z\, x\, y\, c^4 t^4 - 12\, c^2 t^2 y\, x^2 z^2 - 6\, \mu\, x\, y^2 c^2 t^2 - 2\, \mu\, x\, z^2 c^2 t^2 - 4\, \mu\, c\, t\, y^3 z - 4\, \mu\, c\, t\, y\, z^3$$

$$- 4\, \mu\, c^3 t^3 y\, z - 4\, \mu\, c\, t\, y\, x^3 - 4\, \mu\, c\, t\, y^3 x - 4\, \mu\, c^3 t^3 y\, x - 6\, \mu\, y^2 c^2 t^2 z - 2\, \mu\, c^2 t^2 x^2 z$$

$$+ 8\, x\, y\, z^3 c^2 t^2 - 8\, c\, t\, y^2 z\, x^3 - 4\, c\, t\, y^4 z\, x - 8\, c\, t\, y^2 z^3 x - 4\, \mu\, c\, t\, y\, x\, z^2 - 4\, \mu\, c\, t\, y\, z\, x^2)\,)\,,$$

$$-\frac{1}{(-x^2 - y^2 - z^2 + c^2 t^2)^8\, \mu} \left(8\, (\varepsilon\, c^2 \mu - 1)\, (-33\, y^4 z\, c^2 t^2 - 46\, y^2 z^3 c^2 t^2 + \mu\, c\, t\, x^4\right.$$
$$+ 2\, \mu\, c^3 t^3 x^2 + 10\, \mu\, y^2 c^3 t^3 + 10\, \mu\, c^2 t^2 y^3 + 5\, \mu\, c^4 t^4 y + 2\, \mu\, y\, x^2 z^2 + 5\, \mu\, y^4 c\, t + 2\, \mu\, z^2 c^3 t^3$$
$$+ \mu\, z^4 c\, t - 12\, c^3 t^3 y^3 z - 18\, c\, t\, y\, z^5 + 7\, x\, z^4 c^2 t^2 - 5\, x\, z^2 c^4 t^4 - 18\, c\, t\, y^5 z - 36\, c\, t\, y^3 z^3$$
$$- 12\, c^3 t^3 y\, z^3 - 2\, x^3 y^2 c^2 t^2 - x\, y^4 c^2 t^2 - x\, y^2 c^4 t^4 + 6\, x^3 z^2 c^2 t^2 + 30\, c^5 t^5 y\, z + 5\, z\, c^4 t^4 x^2$$
$$+ 27\, z\, c^4 t^4 y^2 + 2\, c\, t\, y\, x^5 + 4\, c\, t\, y^3 x^3 - 4\, c^3 t^3 y\, x^3 + 2\, c\, t\, y^5 x - 4\, c^3 t^3 y^3 x + 2\, c^5 t^5 y\, x$$
$$- 13\, c^2 t^2 x^4 z - 26\, c^2 t^2 x^2 z^3 - 5\, z^5 y^2 + 3\, x^3 y^4 - 5\, x^3 z^4 + x\, y^6 - 3\, x\, z^6 - 3\, y^6 z - 7\, y^4 z^3$$
$$- x^6 z - 3\, x^4 z^3 + \mu\, y^5 + x^7 + 3\, x^5 y^2 - x^5 z^2 - 3\, z^5 x^2 + \mu\, c^5 t^5 + \mu\, y\, x^4 + 2\, \mu\, y^3 x^2 + 2\, \mu\, y^3 z^2$$
$$+ \mu\, y\, z^4 - 10\, y^2 x^2 z^3 - 7\, y^4 x^2 z - x^3 c^4 t^4 - 5\, y^2 x^4 z - x\, y^4 z^2 - 5\, x\, z^4 y^2 - 2\, x^3 y^2 z^2$$
$$+ 5\, z^3 c^4 t^4 + 9\, z\, c^6 t^6 - 13\, z^5 c^2 t^2 + c^6 t^6 x - z^7 - 36\, c\, t\, y^3 z\, x^2 - 36\, c\, t\, y\, z^3 x^2 - 12\, c^3 t^3 y\, z\, x^2$$
$$- 46\, y^2 x^2 z\, c^2 t^2 + 4\, c\, t\, y\, x^3 z^2 + 4\, c\, t\, y^3 x\, z^2 - 4\, c^3 t^3 y\, x\, z^2 + 2\, c\, t\, y\, x\, z^4 - 18\, c\, t\, y\, z\, x^4$$
$$+ 6\, \mu\, y^2 x^2 c\, t + 6\, \mu\, y^2 z^2 c\, t + 2\, \mu\, c\, t\, x^2 z^2 + 6\, \mu\, c^2 t^2 y\, x^2 + 6\, \mu\, c^2 t^2 y\, z^2 + 6\, x\, y^2 c^2 t^2 z^2$$
$$- x^5 c^2 t^2)\,)\,]$$

$$\mathit{Dissipation} = \frac{4\, \mu\, (c\, t + y)\, \varepsilon\, c\, (z^2 + x^2 + y^2 + 2\, c\, t\, y + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5}$$

****************************** *END PROCEDURE* ****************************** **(34)**

Example Saturn's Rings Plasma Accretion Disc Hedge Hog Solution

****

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`Example 10a  Saturns Rings  -- a Plasma Accretion disc from a Hedge Hog
  solution.  p201 vol4`:
> Holder:=(1*x^2+1*y^2+1*c*z^2)^(1/2);Holder2:=(1*x^2+1*y^2);
> A1:=(alpha*z*m/Holder2/Holder*(-y));A2:=(alpha*z*m/Holder2/Holder*x);
> A3:=0; phi:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,c,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

$$Holder := \sqrt{x^2 + y^2 + c\,z^2}$$

$$Holder2 := x^2 + y^2$$

$$A1 := -\frac{\alpha\, z\, m\, y}{(x^2+y^2)\sqrt{x^2+y^2+c\,z^2}}$$

$$A2 := \frac{\alpha\, z\, m\, x}{(x^2+y^2)\sqrt{x^2+y^2+c\,z^2}}$$

$$A3 := 0$$

$$\phi := 0$$

Example 10a Saturns Rings -- a Plasma Accretion disc from a Hedge Hog solution. p201 vol4

********************************* *Differential Form Format* ********************
****

$$Action\ 1\text{-}form = -\frac{\alpha\, z\, m\, y\, d(x)}{(x^2+y^2)\sqrt{x^2+y^2+c\,z^2}} + \frac{\alpha\, z\, m\, x\, d(y)}{(x^2+y^2)\sqrt{x^2+y^2+c\,z^2}}$$

$$Intensity\ 2\text{-}form\ F{=}dA \;= \left(\frac{\alpha\, z\, m\,(-x^2 y^2 - 2\,y^4 - z^2 y^2 c + x^4 + z^2 x^2 c)}{(x^2+y^2)^2\,(x^2+y^2+c\,z^2)^{3/2}} - \frac{\alpha\, z\, m\,(2\,x^4 + x^2 y^2 + z^2 x^2 c - y^4 - z^2 y^2 c)}{(x^2+y^2)^2\,(x^2+y^2+c\,z^2)^{3/2}}\right)(d(x))\ \&\wedge\ (d(y)) + \frac{\alpha\, m\, y\,(d(x))\ \&\wedge\ (d(z))}{(x^2+y^2+c\,z^2)^{3/2}} - \frac{\alpha\, m\, x\,(d(y))\ \&\wedge\ (d(z))}{(x^2+y^2+c\,z^2)^{3/2}}$$

Topological Torsion 3-form $A\wedge F = 0$

Topological Parity 4-form $F\wedge F = 0$

********************************* *Using EM format* *****************************
****

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[-\frac{\alpha m x}{(x^2+y^2+c z^2)^{3/2}}, -\frac{\alpha m y}{(x^2+y^2+c z^2)^{3/2}}, -\frac{\alpha z m}{(x^2+y^2+c z^2)^{3/2}} \right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension PTD = 2

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

Xm or linear (Mean) curvature = 0

$$Yg\ or\ quadratic\ (GAUSS)\ curvature = -\frac{(2x^2+2y^2+c z^2)\,\alpha^2 z^2 m^2}{(x^2+y^2+c z^2)^2\,(x^2+y^2)^2}$$

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

D field = [0, 0, 0]

$$H\ field = \left[-\frac{\alpha m x}{(x^2+y^2+c z^2)^{3/2}\mu}, -\frac{\alpha m y}{(x^2+y^2+c z^2)^{3/2}\mu}, -\frac{\alpha z m}{(x^2+y^2+c z^2)^{3/2}\mu} \right]$$

Poynting vector ExH = *EXH*

$$Amperian\ Current\ 4Vector\quad curlH\text{-}dD/dt=J4 = \left[-\frac{3\alpha z m y(-1+c)}{(x^2+y^2+c z^2)^{5/2}\mu}, \frac{3\alpha z m x(-1+c)}{(x^2+y^2+c z^2)^{5/2}\mu}, 0, 0 \right]$$

Amerian charge density divD = rho = 0

divergence Lorentz Current 4Vector, 4div(J4) = 0

$$Topological\ SPIN\ 4\ vector\ S4 = \left[-\frac{\alpha^2 z^2 m^2 x}{(x^2+y^2)(x^2+y^2+c z^2)^2\mu}, -\frac{\alpha^2 z^2 m^2 y}{(x^2+y^2)(x^2+y^2+c z^2)^2\mu}, \frac{\alpha^2 z m^2}{(x^2+y^2+c z^2)^2\mu}, 0 \right]$$

$$Topological\ SPIN\ 3\text{-}form = -\frac{\alpha^2 z^2 m^2 x\ \&\wedge(d(y), d(z), d(t))}{(x^2+y^2)(x^2+y^2+c z^2)^2\mu}$$

$$+\frac{\alpha^2 z^2 m^2 y \,\&\wedge(d(x), d(z), d(t))}{(x^2+y^2)\,(x^2+y^2+c z^2)^2 \mu} + \frac{\alpha^2 z\, m^2 \,\&\wedge(d(x), d(y), d(t))}{(x^2+y^2+c z^2)^2 \mu}$$

Spin density rho_spin = 0

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{\alpha^2 m^2 (x^2+y^2+z^2)}{(x^2+y^2+c z^2)^3 \mu}$$

$$B.H = \frac{\alpha^2 m^2 (x^2+y^2+z^2)}{(x^2+y^2+c z^2)^3 \mu}$$

$$D.E = 0$$

$$A.J = \frac{3 \alpha^2 z^2 m^2 (-1+c)}{(x^2+y^2+c z^2)^3 \mu}$$

$$-rho.phi = 0$$

$$\textit{Poincare I} \qquad \textit{(B.H - D.E)-(A.J - rho.phi)} = -\frac{\alpha^2 m^2 (-x^2-y^2-4 z^2+3 c z^2)}{(x^2+y^2+c z^2)^3 \mu}$$

$$\textit{London Coefficient} \qquad LC = \frac{3 (-1+c) (x^2+y^2)}{(x^2+y^2+c z^2)^2 \mu}$$

$$\textit{PROCA coefficient curlcurlB} = \left[\frac{3 \alpha m x (-1+c) (4 c z^2 - y^2 - x^2)}{(x^2+y^2+c z^2)^{7/2}}, \frac{3 \alpha m y (-1+c) (4 c z^2 - y^2 - x^2)}{(x^2+y^2+c z^2)^{7/2}}, \frac{3 \alpha z m (-1+c) (2 c z^2 - 3 y^2 - 3 x^2)}{(x^2+y^2+c z^2)^{7/2}} \right]$$

$$\textit{Amperian Current 4Vector} \qquad \textit{curlH-dD/dt=J4} = \left[-\frac{3 \alpha z m y (-1+c)}{(x^2+y^2+c z^2)^{5/2} \mu}, \frac{3 \alpha z m x (-1+c)}{(x^2+y^2+c z^2)^{5/2} \mu}, 0, 0 \right]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B)

$$= \left[\frac{3 \alpha^2 z^2 m^2 x (-1+c)}{(x^2+y^2+c z^2)^4 \mu}, \frac{3 \alpha^2 z^2 m^2 y (-1+c)}{(x^2+y^2+c z^2)^4 \mu}, -\frac{3 \alpha^2 z m^2 (-1+c) (x^2+y^2)}{(x^2+y^2+c z^2)^4 \mu} \right]$$

Amperian Dissipation Jampere dot E = 0

$$\textit{Lorentz Force Spin factor LFSPIN} = -\frac{1}{3} \frac{(x^2+y^2+c z^2)^2}{(x^2+y^2) (-1+c)}$$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD] =

$$\left[-\frac{\alpha^2 z^2 m^2 x}{(x^2+y^2)\,(x^2+y^2+c\,z^2)^2\,\mu}, -\frac{\alpha^2 z^2 m^2 y}{(x^2+y^2)\,(x^2+y^2+c\,z^2)^2\,\mu}, \frac{\alpha^2 z\, m^2}{(x^2+y^2+c\,z^2)^2\,\mu}, 0 \right]$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B) =

$$\left[-\frac{\alpha^3 z\, m^3 y\,(x^2+y^2+z^2)}{(x^2+y^2+c\,z^2)^{7/2}\,(x^2+y^2)\,\mu}, \frac{\alpha^3 z\, m^3 x\,(x^2+y^2+z^2)}{(x^2+y^2+c\,z^2)^{7/2}\,(x^2+y^2)\,\mu}, 0 \right]$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector =

$$\left[\frac{1}{(x^2+y^2+c\,z^2)^{15/2}\,\mu\,(x^2+y^2)} \left(\alpha^2 z\, m^2 \left(3\,z\,x\,(x^2+y^2+c\,z^2)^{7/2}\, c\, y^2 - 5\,\alpha\, m\, y^9\,\mu\, x^2 - 10\,\alpha\, m\, y^7\,\mu\, x^4 - 5\,\alpha\, m\, y^3\,\mu\, x^8 - 10\,\alpha\, m\, y^5\,\mu\, x^6 - \alpha\, m\, y^9\,\mu\, z^2 \right.\right.\right.$$

$$- 4\,\alpha\, m\, y\,\mu\, x^8\, c\, z^2 - \alpha\, m\, y\,\mu\, x^{10} - 3\,z\,x^3\,(x^2+y^2+c\,z^2)^{7/2} - 4\,\alpha\, m\, y^9\,\mu\, c\, z^2$$

$$- 6\,\alpha\, m\, y^7\,\mu\, c^2 z^4 - 4\,\alpha\, m\, y^5\,\mu\, c^3 z^6 - \alpha\, m\, y^3\,\mu\, c^4 z^8 - 4\,\alpha\, m\, y^7\,\mu\, z^4\, c - 6\,\alpha\, m\, y^5\,\mu\, z^6\, c^2$$

$$- 4\,\alpha\, m\, y^3\,\mu\, z^8\, c^3 - 4\,\alpha\, m\, y^7\,\mu\, z^2\, x^2 - 4\,\alpha\, m\, y^3\,\mu\, z^2\, x^6 - 6\,\alpha\, m\, y^5\,\mu\, z^2\, x^4 - \alpha\, m\, y\,\mu\, z^2\, x^8$$

$$- \alpha\, m\, y\,\mu\, z^{10}\, c^4 - 3\,z\,x\,(x^2+y^2+c\,z^2)^{7/2}\, y^2 + 3\,z\,x^3\,(x^2+y^2+c\,z^2)^{7/2}\, c - \alpha\, m\, y^{11}\,\mu$$

$$- 6\,\alpha\, m\, y\,\mu\, x^6\, c^2 z^4 - 4\,\alpha\, m\, y\,\mu\, x^4\, c^3 z^6 - \alpha\, m\, y\,\mu\, x^2\, c^4 z^8 - 4\,\alpha\, m\, y\,\mu\, z^4\, x^6\, c$$

$$- 6\,\alpha\, m\, y\,\mu\, z^6\, x^4\, c^2 - 4\,\alpha\, m\, y\,\mu\, z^8\, x^2\, c^3 - 16\,\alpha\, m\, y^7\,\mu\, x^2\, c\, z^2 - 18\,\alpha\, m\, y^5\,\mu\, x^2\, c^2 z^4$$

$$- 8\,\alpha\,m\,y^3\,\mu\,x^2\,c^3\,z^6 - 16\,\alpha\,m\,y^3\,\mu\,x^6\,c\,z^2 - 24\,\alpha\,m\,y^5\,\mu\,x^4\,c\,z^2 - 18\,\alpha\,m\,y^3\,\mu\,x^4\,c^2\,z^4$$

$$- 12\,\alpha\,m\,y^3\,\mu\,z^4\,x^4\,c - 12\,\alpha\,m\,y^5\,\mu\,z^4\,x^2\,c - 12\,\alpha\,m\,y^3\,\mu\,z^6\,x^2\,c^2)\big)\big),$$

$$\frac{1}{(x^2+y^2+c\,z^2)^{15/2}\,\mu\,(x^2+y^2)}\big(\alpha^2\,z\,m^2\,\big(\alpha\,m\,x\,\mu\,y^{10} + 5\,\alpha\,m\,x^3\,\mu\,y^8 + 10\,\alpha\,m\,x^5\,\mu\,y^6$$

$$+ 5\,\alpha\,m\,x^9\,\mu\,y^2 + 10\,\alpha\,m\,x^7\,\mu\,y^4 + \alpha\,m\,x^9\,\mu\,z^2 + 18\,\alpha\,m\,x^5\,\mu\,y^2\,c^2\,z^4 + 12\,\alpha\,m\,x^5\,\mu\,z^4\,y^2\,c$$

$$+ 12\,\alpha\,m\,x^3\,\mu\,z^4\,c\,y^4 + 12\,\alpha\,m\,x^3\,\mu\,z^6\,y^2\,c^2 + 6\,\alpha\,m\,x\,\mu\,y^6\,c^2\,z^4 + 4\,\alpha\,m\,x\,\mu\,y^4\,c^3\,z^6$$

$$+ \alpha\,m\,x\,\mu\,y^2\,c^4\,z^8 + 4\,\alpha\,m\,x\,\mu\,z^4\,y^6\,c + 6\,\alpha\,m\,x\,\mu\,z^6\,y^4\,c^2 + 4\,\alpha\,m\,x\,\mu\,z^8\,y^2\,c^3$$

$$+ 16\,\alpha\,m\,x^3\,\mu\,y^6\,c\,z^2 + 18\,\alpha\,m\,x^3\,\mu\,y^4\,c^2\,z^4 + 8\,\alpha\,m\,x^3\,\mu\,y^2\,c^3\,z^6 + 16\,\alpha\,m\,x^7\,\mu\,y^2\,c\,z^2$$

$$+ 24\,\alpha\,m\,x^5\,\mu\,c\,y^4\,z^2 + 4\,\alpha\,m\,x\,\mu\,y^8\,c\,z^2 - 3\,z\,y^3\,(x^2+y^2+c\,z^2)^{7/2} - 3\,z\,y\,(x^2+y^2$$

$$+ c\,z^2)^{7/2}\,x^2 + 3\,z\,y^3\,(x^2+y^2+c\,z^2)^{7/2}\,c + 3\,z\,y\,(x^2+y^2+c\,z^2)^{7/2}\,c\,x^2 + \alpha\,m\,x^{11}\,\mu$$

$$+ 4\,\alpha\,m\,x^9\,\mu\,c\,z^2 + 6\,\alpha\,m\,x^7\,\mu\,c^2\,z^4 + 4\,\alpha\,m\,x^5\,\mu\,c^3\,z^6 + \alpha\,m\,x^3\,\mu\,c^4\,z^8 + 4\,\alpha\,m\,x^7\,\mu\,z^4\,c$$

$$+ 6\,\alpha\,m\,x^5\,\mu\,z^6\,c^2 + 4\,\alpha\,m\,x^3\,\mu\,z^8\,c^3 + 4\,\alpha\,m\,x^3\,\mu\,z^2\,y^6 + 4\,\alpha\,m\,x^7\,\mu\,z^2\,y^2 + 6\,\alpha\,m\,x^5\,\mu\,z^2\,y^4$$

$$+ \alpha\,m\,x\,\mu\,z^2\,y^8 + \alpha\,m\,x\,\mu\,z^{10}\,c^4\big)\big),\ -\frac{3\,\alpha^2\,z\,m^2\,(-1+c)\,(x^2+y^2)}{(x^2+y^2+c\,z^2)^4\,\mu}\Bigg]$$

$$\mathit{Dissipation} = 0$$

*****************************   *END PROCEDURE*   ****************************   **(35)**

****

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`Example 10b  Dirac Type magnetic HedgeHog solution.  p201 vol4`:
> Holder:=(1*x^2+1*y^2+1*c*z^2)^(2/2);Holder2:=(1*x^2+1*y^2)^0;
> A1:=(alpha*m/Holder2/Holder*(-y));A2:=(alpha*m/Holder2/Holder*x);
> A3:=0; phi:=0;ee:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,c,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

$$Holder := x^2 + y^2 + c\,z^2$$

$$Holder2 := 1$$

$$A1 := -\frac{\alpha\, m\, y}{x^2 + y^2 + c\,z^2}$$

$$A2 := \frac{\alpha\, m\, x}{x^2 + y^2 + c\,z^2}$$

$$A3 := 0$$

$$\phi := 0$$

$$ee := 0$$

Example 10b Dirac Type magnetic HedgeHog solution. p201 vol4

********************************   *Differential Form Format*   ********************
****

$$Action\ 1\text{-}form = -\frac{\alpha\, m\, y\, d(x)}{x^2 + y^2 + c\,z^2} + \frac{\alpha\, m\, x\, d(y)}{x^2 + y^2 + c\,z^2}$$

$$Intensity\ 2\text{-}form\ F{=}dA = \left(\frac{m\,\alpha\,(-y^2 + x^2 + c\,z^2)}{(x^2 + y^2 + c\,z^2)^2} + \frac{m\,\alpha\,(-x^2 + y^2 + c\,z^2)}{(x^2 + y^2 + c\,z^2)^2} \right) (d(x))\ \&\wedge\ (d(y)) - \frac{2\,\alpha\, m\, y\, z\, c\ (d(x))\ \&\wedge\ (d(z))}{(x^2 + y^2 + c\,z^2)^2} + \frac{2\,\alpha\, m\, x\, z\, c\ (d(y))\ \&\wedge\ (d(z))}{(x^2 + y^2 + c\,z^2)^2}$$

Topological Torsion 3-form $A\wedge F = 0$

Topological Parity 4-form $F\wedge F = 0$

******************************** *Using EM format* ****************************
****

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[\frac{2\,\alpha\,m\,x\,z\,c}{(x^2+y^2+c\,z^2)^2}, \frac{2\,\alpha\,m\,y\,z\,c}{(x^2+y^2+c\,z^2)^2}, \frac{2\,m\,\alpha\,c\,z^2}{(x^2+y^2+c\,z^2)^2} \right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= [0, 0, 0, 0]$

Helicity AdotB $= 0$

Poincare II =2(E.B) $= 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension   PTD $= 2$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

Xm or linear (Mean) curvature $= 0$

$$Yg\ or\ quadratic\ (GAUSS)\ curvature\ = \frac{\alpha^2\,m^2\,(c\,z^2-y^2-x^2)}{(x^2+y^2+c\,z^2)^3}$$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH $= 0$

D field $= [0, 0, 0]$

$$H\ field = \left[\frac{2\,\alpha\,m\,x\,z\,c}{(x^2+y^2+c\,z^2)^2\,\mu}, \frac{2\,\alpha\,m\,y\,z\,c}{(x^2+y^2+c\,z^2)^2\,\mu}, \frac{2\,m\,\alpha\,c\,z^2}{(x^2+y^2+c\,z^2)^2\,\mu} \right]$$

Poynting vector ExH = EXH

$$Amperian\ Current\ 4Vector\quad curlH\text{-}dD/dt\text{=}J4\ = \left[\frac{2\,\alpha\,m\,y\,c\,(-x^2-y^2-4\,z^2+3\,c\,z^2)}{(x^2+y^2+c\,z^2)^3\,\mu}, -\frac{2\,\alpha\,m\,x\,c\,(-x^2-y^2-4\,z^2+3\,c\,z^2)}{(x^2+y^2+c\,z^2)^3\,\mu}, 0, 0 \right]$$

Amerian charge density   divD = rho $= 0$

divergence Lorentz Current 4Vector, 4div(J4) $= 0$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[\frac{2\,\alpha^2\,m^2\,x\,c\,z^2}{(x^2+y^2+c\,z^2)^3\,\mu}, \frac{2\,\alpha^2\,m^2\,y\,c\,z^2}{(x^2+y^2+c\,z^2)^3\,\mu}, -\frac{2\,\alpha^2\,m^2\,z\,c\,(x^2+y^2)}{(x^2+y^2+c\,z^2)^3\,\mu}, 0 \right]$$

$$Topological\ SPIN\ 3\text{-}form = \frac{2\,\alpha^2\,m^2\,x\,c\,z^2\,\&\text{^}(d(y), d(z), d(t))}{(x^2+y^2+c\,z^2)^3\,\mu}$$

$$-\frac{2\alpha^2 m^2 y c z^2 \,\&\wedge(d(x), d(z), d(t))}{(x^2+y^2+c z^2)^3 \mu} - \frac{2\alpha^2 m^2 z c (x^2+y^2) \,\&\wedge(d(x), d(y), d(t))}{(x^2+y^2+c z^2)^3 \mu}$$

Spin density rho_spin = 0

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{4\alpha^2 m^2 z^2 c^2 (x^2+y^2+z^2)}{(x^2+y^2+c z^2)^4 \mu}$$

$$B.H = \frac{4\alpha^2 m^2 z^2 c^2 (x^2+y^2+z^2)}{(x^2+y^2+c z^2)^4 \mu}$$

$$D.E = 0$$

$$A.J = -\frac{2\alpha^2 m^2 c (-x^2-y^2-4z^2+3cz^2)(x^2+y^2)}{(x^2+y^2+c z^2)^4 \mu}$$

-rho.phi = 0

Poincare I (B.H - D.E)-(A.J - rho.phi)

$$= \frac{2c\alpha^2 m^2 (5z^2x^2c + 5z^2y^2c + 2z^4c - x^4 - 2x^2y^2 - 4x^2z^2 - 4y^2z^2 - y^4)}{(x^2+y^2+c z^2)^4 \mu}$$

London Coefficient $LC = -\frac{2c(-x^2-y^2-4z^2+3cz^2)}{(x^2+y^2+c z^2)^2 \mu}$

$$\textit{PROCA coefficient curlcurlB} = \left[-\frac{8\alpha m x z c (3c^2z^2 - 3cx^2 - 3cy^2 + 2x^2 + 2y^2 - 4cz^2)}{(x^2+y^2+c z^2)^4}, -\frac{8\alpha m y z c (3c^2z^2 - 3cx^2 - 3cy^2 + 2x^2 + 2y^2 - 4cz^2)}{(x^2+y^2+c z^2)^4}, -\frac{4\alpha m c (-8z^2x^2c - 8z^2y^2c + 3c^2z^4 + x^4 + 2x^2y^2 + y^4 + 8x^2z^2 + 8y^2z^2 - 4z^4c)}{(x^2+y^2+c z^2)^4} \right]$$

$$\textit{Amperian Current 4Vector} \quad \textit{curlH-dD/dt=J4} = \left[\frac{2\alpha m y c (-x^2-y^2-4z^2+3cz^2)}{(x^2+y^2+c z^2)^3 \mu}, -\frac{2\alpha m x c (-x^2-y^2-4z^2+3cz^2)}{(x^2+y^2+c z^2)^3 \mu}, 0, 0 \right]$$

Lorentz Force 3 vector due to Ampere current FL = -(rho_ampere E + J_ampere x B)

$$= \left[\frac{4\alpha^2 m^2 x c^2 (-x^2-y^2-4z^2+3cz^2) z^2}{(x^2+y^2+c z^2)^5 \mu}, \frac{4\alpha^2 m^2 y c^2 (-x^2-y^2-4z^2+3cz^2) z^2}{(x^2+y^2+c z^2)^5 \mu}, \right.$$

$$-\frac{4\alpha^2 m^2 c^2 (-x^2-y^2-4z^2+3cz^2)\, z\, (x^2+y^2)}{(x^2+y^2+cz^2)^5 \mu}\Bigg]$$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $= \frac{1}{2}\,\frac{(x^2+y^2+cz^2)^2}{c\,(-x^2-y^2-4z^2+3cz^2)}$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $=0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \Bigg[\frac{2\alpha^2 m^2 x c z^2}{(x^2+y^2+cz^2)^3 \mu}, \frac{2\alpha^2 m^2 y c z^2}{(x^2+y^2+cz^2)^3 \mu}, -\frac{2\alpha^2 m^2 z c (x^2+y^2)}{(x^2+y^2+cz^2)^3 \mu}, 0\Bigg]$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= \Bigg[-\frac{4\alpha^3 m^3 y c^2 z^2 (x^2+y^2+z^2)}{(x^2+y^2+cz^2)^5 \mu}, \frac{4\alpha^3 m^3 z^2 c^2 x (x^2+y^2+z^2)}{(x^2+y^2+cz^2)^5 \mu}, 0\Bigg]$

Spin Dissipation J_spin dot E $=0$

Disspative Force 3 vector

$$= \Bigg[\frac{4\alpha^2 m^2 z^2 c^2 (3z^2 c x - 4xz^2 - x^3 - xy^2 - \alpha m y \mu x^2 - \alpha m y^3 \mu - \alpha m y \mu z^2)}{(x^2+y^2+cz^2)^5 \mu},$$
$$\frac{4\alpha^2 m^2 z^2 c^2 (3z^2 y c - 4yz^2 - yx^2 - y^3 + \alpha m x^3 \mu + \alpha m x \mu y^2 + \alpha m x \mu z^2)}{(x^2+y^2+cz^2)^5 \mu},$$
$$-\frac{4\alpha^2 m^2 c^2 (-x^2-y^2-4z^2+3cz^2)\, z\, (x^2+y^2)}{(x^2+y^2+cz^2)^5 \mu}\Bigg]$$

Dissipation $=0$

****************************** *END PROCEDURE* ****************************** **(36)**

****
Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`Example 10c  Dirac Type magnetic HedgeHog solution.  p201 vol4`:
> Holder:=(1*x^2+1*y^2+1*c*z^2)^(4/2);Holder2:=(1*x^2+1*y^2)^0;
> A1:=(alpha*m*z^2/Holder2/Holder*(-y));A2:=(alpha*m*z^2/Holder2/Holder*x);
> A3:=0; phi:=0;ee:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,c,1,2,0,0,0*alpha*(g+I*gamma),0):
```

$$Holder := (x^2 + y^2 + c\,z^2)^2$$

$$Holder2 := 1$$

$$A1 := -\frac{\alpha\, m\, z^2\, y}{(x^2 + y^2 + c\,z^2)^2}$$

$$A2 := \frac{\alpha\, m\, z^2\, x}{(x^2 + y^2 + c\,z^2)^2}$$

$$A3 := 0$$

$$\phi := 0$$

$$ee := 0$$

Example 10c Dirac Type magnetic HedgeHog solution. p201 vol4

********************************* *Differential Form Format* ********************
****

$$\text{Action 1-form} = -\frac{\alpha\, m\, z^2\, y\, d(x)}{(x^2 + y^2 + c\,z^2)^2} + \frac{\alpha\, m\, z^2\, x\, d(y)}{(x^2 + y^2 + c\,z^2)^2}$$

$$\text{Intensity 2-form } F{=}dA \; = \left(\frac{\alpha\, m\, z^2\,(-3\,y^2 + x^2 + c\,z^2)}{(x^2 + y^2 + c\,z^2)^3} + \frac{\alpha\, m\, z^2\,(-3\,x^2 + y^2 + c\,z^2)}{(x^2 + y^2 + c\,z^2)^3} \right) (d(x))\ \&\wedge\ (d(y)) - \frac{2\,\alpha\, z\, m\, y\,(c\,z^2 - y^2 - x^2)\ (d(x))\ \&\wedge\ (d(z))}{(x^2 + y^2 + c\,z^2)^3} + \frac{2\,\alpha\, z\, m\, x\,(c\,z^2 - y^2 - x^2)\ (d(y))\ \&\wedge\ (d(z))}{(x^2 + y^2 + c\,z^2)^3}$$

$$\text{Topological Torsion 3-form} \quad A\wedge F = 0$$

$$\text{Topological Parity 4-form} \quad F\wedge F = 0$$

********************************* *Using EM format* *****************************
****

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[\frac{2\,\alpha\,z\,m\,x\,(c\,z^2 - y^2 - x^2)}{(x^2 + y^2 + c\,z^2)^3},\ \frac{2\,\alpha\,z\,m\,y\,(c\,z^2 - y^2 - x^2)}{(x^2 + y^2 + c\,z^2)^3},\ \frac{2\,\alpha\,m\,z^2\,(c\,z^2 - y^2 - x^2)}{(x^2 + y^2 + c\,z^2)^3} \right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension PTD = 2

******************** Correlation Similarity Invariants of Jacobian of (Ak/lambda_N) ****** **********************

Xm or linear (Mean) curvature = 0

$$Yg\ or\ quadratic\ (GAUSS)\ curvature = \frac{\alpha^2\,m^2\,z^4\,(-3\,x^2 - 3\,y^2 + c\,z^2)}{(x^2 + y^2 + c\,z^2)^5}$$

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

*********** Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH ************

Chirality factor CH = 0

D field = [0, 0, 0]

$$H\ field = \left[\frac{2\,\alpha\,z\,m\,x\,(c\,z^2 - y^2 - x^2)}{(x^2 + y^2 + c\,z^2)^3\,\mu},\ \frac{2\,\alpha\,z\,m\,y\,(c\,z^2 - y^2 - x^2)}{(x^2 + y^2 + c\,z^2)^3\,\mu},\ \frac{2\,\alpha\,m\,z^2\,(c\,z^2 - y^2 - x^2)}{(x^2 + y^2 + c\,z^2)^3\,\mu} \right]$$

Poynting vector ExH = *EXH*

Amperian Current 4Vector curlH-dD/dt=J4

$$= \left[\frac{2\,\alpha\,m\,y\,(4\,x^2\,z^2 + 4\,y^2\,z^2 - 8\,z^4\,c + 2\,x^2\,y^2 + y^4 + x^4 + 3\,c^2\,z^4 - 8\,z^2\,x^2\,c - 8\,z^2\,y^2\,c)}{(x^2 + y^2 + c\,z^2)^4\,\mu}, \right.$$
$$\left. - \frac{2\,\alpha\,m\,x\,(4\,x^2\,z^2 + 4\,y^2\,z^2 - 8\,z^4\,c + 2\,x^2\,y^2 + y^4 + x^4 + 3\,c^2\,z^4 - 8\,z^2\,x^2\,c - 8\,z^2\,y^2\,c)}{(x^2 + y^2 + c\,z^2)^4\,\mu}, 0, 0 \right]$$

Amerian charge density divD = rho = 0

divergence Lorentz Current 4Vector, 4div(J4) = 0

$$Topological\ SPIN\ 4\ vector\ S4 = \left[\frac{2\,\alpha^2\,m^2\,z^4\,x\,(c\,z^2 - y^2 - x^2)}{(x^2 + y^2 + c\,z^2)^5\,\mu},\ \frac{2\,\alpha^2\,m^2\,z^4\,y\,(c\,z^2 - y^2 - x^2)}{(x^2 + y^2 + c\,z^2)^5\,\mu}, \right.$$
$$\left. - \frac{2\,\alpha^2\,m^2\,z^3\,(c\,z^2 - y^2 - x^2)\,(x^2 + y^2)}{(x^2 + y^2 + c\,z^2)^5\,\mu}, 0 \right]$$

$$\textit{Topological SPIN 3-form} = \frac{2\,\alpha^2\,m^2\,z^4\,x\,(c\,z^2 - y^2 - x^2)\ \&\wedge(d(y), d(z), d(t))}{(x^2 + y^2 + c\,z^2)^5\,\mu}$$

$$- \frac{2\,\alpha^2\,m^2\,z^4\,y\,(c\,z^2 - y^2 - x^2)\ \&\wedge(d(x), d(z), d(t))}{(x^2 + y^2 + c\,z^2)^5\,\mu}$$

$$- \frac{2\,\alpha^2\,m^2\,z^3\,(c\,z^2 - y^2 - x^2)\,(x^2 + y^2)\ \&\wedge(d(x), d(y), d(t))}{(x^2 + y^2 + c\,z^2)^5\,\mu}$$

$$\textit{Spin density rho\_spin} = 0$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{4\,\alpha^2\,z^2\,m^2\,(c\,z^2 - y^2 - x^2)^2\,(x^2 + y^2 + z^2)}{(x^2 + y^2 + c\,z^2)^6\,\mu}$$

$$B.H = \frac{4\,\alpha^2\,z^2\,m^2\,(c\,z^2 - y^2 - x^2)^2\,(x^2 + y^2 + z^2)}{(x^2 + y^2 + c\,z^2)^6\,\mu}$$

$$D.E = 0$$

$$A.J = -\frac{1}{(x^2 + y^2 + c\,z^2)^6\,\mu}\Big(2\,\alpha^2\,m^2\,z^2\,(4\,x^2\,z^2 + 4\,y^2\,z^2 - 8\,z^4\,c + 2\,x^2\,y^2 + y^4 + x^4 + 3\,c^2\,z^4$$
$$- 8\,z^2\,x^2\,c - 8\,z^2\,y^2\,c)\,(x^2 + y^2)\Big)$$

$$\textit{-rho.phi} = 0$$

$$\textit{Poincare I} \qquad \textit{(B.H - D.E)-(A.J - rho.phi)} = \frac{1}{(x^2 + y^2 + c\,z^2)^6\,\mu}\Big(2\,\alpha^2\,z^2\,m^2\,(3\,y^6 + 3\,x^6$$
$$+ 6\,y^4\,z^2 + 6\,x^4\,z^2 - 12\,z^4\,y^2\,c + 5\,x^2\,c^2\,z^4 + 5\,y^2\,c^2\,z^4 + 2\,z^6\,c^2 - 24\,x^2\,y^2\,c\,z^2 + 9\,x^4\,y^2$$
$$+ 9\,x^2\,y^4 - 12\,c\,y^4\,z^2 - 12\,c\,x^4\,z^2 - 12\,c\,x^2\,z^4 + 12\,x^2\,y^2\,z^2)\Big)$$

$$\textit{London Coefficient} \qquad LC =$$
$$-\frac{2\,(4\,x^2\,z^2 + 4\,y^2\,z^2 - 8\,z^4\,c + 2\,x^2\,y^2 + y^4 + x^4 + 3\,c^2\,z^4 - 8\,z^2\,x^2\,c - 8\,z^2\,y^2\,c)}{(x^2 + y^2 + c\,z^2)^2\,\mu\,z^2}$$

$$\textit{PROCA coefficient curlcurlB} = \Big[-\frac{1}{(x^2 + y^2 + c\,z^2)^5}\big(8\,\alpha\,z\,m\,x\,(-2\,x^4 + 3\,c^3\,z^4 + 14\,z^2\,y^2\,c$$
$$- 4\,x^2\,y^2 - 2\,y^4 - 8\,c^2\,z^4 - 15\,c^2\,z^2\,x^2 + 14\,z^2\,x^2\,c + 12\,y^2\,x^2\,c - 15\,c^2\,y^2\,z^2 + 6\,y^4\,c$$
$$+ 6\,c\,x^4)\big),\ -\frac{1}{(x^2 + y^2 + c\,z^2)^5}\big(8\,\alpha\,z\,m\,y\,(-2\,x^4 + 3\,c^3\,z^4 + 14\,z^2\,y^2\,c - 4\,x^2\,y^2 - 2\,y^4$$
$$- 8\,c^2\,z^4 - 15\,c^2\,z^2\,x^2 + 14\,z^2\,x^2\,c + 12\,y^2\,x^2\,c - 15\,c^2\,y^2\,z^2 + 6\,y^4\,c + 6\,c\,x^4)\big),$$

$$- \frac{1}{(x^2+y^2+c z^2)^5} \left(4 m \alpha \left(-y^6 - x^6 + 3 c^3 z^6 - 8 y^4 z^2 - 8 x^4 z^2 + 32 z^4 y^2 c - 25 x^2 c^2 z^4 - 25 y^2 c^2 z^4 - 8 z^6 c^2 + 38 x^2 y^2 c z^2 - 3 x^4 y^2 - 3 x^2 y^4 + 19 c y^4 z^2 + 19 c x^4 z^2 + 32 c x^2 z^4 - 16 x^2 y^2 z^2\right)\right)]$$

Amperian Current 4Vector   curlH-dD/dt=J4

$$= \left[\frac{2 \alpha m y (4 x^2 z^2 + 4 y^2 z^2 - 8 z^4 c + 2 x^2 y^2 + y^4 + x^4 + 3 c^2 z^4 - 8 z^2 x^2 c - 8 z^2 y^2 c)}{(x^2+y^2+c z^2)^4 \mu}, - \frac{2 \alpha m x (4 x^2 z^2 + 4 y^2 z^2 - 8 z^4 c + 2 x^2 y^2 + y^4 + x^4 + 3 c^2 z^4 - 8 z^2 x^2 c - 8 z^2 y^2 c)}{(x^2+y^2+c z^2)^4 \mu}, 0, 0 \right]$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B)

$$= \left[\frac{1}{(x^2+y^2+c z^2)^7 \mu} \left(4 \alpha^2 m^2 x (4 x^2 z^2 + 4 y^2 z^2 - 8 z^4 c + 2 x^2 y^2 + y^4 + x^4 + 3 c^2 z^4 - 8 z^2 x^2 c - 8 z^2 y^2 c) z^2 (c z^2 - y^2 - x^2)\right), \frac{1}{(x^2+y^2+c z^2)^7 \mu} \left(4 \alpha^2 m^2 y (4 x^2 z^2 + 4 y^2 z^2 - 8 z^4 c + 2 x^2 y^2 + y^4 + x^4 + 3 c^2 z^4 - 8 z^2 x^2 c - 8 z^2 y^2 c) z^2 (c z^2 - y^2 - x^2)\right), - \frac{1}{(x^2+y^2+c z^2)^7 \mu} \left(4 \alpha^2 m^2 (4 x^2 z^2 + 4 y^2 z^2 - 8 z^4 c + 2 x^2 y^2 + y^4 + x^4 + 3 c^2 z^4 - 8 z^2 x^2 c - 8 z^2 y^2 c) z (c z^2 - y^2 - x^2) (x^2 + y^2)\right) \right]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN

$$= \frac{1}{2} \frac{z^2 (x^2+y^2+c z^2)^2}{4 x^2 z^2 + 4 y^2 z^2 - 8 z^4 c + 2 x^2 y^2 + y^4 + x^4 + 3 c^2 z^4 - 8 z^2 x^2 c - 8 z^2 y^2 c}$$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{2\,\alpha^2\, m^2\, z^4\, x\,(c\,z^2 - y^2 - x^2)}{(x^2 + y^2 + c\,z^2)^5\,\mu}, \frac{2\,\alpha^2\, m^2\, z^4\, y\,(c\,z^2 - y^2 - x^2)}{(x^2 + y^2 + c\,z^2)^5\,\mu}, - \frac{2\,\alpha^2\, m^2\, z^3\,(c\,z^2 - y^2 - x^2)\,(x^2 + y^2)}{(x^2 + y^2 + c\,z^2)^5\,\mu}, 0 \right]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= \Big[$

$$- \frac{4\,\alpha^3\, m^3\, z^4\, y\,(c\,z^2 - y^2 - x^2)^2\,(x^2 + y^2 + z^2)}{(x^2 + y^2 + c\,z^2)^8\,\mu}, \frac{4\,\alpha^3\, m^3\, z^4\,(c\,z^2 - y^2 - x^2)^2\, x\,(x^2 + y^2 + z^2)}{(x^2 + y^2 + c\,z^2)^8\,\mu}, 0 \Big]$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \Big[\frac{1}{(x^2 + y^2 + c\,z^2)^8\,\mu} \big(4\,\alpha^2\, m^2\, z^2\,(c\,z^2 - y^2 - x^2)\,(x\,y^6 + x^7$

$$+ 3\,c^3\, z^6\, x + 4\,x\,y^4\, z^2 + 4\,x^5\, z^2 - 4\,z^4\, x\,c\,y^2 - 5\,x^3\, c^2\, z^4 - 5\,x\,y^2\, c^2\, z^4 - 8\,c^2\, z^6\, x - 14\,c\,z^2\, x^3\, y^2$$

$$+ 3\,x^5\, y^2 + 3\,x^3\, y^4 - 7\,c\,z^2\, x\,y^4 - 7\,c\,z^2\, x^5 - 4\,z^4\, x^3\, c + 8\,x^3\, y^2\, z^2 - \alpha\, m\, y\,\mu\, z^4\, x^2\, c$$

$$- \alpha\, m\, y^3\,\mu\, z^4\, c - \alpha\, m\, y\,\mu\, z^6\, c + 2\,\alpha\, m\, y^3\,\mu\, z^2\, x^2 + \alpha\, m\, y^5\,\mu\, z^2 + \alpha\, m\, y^3\,\mu\, z^4 + \alpha\, m\, y\,\mu\, z^2\, x^4$$

$$+ \alpha\, m\, y\,\mu\, z^4\, x^2)\big), \frac{1}{(x^2 + y^2 + c\,z^2)^8\,\mu} \big(4\,\alpha^2\, m^2\, z^2\,(c\,z^2 - y^2 - x^2)\,(y^7 + y\,x^6 + 3\,c^3\, z^6\, y$$

$$+ 4\,y^5\, z^2 + 4\,z^2\, y\,x^4 - 4\,z^4\, y^3\, c - 5\,x^2\, y\,c^2\, z^4 - 5\,y^3\, c^2\, z^4 - 8\,c^2\, z^6\, y - 14\,c\,z^2\, y^3\, x^2 + 3\,y^3\, x^4$$

$$+ 3\,y^5\, x^2 - 7\,c\,z^2\, y^5 - 7\,c\,z^2\, y\,x^4 - 4\,z^4\, x^2\, y\,c + 8\,y^3\, x^2\, z^2 + \alpha\, m\, x^3\,\mu\, z^4\, c + \alpha\, m\, x\,\mu\, z^4\, y^2\, c$$

$$+ \alpha\, m\, x\,\mu\, z^6\, c - 2\,\alpha\, m\, x^3\,\mu\, z^2\, y^2 - \alpha\, m\, x\,\mu\, z^2\, y^4 - \alpha\, m\, x\,\mu\, z^4\, y^2 - \alpha\, m\, x^5\,\mu\, z^2 - \alpha\, m\, x^3\,\mu\, z^4)\big)$$

$$, - \frac{1}{(x^2 + y^2 + c\,z^2)^7\,\mu} \big(4\,\alpha^2\, m^2\,(4\,x^2\, z^2 + 4\,y^2\, z^2 - 8\,z^4\, c + 2\,x^2\, y^2 + y^4 + x^4 + 3\,c^2\, z^4$$

$$-8z^2x^2c - 8z^2y^2c)\, z\,(c z^2 - y^2 - x^2)\,(x^2+y^2)\,)\Big]$$

$$Dissipation \;=0$$

$$\text{******************************} \quad END\ PROCEDURE \quad \text{****************************} \qquad \textbf{(37)}$$

Enter the name of the problem, and the components of the 4 potential
Q = charge, Omega = strength and sign of rotation

```
> NAME:=`Examplle 11a - An Electromagnetic Pump    `;

> A1:=0;A2:=-1/2*Bx*z;
> A3:=+1/2*Bx*y+Bx*z/2; phi:=+0*Ex*x+0*Ez*z+Ey*y;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

$$NAME := Examplle\ 11a\text{ - }An\ Electromagnetic\ Pump$$

$$A1 := 0$$

$$A2 := -\frac{1}{2}\, Bx\, z$$

$$A3 := \frac{1}{2}\, Bx\, y + \frac{1}{2}\, Bx\, z$$

$$\phi := Ey\, y$$

$$Examplle\ 11a\text{ - }An\ Electromagnetic\ Pump$$

$$\text{********************************} \quad Differential\ Form\ Format \quad \text{*******************} \text{*************}$$

$$Action\ 1\text{-}form = -\frac{1}{2}\, Bx\, z\, d(y) + \left(\frac{1}{2}\, Bx\, y + \frac{1}{2}\, Bx\, z\right) d(z) - Ey\, y\, d(t)$$

$$Intensity\ 2\text{-}form\ F{=}dA \;= Bx\,(d(y))\ \&\wedge\ (d(z)) - Ey\,(d(y))\ \&\wedge\ (d(t))$$

$$Topological\ Torsion\ 3\text{-}form \quad A\wedge F = \left(-Ey\, y\, Bx + \frac{1}{2}\, Bx\,(y+z)\, Ey\right) \&\wedge(d(y), d(z), d(t))$$

$$Topological\ Parity\ 4\text{-}form \quad F\wedge F = 0$$

$$\text{********************************}\ Using\ EM\ format\ \text{****************************} \text{****}$$

$E\ field = [0, -Ey, 0]$

$B\ field = [Bx, 0, 0]$

$Topological\ TORSION\ 4\ vector\quad T4 = -[ExA + Bphi,AdotB] = \left[-\frac{1}{2}\ Bx\, Ey\, (y - z), 0, 0, 0 \right]$

$Helicity\ AdotB = 0$

$Poincare\ II = 2(E.B) = 0$

$coefficient\ of\ Topological\ Parity\ 4\text{-}form = 0$

$Pfaff\ Topological\ Dimension \quad PTD = 3$

******************** Correlation Similarity Invariants of Jacobian of (Ak/lambda_N) ****** *********************

$Xm\ or\ linear\ (Mean)\ curvature = \frac{1}{2}\ Bx$

$Yg\ or\ quadratic\ (GAUSS)\ curvature = \frac{1}{4}\ Bx^2$

$Za\ or\ Cubic\ (Interaction\ internal\ energy)\ curvature = 0$

$Tk\ or\ quartic\ (4D\ expansion)\ curvature = 0$

********** Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH ***********

$Chirality\ factor\ CH = 0$

$D\ field = [0, -\varepsilon\, Ey, 0]$

$H\ field = \left[\frac{Bx}{\mu}, 0, 0 \right]$

$Poynting\ vector\ ExH = EXH$

$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt=J4 = [0, 0, 0, 0]$

$Amerian\ charge\ density \quad divD = rho = 0$

$divergence\ Lorentz\ Current\ 4Vector, \quad 4div(J4) = 0$

$Topological\ SPIN\ 4\ vector\ S4 = \left[0, \frac{1}{2}\ \frac{Bx^2\, y + Bx^2\, z - 2\, \varepsilon\, Ey^2\, y\, \mu}{\mu}, \frac{1}{2}\ \frac{Bx^2\, z}{\mu}, \frac{1}{2}\ Bx\, z\, \varepsilon\, Ey \right]$

$$Topological\ SPIN\ 3\text{-}form = -\frac{1}{2}\ \frac{\left(Bx^2\, y + Bx^2\, z - 2\, \varepsilon\, Ey^2\, y\, \mu\right)\ \&\text{^}(d(x), d(z), d(t))}{\mu} + \frac{1}{2}\ \frac{Bx^2\, z\ \&\text{^}(d(x), d(y), d(t))}{\mu} - \frac{1}{2}\ Bx\, z\, \varepsilon\, Ey\ \&\text{^}(d(x), d(y), d(z))$$

$Spin\ density\ rho\_spin = \frac{1}{2}\ Bx\, z\, \varepsilon\, Ey$

$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = \frac{Bx^2 - \varepsilon\, Ey^2\, \mu}{\mu}$

$$B.H = \frac{Bx^2}{\mu}$$

$$D.E = \varepsilon\, Ey^2$$

$$A.J = 0$$

$$\text{-}rho.phi = 0$$

Poincare I $\quad (B.H - D.E)\text{-}(A.J - rho.phi) = \dfrac{Bx^2 - \varepsilon\, Ey^2\, \mu}{\mu}$

London Coefficient $\quad LC =$

$$-\frac{2\left(4 x^2 z^2 + 4 y^2 z^2 - 8 z^4 c + 2 x^2 y^2 + y^4 + x^4 + 3 c^2 z^4 - 8 z^2 x^2 c - 8 z^2 y^2 c\right)}{\left(x^2 + y^2 + c z^2\right)^2 \mu z^2}$$

PROCA coefficient curlcurlB $= [0, 0, 0]$

Amperian Current 4Vector $\quad$ *curlH-dD/dt=J4* $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Ampere current $\quad$ *FL = -(rho_ampere E + J_ampere x B)* $= [0, 0, 0]$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= 0$

Topological Torsion current 4 vector $\quad$ *T4 = -[ExA + B.phi,AdotB]* $= \left[-\frac{1}{2}\, Bx\, Ey\, (y - z), 0, 0, 0\right]$

Lorentz Force 3 vector due to Torsion current $\quad$ *TF = -(rho_torsion E + J_torsion x B)* $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector $\quad$ *TS4 = -[A x H + D.phi,AdotD]* $=$

$$\left[0, \frac{1}{2}\,\frac{Bx^2 y + Bx^2 z - 2\,\varepsilon\, Ey^2\, y\, \mu}{\mu}, \frac{1}{2}\,\frac{Bx^2 z}{\mu}, \frac{1}{2}\, Bx\, z\, \varepsilon\, Ey\right]$$

Lorentz Force 3 vector due to Spin current $\quad$ *SF = --(rho_spin E + J_spin x B)* $=$

$$\left[0, -\frac{1}{2}\,\frac{Bx\, z\left(Bx^2 - \varepsilon\, Ey^2\, \mu\right)}{\mu}, \frac{1}{2}\,\frac{\left(Bx^2 y + Bx^2 z - 2\,\varepsilon\, Ey^2\, y\, \mu\right) Bx}{\mu}\right]$$

$$\textit{Spin Dissipation J\_spin dot E} = -\frac{1}{2}\frac{\left(Bx^2 y + Bx^2 z - 2\,\varepsilon\, Ey^2\, y\,\mu\right) Ey}{\mu}$$

$$\textit{Disspative Force 3 vector} = \left[0, -\frac{1}{2}\,Bx\,z\left(Bx^2 - \varepsilon\,Ey^2\,\mu\right), \frac{1}{2}\left(Bx^2 y + Bx^2 z - 2\,\varepsilon\,Ey^2\,y\,\mu\right) Bx\right]$$

$$\textit{Dissipation} = \frac{1}{2}\,Bx\,Ey\left(z\,\varepsilon\,\mu - y + z\right)$$

****************************** *END PROCEDURE* ****************************** **(38)**

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

```
> NAME:=`Example 10a -- Plasma Accretion disc -- Hedge Hog solution.`;
> Gamma:=-z*I/(x^2+y^2)^1*m/(a*x^2+a*y^2+c*z^2)^(1/2);
> Ax:=Gamma*(-y);Ay:=Gamma*x;
> Az:=0; phi:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 10a -- Plasma Accretion disc -- Hedge Hog solution.

$$\Gamma := -\frac{I\,z\,m}{\left(x^2+y^2\right)\sqrt{x^2\,a + a\,y^2 + c\,z^2}}$$

$$Ax := \frac{I\,z\,m\,y}{\left(x^2+y^2\right)\sqrt{x^2\,a + a\,y^2 + c\,z^2}}$$

$$Ay := -\frac{I\,z\,m\,x}{\left(x^2+y^2\right)\sqrt{x^2\,a + a\,y^2 + c\,z^2}}$$

$$Az := 0$$

$$\phi := 0$$

Example 10a -- Plasma Accretion disc -- Hedge Hog solution.

********************************* *Differential Form Format* ********************
****

$$\textit{Action 1-form} = \frac{I\,z\,m\,y\,d(x)}{\left(x^2+y^2\right)\sqrt{x^2\,a + a\,y^2 + c\,z^2}} - \frac{I\,z\,m\,x\,d(y)}{\left(x^2+y^2\right)\sqrt{x^2\,a + a\,y^2 + c\,z^2}}$$

$$\text{Intensity 2-form } F=dA = \left(-\frac{I z m\,(-x^2 a y^2 - 2 a y^4 - z^2 y^2 c + x^4 a + z^2 x^2 c)}{(x^2+y^2)^2\,(x^2 a + a y^2 + c z^2)^{3/2}} + \frac{I z m\,(2 x^4 a + x^2 a y^2 + z^2 x^2 c - a y^4 - z^2 y^2 c)}{(x^2+y^2)^2\,(x^2 a + a y^2 + c z^2)^{3/2}} \right) (d(x))\ \&\wedge\ (d(y)) - \frac{I a m y\,(d(x))\ \&\wedge\ (d(z))}{(x^2 a + a y^2 + c z^2)^{3/2}} + \frac{I a m x\,(d(y))\ \&\wedge\ (d(z))}{(x^2 a + a y^2 + c z^2)^{3/2}}$$

Topological Torsion 3-form $A\wedge F = 0$

Topological Parity 4-form $F\wedge F = 0$

******************************** *Using EM format* ****************************
****

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[\frac{I a m x}{(x^2 a + a y^2 + c z^2)^{3/2}}, \frac{I a m y}{(x^2 a + a y^2 + c z^2)^{3/2}}, \frac{I a z m}{(x^2 a + a y^2 + c z^2)^{3/2}} \right]$$

Topological TORSION 4 vector $T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]$

Helicity $AdotB = 0$

Poincare II $=2(E.B) = 0$

coefficient of Topological Parity 4-form $= 0$

Pfaff Topological Dimension $PTD = 2$

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

Xm or linear (Mean) curvature $= 0$

$$\text{Yg or quadratic (GAUSS) curvature} = \frac{(2 x^2 a + 2 a y^2 + c z^2)\, z^2 m^2}{(x^2 a + a y^2 + c z^2)^2\,(x^2+y^2)^2}$$

Za or Cubic (Interaction internal energy) curvature $= 0$

Tk or quartic (4D expansion) curvature $= 0$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor $CH = 0$

$$D\ field = [0, 0, 0]$$

$$H\ field = \left[\frac{I a m x}{(x^2 a + a y^2 + c z^2)^{3/2}\,\mu}, \frac{I a m y}{(x^2 a + a y^2 + c z^2)^{3/2}\,\mu}, \frac{I a z m}{(x^2 a + a y^2 + c z^2)^{3/2}\,\mu} \right]$$

Poynting vector $ExH = EXH$

$$\text{Amperian Current 4Vector}\quad curlH\text{-}dD/dt=J4 = \left[-\frac{3 I a z m y\,(a-c)}{(x^2 a + a y^2 + c z^2)^{5/2}\,\mu}, \right.$$

$$\frac{3\,\mathrm{I}\,a\,m\,x\,z\,(a-c)}{(x^2 a + a y^2 + c z^2)^{5/2}\,\mu},\,0,\,0\Big]$$

Amerian charge density $\quad divD = rho = 0$

divergence Lorentz Current 4Vector, $\quad 4div(J4) = 0$

$$\text{Topological SPIN 4 vector } S4 = \left[\frac{z^2 m^2 x\,a}{(x^2+y^2)\,(x^2 a + a y^2 + c z^2)^2\,\mu},\right.$$

$$\left.\frac{z^2 m^2 y\,a}{(x^2+y^2)\,(x^2 a + a y^2 + c z^2)^2\,\mu},\ -\frac{z\,m^2 a}{(x^2 a + a y^2 + c z^2)^2\,\mu},\ 0\right]$$

$$\text{Topological SPIN 3-form} = \frac{z^2 m^2 x\,a\ \&\text{^}(d(y),d(z),d(t))}{(x^2+y^2)\,(x^2 a + a y^2 + c z^2)^2\,\mu}$$

$$-\frac{z^2 m^2 y\,a\ \&\text{^}(d(x),d(z),d(t))}{(x^2+y^2)\,(x^2 a + a y^2 + c z^2)^2\,\mu} - \frac{z\,m^2 a\ \&\text{^}(d(x),d(y),d(t))}{(x^2 a + a y^2 + c z^2)^2\,\mu}$$

Spin density rho_spin $= 0$

$$\text{LaGrange field energy density (B.H-D.E)} = -\frac{a^2 m^2 (x^2+y^2+z^2)}{(x^2 a + a y^2 + c z^2)^3\,\mu}$$

$$B.H = -\frac{a^2 m^2 (x^2+y^2+z^2)}{(x^2 a + a y^2 + c z^2)^3\,\mu}$$

$$D.E = 0$$

$$A.J = \frac{3 z^2 m^2 a\,(a-c)}{(x^2 a + a y^2 + c z^2)^3\,\mu}$$

$$\text{-rho.phi} = 0$$

$$\text{Poincare I} \qquad \text{(B.H - D.E)-(A.J - rho.phi)} = -\frac{a\,m^2 (x^2 a + a y^2 + 4 a z^2 - 3 c z^2)}{(x^2 a + a y^2 + c z^2)^3\,\mu}$$

$$\text{London Coefficient} \qquad LC = -\frac{3 a\,(a-c)\,(x^2+y^2)}{(x^2 a + a y^2 + c z^2)^2\,\mu}$$

$$\text{PROCA coefficient curlcurlB} = \left[-\frac{3\,\mathrm{I}\,(x^2 a + a y^2 - 4 c z^2)\,(a-c)\,a\,m\,x}{(x^2 a + a y^2 + c z^2)^{7/2}},\right.$$

$$\left.-\frac{3\,\mathrm{I}\,(x^2 a + a y^2 - 4 c z^2)\,(a-c)\,a\,m\,y}{(x^2 a + a y^2 + c z^2)^{7/2}},\ -\frac{3\,\mathrm{I}\,(3 a y^2 + 3 x^2 a - 2 c z^2)\,(a-c)\,a\,m\,z}{(x^2 a + a y^2 + c z^2)^{7/2}}\right]$$

$$\text{Amperian Current 4Vector} \qquad \text{curlH-dD/dt=J4} = \left[-\frac{3\,\mathrm{I}\,a\,z\,m\,y\,(a-c)}{(x^2 a + a y^2 + c z^2)^{5/2}\,\mu},\right.$$

$$\frac{3\,I\,a\,m\,x\,z\,(a-c)}{(x^2 a + a y^2 + c z^2)^{5/2}\,\mu}, 0, 0 \Big]$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B)

$$= \left[\frac{3 a^2 m^2 x z^2 (a-c)}{(x^2 a + a y^2 + c z^2)^4 \mu}, \frac{3 a^2 z^2 m^2 y (a-c)}{(x^2 a + a y^2 + c z^2)^4 \mu}, -\frac{3 a^2 z m^2 (a-c) (x^2+y^2)}{(x^2 a + a y^2 + c z^2)^4 \mu} \right]$$

Amperian Dissipation Jampere dot E $= 0$

$$\textit{Lorentz Force Spin factor}\ \ LFSPIN = \frac{1}{3}\,\frac{(x^2 a + a y^2 + c z^2)^2}{(x^2+y^2)\,a\,(a-c)}$$

Topological Torsion current 4 vector   T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current   TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector   TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{z^2 m^2 x a}{(x^2+y^2)(x^2 a + a y^2 + c z^2)^2 \mu}, \frac{z^2 m^2 y a}{(x^2+y^2)(x^2 a + a y^2 + c z^2)^2 \mu}, -\frac{z m^2 a}{(x^2 a + a y^2 + c z^2)^2 \mu}, 0 \right]$$

Lorentz Force 3 vector due to Spin current   SF = --(rho_spin E + J_spin x B)

$$= \left[\frac{I (x^2+y^2+z^2)\, y z a^2 m^3}{(x^2 a + a y^2 + c z^2)^{7/2} \mu (x + I y)(-x + I y)}, -\frac{I (x^2+y^2+z^2)\, x z a^2 m^3}{(x^2 a + a y^2 + c z^2)^{7/2} \mu (x + I y)(-x + I y)}, 0 \right]$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector

$$= \Big[\frac{1}{(x^2 a + a y^2 + c z^2)^{15/2} \mu (x + I y)(-x + I y)} \Big(I \big(24 y^5 m \mu x^4 a^3 c z^2$$

$$+ 18 y^3 m \mu x^4 a^2 c^2 z^4 + 16 y^7 m \mu x^2 a^3 c z^2 + 18 y^5 m \mu x^2 a^2 c^2 z^4 + 8 y^3 m \mu x^2 a c^3 z^6$$

$$+ 12 y^3 m \mu z^4 x^4 a^3 c + 12 y^5 m \mu z^4 x^2 a^3 c + 12 y^3 m \mu z^6 x^2 a^2 c^2 + 16 y^3 m \mu x^6 a^3 c z^2$$

$+ 4\, y\, m\, \mu\, x^8\, a^3\, c\, z^2 + 6\, y\, m\, \mu\, x^6\, a^2\, c^2\, z^4 + 4\, y\, m\, \mu\, x^4\, a\, c^3\, z^6 + 4\, y\, m\, \mu\, z^4\, x^6\, a^3\, c$

$+ 6\, y\, m\, \mu\, z^6\, x^4\, a^2\, c^2 + 4\, y\, m\, \mu\, z^8\, x^2\, a\, c^3 + 3\, \mathrm{I}\, x\, z\, \left(x^2\, a + a\, y^2 + c\, z^2\right)^{7/2}\, a\, y^2$

$- 3\, \mathrm{I}\, x\, z\, \left(x^2\, a + a\, y^2 + c\, z^2\right)^{7/2}\, c\, y^2 + m\, y^3\, \mu\, c^4\, z^8 + 10\, y^7\, m\, \mu\, x^4\, a^4 + 10\, y^5\, m\, \mu\, x^6\, a^4$

$+ 5\, y^3\, m\, \mu\, x^8\, a^4 + 5\, y^9\, m\, \mu\, x^2\, a^4 + y^9\, m\, \mu\, z^2\, a^4 + y\, m\, \mu\, x^{10}\, a^4 + y\, m\, \mu\, z^2\, x^8\, a^4$

$+ 4\, y^9\, m\, \mu\, a^3\, c\, z^2 + 6\, y^7\, m\, \mu\, a^2\, c^2\, z^4 + 4\, y^5\, m\, \mu\, a\, c^3\, z^6 + 4\, y^7\, m\, \mu\, z^2\, x^2\, a^4 + 6\, y^5\, m\, \mu\, z^2\, x^4\, a^4$

$+ 4\, y^3\, m\, \mu\, z^2\, x^6\, a^4 + 4\, y^7\, m\, \mu\, z^4\, a^3\, c + 6\, y^5\, m\, \mu\, z^6\, a^2\, c^2 + 4\, y^3\, m\, \mu\, z^8\, a\, c^3 + m\, y\, \mu\, x^2\, c^4\, z^8$

$+ 3\, \mathrm{I}\, x^3\, z\, \left(x^2\, a + a\, y^2 + c\, z^2\right)^{7/2}\, a - 3\, \mathrm{I}\, x^3\, z\, \left(x^2\, a + a\, y^2 + c\, z^2\right)^{7/2}\, c + m\, y\, \mu\, z^{10}\, c^4$

$+ y^{11}\, m\, \mu\, a^4\big)\, z\, a^2\, m^2\big),\ \dfrac{1}{\left(x^2\, a + a\, y^2 + c\, z^2\right)^{15/2}\, \mu\, (x + \mathrm{I}\, y)\, (-x + \mathrm{I}\, y)}\Big(\mathrm{I}\,\Big($

$-16\, y^6\, m\, \mu\, x^3\, a^3\, c\, z^2 - 18\, y^4\, m\, \mu\, x^3\, a^2\, c^2\, z^4 - 24\, y^4\, m\, \mu\, x^5\, a^3\, c\, z^2 - 12\, y^4\, m\, \mu\, z^4\, x^3\, a^3\, c$

$- 12\, y^2\, m\, \mu\, z^4\, x^5\, a^3\, c - 18\, y^2\, m\, \mu\, x^5\, a^2\, c^2\, z^4 - 12\, y^2\, m\, \mu\, z^6\, x^3\, a^2\, c^2 - 8\, y^2\, m\, \mu\, x^3\, a\, c^3\, z^6$

$- 16\, y^2\, m\, \mu\, x^7\, a^3\, c\, z^2 - 4\, x\, y^8\, m\, \mu\, a^3\, c\, z^2 - 6\, x\, y^6\, m\, \mu\, a^2\, c^2\, z^4 - 4\, x\, y^6\, m\, \mu\, z^4\, a^3\, c$

$- 6\, x\, y^4\, m\, \mu\, z^6\, a^2\, c^2 - 4\, x\, y^4\, m\, \mu\, a\, c^3\, z^6 - 4\, x\, y^2\, m\, \mu\, z^8\, a\, c^3 + 3\, \mathrm{I}\, z\, y^3\, \left(x^2\, a + a\, y^2 + c\, z^2\right)^{7/2}\, a$

$- 3\, \mathrm{I}\, z\, y^3\, \left(x^2\, a + a\, y^2 + c\, z^2\right)^{7/2}\, c - m\, \mu\, z^2\, x^9\, a^4 - 5\, y^8\, m\, \mu\, x^3\, a^4 - 10\, y^6\, m\, \mu\, x^5\, a^4$

$- 10\, y^4\, m\, \mu\, x^7\, a^4 - 5\, y^2\, m\, \mu\, x^9\, a^4 - x\, m\, \mu\, a^4\, y^{10} - x\, y^8\, m\, \mu\, z^2\, a^4 - 4\, m\, \mu\, x^9\, a^3\, c\, z^2$

$- 6\, m\, \mu\, x^7\, a^2\, c^2\, z^4 - 4\, m\, \mu\, x^5\, a\, c^3\, z^6 - 4\, m\, \mu\, z^4\, x^7\, a^3\, c - 6\, m\, \mu\, z^6\, x^5\, a^2\, c^2 - 4\, m\, \mu\, z^8\, x^3\, a\, c^3$

$$-4y^6 m\mu z^2 x^3 a^4 - 6y^4 m\mu z^2 x^5 a^4 - 4y^2 m\mu z^2 x^7 a^4 + 3\,\mathrm{I}\,y x^2 z\,(x^2 a + a y^2 + c z^2)^{7/2} a$$

$$-3\,\mathrm{I}\,y x^2 z\,(x^2 a + a y^2 + c z^2)^{7/2} c - m x\mu y^2 c^4 z^8 - m x^3 \mu c^4 z^8 - m x \mu z^{10} c^4$$

$$- m\mu x^{11} a^4\big) z a^2 m^2\Big), -\frac{3 a^2 z m^2 (a-c)(x^2+y^2)}{(x^2 a + a y^2 + c z^2)^4 \mu}\Bigg]$$

$$Dissipation\ = 0$$

****************************** *END PROCEDURE* **************************** **(39)**

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`Example 12 -- Black Hole 2 singular vortex ring `;
```

```
> phi := 1; A1:=a*y/(x^2+y^2+z^2);A2 := -a*x/(x^2+y^2+z^2);A3:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(A1,A2,A3,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

****

NAME := Example 12 -- Black Hole 2 singular vortex ring

$$\phi := 1$$

$$A1 := \frac{a\,y}{x^2+y^2+z^2}$$

$$A2 := -\frac{a\,x}{x^2+y^2+z^2}$$

$$A3 := 0$$

Example 12 -- Black Hole 2 singular vortex ring

********************************* *Differential Form Format* ********************
****

$$Action\ 1\text{-}form = \frac{(-x^2-y^2-z^2)\,d(t)}{x^2+y^2+z^2} + \frac{a\,y\,d(x)}{x^2+y^2+z^2} - \frac{a\,x\,d(y)}{x^2+y^2+z^2}$$

$$Intensity\ 2\text{-}form\ F{=}dA\ = \left(-\frac{a\,(x^2-y^2+z^2)}{(x^2+y^2+z^2)^2} + \frac{a\,(x^2-y^2-z^2)}{(x^2+y^2+z^2)^2}\right)(d(x))\ \&\text{^}\ (d(y))$$

$$+\frac{2\,a\,y\,z\,(d(x))\ \&\text{^}\ (d(z))}{(x^2+y^2+z^2)^2} - \frac{2\,a\,x\,z\,(d(y))\ \&\text{^}\ (d(z))}{(x^2+y^2+z^2)^2}$$

Topological Torsion 3-form $A\wedge F = \dfrac{2\,a\,z^2\,\&\wedge(d(x),d(y),d(t))}{(x^2+y^2+z^2)^2}$

$$-\frac{2\,a\,y\,z\,\&\wedge(d(x),d(z),d(t))}{(x^2+y^2+z^2)^2}+\frac{2\,a\,x\,z\,\&\wedge(d(y),d(z),d(t))}{(x^2+y^2+z^2)^2}$$

Topological Parity 4-form $F\wedge F = 0$

********************************** *Using EM format* ******************************
****

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[-\frac{2\,a\,x\,z}{(x^2+y^2+z^2)^2}, -\frac{2\,a\,y\,z}{(x^2+y^2+z^2)^2}, -\frac{2\,a\,z^2}{(x^2+y^2+z^2)^2}\right]$$

Topological TORSION 4 vector *T4 = -[ExA + Bphi,AdotB]* $= \left[\dfrac{2\,a\,x\,z}{(x^2+y^2+z^2)^2},\right.$

$$\left.\frac{2\,a\,y\,z}{(x^2+y^2+z^2)^2}, \frac{2\,a\,z^2}{(x^2+y^2+z^2)^2}, 0\right]$$

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension *PTD* = 3

******************** *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ******
****

Xm or linear (Mean) curvature = 0

Yg or quadratic (GAUSS) curvature $= -\dfrac{a^2\,(x^2+y^2-z^2)}{(x^2+y^2+z^2)^3}$

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

$$D\ field = [0, 0, 0]$$

$$H\ field = \left[-\frac{2\,a\,x\,z}{(x^2+y^2+z^2)^2\,\mu}, -\frac{2\,a\,y\,z}{(x^2+y^2+z^2)^2\,\mu}, -\frac{2\,a\,z^2}{(x^2+y^2+z^2)^2\,\mu}\right]$$

Poynting vector ExH = *EXH*

Amperian Current 4Vector *curlH-dD/dt=J4* $= \left[\dfrac{2\,a\,y}{(x^2+y^2+z^2)^2\,\mu}, -\dfrac{2\,a\,x}{(x^2+y^2+z^2)^2\,\mu}, 0,\right.$

$0]$

Amerian charge density $\quad divD = rho = 0$

divergence Lorentz Current 4Vector, $\quad 4div(J4) = 0$

$$Topological\ SPIN\ 4\ vector\ S4 = \left[\frac{2\,a^2\,x\,z^2}{(x^2+y^2+z^2)^3\,\mu}, \frac{2\,a^2\,y\,z^2}{(x^2+y^2+z^2)^3\,\mu}, -\frac{2\,z\,a^2\,(x^2+y^2)}{\mu\,(x^2+y^2+z^2)^3}, 0\right]$$

$$Topological\ SPIN\ 3\text{-}form = \frac{2\,a^2\,x\,z^2\ \&\wedge(d(y), d(z), d(t))}{(x^2+y^2+z^2)^3\,\mu} - \frac{2\,a^2\,y\,z^2\ \&\wedge(d(x), d(z), d(t))}{(x^2+y^2+z^2)^3\,\mu}$$
$$- \frac{2\,z\,a^2\,(x^2+y^2)\ \&\wedge(d(x), d(y), d(t))}{\mu\,(x^2+y^2+z^2)^3}$$

Spin density rho_spin $= 0$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = \frac{4\,a^2\,z^2}{\mu\,(x^2+y^2+z^2)^3}$$

$$B.H = \frac{4\,a^2\,z^2}{\mu\,(x^2+y^2+z^2)^3}$$

$$D.E = 0$$

$$A.J = \frac{2\,a^2\,(x^2+y^2)}{\mu\,(x^2+y^2+z^2)^3}$$

$$\text{-}rho.phi = 0$$

$$Poincare\ I \quad (B.H - D.E)\text{-}(A.J - rho.phi) = -\frac{2\,a^2\,(-2\,z^2+x^2+y^2)}{\mu\,(x^2+y^2+z^2)^3}$$

$$London\ Coefficient \quad LC = \frac{2}{(x^2+y^2+z^2)\,\mu}$$

$$PROCA\ coefficient\ curlcurlB = \left[-\frac{8\,a\,x\,z}{(x^2+y^2+z^2)^3}, -\frac{8\,a\,y\,z}{(x^2+y^2+z^2)^3}, \frac{4\,a\,(x^2+y^2-z^2)}{(x^2+y^2+z^2)^3}\right]$$

$$Amperian\ Current\ 4Vector \quad curlH\text{-}dD/dt\text{=}J4 = \left[\frac{2\,a\,y}{(x^2+y^2+z^2)^2\,\mu}, -\frac{2\,a\,x}{(x^2+y^2+z^2)^2\,\mu}, 0,\right.$$
$$\left.0\right]$$

Lorentz Force 3 vector due to Ampere current $\quad FL = \text{-}(rho\_ampere\ E + J\_ampere\ x\ B) = [$

$$-\frac{4 a^2 x z^2}{(x^2+y^2+z^2)^4 \mu}, -\frac{4 a^2 y z^2}{(x^2+y^2+z^2)^4 \mu}, \frac{4 z a^2 (x^2+y^2)}{(x^2+y^2+z^2)^4 \mu}\Bigg]$$

Amperian Dissipation Jampere dot E $= 0$

Lorentz Force Spin factor LFSPIN $= -\frac{1}{2} x^2 - \frac{1}{2} y^2 - \frac{1}{2} z^2$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= \Bigg[\frac{2 a x z}{(x^2+y^2+z^2)^2},$

$$\frac{2 a y z}{(x^2+y^2+z^2)^2}, \frac{2 a z^2}{(x^2+y^2+z^2)^2}, 0\Bigg]$$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] $= \Bigg[\frac{2 a^2 x z^2}{(x^2+y^2+z^2)^3 \mu},$

$$\frac{2 a^2 y z^2}{(x^2+y^2+z^2)^3 \mu}, -\frac{2 z a^2 (x^2+y^2)}{\mu (x^2+y^2+z^2)^3}, 0\Bigg]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B)

$$= \Bigg[\frac{4 a^3 y z^2}{(x^2+y^2+z^2)^4 \mu}, -\frac{4 z^2 a^3 x}{(x^2+y^2+z^2)^4 \mu}, 0\Bigg]$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \Bigg[\frac{4 a^2 z^2 (-x + a y \mu)}{(x^2+y^2+z^2)^4 \mu}, -\frac{4 a^2 z^2 (y + a x \mu)}{(x^2+y^2+z^2)^4 \mu}, \frac{4 z a^2 (x^2+y^2)}{(x^2+y^2+z^2)^4 \mu}\Bigg]$

Dissipation $= \frac{2 a x z}{(x^2+y^2+z^2)^2}$

*********************************** END PROCEDURE ******************************** **(40)**

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`Example 10c -- Dirac Hedge Hog solution.   J
```

```
   ~ A      S ~ Lorentz Force `;
> Gamma:=I*m*(1/(2*(x^2+y^2+z^2)^(1/2)));
> Ax:=Gamma*(-y);Ay:=Gamma*x;
> Az:=0; phi:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+I*gamma),0):
```

NAME := Example 10c -- Dirac Hedge Hog solution.   J

~ A   S ~ Lorentz Force

$$\Gamma := \frac{\frac{1}{2}\,\mathrm{I}\,m}{\sqrt{x^2+y^2+z^2}}$$

$$Ax := -\frac{\frac{1}{2}\,\mathrm{I}\,m\,y}{\sqrt{x^2+y^2+z^2}}$$

$$Ay := \frac{\frac{1}{2}\,\mathrm{I}\,m\,x}{\sqrt{x^2+y^2+z^2}}$$

$$Az := 0$$

$$\phi := 0$$

Example 10c -- Dirac Hedge Hog solution.   J

~ A   S ~ Lorentz Force

*********************************   *Differential Form Format*   ********************
****

$$\text{Action 1-form} = -\frac{\frac{1}{2}\,\mathrm{I}\,m\,y\,d(x)}{\sqrt{x^2+y^2+z^2}} + \frac{\frac{1}{2}\,\mathrm{I}\,m\,x\,d(y)}{\sqrt{x^2+y^2+z^2}}$$

$$\text{Intensity 2-form } F=dA \;=\; \left(\frac{\frac{1}{2}\,\mathrm{I}\,m\,(x^2+z^2)}{(x^2+y^2+z^2)^{3/2}} + \frac{\frac{1}{2}\,\mathrm{I}\,m\,(y^2+z^2)}{(x^2+y^2+z^2)^{3/2}}\right)(d(x))\ \&\wedge\ (d(y))$$
$$-\frac{\frac{1}{2}\,\mathrm{I}\,m\,y\,z\,(d(x))\ \&\wedge\ (d(z))}{(x^2+y^2+z^2)^{3/2}} + \frac{\frac{1}{2}\,\mathrm{I}\,m\,x\,z\,(d(y))\ \&\wedge\ (d(z))}{(x^2+y^2+z^2)^{3/2}}$$

Topological Torsion 3-form $A\wedge F = 0$

Topological Parity 4-form $F\wedge F = 0$

********************************* *Using EM format* ******************************
****

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[\frac{\frac{1}{2} I m x z}{(x^2+y^2+z^2)^{3/2}}, \frac{\frac{1}{2} I m y z}{(x^2+y^2+z^2)^{3/2}}, \frac{\frac{1}{2} I m (x^2+y^2+2z^2)}{(x^2+y^2+z^2)^{3/2}} \right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] = [0, 0, 0, 0]

Helicity AdotB = 0

Poincare II =2(E.B) = 0

coefficient of Topological Parity 4-form = 0

Pfaff Topological Dimension PTD = 2

******************* *Correlation Similarity Invariants of Jacobian of (Ak/lambda_N)* ************************

Xm or linear (Mean) curvature = 0

$$Yg\ or\ quadratic\ (GAUSS)\ curvature = -\frac{1}{4} \frac{m^2 z^2}{(x^2+y^2+z^2)^2}$$

Za or Cubic (Interaction internal energy) curvature = 0

Tk or quartic (4D expansion) curvature = 0

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

Chirality factor CH = 0

D field = [0, 0, 0]

$$H\ field = \left[\frac{\frac{1}{2} I m x z}{(x^2+y^2+z^2)^{3/2} \mu}, \frac{\frac{1}{2} I m y z}{(x^2+y^2+z^2)^{3/2} \mu}, \frac{\frac{1}{2} I m (x^2+y^2+2z^2)}{(x^2+y^2+z^2)^{3/2} \mu} \right]$$

Poynting vector ExH = *EXH*

$$Amperian\ Current\ 4Vector\ \ curlH\text{-}dD/dt\text{=}J4 = \left[-\frac{I m y}{(x^2+y^2+z^2)^{3/2} \mu}, \frac{I m x}{(x^2+y^2+z^2)^{3/2} \mu}, 0, 0 \right]$$

Amerian charge density divD = rho = 0

divergence Lorentz Current 4Vector, 4div(J4) = 0

$$Topological\ SPIN\ 4\ vector\ S4 = \left[-\frac{1}{4} \frac{m^2 x (x^2+y^2+2z^2)}{(x^2+y^2+z^2)^2 \mu}, -\frac{1}{4} \frac{m^2 y (x^2+y^2+2z^2)}{(x^2+y^2+z^2)^2 \mu}, \frac{1}{4} \frac{m^2 z (x^2+y^2)}{(x^2+y^2+z^2)^2 \mu}, 0 \right]$$

$$Topological\ SPIN\ 3\text{-}form = -\frac{1}{4}\frac{m^2 x (x^2+y^2+2z^2)\ \&\wedge(d(y), d(z), d(t))}{(x^2+y^2+z^2)^2\mu}$$
$$+\frac{1}{4}\frac{m^2 y (x^2+y^2+2z^2)\ \&\wedge(d(x), d(z), d(t))}{(x^2+y^2+z^2)^2\mu}$$
$$+\frac{1}{4}\frac{m^2 z (x^2+y^2)\ \&\wedge(d(x), d(y), d(t))}{(x^2+y^2+z^2)^2\mu}$$

$$Spin\ density\ rho\_spin = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{1}{4}\frac{(x^2+y^2+4z^2)\,m^2}{(x^2+y^2+z^2)^2\mu}$$

$$B.H = -\frac{1}{4}\frac{(x^2+y^2+4z^2)\,m^2}{(x^2+y^2+z^2)^2\mu}$$

$$D.E = 0$$

$$A.J = -\frac{1}{2}\frac{m^2(x^2+y^2)}{(x^2+y^2+z^2)^2\mu}$$

$$\text{-}rho.phi = 0$$

$$Poincare\ I \qquad (B.H - D.E)\text{-}(A.J - rho.phi) = \frac{1}{4}\frac{m^2(x^2+y^2-4z^2)}{(x^2+y^2+z^2)^2\mu}$$

$$London\ Coefficient \qquad LC = \frac{2}{(x^2+y^2+z^2)\mu}$$

$$PROCA\ coefficient\ curlcurlB = \left[\frac{3\,I\,m\,x\,z}{(x^2+y^2+z^2)^{5/2}}, \frac{3\,I\,m\,y\,z}{(x^2+y^2+z^2)^{5/2}}, -\frac{I(-2z^2+x^2+y^2)\,m}{(x^2+y^2+z^2)^{5/2}}\right]$$

$$Amperian\ Current\ 4Vector \qquad curlH\text{-}dD/dt\text{=}J4 \quad = \left[-\frac{I\,m\,y}{(x^2+y^2+z^2)^{3/2}\mu}, \frac{I\,m\,x}{(x^2+y^2+z^2)^{3/2}\mu}, 0, 0\right]$$

$$Lorentz\ Force\ 3\ vector\ due\ to\ Ampere\ current \quad FL = \text{-}(rho\_ampere\ E + J\_ampere\ x\ B)$$
$$= \left[\frac{1}{2}\frac{m^2 x (x^2+y^2+2z^2)}{(x^2+y^2+z^2)^3\mu}, \frac{1}{2}\frac{m^2 y (x^2+y^2+2z^2)}{(x^2+y^2+z^2)^3\mu}, -\frac{1}{2}\frac{m^2 z (x^2+y^2)}{\mu (x^2+y^2+z^2)^3}\right]$$

$$Amperian\ Dissipation\ \ Jampere\ dot\ E \quad = 0$$

Lorentz Force Spin factor LFSPIN $= -\frac{1}{2}x^2 - \frac{1}{2}y^2 - \frac{1}{2}z^2$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $= [0, 0, 0, 0]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $= [0, 0, 0]$

Torsion Dissipation Jtorsion dot E $= 0$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD] =

$$\left[-\frac{1}{4}\frac{m^2 x (x^2+y^2+2z^2)}{(x^2+y^2+z^2)^2 \mu}, -\frac{1}{4}\frac{m^2 y (x^2+y^2+2z^2)}{(x^2+y^2+z^2)^2 \mu}, \frac{1}{4}\frac{m^2 z (x^2+y^2)}{(x^2+y^2+z^2)^2 \mu}, 0\right]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B)

$$= \left[\frac{\frac{1}{8} I (x^2+y^2+4z^2) y m^3}{(x^2+y^2+z^2)^{5/2} \mu}, -\frac{\frac{1}{8} I (x^2+y^2+4z^2) x m^3}{(x^2+y^2+z^2)^{5/2} \mu}, 0\right]$$

Spin Dissipation J_spin dot E $= 0$

Disspative Force 3 vector $= \Big[\frac{1}{8}\frac{1}{\mu (x^2+y^2+z^2)^{11/2}}\Big(m^2\Big(4x^3(x^2+y^2+z^2)^{5/2} + 4x(x^2$

$+ y^2 + z^2)^{5/2} y^2 + 8x(x^2+y^2+z^2)^{5/2} z^2 + 30 I y^3 m \mu x^2 z^4 + 6 I y^5 m \mu x^4 + 13 I y m \mu x^2 z^6$

$+ 7 I y m \mu x^6 z^2 + I y m \mu x^8 + 4 I y m \mu z^8 + I y^9 m \mu + 15 I y m \mu x^4 z^4 + 4 I y^7 m \mu x^2$

$+ 15 I y^5 m \mu z^4 + 7 I y^7 m \mu z^2 + 21 I y^5 m \mu x^2 z^2 + 21 I y^3 m \mu x^4 z^2 + 13 I y^3 m \mu z^6$

$+ 4 I y^3 m \mu x^6\Big)\Big), -\frac{1}{8}\frac{1}{(x^2+y^2+z^2)^{11/2} \mu}\Big(m^2\Big(-4y(x^2+y^2+z^2)^{5/2} x^2$

$- 4y^3(x^2+y^2+z^2)^{5/2} - 8y(x^2+y^2+z^2)^{5/2} z^2 + 6 I x^5 m \mu y^4 + 15 I x m \mu y^4 z^4$

$+ 21 I x^3 m \mu y^4 z^2 + 21 I x^5 m \mu y^2 z^2 + I x^9 m \mu + 4 I x^7 m \mu y^2 + I x m \mu y^8 + 30 I x^3 m \mu y^2 z^4$

$$+ 7\,\mathrm{I}\,x^7\,m\,\mu\,z^2 + 7\,\mathrm{I}\,x\,m\,\mu\,y^6\,z^2 + 13\,\mathrm{I}\,x\,m\,\mu\,z^6\,y^2 + 4\,\mathrm{I}\,x^3\,m\,\mu\,y^6 + 15\,\mathrm{I}\,x^5\,m\,\mu\,z^4 + 4\,\mathrm{I}\,x\,m\,\mu\,z^8$$

$$+ 13\,\mathrm{I}\,x^3\,m\,\mu\,z^6\big)\big),\ -\frac{1}{2}\,\frac{m^2\,z\,(x^2+y^2)}{\mu\,(x^2+y^2+z^2)^3}\Bigg]$$

$$Dissipation\ = 0$$

$$****************************** \quad END\ PROCEDURE \quad ****************************** \qquad (41)$$

Enter the name of the problem, and the components of the 4 potential

```
> NAME:=`Example 13 -- Bateman`;
```

```
> phi:=AAa(x,y,z,t); Az:=AAb(x,y,z,t);Ax:=0;Ay:=0;
```

Then call the procedure JCM(A1,A2,A3,phi,a,b,c,e,p,N,H,chirality,sigma)

```
> JCM(Ax,Ay,Az,phi,1,1,1,1,2,0,0,0*alpha*(g+i*gamma),0);
```

****

$$NAME := Example\ 13\ \text{--}\ Bateman$$

$$\phi := AAa(x, y, z, t)$$

$$Az := AAb(x, y, z, t)$$

$$Ax := 0$$

$$Ay := 0$$

$$Example\ 13\ \text{--}\ Bateman$$

$$********************************* \quad Differential\ Form\ Format \quad ********************\ *************$$

$$Action\ 1\text{-}form = AAb(x, y, z, t)\ d(z) - AAa(x, y, z, t)\ d(t)$$

$$Intensity\ 2\text{-}form\ F{=}dA\ = -\left(\frac{\partial}{\partial x} AAa(x, y, z, t)\right) (d(x))\ \&\wedge\ (d(t)) + \left(\frac{\partial}{\partial x} AAb(x, y, z, t)\right) (d(x))\ \&\wedge\ (d(z)) - \left(\frac{\partial}{\partial y} AAa(x, y, z, t)\right) (d(y))\ \&\wedge\ (d(t)) + \left(\frac{\partial}{\partial y} AAb(x, y, z, t)\right) (d(y))\ \&\wedge\ (d(z)) + \left(-\left(\frac{\partial}{\partial t} AAb(x, y, z, t)\right) - \left(\frac{\partial}{\partial z} AAa(x, y, z, t)\right)\right) (d(z))\ \&\wedge\ (d(t))$$

Topological Torsion 3-form $A\wedge F = \left(AAb(x,y,z,t)\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right) - AAa(x,y,z,t)\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)\right) \&\wedge(d(x), d(z), d(t)) + \left(AAb(x,y,z,t)\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right) - AAa(x,y,z,t)\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\right) \&\wedge(d(y), d(z), d(t))$

Topological Parity 4-form $F\wedge F = -2\left(\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right) - \left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)\right) \&\wedge(d(x), d(y), d(z), d(t))$

********************************* Using EM format ******************************

$$E\ field = \left[-\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right), -\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right), -\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right) - \left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)\right]$$

$$B\ field = \left[\frac{\partial}{\partial y}AAb(x,y,z,t), -\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right), 0\right]$$

Topological TORSION 4 vector T4 = -[ExA + Bphi,AdotB] $= \left[\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)AAb(x,y,z,t) - \left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)AAa(x,y,z,t), -\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right)AAb(x,y,z,t) + \left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)AAa(x,y,z,t), 0, 0\right]$

$$Helicity\ AdotB = 0$$

Poincare II =2(E.B) $= 2\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right) - 2\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)$

coefficient of Topological Parity 4-form $= 2\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right) - 2\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)$

Pfaff Topological Dimension $PTD = 4$

****************** Correlation Similarity Invariants of Jacobian of (Ak/lambda_N) ****** ********************

Xm or linear (Mean) curvature $= \frac{\partial}{\partial z}AAb(x,y,z,t) - \left(\frac{\partial}{\partial t}AAa(x,y,z,t)\right)$

$$\textit{Yg or quadratic (GAUSS) curvature} = -\left(\frac{\partial}{\partial z} AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial t} AAa(x,y,z,t)\right) + \left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right)$$

$$\textit{Za or Cubic (Interaction internal energy) curvature} = 0$$

$$\textit{Tk or quartic (4D expansion) curvature} = 0$$

********** *Compute Current using from Maxell-Ampere equations for constitutive equations with chirality CH* ***********

$$\textit{Chirality factor CH} = 0$$

$$\textit{D field} = \left[-\varepsilon\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right), -\varepsilon\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right), -\varepsilon\left(\frac{\partial}{\partial t} AAb(x,y,z,t) + \frac{\partial}{\partial z} AAa(x,y,z,t)\right)\right]$$

$$\textit{H field} = \left[\frac{\frac{\partial}{\partial y} AAb(x,y,z,t)}{\mu}, -\frac{\frac{\partial}{\partial x} AAb(x,y,z,t)}{\mu}, 0\right]$$

$$\textit{Poynting vector ExH} = \left[-\frac{\left(\frac{\partial}{\partial t} AAb(x,y,z,t) + \frac{\partial}{\partial z} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right)}{\mu}, -\frac{\left(\frac{\partial}{\partial t} AAb(x,y,z,t) + \frac{\partial}{\partial z} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)}{\mu}, \frac{\left(\frac{\partial}{\partial x} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial x} AAb(x,y,z,t)\right) + \left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)}{\mu}\right]$$

Amperian Current 4Vector *curlH-dD/dt=J4*

$$= \left[\frac{\frac{\partial^2}{\partial z\,\partial x} AAb(x,y,z,t) + \varepsilon\left(\frac{\partial^2}{\partial x\,\partial t} AAa(x,y,z,t)\right)\mu}{\mu}, \frac{\frac{\partial^2}{\partial z\,\partial y} AAb(x,y,z,t) + \varepsilon\left(\frac{\partial^2}{\partial y\,\partial t} AAa(x,y,z,t)\right)\mu}{\mu}, \frac{1}{\mu}\left(-\left(\frac{\partial^2}{\partial x^2} AAb(x,y,z,t)\right) - \left(\frac{\partial^2}{\partial y^2} AAb(x,y,z,t)\right) + \varepsilon\,\mu\left(\frac{\partial^2}{\partial t^2} AAb(x,y,z,t)\right) + \varepsilon\,\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAa(x,y,z,t)\right)\right), -\varepsilon\left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t) + \frac{\partial^2}{\partial y^2} AAa(x,y,z,t) + \frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,t) + \frac{\partial^2}{\partial z^2} AAa(x,y,z,t)\right)\right]$$

Amerian charge density $\quad divD = rho = -\varepsilon \left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t) + \frac{\partial^2}{\partial y^2} AAa(x,y,z,t) + \frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,t) + \frac{\partial^2}{\partial z^2} AAa(x,y,z,t) \right)$

divergence Lorentz Current 4Vector, $\quad$ *4div(J4)* $= 0$

Topological SPIN 4 vector S4

$$= \left[\frac{AAb(x,y,z,t) \left(\frac{\partial}{\partial x} AAb(x,y,z,t) \right) - \varepsilon \left(\frac{\partial}{\partial x} AAa(x,y,z,t) \right) AAa(x,y,z,t)\, \mu}{\mu}, \frac{AAb(x,y,z,t) \left(\frac{\partial}{\partial y} AAb(x,y,z,t) \right) - \varepsilon \left(\frac{\partial}{\partial y} AAa(x,y,z,t) \right) AAa(x,y,z,t)\, \mu}{\mu}, -\varepsilon \left(\frac{\partial}{\partial t} AAb(x,y,z,t) + \frac{\partial}{\partial z} AAa(x,y,z,t) \right) AAa(x,y,z,t), -AAb(x,y,z,t)\, \varepsilon \left(\frac{\partial}{\partial t} AAb(x,y,z,t) + \frac{\partial}{\partial z} AAa(x,y,z,t) \right) \right]$$

Topological SPIN 3-form $= \frac{1}{\mu} \left(\left(AAb(x,y,z,t) \left(\frac{\partial}{\partial x} AAb(x,y,z,t) \right) - \varepsilon \left(\frac{\partial}{\partial x} AAa(x,y,z,t) \right) AAa(x,y,z,t)\, \mu \right) \&\hat{}(d(y), d(z), d(t)) \right) - \frac{1}{\mu} \left(\left(AAb(x,y,z,t) \left(\frac{\partial}{\partial y} AAb(x,y,z,t) \right) - \varepsilon \left(\frac{\partial}{\partial y} AAa(x,y,z,t) \right) AAa(x,y,z,t)\, \mu \right) \&\hat{}(d(x), d(z), d(t)) \right) - \varepsilon \left(\frac{\partial}{\partial t} AAb(x,y,z,t) + \frac{\partial}{\partial z} AAa(x,y,z,t) \right) AAa(x,y,z,t) \&\hat{}(d(x), d(y), d(t)) + AAb(x,y,z,t)\, \varepsilon \left(\frac{\partial}{\partial t} AAb(x,y,z,t) + \frac{\partial}{\partial z} AAa(x,y,z,t) \right) \&\hat{}(d(x), d(y), d(z))$

Spin density rho_spin $= -AAb(x,y,z,t)\, \varepsilon \left(\frac{\partial}{\partial t} AAb(x,y,z,t) + \frac{\partial}{\partial z} AAa(x,y,z,t) \right)$

LaGrange field energy density (B.H-D.E) $= -\frac{1}{\mu} \left(-\left(\frac{\partial}{\partial y} AAb(x,y,z,t) \right)^2 - \left(\frac{\partial}{\partial x} AAb(x,y,z,t) \right)^2 + \varepsilon\mu \left(\frac{\partial}{\partial x} AAa(x,y,z,t) \right)^2 + \varepsilon\mu \left(\frac{\partial}{\partial y} AAa(x,y,z,t) \right)^2 + \varepsilon\mu \left(\frac{\partial}{\partial t} AAb(x,y,z,t) \right)^2 + 2\,\varepsilon\mu \left(\frac{\partial}{\partial t} AAb(x,y,z,t) \right) \left(\frac{\partial}{\partial z} AAa(x,y,z,t) \right) + \varepsilon\mu \left(\frac{\partial}{\partial z} AAa(x,y,z,t) \right)^2 \right)$

$$B.H = \frac{\left(\frac{\partial}{\partial y} AAb(x,y,z,t) \right)^2 + \left(\frac{\partial}{\partial x} AAb(x,y,z,t) \right)^2}{\mu}$$

$$D.E = \varepsilon \left(\left(\frac{\partial}{\partial x} AAa(x, y, z, t) \right)^2 + \left(\frac{\partial}{\partial y} AAa(x, y, z, t) \right)^2 + \left(\frac{\partial}{\partial t} AAb(x, y, z, t) \right)^2 + 2 \left(\frac{\partial}{\partial t} AAb(x, y, z, t) \right) \left(\frac{\partial}{\partial z} AAa(x, y, z, t) \right) + \left(\frac{\partial}{\partial z} AAa(x, y, z, t) \right)^2 \right)$$

$$A.J = \frac{1}{\mu} \left(AAb(x, y, z, t) \left(- \left(\frac{\partial^2}{\partial x^2} AAb(x, y, z, t) \right) - \left(\frac{\partial^2}{\partial y^2} AAb(x, y, z, t) \right) + \varepsilon\, \mu \left(\frac{\partial^2}{\partial t^2} AAb(x, y, z, t) \right) + \varepsilon\, \mu \left(\frac{\partial^2}{\partial z\, \partial t} AAa(x, y, z, t) \right) \right) \right)$$

$$\textit{-rho.phi} = -AAa(x, y, z, t)\, \varepsilon \left(\frac{\partial^2}{\partial x^2} AAa(x, y, z, t) + \frac{\partial^2}{\partial y^2} AAa(x, y, z, t) + \frac{\partial^2}{\partial z\, \partial t} AAb(x, y, z, t) + \frac{\partial^2}{\partial z^2} AAa(x, y, z, t) \right)$$

Poincare I *(B.H - D.E)-(A.J - rho.phi)*

$$= -\frac{1}{\mu} \left(- \left(\frac{\partial}{\partial y} AAb(x, y, z, t) \right)^2 - \left(\frac{\partial}{\partial x} AAb(x, y, z, t) \right)^2 + \varepsilon\, \mu \left(\frac{\partial}{\partial x} AAa(x, y, z, t) \right)^2 + \varepsilon\, \mu \left(\frac{\partial}{\partial y} AAa(x, y, z, t) \right)^2 + \varepsilon\, \mu \left(\frac{\partial}{\partial t} AAb(x, y, z, t) \right)^2 + 2\, \varepsilon\, \mu \left(\frac{\partial}{\partial t} AAb(x, y, z, t) \right) \left(\frac{\partial}{\partial z} AAa(x, y, z, t) \right) + \varepsilon\, \mu \left(\frac{\partial}{\partial z} AAa(x, y, z, t) \right)^2 - AAb(x, y, z, t) \left(\frac{\partial^2}{\partial x^2} AAb(x, y, z, t) \right) - AAb(x, y, z, t) \left(\frac{\partial^2}{\partial y^2} AAb(x, y, z, t) \right) + AAb(x, y, z, t)\, \varepsilon\, \mu \left(\frac{\partial^2}{\partial t^2} AAb(x, y, z, t) \right) + AAb(x, y, z, t)\, \varepsilon\, \mu \left(\frac{\partial^2}{\partial z\, \partial t} AAa(x, y, z, t) \right) + AAa(x, y, z, t)\, \varepsilon\, \mu \left(\frac{\partial^2}{\partial x^2} AAa(x, y, z, t) \right) + AAa(x, y, z, t)\, \varepsilon\, \mu \left(\frac{\partial^2}{\partial y^2} AAa(x, y, z, t) \right) + AAa(x, y, z, t)\, \varepsilon\, \mu \left(\frac{\partial^2}{\partial z\, \partial t} AAb(x, y, z, t) \right) + AAa(x, y, z, t)\, \varepsilon\, \mu \left(\frac{\partial^2}{\partial z^2} AAa(x, y, z, t) \right) \right)$$

London Coefficient $LC = \frac{2}{(x^2 + y^2 + z^2)\, \mu}$

PROCA coefficient curlcurlB $= \left[- \left(\frac{\partial^3}{\partial y\, \partial x^2} AAb(x, y, z, t) \right) - \left(\frac{\partial^3}{\partial y^3} AAb(x, y, z, t) \right) - \left(\frac{\partial^3}{\partial z^2\, \partial y} AAb(x, y, z, t) \right), \frac{\partial^3}{\partial z^2\, \partial x} AAb(x, y, z, t) + \frac{\partial^3}{\partial x^3} AAb(x, y, z, t) + \frac{\partial^3}{\partial y^2\, \partial x} AAb(x, y, z, t), 0 \right]$

Amperian Current 4Vector *curlH-dD/dt=J4*

$$= \left[\frac{\frac{\partial^2}{\partial z\,\partial x} AAb(x, y, z, t) + \varepsilon \left(\frac{\partial^2}{\partial x\,\partial t} AAa(x, y, z, t) \right) \mu}{\mu}, \right.$$

$$\frac{\frac{\partial^2}{\partial z\,\partial y} AAb(x, y, z, t) + \varepsilon \left(\frac{\partial^2}{\partial y\,\partial t} AAa(x, y, z, t) \right) \mu}{\mu}, \frac{1}{\mu} \left(- \left(\frac{\partial^2}{\partial x^2} AAb(x, y, z, t) \right) \right.$$

$$\left. - \left(\frac{\partial^2}{\partial y^2} AAb(x, y, z, t) \right) + \varepsilon\,\mu \left(\frac{\partial^2}{\partial t^2} AAb(x, y, z, t) \right) + \varepsilon\,\mu \left(\frac{\partial^2}{\partial z\,\partial t} AAa(x, y, z, t) \right) \right),$$

$$-\varepsilon \left(\frac{\partial^2}{\partial x^2} AAa(x, y, z, t) + \frac{\partial^2}{\partial y^2} AAa(x, y, z, t) + \frac{\partial^2}{\partial z\,\partial t} AAb(x, y, z, t) + \frac{\partial^2}{\partial z^2} AAa(x, y, z,$$

$$\left. t) \right) \right]$$

Lorentz Force 3 vector due to Ampere current   FL = -(rho_ampere E + J_ampere x B) $= \Bigg[$

$$-\frac{1}{\mu} \left(\varepsilon \left(\frac{\partial}{\partial x} AAa(x, y, z, t) \right) \mu \left(\frac{\partial^2}{\partial x^2} AAa(x, y, z, t) \right) + \varepsilon \left(\frac{\partial}{\partial x} AAa(x, y, z, \right.\right.$$

$$\left. t) \right) \mu \left(\frac{\partial^2}{\partial y^2} AAa(x, y, z, t) \right) + \varepsilon \left(\frac{\partial}{\partial x} AAa(x, y, z, t) \right) \mu \left(\frac{\partial^2}{\partial z\,\partial t} AAb(x, y, z, t) \right)$$

$$+ \varepsilon \left(\frac{\partial}{\partial x} AAa(x, y, z, t) \right) \mu \left(\frac{\partial^2}{\partial z^2} AAa(x, y, z, t) \right) - \left(\frac{\partial}{\partial x} AAb(x, y, z, t) \right) \left(\frac{\partial^2}{\partial x^2} AAb(x, y, \right.$$

$$\left. z, t) \right) - \left(\frac{\partial}{\partial x} AAb(x, y, z, t) \right) \left(\frac{\partial^2}{\partial y^2} AAb(x, y, z, t) \right) + \left(\frac{\partial}{\partial x} AAb(x, y, z, \right.$$

$$\left.\left. t) \right) \varepsilon\,\mu \left(\frac{\partial^2}{\partial t^2} AAb(x, y, z, t) \right) + \left(\frac{\partial}{\partial x} AAb(x, y, z, t) \right) \varepsilon\,\mu \left(\frac{\partial^2}{\partial z\,\partial t} AAa(x, y, z, t) \right) \right),$$

$$-\frac{1}{\mu} \left(\varepsilon \left(\frac{\partial}{\partial y} AAa(x, y, z, t) \right) \mu \left(\frac{\partial^2}{\partial x^2} AAa(x, y, z, t) \right) + \varepsilon \left(\frac{\partial}{\partial y} AAa(x, y, z, \right.\right.$$

$$\left. t) \right) \mu \left(\frac{\partial^2}{\partial y^2} AAa(x, y, z, t) \right) + \varepsilon \left(\frac{\partial}{\partial y} AAa(x, y, z, t) \right) \mu \left(\frac{\partial^2}{\partial z\,\partial t} AAb(x, y, z, t) \right)$$

$$+\varepsilon\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)\mu\left(\frac{\partial^2}{\partial z^2}AAa(x,y,z,t)\right)-\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial x^2}AAb(x,y,z,t)\right)-\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial y^2}AAb(x,y,z,t)\right)+\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\varepsilon\mu\left(\frac{\partial^2}{\partial t^2}AAb(x,y,z,t)\right)+\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\varepsilon\mu\left(\frac{\partial^2}{\partial z\,\partial t}AAa(x,y,z,t)\right)\Bigg),$$

$$-\frac{1}{\mu}\Bigg(\varepsilon\mu\left(\frac{\partial^2}{\partial x^2}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)+\varepsilon\mu\left(\frac{\partial^2}{\partial x^2}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)+\varepsilon\mu\left(\frac{\partial^2}{\partial y^2}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)$$

$$+\varepsilon\mu\left(\frac{\partial^2}{\partial y^2}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)+\varepsilon\mu\left(\frac{\partial^2}{\partial z\,\partial t}AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)+\varepsilon\mu\left(\frac{\partial^2}{\partial z\,\partial t}AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)$$

$$+\varepsilon\mu\left(\frac{\partial^2}{\partial z^2}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)+\varepsilon\mu\left(\frac{\partial^2}{\partial z^2}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)-\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial z\,\partial x}AAb(x,y,z,t)\right)-\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)\varepsilon\left(\frac{\partial^2}{\partial x\,\partial t}AAa(x,y,z,t)\right)\mu-\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial z\,\partial y}AAb(x,y,z,t)\right)$$

$$-\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\varepsilon\left(\frac{\partial^2}{\partial y\,\partial t}AAa(x,y,z,t)\right)\mu\Bigg)\Bigg]$$

Amperian Dissipation Jampere dot E $=0$

Lorentz Force Spin factor LFSPIN $=0$

Topological Torsion current 4 vector T4 = -[ExA + B.phi,AdotB] $=\left[\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)AAb(x,y,z,t)-\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)AAa(x,y,z,t),\ -\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right)AAb(x,y,z,t)+\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)AAa(x,y,z,t),0,0\right]$

Lorentz Force 3 vector due to Torsion current TF = -(rho_torsion E + J_torsion x B) $=\Bigg[0,0,AAb(x,y,z,t)\Bigg(\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)-\left(\frac{\partial}{\partial y}AAa(x,y,z,$

$t)\Big)\left(\frac{\partial}{\partial x} AAb(x, y, z, t)\right)\Big)\Big]$

Torsion Dissipation Jtorsion dot E $= -AAa(x, y, z, t)\left(\left(\frac{\partial}{\partial x} AAa(x, y, z, t)\right)\left(\frac{\partial}{\partial y} AAb(x, y, z, t)\right) - \left(\frac{\partial}{\partial y} AAa(x, y, z, t)\right)\left(\frac{\partial}{\partial x} AAb(x, y, z, t)\right)\right)$

Topological Spin current 4 vector TS4 = -[A x H + D.phi,AdotD]

$$= \left[\frac{AAb(x, y, z, t)\left(\frac{\partial}{\partial x} AAb(x, y, z, t)\right) - \varepsilon\left(\frac{\partial}{\partial x} AAa(x, y, z, t)\right) AAa(x, y, z, t)\,\mu}{\mu},\right.$$

$$\frac{AAb(x, y, z, t)\left(\frac{\partial}{\partial y} AAb(x, y, z, t)\right) - \varepsilon\left(\frac{\partial}{\partial y} AAa(x, y, z, t)\right) AAa(x, y, z, t)\,\mu}{\mu},$$

$$-\varepsilon\left(\frac{\partial}{\partial t} AAb(x, y, z, t) + \frac{\partial}{\partial z} AAa(x, y, z, t)\right) AAa(x, y, z, t),\ -AAb(x, y, z, t)\,\varepsilon\left(\frac{\partial}{\partial t} AAb(x, y, z, t) + \frac{\partial}{\partial z} AAa(x, y, z, t)\right)\Big]$$

Lorentz Force 3 vector due to Spin current SF = --(rho_spin E + J_spin x B) $= \Big[$

$$-\varepsilon\left(\frac{\partial}{\partial t} AAb(x, y, z, t) + \frac{\partial}{\partial z} AAa(x, y, z, t)\right)\left(\left(\frac{\partial}{\partial x} AAa(x, y, z, t)\right) AAb(x, y, z, t) - \left(\frac{\partial}{\partial x} AAb(x, y, z, t)\right) AAa(x, y, z, t)\right),$$

$$-\varepsilon\left(\frac{\partial}{\partial t} AAb(x, y, z, t) + \frac{\partial}{\partial z} AAa(x, y, z, t)\right)\left(\left(\frac{\partial}{\partial y} AAa(x, y, z, t)\right) AAb(x, y, z, t) - \left(\frac{\partial}{\partial y} AAb(x, y, z, t)\right) AAa(x, y, z, t)\right),$$

$$-\frac{1}{\mu}\Bigg(AAb(x, y, z, t)\,\varepsilon\,\mu\left(\frac{\partial}{\partial t} AAb(x, y, z, t)\right)^2 + 2\,AAb(x, y, z, t)\,\varepsilon\,\mu\left(\frac{\partial}{\partial t} AAb(x, y, z, t)\right)\left(\frac{\partial}{\partial z} AAa(x, y, z, t)\right) + AAb(x, y, z, t)\,\varepsilon\,\mu\left(\frac{\partial}{\partial z} AAa(x, y, z, t)\right)^2 - AAb(x, y, z, t)\left(\frac{\partial}{\partial x} AAb(x, y, z, t)\right)^2 + \left(\frac{\partial}{\partial x} AAb(x, y, z, t)\right)\varepsilon\left(\frac{\partial}{\partial x} AAa(x, y, z, t)\right) AAa(x, y, z, t)\,\mu$$

$$- AAb(x, y, z, t)\left(\frac{\partial}{\partial y} AAb(x, y, z, t)\right)^2 + \left(\frac{\partial}{\partial y} AAb(x, y, z, t)\right)\varepsilon\left(\frac{\partial}{\partial y} AAa(x, y, z,$$

$$t)\Big) AAa(x, y, z, t)\,\mu\Big)\Big]$$

Spin Dissipation J_spin dot E $= -\frac{1}{\mu}\Bigg(\Big(\Big(\frac{\partial}{\partial x} AAa(x, y, z, t)\Big) AAb(x, y, z, t)\,\Big(\frac{\partial}{\partial x} AAb(x, y, z,$

$$t)\Big) - \varepsilon\Big(\frac{\partial}{\partial x} AAa(x, y, z, t)\Big)^2 AAa(x, y, z, t)\,\mu + \Big(\frac{\partial}{\partial y} AAa(x, y, z, t)\Big) AAb(x, y, z,$$

$$t)\,\Big(\frac{\partial}{\partial y} AAb(x, y, z, t)\Big) - \varepsilon\Big(\frac{\partial}{\partial y} AAa(x, y, z, t)\Big)^2 AAa(x, y, z, t)\,\mu - AAa(x, y, z,$$

$$t)\,\varepsilon\,\mu\Big(\frac{\partial}{\partial t} AAb(x, y, z, t)\Big)^2 - 2\,AAa(x, y, z, t)\,\varepsilon\,\mu\Big(\frac{\partial}{\partial t} AAb(x, y, z, t)\Big)\Big(\frac{\partial}{\partial z} AAa(x, y, z,$$

$$t)\Big) - AAa(x, y, z, t)\,\varepsilon\,\mu\Big(\frac{\partial}{\partial z} AAa(x, y, z, t)\Big)^2\Bigg)$$

Disspative Force 3 vector $= \Bigg[-\frac{1}{\mu}\Bigg(\varepsilon\Big(\frac{\partial}{\partial x} AAa(x, y, z, t)\Big)\,\mu\Big(\frac{\partial^2}{\partial x^2} AAa(x, y, z, t)\Big)$

$$+\varepsilon\Big(\frac{\partial}{\partial x} AAa(x, y, z, t)\Big)\,\mu\Big(\frac{\partial^2}{\partial y^2} AAa(x, y, z, t)\Big) + \varepsilon\Big(\frac{\partial}{\partial x} AAa(x, y, z,$$

$$t)\Big)\,\mu\Big(\frac{\partial^2}{\partial z\,\partial t} AAb(x, y, z, t)\Big) + \varepsilon\Big(\frac{\partial}{\partial x} AAa(x, y, z, t)\Big)\,\mu\Big(\frac{\partial^2}{\partial z^2} AAa(x, y, z, t)\Big)$$

$$-\Big(\frac{\partial}{\partial x} AAb(x, y, z, t)\Big)\Big(\frac{\partial^2}{\partial x^2} AAb(x, y, z, t)\Big) - \Big(\frac{\partial}{\partial x} AAb(x, y, z, t)\Big)\Big(\frac{\partial^2}{\partial y^2} AAb(x, y, z,$$

$$t)\Big) + \Big(\frac{\partial}{\partial x} AAb(x, y, z, t)\Big)\,\varepsilon\,\mu\Big(\frac{\partial^2}{\partial t^2} AAb(x, y, z, t)\Big) + \Big(\frac{\partial}{\partial x} AAb(x, y, z,$$

$$t)\Big)\,\varepsilon\,\mu\Big(\frac{\partial^2}{\partial z\,\partial t} AAa(x, y, z, t)\Big) + \mu^2\,\varepsilon\Big(\frac{\partial}{\partial t} AAb(x, y, z, t)\Big)\Big(\frac{\partial}{\partial x} AAa(x, y, z, t)\Big) AAb(x, y,$$

$$z, t) - \mu^2\,\varepsilon\Big(\frac{\partial}{\partial t} AAb(x, y, z, t)\Big)\Big(\frac{\partial}{\partial x} AAb(x, y, z, t)\Big) AAa(x, y, z, t) + \mu^2\,\varepsilon\Big(\frac{\partial}{\partial z} AAa(x, y,$$

$$z, t)\Big)\Big(\frac{\partial}{\partial x} AAa(x, y, z, t)\Big) AAb(x, y, z, t) - \mu^2\,\varepsilon\Big(\frac{\partial}{\partial z} AAa(x, y, z, t)\Big)\Big(\frac{\partial}{\partial x} AAb(x, y, z,$$

$$t)\Big) AAa(x,y,z,t)\Big), -\frac{1}{\mu}\Bigg(\varepsilon\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)\mu\left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t)\right)$$

$$+\varepsilon\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)\mu\left(\frac{\partial^2}{\partial y^2} AAa(x,y,z,t)\right)+\varepsilon\left(\frac{\partial}{\partial y} AAa(x,y,z,\right.$$

$$\left.t)\right)\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,t)\right)+\varepsilon\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right)\mu\left(\frac{\partial^2}{\partial z^2} AAa(x,y,z,t)\right)$$

$$-\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial x^2} AAb(x,y,z,t)\right)-\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial y^2} AAb(x,y,z,\right.$$

$$\left.t)\right)+\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right)\varepsilon\,\mu\left(\frac{\partial^2}{\partial t^2} AAb(x,y,z,t)\right)+\left(\frac{\partial}{\partial y} AAb(x,y,z,\right.$$

$$\left.t)\right)\varepsilon\,\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAa(x,y,z,t)\right)+\mu^2\varepsilon\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right) AAb(x,y,$$

$$z,t)-\mu^2\varepsilon\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial y} AAb(x,y,z,t)\right) AAa(x,y,z,t)+\mu^2\varepsilon\left(\frac{\partial}{\partial z} AAa(x,y,\right.$$

$$\left.z,t)\right)\left(\frac{\partial}{\partial y} AAa(x,y,z,t)\right) AAb(x,y,z,t)-\mu^2\varepsilon\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial y} AAb(x,y,z,\right.$$

$$t)\Big) AAa(x,y,z,t)\Big), -\frac{1}{\mu}\Bigg(\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)$$

$$+\varepsilon\,\mu\left(\frac{\partial^2}{\partial x^2} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right)+\varepsilon\,\mu\left(\frac{\partial^2}{\partial y^2} AAa(x,y,z,\right.$$

$$\left.t)\right)\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)+\varepsilon\,\mu\left(\frac{\partial^2}{\partial y^2} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right)$$

$$+\varepsilon\,\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)+\varepsilon\,\mu\left(\frac{\partial^2}{\partial z\,\partial t} AAb(x,y,z,\right.$$

$$\left.t)\right)\left(\frac{\partial}{\partial z} AAa(x,y,z,t)\right)+\varepsilon\,\mu\left(\frac{\partial^2}{\partial z^2} AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial t} AAb(x,y,z,t)\right)$$

$$+\varepsilon\,\mu\left(\frac{\partial^2}{\partial z^2}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)-\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial z\,\partial x}AAb(x,y,z,t)\right)-\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)\varepsilon\left(\frac{\partial^2}{\partial x\,\partial t}AAa(x,y,z,t)\right)\mu-\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\left(\frac{\partial^2}{\partial z\,\partial y}AAb(x,y,z,t)\right)-\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\varepsilon\left(\frac{\partial^2}{\partial y\,\partial t}AAa(x,y,z,t)\right)\mu+AAb(x,y,z,t)\,\varepsilon\,\mu^2\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)^2+2\,AAb(x,y,z,t)\,\varepsilon\,\mu^2\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)+AAb(x,y,z,t)\,\varepsilon\,\mu^2\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)^2-AAb(x,y,z,t)\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)^2\mu+\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)\varepsilon\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right)AAa(x,y,z,t)\,\mu^2-AAb(x,y,z,t)\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)^2\mu+\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)\varepsilon\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)AAa(x,y,z,t)\,\mu^2-AAb(x,y,z,t)\,\mu\left(\frac{\partial}{\partial x}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)+AAb(x,y,z,t)\,\mu\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)\left(\frac{\partial}{\partial x}AAb(x,y,z,t)\right)\Bigg)\Bigg]$$

$$Dissipation\ = -\varepsilon\left(\frac{\partial^2}{\partial x^2}AAa(x,y,z,t)\right)-\varepsilon\left(\frac{\partial^2}{\partial y^2}AAa(x,y,z,t)\right)-\varepsilon\left(\frac{\partial^2}{\partial z\,\partial t}AAb(x,y,z,t)\right)-\varepsilon\left(\frac{\partial^2}{\partial z^2}AAa(x,y,z,t)\right)-AAb(x,y,z,t)\,\varepsilon\,\mu\left(\frac{\partial}{\partial t}AAb(x,y,z,t)\right)-AAb(x,y,z,t)\,\varepsilon\,\mu\left(\frac{\partial}{\partial z}AAa(x,y,z,t)\right)+\left(\frac{\partial}{\partial y}AAa(x,y,z,t)\right)AAb(x,y,z,t)-\left(\frac{\partial}{\partial y}AAb(x,y,z,t)\right)AAa(x,y,z,t)$$

****************************** *END PROCEDURE* ***************************** **(42)**