```
> restart:
```

MapleEM.mws
from maxwell/mws and maxwellplasma.mws
Updated 12/12/97, 11/5/98, 10/24/2002 Correcting sign of T4 and d(A^F), 11/09/2003,
Last update: November 06,2008
-- R. M. Kiehn

NOTES:
The fundamental references are my monographs Vol1 and Vol4, which can be found at http://www.lulu.com/kiehn
*
This Maple program computes Maxwell-Faraday formulas from the postulate of potentials F-dA=0.
Given a 1-form of Action on 4D space time, the E and B fields follow by exterior differentiation.
The 2-form F is the set of limit points for the 1-form, A.
The Maxwell Ampere equations are computed from the postulate of charge currents, J-dG=0.
The 2-form density, G, with components D and H, is constructed in several ways
1. The most simple assumption selects the Lorentz vacuum constitutive equations, D = epsilon E B = mu H.
2. A more complicated procedure selects the complex 6x6 constitutive matrix formualted by Post (see Vol 4)
3. Another procedure selects a chiral formulation for a the constitutive matrix. (see Vol4)
*
The 1-form of Action not only encodes the electromagnetic potentials,
but also topologically encodes a thermodynamic system. (see Vol1).
The Potentials, not the charge current densities, are used as the computational starting point,
with functions defined on a basis variety if 4 dimensions (x,y,z,t).
This topological approach is more useful for the constuction of field, not necessarily particle, properties
of Plasma systems, where charge currents can be associated with collective states, not individual particles
*
The program also permits the study of homogeneous systems of various degrees, through the use of
Holder Norm divisors.
*
The procedure has as input the 4 potential, and computes
E,B,D,H,Jamperian,Jdisplacement,Jtotal, and the Charge density, rho,
as well as
the Torsion vector = -{ ExA+Bphi, AdotB]
the Spin Vector = A x H + Dphi, AdotD,
the First Poincare invariant = F^G -A^J = (BdotH-DdotE) -(AdotJ-rho.phi),
the second Poincare Invariant = F^F = +2EdotB
(see vol4)

```
>  with(liesymm):with(linalg):with(plots):
> setup(x,y,z,t):defform(x=0,y=0,z=0,t=0,a=const,b=const,c=const,k=const,mu=
  const,m=const,alpha=const,beta=const,m=const,n=const,omega=const,epsilon=
  const,pi=const,p-const,e=const,N=const,H=const,Az=0,phi=0,Ax=0,Ay=0,Gamma=
  const,Omega=const):
```

The main procedure

```
JCM:=proc(Ax,Ay,Az,phi,e,p,N,H)
     local A,A1,A2,A3,A4,BFC,TFC,EF1,EF2,EF3,JAC,JDC,SFC,ExBC:
global A1form,BF,EF,TF,HEL,P1,P2,DF,HF,CD,JA,JD,SPD,SF,JTOT,ExB,NAME,LAGF,AJ,VW,
DISS,lambda,lambdaH,AH,JACOB,ADJACOB,AN,CURR:
lambda:=(x^p+y^p+z^p+e*(c*t)^p)^(N/p):
> A1:=Ax/lambda;A2:=Ay/lambda;A3:=Az/lambda;A4:=phi/lambda;  A:=[A1,A2,A3]:
  A1form:=A1*d(x)+A2*d(y)+A3*d(z)-A4*d(t):
> lambdaH:=(x^p+y^p+z^p+e*(c*t)^p)^(H/p): AH:=[Ax/lambdaH,Ay/lambdaH,
  Az/lambdaH,phi/lambdaH];AN:=[A1,A2,A3,A4];
   EF1:=evalm(-grad(A4,[x,y,z])):
    EF2:=-[diff(A1,t),diff(A2,t),diff(A3,t)];                         EF3:=[EF1[1]+
EF2[1],EF1[2]+EF2[2],EF1[3]+EF2[3]];
```

```
     EF:=[factor(simplify(EF3[1])),factor(simplify(EF3[2])),factor(simplify(EF3[3])
)];
     BFC:=(curl([A1,A2,A3],[x,y,z])):
     BF:=[factor(simplify(BFC[1])),factor(simplify(BFC[2])),factor(simplify(BFC[3])
)];
     HEL:=-factor(simplify(innerprod(A,BF)));
     TFC:=-[crossprod(EF,A)[1]+BF[1]*A4,crossprod(EF,A)[2]+BF[2]*A4,crossprod(EF,A)
[3]+BF[3]*A4];
     TF:=[factor(simplify(TFC[1])),factor(simplify(TFC[2])),factor(simplify(TFC[3])
),factor(HEL)];
     P2:=factor(simplify(2*innerprod(EF,BF)));

HF:=[factor(simplify(BFC[1]/mu)),factor(simplify(BFC[2]/mu)),factor(simplify(BFC[3]
/mu))];
DF:=[factor(simplify(epsilon*EF3[1])),factor(simplify(epsilon*EF3[2])),factor
(simplify(epsilon*EF3[3]))];
CD:=factor(diverge([DF[1],DF[2],DF[3]],[x,y,z]));
JAC:=curl([HF[1],HF[2],HF[3]],[x,y,z]);JA:=[JAC[1],JAC[2],JAC[3]];
JDC:=-[diff(DF[1],t),diff(DF[2],t),diff(DF[3],t)];JD:=[factor(JDC[1]),factor(JDC[2]
),factor(JDC[3])];
JTOT:=[factor(simplify(JA[1]+JD[1])),factor(simplify(JA[2]+JD[2])),factor(simplify
(JA[3]+JD[3]))];
DISS:=factor(simplify(innerprod(JTOT,EF)));
SPD:=factor(simplify(innerprod(A,DF)));
VW:=[factor(simplify(CD*EF[1]+crossprod(JTOT,BF)[1])),factor(simplify(CD*EF[2]+
crossprod(JTOT,BF)[2])),factor(simplify(CD*EF[3]+crossprod(JTOT,BF)[3]))];
   SFC:=[crossprod(A,HF)[1]+DF[1]*A4,crossprod(A,HF)[2]+DF[2]*A4,crossprod(A,HF)[3]
+DF[3]*A4];
     SF:=[factor(simplify(SFC[1])),factor(simplify(SFC[2])),factor(simplify(SFC[3])
),SPD];
AJ:=factor(simplify(innerprod(A,JTOT)-CD*A4));
LAGF:=factor(simplify(innerprod(BF,HF)-innerprod(DF,EF)));P1:=factor(simplify(LAGF-
AJ));
JACOB:=simplify(jacobian(AN,[x,y,z,t]));ADJACOB:=adjoint(JACOB);
CURR:=simplify(innerprod(ADJACOB,AH));
ExBC:=crossprod(EF,HF);
ExB:=[factor(simplify(ExBC[1])),factor(simplify(ExBC[2])),factor(simplify(ExBC[3]))
];
print(`Lambda `=lambda);print(`LambdaH`=lambdaH);print(`N ` = N);print(`H ` = H);
print(NAME);
print(`Action`= A1form);print(`E field`= EF);print(`B field`= BF);
print(`Topological Torsion`= TF);print(`Helicity AdotB`= HEL);print(`Poincare 2 E.
B`= P2);
print(`D field`= DF);print(`H field`= HF);print(`Poynting vector ExH`=ExB);
print(`********************  Lorenz constitutive equations, B = mu H, D = epsilon
E***************`);
print(`Lorenz 4Current`= [simplify(JTOT[1]),JTOT[2],JTOT[3],CD]);
print(`Lorenz charge density 3D`= CD);
print(`divergence Lorenz 4Current `= factor(diverge([JTOT[1],JTOT[2],JTOT[3],CD],
```

```
[x,y,z,t])));
print(`**************************Adjoint Current****************************`);
print(`N ` = N);print(`H ` = H);
print(`Adjoint4current `= simplify(CURR));
print(`Adjointchargedensity `= simplify(CURR[4]));
print(`divergenceAdjoint4current `= factor(diverge(CURR,[x,y,z,t])));
print(`*************************************************************`);
print(`Topological SPIN`=SF);
print(`chiralty AdotD`= SPD);
print(`LaGrange field energy density (B.H-D.E)`= LAGF);
print(`Interaction energy density (A.J-rho.phi)`= AJ);
print(`Poincare 1 (B.H-D.E)-(A.J-rho.phi)`= P1);
print(`Virtual work `=VW);
print(`JdotE power `=DISS);
  end:
```

****

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

```
> NAME:=`Example 8a-- Index 1 Irreversible solution EdotB < 0 (kinematic out) Type 1`;

> Ax:=Omega*y;Ay:=-Omega*x;Az:=Gamma*c*t;phi:=+Gamma*c*z;
```

Then call the procedure JCM(Ax,Ay,Az,phi)

```
> JCM(Ax,Ay,Az,phi,-1,2,0,0);
```

****

NAME := *Example 8a-- Index 1 Irreversible solution EdotB < 0 (kinematic out) Type 1*

$$Ax := \Omega\, y$$

$$Ay := -\Omega\, x$$

$$Az := \Gamma\, c\, t$$

$$\phi := \Gamma\, c\, z$$

$$Lambda\ = 1$$

$$LambdaH = 1$$

$$N\ = 0$$

$$H\ = 0$$

Example 8a-- Index 1 Irreversible solution EdotB < 0 (kinematic out) Type 1

$$Action = \Omega\, y\, d(x) - \Omega\, x\, d(y) + \Gamma\, c\, t\, d(z) - \Gamma\, c\, z\, d(t)$$

$$E\ field = [0, 0, -2\,\Gamma\, c]$$

$$B\ field = [0, 0, -2\,\Omega]$$

$$\textit{Topological Torsion} = [2\,\Gamma\,c\,\Omega\,x, 2\,\Gamma\,c\,\Omega\,y, 2\,\Omega\,\Gamma\,c\,z, 2\,\Gamma\,c\,t\,\Omega]$$

$$\textit{Helicity AdotB} = 2\,\Gamma\,c\,t\,\Omega$$

$$\textit{Poincare 2 E.B} = 8\,\Gamma\,c\,\Omega$$

$$\textit{D field} = [0, 0, -2\,\varepsilon\,\Gamma\,c]$$

$$\textit{H field} = \left[0, 0, -\frac{2\,\Omega}{\mu}\right]$$

$$\textit{Poynting vector ExH} = [0, 0, 0]$$

******************** *Lorenz constitutive equations, B = mu H, D = epsilon E***************

$$\textit{Lorenz 4Current} = [0, 0, 0, 0]$$

$$\textit{Lorenz charge density 3D} = 0$$

$$\textit{divergence Lorenz 4Current} = 0$$

***************************Adjoint Current****************************

$$N = 0$$

$$H = 0$$

$$\textit{Adjoint4current} = CURR$$

$$\textit{Adjointchargedensity} = -\Omega^2\,\Gamma^2\,c^2\,t$$

$$\textit{divergenceAdjoint4current} = -4\,\Omega^2\,\Gamma^2\,c^2$$

****

$$\textit{Topological SPIN} = \left[\frac{2\,\Omega^2\,x}{\mu}, \frac{2\,\Omega^2\,y}{\mu}, -2\,\varepsilon\,\Gamma^2\,c^2\,z, -2\,\Gamma^2\,c^2\,t\,\varepsilon\right]$$

$$\textit{chiralty AdotD} = -2\,\Gamma^2\,c^2\,t\,\varepsilon$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{4\,(-\Omega^2 + \varepsilon\,\Gamma^2\,c^2\,\mu)}{\mu}$$

$$\textit{Interaction energy density (A.J-rho.phi)} = 0$$

$$\textit{Poincare 1 (B.H-D.E)-(A.J-rho.phi)} = -\frac{4\,(-\Omega^2 + \varepsilon\,\Gamma^2\,c^2\,\mu)}{\mu}$$

$$\textit{Virtual work} = [0, 0, 0]$$

$$\textit{JdotE power} = 0 \qquad \textbf{(1)}$$

```
> SS:=simplify(JACOB);
> Xm:=factor(trace(SS));S2:=factor(trace(innerprod(SS,SS))): Yg:=factor((-(1/2)
  *((-trace(SS)*trace(SS)+S2))));Za:=factor((trace(adjoint(SS))));Tk:=det(SS);
> subs(Omega=1,Gamma=1,Yg);
```

$$SS := JACOB$$

$$Xm := 0$$

$$Yg := -(\Gamma c - \Omega)(\Gamma c + \Omega)$$

$$Za := 0$$

$$Tk := -\Omega^2 \Gamma^2 c^2$$

$$-(c-1)(c+1) \quad \textbf{(2)}$$

```
> d(A1form);
```

$$-2\,\Omega\,(d(x))\ \&\wedge\ (d(y)) - 2\,\Gamma\,c\,(d(z))\ \&\wedge\ (d(t)) \quad \textbf{(3)}$$

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

```
> NAME:=`Example 7b-- Index 1 Irreversible solution EdotB < 0 t (kinematic in)
  t goes to -t in coefficients, not dt`;

> Ax:=y;Ay:=-x;Az:=-c*t;phi:=+z*c;
```

Then call the procedure JCM(Ax,Ay,Az,phi)

```
> JCM(Ax,Ay,Az,phi,-1,2,4,H):
```

****

NAME :=

Example 7b-- Index 1 Irreversible solution EdotB < 0 *t (kinematic in) t goes to -t in coefficients, not dt*

$$Ax := y$$

$$Ay := -x$$

$$Az := -c\,t$$

$$\phi := z\,c$$

$$Lambda = (x^2 + y^2 + z^2 - c^2 t^2)^2$$

$$LambdaH = (x^2 + y^2 + z^2 - c^2 t^2)^{\frac{1}{2}H}$$

$$N = 4$$

$$H = H$$

Example 7b-- Index 1 Irreversible solution EdotB < 0 *t (kinematic in) t goes to -t in coefficients, not dt*

$$Action = \frac{y\,d(x)}{(x^2+y^2+z^2-c^2t^2)^2} - \frac{x\,d(y)}{(x^2+y^2+z^2-c^2t^2)^2} - \frac{c\,t\,d(z)}{(x^2+y^2+z^2-c^2t^2)^2}$$

$$-\frac{z\,c\,d(t)}{(x^2+y^2+z^2-c^2t^2)^2}$$

$$E\ field = \left[\frac{4\,c\,(-z\,x+y\,c\,t)}{(-x^2-y^2-z^2+c^2t^2)^3}, -\frac{4\,c\,(z\,y+x\,c\,t)}{(-x^2-y^2-z^2+c^2t^2)^3}, -\frac{4\,c\,(z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3}\right]$$

$$B\ field = \left[-\frac{4\,(-z\,x+y\,c\,t)}{(-x^2-y^2-z^2+c^2t^2)^3}, \frac{4\,(z\,y+x\,c\,t)}{(-x^2-y^2-z^2+c^2t^2)^3}, -\frac{2\,(x^2+y^2-z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3}\right]$$

$$Topological\ Torsion = \left[0, 0, \frac{2\,z\,c}{(-x^2-y^2-z^2+c^2t^2)^4}, -\frac{2\,c\,t}{(-x^2-y^2-z^2+c^2t^2)^4}\right]$$

$$Helicity\ AdotB = -\frac{2\,c\,t}{(-x^2-y^2-z^2+c^2t^2)^4}$$

$$Poincare\ 2\ E.B = \frac{16\,c\,(z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^5}$$

$$D\ field = \left[\frac{4\,\varepsilon\,c\,(-z\,x+y\,c\,t)}{(-x^2-y^2-z^2+c^2t^2)^3}, -\frac{4\,\varepsilon\,c\,(z\,y+x\,c\,t)}{(-x^2-y^2-z^2+c^2t^2)^3}, -\frac{4\,\varepsilon\,c\,(z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3}\right]$$

$$H\ field = \left[-\frac{4\,(-z\,x+y\,c\,t)}{(-x^2-y^2-z^2+c^2t^2)^3\,\mu}, \frac{4\,(z\,y+x\,c\,t)}{(-x^2-y^2-z^2+c^2t^2)^3\,\mu}, -\frac{2\,(x^2+y^2-z^2+c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^3\,\mu}\right]$$

$$Poynting\ vector\ ExH = \left[\frac{8\,c\,(z\,y+x\,c\,t)\,(x^2+y^2+z^2+3\,c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^6\,\mu}, \frac{8\,c\,(-z\,x+y\,c\,t)\,(x^2+y^2+z^2+3\,c^2t^2)}{(-x^2-y^2-z^2+c^2t^2)^6\,\mu}, 0\right]$$

******************* *Lorenz constitutive equations, B = mu H, D = epsilon E***************

$$Lorenz\ 4Current = \left[\frac{1}{(-x^2-y^2-z^2+c^2t^2)^4\,\mu}\left(4\left(-y\,x^2-y^3-y\,z^2-5\,y\,c^2t^2-6\,c\,t\,z\,x + \varepsilon\,c^2\mu\,y\,x^2+\varepsilon\,c^2\mu\,y^3+\varepsilon\,c^2\mu\,y\,z^2+5\,\varepsilon\,c^4\mu\,y\,t^2-6\,\varepsilon\,c^3\mu\,t\,z\,x\right)\right),\right.$$

$$-\frac{1}{(-x^2-y^2-z^2+c^2t^2)^4\,\mu}\left(4\left(-x^3-x\,y^2-x\,z^2-5\,c^2t^2x+6\,c\,t\,z\,y+\varepsilon\,c^2\mu\,x^3 + \varepsilon\,c^2\mu\,x\,y^2+\varepsilon\,c^2\mu\,x\,z^2+5\,\varepsilon\,c^4\mu\,t^2x+6\,\varepsilon\,c^3\mu\,t\,z\,y\right)\right),$$

$$-\frac{8\,c\,t\,(-2\,x^2-2\,y^2+z^2-c^2\,t^2+\varepsilon\,c^2\,\mu\,x^2+\varepsilon\,c^2\,\mu\,y^2+4\,\varepsilon\,c^2\,\mu\,z^2+2\,\varepsilon\,c^4\,t^2\,\mu)}{(-x^2-y^2-z^2+c^2\,t^2)^4\,\mu},$$

$$-\frac{8\,\varepsilon\,c\,z\,(x^2+y^2+z^2+5\,c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^4}\Bigg]$$

$$\textit{Lorenz charge density 3D} = -\frac{8\,\varepsilon\,c\,z\,(x^2+y^2+z^2+5\,c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^4}$$

$$\textit{divergence Lorenz 4Current} = 0$$

*************************Adjoint Current***************************

$$N = 4$$

$$H = H$$

$$\textit{Adjoint4current} = CURR$$

$$\textit{Adjointchargedensity} = c^2\,t\,(x^2+y^2+z^2-c^2\,t^2)^{-\frac{1}{2}H-6}$$

$$\textit{divergenceAdjoint4current} = -(8+H)\,(x^2+y^2+z^2-c^2\,t^2)^{-\frac{1}{2}H-6}\,c^2$$

****

$$\textit{Topological SPIN} = \Bigg[\frac{2\,(x^3+x\,y^2-x\,z^2+3\,c^2\,t^2\,x+2\,c\,t\,z\,y-2\,\varepsilon\,c^2\,\mu\,x\,z^2+2\,\varepsilon\,c^3\,\mu\,t\,z\,y)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu},$$

$$-\frac{2\,(2\,c\,t\,z\,x-3\,y\,c^2\,t^2-y\,x^2-y^3+y\,z^2+2\,\varepsilon\,c^2\,\mu\,y\,z^2+2\,\varepsilon\,c^3\,\mu\,t\,z\,x)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu},$$

$$-\frac{4\,z\,(-y^2-x^2+\varepsilon\,c^2\,\mu\,z^2+\varepsilon\,c^4\,t^2\,\mu)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu},\ \frac{4\,\varepsilon\,c^2\,t\,(y^2+x^2+z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5}\Bigg]$$

$$\textit{chiralty AdotD} = \frac{4\,\varepsilon\,c^2\,t\,(y^2+x^2+z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5}$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}\,(4\,(-2\,z^2\,x^2$$
$$-6\,y^2\,c^2\,t^2-2\,z^2\,y^2-6\,x^2\,c^2\,t^2-x^4-2\,y^2\,x^2-y^4-z^4+2\,z^2\,c^2\,t^2-c^4\,t^4+4\,\varepsilon\,c^2\,\mu\,x^2\,z^2$$
$$+4\,\varepsilon\,c^4\,\mu\,y^2\,t^2+4\,\varepsilon\,c^2\,\mu\,y^2\,z^2+4\,\varepsilon\,c^4\,\mu\,x^2\,t^2+4\,\varepsilon\,c^2\,\mu\,z^4+8\,\varepsilon\,c^4\,\mu\,z^2\,t^2+4\,\varepsilon\,c^6\,t^4\,\mu))$$

$$\textit{Interaction energy density (A.J-rho.phi)} = \frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}\,(4\,(-z^2\,x^2-z^2\,y^2-2\,c^4\,t^4$$
$$-9\,y^2\,c^2\,t^2-9\,x^2\,c^2\,t^2+2\,z^2\,c^2\,t^2+4\,\varepsilon\,c^6\,t^4\,\mu-2\,y^2\,x^2-y^4-x^4+3\,\varepsilon\,c^2\,\mu\,x^2\,z^2$$
$$+7\,\varepsilon\,c^4\,\mu\,x^2\,t^2+3\,\varepsilon\,c^2\,\mu\,y^2\,z^2+7\,\varepsilon\,c^4\,\mu\,y^2\,t^2+18\,\varepsilon\,c^4\,\mu\,z^2\,t^2+\varepsilon\,c^2\,\mu\,y^4+\varepsilon\,c^2\,\mu\,x^4$$

$+ 2\,\varepsilon\,c^2\,\mu\,y^2\,x^2 + 2\,\varepsilon\,c^2\,\mu\,z^4))$

$$\mathit{Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi)} = -\frac{1}{(-x^2-y^2-z^2+c^2t^2)^6\,\mu}\Big(4\,\big(-3\,z^2x^2 - 3\,z^2y^2 - 3\,c^4t^4 - 15\,y^2c^2t^2 - 15\,x^2c^2t^2 + 4\,z^2c^2t^2 - z^4 + 6\,\varepsilon\,c^2\,\mu\,z^4 + 8\,\varepsilon\,c^6\,t^4\,\mu - 4\,y^2x^2 - 2\,y^4 - 2\,x^4 + 7\,\varepsilon\,c^2\,\mu\,x^2z^2 + 11\,\varepsilon\,c^4\,\mu\,x^2t^2 + 7\,\varepsilon\,c^2\,\mu\,y^2z^2 + 11\,\varepsilon\,c^4\,\mu\,y^2t^2 + 26\,\varepsilon\,c^4\,\mu\,z^2t^2 + \varepsilon\,c^2\,\mu\,y^4 + \varepsilon\,c^2\,\mu\,x^4 + 2\,\varepsilon\,c^2\,\mu\,y^2x^2\big)\Big)$$

$$\mathit{Virtual\ work} = \Bigg[\frac{1}{(-x^2-y^2-z^2+c^2t^2)^7\,\mu}\Big(8\,\big(4\,\varepsilon\,c^2z^2\,\mu\,x^3 + 3\,\varepsilon\,c^2z^4\,\mu\,x - 2\,c\,t\,z\,y\,x^2 + 2\,\varepsilon\,c^2\,\mu\,x^3y^2 + 10\,\varepsilon\,c^4\,\mu\,x^3t^2 + \varepsilon\,c^2\,\mu\,x\,y^4 + 13\,\varepsilon\,c^6\,\mu\,t^4x + 4\,\varepsilon\,c^2z^2\,\mu\,y^2x + 6\,\varepsilon\,c^3\,z\,\mu\,y^3t + 6\,\varepsilon\,c^3z^3\,\mu\,y\,t + 32\,\varepsilon\,c^4z^2\,\mu\,t^2x - 6\,\varepsilon\,c^5\,z\,\mu\,t^3y + 10\,\varepsilon\,c^4\,\mu\,x\,y^2t^2 - 14\,x^3c^2t^2 - 9\,c^4t^4x - 2\,x^3y^2 - x\,y^4 + x\,z^4 - 14\,x\,y^2c^2t^2 + 8\,x\,z^2c^2t^2 - 2\,c\,t\,z\,y^3 - 2\,c\,t\,z^3y + 2\,c^3t^3z\,y + \varepsilon\,c^2\,\mu\,x^5 - x^5 + 6\,\varepsilon\,c^3\,z\,\mu\,x^2y\,t\big)\Big), \frac{1}{(-x^2-y^2-z^2+c^2t^2)^7\,\mu}\Big(8\,\big(4\,\varepsilon\,c^2z^2\,\mu\,y^3 + 3\,\varepsilon\,c^2z^4\,\mu\,y + 2\,c\,t\,y^2z\,x + 10\,c^4t^2\,\varepsilon\,\mu\,y^3 + 13\,c^6t^4\,\varepsilon\,\mu\,y + \varepsilon\,c^2\,\mu\,y\,x^4 + 2\,\varepsilon\,c^2\,\mu\,y^3x^2 + 4\,\varepsilon\,c^2z^2\,\mu\,x^2y - 6\,\varepsilon\,c^3\,z\,\mu\,x^3t - 6\,\varepsilon\,c^3z^3\,\mu\,x\,t + 32\,\varepsilon\,c^4z^2\,\mu\,t^2y + 6\,\varepsilon\,c^5\,z\,\mu\,t^3x + 10\,c^4t^2\,\varepsilon\,\mu\,x^2y - y\,x^4 - 2\,y^3x^2 + y\,z^4 + 2\,c\,t\,x^3z - 14\,c^2t^2x^2y + 2\,c\,t\,z^3x + 8\,c^2t^2z^2y - 2\,c^3t^3z\,x + \varepsilon\,c^2\,\mu\,y^5 - 14\,c^2t^2y^3 - 9\,c^4t^4y - y^5 - 6\,\varepsilon\,c^3\,z\,\mu\,y^2x\,t\big)\Big), \frac{1}{(-x^2-y^2-z^2+c^2t^2)^7\,\mu}\Big(16\,z\,\big(\varepsilon\,c^4\,\mu\,x^2t^2 + \varepsilon\,c^4\,\mu\,y^2t^2 + 2\,\varepsilon\,c^2\,\mu\,y^2x^2 + 2\,\varepsilon\,c^2\,\mu\,z^4 - 11\,y^2c^2t^2 - 11\,x^2c^2t^2 - 2\,y^2x^2 + 3\,\varepsilon\,c^2\,\mu\,x^2z^2 + 3\,\varepsilon\,c^2\,\mu\,y^2z^2 + 12\,\varepsilon\,c^4\,\mu\,z^2t^2 + 10\,\varepsilon\,c^6\,t^4\,\mu + \varepsilon\,c^2\,\mu\,y^4 + \varepsilon\,c^2\,\mu\,x^4 - y^4 - z^2y^2 - x^4 - z^2x^2\big)\Big)\Bigg]$$

$$\mathit{JdotE\ power} = \frac{1}{(-x^2-y^2-z^2+c^2t^2)^7\,\mu}\Big(16\,c^2\,t\,\big(-2\,y^2x^2 + z^2y^2 + z^2x^2 + 4\,\varepsilon\,c^6\,t^4\,\mu - y^4 - 9\,y^2c^2t^2 - x^4 - 9\,x^2c^2t^2 + 2\,z^4 + \varepsilon\,c^2\,\mu\,y^4 + 7\,\varepsilon\,c^4\,\mu\,y^2t^2 + \varepsilon\,c^2\,\mu\,x^4 + 7\,\varepsilon\,c^4\,\mu\,x^2t^2 \tag{4}$$

$$+8\,\varepsilon\,c^2\,\mu\,z^4+12\,\varepsilon\,c^4\,\mu\,z^2\,t^2+2\,\varepsilon\,c^2\,\mu\,y^2\,x^2+9\,\varepsilon\,c^2\,\mu\,y^2\,z^2+9\,\varepsilon\,c^2\,\mu\,x^2\,z^2-2\,c^4\,t^4))$$

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

```
> NAME:=`Example 8b-- Index 1 Irreversible solution EdotB >0 (kinematic out)
  Type 2`;
```

Then call the procedure JCM(Ax,Ay,Az,phi)

```
> JCM(Ax,Ay,Az,phi,-1,2,4,4):
```

****

$$NAME := Example\ 8b\text{-- } Index\ 1\ Irreversible\ solution\ EdotB\ >0\ (kinematic\ out)\ Type\ 2$$

$$Lambda\ = (x^2+y^2+z^2-c^2\,t^2)^2$$

$$LambdaH = (x^2+y^2+z^2-c^2\,t^2)^2$$

$$N\ = 4$$

$$H\ = 4$$

$$Example\ 8b\text{-- } Index\ 1\ Irreversible\ solution\ EdotB\ >0\ (kinematic\ out)\ Type\ 2$$

$$Action = \frac{y\,d(x)}{(x^2+y^2+z^2-c^2\,t^2)^2}-\frac{x\,d(y)}{(x^2+y^2+z^2-c^2\,t^2)^2}-\frac{c\,t\,d(z)}{(x^2+y^2+z^2-c^2\,t^2)^2}-\frac{z\,c\,d(t)}{(x^2+y^2+z^2-c^2\,t^2)^2}$$

$$E\ field = \left[\frac{4\,c\,(-z\,x+y\,c\,t)}{(-x^2-y^2-z^2+c^2\,t^2)^3},\ -\frac{4\,c\,(z\,y+x\,c\,t)}{(-x^2-y^2-z^2+c^2\,t^2)^3},\ -\frac{4\,c\,(z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^3}\right]$$

$$B\ field = \left[-\frac{4\,(-z\,x+y\,c\,t)}{(-x^2-y^2-z^2+c^2\,t^2)^3},\ \frac{4\,(z\,y+x\,c\,t)}{(-x^2-y^2-z^2+c^2\,t^2)^3},\ -\frac{2\,(x^2+y^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^3}\right]$$

$$Topological\ Torsion = \left[0,\ 0,\ \frac{2\,z\,c}{(-x^2-y^2-z^2+c^2\,t^2)^4},\ -\frac{2\,c\,t}{(-x^2-y^2-z^2+c^2\,t^2)^4}\right]$$

$$Helicity\ AdotB = -\frac{2\,c\,t}{(-x^2-y^2-z^2+c^2\,t^2)^4}$$

$$Poincare\ 2\ E.B = \frac{16\,c\,(z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5}$$

$$D\ field = \left[\frac{4\,\varepsilon\,c\,(-z\,x+y\,c\,t)}{(-x^2-y^2-z^2+c^2\,t^2)^3},\ -\frac{4\,\varepsilon\,c\,(z\,y+x\,c\,t)}{(-x^2-y^2-z^2+c^2\,t^2)^3},\ -\frac{4\,\varepsilon\,c\,(z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^3}\right]$$

$$H\ field = \left[-\frac{4\,(-z\,x+y\,c\,t)}{(-x^2-y^2-z^2+c^2\,t^2)^3\,\mu},\ \frac{4\,(z\,y+x\,c\,t)}{(-x^2-y^2-z^2+c^2\,t^2)^3\,\mu},\ -\frac{2\,(x^2+y^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^3\,\mu}\right]$$

$$Poynting\ vector\ ExH = \left[\frac{8\,c\,(z\,y + x\,c\,t)\,(x^2+y^2+z^2+3\,c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}, \frac{8\,c\,(-z\,x + y\,c\,t)\,(x^2+y^2+z^2+3\,c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}, 0 \right]$$

******************* Lorenz constitutive equations, B = mu H, D = epsilon E***************

$$Lorenz\ 4Current = \left[\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^4\,\mu}\big(4\,(-y\,x^2-y^3-y\,z^2-5\,y\,c^2\,t^2-6\,c\,t\,z\,x + \varepsilon\,c^2\,\mu\,y\,x^2+\varepsilon\,c^2\,\mu\,y^3+\varepsilon\,c^2\,\mu\,y\,z^2+5\,\varepsilon\,c^4\,\mu\,y\,t^2-6\,\varepsilon\,c^3\,\mu\,t\,z\,x)\big), -\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^4\,\mu}\big(4\,(-x^3-x\,y^2-x\,z^2-5\,c^2\,t^2\,x+6\,c\,t\,z\,y+\varepsilon\,c^2\,\mu\,x^3 + \varepsilon\,c^2\,\mu\,x\,y^2+\varepsilon\,c^2\,\mu\,x\,z^2+5\,\varepsilon\,c^4\,\mu\,t^2\,x+6\,\varepsilon\,c^3\,\mu\,t\,z\,y)\big), -\frac{8\,c\,t\,(-2\,x^2-2\,y^2+z^2-c^2\,t^2+\varepsilon\,c^2\,\mu\,x^2+\varepsilon\,c^2\,\mu\,y^2+4\,\varepsilon\,c^2\,\mu\,z^2+2\,\varepsilon\,c^4\,t^2\,\mu)}{(-x^2-y^2-z^2+c^2\,t^2)^4\,\mu}, -\frac{8\,\varepsilon\,c\,z\,(x^2+y^2+z^2+5\,c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^4} \right]$$

$$Lorenz\ charge\ density\ 3D = -\frac{8\,\varepsilon\,c\,z\,(x^2+y^2+z^2+5\,c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^4}$$

$$divergence\ Lorenz\ 4Current\ = 0$$

**************************Adjoint Current**************************

$$N\ = 4$$

$$H\ = 4$$

$$Adjoint4current\ = CURR$$

$$Adjointchargedensity\ = \frac{c^2\,t}{(-x^2-y^2-z^2+c^2\,t^2)^8}$$

$$divergenceAdjoint4current\ = -\frac{12\,c^2}{(-x^2-y^2-z^2+c^2\,t^2)^8}$$

****

$$Topological\ SPIN = \left[\frac{2\,(x^3+x\,y^2-x\,z^2+3\,c^2\,t^2\,x+2\,c\,t\,z\,y-2\,\varepsilon\,c^2\,\mu\,x\,z^2+2\,\varepsilon\,c^3\,\mu\,t\,z\,y)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu}, \right.$$

$$-\frac{2\left(2ctzx-3yc^2t^2-yx^2-y^3+yz^2+2\varepsilon c^2\mu yz^2+2\varepsilon c^3\mu tzx\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5\mu},$$

$$\left.-\frac{4z\left(-y^2-x^2+\varepsilon c^2\mu z^2+\varepsilon c^4t^2\mu\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5\mu},\ \frac{4\varepsilon c^2t\left(y^2+x^2+z^2+c^2t^2\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}\right]$$

$$\text{chiralty AdotD} = \frac{4\varepsilon c^2t\left(y^2+x^2+z^2+c^2t^2\right)}{\left(-x^2-y^2-z^2+c^2t^2\right)^5}$$

$$\text{LaGrange field energy density (B.H-D.E)} = -\frac{1}{\left(-x^2-y^2-z^2+c^2t^2\right)^6\mu}\left(4\left(-2z^2x^2\right.\right.$$
$$-6y^2c^2t^2-2z^2y^2-6x^2c^2t^2-x^4-2y^2x^2-y^4-z^4+2z^2c^2t^2-c^4t^4+4\varepsilon c^2\mu x^2z^2$$
$$\left.\left.+4\varepsilon c^4\mu y^2t^2+4\varepsilon c^2\mu y^2z^2+4\varepsilon c^4\mu x^2t^2+4\varepsilon c^2\mu z^4+8\varepsilon c^4\mu z^2t^2+4\varepsilon c^6t^4\mu\right)\right)$$

$$\text{Interaction energy density (A.J-rho.phi)} = \frac{1}{\left(-x^2-y^2-z^2+c^2t^2\right)^6\mu}\left(4\left(-z^2x^2-z^2y^2-2c^4t^4\right.\right.$$
$$-9y^2c^2t^2-9x^2c^2t^2+2z^2c^2t^2+4\varepsilon c^6t^4\mu-2y^2x^2-y^4-x^4+3\varepsilon c^2\mu x^2z^2$$
$$+7\varepsilon c^4\mu x^2t^2+3\varepsilon c^2\mu y^2z^2+7\varepsilon c^4\mu y^2t^2+18\varepsilon c^4\mu z^2t^2+\varepsilon c^2\mu y^4+\varepsilon c^2\mu x^4$$
$$\left.\left.+2\varepsilon c^2\mu y^2x^2+2\varepsilon c^2\mu z^4\right)\right)$$

$$\text{Poincare 1 (B.H-D.E)-(A.J-rho.phi)} = -\frac{1}{\left(-x^2-y^2-z^2+c^2t^2\right)^6\mu}\left(4\left(-3z^2x^2-3z^2y^2\right.\right.$$
$$-3c^4t^4-15y^2c^2t^2-15x^2c^2t^2+4z^2c^2t^2-z^4+6\varepsilon c^2\mu z^4+8\varepsilon c^6t^4\mu-4y^2x^2-2y^4$$
$$-2x^4+7\varepsilon c^2\mu x^2z^2+11\varepsilon c^4\mu x^2t^2+7\varepsilon c^2\mu y^2z^2+11\varepsilon c^4\mu y^2t^2+26\varepsilon c^4\mu z^2t^2$$
$$\left.\left.+\varepsilon c^2\mu y^4+\varepsilon c^2\mu x^4+2\varepsilon c^2\mu y^2x^2\right)\right)$$

$$\text{Virtual work} = \left[\frac{1}{\left(-x^2-y^2-z^2+c^2t^2\right)^7\mu}\left(8\left(4\varepsilon c^2z^2\mu x^3+3\varepsilon c^2z^4\mu x-2ctzyx^2\right.\right.\right.$$

$$+2\varepsilon c^2\mu x^3y^2+10\varepsilon c^4\mu x^3t^2+\varepsilon c^2\mu xy^4+13\varepsilon c^6\mu t^4x+4\varepsilon c^2z^2\mu y^2x+6\varepsilon c^3z\mu y^3t$$

$$+6\varepsilon c^3z^3\mu yt+32\varepsilon c^4z^2\mu t^2x-6\varepsilon c^5z\mu t^3y+10\varepsilon c^4\mu xy^2t^2-14x^3c^2t^2-9c^4t^4x$$

$$-2x^3y^2-xy^4+xz^4-14xy^2c^2t^2+8xz^2c^2t^2-2ctzy^3-2ctz^3y+2c^3t^3zy$$

$$\left.\left.+\varepsilon c^2\mu x^5-x^5+6\varepsilon c^3z\mu x^2yt\right)\right),\ \frac{1}{\left(-x^2-y^2-z^2+c^2t^2\right)^7\mu}\left(8\left(4\varepsilon c^2z^2\mu y^3\right.\right.$$

$$+3\,\varepsilon\,c^2\,z^4\,\mu\,y+2\,c\,t\,y^2\,z\,x+10\,c^4\,t^2\,\varepsilon\,\mu\,y^3+13\,c^6\,t^4\,\varepsilon\,\mu\,y+\varepsilon\,c^2\,\mu\,y\,x^4+2\,\varepsilon\,c^2\,\mu\,y^3\,x^2$$

$$+4\,\varepsilon\,c^2\,z^2\,\mu\,x^2\,y-6\,\varepsilon\,c^3\,z\,\mu\,x^3\,t-6\,\varepsilon\,c^3\,z^3\,\mu\,x\,t+32\,\varepsilon\,c^4\,z^2\,\mu\,t^2\,y+6\,\varepsilon\,c^5\,z\,\mu\,t^3\,x$$

$$+10\,c^4\,t^2\,\varepsilon\,\mu\,x^2\,y-y\,x^4-2\,y^3\,x^2+y\,z^4+2\,c\,t\,x^3\,z-14\,c^2\,t^2\,x^2\,y+2\,c\,t\,z^3\,x+8\,c^2\,t^2\,z^2\,y$$

$$-2\,c^3\,t^3\,z\,x+\varepsilon\,c^2\,\mu\,y^5-14\,c^2\,t^2\,y^3-9\,c^4\,t^4\,y-y^5-6\,\varepsilon\,c^3\,z\,\mu\,y^2\,x\,t)),$$

$$\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^7\,\mu}\,(16\,z\,(\varepsilon\,c^4\,\mu\,x^2\,t^2+\varepsilon\,c^4\,\mu\,y^2\,t^2+2\,\varepsilon\,c^2\,\mu\,y^2\,x^2+2\,\varepsilon\,c^2\,\mu\,z^4$$
$$-11\,y^2\,c^2\,t^2-11\,x^2\,c^2\,t^2-2\,y^2\,x^2+3\,\varepsilon\,c^2\,\mu\,x^2\,z^2+3\,\varepsilon\,c^2\,\mu\,y^2\,z^2+12\,\varepsilon\,c^4\,\mu\,z^2\,t^2$$
$$+10\,\varepsilon\,c^6\,t^4\,\mu+\varepsilon\,c^2\,\mu\,y^4+\varepsilon\,c^2\,\mu\,x^4-y^4-z^2\,y^2-x^4-z^2\,x^2))]$$

$$JdotE\ power\ =\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^7\,\mu}\,(16\,c^2\,t\,(-2\,y^2\,x^2+z^2\,y^2+z^2\,x^2+4\,\varepsilon\,c^6\,t^4\,\mu-y^4$$
$$-9\,y^2\,c^2\,t^2-x^4-9\,x^2\,c^2\,t^2+2\,z^4+\varepsilon\,c^2\,\mu\,y^4+7\,\varepsilon\,c^4\,\mu\,y^2\,t^2+\varepsilon\,c^2\,\mu\,x^4+7\,\varepsilon\,c^4\,\mu\,x^2\,t^2$$
$$+8\,\varepsilon\,c^2\,\mu\,z^4+12\,\varepsilon\,c^4\,\mu\,z^2\,t^2+2\,\varepsilon\,c^2\,\mu\,y^2\,x^2+9\,\varepsilon\,c^2\,\mu\,y^2\,z^2+9\,\varepsilon\,c^2\,\mu\,x^2\,z^2-2\,c^4\,t^4))\qquad(5)$$

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

```
> NAME:=`Example 9-- Index 1 Irreversible solution EdotB =0 Type 1 + Type 2`;
```

```
> Ax:=(c*t+y);Ay:=(-z-x);Az:=(c*t+y);phi:=(+x+z)*c;
```

Then call the procedure JCM(Ax,Ay,Az,phi)

```
> JCM(Ax,Ay,Az,phi,1,2,4,4):
```

****

NAME := Example 9-- Index 1 Irreversible solution EdotB =0 Type 1 + Type 2

$$Ax := c\,t+y$$

$$Ay := -z-x$$

$$Az := c\,t+y$$

$$\phi := (x+z)\,c$$

$$Lambda\ =(y^2+x^2+z^2+c^2\,t^2)^2$$

$$LambdaH=(y^2+x^2+z^2+c^2\,t^2)^2$$

$$N\ =4$$

$$H\ =4$$

Example 9-- Index 1 Irreversible solution EdotB =0 Type 1 + Type 2

$$Action = \frac{(c\,t+y)\,d(x)}{(y^2+x^2+z^2+c^2\,t^2)^2} + \frac{(-z-x)\,d(y)}{(y^2+x^2+z^2+c^2\,t^2)^2} + \frac{(c\,t+y)\,d(z)}{(y^2+x^2+z^2+c^2\,t^2)^2} - \frac{(x+z)\,c\,d(t)}{(y^2+x^2+z^2+c^2\,t^2)^2}$$

$$E\ field = \left[\frac{2\,c\,(-y^2+x^2-z^2+c^2\,t^2+2\,z\,x+2\,y\,c\,t)}{(y^2+x^2+z^2+c^2\,t^2)^3}, -\frac{4\,(x+z)\,c\,(-y+c\,t)}{(y^2+x^2+z^2+c^2\,t^2)^3}, \frac{2\,c\,(-y^2-x^2+z^2+c^2\,t^2+2\,z\,x+2\,y\,c\,t)}{(y^2+x^2+z^2+c^2\,t^2)^3}\right]$$

$$B\ field = \left[\frac{2\,(-y^2+x^2-z^2+c^2\,t^2-2\,y\,c\,t-2\,z\,x)}{(y^2+x^2+z^2+c^2\,t^2)^3}, \frac{4\,(c\,t+y)\,(-z+x)}{(y^2+x^2+z^2+c^2\,t^2)^3}, -\frac{2\,(-y^2-x^2+z^2+c^2\,t^2-2\,z\,x-2\,y\,c\,t)}{(y^2+x^2+z^2+c^2\,t^2)^3}\right]$$

$$Topological\ Torsion = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = \left[\frac{2\,\varepsilon\,c\,(-y^2+x^2-z^2+c^2\,t^2+2\,z\,x+2\,y\,c\,t)}{(y^2+x^2+z^2+c^2\,t^2)^3}, -\frac{4\,\varepsilon\,(x+z)\,c\,(-y+c\,t)}{(y^2+x^2+z^2+c^2\,t^2)^3}, \frac{2\,\varepsilon\,c\,(-y^2-x^2+z^2+c^2\,t^2+2\,z\,x+2\,y\,c\,t)}{(y^2+x^2+z^2+c^2\,t^2)^3}\right]$$

$$H\ field = \left[\frac{2\,(-y^2+x^2-z^2+c^2\,t^2-2\,y\,c\,t-2\,z\,x)}{(y^2+x^2+z^2+c^2\,t^2)^3\,\mu}, \frac{4\,(c\,t+y)\,(-z+x)}{(y^2+x^2+z^2+c^2\,t^2)^3\,\mu}, -\frac{2\,(-y^2-x^2+z^2+c^2\,t^2-2\,z\,x-2\,y\,c\,t)}{(y^2+x^2+z^2+c^2\,t^2)^3\,\mu}\right]$$

$$Poynting\ vector\ ExH = \left[\frac{16\,c\,(z\,y^2\,c\,t+y\,x\,z^2-3\,x^2\,c\,t\,z-3\,y\,x\,c^2\,t^2+z^3\,c\,t+z\,c^3\,t^3+y\,x^3+y^3\,x)}{(y^2+x^2+z^2+c^2\,t^2)^6\,\mu}, \frac{8\,c\,(-8\,z\,x\,y\,c\,t-2\,z^2\,x^2+c^4\,t^4-6\,y^2\,c^2\,t^2-z^4+y^4-x^4)}{(y^2+x^2+z^2+c^2\,t^2)^6\,\mu}, \frac{16\,c\,(x^2\,z\,y+x^3\,c\,t+c^3\,t^3\,x+y^3\,z+z^3\,y+y^2\,x\,c\,t-3\,z^2\,x\,c\,t-3\,c^2\,t^2\,z\,y)}{(y^2+x^2+z^2+c^2\,t^2)^6\,\mu}\right]$$

******************* *Lorenz constitutive equations, B = mu H, D = epsilon E***************

Lorenz 4Current

$$= \left[\frac{1}{(y^2+x^2+z^2+c^2t^2)^4\mu} \left(4\left(-4y^2ct+2x^2ct-4ctz^2+2c^3t^3-y^3-yx^2 -yz^2+5yc^2t^2+6ctzx\right)\left(\varepsilon c^2\mu+1\right)\right), -\frac{4(x+z)\left(-y^2-x^2-z^2+5c^2t^2-6yct\right)\left(\varepsilon c^2\mu+1\right)}{(y^2+x^2+z^2+c^2t^2)^4\mu}, \frac{1}{(y^2+x^2+z^2+c^2t^2)^4\mu}\left(4\left(-4y^2ct-4x^2ct+2ctz^2+2c^3t^3-y^3-yx^2-yz^2 +5yc^2t^2+6ctzx\right)\left(\varepsilon c^2\mu+1\right)\right), 0 \right]$$

Lorenz charge density 3D = 0

divergence Lorenz 4Current = 0

**************************Adjoint Current*****************************

$N = 4$

$H = 4$

Adjoint4current = *CURR*

Adjointchargedensity = 0

divergenceAdjoint4current = 0

****

$$\textit{Topological SPIN} = \left[\frac{1}{(y^2+x^2+z^2+c^2t^2)^5\mu}\left(2\left(2\varepsilon c^3\mu tzy+2ctzy-6xyct -\varepsilon c^2\mu xy^2+\varepsilon c^2\mu xz^2+\varepsilon c^4\mu t^2x-xz^2-3zx^2+zy^2-c^2t^2x+3zc^2t^2+z^3 +\varepsilon c^2\mu x^3-3xy^2-x^3-\varepsilon c^2z^3\mu+\varepsilon c^4z\mu t^2-\varepsilon c^2\mu zy^2+3\varepsilon c^2\mu zx^2+2\varepsilon c^3\mu xyt\right)\right), -\frac{1}{(y^2+x^2+z^2+c^2t^2)^5\mu}\left(4\left(2\varepsilon c^3\mu tzx+2ctzx+2yzx-\varepsilon c^2\mu yx^2 -\varepsilon c^2\mu yz^2-c^3t^3+3y^2ct+yc^2t^2+y^3+\varepsilon c^3t\mu x^2+\varepsilon c^3t\mu z^2-2\varepsilon c^2\mu yzx\right)\right), \frac{1}{(y^2+x^2+z^2+c^2t^2)^5\mu}\left(2\left(2\varepsilon c^3\mu tzy-6ctzy+2xyct-\varepsilon c^2\mu xy^2+3\varepsilon c^2\mu xz^2\right.\right.$$

$$+\varepsilon c^4 \mu t^2 x - 3 x z^2 - z x^2 - 3 z y^2 + 3 c^2 t^2 x - z c^2 t^2 - z^3 - \varepsilon c^2 \mu x^3 + x y^2 + x^3 + \varepsilon c^2 z^3 \mu$$

$$+\varepsilon c^4 z \mu t^2 - \varepsilon c^2 \mu z y^2 + \varepsilon c^2 \mu z x^2 + 2 \varepsilon c^3 \mu x y t)),$$

$$\left.\frac{4 \varepsilon c \left(4 c t z x - y z^2 + c^3 t^3 + y^2 c t + x^2 c t + c t z^2 + 3 y c^2 t^2 - y x^2 - y^3\right)}{\left(y^2 + x^2 + z^2 + c^2 t^2\right)^5}\right]$$

$$\textit{chiralty AdotD} = \frac{4 \varepsilon c \left(4 c t z x - y z^2 + c^3 t^3 + y^2 c t + x^2 c t + c t z^2 + 3 y c^2 t^2 - y x^2 - y^3\right)}{\left(y^2 + x^2 + z^2 + c^2 t^2\right)^5}$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{1}{\left(y^2 + x^2 + z^2 + c^2 t^2\right)^6 \mu} \left(8 \left(-2 z^2 x^2 - 2 z^2 y^2\right.\right.$$

$$- c^4 t^4 - 2 y^2 c^2 t^2 - 2 x^2 c^2 t^2 - 2 z^2 c^2 t^2 - z^4 + \varepsilon c^2 \mu z^4 + \varepsilon c^6 t^4 \mu - 2 y^2 x^2 - y^4 - x^4$$

$$+ 2 \varepsilon c^2 \mu x^2 z^2 + 2 \varepsilon c^4 \mu x^2 t^2 + 2 \varepsilon c^2 \mu y^2 z^2 + 2 \varepsilon c^4 \mu y^2 t^2 + 2 \varepsilon c^4 \mu z^2 t^2 + \varepsilon c^2 \mu y^4$$

$$+ \varepsilon c^2 \mu x^4 - 4 \varepsilon c^3 \mu y^3 t + 4 \varepsilon c^5 t^3 \mu y + 2 \varepsilon c^2 \mu y^2 x^2 - 4 \varepsilon c^3 \mu x^2 y t - 4 \varepsilon c^3 \mu z^2 y t$$

$$\left.\left. + 8 \varepsilon c^4 t^2 \mu z x - 4 y^3 c t + 4 c^3 t^3 y - 4 x^2 y c t - 4 z^2 y c t + 8 c^2 t^2 z x\right)\right)$$

$$\textit{Interaction energy density (A.J-rho.phi)} = \frac{1}{\left(y^2 + x^2 + z^2 + c^2 t^2\right)^6 \mu} \left(4 \left(\varepsilon c^2 \mu + 1\right) \left(-2 z^2 x^2\right.\right.$$

$$- 3 z^2 y^2 + 4 c^4 t^4 + 2 y^2 c^2 t^2 + 3 x^2 c^2 t^2 + 3 z^2 c^2 t^2 - z^4 - 3 y^2 x^2 - 2 y^4 - x^4 - 2 y^2 z x$$

$$\left.\left. - 10 y^3 c t + 14 c^3 t^3 y - 2 x^3 z - 2 z^3 x - 10 x^2 y c t - 10 z^2 y c t + 22 c^2 t^2 z x\right)\right)$$

$$\textit{Poincare 1 (B.H-D.E)-(A.J-rho.phi)} = -\frac{1}{\left(y^2 + x^2 + z^2 + c^2 t^2\right)^6 \mu} \left(4 \left(-6 z^2 x^2 - 7 z^2 y^2 + 2 c^4 t^4\right.\right.$$

$$- 2 y^2 c^2 t^2 - x^2 c^2 t^2 - z^2 c^2 t^2 - 3 z^4 + \varepsilon c^2 \mu z^4 + 6 \varepsilon c^6 t^4 \mu - 7 y^2 x^2 - 4 y^4 - 3 x^4$$

$$+ 2 \varepsilon c^2 \mu x^2 z^2 + 7 \varepsilon c^4 \mu x^2 t^2 + \varepsilon c^2 \mu y^2 z^2 + 6 \varepsilon c^4 \mu y^2 t^2 + 7 \varepsilon c^4 \mu z^2 t^2 + \varepsilon c^2 \mu x^4$$

$$- 2 \varepsilon c^2 \mu x^3 z - 18 \varepsilon c^3 \mu y^3 t - 2 \varepsilon c^2 \mu z^3 x + 22 \varepsilon c^5 t^3 \mu y + \varepsilon c^2 \mu y^2 x^2 - 18 \varepsilon c^3 \mu x^2 y t$$

$$- 2 \varepsilon c^2 \mu y^2 z x - 18 \varepsilon c^3 \mu z^2 y t + 38 \varepsilon c^4 t^2 \mu z x - 2 y^2 z x - 18 y^3 c t + 22 c^3 t^3 y - 2 x^3 z$$

$$\left.\left. - 2 z^3 x - 18 x^2 y c t - 18 z^2 y c t + 38 c^2 t^2 z x\right)\right)$$

$$\textit{Virtual work} = \left[\frac{1}{\left(y^2 + x^2 + z^2 + c^2 t^2\right)^7 \mu} \left(8 \left(\varepsilon c^2 \mu + 1\right) \left(-2 c t z y x^2 + 2 x^3 c^2 t^2 + 4 x y^2 z^2\right.\right.\right.$$

$$+ c^4 t^4 x + 4 x^3 y^2 + 2 x^3 z^2 + 3 x y^4 + x z^4 + 4 x y^2 c^2 t^2 + 2 x z^2 c^2 t^2 - 2 c t z y^3 - 2 c t z^3 y$$

$$- 2 c^3 t^3 z y + 18 y x^3 c t + 18 y^3 x c t + 8 y^2 c^2 t^2 z - 30 y c^3 t^3 x - 36 c^2 t^2 z x^2 + x^5$$

$$+2y^2x^2z+18yz^2xct-y^4z-2y^2z^3+3x^4z+2x^2z^3+9zc^4t^4+8z^3c^2t^2-z^5))$$

$$,\frac{1}{(y^2+x^2+z^2+c^2t^2)^7\mu}(16(\varepsilon c^2\mu+1)(3y^2x^2ct+3y^2z^2ct-6ctz^2x^2+2yx^3z$$

$$+2y^3zx+6y^4ct+2yz^3x-16y^2c^3t^3-3x^4ct-c^3t^3x^2-3ctz^4-22yc^2t^2zx$$

$$+2cty^2zx+2c^5t^5+y^3x^2+y^3z^2+2ctx^3z+c^2t^2x^2y+2ctz^3x+c^2t^2z^2y$$

$$+2c^3t^3zx+2c^2t^2y^3+c^4t^4y+y^5-c^3t^3z^2)),\frac{1}{(y^2+x^2+z^2+c^2t^2)^7\mu}(8(\varepsilon c^2\mu$$

$$+1)(18ctzyx^2+8x^3c^2t^2+2xy^2z^2+9c^4t^4x-2x^3y^2+2x^3z^2-xy^4+3xz^4$$

$$+8xy^2c^2t^2-36xz^2c^2t^2+18ctzy^3+18ctz^3y-30c^3t^3zy-2yx^3ct-2y^3xct$$

$$+4y^2c^2t^2z-2yc^3t^3x+2c^2t^2zx^2-x^5+4y^2x^2z-2yz^2xct+3y^4z+4y^2z^3+x^4z$$

$$+2x^2z^3+zc^4t^4+2z^3c^2t^2+z^5))]$$

$$JdotE\ power=\frac{1}{(y^2+x^2+z^2+c^2t^2)^7\mu}(16(\varepsilon c^2\mu+1)c(4y^2x^2ct+4y^2z^2ct+4ctz^2x^2 \tag{6}$$

$$+2y^4ct+4y^2c^3t^3+2x^4ct+4c^3t^3x^2+2ctz^4-4cty^2zx+2c^5t^5+yx^4+2y^3x^2$$

$$+2y^3z^2+yz^4-4ctx^3z-14c^2t^2x^2y-4ctz^3x-14c^2t^2z^2y+20c^3t^3zx-14c^2t^2y^3$$

$$+9c^4t^4y+2yx^2z^2+y^5+4c^3t^3z^2))$$

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

```
> NAME:=`Example 10a -- Plasma Accretion disc -- Hedge Hog solution.`;
> Gamma:=-z*I/(x^2+y^2)^1*m/(a*x^2+a*y^2+c*z^2)^(1/2);
> Ax:=Gamma*(-y);Ay:=Gamma*x;
> Az:=0; phi:=0;
```

Then call the procedure JCM(Ax,Ay,Az,phi)

```
> JCM(Ax,Ay,Az,phi,1,2,0,0):
```

****

NAME := Example 10a -- Plasma Accretion disc -- Hedge Hog solution.

$$\Gamma:=-\frac{I\,z\,m}{(x^2+y^2)\sqrt{a\,x^2+a\,y^2+z^2c}}$$

$$Ax:=\frac{I\,z\,m\,y}{(x^2+y^2)\sqrt{a\,x^2+a\,y^2+z^2c}}$$

$$Ay:=-\frac{I\,z\,m\,x}{(x^2+y^2)\sqrt{a\,x^2+a\,y^2+z^2c}}$$

$$Az := 0$$

$$\phi := 0$$

$$Lambda \ = 1$$

$$LambdaH = 1$$

$$N \ = 0$$

$$H \ = 0$$

Example 10a -- Plasma Accretion disc -- Hedge Hog solution.

$$Action = \frac{I\,z\,m\,y\,d(x)}{(x^2+y^2)\sqrt{a\,x^2+a\,y^2+z^2\,c}} - \frac{I\,z\,m\,x\,d(y)}{(x^2+y^2)\sqrt{a\,x^2+a\,y^2+z^2\,c}}$$

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[\frac{I\,a\,m\,x}{(a\,x^2+a\,y^2+z^2\,c)^{3/2}}, \frac{I\,a\,m\,y}{(a\,x^2+a\,y^2+z^2\,c)^{3/2}}, \frac{I\,a\,z\,m}{(a\,x^2+a\,y^2+z^2\,c)^{3/2}}\right]$$

$$Topological\ Torsion = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = [0, 0, 0]$$

$$H\ field = \left[\frac{I\,a\,m\,x}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^{3/2}}, \frac{I\,a\,m\,y}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^{3/2}}, \frac{I\,a\,z\,m}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^{3/2}}\right]$$

$$Poynting\ vector\ ExH = [0, 0, 0]$$

******************** *Lorenz constitutive equations, B = mu H, D = epsilon E*************
***

$$Lorenz\ 4Current = \left[-\frac{3\,I\,a\,z\,m\,y\,(a-c)}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^{5/2}}, \frac{3\,I\,a\,m\,x\,z\,(a-c)}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^{5/2}}, 0, 0\right]$$

$$Lorenz\ charge\ density\ 3D = 0$$

$$divergence\ Lorenz\ 4Current \ = 0$$

***************************Adjoint Current***************************

$$N \ = 0$$

$$H \ = 0$$

$$Adjoint4current \ = CURR$$

$$Adjointchargedensity \ = 0$$

$$divergenceAdjoint4current \ = 0$$

****

$$Topological\ SPIN = \left[\frac{z^2\,m^2\,x\,a}{(x^2+y^2)\,(a\,x^2+a\,y^2+z^2\,c)^2\,\mu}, \frac{z^2\,m^2\,y\,a}{(x^2+y^2)\,(a\,x^2+a\,y^2+z^2\,c)^2\,\mu},\right.$$

$$-\frac{z\,m^2\,a}{\left(a\,x^2+a\,y^2+z^2\,c\right)^2\,\mu},\,0\Bigg]$$

$$\mathit{chiralty\ AdotD}=0$$

$$\mathit{LaGrange\ field\ energy\ density\ (B.H\text{-}D.E)}=-\frac{m^2\,a^2\,\left(x^2+y^2+z^2\right)}{\mu\,\left(a\,x^2+a\,y^2+z^2\,c\right)^3}$$

$$\mathit{Interaction\ energy\ density\ (A.J\text{-}rho.phi)}=\frac{3\,z^2\,m^2\,a\,\left(a-c\right)}{\mu\,\left(a\,x^2+a\,y^2+z^2\,c\right)^3}$$

$$\mathit{Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi)}=-\frac{m^2\,a\,\left(a\,x^2+a\,y^2+4\,a\,z^2-3\,z^2\,c\right)}{\mu\,\left(a\,x^2+a\,y^2+z^2\,c\right)^3}$$

$$\mathit{Virtual\ work}\ =\Bigg[-\frac{3\,a^2\,m^2\,x\,z^2\,\left(a-c\right)}{\mu\,\left(a\,x^2+a\,y^2+z^2\,c\right)^4},\,-\frac{3\,a^2\,z^2\,m^2\,y\,\left(a-c\right)}{\mu\,\left(a\,x^2+a\,y^2+z^2\,c\right)^4},$$

$$\frac{3\,a^2\,z\,m^2\,\left(a-c\right)\,\left(x^2+y^2\right)}{\mu\,\left(a\,x^2+a\,y^2+z^2\,c\right)^4}\Bigg]$$

$$\mathit{JdotE\ power}\ =0 \tag{7}$$

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

```
> NAME:=`Example 10b -- Dirac Hedge Hog solution.`;
> r:=(x^2+y^2+1*z^2)^(1/2);Gamma:=factor(I*(m/2)/(r*(z-r)));
> Ax:=Gamma*(-y);Ay:=Gamma*x;
> Az:=0; phi:=0;
```

Then call the procedure JCM(Ax,Ay,Az,phi)

```
> JCM(Ax,Ay,Az,phi,1,2,0,0):
```

****

$$\mathit{NAME} := \mathit{Example\ 10b\ \text{--}\ Dirac\ Hedge\ Hog\ solution.}$$

$$r:=\sqrt{x^2+y^2+z^2}$$

$$\Gamma:=-\frac{\frac{1}{2}\,\mathrm{I}\,m}{\sqrt{x^2+y^2+z^2}\,\left(-z+\sqrt{x^2+y^2+z^2}\right)}$$

$$Ax:=\frac{\frac{1}{2}\,\mathrm{I}\,m\,y}{\sqrt{x^2+y^2+z^2}\,\left(-z+\sqrt{x^2+y^2+z^2}\right)}$$

$$Ay:=-\frac{\frac{1}{2}\,\mathrm{I}\,m\,x}{\sqrt{x^2+y^2+z^2}\,\left(-z+\sqrt{x^2+y^2+z^2}\right)}$$

$$Az:=0$$

$$\phi:=0$$

$$Lambda = 1$$

$$LambdaH = 1$$

$$N = 0$$

$$H = 0$$

Example 10b -- Dirac Hedge Hog solution.

$$Action = \frac{\frac{1}{2} \mathrm{I}\, m\, y\, d(x)}{\sqrt{x^2+y^2+z^2}\,\left(-z+\sqrt{x^2+y^2+z^2}\right)} - \frac{\frac{1}{2} \mathrm{I}\, m\, x\, d(y)}{\sqrt{x^2+y^2+z^2}\,\left(-z+\sqrt{x^2+y^2+z^2}\right)}$$

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[\frac{\frac{1}{2} \mathrm{I}\, m\, x}{(x^2+y^2+z^2)^{3/2}}, \frac{\frac{1}{2} \mathrm{I}\, m\, y}{(x^2+y^2+z^2)^{3/2}}, -\frac{\frac{1}{2} \mathrm{I} \left(-x^2-y^2-2 z^2+2\sqrt{x^2+y^2+z^2}\, z\right) z\, m}{(x^2+y^2+z^2)^{3/2} \left(-z+\sqrt{x^2+y^2+z^2}\right)^2} \right]$$

$$Topological\ Torsion = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = [0, 0, 0]$$

$$H\ field = \left[\frac{\frac{1}{2} \mathrm{I}\, m\, x}{\mu\,(x^2+y^2+z^2)^{3/2}}, \frac{\frac{1}{2} \mathrm{I}\, m\, y}{\mu\,(x^2+y^2+z^2)^{3/2}}, -\frac{\frac{1}{2} \mathrm{I} \left(-x^2-y^2-2 z^2+2\sqrt{x^2+y^2+z^2}\, z\right) z\, m}{(x^2+y^2+z^2)^{3/2} \left(-z+\sqrt{x^2+y^2+z^2}\right)^2 \mu} \right]$$

$$Poynting\ vector\ ExH = [0, 0, 0]$$

******************** *Lorenz constitutive equations, B = mu H, D = epsilon E**************

$$Lorenz\ 4Current = [0, 0, 0, 0]$$

$$Lorenz\ charge\ density\ 3D = 0$$

$$divergence\ Lorenz\ 4Current = 0$$

***************************Adjoint Current*****************************

$$N = 0$$

$$H = 0$$

$$Adjoint4current = CURR$$

$$Adjointchargedensity = 0$$

$$divergenceAdjoint4current = 0$$

$$**$$

$$Topological\ SPIN = \left[-\frac{1}{4} \frac{m^2 x \left(-x^2 - y^2 - 2 z^2 + 2 \sqrt{x^2+y^2+z^2}\, z\right) z}{(x^2+y^2+z^2)^2 \left(-z + \sqrt{x^2+y^2+z^2}\right)^3 \mu}, -\frac{1}{4} \frac{m^2 y \left(-x^2 - y^2 - 2 z^2 + 2 \sqrt{x^2+y^2+z^2}\, z\right) z}{(x^2+y^2+z^2)^2 \left(-z + \sqrt{x^2+y^2+z^2}\right)^3 \mu}, -\frac{1}{4} \frac{m^2 (x^2+y^2)}{(x^2+y^2+z^2)^2 \left(-z + \sqrt{x^2+y^2+z^2}\right) \mu}, 0 \right]$$

$$chiralty\ AdotD = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = \frac{1}{4} \frac{1}{\mu \left(-z + \sqrt{x^2+y^2+z^2}\right)^4 (x^2+y^2+z^2)^2} \Big(\big(-x^4 - 8 z^2 x^2 + 4 x^2 z \sqrt{x^2+y^2+z^2} - 2 y^2 x^2 - 8 z^2 y^2 - 8 z^4 + 4 y^2 z \sqrt{x^2+y^2+z^2} - y^4 + 8 \sqrt{x^2+y^2+z^2}\, z^3 \big) m^2 \Big)$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = 0$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = \frac{1}{4} \frac{1}{\mu \left(-z + \sqrt{x^2+y^2+z^2}\right)^4 (x^2+y^2+z^2)^2} \Big(\big(-x^4 - 8 z^2 x^2 + 4 x^2 z \sqrt{x^2+y^2+z^2} - 2 y^2 x^2 - 8 z^2 y^2 - 8 z^4 + 4 y^2 z \sqrt{x^2+y^2+z^2} - y^4 + 8 \sqrt{x^2+y^2+z^2}\, z^3 \big) m^2 \Big)$$

$$Virtual\ work = [0, 0, 0]$$

$$JdotE\ power = 0 \tag{8}$$

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

```
> NAME:=`Example 11-- Coulomb plus Bohm-Aharanov singular vortex string`;
> lambda:=(x^2+y^2)^(2/2);
> Ax:=y/lambda;Ay:=-x/lambda;Az:=0;phi:=m/((4*pi*epsilon)*(x^2+y^2+z^2)^(1/2));
```

Then call the procedure JCM(Ax,Ay,Az,phi)

```
> JCM(Ax,Ay,Az,phi,1,2,0,0):
```

****

NAME := *Example 11-- Coulomb plus Bohm-Aharanov singular vortex string*

$$\lambda := x^2 + y^2$$

$$Ax := \frac{y}{x^2 + y^2}$$

$$Ay := -\frac{x}{x^2+y^2}$$

$$Az := 0$$

$$\phi := \frac{1}{4}\,\frac{m}{\pi\,\varepsilon\,\sqrt{x^2+y^2+z^2}}$$

$$Lambda\ = 1$$

$$LambdaH = 1$$

$$N\ = 0$$

$$H\ = 0$$

Example 11-- Coulomb plus Bohm-Aharanov singular vortex string

$$Action = \frac{y\,d(x)}{x^2+y^2} - \frac{x\,d(y)}{x^2+y^2} - \frac{1}{4}\,\frac{m\,d(t)}{\pi\,\varepsilon\,\sqrt{x^2+y^2+z^2}}$$

$$E\ field = \left[\frac{1}{4}\,\frac{m\,x}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}},\ \frac{1}{4}\,\frac{m\,y}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}},\ \frac{1}{4}\,\frac{m\,z}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}}\right]$$

$$B\ field = [0, 0, 0]$$

$$Topological\ Torsion = \left[-\frac{1}{4}\,\frac{m\,z\,x}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}\,(x^2+y^2)},\right.$$

$$\left. -\frac{1}{4}\,\frac{m\,z\,y}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}\,(x^2+y^2)},\ \frac{1}{4}\,\frac{m}{\pi\,\varepsilon\,(x^2+y^2+z^2)^{3/2}},\ 0\right]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = \left[\frac{1}{4}\,\frac{m\,x}{\pi\,(x^2+y^2+z^2)^{3/2}},\ \frac{1}{4}\,\frac{m\,y}{\pi\,(x^2+y^2+z^2)^{3/2}},\ \frac{1}{4}\,\frac{m\,z}{\pi\,(x^2+y^2+z^2)^{3/2}}\right]$$

$$H\ field = [0, 0, 0]$$

$$Poynting\ vector\ ExH = [0, 0, 0]$$

******************* *Lorenz constitutive equations, B = mu H, D = epsilon E*************
***

$$Lorenz\ 4Current = [0, 0, 0, 0]$$

$$Lorenz\ charge\ density\ 3D = 0$$

$$divergence\ Lorenz\ 4Current\ = 0$$

***************************Adjoint Current*****************************

$$N\ = 0$$

$$H\ = 0$$

$$Adjoint4current\ = CURR$$

$$Adjointchargedensity\ = 0$$

$$divergenceAdjoint4current = 0$$

****

$$Topological\ SPIN = \left[\frac{1}{16} \frac{m^2 x}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, \frac{1}{16} \frac{m^2 y}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, \frac{1}{16} \frac{m^2 z}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}, 0 \right]$$

$$chiralty\ AdotD = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{1}{16} \frac{m^2}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = 0$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = -\frac{1}{16} \frac{m^2}{\pi^2 (x^2+y^2+z^2)^2 \varepsilon}$$

$$Virtual\ work = [0, 0, 0]$$

$$JdotE\ power = 0 \qquad \textbf{(9)}$$

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

```
> NAME:=`Example A-- Hopf signature index 0. The 1-form is divided by the
  Holder norm p=2, n=4 `;

> Ax:=y;Ay:=-x;Az:=c*t;phi:=z*c;
```

Then call the procedure JCM(Ax,Ay,Az,phi)

```
> JCM(Ax,Ay,Az,phi,1,2,4,4):
```

****

NAME :=

Example A-- Hopf signature index 0. The 1-form is divided by the Holder norm p=2, n=4

$$Ax := y$$

$$Ay := -x$$

$$Az := c\,t$$

$$\phi := z\,c$$

$$Lambda = (y^2+x^2+z^2+c^2 t^2)^2$$

$$LambdaH = (y^2+x^2+z^2+c^2 t^2)^2$$

$$N = 4$$

$$H = 4$$

Example A-- Hopf signature index 0. The 1-form is divided by the Holder norm p=2, n=4

$$Action = \frac{y\,d(x)}{(y^2+x^2+z^2+c^2 t^2)^2} - \frac{x\,d(y)}{(y^2+x^2+z^2+c^2 t^2)^2} + \frac{c\,t\,d(z)}{(y^2+x^2+z^2+c^2 t^2)^2}$$

$$-\frac{z\,c\,d(t)}{(y^2+x^2+z^2+c^2t^2)^2}$$

$$E\ field = \left[\frac{4\,c\,(z\,x+y\,c\,t)}{(y^2+x^2+z^2+c^2t^2)^3}, -\frac{4\,c\,(-z\,y+x\,c\,t)}{(y^2+x^2+z^2+c^2t^2)^3}, \frac{2\,c\,(-y^2-x^2+z^2+c^2t^2)}{(y^2+x^2+z^2+c^2t^2)^3}\right]$$

$$B\ field = \left[-\frac{4\,(z\,x+y\,c\,t)}{(y^2+x^2+z^2+c^2t^2)^3}, \frac{4\,(-z\,y+x\,c\,t)}{(y^2+x^2+z^2+c^2t^2)^3}, -\frac{2\,(-y^2-x^2+z^2+c^2t^2)}{(y^2+x^2+z^2+c^2t^2)^3}\right]$$

$$Topological\ Torsion = \left[\frac{2\,x\,c}{(y^2+x^2+z^2+c^2t^2)^4}, \frac{2\,y\,c}{(y^2+x^2+z^2+c^2t^2)^4}, \frac{2\,z\,c}{(y^2+x^2+z^2+c^2t^2)^4}, \frac{2\,c\,t}{(y^2+x^2+z^2+c^2t^2)^4}\right]$$

$$Helicity\ AdotB = \frac{2\,c\,t}{(y^2+x^2+z^2+c^2t^2)^4}$$

$$Poincare\ 2\ E.B = -\frac{8\,c}{(y^2+x^2+z^2+c^2t^2)^4}$$

$$D\ field = \left[\frac{4\,\varepsilon\,c\,(z\,x+y\,c\,t)}{(y^2+x^2+z^2+c^2t^2)^3}, -\frac{4\,\varepsilon\,c\,(-z\,y+x\,c\,t)}{(y^2+x^2+z^2+c^2t^2)^3}, \frac{2\,\varepsilon\,c\,(-y^2-x^2+z^2+c^2t^2)}{(y^2+x^2+z^2+c^2t^2)^3}\right]$$

$$H\ field = \left[-\frac{4\,(z\,x+y\,c\,t)}{(y^2+x^2+z^2+c^2t^2)^3\,\mu}, \frac{4\,(-z\,y+x\,c\,t)}{(y^2+x^2+z^2+c^2t^2)^3\,\mu}, -\frac{2\,(-y^2-x^2+z^2+c^2t^2)}{(y^2+x^2+z^2+c^2t^2)^3\,\mu}\right]$$

$$Poynting\ vector\ ExH = [0, 0, 0]$$

******************* Lorenz constitutive equations, B = mu H, D = epsilon E***************

$$Lorenz\ 4Current = \left[\frac{4\,(-y^3-y\,x^2-y\,z^2+5\,y\,c^2t^2+6\,c\,t\,z\,x)\,(\varepsilon\,c^2\mu+1)}{(y^2+x^2+z^2+c^2t^2)^4\,\mu}, -\frac{4\,(-x\,y^2-x^3-x\,z^2+5\,c^2t^2x-6\,c\,t\,z\,y)\,(\varepsilon\,c^2\mu+1)}{(y^2+x^2+z^2+c^2t^2)^4\,\mu}, \frac{8\,c\,t\,(-2\,y^2-2\,x^2+z^2+c^2t^2)\,(\varepsilon\,c^2\mu+1)}{(y^2+x^2+z^2+c^2t^2)^4\,\mu}, 0\right]$$

$$Lorenz\ charge\ density\ 3D = 0$$

$$divergence\ Lorenz\ 4Current\ = 0$$

**************************Adjoint Current****************************

$$N\ = 4$$

$$H\ = 4$$

$$Adjoint4current\ = CURR$$

$$Adjointchargedensity = -\frac{c^2 t}{(y^2+x^2+z^2+c^2 t^2)^8}$$

$$divergenceAdjoint4current = \frac{12 c^2}{(y^2+x^2+z^2+c^2 t^2)^8}$$

****

$$Topological\ SPIN = \left[\frac{2(-x y^2 - x^3 + x z^2 - c^2 t^2 x + 2 c t z y + 2 \varepsilon c^2 \mu x z^2 + 2 \varepsilon c^3 \mu t z y)}{(y^2+x^2+z^2+c^2 t^2)^5 \mu}, -\frac{2(2 c t z x + y c^2 t^2 + y^3 + y x^2 - y z^2 - 2 \varepsilon c^2 \mu y z^2 + 2 \varepsilon c^3 \mu t z x)}{(y^2+x^2+z^2+c^2 t^2)^5 \mu}, \frac{2 z(-2 y^2 - 2 x^2 - \varepsilon c^2 \mu y^2 - \varepsilon c^2 \mu x^2 + \varepsilon c^2 \mu z^2 + \varepsilon c^4 t^2 \mu)}{(y^2+x^2+z^2+c^2 t^2)^5 \mu}, \frac{2 \varepsilon c^2 t}{(y^2+x^2+z^2+c^2 t^2)^4}\right]$$

$$chiralty\ AdotD = \frac{2 \varepsilon c^2 t}{(y^2+x^2+z^2+c^2 t^2)^4}$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{4(-1+\varepsilon c^2 \mu)}{(y^2+x^2+z^2+c^2 t^2)^4 \mu}$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = \frac{4(2 c^2 t^2 - y^2 - x^2)(\varepsilon c^2 \mu + 1)}{(y^2+x^2+z^2+c^2 t^2)^5 \mu}$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = -\frac{4(-2 y^2 - 2 x^2 - z^2 + c^2 t^2 + \varepsilon c^2 \mu z^2 + 3 \varepsilon c^4 t^2 \mu)}{(y^2+x^2+z^2+c^2 t^2)^5 \mu}$$

$$Virtual\ work = \left[\frac{8(c^2 t^2 x - 2 c t z y + x^3 + x y^2 - x z^2)(\varepsilon c^2 \mu + 1)}{(y^2+x^2+z^2+c^2 t^2)^6 \mu}, \frac{8(y c^2 t^2 + 2 c t z x - y z^2 + y x^2 + y^3)(\varepsilon c^2 \mu + 1)}{(y^2+x^2+z^2+c^2 t^2)^6 \mu}, \frac{16(x^2+y^2) z(\varepsilon c^2 \mu + 1)}{(y^2+x^2+z^2+c^2 t^2)^6 \mu}\right]$$

$$JdotE\ power = \frac{16 t c^2(\varepsilon c^2 \mu + 1)}{(y^2+x^2+z^2+c^2 t^2)^5 \mu} \quad (10)$$

Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.

```
> NAME:=`Example B-- Hopf signature index 1. The 1-form is divided by the
  Holder norm p=2, n=4 `;
> ff:=1;p:=2;n:=4;
> lambda:=(x^p+y^p+z^p-(c*t)^p)^(n/p);
> Ax:=y;Ay:=-x;Az:=c*t;phi:=z*c;
```

Then call the procedure JCM(Ax,Ay,Az,phi)

```
> JCM(Ax,Ay,Az,phi,-1,2,4,4):
```

****

NAME :=

Example B-- Hopf signature index 1. The 1-form is divided by the Holder norm p=2, n=4

$$ff := 1$$

$$p := 2$$

$$n := 4$$

$$\lambda := (x^2 + y^2 + z^2 - c^2 t^2)^2$$

$$Ax := y$$

$$Ay := -x$$

$$Az := c\,t$$

$$\phi := z\,c$$

$$Lambda = (x^2 + y^2 + z^2 - c^2 t^2)^2$$

$$LambdaH = (x^2 + y^2 + z^2 - c^2 t^2)^2$$

$$N = 4$$

$$H = 4$$

Example B-- Hopf signature index 1. The 1-form is divided by the Holder norm p=2, n=4

$$Action = \frac{y\,d(x)}{(x^2 + y^2 + z^2 - c^2 t^2)^2} - \frac{x\,d(y)}{(x^2 + y^2 + z^2 - c^2 t^2)^2} + \frac{c\,t\,d(z)}{(x^2 + y^2 + z^2 - c^2 t^2)^2} - \frac{z\,c\,d(t)}{(x^2 + y^2 + z^2 - c^2 t^2)^2}$$

$$E\ field = \left[\frac{4\,c\,(-z\,x + y\,c\,t)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, -\frac{4\,c\,(z\,y + x\,c\,t)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, \frac{2\,c\,(x^2 + y^2 - z^2 + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3} \right]$$

$$B\ field = \left[\frac{4\,(z\,x + y\,c\,t)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, -\frac{4\,(-z\,y + x\,c\,t)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, -\frac{2\,(x^2 + y^2 - z^2 + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3} \right]$$

$$Topological\ Torsion = \left[\frac{2\,x\,c}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}, \frac{2\,y\,c}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}, \frac{2\,z\,c}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}, \frac{2\,c\,t}{(-x^2 - y^2 - z^2 + c^2 t^2)^4} \right]$$

$$Helicity\ AdotB = \frac{2\,c\,t}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}$$

$$Poincare\ 2\ E.B = -\frac{8\,c}{(-x^2 - y^2 - z^2 + c^2 t^2)^4}$$

$$D\ field = \left[\frac{4\,\varepsilon\,c\,(-z\,x + y\,c\,t)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, -\frac{4\,\varepsilon\,c\,(z\,y + x\,c\,t)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3}, \frac{2\,\varepsilon\,c\,(x^2 + y^2 - z^2 + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3} \right]$$

$$H\ field = \left[\frac{4\,(z\,x + y\,c\,t)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3\,\mu},\ -\frac{4\,(-z\,y + x\,c\,t)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3\,\mu},\ -\frac{2\,(x^2 + y^2 - z^2 + c^2 t^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^3\,\mu} \right]$$

$$Poynting\ vector\ ExH = \left[\frac{16\,c^2\,(x^2 + y^2 - z^2 + c^2 t^2)\,x\,t}{(-x^2 - y^2 - z^2 + c^2 t^2)^6\,\mu},\ \frac{16\,c^2\,(x^2 + y^2 - z^2 + c^2 t^2)\,y\,t}{(-x^2 - y^2 - z^2 + c^2 t^2)^6\,\mu},\ \frac{32\,c^2\,z\,t\,(x^2 + y^2)}{(-x^2 - y^2 - z^2 + c^2 t^2)^6\,\mu} \right]$$

******************** *Lorenz constitutive equations, B = mu H, D = epsilon E****************

$$Lorenz\ 4Current = \left[\frac{4\,(y\,x^2 + y^3 + y\,z^2 + 5\,y\,c^2 t^2 - 6\,c\,t\,z\,x)\,(-1 + \varepsilon\,c^2\,\mu)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4\,\mu},\ -\frac{4\,(x^3 + x\,y^2 + x\,z^2 + 5\,c^2 t^2 x + 6\,c\,t\,z\,y)\,(-1 + \varepsilon\,c^2\,\mu)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4\,\mu},\ \frac{8\,c\,t\,(2\,x^2 + 2\,y^2 - z^2 + c^2 t^2)\,(-1 + \varepsilon\,c^2\,\mu)}{(-x^2 - y^2 - z^2 + c^2 t^2)^4\,\mu},\ 0 \right]$$

$$Lorenz\ charge\ density\ 3D = 0$$

$$divergence\ Lorenz\ 4Current\ = 0$$

**************************Adjoint Current***************************

$$N\ = 4$$

$$H\ = 4$$

$$Adjoint4current\ = CURR$$

$$Adjointchargedensity\ = -\frac{c^2\,t}{(-x^2 - y^2 - z^2 + c^2 t^2)^8}$$

$$divergenceAdjoint4current\ = \frac{12\,c^2}{(-x^2 - y^2 - z^2 + c^2 t^2)^8}$$

****

$$Topological\ SPIN = \left[\frac{2\,(x^3 + x\,y^2 - x\,z^2 + 3\,c^2 t^2 x - 2\,c\,t\,z\,y - 2\,\varepsilon\,c^2\,\mu\,x\,z^2 + 2\,\varepsilon\,c^3\,\mu\,t\,z\,y)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5\,\mu},\ -\frac{2\,(-2\,c\,t\,z\,x - 3\,y\,c^2 t^2 - y\,x^2 - y^3 + y\,z^2 + 2\,\varepsilon\,c^2\,\mu\,y\,z^2 + 2\,\varepsilon\,c^3\,\mu\,t\,z\,x)}{(-x^2 - y^2 - z^2 + c^2 t^2)^5\,\mu}, \right.$$

$$\frac{2\,z\,(2\,y^2+2\,x^2+\varepsilon\,c^2\,\mu\,x^2+\varepsilon\,c^2\,\mu\,y^2-\varepsilon\,c^2\,\mu\,z^2+\varepsilon\,c^4\,t^2\,\mu)}{(-x^2-y^2-z^2+c^2\,t^2)^5\,\mu},$$

$$\left.\frac{2\,\varepsilon\,c^2\,t\,(3\,y^2+3\,x^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5}\right]$$

$$\textit{chiralty AdotD} = \frac{2\,\varepsilon\,c^2\,t\,(3\,y^2+3\,x^2-z^2+c^2\,t^2)}{(-x^2-y^2-z^2+c^2\,t^2)^5}$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}\left(4\,(2\,z^2\,x^2+6\,y^2\,c^2\,t^2 + 2\,z^2\,y^2+6\,x^2\,c^2\,t^2+x^4+2\,y^2\,x^2+y^4+z^4-2\,z^2\,c^2\,t^2+c^4\,t^4)\,(-1+\varepsilon\,c^2\,\mu)\right)$$

$$\textit{Interaction energy density (A.J-rho.phi)} = \frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}\left(4\,(-1+\varepsilon\,c^2\,\mu)\,(2\,y^2\,x^2+y^4+z^2\,y^2+9\,y^2\,c^2\,t^2+x^4 + z^2\,x^2+9\,x^2\,c^2\,t^2-2\,z^2\,c^2\,t^2+2\,c^4\,t^4)\right)$$

$$\textit{Poincare 1 (B.H-D.E)-(A.J-rho.phi)} = -\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^6\,\mu}\left(4\,(-1+\varepsilon\,c^2\,\mu)\,(3\,z^2\,x^2 + 3\,z^2\,y^2+3\,c^4\,t^4+15\,y^2\,c^2\,t^2+15\,x^2\,c^2\,t^2-4\,z^2\,c^2\,t^2+z^4+2\,x^4+4\,y^2\,x^2+2\,y^4)\right)$$

$$\textit{Virtual work} = \left[\frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^7\,\mu}\left(8\,(-1+\varepsilon\,c^2\,\mu)\,(x^5+2\,x^3\,y^2+14\,x^3\,c^2\,t^2+x\,y^4 + 14\,x\,y^2\,c^2\,t^2-x\,z^4-8\,x\,z^2\,c^2\,t^2+9\,c^4\,t^4\,x-2\,c\,t\,z\,y\,x^2-2\,c\,t\,z\,y^3-2\,c\,t\,z^3\,y + 2\,c^3\,t^3\,z\,y)\right), \frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^7\,\mu}\left(8\,(-1+\varepsilon\,c^2\,\mu)\,(2\,c\,t\,x^3\,z+14\,c^2\,t^2\,x^2\,y + 2\,c\,t\,y^2\,z\,x+14\,c^2\,t^2\,y^3+2\,c\,t\,z^3\,x-8\,c^2\,t^2\,z^2\,y-2\,c^3\,t^3\,z\,x+9\,c^4\,t^4\,y+y\,x^4+2\,y^3\,x^2 + y^5-y\,z^4)\right), \frac{16\,(-1+\varepsilon\,c^2\,\mu)\,z\,(x^2+y^2)\,(x^2+y^2+11\,c^2\,t^2+z^2)}{(-x^2-y^2-z^2+c^2\,t^2)^7\,\mu}\right]$$

$$\textit{JdotE power} = \frac{1}{(-x^2-y^2-z^2+c^2\,t^2)^7\,\mu}\left(16\,(-1+\varepsilon\,c^2\,\mu)\,c^2\,t\,(6\,y^2\,x^2+4\,z^2\,y^2+4\,z^2\,x^2 + 3\,y^4+8\,y^2\,c^2\,t^2+3\,x^4+8\,x^2\,c^2\,t^2+z^4+c^4\,t^4-2\,z^2\,c^2\,t^2)\right) \qquad \textbf{(11)}$$

```
> NAME:=`Example 12 -- Black Hole 2 singular vortex ring `;
```

```
> phi := 1; Ax:=a*y/(x^2+y^2+z^2);Ay := -a*x/(x^2+y^2+z^2);Az:=0;
```

Then call the procedure JCM(Ax,Ay,Az,phi)

```
> JCM(Ax,Ay,Az,phi,1,2,0,0):
```

****

NAME := Example 12 -- Black Hole 2 singular vortex ring

$$\phi := 1$$

$$Ax := \frac{a\,y}{x^2+y^2+z^2}$$

$$Ay := -\frac{a\,x}{x^2+y^2+z^2}$$

$$Az := 0$$

$$Lambda = 1$$

$$LambdaH = 1$$

$$N = 0$$

$$H = 0$$

Example 12 -- Black Hole 2 singular vortex ring

$$Action = \frac{a\,y\,d(x)}{x^2+y^2+z^2} - \frac{a\,x\,d(y)}{x^2+y^2+z^2} - d(t)$$

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[-\frac{2\,a\,x\,z}{(x^2+y^2+z^2)^2}, -\frac{2\,a\,y\,z}{(x^2+y^2+z^2)^2}, -\frac{2\,a\,z^2}{(x^2+y^2+z^2)^2} \right]$$

$$Topological\ Torsion = \left[\frac{2\,a\,x\,z}{(x^2+y^2+z^2)^2}, \frac{2\,a\,y\,z}{(x^2+y^2+z^2)^2}, \frac{2\,a\,z^2}{(x^2+y^2+z^2)^2}, 0 \right]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = [0, 0, 0]$$

$$H\ field = \left[-\frac{2\,a\,x\,z}{(x^2+y^2+z^2)^2\,\mu}, -\frac{2\,a\,y\,z}{(x^2+y^2+z^2)^2\,\mu}, -\frac{2\,a\,z^2}{(x^2+y^2+z^2)^2\,\mu} \right]$$

$$Poynting\ vector\ ExH = [0, 0, 0]$$

******************* *Lorenz constitutive equations, B = mu H, D = epsilon E***************

$$Lorenz\ 4Current = \left[\frac{2\,a\,y}{(x^2+y^2+z^2)^2\,\mu}, -\frac{2\,a\,x}{(x^2+y^2+z^2)^2\,\mu}, 0, 0 \right]$$

$$Lorenz\ charge\ density\ 3D = 0$$

$$divergence\ Lorenz\ 4Current = 0$$

***************************Adjoint Current*****************************

$N = 0$

$H = 0$

$Adjoint4current = CURR$

$Adjointchargedensity = 0$

$divergenceAdjoint4current = 0$

****

$$Topological\ SPIN = \left[\frac{2\,a^2\,x\,z^2}{(x^2+y^2+z^2)^3\,\mu}, \frac{2\,a^2\,y\,z^2}{(x^2+y^2+z^2)^3\,\mu}, -\frac{2\,z\,a^2\,(x^2+y^2)}{(x^2+y^2+z^2)^3\,\mu}, 0 \right]$$

$chiralty\ AdotD = 0$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = \frac{4\,z^2\,a^2}{(x^2+y^2+z^2)^3\,\mu}$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = \frac{2\,a^2\,(x^2+y^2)}{(x^2+y^2+z^2)^3\,\mu}$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = -\frac{2\,a^2\,(-2\,z^2+x^2+y^2)}{(x^2+y^2+z^2)^3\,\mu}$$

$$Virtual\ work = \left[\frac{4\,a^2\,x\,z^2}{(x^2+y^2+z^2)^4\,\mu}, \frac{4\,a^2\,y\,z^2}{(x^2+y^2+z^2)^4\,\mu}, -\frac{4\,z\,a^2\,(x^2+y^2)}{(x^2+y^2+z^2)^4\,\mu} \right]$$

$JdotE\ power = 0$ **(12)**

```
> NAME:=`Example 13 -- `;

> phi:=alpha(x,y,z); Az:=beta(x,y,z);Ax:=0;Ay:=0;
```

Then call the procedure JCM(Ax,Ay,Az,phi)

```
> JCM(Ax,Ay,Az,phi,1,2,0,0):
```

****

$NAME := Example\ 13\ \text{--}$

$\phi := \alpha(x, y, z)$

$Az := \beta(x, y, z)$

$Ax := 0$

$Ay := 0$

$Lambda = 1$

$LambdaH = 1$

$N = 0$

$$H = 0$$

Example 13 --

$$Action = \beta(x, y, z)\, d(z) - \alpha(x, y, z)\, d(t)$$

$$E\ field = \left[-\left(\frac{\partial}{\partial x}\alpha(x, y, z)\right), -\left(\frac{\partial}{\partial y}\alpha(x, y, z)\right), -\left(\frac{\partial}{\partial z}\alpha(x, y, z)\right)\right]$$

$$B\ field = \left[\frac{\partial}{\partial y}\beta(x, y, z), -\left(\frac{\partial}{\partial x}\beta(x, y, z)\right), 0\right]$$

$$Topological\ Torsion = \left[\left(\frac{\partial}{\partial y}\alpha(x, y, z)\right)\beta(x, y, z) - \left(\frac{\partial}{\partial y}\beta(x, y, z)\right)\alpha(x, y, z), -\left(\frac{\partial}{\partial x}\alpha(x, y, z)\right)\beta(x, y, z) + \left(\frac{\partial}{\partial x}\beta(x, y, z)\right)\alpha(x, y, z), 0, 0\right]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = -2\left(\frac{\partial}{\partial x}\alpha(x, y, z)\right)\left(\frac{\partial}{\partial y}\beta(x, y, z)\right) + 2\left(\frac{\partial}{\partial y}\alpha(x, y, z)\right)\left(\frac{\partial}{\partial x}\beta(x, y, z)\right)$$

$$D\ field = \left[-\varepsilon\left(\frac{\partial}{\partial x}\alpha(x, y, z)\right), -\varepsilon\left(\frac{\partial}{\partial y}\alpha(x, y, z)\right), -\varepsilon\left(\frac{\partial}{\partial z}\alpha(x, y, z)\right)\right]$$

$$H\ field = \left[\frac{\frac{\partial}{\partial y}\beta(x, y, z)}{\mu}, -\frac{\frac{\partial}{\partial x}\beta(x, y, z)}{\mu}, 0\right]$$

$$Poynting\ vector\ ExH = \left[-\frac{\left(\frac{\partial}{\partial z}\alpha(x, y, z)\right)\left(\frac{\partial}{\partial x}\beta(x, y, z)\right)}{\mu}, -\frac{\left(\frac{\partial}{\partial z}\alpha(x, y, z)\right)\left(\frac{\partial}{\partial y}\beta(x, y, z)\right)}{\mu}, \frac{\left(\frac{\partial}{\partial x}\alpha(x, y, z)\right)\left(\frac{\partial}{\partial x}\beta(x, y, z)\right) + \left(\frac{\partial}{\partial y}\alpha(x, y, z)\right)\left(\frac{\partial}{\partial y}\beta(x, y, z)\right)}{\mu}\right]$$

******************* *Lorenz constitutive equations, B = mu H, D = epsilon E**************

$$Lorenz\ 4Current = \left[\frac{\frac{\partial^2}{\partial z\,\partial x}\beta(x, y, z)}{\mu}, \frac{\frac{\partial^2}{\partial z\,\partial y}\beta(x, y, z)}{\mu}, -\frac{\frac{\partial^2}{\partial x^2}\beta(x, y, z) + \frac{\partial^2}{\partial y^2}\beta(x, y, z)}{\mu}, -\varepsilon\left(\frac{\partial^2}{\partial x^2}\alpha(x, y, z) + \frac{\partial^2}{\partial y^2}\alpha(x, y, z) + \frac{\partial^2}{\partial z^2}\alpha(x, y, z)\right)\right]$$

$$Lorenz\ charge\ density\ 3D = -\varepsilon\left(\frac{\partial^2}{\partial x^2}\alpha(x, y, z) + \frac{\partial^2}{\partial y^2}\alpha(x, y, z) + \frac{\partial^2}{\partial z^2}\alpha(x, y, z)\right)$$

$$divergence\ Lorenz\ 4Current\ = 0$$

$$*************************** Adjoint\ Current ****************************$$

$$N\ = 0$$

$$H\ = 0$$

$$Adjoint4current\ = CURR$$

$$Adjointchargedensity\ = 0$$

$$divergenceAdjoint4current\ = 0$$

$$***$$

$$Topological\ SPIN = \left[-\frac{-\beta(x,y,z)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)+\varepsilon\left(\frac{\partial}{\partial x}\alpha(x,y,z)\right)\alpha(x,y,z)\,\mu}{\mu}, -\frac{-\beta(x,y,z)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)+\varepsilon\left(\frac{\partial}{\partial y}\alpha(x,y,z)\right)\alpha(x,y,z)\,\mu}{\mu}, -\varepsilon\left(\frac{\partial}{\partial z}\alpha(x,y,z)\right)\alpha(x,y,z), -\beta(x,y,z)\,\varepsilon\left(\frac{\partial}{\partial z}\alpha(x,y,z)\right)\right]$$

$$chiralty\ AdotD = -\beta(x,y,z)\,\varepsilon\left(\frac{\partial}{\partial z}\alpha(x,y,z)\right)$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{1}{\mu}\left(-\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)^2-\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)^2 +\varepsilon\left(\frac{\partial}{\partial x}\alpha(x,y,z)\right)^2\mu+\varepsilon\left(\frac{\partial}{\partial y}\alpha(x,y,z)\right)^2\mu+\varepsilon\left(\frac{\partial}{\partial z}\alpha(x,y,z)\right)^2\mu\right)$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = -\frac{1}{\mu}\left(\beta(x,y,z)\left(\frac{\partial^2}{\partial x^2}\beta(x,y,z)\right)+\beta(x,y,z)\left(\frac{\partial^2}{\partial y^2}\beta(x,y,z)\right)-\varepsilon\,\alpha(x,y,z)\,\mu\left(\frac{\partial^2}{\partial x^2}\alpha(x,y,z)\right)-\varepsilon\,\alpha(x,y,z)\,\mu\left(\frac{\partial^2}{\partial y^2}\alpha(x,y,z)\right) -\varepsilon\,\alpha(x,y,z)\,\mu\left(\frac{\partial^2}{\partial z^2}\alpha(x,y,z)\right)\right)$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = -\frac{1}{\mu}\left(-\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)^2-\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)^2 +\varepsilon\left(\frac{\partial}{\partial x}\alpha(x,y,z)\right)^2\mu+\varepsilon\left(\frac{\partial}{\partial y}\alpha(x,y,z)\right)^2\mu+\varepsilon\left(\frac{\partial}{\partial z}\alpha(x,y,z)\right)^2\mu-\beta(x,y,z)\left(\frac{\partial^2}{\partial x^2}\beta(x,y,z)\right)-\beta(x,y,z)\left(\frac{\partial^2}{\partial y^2}\beta(x,y,z)\right)+\varepsilon\,\alpha(x,y,z)\,\mu\left(\frac{\partial^2}{\partial x^2}\alpha(x,y,z)\right)\right.$$

$$+\varepsilon\,\alpha(x,y,z)\,\mu\left(\frac{\partial^2}{\partial y^2}\alpha(x,y,z)\right)+\varepsilon\,\alpha(x,y,z)\,\mu\left(\frac{\partial^2}{\partial z^2}\alpha(x,y,z)\right)\bigg)\bigg)$$

$$\mathit{Virtual\ work}\ =\left[\frac{1}{\mu}\left(\varepsilon\left(\frac{\partial}{\partial x}\alpha(x,y,z)\right)\mu\left(\frac{\partial^2}{\partial x^2}\alpha(x,y,z)\right)+\varepsilon\left(\frac{\partial}{\partial x}\alpha(x,y,z)\right)\mu\left(\frac{\partial^2}{\partial y^2}\alpha(x,y,z)\right)+\varepsilon\left(\frac{\partial}{\partial x}\alpha(x,y,z)\right)\mu\left(\frac{\partial^2}{\partial z^2}\alpha(x,y,z)\right)-\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial x^2}\beta(x,y,z)\right)-\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial y^2}\beta(x,y,z)\right)\right),\frac{1}{\mu}\left(\varepsilon\left(\frac{\partial}{\partial y}\alpha(x,y,z)\right)\mu\left(\frac{\partial^2}{\partial x^2}\alpha(x,y,z)\right)+\varepsilon\left(\frac{\partial}{\partial y}\alpha(x,y,z)\right)\mu\left(\frac{\partial^2}{\partial y^2}\alpha(x,y,z)\right)+\varepsilon\left(\frac{\partial}{\partial y}\alpha(x,y,z)\right)\mu\left(\frac{\partial^2}{\partial z^2}\alpha(x,y,z)\right)-\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial x^2}\beta(x,y,z)\right)-\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial y^2}\beta(x,y,z)\right)\right),\frac{1}{\mu}\left(\varepsilon\left(\frac{\partial}{\partial z}\alpha(x,y,z)\right)\mu\left(\frac{\partial^2}{\partial x^2}\alpha(x,y,z)\right)+\varepsilon\left(\frac{\partial}{\partial z}\alpha(x,y,z)\right)\mu\left(\frac{\partial^2}{\partial y^2}\alpha(x,y,z)\right)+\varepsilon\left(\frac{\partial}{\partial z}\alpha(x,y,z)\right)\mu\left(\frac{\partial^2}{\partial z^2}\alpha(x,y,z)\right)-\left(\frac{\partial^2}{\partial z\,\partial x}\beta(x,y,z)\right)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)-\left(\frac{\partial^2}{\partial z\,\partial y}\beta(x,y,z)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)\right)\right]$$

$$\mathit{JdotE\ power}\ = -\frac{1}{\mu}\left(\left(\frac{\partial^2}{\partial z\,\partial x}\beta(x,y,z)\right)\left(\frac{\partial}{\partial x}\alpha(x,y,z)\right)+\left(\frac{\partial^2}{\partial z\,\partial y}\beta(x,y,z)\right)\left(\frac{\partial}{\partial y}\alpha(x,y,z)\right)-\left(\frac{\partial}{\partial z}\alpha(x,y,z)\right)\left(\frac{\partial^2}{\partial x^2}\beta(x,y,z)\right)-\left(\frac{\partial}{\partial z}\alpha(x,y,z)\right)\left(\frac{\partial^2}{\partial y^2}\beta(x,y,z)\right)\right)\qquad\mathbf{(13)}$$

```
> Action:=Az*d(z)-phi*d(t);
```

$$\mathit{Action} := \beta(x,y,z)\,d(z)-\alpha(x,y,z)\,d(t)\qquad\mathbf{(14)}$$

```
> F:=d(Action);
```

$$F := \left(\frac{\partial}{\partial x}\beta(x,y,z)\right)(d(x))\ \&\wedge\ (d(z))+\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)(d(y))\ \&\wedge\ (d(z))-\left(\frac{\partial}{\partial x}\alpha(x,y,z)\right)(d(x))\ \&\wedge\ (d(t))-\left(\frac{\partial}{\partial y}\alpha(x,y,z)\right)(d(y))\ \&\wedge\ (d(t))-\left(\frac{\partial}{\partial z}\alpha(x,y,z)\right)(d(z))\ \&\wedge\ (d(t))\qquad\mathbf{(15)}$$

```
> F&^F;
```

$$\left(2\left(\frac{\partial}{\partial y}\alpha(x,y,z)\right)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)-2\left(\frac{\partial}{\partial x}\alpha(x,y,z)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)\right)\&\wedge(d(x),d(y),d(z),d(t)) \quad (16)$$

```
> Action&^F;
```

$$\left(-\alpha(x,y,z)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)+\beta(x,y,z)\left(\frac{\partial}{\partial x}\alpha(x,y,z)\right)\right)\&\wedge(d(x),d(z),d(t))+\left(-\alpha(x,y,z)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)+\beta(x,y,z)\left(\frac{\partial}{\partial y}\alpha(x,y,z)\right)\right)\&\wedge(d(y),d(z),d(t)) \quad (17)$$