```
> restart: with (linalg):with(liesymm):with(difforms):
> setup(r,theta,phi,tau):defform(x=0,y=0,z=0,t=0,Vx=0,Vy=0,Vz=0,D1=0,D2=0,D3=0,Ax=
  0,Ay=0,Az=0,C=0,Phi=0,phi=0,theta=0,r=0,tau=0,a=const,b=const,c=const,aa=const,b
  b=const,cc=const,cc0=const,aa0=const,Lx=0,Ly=0,Lz=0);
```

Warning, the protected names norm and trace have been redefined and unprotected

Warning, the protected name close has been redefined and unprotected

Warning, the names &^, d and wdegree have been redefined

CARTAN CONNECTIONS, RICCI ROTATION, AFFINE TORSION,
METRIC, CHRISTOFFEL CONNECTIONS, CURVATURE
and
CARTAN's EQUATIONS of STRUCTURE

R. M. Kiehn
July 3, 2001 last update Nov 25, 2005

rkiehn2352@aol.com

Coefficient procedure

```
> COEF:=proc(aaa,bbb,ccc,aaa0,ccc0,eee)
          local AA,a,b,j,i,k;
           global FF,GG,CCCC,C2S,SHIPTR,TTCCCC,finalmetric:
  print(`  `);print(`Basis Frame for given pertubations`);print(`
 `);print(subs(aa=aaa,bb=bbb,cc=ccc,aa0=aaa0,cc0=ccc0,evalm(FF)));
  d(GG):print(`  `);print(`---------- Non Zero Cartan Connection Coefficients {C}
 for the given perturbations ----------`);print(`  `);
   for b from 1 to dim do for a from 1 to dim do for k from 1 to dim do if
 subs(aa=aaa,bb=bbb,cc=ccc,aa0=aaa0,cc0=ccc0,CCCC[a,b,k]=0) then else
 print(`Cartan_RIGHT`(a,b,k)=factor(CCCC[a,b,k])) fi od od od ;
 print(`  `);print(`----------  Non Zero Christoffel Connection Coefficients
 {Gamma} for the perturbations ----------`);print(`  `);

 for i from 1 to dim do for j from 1 to dim do for k from 1 to dim do if
 simplify(subs(aa=aaa,bb=bbb,cc=ccc,aa0=aaa0,cc0=ccc0,C2S[i,j,k]))=0 then  else
 print(`Christoffel_Gamma2`(i,j,k)=simplify(subs(aa=aaa,bb=bbb,cc=ccc,aa0=aaa0,cc
 0=ccc0,C2S[i,j,k]))) fi od od od;
 print(`  `);print(`-----------  Non Zero Residue Connection Coefficients {T}=
 {C} - {Gamma} for the given perturbations ----------`);print(`  `);
 for i from 1 to dim do for j from 1 to dim do for k from 1 to dim do if
 subs(aa=aaa,bb=bbb,cc=ccc,aa0=aaa0,cc0=ccc0,(SHIPTR[i,j,k]))=0 then else
 print(`T`(i,j,k)=simplify(subs(aa=aaa,bb=bbb,cc=ccc,aa0=aaa0,cc0=ccc0,SHIPTR[i,j
 ,k]))) fi od od od ;

 print(`  `);print(`----------  Non Zero Affine Torsion  for the perturbations
```

```
       ----------`);print(`  `);

   for i from 1 to dim do for j from 1 to dim do for k from 1 to dim do if
   subs(aa=aaa,bb=bbb,cc=ccc,aa0=aaa0,cc0=ccc0,TTCCCC[i,j,k]=0) then else
   print(`CartanaffineTorsion`(i,k,j)=factor(subs(aa=aaa,bb=bbb,cc=ccc,aa0=aaa0,cc0
   =ccc0,TTCCCC[i,k,j]))) fi od od od ;

       end:
```

Perturbation procedure

```
>  PERT:=proc(aaa,bbb,ccc,aaa0,ccc0,eee)
        local
   AA,AT,ATr,ATtheta,ATphi,ATtau,AffineTorsion,ATT,Cartan_AffineTorsion,metricpullb
   ack;
           global FF,GG,CCC,finalmetric:
    print(`  `);print(`Basis Frame for given pertubations`);print(`
    `);print(subs(aa=aaa,bb=bbb,cc=ccc,aa0=aaa0,cc0=ccc0,evalm(FF)));
    d(GG):print(`  `);print(`---------- Perturbed Cartan Connection matrix of
   1-forms for the given perturbations ----------`);print(`  `);

   CCC:=simpform(simplify(factor(subs(aa=aaa,bb=bbb,cc=ccc,aa0=aaa0,cc0=ccc0,innerp
   rod(-d(evalm(GG)),evalm(FF))))));print(evalm(CCC));

    print(`  `);print(`---------- Perturbed pullback metric for the given
   perturbations ----------`);print(`  `);

   metricpullback:=simplify(subs(aa=aaa,bb=bbb,cc=ccc,aa0=aaa0,cc0=ccc0,ee=eee,inne
   rprod(transpose(FF),finalmetric,FF)));

   print(metricpullback);
    print(`  `);print(`-----------Perturbed Cartan Vector of Affine Torsion 2-forms
   for the given perturbations ----------`);print(`  `);
   AA:=array([[0,0,0,d(r)],[0,0,0,d(theta)],[0,0,0,d(phi)],[0,0,0,d(tau)]]):


   AT:=CCC&^AA:ATr:=simpform(simplify(factor(AT[1,4]))):ATtheta:=simpform(simplify(
   factor(AT[2,4]))):ATphi:=simpform(simplify(factor(AT[3,4]))):ATtau:=simpform(sim
   plify(factor(AT[4,4]))):


   AffineTorsion:=array([[ATr],[ATtheta],[ATphi],[ATtau]]):ATT:=evalm(AffineTorsion
   ):
   Cartan_AffineTorsion:=subs(aa=aaa,bb=bbb,cc=ccc,aa0=aaa0,cc0=ccc0,evalm(ATT));
   print (Cartan_AffineTorsion);
       end:


>
```

INTRODUCTION

Examples will be given for matrix Basis Frame fields of C2 functions, [F] ,

acting algebraically on a column vector of coordinate differentials of an initial domain, R = {r,theta,phi,tau}, to produce a column vector of 1-forms |sigma> on a final range.

$$[F]|dR> = |sigma>$$

If the vector of 1-forms |sigma> on the final state are exact, then there exist integrals uniquely defining the coordinates {x,y,z,t} of the final state in terms of functions of the variables of the initial state.   On the restricted region of the initial domain where the columns (contravariant vectors) of the Frame field are linearly independent, then the Frame field can be used as a basis for a vector space.   On the restricted region, an inverse matrix of functions [G] can be deduced algebraically. The Frame field is then defined as a Basis Frame.   All Basis Frames composed of C1 functions admit, via differential and algebraic processes, a (right) Cartan matrix [C] of connection 1-forms, such that the differential of any column vector of the Frame of basis vectors is a linear combination of all of the column vectors of the Frame.

$$d[F] = [F][C]$$

The differential process is said to be closed.   The right Cartan matrix of connection 1-forms can be computed explicitly from the matrix formulas

$$[C] = - [dG][F] = + [G][dF]$$

Application of the connection rule to the exterior differentials of the column vector of 1-forms |sigma> yields

$$d|sigma> = [F] \{ [C] \char94 |dR>\}$$

If every Pfaffian component of the column vector of 1-forms, |sigma>, is integrable, the exterior derivative of each component must vanish. Hence the column vector of (affine torsion) 2-forms, [C] ^ |dR>, must vanish in the integrable case.   When |sigma> is not uniquely integrable, the associated right Cartan matrix of connection 1-forms [C] is said to admit Torsion. The lack of integrability implies that there does not exist a unique function of the independent variables on the initial state that maps to the associated coordinate on the final state. There may be more than one such function (!), or none. In certain (lucky) instances, the torsion components can be made to vanish by the use of suitable integrating factors, which in effect redefine the

matrix elements of the Basis Frame field [F]. In the most general case, such integrating factors do not exist for domains of dimension > 2. A component 1-form of |sigma> which admits affine torsion but does not admit an integrating factor is said to exhibit topological torsion. It is the irreducible topological torsion situations that are the most interesting.

A metric [metricfinal] will (can) be imposed arbitrarily on the final range, and its compatible preimage [pullbackmetric] on the initial domain will be computed relative to and compatible with the linear map, [F], used as a congruent transformation:

[pullbackmetric] = [F transpose][metricfinal][F]

A special symmetric Christoffel Connection of the second kind, {Gamma}, can be defined on the initial state in terms of algebraic processes and exterior differentiations of the pullbackmetric, and leads to a matrix of Christoffel connection 1-forms, [Gamma]. If the metric on the final state is a set of constants, then it follows that

d[pullbackmetric] = [C transpose][pullbackmetric] + [pullbackmetric][C]

This condition is related to Christoffel (Riemannian) Curvatures on the initial state. These Christoffel curvature terms can also be deduced in terms of another exterior differential process applied to the Christoffel connection. The result is a matrix of Christoffel - metric curvature 2-forms produced from the formula:

Christoffel (metric induced) matrix of curvature 2-forms = [Gamma]^[Gamma]+d[Gamma].

These matrix constructions agree with the usual tensor formulation.

These concepts will be compared to the production of a matrix Cartan Connection deduced from differential and algebraic processes applied to the Basis Frame matrix, not the symmetric conguent metric. The right Cartan matrix of Connection 1-forms [C] is also defined on the initial state, and also can be used to compute a matrix of curvature 2-forms, according to the formula:

[Cartan matrix of curvature 2-forms] = [C]^[C]+d[C] .

The Cartan matrix of Curvature 2-forms is not the same as Christoffel matrix of curvature 2-forms.
In fact, for C2 Basis Functions, the matrix of Cartan Curvature 2-forms is always zero, where the matrix based matrix of curvature 2-forms need not be zero.

$$[C]^\wedge[C]+d[C]=0$$

In effect, it will be assumed that metric induced matrix of curvature 2-forms will be associated with the stress energy of the graviational field.

$$[\text{gravity stress energy}] = [Gamma]^\wedge[Gamma]+d[Gamma]$$

The Christoffel connection is always free of Affine Torsion, the Cartan connection can support anti-symmetries defined as Affine Torsion.

It is classic to decompose the matrix of Cartan Connection 1-forms into the sum of the matrix of Christoffel connection 1-forms, and another residue matrix of 1-forms [T], whose coefficient functions are sometimes defined as the Ricci rotation coefficients .

$$[C] = [Gamma] + [T]$$

As the Christoffel connections are symmetric in the lower two indices, any anti-symmetries leading to "affine torsion" of the Cartan connection must be associated with the Residue matrix, [T].

In matrix language, the anti-symmetries in the lower indices of the connection coefficients leads to a vector of "affine torsion" 2-forms given by the formula

$$|\text{Affine Torsion 2-forms}> = [C]^\wedge|dR>$$

Examples

The examples will start with the classic map of spherical coordinates {r,theta,phi,tau} (the initial state) mapped into the Cartesian coordiantes {x,y,z,t}, which is the final state. The Jacobian matrix of partial derivatives of the mapping function will serve as the primitive definition of a Basis Frame, [F]. The Right Cartan connection matrix for any basis frame is defined as [C] = - [dG][F] = + [G][dF] , where [G] is the inverse of [F]. Realize that the Right Cartan matrix is defined entirely on the initial state in terms of initial state independent variables.

The curvature and the Affine Torsion of the initial state based upon an integrable set of mapping functions are both zero.

Perturbation of the Frame

For the integrable set, the map is 1-- and onto a euclidean space, and the Basis Frame is the Jacobian matrix of the mapping. The 1-forms of the final state are exact differentials [dx,dy,dz,dt].
A pertubation will be made on the original Frame Field in the final state such that the 1-forms are (possibly) not exact. The Cartan Connection constructed on the perturbed Frame Field can admit Affine Torsion. In the example, only one of the 1-forms on the final state will not be closed, and may even be not integrable. If a 1-form is not closed, it is possible to find an integrating factor, redefining the integrable map, when the Frobeneius integrability condition is satisfied (the Pfaff dimension of the 1-form is equal to 2). The integrating factors need not be unique. If the 1-form does not satisfy the Frobenius condition, an integrating factor does not exist. The Pfaff diomension of the 1-form is 3 or greater.

Perturbation of the metric

Initially it will be assumed that the metric on the final state is a set of constants equal to the identity matrix. Then a perturbation of this final state metric will be defined and its effect on curvatures and torsion will be examined. The compatible (pullback) of the perturbed metric in the initial state will be evaluated in terms of the spherical coordinate variables.
*

Then for each pertubation separately, and then in combination, the Cartan Connection (Frame Based), the Christoffel connection (metric based) and the Ricci components will be evaluated and compared.

The classic map from spherical to Cartesian Coordinates is given by the expressions:

```
> x:=r*sin(theta)*cos(phi);y:=r*sin(theta)*sin(phi);z:=r*cos(theta);t:=tau;JAC:=ja
  cobian([x,y,z,t],[r,theta,phi,tau]);DETJAC:=simplify(det(JAC));g0:=simplify(inne
  rprod(transpose(JAC),JAC)):InverseJacobian:=evalm(g0);DR:=innerprod(JAC,[d(r),d(
  theta),d(phi),d(tau)]):Unperturbed_dx:=DR[1];Unperturbed_dy:=DR[2];Unperturbed_d
  z:=DR[3];Unperturbed_dt:=DR[4];
>
```

$$x := r \sin(\theta) \cos(\phi)$$
$$y := r \sin(\theta) \sin(\phi)$$
$$z := r \cos(\theta)$$
$$t := \tau$$

$$JAC := \begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$DETJAC := \sin(\theta)\, r^2$$

$$InverseJacobian := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & -r^2(-1+\cos(\theta)^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Unperturbed\_dx := \sin(\theta)\cos(\phi)\,\mathrm{d}(r) + r\cos(\theta)\cos(\phi)\,\mathrm{d}(\theta) - r\sin(\theta)\sin(\phi)\,\mathrm{d}(\phi)$$
$$Unperturbed\_dy := \sin(\theta)\sin(\phi)\,\mathrm{d}(r) + r\cos(\theta)\sin(\phi)\,\mathrm{d}(\theta) + r\sin(\theta)\cos(\phi)\,\mathrm{d}(\phi)$$

$$Unperturbed\_dz := \cos(\theta)\,d(r) - r\sin(\theta)\,d(\theta)$$
$$Unperturbed\_dt := d(\tau)$$

Note that the Jacobian matrix of the given transformation is a member of the 10 parameter sub group of the general linear group, known in the mathematics literature as an "affine matrix with a fixed point".

The induced 1-forms |sigma> are exact differentials if generated by the unperturbed (Jacobian) Frame matrix according to the formula:

[F]|dR> = |sigma>

****

The Frame [F] matrix will be perturbed in several ways. The first perturbation will be where the differential dt, as generated by the perturbed frame, is modified such that sigma_t = {(1+bb/r)^(1/2)}dt . This perturbation does not change the Group structure of the unperturbed frame, but does effect the notion of time.

The second perturbation will be such that the exact differential dz is modified such that sigma_z = dz + aa(ydx-xdy)+aa0(ydx-xdy)/(x^2+y^2), a "twist" term for the aa component, and a closed component for the aa0 term. This perturbation changes the group structure of the Basis Frame such that the unperturbed frame becomes an element of the 13 parameter Transitive group defined as the P-Affine group. However, the pertubation does not change the differential element of time. Time is universal in a global Newtonian (absolute) sense.

A third pertubation will be such that the differential dt is modified such that sigma_t = dt + cc(ydx-xdy) + cc0(ydx-xdy)/(x^2+y^2), This perturbation also changes the group structure of the unperturbed frame such that the perturbed frame becomes an element of element of the 13 parameter Intransitive group, define as the W-Affine group. The cc term represents a vorticity component and the cc0 term represents circulation without vorticity.

```
> Perta:=(wcollect(((y*d(x)-x*d(y)))));FF33Ppurt:=simplify(getcoeff((aa+aa0/(x^2+y
  ^2))*Perta&^d(r)&^d(theta)));FF33Wpurt:=simplify(getcoeff((cc+cc0/(x^2+y^2))*Per
  ta&^d(r)&^d(theta)));
```

$$Perta := (-r^2 \sin(\theta)^2 \sin(\phi)^2 - r^2 \sin(\theta)^2 \cos(\phi)^2)\, d(\phi)$$

$$FF33Ppurt := -aa\, r^2 + aa\, r^2 \cos(\theta)^2 - aa0$$

$$FF33Wpurt := -cc\, r^2 + cc\, r^2 \cos(\theta)^2 - cc0$$

> `` ` `` `;`ALL pertubation Coefficients ON`;`` ` `` `;`
> `FF:=evalm(evalm(JAC)+array([[0,0,0,0],[0,0,0,0],[0,0,0,FF33Ppurt],[0,0,FF33Wpurt`
  `,((1+bb/r)^(1/2)-1)]])):Perturbed_Frame_matrix:=evalm(FF);GGT:=transpose(FF):`
> `finalmetric := array([[1, 0, 0,0], [0,1, 0,0], [0, 0, 1,0],[0,0,0,ee]]):`
> `pullbackmetric:=simplify((innerprod(transpose(FF),finalmetric,FF)));metric:=pull`
  `backmetric:perturbed_metric:=evalm(metric):`

ALL pertubation Coefficients ON

Perturbed_Frame_matrix :=

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & -aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0 \\ 0 & 0 & -cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0 & \sqrt{1+\dfrac{bb}{r}} \end{bmatrix}$$

pullbackmetric :=

$$[1, 0, 0, -\cos(\theta)\,aa\,r^2 + \cos(\theta)^3\,aa\,r^2 - \cos(\theta)\,aa0]$$

$$[0, r^2, 0, -r\sin(\theta)\,(-aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0)]$$

$$\left[0, 0,\right.$$

$$r^2 - r^2\cos(\theta)^2 + ee\,cc^2\,r^4 - 2\,ee\,cc^2\,r^4\cos(\theta)^2 + 2\,ee\,cc\,r^2\,cc0 + ee\,cc^2\,r^4\cos(\theta)^4 - 2\,ee\,cc\,r^2\cos(\theta)^2\,cc0 + ee\,cc0^2,$$

$$\left.-\sqrt{\dfrac{r+bb}{r}}\,ee\,cc\,r^2 + \sqrt{\dfrac{r+bb}{r}}\,ee\,cc\,r^2\cos(\theta)^2 - \sqrt{\dfrac{r+bb}{r}}\,ee\,cc0\right]$$

$$\left[-\cos(\theta)\,aa\,r^2 + \cos(\theta)^3\,aa\,r^2 - \cos(\theta)\,aa0, -r\sin(\theta)\,(-aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0),\right.$$

$$-\sqrt{\dfrac{r+bb}{r}}\,ee\,cc\,r^2 + \sqrt{\dfrac{r+bb}{r}}\,ee\,cc\,r^2\cos(\theta)^2 - \sqrt{\dfrac{r+bb}{r}}\,ee\,cc0,$$

$$\left.\dfrac{aa^2\,r^5 - 2\,aa^2\,r^5\cos(\theta)^2 + 2\,aa\,r^3\,aa0 + aa^2\,r^5\cos(\theta)^4 - 2\,aa\,r^3\cos(\theta)^2\,aa0 + r\,aa0^2 + ee\,r + ee\,bb}{r}\right]$$

>

> `GG:=evalm(simplify(evalm(inverse(FF)))):DETFF:=simplify(det(FF));`

$$DETFF := \sin(\theta)\,r^2\,\sqrt{\dfrac{r+bb}{r}}$$

In these examples, the determinant of the Frame matrix is affected only by the "metric" perturbation bb.

The next computation checks to see that the Perturbed Basis Frame produces the desired differential structures:

$$|sigma> = [FF]|dR>.$$

The integrability of the various 1-forms is computed. It the Topogical Torsion is not zero, the 1-form is NOT integrable.

The homogeneous twists (aa0 not zero) and rotations (cc0 not zero) do not contribute to the topological torsion terms, but they do form the basis of period integrals, involving topological defect structures and path dependent effects.

```
> sigma:=innerprod(FF,[d(r),d(theta),d(phi),d(tau)]):
> sigma_x:=sigma[1];perurbation_of_dx:=sigma_x-DR[1];sigma_y:=sigma[2];perurbation
  _of_dy:=sigma_y-DR[2];sigma_z:=sigma[3];perurbation_of_dz:=factor(sigma_z-DR[3])
  ;sigma_t:=sigma[4];perurbation_of_dt:=factor(sigma_t-DR[4]);dsigma_x:=d(sigma[1]
  );dsigma_y:=d(sigma[2]);dsigma_z:=d(sigma[3]);dsigma_t:=d(sigma[4]);TOPTOR1:=sim
  pform(sigma_x&^dsigma_x);TOPTOR2:=sigma_y&^dsigma_y;TOPTOR3:=sigma_z&^dsigma_z;T
  OPTOR4:=sigma_t&^dsigma_t;
```

$$sigma\_x := \sin(\theta)\cos(\phi)\,d(r) + r\cos(\theta)\cos(\phi)\,d(\theta) - r\sin(\theta)\sin(\phi)\,d(\phi)$$

$$perurbation\_of\_dx := 0$$

$$sigma\_y := \sin(\theta)\sin(\phi)\,d(r) + r\cos(\theta)\sin(\phi)\,d(\theta) + r\sin(\theta)\cos(\phi)\,d(\phi)$$

$$perurbation\_of\_dy := 0$$

$$sigma\_z := \cos(\theta)\,d(r) - r\sin(\theta)\,d(\theta) - d(\tau)\,aa\,r^2 + d(\tau)\,aa\,r^2\cos(\theta)^2 - d(\tau)\,aa0$$

$$perurbation\_of\_dz := (-aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0)\,d(\tau)$$

$$sigma\_t := -d(\phi)\,cc\,r^2 + d(\phi)\,cc\,r^2\cos(\theta)^2 - d(\phi)\,cc0 + \sqrt{\frac{r+bb}{r}}\,d(\tau)$$

$$perurbation\_of\_dt := -d(\phi)\,cc\,r^2 + d(\phi)\,cc\,r^2\cos(\theta)^2 - d(\phi)\,cc0 + \sqrt{\frac{r+bb}{r}}\,d(\tau) - d(\tau)$$

$$dsigma\_x := 0$$

$$dsigma\_y := 0$$

$$dsigma\_z := (-2\,aa\,r + 2\,aa\,r\cos(\theta)^2)\,(d(r)\,\&^\wedge d(\tau)) - 2\,aa\,r^2\cos(\theta)\sin(\theta)\,(d(\theta)\,\&^\wedge d(\tau))$$

$$dsigma\_t :=$$
$$-2\,cc\,r^2\cos(\theta)\sin(\theta)\,(d(\theta)\,\&^\wedge d(\phi)) + (-2\,cc\,r + 2\,cc\,r\cos(\theta)^2)\,(d(r)\,\&^\wedge d(\phi)) - \frac{1}{2}\frac{bb\,(d(r)\,\&^\wedge d(\tau))}{\sqrt{\frac{r+bb}{r}}\,r^2}$$

$$TOPTOR1 := 0$$

$$TOPTOR2 := 0$$

$$TOPTOR3 := -2\,r^2\sin(\theta)\,aa\,\&^\wedge(d(r),d(\theta),d(\tau))$$

$$TOPTOR4 := -2\sqrt{\frac{r+bb}{r}}\,cc\,r^2\cos(\theta)\sin(\theta)\,\&^\wedge(d(\tau),d(\theta),d(\phi))$$

$$-\frac{1}{2}\frac{(-5\,cc\,bb\,r^2 + 5\,cc\,\cos(\theta)^2\,bb\,r^2 - cc0\,bb - 4\,cc\,r^3 + 4\,cc\,r^3\,\cos(\theta)^2)\,\&^{\wedge}(\mathrm{d}(\phi),\mathrm{d}(r),\mathrm{d}(\tau))}{\sqrt{\dfrac{r+bb}{r}}\,r^2}$$

[>

For the given perturbations, the "twist" perturbation, aa, (in the absence of the other perturbations) makes the perturbed 1-form sigma_z non integrable. The 3-form of Topological Torsion 3 not is zero. The Affine torsion is not zero.

The "metric" pertubation, bb, (in the absence of the other perturbations) makes the perturbed 1-form sigma_t not closed, but it admits an intgrating factor, and so can be made integrable.

The Topological Torsion4 is zero, but the affine torsion is not zero.

This result is an example that demonstrates that Topological Torsion is not the same as Afiine Torsion.

The "rotational pertubation", cc, (in the absence of the other perturbations) makes the perturbed 1-form sigma_t non integrable. The Topological Torsion4 is not zero, AND the affine torsion is not zero.

Enter Perturbation coefficients here. Either 0, or aaa:=aa, ccc:=cc, bbb:=bb, aaa0:=aa0,ccc0=cc0.

aa gives a P-Affine twist perturbation,

cc gives a W-affine rotation perturbation,

bb gives a metric perturbation,

cc0 gives a closed rotation (without vorticity) W-affine, and depends upon metric signature

aa0 gives a closed twist p-affine and depends upon the metric signature

ee is the signature (either 1 or -1) ,

>

THE CARTAN CONNECTION MATRIX

of 1-forms for various perturbations

Enter Perturbation coefficients here.

Enter all as 0, or any combination of aaa:=aa, bbb:=bb, ccc:=cc, aaa0:=aa0, or ccc0=cc0 eee=ee,

in the order aaa, bbb, ccc, aaa0,ccc0,eee

```
> 
************************************************
******
> `---------------- Perturbation Coefficients  ------------------`;` `;
aaa:=0;bbb:=0;ccc:=0;aaa0:=0;ccc0:=0;eee:=ee;`All perturbations OFF   No affine
torsion`;
> PERT(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- Perturbation Coefficients ------------------

$$aaa := 0$$
$$bbb := 0$$
$$ccc := 0$$
$$aaa0 := 0$$
$$ccc0 := 0$$
$$eee := ee$$

All perturbations OFF   No affine torsion

Basis Frame for given pertubations

$$
\begin{bmatrix}
\sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\
\sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\
\cos(\theta) & -r\sin(\theta) & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

---------- Perturbed Cartan Connection matrix of 1-forms for the given perturbations ----------

$$\begin{bmatrix} 0 & -\mathrm{d}(\theta)\,r & \mathrm{d}(\phi)\,r\,(-1+\cos(\theta)^2) & 0 \\ \dfrac{\mathrm{d}(\theta)}{r} & \dfrac{\mathrm{d}(r)}{r} & -\sin(\theta)\,\mathrm{d}(\phi)\cos(\theta) & 0 \\ \dfrac{\mathrm{d}(\phi)}{r} & \dfrac{\cos(\theta)\,\mathrm{d}(\phi)}{\sin(\theta)} & \dfrac{\mathrm{d}(r)}{r}+\dfrac{\cos(\theta)\,\mathrm{d}(\theta)}{\sin(\theta)} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

---------- Perturbed pullback metric for the given perturbations ----------

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & -r^2\,(-1+\cos(\theta)^2) & 0 \\ 0 & 0 & 0 & ee \end{bmatrix}$$

-----------Perturbed Cartan Vector of Affine Torsion 2-forms for the given perturbations ----------

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
> `----------------- Perturbation Coefficients  ------------------`;`  `;
aaa:=aa;bbb:=0;ccc:=0;aaa0:=0;ccc0:=0;eee:=ee;`Transitive P-Affine Basis Frame
with twist and Affine Torsion`;
> PERT(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- Perturbation Coefficients -------------------

$$aaa := aa$$
$$bbb := 0$$
$$ccc := 0$$
$$aaa0 := 0$$
$$ccc0 := 0$$
$$eee := ee$$

Transitive P-Affine Basis Frame with twist and Affine Torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & -aa\,r^2+aa\,r^2\cos(\theta)^2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

---------- Perturbed Cartan Connection matrix of 1-forms for the given perturbations ----------

$$\begin{bmatrix} 0, & -d(\theta)\,r, & d(\phi)\,r\,(-1+\cos(\theta)^2), & (-2\cos(\theta)\,aa\,r+2\cos(\theta)^3\,aa\,r)\,d(r)-2\cos(\theta)^2\,aa\,r^2\sin(\theta)\,d(\theta) \\ \dfrac{d(\theta)}{r}, & \dfrac{d(r)}{r}, & -\sin(\theta)\,d(\phi)\cos(\theta), & (2\sin(\theta)\,aa-2\sin(\theta)\,aa\cos(\theta)^2)\,d(r)+2\sin(\theta)^2\,aa\,r\cos(\theta)\,d(\theta) \\ \dfrac{d(\phi)}{r}, & \dfrac{\cos(\theta)\,d(\phi)}{\sin(\theta)}, & \dfrac{d(r)}{r}+\dfrac{\cos(\theta)\,d(\theta)}{\sin(\theta)}, & 0 \\ 0, & 0, & 0, & 0 \end{bmatrix}$$

---------- *Perturbed pullback metric for the given perturbations* ----------

$$\begin{bmatrix} 1, & 0, & 0, & -\cos(\theta)\,aa\,r^2+\cos(\theta)^3\,aa\,r^2 \\ 0, & r^2, & 0, & -r^3\sin(\theta)\,aa\,(-1+\cos(\theta)^2) \\ 0, & 0, & -r^2\,(-1+\cos(\theta)^2), & 0 \\ -\cos(\theta)\,aa\,r^2+\cos(\theta)^3\,aa\,r^2, & -r^3\sin(\theta)\,aa\,(-1+\cos(\theta)^2), & 0, & aa^2\,r^4-2\,aa^2\,r^4\cos(\theta)^2+aa^2\,r^4\cos(\theta)^4+ee \end{bmatrix}$$

----------*Perturbed Cartan Vector of Affine Torsion 2-forms for the given perturbations* ----------

$$\begin{bmatrix} (-2\cos(\theta)\,aa\,r+2\cos(\theta)^3\,aa\,r)\,(d(r)\,\&\wedge\,d(\tau))-2\cos(\theta)^2\,aa\,r^2\sin(\theta)\,(d(\theta)\,\&\wedge\,d(\tau)) \\ (2\sin(\theta)\,aa-2\sin(\theta)\,aa\cos(\theta)^2)\,(d(r)\,\&\wedge\,d(\tau))+(2\cos(\theta)\,aa\,r-2\cos(\theta)^3\,aa\,r)\,(d(\theta)\,\&\wedge\,d(\tau)) \\ 0 \\ 0 \end{bmatrix}$$

```
> `---------------- Perturbation Coefficients -------------------`;`  `;
aaa:=0;bbb:=0;ccc:=cc;aaa0:=0;ccc0:=0;eee:=ee;`Intransitive W-Affine Basis Frame
with vorticity and affine torsion`;
> PERT(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- *Perturbation Coefficients* -------------------

$$aaa := 0$$
$$bbb := 0$$
$$ccc := cc$$
$$aaa0 := 0$$
$$ccc0 := 0$$
$$eee := ee$$

Intransitive W-Affine Basis Frame with vorticity and affine torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & 0 \\ 0 & 0 & -cc\,r^2+cc\,r^2\cos(\theta)^2 & 1 \end{bmatrix}$$

---------- *Perturbed Cartan Connection matrix of 1-forms for the given perturbations* ----------

$$
\begin{bmatrix}
0 & -d(\theta)\,r & d(\phi)\,r\,(-1+\cos(\theta)^2) & 0 \\
\dfrac{d(\theta)}{r} & \dfrac{d(r)}{r} & -\sin(\theta)\,d(\phi)\,\cos(\theta) & 0 \\
\dfrac{d(\phi)}{r} & \dfrac{\cos(\theta)\,d(\phi)}{\sin(\theta)} & \dfrac{d(r)}{r}+\dfrac{\cos(\theta)\,d(\theta)}{\sin(\theta)} & 0 \\
-d(\phi)\,cc\,r\,(-1+\cos(\theta)^2) & \cos(\theta)\,d(\phi)\,cc\,r^2\sin(\theta) & -\sin(\theta)^2\,d(r)\,cc\,r-cc\,r^2\cos(\theta)\sin(\theta)\,d(\theta) & 0
\end{bmatrix}
$$

---------- Perturbed pullback metric for the given perturbations ----------

$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & r^2 & 0 & 0 \\
0 & 0 & r^2\,(1-\cos(\theta)^2+ee\,cc^2\,r^2-2\,ee\,cc^2\,r^2\cos(\theta)^2+ee\,cc^2\,r^2\cos(\theta)^4) & -cc\,r^2\,ee+cc\,r^2\,ee\cos(\theta)^2 \\
0 & 0 & -cc\,r^2\,ee+cc\,r^2\,ee\cos(\theta)^2 & ee
\end{bmatrix}
$$

----------Perturbed Cartan Vector of Affine Torsion 2-forms for the given perturbations ----------

$$
\begin{bmatrix}
0 \\
0 \\
0 \\
2\,cc\,r^2\cos(\theta)\sin(\theta)\,(d(\phi)\,\&^\wedge\,d(\theta))+(2\,cc\,r-2\,cc\,r\cos(\theta)^2)\,(d(\phi)\,\&^\wedge\,d(r))
\end{bmatrix}
$$

```
> `---------------- Perturbation Coefficients  ------------------`;`   `;
aaa:=0;bbb:=bb;ccc:=0;aaa0:=0;ccc0:=0;eee:=ee;`Metric perturbation only. Non zero
affine torsion 10 parameter group`;
> PERT(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- Perturbation Coefficients ------------------

$$aaa := 0$$
$$bbb := bb$$
$$ccc := 0$$
$$aaa0 := 0$$
$$ccc0 := 0$$
$$eee := ee$$

Metric perturbation only. Non zero affine torsion 10 parameter group

Basis Frame for given pertubations

$$
\begin{bmatrix}
\sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\
\sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\
\cos(\theta) & -r\sin(\theta) & 0 & 0 \\
0 & 0 & 0 & \sqrt{1+\dfrac{bb}{r}}
\end{bmatrix}
$$

---------- Perturbed Cartan Connection matrix of 1-forms for the given perturbations ----------

$$\begin{bmatrix} 0 & -d(\theta)\,r & d(\phi)\,r\,(-1+\cos(\theta)^2) & 0 \\[2mm] \dfrac{d(\theta)}{r} & \dfrac{d(r)}{r} & -\sin(\theta)\,d(\phi)\,\cos(\theta) & 0 \\[2mm] \dfrac{d(\phi)}{r} & \dfrac{\cos(\theta)\,d(\phi)}{\sin(\theta)} & \dfrac{d(r)}{r}+\dfrac{\cos(\theta)\,d(\theta)}{\sin(\theta)} & 0 \\[2mm] 0 & 0 & 0 & -\dfrac{1}{2}\dfrac{bb\,d(r)}{(r+bb)\,r} \end{bmatrix}$$

---------- Perturbed pullback metric for the given perturbations ----------

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & -r^2\,(-1+\cos(\theta)^2) & 0 \\ 0 & 0 & 0 & \dfrac{(r+bb)\,ee}{r} \end{bmatrix}$$

----------Perturbed Cartan Vector of Affine Torsion 2-forms for the given perturbations ----------

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ -\dfrac{1}{2}\dfrac{bb\,(d(r)\,\&{}^{\wedge}\,d(\tau))}{(r+bb)\,r} \end{bmatrix}$$

```
> `----------------- Perturbation Coefficients -------------------`;`  `;
aaa:=0;bbb:=0;ccc:=0;aaa0:=aa0;ccc0:=0;eee:=ee;`Closed Twist perturbation,
P-Affine Frame No affine Torsion`;
> PERT(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- Perturbation Coefficients ------------------

$$aaa := 0$$
$$bbb := 0$$
$$ccc := 0$$
$$aaa0 := aa0$$
$$ccc0 := 0$$
$$eee := ee$$

Closed Twist perturbation, P-Affine Frame No affine Torsion

Basis Frame for given perturbations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & -aa0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

---------- Perturbed Cartan Connection matrix of 1-forms for the given perturbations ----------

$$\begin{bmatrix} 0 & -\mathrm{d}(\theta)\,r & \mathrm{d}(\phi)\,r\,(-1+\cos(\theta)^2) & 0 \\ \dfrac{\mathrm{d}(\theta)}{r} & \dfrac{\mathrm{d}(r)}{r} & -\sin(\theta)\,\mathrm{d}(\phi)\cos(\theta) & 0 \\ \dfrac{\mathrm{d}(\phi)}{r} & \dfrac{\cos(\theta)\,\mathrm{d}(\phi)}{\sin(\theta)} & \dfrac{\mathrm{d}(r)}{r}+\dfrac{\cos(\theta)\,\mathrm{d}(\theta)}{\sin(\theta)} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

---------- Perturbed pullback metric for the given perturbations ----------

$$\begin{bmatrix} 1 & 0 & 0 & -\cos(\theta)\,aa0 \\ 0 & r^2 & 0 & r\sin(\theta)\,aa0 \\ 0 & 0 & -r^2\,(-1+\cos(\theta)^2) & 0 \\ -\cos(\theta)\,aa0 & r\sin(\theta)\,aa0 & 0 & aa0^2+ee \end{bmatrix}$$

----------Perturbed Cartan Vector of Affine Torsion 2-forms for the given perturbations ----------

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
> `----------------- Perturbation Coefficients  ------------------`;`   `;
aaa:=0;bbb:=0;ccc:=0;aaa0:=0;ccc0:=cc0;eee:=ee;`Closed Rotation pertubation,
W-Affine Frame No Affine Torsion`;
> PERT(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- Perturbation Coefficients -------------------

$$aaa := 0$$
$$bbb := 0$$
$$ccc := 0$$
$$aaa0 := 0$$
$$ccc0 := cc0$$
$$eee := ee$$

Closed Rotation pertubation, W-Affine Frame No Affine Torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & 0 \\ 0 & 0 & -cc0 & 1 \end{bmatrix}$$

---------- Perturbed Cartan Connection matrix of 1-forms for the given perturbations ----------

$$\begin{bmatrix} 0 & -d(\theta)\,r & d(\phi)\,r\,(-1+\cos(\theta)^2) & 0 \\[6pt] \dfrac{d(\theta)}{r} & \dfrac{d(r)}{r} & -\sin(\theta)\,d(\phi)\,\cos(\theta) & 0 \\[6pt] \dfrac{d(\phi)}{r} & \dfrac{\cos(\theta)\,d(\phi)}{\sin(\theta)} & \dfrac{d(r)}{r}+\dfrac{\cos(\theta)\,d(\theta)}{\sin(\theta)} & 0 \\[6pt] \dfrac{d(\phi)\,cc0}{r} & \dfrac{\cos(\theta)\,d(\phi)\,cc0}{\sin(\theta)} & \dfrac{cc0\,d(r)}{r}+\dfrac{cc0\,\cos(\theta)\,d(\theta)}{\sin(\theta)} & 0 \end{bmatrix}$$

---------- *Perturbed pullback metric for the given perturbations* ----------

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2 - r^2\cos(\theta)^2 + ee\,cc0^2 & -cc0\,ee \\ 0 & 0 & -cc0\,ee & ee \end{bmatrix}$$

----------*Perturbed Cartan Vector of Affine Torsion 2-forms for the given perturbations* ----------

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
> `---------------- Perturbation Coefficients  ------------------`;`   `;
aaa:=0;bbb:=bb;ccc:=0;aaa0:=aa0;ccc0:=0;eee:=ee;`Closed Twist with metric,
P-Affine Frame, Non Zero Affine Torsion`;
> PERT(aaa,bbb,ccc,aaa0,ccc0,eee);
```
---------------- *Perturbation Coefficients* ------------------

$$aaa := 0$$
$$bbb := bb$$
$$ccc := 0$$
$$aaa0 := aa0$$
$$ccc0 := 0$$
$$eee := ee$$

Closed Twist with metric, P-Affine Frame, Non Zero Affine Torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & -aa0 \\ 0 & 0 & 0 & \sqrt{1+\dfrac{bb}{r}} \end{bmatrix}$$

---------- *Perturbed Cartan Connection matrix of 1-forms for the given perturbations* ----------

$$\begin{bmatrix} 0 & -\mathrm{d}(\theta)\,r & \mathrm{d}(\phi)\,r\,(-1+\cos(\theta)^2) & -\dfrac{1}{2}\dfrac{\cos(\theta)\,\mathrm{d}(r)\,bb\,aa0}{(r+bb)\,r} \\[2mm] \dfrac{\mathrm{d}(\theta)}{r} & \dfrac{\mathrm{d}(r)}{r} & -\sin(\theta)\,\mathrm{d}(\phi)\cos(\theta) & \dfrac{1}{2}\dfrac{\sin(\theta)\,\mathrm{d}(r)\,bb\,aa0}{(r+bb)\,r^2} \\[2mm] \dfrac{\mathrm{d}(\phi)}{r} & \dfrac{\cos(\theta)\,\mathrm{d}(\phi)}{\sin(\theta)} & \dfrac{\mathrm{d}(r)}{r}+\dfrac{\cos(\theta)\,\mathrm{d}(\theta)}{\sin(\theta)} & 0 \\[2mm] 0 & 0 & 0 & -\dfrac{1}{2}\dfrac{bb\,\mathrm{d}(r)}{(r+bb)\,r} \end{bmatrix}$$

---------- Perturbed pullback metric for the given perturbations ----------

$$\begin{bmatrix} 1 & 0 & 0 & -\cos(\theta)\,aa0 \\ 0 & r^2 & 0 & r\sin(\theta)\,aa0 \\ 0 & 0 & -r^2\,(-1+\cos(\theta)^2) & 0 \\ -\cos(\theta)\,aa0 & r\sin(\theta)\,aa0 & 0 & \dfrac{r\,aa0^2+ee\,r+ee\,bb}{r} \end{bmatrix}$$

-----------Perturbed Cartan Vector of Affine Torsion 2-forms for the given perturbations ----------

$$\begin{bmatrix} -\dfrac{1}{2}\dfrac{\cos(\theta)\,bb\,aa0\,(\mathrm{d}(r)\,\&^\wedge\,\mathrm{d}(\tau))}{(r+bb)\,r} \\[2mm] \dfrac{1}{2}\dfrac{\sin(\theta)\,bb\,aa0\,(\mathrm{d}(r)\,\&^\wedge\,\mathrm{d}(\tau))}{(r+bb)\,r^2} \\[2mm] 0 \\[2mm] -\dfrac{1}{2}\dfrac{bb\,(\mathrm{d}(r)\,\&^\wedge\,\mathrm{d}(\tau))}{(r+bb)\,r} \end{bmatrix}$$

```
> `---------------- Perturbation Coefficients  ------------------`;`   `;
aaa:=0;bbb:=bb;ccc:=0;aaa0:=0;ccc0:=cc0;eee:=ee;`Closed circulation with metric
pertubation, W-Affine Frame, Non zero Affine Torsion`;
> PERT(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- Perturbation Coefficients ------------------

$$aaa := 0$$
$$bbb := bb$$
$$ccc := 0$$
$$aaa0 := 0$$
$$ccc0 := cc0$$
$$eee := ee$$

Closed circulation with metric pertubation, W-Affine Frame, Non zero Affine Torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & 0 \\ 0 & 0 & -cc0 & \sqrt{1+\dfrac{bb}{r}} \end{bmatrix}$$

---------- *Perturbed Cartan Connection matrix of 1-forms for the given perturbations* ----------

$$\begin{bmatrix} 0 & -\mathrm{d}(\theta)\,r & \mathrm{d}(\phi)\,r\,(-1+\cos(\theta)^2) & 0 \\ \dfrac{\mathrm{d}(\theta)}{r} & \dfrac{\mathrm{d}(r)}{r} & -\sin(\theta)\,\mathrm{d}(\phi)\cos(\theta) & 0 \\ \dfrac{\mathrm{d}(\phi)}{r} & \dfrac{\cos(\theta)\,\mathrm{d}(\phi)}{\sin(\theta)} & \dfrac{\mathrm{d}(r)}{r}+\dfrac{\cos(\theta)\,\mathrm{d}(\theta)}{\sin(\theta)} & 0 \\ \dfrac{\mathrm{d}(\phi)\,cc0}{r\sqrt{\dfrac{r+bb}{r}}} & \dfrac{\cos(\theta)\,\mathrm{d}(\phi)\,cc0}{\sin(\theta)\sqrt{\dfrac{r+bb}{r}}} & \dfrac{\mathrm{d}(r)\,cc0}{\sqrt{\dfrac{r+bb}{r}}\,r}+\dfrac{\cos(\theta)\,\mathrm{d}(\theta)\,cc0}{\sqrt{\dfrac{r+bb}{r}}\,\sin(\theta)} & -\dfrac{1}{2}\dfrac{bb\,\mathrm{d}(r)}{(r+bb)\,r} \end{bmatrix}$$

---------- *Perturbed pullback metric for the given perturbations* ----------

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2-r^2\cos(\theta)^2+ee\,cc0^2 & -\sqrt{\dfrac{r+bb}{r}}\,ee\,cc0 \\ 0 & 0 & -\sqrt{\dfrac{r+bb}{r}}\,ee\,cc0 & \dfrac{(r+bb)\,ee}{r} \end{bmatrix}$$

----------*Perturbed Cartan Vector of Affine Torsion 2-forms for the given perturbations* ----------

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{2}\dfrac{bb\,(\mathrm{d}(r)\,\&^{\wedge}\,\mathrm{d}(\tau))\sin(\theta)^2}{(-bb+bb\cos(\theta)^2-r+r\cos(\theta)^2)\,r} \end{bmatrix}$$

```
> `----------------- Perturbation Coefficients  ------------------`;`   `;
aaa:=aa;bbb:=0;ccc:=0;aaa0:=0;ccc0:=0;eee:=ee;`Twist with metric pertubation,
P-Affine Frame, Non Zero Affine Torsion`;
> PERT(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- *Perturbation Coefficients* -------------------

$$aaa := aa$$
$$bbb := 0$$
$$ccc := 0$$
$$aaa0 := 0$$
$$ccc0 := 0$$
$$eee := ee$$

Twist with metric pertubation, P-Affine Frame, Non Zero Affine Torsion

Basis Frame for given pertubations

$$
\begin{bmatrix}
\sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\
\sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\
\cos(\theta) & -r\sin(\theta) & 0 & -aa\,r^2 + aa\,r^2\cos(\theta)^2 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

---------- *Perturbed Cartan Connection matrix of 1-forms for the given perturbations* ----------

$$
\begin{bmatrix}
0, & -d(\theta)\,r, & d(\phi)\,r\,(-1+\cos(\theta)^2), & (-2\cos(\theta)\,aa\,r + 2\cos(\theta)^3\,aa\,r)\,d(r) - 2\cos(\theta)^2\,aa\,r^2\sin(\theta)\,d(\theta) \\
\dfrac{d(\theta)}{r}, & \dfrac{d(r)}{r}, & -\sin(\theta)\,d(\phi)\cos(\theta), & (2\sin(\theta)\,aa - 2\sin(\theta)\,aa\cos(\theta)^2)\,d(r) + 2\sin(\theta)^2\,aa\,r\cos(\theta)\,d(\theta) \\
\dfrac{d(\phi)}{r}, & \dfrac{\cos(\theta)\,d(\phi)}{\sin(\theta)}, & \dfrac{d(r)}{r} + \dfrac{\cos(\theta)\,d(\theta)}{\sin(\theta)}, & 0 \\
0, & 0, & 0, & 0
\end{bmatrix}
$$

---------- *Perturbed pullback metric for the given perturbations* ----------

$$
\begin{bmatrix}
1, & 0, & 0, & -\cos(\theta)\,aa\,r^2 + \cos(\theta)^3\,aa\,r^2 \\
0, & r^2, & 0, & -r^3\sin(\theta)\,aa\,(-1+\cos(\theta)^2) \\
0, & 0, & -r^2(-1+\cos(\theta)^2), & 0 \\
-\cos(\theta)\,aa\,r^2 + \cos(\theta)^3\,aa\,r^2, & -r^3\sin(\theta)\,aa\,(-1+\cos(\theta)^2), & 0, & aa^2\,r^4 - 2\,aa^2\,r^4\cos(\theta)^2 + aa^2\,r^4\cos(\theta)^4 + ee
\end{bmatrix}
$$

-----------*Perturbed Cartan Vector of Affine Torsion 2-forms for the given perturbations* ----------

$$
\begin{bmatrix}
(-2\cos(\theta)\,aa\,r + 2\cos(\theta)^3\,aa\,r)\,(d(r)\,\&^\wedge\,d(\tau)) - 2\cos(\theta)^2\,aa\,r^2\sin(\theta)\,(d(\theta)\,\&^\wedge\,d(\tau)) \\
(2\sin(\theta)\,aa - 2\sin(\theta)\,aa\cos(\theta)^2)\,(d(r)\,\&^\wedge\,d(\tau)) + (2\cos(\theta)\,aa\,r - 2\cos(\theta)^3\,aa\,r)\,(d(\theta)\,\&^\wedge\,d(\tau)) \\
0 \\
0
\end{bmatrix}
$$

> `---------------- Perturbation Coefficients  ------------------`;` `;
aaa:=0;bbb:=bb;ccc:=cc;aaa0:=0;ccc0:=0;eee:=ee;`Rotation with metric pertubation, W-Affine Frame, Non Zero Affine Torsion`;
> PERT(aaa,bbb,ccc,aaa0,ccc0,eee);

---------------- *Perturbation Coefficients* ------------------

$$aaa := 0$$
$$bbb := bb$$
$$ccc := cc$$
$$aaa0 := 0$$
$$ccc0 := 0$$
$$eee := ee$$

Rotation with metric pertubation, W-Affine Frame, Non Zero Affine Torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & 0 \\ 0 & 0 & -cc\,r^2 + cc\,r^2\cos(\theta)^2 & \sqrt{1+\dfrac{bb}{r}} \end{bmatrix}$$

---------- Perturbed Cartan Connection matrix of 1-forms for the given perturbations ----------

$[\,0\,,\,-d(\theta)\,r\,,\,d(\phi)\,r\,(-1+\cos(\theta)^2)\,,\,0\,]$

$\left[\dfrac{d(\theta)}{r}\,,\,\dfrac{d(r)}{r}\,,\,-\sin(\theta)\,d(\phi)\cos(\theta)\,,\,0\right]$

$\left[\dfrac{d(\phi)}{r}\,,\,\dfrac{\cos(\theta)\,d(\phi)}{\sin(\theta)}\,,\,\dfrac{d(r)}{r}+\dfrac{\cos(\theta)\,d(\theta)}{\sin(\theta)}\,,\,0\right]$

$$\left[\,-\dfrac{d(\phi)\,cc\,r\,(-1+\cos(\theta)^2)}{\sqrt{\dfrac{r+bb}{r}}}\,,\,\dfrac{\sqrt{\dfrac{r+bb}{r}}\,\sin(\theta)\,r^3\,cc\,d(\phi)\cos(\theta)}{r+bb}\,,\right.$$

$$\left.-\dfrac{d(r)\sin(\theta)^2\,cc\,r^2\sqrt{\dfrac{r+bb}{r}}}{r+bb}-\dfrac{\cos(\theta)\,d(\theta)\,r^3\,cc\,\sin(\theta)\sqrt{\dfrac{r+bb}{r}}}{r+bb}\,,\,-\dfrac{1}{2}\dfrac{bb\,d(r)}{(r+bb)\,r}\right]$$

---------- Perturbed pullback metric for the given perturbations ----------

$[\,1\,,\,0\,,\,0\,,\,0\,]$

$\quad[\,0\,,\,r^2\,,\,0\,,\,0\,]$

$$\left[\,0\,,\,0\,,\,r^2\,(1-\cos(\theta)^2+ee\,cc^2\,r^2-2\,ee\,cc^2\,r^2\cos(\theta)^2+ee\,cc^2\,r^2\cos(\theta)^4)\,,\right.$$

$$\left.-\sqrt{\dfrac{r+bb}{r}}\,ee\,cc\,r^2+\sqrt{\dfrac{r+bb}{r}}\,ee\,cc\,r^2\cos(\theta)^2\right]$$

$$\left[\,0\,,\,0\,,\,-\sqrt{\dfrac{r+bb}{r}}\,ee\,cc\,r^2+\sqrt{\dfrac{r+bb}{r}}\,ee\,cc\,r^2\cos(\theta)^2\,,\,\dfrac{(r+bb)\,ee}{r}\right]$$

----------Perturbed Cartan Vector of Affine Torsion 2-forms for the given perturbations ----------

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 2\,\dfrac{r^2\sin(\theta)\,cc\,\cos(\theta)\,(d(\phi)\,\&^\wedge\,d(\theta))}{\sqrt{\dfrac{r+bb}{r}}}-\dfrac{2\,cc\,r\,(-1+\cos(\theta)^2)\,(d(\phi)\,\&^\wedge\,d(r))}{\sqrt{\dfrac{r+bb}{r}}}-\dfrac{1}{2}\dfrac{bb\,(d(r)\,\&^\wedge\,d(\tau))}{(r+bb)\,r} \end{bmatrix}$$

> `---------------- Perturbation Coefficients  ------------------`;` `;

```
aaa:=aa;bbb:=bb;ccc:=cc;aaa0:=aa0;ccc0:=cc0;eee:=ee;`All Perturbations On, Non
Zero Affine Torsion`;
> PERT(aaa,bbb,ccc,aaa0,ccc0,eee);
```

----------------- *Perturbation Coefficients* -------------------

$$aaa := aa$$
$$bbb := bb$$
$$ccc := cc$$
$$aaa0 := aa0$$
$$ccc0 := cc0$$
$$eee := ee$$

All Perturbations On, Non Zero Affine Torsion

Basis Frame for given pertubations

$$
\begin{bmatrix}
\sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\
\sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\
\cos(\theta) & -r\sin(\theta) & 0 & -aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0 \\
0 & 0 & -cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0 & \sqrt{1+\dfrac{bb}{r}}
\end{bmatrix}
$$

--------- *Perturbed Cartan Connection matrix of 1-forms for the given perturbations* ----------

$$
\left[d(\phi)\sqrt{\frac{r+bb}{r}}\,(aa0\,cc0 + r^2\,aa0\,cc + r^2\,aa\,cc0 + r^4\,aa\,cc - r^2\cos(\theta)^2\,aa\,cc0 + r^4\cos(\theta)^4\,aa\,cc \right.
$$

$$
- 2r^4\cos(\theta)^2\,aa\,cc - r^2\cos(\theta)^2\,aa0\,cc)\cos(\theta)/(r+bb)\,,\, -r\sin(\theta)\sqrt{\frac{r+bb}{r}}\cos(\theta)^2\,(aa0\,cc0 + r^2\,aa0\,cc
$$

$$
+ r^2\,aa\,cc0 + r^4\,aa\,cc - r^2\cos(\theta)^2\,aa\,cc0 + r^4\cos(\theta)^4\,aa\,cc - 2r^4\cos(\theta)^2\,aa\,cc - r^2\cos(\theta)^2\,aa0\,cc)\,d(\phi)\Big/\Big(
$$

$$
(-1+\cos(\theta)^2)(r+bb)\Big) - d(\theta)\,r\,,\, -r\sin(\theta)^2\,d(\phi) + \sin(\theta)^2\sqrt{\frac{r+bb}{r}}\cos(\theta)(r^2\,aa0\,cc - r^2\cos(\theta)^2\,aa0\,cc
$$

$$
- aa0\,cc0 + r^4\,aa\,cc - r^2\,aa\,cc0 + r^2\cos(\theta)^2\,aa\,cc0 - 2r^4\cos(\theta)^2\,aa\,cc + r^4\cos(\theta)^4\,aa\,cc)\,d(r)\Big/\Big(
$$

$$
-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2) + r\sin(\theta)\sqrt{\frac{r+bb}{r}}\cos(\theta)^2\,(r^2\,aa0\,cc - r^2\cos(\theta)^2\,aa0\,cc - aa0\,cc0 + r^4\,aa\,cc
$$

$$
- r^2\,aa\,cc0 + r^2\cos(\theta)^2\,aa\,cc0 - 2r^4\cos(\theta)^2\,aa\,cc + r^4\cos(\theta)^4\,aa\,cc)\,d(\theta)\Big/(-bb+bb\cos(\theta)^2 - r + r\cos(\theta)^2)\,,
$$

$$
\left.\frac{1}{2}\frac{\cos(\theta)(-4\,aa\,r^3 - 5\,bb\,aa\,r^2 + 4\,aa\,r^3\cos(\theta)^2 + 5\,bb\,aa\,r^2\cos(\theta)^2 - bb\,aa0)\,d(r)}{r(r+bb)} - 2\cos(\theta)^2\,aa\,r^2\sin(\theta)\,d(\theta) \right]
$$

$$
\left[-\sin(\theta)\sqrt{\frac{r+bb}{r}}\,(aa0\,cc0 + r^2\,aa0\,cc + r^2\,aa\,cc0 + r^4\,aa\,cc - r^2\cos(\theta)^2\,aa\,cc0 + r^4\cos(\theta)^4\,aa\,cc \right.
$$

$$
- 2r^4\cos(\theta)^2\,aa\,cc - r^2\cos(\theta)^2\,aa0\,cc)\,d(\phi)/(r(r+bb)) + \frac{d(\theta)}{r}\,,\, -d(\phi)\sqrt{\frac{r+bb}{r}}\,(aa0\,cc0 + r^2\,aa0\,cc
$$

$$
+ r^2\,aa\,cc0 + r^4\,aa\,cc - r^2\cos(\theta)^2\,aa\,cc0 + r^4\cos(\theta)^4\,aa\,cc - 2r^4\cos(\theta)^2\,aa\,cc - r^2\cos(\theta)^2\,aa0\,cc)\cos(\theta)/(
$$

$$
r+bb) + \frac{d(r)}{r}\,,\, -\sin(\theta)\,d(\phi)\cos(\theta) + \sin(\theta)(r^2\,aa0\,cc - r^2\cos(\theta)^2\,aa0\,cc - aa0\,cc0 + r^4\,aa\,cc - r^2\,aa\,cc0
$$

$+ r^2 \cos(\theta)^2 \, aa \, cc0 - 2 \, r^4 \cos(\theta)^2 \, aa \, cc + r^4 \cos(\theta)^4 \, aa \, cc) \, \mathrm{d}(r) \Big/ \left(\sqrt{\dfrac{r+bb}{r}} \, r^2 \right) + \cos(\theta) \, (r^2 \, aa0 \, cc$

$- r^2 \cos(\theta)^2 \, aa0 \, cc - aa0 \, cc0 + r^4 \, aa \, cc - r^2 \, aa \, cc0 + r^2 \cos(\theta)^2 \, aa \, cc0 - 2 \, r^4 \cos(\theta)^2 \, aa \, cc + r^4 \cos(\theta)^4 \, aa \, cc) \, \mathrm{d}(\theta)$

$\Big/ \left(r \sqrt{\dfrac{r+bb}{r}} \right),$

$-\dfrac{1}{2} \dfrac{\sin(\theta) \, (-4 \, aa \, r^3 - 5 \, bb \, aa \, r^2 + 4 \, aa \, r^3 \cos(\theta)^2 + 5 \, bb \, aa \, r^2 \cos(\theta)^2 - bb \, aa0) \, \mathrm{d}(r)}{(r+bb) \, r^2} + 2 \sin(\theta)^2 \, aa \, r \cos(\theta) \, \mathrm{d}(\theta)$

$\Bigg]$

$\left[\dfrac{\mathrm{d}(\phi)}{r} , \dfrac{\cos(\theta) \, \mathrm{d}(\phi)}{\sin(\theta)} , \dfrac{\mathrm{d}(r)}{r} + \dfrac{\cos(\theta) \, \mathrm{d}(\theta)}{\sin(\theta)} , 0 \right]$

$\Bigg[$

$-\dfrac{\mathrm{d}(\phi) \, (-cc \, r^2 + cc \, r^2 \cos(\theta)^2 - cc0)}{r \sqrt{\dfrac{r+bb}{r}}} , -\dfrac{\cos(\theta) \, \mathrm{d}(\phi) \, (-cc \, r^2 + cc \, r^2 \cos(\theta)^2 - cc0)}{\sin(\theta) \sqrt{\dfrac{r+bb}{r}}} ,$

$-\dfrac{\sqrt{\dfrac{r+bb}{r}} \, \sin(\theta)^2 \, (cc0 + cc \, r^2 \cos(\theta)^2 - cc \, r^2) \, \mathrm{d}(r)}{-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2}$

$-\dfrac{\sqrt{\dfrac{r+bb}{r}} \, r \cos(\theta) \sin(\theta) \, (cc0 + cc \, r^2 \cos(\theta)^2 - cc \, r^2) \, \mathrm{d}(\theta)}{-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2} , -\dfrac{1}{2} \dfrac{bb \, \mathrm{d}(r)}{(r+bb) \, r} \Bigg]$

---------- *Perturbed pullback metric for the given perturbations* ----------

$[1, 0, 0, -\cos(\theta) \, aa \, r^2 + \cos(\theta)^3 \, aa \, r^2 - \cos(\theta) \, aa0]$

$\quad [0, r^2, 0, -r \sin(\theta) \, (-aa \, r^2 + aa \, r^2 \cos(\theta)^2 - aa0)]$

$\quad \Bigg[0, 0,$

$r^2 - r^2 \cos(\theta)^2 + ee \, cc^2 \, r^4 - 2 \, ee \, cc^2 \, r^4 \cos(\theta)^2 + 2 \, ee \, cc \, r^2 \, cc0 + ee \, cc^2 \, r^4 \cos(\theta)^4 - 2 \, ee \, cc \, r^2 \cos(\theta)^2 \, cc0 + ee \, cc0^2 ,$

$-\sqrt{\dfrac{r+bb}{r}} \, ee \, cc \, r^2 + \sqrt{\dfrac{r+bb}{r}} \, ee \, cc \, r^2 \cos(\theta)^2 - \sqrt{\dfrac{r+bb}{r}} \, ee \, cc0 \Bigg]$

$\quad \Big[-\cos(\theta) \, aa \, r^2 + \cos(\theta)^3 \, aa \, r^2 - \cos(\theta) \, aa0 , -r \sin(\theta) \, (-aa \, r^2 + aa \, r^2 \cos(\theta)^2 - aa0) ,$

$-\sqrt{\dfrac{r+bb}{r}} \, ee \, cc \, r^2 + \sqrt{\dfrac{r+bb}{r}} \, ee \, cc \, r^2 \cos(\theta)^2 - \sqrt{\dfrac{r+bb}{r}} \, ee \, cc0 ,$

$\dfrac{aa^2 \, r^5 - 2 \, aa^2 \, r^5 \cos(\theta)^2 + 2 \, aa \, r^3 \, aa0 + aa^2 \, r^5 \cos(\theta)^4 - 2 \, aa \, r^3 \cos(\theta)^2 \, aa0 + r \, aa0^2 + ee \, r + ee \, bb}{r} \Big]$

----------*Perturbed Cartan Vector of Affine Torsion 2-forms for the given perturbations* ----------

$$\left[\begin{array}{c} 2\,\dfrac{(-aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0)\cos(\theta)^2\,r^3\sin(\theta)\sqrt{\dfrac{r+bb}{r}}\,cc\,(\mathrm{d}(\theta)\,\&^\wedge\,\mathrm{d}(\phi))}{r+bb} \\[3mm] +\,\dfrac{2\,(\cos(\theta)^4\,r^2\,aa - 2\,aa\,r^2\cos(\theta)^2 + aa0 + aa\,r^2 - \cos(\theta)^2\,aa0)\cos(\theta)\sqrt{\dfrac{r+bb}{r}}\,r^2\,cc\,(\mathrm{d}(\phi)\,\&^\wedge\,\mathrm{d}(r))}{r+bb} \\[3mm] +\,\dfrac{\dfrac{1}{2}\cos(\theta)(-4\,aa\,r^3 - 5\,bb\,aa\,r^2 + 4\,aa\,r^3\cos(\theta)^2 + 5\,bb\,aa\,r^2\cos(\theta)^2 - bb\,aa0)(\mathrm{d}(r)\,\&^\wedge\,\mathrm{d}(\tau))}{r\,(r+bb)} \\[3mm] -\,2\cos(\theta)^2\,aa\,r^2\sin(\theta)\,(\mathrm{d}(\theta)\,\&^\wedge\,\mathrm{d}(\tau)) \end{array}\right]$$

$$\left[\begin{array}{c} 2\,\dfrac{(\cos(\theta)^4\,r^2\,aa - 2\,aa\,r^2\cos(\theta)^2 + aa0 + aa\,r^2 - \cos(\theta)^2\,aa0)\,r\cos(\theta)\,cc\,(\mathrm{d}(\theta)\,\&^\wedge\,\mathrm{d}(\phi))}{\sqrt{\dfrac{r+bb}{r}}} \\[3mm] +\,\dfrac{2\,(\cos(\theta)^4\,r^2\,aa - 2\,aa\,r^2\cos(\theta)^2 + aa0 + aa\,r^2 - \cos(\theta)^2\,aa0)\sin(\theta)\,cc\,(\mathrm{d}(r)\,\&^\wedge\,\mathrm{d}(\phi))}{\sqrt{\dfrac{r+bb}{r}}} \\[3mm] -\,\dfrac{1}{2}\dfrac{\sin(\theta)(-4\,aa\,r^3 - 5\,bb\,aa\,r^2 + 4\,aa\,r^3\cos(\theta)^2 + 5\,bb\,aa\,r^2\cos(\theta)^2 - bb\,aa0)(\mathrm{d}(r)\,\&^\wedge\,\mathrm{d}(\tau))}{(r+bb)\,r^2} \\[3mm] -\,2\,(-1+\cos(\theta)^2)\cos(\theta)\,aa\,r\,(\mathrm{d}(\theta)\,\&^\wedge\,\mathrm{d}(\tau)) \end{array}\right]$$

$$[0]$$

$$\left[\begin{array}{c} -2\,\dfrac{\sqrt{\dfrac{r+bb}{r}}\sin(\theta)\,r^3\,cc\cos(\theta)\,(\mathrm{d}(\theta)\,\&^\wedge\,\mathrm{d}(\phi))}{r+bb} - \dfrac{2\sin(\theta)^2\,cc\,r^2\sqrt{\dfrac{r+bb}{r}}\,(\mathrm{d}(r)\,\&^\wedge\,\mathrm{d}(\phi))}{r+bb} \\[3mm] +\,\dfrac{\dfrac{1}{2}\,bb\,(\mathrm{d}(r)\,\&^\wedge\,\mathrm{d}(\tau))\sin(\theta)^2}{(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)\,r} \end{array}\right]$$

```
>
>
```

****

        ****

The Cartan Connection [C] coefficients, the Christoffel Coefficients [Gamma] and the Residue Cartan Connection coefficients [T] can be computed by tensor methods.

```
> dim:=4:coord:=[r,theta,phi,tau]:
```

```
> for i from 1 to dim do for j from 1 to dim do for k from 1 to dim do d1GG[i,j,k]
  := (diff(GG[i,j],coord[k])) od od od:
for b from 1 to dim do for a from 1 to dim do for k from 1 to dim do  ss:=0;for m
from 1 to dim do ss := ss+(d1GG[a,m,k]*FF[m,b]); CCCC[a,b,k]:=simplify(subs(-ss))
od od od od ;
> #for b from 1 to dim do for a from 1 to dim do for k from 1 to dim do if
  CCCC[a,b,k]=0 then else print(`Cartan_RIGHT`(a,b,k)=factor(CCCC[a,b,k])) fi od
  od od :
```

Compute the Affine Torsion coefficients

```
> for j from 1 to dim do for i from 1 to dim do for k from 1 to dim do  s :=
  (CCCC[i,j,k]-CCCC[i,k,j])/2; TTCCCC[i,j,k]:=s od od od   ;
> #for i from 1 to dim do for j from 1 to dim do for k from 1 to dim do if
  TTCCCC[i,j,k]=0 then else
  print(`CartanaffineTorsion`(i,k,j)=factor(TTCCCC[i,k,j])) fi od od od :

>
>
```

The metric on the final state induces a metric on the initial state via the congruent mapping [pullbackmetric] = [Frame_transpose][finalmetric][Frame]. For the example, the coefficient bb/r gives a measure of the assumed perturbation of the Euclidean metric on the final state.

Non Zero Christoffel Connection coefficients from the induced metric
Gamma2(i,j,k) index (1,-1,-1)
The algebra can be quite tedious.

```
> finalmetric := array([[1, 0, 0,0], [0,1, 0,0], [0, 0, 1,0],[0,0,0,ee]]):
> pullbackmetric:=simplify((innerprod(transpose(FF),finalmetric,FF))):

> dim:=4:coord:=[r,theta,phi,tau]:
> metric:= simplify(subs(aa=aa,bb=bb,cc=cc,(evalm(pullbackmetric)))):
  Metric_induced_by_congruent_pullback:=evalm(metric):

> metricinverse:=inverse(metric):
> for i from 1 to dim do for j from 1 to dim do for k from 1 to dim do
  d1gun[i,j,k] := (diff(metric[i,j],coord[k])) od od od:
>
> for i from 1 to dim do for j from i to dim do for k from 1 to dim do C1S[i,j,k]
  := 0  od od od; for i from 1 to dim do for j from 1 to dim do for k from 1 to
  dim do C1S[i,j,k] := 1/2*d1gun[i,k,j]+1/2*d1gun[j,k,i]-1/2*d1gun[i,j,k]  od od
  od;
>  for k from 1 to dim do for i from 1 to dim do for j from 1 to dim do ss := 0;
  for m to dim do ss := ss+metricinverse[k,m]*C1S[i,j,m] od; C2S[k,i,j] :=
  simplify(factor(ss),trig) od od od;
> #for i from 1 to dim do for j from 1 to dim do for k from 1 to dim do if
  C2S[i,j,k]=0 then  else print(`Christoffel_Gamma2`(i,j,k)=simplify(C2S[i,j,k]))
  fi od od od:
>
```

The Right Cartan coefficients can be decomposed into the sum of Christoffel Symbols plus of the Residue coefficients, T(i,j,k)

Right Cartan(i,jk) = ChristoffelGamma(i,jk) + T(i,jk)

If there is no difference between the Christoffel symbols and the Cartan connection symbols, then the T(i,j,k) are zero.

COMPUTE THE T(i,jk)

for various perturbations

```
> for i from 1 to dim do for j from 1 to dim do for k from 1 to dim do ss:=0; ss
  :=
  (subs(aa=aa,bb=bb,cc=cc,aa0=aa0,cc0=cc0,CCCC[i,j,k])-subs(aa=aa,bb=bb,cc=cc,aa0=
  aa0,cc0=cc0,C2S[i,j,k])); SHIPTR[i,j,k]:=simplify(ss)  od od od   ;
> #for i from 1 to dim do for j from 1 to dim do for k from 1 to dim do if
  subs(aa=aa,bb=bb,cc=cc,aa0=aa0,cc0=cc0,C2S[i,j,k])=0 and
  subs(aa=aa,bb=bb,cc=cc,aa0=aa0,cc0=cc0,CCCC[i,j,k])=0 then else
  print(`T`(i,j,k)=simplify(SHIPTR[i,j,k])) fi od od od :
>
>
>
> `----------------- Perturbation Coefficients  -------------------`;`   `;
aaa:=0;bbb:=0;ccc:=0;aaa0:=0;ccc0:=0;eee:=ee;`All perturbations OFF    No affine
torsion`;
> COEF(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- Perturbation Coefficients -------------------

$$aaa := 0$$
$$bbb := 0$$
$$ccc := 0$$
$$aaa0 := 0$$
$$ccc0 := 0$$
$$eee := ee$$

All perturbations OFF   No affine torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

--------- Non Zero Cartan Connection Coefficients {C} for the given perturbations ----------

$$\text{Cartan\_RIGHT}(2, 1, 2) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3, 1, 3) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(1, 2, 2) = -r$$

$$\text{Cartan\_RIGHT}(2, 2, 1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3, 2, 3) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(1, 3, 3) = r\,(-1 + \cos(\theta))\,(\cos(\theta) + 1)$$

$$\text{Cartan\_RIGHT}(2, 3, 3) = -\cos(\theta)\,\sin(\theta)$$

$$\text{Cartan\_RIGHT}(3, 3, 1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3, 3, 2) = \frac{\cos(\theta)}{\sin(\theta)}$$

--------- Non Zero Christoffel Connection Coefficients {Gamma} for the perturbations ----------

$$\text{Christoffel\_Gamma2}(1, 2, 2) = -r$$

$$\text{Christoffel\_Gamma2}(1, 3, 3) = -r + r\,\cos(\theta)^2$$

$$\text{Christoffel\_Gamma2}(2, 1, 2) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2, 2, 1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2, 3, 3) = -\cos(\theta)\,\sin(\theta)$$

$$\text{Christoffel\_Gamma2}(3, 1, 3) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 2, 3) = -\frac{\sin(\theta)\,\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3, 3, 1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 3, 2) = -\frac{\sin(\theta)\,\cos(\theta)}{-1 + \cos(\theta)^2}$$

---------- Non Zero Residue Connection Coefficients {T}= {C} - {Gamma} for the given perturbations ----------

--------- Non Zero Affine Torsion for the perturbations ----------

> `----------------- Perturbation Coefficients  -------------------`;` `;

```
aaa:=aa;bbb:=0;ccc:=0;aaa0:=0;ccc0:=0;eee:=ee;`Transitive P-Affine Basis Frame
with twist and Affine Torsion`;
> COEF(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- Perturbation Coefficients -------------------

$$aaa := aa$$
$$bbb := 0$$
$$ccc := 0$$
$$aaa0 := 0$$
$$ccc0 := 0$$
$$eee := ee$$

Transitive P-Affine Basis Frame with twist and Affine Torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & -aa\,r^2 + aa\,r^2\cos(\theta)^2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

---------- Non Zero Cartan Connection Coefficients {C} for the given perturbations ----------

$$\text{Cartan\_RIGHT}(2,1,2) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,1,3) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(1,2,2) = -r$$

$$\text{Cartan\_RIGHT}(2,2,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,2,3) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(1,3,3) = r\,(\cos(\theta)-1)\,(\cos(\theta)+1)$$

$$\text{Cartan\_RIGHT}(2,3,3) = -\cos(\theta)\sin(\theta)$$

$$\text{Cartan\_RIGHT}(3,3,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,3,2) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(1,4,1) = \frac{1}{2}\frac{\cos(\theta)\,(-4\,aa\,r^3 - 5\,bb\,aa\,r^2 + 4\,aa\,r^3\cos(\theta)^2 + 5\,bb\,aa\,r^2\cos(\theta)^2 - bb\,aa0)}{r\,(r+bb)}$$

$$\text{Cartan\_RIGHT}(1,4,2) = -2\cos(\theta)^2\,aa\,r^2\sin(\theta)$$

$$\text{Cartan\_RIGHT}(2,4,1) = -\frac{1}{2}\frac{\sin(\theta)\,(-4\,aa\,r^3 - 5\,bb\,aa\,r^2 + 4\,aa\,r^3\cos(\theta)^2 + 5\,bb\,aa\,r^2\cos(\theta)^2 - bb\,aa0)}{(r+bb)\,r^2}$$

$$\text{Cartan\_RIGHT}(2,4,2) = -2\cos(\theta)\,aa\,r\,(\cos(\theta)-1)\,(\cos(\theta)+1)$$

$$\text{Christoffel\_Gamma2}(1,1,1) = -2\,\frac{aa^2\,r^3\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\cos(\theta)^2}{ee}$$

$$\text{Christoffel\_Gamma2}(1,1,2) = \frac{(2\cos(\theta)^4 - 3\cos(\theta)^2 + 1)\,r^4\,aa^2\,\sin(\theta)\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(1,1,4) = -\frac{(\cos(\theta)^6 - 3\cos(\theta)^4 + 3\cos(\theta)^2 - 1)\,r^5\,aa^3\,\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(1,2,1) = \frac{(2\cos(\theta)^4 - 3\cos(\theta)^2 + 1)\,r^4\,aa^2\,\sin(\theta)\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(1,2,2) = \frac{r\,(2\,r^4\cos(\theta)^6\,aa^2 - 4\,aa^2\,r^4\cos(\theta)^4 + 2\,aa^2\,r^4\cos(\theta)^2 - ee)}{ee}$$

$$\text{Christoffel\_Gamma2}(1,2,4) = \frac{(r^4\cos(\theta)^6\,aa^2 - 2\,aa^2\,r^4\cos(\theta)^4 + aa^2\,r^4\cos(\theta)^2 - ee)\,r^2\,aa\,\sin(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(1,3,3) = -r + r\cos(\theta)^2$$

$$\text{Christoffel\_Gamma2}(1,4,1) = -\frac{(\cos(\theta)^6 - 3\cos(\theta)^4 + 3\cos(\theta)^2 - 1)\,r^5\,aa^3\,\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(1,4,2) = \frac{(aa^2\,r^4\cos(\theta)^6 - 2\,aa^2\,r^4\cos(\theta)^4 + aa^2\,r^4\cos(\theta)^2 - ee)\,r^2\,aa\,\sin(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(1,4,4) = -2\,r^3\,aa^2\cos(\theta)^4 + 4\,r^3\,aa^2\cos(\theta)^2 - 2\,r^3\,aa^2$$

$$\text{Christoffel\_Gamma2}(2,1,1) = 2\,\frac{aa^2\,r^2\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\sin(\theta)\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(2,1,2) = \frac{-aa^2\,r^4 + ee + 2\,aa^2\,r^4\cos(\theta)^6 - 5\,aa^2\,r^4\cos(\theta)^4 + 4\,aa^2\,r^4\cos(\theta)^2}{r\,ee}$$

$$\text{Christoffel\_Gamma2}(2,1,4) = \frac{(aa^2\,r^4\cos(\theta)^6 - 3\,aa^2\,r^4\cos(\theta)^4 + 3\,aa^2\,r^4\cos(\theta)^2 + ee - aa^2\,r^4)\,aa\,\sin(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(2,2,1) = \frac{-aa^2\,r^4 + ee + 2\,aa^2\,r^4\cos(\theta)^6 - 5\,aa^2\,r^4\cos(\theta)^4 + 4\,aa^2\,r^4\cos(\theta)^2}{r\,ee}$$

$$\text{Christoffel\_Gamma2}(2,2,2) = -2\,\frac{aa^2\,r^4\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\sin(\theta)\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(2,2,4) = \frac{(\cos(\theta)^6 - 3\cos(\theta)^4 + 3\cos(\theta)^2 - 1)\,r^5\,aa^3\,\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(2,3,3) = -\cos(\theta)\sin(\theta)$$

$$\text{Christoffel\_Gamma2}(2,4,1) = \frac{(aa^2\,r^4\cos(\theta)^6 - 3\,aa^2\,r^4\cos(\theta)^4 + 3\,aa^2\,r^4\cos(\theta)^2 + ee - aa^2\,r^4)\,aa\,\sin(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(2,4,2) = \frac{(\cos(\theta)^6 - 3\cos(\theta)^4 + 3\cos(\theta)^2 - 1)\,r^5\,aa^3\,\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(2,4,4) = 2\,aa^2\,r^2\,(-1 + \cos(\theta)^2)\sin(\theta)\cos(\theta)$$

$$\text{Christoffel\_Gamma2}(3,1,3) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3,2,3) = -\frac{\sin(\theta)\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3, 3, 1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 3, 2) = -\frac{\sin(\theta)\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(4, 1, 1) = 2\,\frac{aa\cos(\theta)\,r\,(-1 + \cos(\theta)^2)}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 1, 2) = -\frac{(2\cos(\theta)^2 - 1)\,r^2\,aa\,\sin(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 1, 4) = \frac{(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\,r^3\,aa^2}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 2, 1) = -\frac{(2\cos(\theta)^2 - 1)\,r^2\,aa\,\sin(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 2, 2) = -2\,\frac{aa\,r^3\,(-1 + \cos(\theta)^2)\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 2, 4) = -\frac{(-1 + \cos(\theta)^2)\,r^4\,aa^2\,\sin(\theta)\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 4, 1) = \frac{(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\,r^3\,aa^2}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 4, 2) = -\frac{(-1 + \cos(\theta)^2)\,r^4\,aa^2\,\sin(\theta)\cos(\theta)}{ee}$$

---------- Non Zero Residue Connection Coefficients {T}= {C} - {Gamma} for the given perturbations ----------

$$T(1, 1, 1) = 2\,\frac{aa^2\,r^3\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\cos(\theta)^2}{ee}$$

$$T(1, 1, 2) = -\frac{(1 - 3\cos(\theta)^2 + 2\cos(\theta)^4)\,r^4\,aa^2\,\sin(\theta)\cos(\theta)}{ee}$$

$$T(1, 1, 4) = \frac{(-1 + \cos(\theta)^6 - 3\cos(\theta)^4 + 3\cos(\theta)^2)\,r^5\,aa^3\,\cos(\theta)}{ee}$$

$$T(1, 2, 1) = -\frac{(1 - 3\cos(\theta)^2 + 2\cos(\theta)^4)\,r^4\,aa^2\,\sin(\theta)\cos(\theta)}{ee}$$

$$T(1, 2, 2) = -2\,\frac{r^5\,\cos(\theta)^2\,aa^2\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)}{ee}$$

$$T(1, 2, 4) = -\frac{(aa^2\,r^4\cos(\theta)^6 - 2\,aa^2\,r^4\cos(\theta)^4 + aa^2\,r^4\cos(\theta)^2 - ee)\,r^2\,aa\,\sin(\theta)}{ee}$$

$$T(1, 4, 1) = \frac{\cos(\theta)\,(\cos(\theta)^6\,aa^2\,r^4 - 3\,aa^2\,r^4\cos(\theta)^4 + 2\cos(\theta)^2\,ee + 3\,aa^2\,r^4\cos(\theta)^2 - 2\,ee - aa^2\,r^4)\,aa\,r}{ee}$$

$$T(1, 4, 2) = -\frac{(\cos(\theta)^6\,aa^2\,r^4 - 2\,aa^2\,r^4\cos(\theta)^4 + aa^2\,r^4\cos(\theta)^2 + 2\cos(\theta)^2\,ee - ee)\,r^2\,aa\,\sin(\theta)}{ee}$$

$$T(1, 4, 4) = 2\,r^3\,aa^2\cos(\theta)^4 - 4\,r^3\,aa^2\cos(\theta)^2 + 2\,r^3\,aa^2$$

$$T(2, 1, 1) = -2\,\frac{aa^2\,r^2\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\sin(\theta)\cos(\theta)}{ee}$$

$$T(2, 1, 2) = -\frac{aa^2\, r^3\, (-1 + 2\cos(\theta)^6 + 4\cos(\theta)^2 - 5\cos(\theta)^4)}{ee}$$

$$T(2, 1, 4) = -\frac{(\cos(\theta)^6\, aa^2\, r^4 - 3\, aa^2\, r^4 \cos(\theta)^4 + 3\, aa^2\, r^4 \cos(\theta)^2 + ee - aa^2\, r^4)\, aa\, \sin(\theta)}{ee}$$

$$T(2, 2, 1) = -\frac{aa^2\, r^3\, (-1 + 2\cos(\theta)^6 + 4\cos(\theta)^2 - 5\cos(\theta)^4)}{ee}$$

$$T(2, 2, 2) = 2\,\frac{aa^2\, r^4\, (\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\, \sin(\theta)\, \cos(\theta)}{ee}$$

$$T(2, 2, 4) = -\frac{aa^3\, r^5\, (-1 + \cos(\theta)^6 - 3\cos(\theta)^4 + 3\cos(\theta)^2)\, \cos(\theta)}{ee}$$

$$T(2, 4, 1) = -\frac{\sin(\theta)\,(\cos(\theta)^6\, aa^2\, r^4 - 3\, aa^2\, r^4 \cos(\theta)^4 + 2\cos(\theta)^2\, ee + 3\, aa^2\, r^4 \cos(\theta)^2 - ee - aa^2\, r^4)\, aa}{ee}$$

$$T(2, 4, 2) = -\frac{\cos(\theta)\,(\cos(\theta)^6\, aa^2\, r^4 - 3\, aa^2\, r^4 \cos(\theta)^4 + 2\cos(\theta)^2\, ee + 3\, aa^2\, r^4 \cos(\theta)^2 - 2\, ee - aa^2\, r^4)\, aa\, r}{ee}$$

$$T(2, 4, 4) = -2\,(-1 + \cos(\theta)^2)\, aa^2\, r^2 \sin(\theta)\, \cos(\theta)$$

$$T(4, 1, 1) = -2\,\frac{aa\, \cos(\theta)\, r\, (-1 + \cos(\theta)^2)}{ee}$$

$$T(4, 1, 2) = \frac{(2\cos(\theta)^2 - 1)\, r^2\, aa\, \sin(\theta)}{ee}$$

$$T(4, 1, 4) = -\frac{r^3\, aa^2\, (\cos(\theta)^4 - 2\cos(\theta)^2 + 1)}{ee}$$

$$T(4, 2, 1) = \frac{(2\cos(\theta)^2 - 1)\, r^2\, aa\, \sin(\theta)}{ee}$$

$$T(4, 2, 2) = 2\,\frac{aa\, r^3\, (-1 + \cos(\theta)^2)\, \cos(\theta)}{ee}$$

$$T(4, 2, 4) = \frac{(-1 + \cos(\theta)^2)\, r^4\, aa^2\, \sin(\theta)\, \cos(\theta)}{ee}$$

$$T(4, 4, 1) = -\frac{r^3\, aa^2\, (\cos(\theta)^4 - 2\cos(\theta)^2 + 1)}{ee}$$

$$T(4, 4, 2) = \frac{(-1 + \cos(\theta)^2)\, r^4\, aa^2\, \sin(\theta)\, \cos(\theta)}{ee}$$

---------- *Non Zero Affine Torsion for the perturbations* ----------

$$\text{CartanaffineTorsion}(1, 4, 1) = aa\, \cos(\theta)\, r\, (\cos(\theta) - 1)\, (\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(1, 4, 2) = -\cos(\theta)^2\, aa\, r^2\, \sin(\theta)$$

$$\text{CartanaffineTorsion}(1, 1, 4) = -aa\, \cos(\theta)\, r\, (\cos(\theta) - 1)\, (\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(1, 2, 4) = \cos(\theta)^2\, aa\, r^2\, \sin(\theta)$$

$$\text{CartanaffineTorsion}(2, 4, 1) = -\sin(\theta)\, aa\, (\cos(\theta) - 1)\, (\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(2, 4, 2) = -aa\, \cos(\theta)\, r\, (\cos(\theta) - 1)\, (\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(2, 1, 4) = \sin(\theta)\, aa\, (\cos(\theta) - 1)\, (\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(2, 2, 4) = aa\, \cos(\theta)\, r\, (\cos(\theta) - 1)\, (\cos(\theta) + 1)$$

```
>  `----------------- Perturbation Coefficients  ------------------`;`  `;
```

```
aaa:=0;bbb:=0;ccc:=cc;aaa0:=0;ccc0:=0;eee:=ee;`Intransitive W-Affine Basis Frame
with vorticity and affine torsion`;
> COEF(aaa,bbb,ccc,aaa0,ccc0,eee);
```

----------------- Perturbation Coefficients -------------------

$$aaa := 0$$
$$bbb := 0$$
$$ccc := cc$$
$$aaa0 := 0$$
$$ccc0 := 0$$
$$eee := ee$$

Intransitive W-Affine Basis Frame with vorticity and affine torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & 0 \\ 0 & 0 & -cc\,r^2 + cc\,r^2\cos(\theta)^2 & 1 \end{bmatrix}$$

--------- Non Zero Cartan Connection Coefficients {C} for the given perturbations ----------

$$\text{Cartan\_RIGHT}(2,1,2) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,1,3) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(4,1,3) = -\frac{-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0}{r\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(1,2,2) = -r$$

$$\text{Cartan\_RIGHT}(2,2,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,2,3) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(4,2,3) = -\frac{\cos(\theta)\,(-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0)}{\sin(\theta)\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(1,3,3) = r\,(\cos(\theta)-1)(\cos(\theta)+1)$$

$$\text{Cartan\_RIGHT}(2,3,3) = -\cos(\theta)\sin(\theta)$$

$$\text{Cartan\_RIGHT}(3,3,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,3,2) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(4, 3, 1) = \frac{cc0 + cc\,r^2\cos(\theta)^2 - cc\,r^2}{\sqrt{\dfrac{r + bb}{r}}\,r}$$

$$\text{Cartan\_RIGHT}(4, 3, 2) = \frac{\cos(\theta)\,(cc0 + cc\,r^2\cos(\theta)^2 - cc\,r^2)}{\sin(\theta)\,\sqrt{\dfrac{r + bb}{r}}}$$

---------- Non Zero Christoffel Connection Coefficients {Gamma} for the perturbations ----------

$$\text{Christoffel\_Gamma2}(1, 2, 2) = -r$$

$$\text{Christoffel\_Gamma2}(1, 3, 3) = -2\,ee\,cc^2\,r^3\cos(\theta)^4 + r\cos(\theta)^2 - r + 4\,ee\,cc^2\,r^3\cos(\theta)^2 - 2\,ee\,cc^2\,r^3$$

$$\text{Christoffel\_Gamma2}(1, 3, 4) = ee\,cc\,r - ee\,cc\,r\cos(\theta)^2$$

$$\text{Christoffel\_Gamma2}(1, 4, 3) = ee\,cc\,r - ee\,cc\,r\cos(\theta)^2$$

$$\text{Christoffel\_Gamma2}(2, 1, 2) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2, 2, 1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2, 3, 3) = (-1 - 2\,ee\,cc^2\,r^2 + 2\,ee\,cc^2\,r^2\cos(\theta)^2)\,\sin(\theta)\cos(\theta)$$

$$\text{Christoffel\_Gamma2}(2, 3, 4) = cc\,ee\,\sin(\theta)\cos(\theta)$$

$$\text{Christoffel\_Gamma2}(2, 4, 3) = cc\,ee\,\sin(\theta)\cos(\theta)$$

$$\text{Christoffel\_Gamma2}(3, 1, 3) = -\frac{ee\,cc^2\,r^2\cos(\theta)^2 - ee\,cc^2\,r^2 - 1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 1, 4) = -\frac{ee\,cc}{r}$$

$$\text{Christoffel\_Gamma2}(3, 2, 3) = \frac{(ee\,cc^2\,r^2\cos(\theta)^2 - ee\,cc^2\,r^2 - 1)\,\sin(\theta)\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3, 2, 4) = \frac{\cos(\theta)\sin(\theta)\,ee\,cc}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3, 3, 1) = -\frac{ee\,cc^2\,r^2\cos(\theta)^2 - ee\,cc^2\,r^2 - 1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 3, 2) = \frac{(ee\,cc^2\,r^2\cos(\theta)^2 - ee\,cc^2\,r^2 - 1)\,\sin(\theta)\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3, 4, 1) = -\frac{ee\,cc}{r}$$

$$\text{Christoffel\_Gamma2}(3, 4, 2) = \frac{\cos(\theta)\sin(\theta)\,ee\,cc}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(4, 1, 3) = ee\,cc^3\,r^3\cos(\theta)^4 - 2\,ee\,cc^3\,r^3\cos(\theta)^2 + ee\,cc^3\,r^3$$

$$\text{Christoffel\_Gamma2}(4, 1, 4) = -ee\,cc^2\,r + ee\,cc^2\,r\cos(\theta)^2$$

$$\text{Christoffel\_Gamma2}(4, 2, 3) = -(-1 + \cos(\theta)^2)\,ee\,cc^3\,r^4\sin(\theta)\cos(\theta)$$

$$\text{Christoffel\_Gamma2}(4, 2, 4) = -\sin(\theta)\cos(\theta)\,ee\,cc^2\,r^2$$

$$\text{Christoffel\_Gamma2}(4, 3, 1) = ee\,cc^3\,r^3\cos(\theta)^4 - 2\,ee\,cc^3\,r^3\cos(\theta)^2 + ee\,cc^3\,r^3$$

$$\text{Christoffel\_Gamma2}(4, 3, 2) = -(-1 + \cos(\theta)^2)\,ee\,cc^3\,r^4\sin(\theta)\cos(\theta)$$

$$\text{Christoffel\_Gamma2}(4, 4, 1) = -ee\ cc^2\ r + ee\ cc^2\ r\cos(\theta)^2$$

$$\text{Christoffel\_Gamma2}(4, 4, 2) = -\sin(\theta)\cos(\theta)\ ee\ cc^2\ r^2$$

---------- Non Zero Residue Connection Coefficients {T}= {C} - {Gamma} for the given perturbations ----------

$$T(1, 3, 3) = 2\ ee\ cc^2\ r^3\cos(\theta)^4 - 4\ ee\ cc^2\ r^3\cos(\theta)^2 + 2\ ee\ cc^2\ r^3$$

$$T(1, 3, 4) = -ee\ cc\ r + ee\ cc\ r\cos(\theta)^2$$

$$T(1, 4, 3) = -ee\ cc\ r + ee\ cc\ r\cos(\theta)^2$$

$$T(2, 3, 3) = -2\cos(\theta)\sin(\theta)\ ee\ cc^2\ r^2\ (-1 + \cos(\theta)^2)$$

$$T(2, 3, 4) = -cc\ ee\ \sin(\theta)\cos(\theta)$$

$$T(2, 4, 3) = -cc\ ee\ \sin(\theta)\cos(\theta)$$

$$T(3, 1, 3) = -r\ ee\ cc^2 + r\ ee\ cc^2\cos(\theta)^2$$

$$T(3, 1, 4) = \frac{ee\ cc}{r}$$

$$T(3, 2, 3) = -\cos(\theta)\sin(\theta)\ ee\ cc^2\ r^2$$

$$T(3, 2, 4) = -\frac{\cos(\theta)\sin(\theta)\ ee\ cc}{-1 + \cos(\theta)^2}$$

$$T(3, 3, 1) = -r\ ee\ cc^2 + r\ ee\ cc^2\cos(\theta)^2$$

$$T(3, 3, 2) = -\cos(\theta)\sin(\theta)\ ee\ cc^2\ r^2$$

$$T(3, 4, 1) = \frac{ee\ cc}{r}$$

$$T(3, 4, 2) = -\frac{\cos(\theta)\sin(\theta)\ ee\ cc}{-1 + \cos(\theta)^2}$$

$$T(4, 1, 3) = -cc^3\ r^3\ ee\cos(\theta)^4 - cc\ r\cos(\theta)^2 + 2\ cc^3\ r^3\ ee\cos(\theta)^2 - cc^3\ r^3\ ee + cc\ r$$

$$T(4, 1, 4) = r\ ee\ cc^2 - r\ ee\ cc^2\cos(\theta)^2$$

$$T(4, 2, 3) = \sin(\theta)\ cc\ (ee\ cc^2\ r^2\cos(\theta)^2 - ee\ cc^2\ r^2 + 1)\ r^2\cos(\theta)$$

$$T(4, 2, 4) = \cos(\theta)\sin(\theta)\ ee\ cc^2\ r^2$$

$$T(4, 3, 1) = -cc^3\ r^3\ ee\cos(\theta)^4 + cc\ r\cos(\theta)^2 + 2\ cc^3\ r^3\ ee\cos(\theta)^2 - cc^3\ r^3\ ee - cc\ r$$

$$T(4, 3, 2) = \sin(\theta)\ cc\ (ee\ cc^2\ r^2\cos(\theta)^2 - 1 - ee\ cc^2\ r^2)\ r^2\cos(\theta)$$

$$T(4, 4, 1) = ee\ cc^2\ r - ee\ cc^2\ r\cos(\theta)^2$$

$$T(4, 4, 2) = r^2\ cc^2\ ee\sin(\theta)\cos(\theta)$$

---------- Non Zero Affine Torsion for the perturbations ----------

$$\text{CartanaffineTorsion}(4, 3, 1) = cc\ r\ (\cos(\theta) - 1)(\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(4, 3, 2) = \frac{\cos(\theta)\ cc\ r^2\ (\cos(\theta) - 1)(\cos(\theta) + 1)}{\sin(\theta)}$$

$$\text{CartanaffineTorsion}(4, 1, 3) = -cc\ r\ (\cos(\theta) - 1)(\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(4, 2, 3) = -\frac{\cos(\theta)\ cc\ r^2\ (\cos(\theta) - 1)(\cos(\theta) + 1)}{\sin(\theta)}$$

```
> `---------------- Perturbation Coefficients  ------------------`;`  `;
aaa:=0;bbb:=bb;ccc:=0;aaa0:=0;ccc0:=0;eee:=ee;`Metric perturbation only. Non zero
affine torsion 10 parameter group`;
```

```
> COEF(aaa,bbb,ccc,aaa0,ccc0,eee);
```

----------------- Perturbation Coefficients -------------------

$$aaa := 0$$
$$bbb := bb$$
$$ccc := 0$$
$$aaa0 := 0$$
$$ccc0 := 0$$
$$eee := ee$$

Metric perturbation only. Non zero affine torsion 10 parameter group

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & 0 \\ 0 & 0 & 0 & \sqrt{1 + \dfrac{bb}{r}} \end{bmatrix}$$

---------- Non Zero Cartan Connection Coefficients {C} for the given perturbations ----------

$$\text{Cartan\_RIGHT}(2,1,2) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,1,3) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(1,2,2) = -r$$

$$\text{Cartan\_RIGHT}(2,2,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,2,3) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(1,3,3) = r(\cos(\theta) - 1)(\cos(\theta) + 1)$$

$$\text{Cartan\_RIGHT}(2,3,3) = -\cos(\theta)\sin(\theta)$$

$$\text{Cartan\_RIGHT}(3,3,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,3,2) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(4,4,1) = -\frac{1}{2}\frac{bb}{(r+bb)r}$$

---------- Non Zero Christoffel Connection Coefficients {Gamma} for the perturbations ----------

$$\text{Christoffel\_Gamma2}(1,2,2) = -r$$

$$\text{Christoffel\_Gamma2}(1,3,3) = -r + r\cos(\theta)^2$$

$$\text{Christoffel\_Gamma2}(1, 4, 4) = \frac{1}{2}\frac{ee\ bb}{r^2}$$

$$\text{Christoffel\_Gamma2}(2, 1, 2) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2, 2, 1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2, 3, 3) = -\cos(\theta)\sin(\theta)$$

$$\text{Christoffel\_Gamma2}(3, 1, 3) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 2, 3) = -\frac{\sin(\theta)\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3, 3, 1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 3, 2) = -\frac{\sin(\theta)\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(4, 1, 4) = -\frac{1}{2}\frac{bb}{(r + bb)\ r}$$

$$\text{Christoffel\_Gamma2}(4, 4, 1) = -\frac{1}{2}\frac{bb}{(r + bb)\ r}$$

---------- Non Zero Residue Connection Coefficients {T}= {C} - {Gamma} for the given perturbations ----------

$$T(1, 4, 4) = -\frac{1}{2}\frac{ee\ bb}{r^2}$$

$$T(4, 1, 4) = \frac{1}{2}\frac{bb}{(r + bb)\ r}$$

---------- Non Zero Affine Torsion for the perturbations ----------

$$\text{CartanaffineTorsion}(4, 4, 1) = -\frac{1}{4}\frac{bb}{(r + bb)\ r}$$

$$\text{CartanaffineTorsion}(4, 1, 4) = \frac{1}{4}\frac{bb}{(r + bb)\ r}$$

```
> `----------------- Perturbation Coefficients  ------------------`;`   `;
aaa:=0;bbb:=0;ccc:=0;aaa0:=aa0;ccc0:=0;eee:=ee;`Closed Twist perturbation,
P-Affine Frame No affine Torsion`;
> COEF(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- Perturbation Coefficients ------------------

$$aaa := 0$$
$$bbb := 0$$
$$ccc := 0$$
$$aaa0 := aa0$$
$$ccc0 := 0$$

eee := *ee*

Closed Twist perturbation, P-Affine Frame No affine Torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & -aa0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

---------- *Non Zero Cartan Connection Coefficients {C} for the given perturbations* ----------

$$\text{Cartan\_RIGHT}(2,1,2) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,1,3) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(1,2,2) = -r$$

$$\text{Cartan\_RIGHT}(2,2,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,2,3) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(1,3,3) = r\,(\cos(\theta)-1)\,(\cos(\theta)+1)$$

$$\text{Cartan\_RIGHT}(2,3,3) = -\cos(\theta)\sin(\theta)$$

$$\text{Cartan\_RIGHT}(3,3,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,3,2) = \frac{\cos(\theta)}{\sin(\theta)}$$

---------- *Non Zero Christoffel Connection Coefficients {Gamma} for the perturbations* ----------

$$\text{Christoffel\_Gamma2}(1,2,2) = -r$$

$$\text{Christoffel\_Gamma2}(1,3,3) = -r + r\cos(\theta)^2$$

$$\text{Christoffel\_Gamma2}(2,1,2) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2,2,1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2,3,3) = -\cos(\theta)\sin(\theta)$$

$$\text{Christoffel\_Gamma2}(3,1,3) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3,2,3) = -\frac{\sin(\theta)\cos(\theta)}{-1+\cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3,3,1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 3, 2) = -\frac{\sin(\theta)\cos(\theta)}{-1 + \cos(\theta)^2}$$

---------- Non Zero Residue Connection Coefficients {T}= {C} - {Gamma} for the given perturbations ----------

---------- Non Zero Affine Torsion for the perturbations ----------

```
> `----------------- Perturbation Coefficients  -------------------`;`   `;
aaa:=0;bbb:=0;ccc:=0;aaa0:=0;ccc0:=cc0;eee:=ee;`Closed Rotation pertubation,
W-Affine Frame No Affine Torsion`;
> COEF(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- Perturbation Coefficients ------------------

$$aaa := 0$$
$$bbb := 0$$
$$ccc := 0$$
$$aaa0 := 0$$
$$ccc0 := cc0$$
$$eee := ee$$

Closed Rotation pertubation, W-Affine Frame No Affine Torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & 0 \\ 0 & 0 & -cc0 & 1 \end{bmatrix}$$

---------- Non Zero Cartan Connection Coefficients {C} for the given perturbations ----------

$$\text{Cartan\_RIGHT}(2, 1, 2) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3, 1, 3) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(4, 1, 3) = -\frac{-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0}{r\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(1, 2, 2) = -r$$

$$\text{Cartan\_RIGHT}(2, 2, 1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3, 2, 3) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(4, 2, 3) = -\frac{\cos(\theta)\,(-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0)}{\sin(\theta)\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(1, 3, 3) = r\,(\cos(\theta) - 1)\,(\cos(\theta) + 1)$$

$$\text{Cartan\_RIGHT}(2, 3, 3) = -\cos(\theta)\sin(\theta)$$

$$\text{Cartan\_RIGHT}(3, 3, 1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3, 3, 2) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(4, 3, 1) = \frac{cc0 + cc\,r^2\cos(\theta)^2 - cc\,r^2}{\sqrt{\dfrac{r+bb}{r}}\;r}$$

$$\text{Cartan\_RIGHT}(4, 3, 2) = \frac{\cos(\theta)\,(cc0 + cc\,r^2\cos(\theta)^2 - cc\,r^2)}{\sin(\theta)\sqrt{\dfrac{r+bb}{r}}}$$

---------- *Non Zero Christoffel Connection Coefficients {Gamma} for the perturbations* ----------

$$\text{Christoffel\_Gamma2}(1, 2, 2) = -r$$

$$\text{Christoffel\_Gamma2}(1, 3, 3) = -r + r\cos(\theta)^2$$

$$\text{Christoffel\_Gamma2}(2, 1, 2) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2, 2, 1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2, 3, 3) = -\cos(\theta)\sin(\theta)$$

$$\text{Christoffel\_Gamma2}(3, 1, 3) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 2, 3) = -\frac{\sin(\theta)\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3, 3, 1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 3, 2) = -\frac{\sin(\theta)\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(4, 1, 3) = \frac{cc0}{r}$$

$$\text{Christoffel\_Gamma2}(4, 2, 3) = -\frac{\cos(\theta)\sin(\theta)\,cc0}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(4, 3, 1) = \frac{cc0}{r}$$

$$\text{Christoffel\_Gamma2}(4, 3, 2) = -\frac{\cos(\theta)\sin(\theta)\,cc0}{-1 + \cos(\theta)^2}$$

---------- *Non Zero Residue Connection Coefficients {T}= {C} - {Gamma} for the given perturbations* ----------

> `----------------- Perturbation Coefficients -------------------`;` `;
aaa:=0;bbb:=bb;ccc:=0;aaa0:=aa0;ccc0:=0;eee:=ee;`Closed Twist with metric,
P-Affine Frame, Non Zero Affine Torsion`;
> COEF(aaa,bbb,ccc,aaa0,ccc0,eee);

---------------- Perturbation Coefficients ------------------

$$aaa := 0$$

$$bbb := bb$$

$$ccc := 0$$

$$aaa0 := aa0$$

$$ccc0 := 0$$

$$eee := ee$$

Closed Twist with metric, P-Affine Frame, Non Zero Affine Torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & -aa0 \\ 0 & 0 & 0 & \sqrt{1+\dfrac{bb}{r}} \end{bmatrix}$$

--------- Non Zero Cartan Connection Coefficients {C} for the given perturbations ----------

$$\text{Cartan\_RIGHT}(2,1,2) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,1,3) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(1,2,2) = -r$$

$$\text{Cartan\_RIGHT}(2,2,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,2,3) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(1,3,3) = r(\cos(\theta)-1)(\cos(\theta)+1)$$

$$\text{Cartan\_RIGHT}(2,3,3) = -\cos(\theta)\sin(\theta)$$

$$\text{Cartan\_RIGHT}(3,3,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,3,2) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(1, 4, 1) = \frac{1}{2} \frac{\cos(\theta)\left(-4\,aa\,r^3 - 5\,bb\,aa\,r^2 + 4\,aa\,r^3 \cos(\theta)^2 + 5\,bb\,aa\,r^2 \cos(\theta)^2 - bb\,aa0\right)}{r\,(r + bb)}$$

$$\text{Cartan\_RIGHT}(2, 4, 1) = -\frac{1}{2} \frac{\sin(\theta)\left(-4\,aa\,r^3 - 5\,bb\,aa\,r^2 + 4\,aa\,r^3 \cos(\theta)^2 + 5\,bb\,aa\,r^2 \cos(\theta)^2 - bb\,aa0\right)}{(r + bb)\,r^2}$$

$$\text{Cartan\_RIGHT}(4, 4, 1) = -\frac{1}{2} \frac{bb}{(r + bb)\,r}$$

---------- *Non Zero Christoffel Connection Coefficients {Gamma} for the perturbations* ----------

$$\text{Christoffel\_Gamma2}(1, 1, 4) = -\frac{1}{2} \frac{\cos(\theta)\,bb\,aa0}{(r + bb)\,r}$$

$$\text{Christoffel\_Gamma2}(1, 2, 2) = -r$$

$$\text{Christoffel\_Gamma2}(1, 3, 3) = -r + r\cos(\theta)^2$$

$$\text{Christoffel\_Gamma2}(1, 4, 1) = -\frac{1}{2} \frac{\cos(\theta)\,bb\,aa0}{(r + bb)\,r}$$

$$\text{Christoffel\_Gamma2}(1, 4, 4) = \frac{1}{2} \frac{\left(r\cos(\theta)^2 aa0^2 + ee\,r + ee\,bb\right)bb}{r^2\,(r + bb)}$$

$$\text{Christoffel\_Gamma2}(2, 1, 2) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2, 1, 4) = \frac{1}{2} \frac{\sin(\theta)\,bb\,aa0}{(r + bb)\,r^2}$$

$$\text{Christoffel\_Gamma2}(2, 2, 1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2, 3, 3) = -\cos(\theta)\sin(\theta)$$

$$\text{Christoffel\_Gamma2}(2, 4, 1) = \frac{1}{2} \frac{\sin(\theta)\,bb\,aa0}{(r + bb)\,r^2}$$

$$\text{Christoffel\_Gamma2}(2, 4, 4) = -\frac{1}{2} \frac{bb\,aa0^2 \sin(\theta)\cos(\theta)}{r^2\,(r + bb)}$$

$$\text{Christoffel\_Gamma2}(3, 1, 3) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 2, 3) = -\frac{\sin(\theta)\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3, 3, 1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 3, 2) = -\frac{\sin(\theta)\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(4, 1, 4) = -\frac{1}{2} \frac{bb}{(r + bb)\,r}$$

$$\text{Christoffel\_Gamma2}(4, 4, 1) = -\frac{1}{2} \frac{bb}{(r + bb)\,r}$$

$$\text{Christoffel\_Gamma2}(4, 4, 4) = \frac{1}{2} \frac{\cos(\theta)\,bb\,aa0}{(r + bb)\,r}$$

$$T(1, 1, 4) = \frac{1}{2} \frac{\cos(\theta)\, bb\, aa0}{(r + bb)\, r}$$

$$T(1, 4, 4) = -\frac{1}{2} \frac{(r \cos(\theta)^2\, aa0^2 + ee\, r + ee\, bb)\, bb}{r^2\, (r + bb)}$$

$$T(2, 1, 4) = -\frac{1}{2} \frac{\sin(\theta)\, bb\, aa0}{(r + bb)\, r^2}$$

$$T(2, 4, 4) = \frac{1}{2} \frac{bb\, aa0^2 \sin(\theta) \cos(\theta)}{r^2\, (r + bb)}$$

$$T(4, 1, 4) = \frac{1}{2} \frac{bb}{(r + bb)\, r}$$

$$T(4, 4, 4) = -\frac{1}{2} \frac{\cos(\theta)\, bb\, aa0}{(r + bb)\, r}$$

---------- Non Zero Affine Torsion for the perturbations ----------

$$\text{CartanaffineTorsion}(1, 4, 1) = -\frac{1}{4} \frac{\cos(\theta)\, bb\, aa0}{(r + bb)\, r}$$

$$\text{CartanaffineTorsion}(1, 1, 4) = \frac{1}{4} \frac{\cos(\theta)\, bb\, aa0}{(r + bb)\, r}$$

$$\text{CartanaffineTorsion}(2, 4, 1) = \frac{1}{4} \frac{\sin(\theta)\, bb\, aa0}{(r + bb)\, r^2}$$

$$\text{CartanaffineTorsion}(2, 1, 4) = -\frac{1}{4} \frac{\sin(\theta)\, bb\, aa0}{(r + bb)\, r^2}$$

$$\text{CartanaffineTorsion}(4, 4, 1) = -\frac{1}{4} \frac{bb}{(r + bb)\, r}$$

$$\text{CartanaffineTorsion}(4, 1, 4) = \frac{1}{4} \frac{bb}{(r + bb)\, r}$$

```
> `---------------- Perturbation Coefficients  -------------------`;`   `;
aaa:=0;bbb:=bb;ccc:=0;aaa0:=0;ccc0:=cc0;eee:=ee;`Closed circulation with metric
pertubation, W-Affine Frame, Non zero Affine Torsion`;
> COEF(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- Perturbation Coefficients -------------------

$$aaa := 0$$

$$bbb := bb$$

$$ccc := 0$$

$$aaa0 := 0$$

$$ccc0 := cc0$$

$$eee := ee$$

Closed circulation with metric pertubation, W-Affine Frame, Non zero Affine Torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & 0 \\ 0 & 0 & -cc0 & \sqrt{1+\dfrac{bb}{r}} \end{bmatrix}$$

---------- Non Zero Cartan Connection Coefficients {C} for the given perturbations ----------

$$\text{Cartan\_RIGHT}(2,1,2) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,1,3) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(4,1,3) = -\frac{-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0}{r\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(1,2,2) = -r$$

$$\text{Cartan\_RIGHT}(2,2,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,2,3) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(4,2,3) = -\frac{\cos(\theta)\,(-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0)}{\sin(\theta)\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(1,3,3) = r\,(-1+\cos(\theta))\,(\cos(\theta)+1)$$

$$\text{Cartan\_RIGHT}(2,3,3) = -\cos(\theta)\sin(\theta)$$

$$\text{Cartan\_RIGHT}(3,3,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,3,2) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(4,3,1) = \frac{cc0 + cc\,r^2\cos(\theta)^2 - cc\,r^2}{\sqrt{\dfrac{r+bb}{r}}\,r}$$

$$\text{Cartan\_RIGHT}(4,3,2) = \frac{\cos(\theta)\,(cc0 + cc\,r^2\cos(\theta)^2 - cc\,r^2)}{\sin(\theta)\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(4,4,1) = -\frac{1}{2}\frac{bb}{(r+bb)\,r}$$

---------- Non Zero Christoffel Connection Coefficients {Gamma} for the perturbations ----------

$$\text{Christoffel\_Gamma2}(1,2,2) = -r$$

$$\text{Christoffel\_Gamma2}(1,3,3) = -r + r\cos(\theta)^2$$

$$\text{Christoffel\_Gamma2}(1,3,4) = -\frac{1}{4}\frac{ee\ bb\ cc0}{r^2\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Christoffel\_Gamma2}(1,4,3) = -\frac{1}{4}\frac{ee\ bb\ cc0}{r^2\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Christoffel\_Gamma2}(1,4,4) = \frac{1}{2}\frac{ee\ bb}{r^2}$$

$$\text{Christoffel\_Gamma2}(2,1,2) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2,2,1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2,3,3) = -\cos(\theta)\sin(\theta)$$

$$\text{Christoffel\_Gamma2}(3,1,3) = \frac{1}{4}\frac{-4\,r^2\,bb + 4\,r^3\cos(\theta)^2 + 4\,r^2\cos(\theta)^2\,bb - 4\,r^3 - ee\,cc0^2\,bb}{r^3\left(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2\right)}$$

$$\text{Christoffel\_Gamma2}(3,1,4) = \frac{1}{4}\frac{bb\ ee\ cc0}{\sqrt{\dfrac{r+bb}{r}}\,(-1+\cos(\theta)^2)\,r^4}$$

$$\text{Christoffel\_Gamma2}(3,2,3) = -\frac{\sin(\theta)\cos(\theta)}{-1+\cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3,3,1) = \frac{1}{4}\frac{-4\,r^2\,bb + 4\,r^3\cos(\theta)^2 + 4\,r^2\cos(\theta)^2\,bb - 4\,r^3 - ee\,cc0^2\,bb}{r^3\left(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2\right)}$$

$$\text{Christoffel\_Gamma2}(3,3,2) = -\frac{\sin(\theta)\cos(\theta)}{-1+\cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3,4,1) = \frac{1}{4}\frac{bb\ ee\ cc0}{\sqrt{\dfrac{r+bb}{r}}\,(-1+\cos(\theta)^2)\,r^4}$$

$$\text{Christoffel\_Gamma2}(4,1,3) = \frac{1}{4}\frac{cc0\left(-ee\,cc0^2\,bb - 5\,r^2\,bb - 4\,r^3 + 5\,r^2\cos(\theta)^2\,bb + 4\,r^3\cos(\theta)^2\right)}{r^3\left(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2\right)\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Christoffel\_Gamma2}(4,1,4) = -\frac{1}{4}\frac{bb\left(-2\,r^2 + 2\,r^2\cos(\theta)^2 - ee\,cc0^2\right)}{r^3\left(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2\right)}$$

$$\text{Christoffel\_Gamma2}(4,2,3) = -\frac{\sqrt{\dfrac{r+bb}{r}}\,r\cos(\theta)\,cc0\,\sin(\theta)}{-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(4,3,1) = \frac{1}{4}\frac{cc0\left(-ee\,cc0^2\,bb - 5\,r^2\,bb - 4\,r^3 + 5\,r^2\cos(\theta)^2\,bb + 4\,r^3\cos(\theta)^2\right)}{r^3\left(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2\right)\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Christoffel\_Gamma2}(4,3,2) = -\frac{\sqrt{\dfrac{r+bb}{r}}\,r\cos(\theta)\,cc0\,\sin(\theta)}{-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(4, 4, 1) = -\frac{1}{4} \frac{bb\,(-2\,r^2 + 2\,r^2 \cos(\theta)^2 - ee\,cc0^2)}{r^3\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)}$$

---------- Non Zero Residue Connection Coefficients {T}= {C} - {Gamma} for the given perturbations ----------

$$T(1, 3, 4) = \frac{1}{4} \frac{ee\,bb\,cc0}{r^2\,\sqrt{\dfrac{r+bb}{r}}}$$

$$T(1, 4, 3) = \frac{1}{4} \frac{ee\,bb\,cc0}{r^2\,\sqrt{\dfrac{r+bb}{r}}}$$

$$T(1, 4, 4) = -\frac{1}{2} \frac{ee\,bb}{r^2}$$

$$T(3, 1, 3) = \frac{1}{4} \frac{ee\,cc0^2\,bb}{r^3\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)}$$

$$T(3, 1, 4) = -\frac{1}{4} \frac{bb\,ee\,cc0}{\sqrt{\dfrac{r+bb}{r}}\,(-1+\cos(\theta)^2)\,r^4}$$

$$T(3, 3, 1) = \frac{1}{4} \frac{ee\,cc0^2\,bb}{r^3\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)}$$

$$T(3, 4, 1) = -\frac{1}{4} \frac{bb\,ee\,cc0}{\sqrt{\dfrac{r+bb}{r}}\,(-1+\cos(\theta)^2)\,r^4}$$

$$T(4, 1, 3) = -\frac{1}{4} \frac{cc0\,bb\,(-ee\,cc0^2 - r^2 + r^2\cos(\theta)^2)}{r^3\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)\,\sqrt{\dfrac{r+bb}{r}}}$$

$$T(4, 1, 4) = \frac{1}{4} \frac{bb\,(-2\,r^2 + 2\,r^2\cos(\theta)^2 - ee\,cc0^2)}{r^3\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)}$$

$$T(4, 3, 1) = -\frac{1}{4} \frac{cc0\,bb\,(-ee\,cc0^2 - r^2 + r^2\cos(\theta)^2)}{r^3\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)\,\sqrt{\dfrac{r+bb}{r}}}$$

$$T(4, 4, 1) = -\frac{1}{4} \frac{ee\,cc0^2\,bb}{r^3\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)}$$

--------- Non Zero Affine Torsion for the perturbations ----------

$$\text{CartanaffineTorsion}(4, 4, 1) = -\frac{1}{4} \frac{bb}{(r+bb)\,r}$$

$$\text{CartanaffineTorsion}(4, 1, 4) = \frac{1}{4} \frac{bb}{(r+bb)\,r}$$

```
> `---------------- Perturbation Coefficients  ------------------`;`  `;
aaa:=aa;bbb:=0;ccc:=0;aaa0:=0;ccc0:=0;eee:=ee;`Twist with metric pertubation,
```

> COEF(aaa,bbb,ccc,aaa0,ccc0,eee);

---------------- *Perturbation Coefficients* ------------------

$$aaa := aa$$
$$bbb := 0$$
$$ccc := 0$$
$$aaa0 := 0$$
$$ccc0 := 0$$
$$eee := ee$$

Twist with metric pertubation, P-Affine Frame, Non Zero Affine Torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & -aa\,r^2 + aa\,r^2\cos(\theta)^2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

--------- *Non Zero Cartan Connection Coefficients {C} for the given perturbations* ----------

$$\text{Cartan\_RIGHT}(2, 1, 2) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3, 1, 3) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(1, 2, 2) = -r$$

$$\text{Cartan\_RIGHT}(2, 2, 1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3, 2, 3) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(1, 3, 3) = r\,(\cos(\theta) - 1)\,(\cos(\theta) + 1)$$

$$\text{Cartan\_RIGHT}(2, 3, 3) = -\cos(\theta)\sin(\theta)$$

$$\text{Cartan\_RIGHT}(3, 3, 1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3, 3, 2) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(1, 4, 1) = \frac{1}{2}\frac{\cos(\theta)\,(-4\,aa\,r^3 - 5\,bb\,aa\,r^2 + 4\,aa\,r^3\cos(\theta)^2 + 5\,bb\,aa\,r^2\cos(\theta)^2 - bb\,aa0)}{r\,(r + bb)}$$

$$\text{Cartan\_RIGHT}(1, 4, 2) = -2\cos(\theta)^2\,aa\,r^2\sin(\theta)$$

$$\text{Cartan\_RIGHT}(2, 4, 1) = -\frac{1}{2}\frac{\sin(\theta)\,(-4\,aa\,r^3 - 5\,bb\,aa\,r^2 + 4\,aa\,r^3\cos(\theta)^2 + 5\,bb\,aa\,r^2\cos(\theta)^2 - bb\,aa0)}{(r + bb)\,r^2}$$

$$\text{Cartan\_RIGHT}(2, 4, 2) = -2\cos(\theta)\,aa\,r\,(\cos(\theta) - 1)\,(\cos(\theta) + 1)$$

--------- *Non Zero Christoffel Connection Coefficients {Gamma} for the perturbations* ----------

$$\text{Christoffel\_Gamma2}(1, 1, 1) = -2\,\frac{aa^2\,r^3\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\,\cos(\theta)^2}{ee}$$

$$\text{Christoffel\_Gamma2}(1, 1, 2) = \frac{(2\cos(\theta)^4 - 3\cos(\theta)^2 + 1)\,r^4\,aa^2\,\sin(\theta)\,\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(1, 1, 4) = -\frac{(\cos(\theta)^6 - 3\cos(\theta)^4 + 3\cos(\theta)^2 - 1)\,r^5\,aa^3\,\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(1, 2, 1) = \frac{(2\cos(\theta)^4 - 3\cos(\theta)^2 + 1)\,r^4\,aa^2\,\sin(\theta)\,\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(1, 2, 2) = \frac{r\,(2\,r^4\cos(\theta)^6\,aa^2 - 4\,aa^2\,r^4\cos(\theta)^4 + 2\,aa^2\,r^4\cos(\theta)^2 - ee)}{ee}$$

$$\text{Christoffel\_Gamma2}(1, 2, 4) = \frac{(r^4\cos(\theta)^6\,aa^2 - 2\,aa^2\,r^4\cos(\theta)^4 + aa^2\,r^4\cos(\theta)^2 - ee)\,r^2\,aa\,\sin(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(1, 3, 3) = -r + r\cos(\theta)^2$$

$$\text{Christoffel\_Gamma2}(1, 4, 1) = -\frac{(\cos(\theta)^6 - 3\cos(\theta)^4 + 3\cos(\theta)^2 - 1)\,r^5\,aa^3\,\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(1, 4, 2) = \frac{(r^4\cos(\theta)^6\,aa^2 - 2\,aa^2\,r^4\cos(\theta)^4 + aa^2\,r^4\cos(\theta)^2 - ee)\,r^2\,aa\,\sin(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(1, 4, 4) = -2\,r^3\,aa^2\cos(\theta)^4 + 4\,r^3\,aa^2\cos(\theta)^2 - 2\,r^3\,aa^2$$

$$\text{Christoffel\_Gamma2}(2, 1, 1) = 2\,\frac{aa^2\,r^2\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\,\sin(\theta)\,\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(2, 1, 2) = \frac{-aa^2\,r^4 + ee + 2\,r^4\cos(\theta)^6\,aa^2 - 5\,aa^2\,r^4\cos(\theta)^4 + 4\,aa^2\,r^4\cos(\theta)^2}{r\,ee}$$

$$\text{Christoffel\_Gamma2}(2, 1, 4) = \frac{(r^4\cos(\theta)^6\,aa^2 - 3\,aa^2\,r^4\cos(\theta)^4 + 3\,aa^2\,r^4\cos(\theta)^2 + ee - aa^2\,r^4)\,aa\,\sin(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(2, 2, 1) = \frac{-aa^2\,r^4 + ee + 2\,r^4\cos(\theta)^6\,aa^2 - 5\,aa^2\,r^4\cos(\theta)^4 + 4\,aa^2\,r^4\cos(\theta)^2}{r\,ee}$$

$$\text{Christoffel\_Gamma2}(2, 2, 2) = -2\,\frac{aa^2\,r^4\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\,\sin(\theta)\,\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(2, 2, 4) = \frac{(\cos(\theta)^6 - 3\cos(\theta)^4 + 3\cos(\theta)^2 - 1)\,r^5\,aa^3\,\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(2, 3, 3) = -\cos(\theta)\,\sin(\theta)$$

$$\text{Christoffel\_Gamma2}(2, 4, 1) = \frac{(r^4\cos(\theta)^6\,aa^2 - 3\,aa^2\,r^4\cos(\theta)^4 + 3\,aa^2\,r^4\cos(\theta)^2 + ee - aa^2\,r^4)\,aa\,\sin(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(2, 4, 2) = \frac{(\cos(\theta)^6 - 3\cos(\theta)^4 + 3\cos(\theta)^2 - 1)\,r^5\,aa^3\,\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(2, 4, 4) = 2\,aa^2\,r^2\,(-1 + \cos(\theta)^2)\,\sin(\theta)\,\cos(\theta)$$

$$\text{Christoffel\_Gamma2}(3, 1, 3) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 2, 3) = -\frac{\sin(\theta)\,\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3, 3, 1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(3, 3, 2) = -\frac{\sin(\theta)\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(4, 1, 1) = 2\,\frac{aa\,\cos(\theta)\,r\,(-1 + \cos(\theta)^2)}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 1, 2) = -\frac{(2\cos(\theta)^2 - 1)\,r^2\,aa\,\sin(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 1, 4) = \frac{(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\,r^3\,aa^2}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 2, 1) = -\frac{(2\cos(\theta)^2 - 1)\,r^2\,aa\,\sin(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 2, 2) = -2\,\frac{aa\,r^3\,(-1 + \cos(\theta)^2)\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 2, 4) = -\frac{(-1 + \cos(\theta)^2)\,r^4\,aa^2\,\sin(\theta)\cos(\theta)}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 4, 1) = \frac{(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\,r^3\,aa^2}{ee}$$

$$\text{Christoffel\_Gamma2}(4, 4, 2) = -\frac{(-1 + \cos(\theta)^2)\,r^4\,aa^2\,\sin(\theta)\cos(\theta)}{ee}$$

---------- Non Zero Residue Connection Coefficients {T}= {C} - {Gamma} for the given perturbations ----------

$$T(1, 1, 1) = 2\,\frac{aa^2\,r^3\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\cos(\theta)^2}{ee}$$

$$T(1, 1, 2) = -\frac{(1 - 3\cos(\theta)^2 + 2\cos(\theta)^4)\,r^4\,aa^2\,\sin(\theta)\cos(\theta)}{ee}$$

$$T(1, 1, 4) = \frac{(-1 + \cos(\theta)^6 - 3\cos(\theta)^4 + 3\cos(\theta)^2)\,r^5\,aa^3\,\cos(\theta)}{ee}$$

$$T(1, 2, 1) = -\frac{(1 - 3\cos(\theta)^2 + 2\cos(\theta)^4)\,r^4\,aa^2\,\sin(\theta)\cos(\theta)}{ee}$$

$$T(1, 2, 2) = -2\,\frac{r^5\,\cos(\theta)^2\,aa^2\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)}{ee}$$

$$T(1, 2, 4) = -\frac{(aa^2\,r^4\,\cos(\theta)^6 - 2\,aa^2\,r^4\,\cos(\theta)^4 + aa^2\,r^4\,\cos(\theta)^2 - ee)\,r^2\,aa\,\sin(\theta)}{ee}$$

$$T(1, 4, 1) = \frac{\cos(\theta)\,(\cos(\theta)^6\,aa^2\,r^4 - 3\,aa^2\,r^4\,\cos(\theta)^4 + 2\cos(\theta)^2\,ee + 3\,aa^2\,r^4\,\cos(\theta)^2 - 2\,ee - aa^2\,r^4)\,aa\,r}{ee}$$

$$T(1, 4, 2) = -\frac{(\cos(\theta)^6\,aa^2\,r^4 - 2\,aa^2\,r^4\,\cos(\theta)^4 + aa^2\,r^4\,\cos(\theta)^2 + 2\cos(\theta)^2\,ee - ee)\,r^2\,aa\,\sin(\theta)}{ee}$$

$$T(1, 4, 4) = 2\,r^3\,aa^2\,\cos(\theta)^4 - 4\,r^3\,aa^2\,\cos(\theta)^2 + 2\,r^3\,aa^2$$

$$T(2, 1, 1) = -2\,\frac{aa^2\,r^2\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\sin(\theta)\cos(\theta)}{ee}$$

$$T(2, 1, 2) = -\frac{aa^2\,r^3\,(-1 + 2\cos(\theta)^6 + 4\cos(\theta)^2 - 5\cos(\theta)^4)}{ee}$$

$$T(2, 1, 4) = -\frac{(\cos(\theta)^6\, aa^2\, r^4 - 3\, aa^2\, r^4 \cos(\theta)^4 + 3\, aa^2\, r^4 \cos(\theta)^2 + ee - aa^2\, r^4)\, aa \sin(\theta)}{ee}$$

$$T(2, 2, 1) = -\frac{aa^2\, r^3\, (-1 + 2\cos(\theta)^6 + 4\cos(\theta)^2 - 5\cos(\theta)^4)}{ee}$$

$$T(2, 2, 2) = 2\,\frac{aa^2\, r^4\, (\cos(\theta)^4 - 2\cos(\theta)^2 + 1)\sin(\theta)\cos(\theta)}{ee}$$

$$T(2, 2, 4) = -\frac{aa^3\, r^5\, (-1 + \cos(\theta)^6 - 3\cos(\theta)^4 + 3\cos(\theta)^2)\cos(\theta)}{ee}$$

$$T(2, 4, 1) = -\frac{\sin(\theta)\,(\cos(\theta)^6\, aa^2\, r^4 - 3\, aa^2\, r^4 \cos(\theta)^4 + 2\cos(\theta)^2\, ee + 3\, aa^2\, r^4 \cos(\theta)^2 - ee - aa^2\, r^4)\, aa}{ee}$$

$$T(2, 4, 2) = -\frac{\cos(\theta)\, aa\, r\,(aa^2\, r^4 \cos(\theta)^6 - 3\, aa^2\, r^4 \cos(\theta)^4 + 3\, aa^2\, r^4 \cos(\theta)^2 + 2\cos(\theta)^2\, ee - 2\, ee - aa^2\, r^4)}{ee}$$

$$T(2, 4, 4) = -2\,(-1 + \cos(\theta)^2)\, aa^2\, r^2 \sin(\theta)\cos(\theta)$$

$$T(4, 1, 1) = -2\,\frac{aa \cos(\theta)\, r\,(-1 + \cos(\theta)^2)}{ee}$$

$$T(4, 1, 2) = \frac{(2\cos(\theta)^2 - 1)\, r^2\, aa \sin(\theta)}{ee}$$

$$T(4, 1, 4) = -\frac{r^3\, aa^2\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)}{ee}$$

$$T(4, 2, 1) = \frac{(2\cos(\theta)^2 - 1)\, r^2\, aa \sin(\theta)}{ee}$$

$$T(4, 2, 2) = 2\,\frac{aa\, r^3\,(-1 + \cos(\theta)^2)\cos(\theta)}{ee}$$

$$T(4, 2, 4) = \frac{(-1 + \cos(\theta)^2)\, r^4\, aa^2 \sin(\theta)\cos(\theta)}{ee}$$

$$T(4, 4, 1) = -\frac{r^3\, aa^2\,(\cos(\theta)^4 - 2\cos(\theta)^2 + 1)}{ee}$$

$$T(4, 4, 2) = \frac{(-1 + \cos(\theta)^2)\, r^4\, aa^2 \sin(\theta)\cos(\theta)}{ee}$$

---------- Non Zero Affine Torsion for the perturbations ----------

$$\text{CartanaffineTorsion}(1, 4, 1) = aa \cos(\theta)\, r\,(\cos(\theta) - 1)(\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(1, 4, 2) = -\cos(\theta)^2\, aa\, r^2 \sin(\theta)$$

$$\text{CartanaffineTorsion}(1, 1, 4) = -aa \cos(\theta)\, r\,(\cos(\theta) - 1)(\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(1, 2, 4) = \cos(\theta)^2\, aa\, r^2 \sin(\theta)$$

$$\text{CartanaffineTorsion}(2, 4, 1) = -\sin(\theta)\, aa\,(\cos(\theta) - 1)(\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(2, 4, 2) = -aa \cos(\theta)\, r\,(\cos(\theta) - 1)(\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(2, 1, 4) = \sin(\theta)\, aa\,(\cos(\theta) - 1)(\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(2, 2, 4) = aa \cos(\theta)\, r\,(\cos(\theta) - 1)(\cos(\theta) + 1)$$

```
> `---------------- Perturbation Coefficients  ------------------`;`   `;
aaa:=0;bbb:=bb;ccc:=cc;aaa0:=0;ccc0:=0;eee:=ee;`Rotation with metric pertubation,
W-Affine Frame, Non Zero Affine Torsion`;
```

```
> COEF(aaa,bbb,ccc,aaa0,ccc0,eee);
```

$$aaa := 0$$
$$bbb := bb$$
$$ccc := cc$$
$$aaa0 := 0$$
$$ccc0 := 0$$
$$eee := ee$$

Rotation with metric pertubation, W-Affine Frame, Non Zero Affine Torsion

Basis Frame for given pertubations

$$
\begin{bmatrix}
\sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\
\sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\
\cos(\theta) & -r\sin(\theta) & 0 & 0 \\
0 & 0 & -cc\,r^2 + cc\,r^2\cos(\theta)^2 & \sqrt{1+\dfrac{bb}{r}}
\end{bmatrix}
$$

--------- *Non Zero Cartan Connection Coefficients {C} for the given perturbations* ----------

$$\text{Cartan\_RIGHT}(2,1,2) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,1,3) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(4,1,3) = -\frac{-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0}{r\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(1,2,2) = -r$$

$$\text{Cartan\_RIGHT}(2,2,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,2,3) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(4,2,3) = -\frac{\cos(\theta)(-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0)}{\sin(\theta)\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(1,3,3) = r(\cos(\theta)-1)(\cos(\theta)+1)$$

$$\text{Cartan\_RIGHT}(2,3,3) = -\cos(\theta)\sin(\theta)$$

$$\text{Cartan\_RIGHT}(3,3,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3,3,2) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(4, 3, 1) = \frac{cc0 + cc\, r^2 \cos(\theta)^2 - cc\, r^2}{\sqrt{\dfrac{r + bb}{r}}\, r}$$

$$\text{Cartan\_RIGHT}(4, 3, 2) = \frac{\cos(\theta)\,(cc0 + cc\, r^2 \cos(\theta)^2 - cc\, r^2)}{\sin(\theta)\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{Cartan\_RIGHT}(4, 4, 1) = -\frac{1}{2}\frac{bb}{(r + bb)\, r}$$

---------- Non Zero Christoffel Connection Coefficients {Gamma} for the perturbations ----------

$$\text{Christoffel\_Gamma2}(1, 2, 2) = -r$$

$$\text{Christoffel\_Gamma2}(1, 3, 3) = -2\, ee\, cc^2\, r^3 \cos(\theta)^4 + r \cos(\theta)^2 - r + 4\, ee\, cc^2\, r^3 \cos(\theta)^2 - 2\, ee\, cc^2\, r^3$$

$$\text{Christoffel\_Gamma2}(1, 3, 4) = -\frac{1}{4}\frac{(3\, bb \cos(\theta)^2 + 4\, r \cos(\theta)^2 - 3\, bb - 4\, r)\, ee\, cc}{\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{Christoffel\_Gamma2}(1, 4, 3) = -\frac{1}{4}\frac{(3\, bb \cos(\theta)^2 + 4\, r \cos(\theta)^2 - 3\, bb - 4\, r)\, ee\, cc}{\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{Christoffel\_Gamma2}(1, 4, 4) = \frac{1}{2}\frac{ee\, bb}{r^2}$$

$$\text{Christoffel\_Gamma2}(2, 1, 2) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2, 2, 1) = \frac{1}{r}$$

$$\text{Christoffel\_Gamma2}(2, 3, 3) = (-1 - 2\, ee\, cc^2\, r^2 + 2\, ee\, cc^2\, r^2 \cos(\theta)^2)\sin(\theta)\cos(\theta)$$

$$\text{Christoffel\_Gamma2}(2, 3, 4) = \frac{(r + bb)\, cc\, ee \sin(\theta)\cos(\theta)}{r\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{Christoffel\_Gamma2}(2, 4, 3) = \frac{(r + bb)\, cc\, ee \sin(\theta)\cos(\theta)}{r\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{Christoffel\_Gamma2}(3, 1, 3) =$$
$$-\frac{1}{4}\frac{4\, ee\, cc^2\, r^3 \cos(\theta)^2 + 5\, r^2\, ee\, bb\, cc^2 \cos(\theta)^2 - 5\, r^2\, ee\, bb\, cc^2 - 4\, r - 4\, ee\, cc^2\, r^3 - 4\, bb}{r\,(r + bb)}$$

$$\text{Christoffel\_Gamma2}(3, 1, 4) = -\frac{1}{4}\frac{(5\, bb + 4\, r)\, ee\, cc}{r^2\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{Christoffel\_Gamma2}(3, 2, 3) = \frac{(-1 - ee\, cc^2\, r^2 + ee\, cc^2\, r^2 \cos(\theta)^2)\sin(\theta)\cos(\theta)}{-1 + \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3, 2, 4) = \frac{ee\, cc\, \sin(\theta)\cos(\theta)\sqrt{\dfrac{r+bb}{r}}}{-1+\cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3, 3, 1) =$$
$$-\frac{1}{4}\frac{4\,ee\, cc^2\, r^3\cos(\theta)^2 + 5\,r^2\, ee\, bb\, cc^2\cos(\theta)^2 - 5\,r^2\, ee\, bb\, cc^2 - 4\,r - 4\,ee\, cc^2\, r^3 - 4\,bb}{r\,(r+bb)}$$

$$\text{Christoffel\_Gamma2}(3, 3, 2) = \frac{(-1 - ee\, cc^2\, r^2 + ee\, cc^2\, r^2\cos(\theta)^2)\sin(\theta)\cos(\theta)}{-1+\cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3, 4, 1) = -\frac{1}{4}\frac{(5\,bb + 4\,r)\,ee\, cc}{r^2\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Christoffel\_Gamma2}(3, 4, 2) = \frac{ee\, cc\, \sin(\theta)\cos(\theta)\sqrt{\dfrac{r+bb}{r}}}{-1+\cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(4, 1, 3) = \frac{1}{4}\big(4\,ee\, cc^2\, r^3\cos(\theta)^4 + 5\,r^2\, ee\, bb\, cc^2\cos(\theta)^4 - 8\,ee\, cc^2\, r^3\cos(\theta)^2 - bb\cos(\theta)^2$$
$$- 10\,r^2\, ee\, bb\, cc^2\cos(\theta)^2 + 5\,r^2\, ee\, bb\, cc^2 + 4\,ee\, cc^2\, r^3 + bb\big)\,cc\, r \Big/ \left(\sqrt{\dfrac{r+bb}{r}}\,(r+bb)\right)$$

$$\text{Christoffel\_Gamma2}(4, 1, 4) = \frac{1}{4}\frac{4\,ee\, cc^2\, r^3\cos(\theta)^2 + 5\,r^2\, ee\, bb\, cc^2\cos(\theta)^2 - 4\,ee\, cc^2\, r^3 - 2\,bb - 5\,r^2\, ee\, bb\, cc^2}{r\,(r+bb)}$$

$$\text{Christoffel\_Gamma2}(4, 2, 3) = -\frac{(-1+\cos(\theta)^2)\,ee\, cc^3\sin(\theta)\cos(\theta)\,r^5\sqrt{\dfrac{r+bb}{r}}}{r+bb}$$

$$\text{Christoffel\_Gamma2}(4, 2, 4) = -r^2\, cc^2\, ee\, \sin(\theta)\cos(\theta)$$

$$\text{Christoffel\_Gamma2}(4, 3, 1) = \frac{1}{4}\big(4\,ee\, cc^2\, r^3\cos(\theta)^4 + 5\,r^2\, ee\, bb\, cc^2\cos(\theta)^4 - 8\,ee\, cc^2\, r^3\cos(\theta)^2 - bb\cos(\theta)^2$$
$$- 10\,r^2\, ee\, bb\, cc^2\cos(\theta)^2 + 5\,r^2\, ee\, bb\, cc^2 + 4\,ee\, cc^2\, r^3 + bb\big)\,cc\, r \Big/ \left(\sqrt{\dfrac{r+bb}{r}}\,(r+bb)\right)$$

$$\text{Christoffel\_Gamma2}(4, 3, 2) = -\frac{(-1+\cos(\theta)^2)\,ee\, cc^3\sin(\theta)\cos(\theta)\,r^5\sqrt{\dfrac{r+bb}{r}}}{r+bb}$$

$$\text{Christoffel\_Gamma2}(4, 4, 1) = \frac{1}{4}\frac{4\,ee\, cc^2\, r^3\cos(\theta)^2 + 5\,r^2\, ee\, bb\, cc^2\cos(\theta)^2 - 4\,ee\, cc^2\, r^3 - 2\,bb - 5\,r^2\, ee\, bb\, cc^2}{r\,(r+bb)}$$

$$\text{Christoffel\_Gamma2}(4, 4, 2) = -r^2\, cc^2\, ee\, \sin(\theta)\cos(\theta)$$

---------- *Non Zero Residue Connection Coefficients {T} = {C} - {Gamma} for the given perturbations* ----------

$$T(1, 3, 3) = 2\,ee\, cc^2\, r^3\cos(\theta)^4 - 4\,ee\, cc^2\, r^3\cos(\theta)^2 + 2\,ee\, cc^2\, r^3$$

$$T(1, 3, 4) = \frac{1}{4}\frac{(3\,bb\cos(\theta)^2 + 4\,r\cos(\theta)^2 - 3\,bb - 4\,r)\,ee\, cc}{\sqrt{\dfrac{r+bb}{r}}}$$

$$T(1, 4, 3) = \frac{1}{4}\frac{(3\,bb\,\cos(\theta)^2 + 4\,r\,\cos(\theta)^2 - 3\,bb - 4\,r)\,ee\,cc}{\sqrt{\dfrac{r+bb}{r}}}$$

$$T(1, 4, 4) = -\frac{1}{2}\frac{ee\,bb}{r^2}$$

$$T(2, 3, 3) = -2\cos(\theta)\sin(\theta)\,ee\,cc^2\,r^2\,(-1 + \cos(\theta)^2)$$

$$T(2, 3, 4) = -\frac{(r+bb)\,cc\,ee\,\sin(\theta)\,\cos(\theta)}{r\sqrt{\dfrac{r+bb}{r}}}$$

$$T(2, 4, 3) = -\frac{(r+bb)\,cc\,ee\,\sin(\theta)\,\cos(\theta)}{r\sqrt{\dfrac{r+bb}{r}}}$$

$$T(3, 1, 3) = \frac{1}{4}\frac{(4\,r\,\cos(\theta)^2 + 5\,bb\,\cos(\theta)^2 - 4\,r - 5\,bb)\,ee\,cc^2\,r}{r+bb}$$

$$T(3, 1, 4) = \frac{1}{4}\frac{(4\,r + 5\,bb)\,ee\,cc}{r^2\sqrt{\dfrac{r+bb}{r}}}$$

$$T(3, 2, 3) = -\cos(\theta)\sin(\theta)\,ee\,cc^2\,r^2$$

$$T(3, 2, 4) = -\frac{ee\,cc\,\sin(\theta)\,\cos(\theta)\sqrt{\dfrac{r+bb}{r}}}{-1 + \cos(\theta)^2}$$

$$T(3, 3, 1) = \frac{1}{4}\frac{(4\,r\,\cos(\theta)^2 + 5\,bb\,\cos(\theta)^2 - 4\,r - 5\,bb)\,ee\,cc^2\,r}{r+bb}$$

$$T(3, 3, 2) = -\cos(\theta)\sin(\theta)\,ee\,cc^2\,r^2$$

$$T(3, 4, 1) = \frac{1}{4}\frac{(4\,r + 5\,bb)\,ee\,cc}{r^2\sqrt{\dfrac{r+bb}{r}}}$$

$$T(3, 4, 2) = -\frac{ee\,cc\,\sin(\theta)\,\cos(\theta)\sqrt{\dfrac{r+bb}{r}}}{-1 + \cos(\theta)^2}$$

$$T(4, 1, 3) = -\frac{1}{4}(5\,ee\,cc^2\,r^2\,\cos(\theta)^4\,bb + 4\,ee\,cc^2\,r^3\,\cos(\theta)^4 + 3\,bb\,\cos(\theta)^2 - 8\,ee\,cc^2\,r^3\,\cos(\theta)^2$$
$$- 10\,ee\,cc^2\,r^2\,\cos(\theta)^2\,bb + 4\,r\,\cos(\theta)^2 + 5\,ee\,cc^2\,r^2\,bb + 4\,ee\,cc^2\,r^3 - 3\,bb - 4\,r)\,cc\,r\left/\left(\sqrt{\frac{r+bb}{r}}\,(r+bb)\right)\right.$$

$$T(4, 1, 4) = -\frac{1}{4}\frac{4\,ee\,cc^2\,r^3\,\cos(\theta)^2 + 5\,ee\,cc^2\,r^2\,\cos(\theta)^2\,bb - 4\,ee\,cc^2\,r^3 - 2\,bb - 5\,ee\,cc^2\,r^2\,bb}{r\,(r+bb)}$$

$$T(4, 2, 3) = \frac{\sqrt{\dfrac{r+bb}{r}}\,\sin(\theta)\,cc\,(ee\,cc^2\,r^2\,\cos(\theta)^2 - ee\,cc^2\,r^2 + 1)\,r^3\,\cos(\theta)}{r+bb}$$

$$T(4, 2, 4) = r^2\,cc^2\,ee\,\sin(\theta)\,\cos(\theta)$$

$$T(4, 3, 1) = -\frac{1}{4}(4\,ee\,cc^2\,r^3\,\cos(\theta)^4 + 5\,ee\,cc^2\,r^2\,\cos(\theta)^4\,bb - 5\,bb\,\cos(\theta)^2 - 8\,ee\,cc^2\,r^3\,\cos(\theta)^2$$

$$- 10\, ee\, cc^2\, r^2 \cos(\theta)^2\, bb - 4\, r \cos(\theta)^2 + 5\, ee\, cc^2\, r^2\, bb + 4\, ee\, cc^2\, r^3 + 5\, bb + 4\, r)\, cc\, r \Bigg/ \left(\sqrt{\frac{r+bb}{r}}\, (r+bb) \right)$$

$$T(4,3,2) = \frac{\sqrt{\dfrac{r+bb}{r}}\, \sin(\theta)\, cc\, (ee\, cc^2\, r^2 \cos(\theta)^2 - 1 - ee\, cc^2\, r^2)\, r^3 \cos(\theta)}{r+bb}$$

$$T(4,4,1) = -\frac{1}{4}\frac{(5\, bb \cos(\theta)^2 + 4\, r \cos(\theta)^2 - 5\, bb - 4\, r)\, ee\, cc^2\, r}{r+bb}$$

$$T(4,4,2) = r^2\, cc^2\, ee\, \sin(\theta) \cos(\theta)$$

---------- *Non Zero Affine Torsion for the perturbations* ----------

$$\text{CartanaffineTorsion}(4,3,1) = \frac{cc\, r\, (-1+\cos(\theta))\, (\cos(\theta)+1)}{\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{CartanaffineTorsion}(4,4,1) = -\frac{1}{4}\frac{bb}{(r+bb)\, r}$$

$$\text{CartanaffineTorsion}(4,3,2) = \frac{\cos(\theta)\, cc\, r^2\, (-1+\cos(\theta))\, (\cos(\theta)+1)}{\sin(\theta)\, \sqrt{\dfrac{r+bb}{r}}}$$

$$\text{CartanaffineTorsion}(4,1,3) = -\frac{cc\, r\, (-1+\cos(\theta))\, (\cos(\theta)+1)}{\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{CartanaffineTorsion}(4,2,3) = -\frac{\cos(\theta)\, cc\, r^2\, (-1+\cos(\theta))\, (\cos(\theta)+1)}{\sin(\theta)\, \sqrt{\dfrac{r+bb}{r}}}$$

$$\text{CartanaffineTorsion}(4,1,4) = \frac{1}{4}\frac{bb}{(r+bb)\, r}$$

```
> `---------------- Perturbation Coefficients  ------------------`;`  `;
aaa:=aa;bbb:=bb;ccc:=cc;aaa0:=aa0;ccc0:=cc0;eee:=ee;`All Perturbations On, Non
Zero Affine Torsion`;
> COEF(aaa,bbb,ccc,aaa0,ccc0,eee);
```

---------------- *Perturbation Coefficients* ------------------

$$aaa := aa$$
$$bbb := bb$$
$$ccc := cc$$
$$aaa0 := aa0$$
$$ccc0 := cc0$$
$$eee := ee$$

All Perturbations On, Non Zero Affine Torsion

Basis Frame for given pertubations

$$\begin{bmatrix} \sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) & 0 \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & -aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0 \\ 0 & 0 & -cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0 & \sqrt{1+\dfrac{bb}{r}} \end{bmatrix}$$

---------- *Non Zero Cartan Connection Coefficients {C} for the given perturbations* ----------

$$\text{Cartan\_RIGHT}(1,1,3) = \frac{\cos(\theta)\,(-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0)\,(-aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0)}{r\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(2,1,2) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(2,1,3) = -\frac{\sin(\theta)\,(-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0)\,(-aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0)}{r^2\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(3,1,3) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(4,1,3) = -\frac{-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0}{r\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(1,2,2) = -r$$

$$\text{Cartan\_RIGHT}(1,2,3) = \frac{\cos(\theta)^2\,(-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0)\,(-aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0)}{\sin(\theta)\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(2,2,1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(2,2,3) = -\frac{\cos(\theta)\,(-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0)\,(-aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0)}{r\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(3,2,3) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(4,2,3) = -\frac{\cos(\theta)\,(-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0)}{\sin(\theta)\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(1,3,1) = -\frac{\cos(\theta)\,(cc0 + cc\,r^2\cos(\theta)^2 - cc\,r^2)\,(-aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0)}{r\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(1,3,2) = -\frac{\cos(\theta)^2\,(cc0 + cc\,r^2\cos(\theta)^2 - cc\,r^2)\,(-aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0)}{\sin(\theta)\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(1,3,3) = r\,(\cos(\theta)-1)\,(\cos(\theta)+1)$$

$$\text{Cartan\_RIGHT}(2, 3, 1) = \frac{\sin(\theta)\,(cc0 + cc\,r^2\cos(\theta)^2 - cc\,r^2)\,(-aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0)}{\sqrt{\dfrac{r+bb}{r}}\,r^2}$$

$$\text{Cartan\_RIGHT}(2, 3, 2) = \frac{\cos(\theta)\,(cc0 + cc\,r^2\cos(\theta)^2 - cc\,r^2)\,(-aa\,r^2 + aa\,r^2\cos(\theta)^2 - aa0)}{r\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(2, 3, 3) = -\cos(\theta)\sin(\theta)$$

$$\text{Cartan\_RIGHT}(3, 3, 1) = \frac{1}{r}$$

$$\text{Cartan\_RIGHT}(3, 3, 2) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\text{Cartan\_RIGHT}(4, 3, 1) = \frac{cc0 + cc\,r^2\cos(\theta)^2 - cc\,r^2}{\sqrt{\dfrac{r+bb}{r}}\,r}$$

$$\text{Cartan\_RIGHT}(4, 3, 2) = \frac{\cos(\theta)\,(cc0 + cc\,r^2\cos(\theta)^2 - cc\,r^2)}{\sin(\theta)\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{Cartan\_RIGHT}(1, 4, 1) = \frac{1}{2}\frac{\cos(\theta)\,(-4\,aa\,r^3 - 5\,bb\,aa\,r^2 + 4\,aa\,r^3\cos(\theta)^2 + 5\,bb\,aa\,r^2\cos(\theta)^2 - bb\,aa0)}{r\,(r+bb)}$$

$$\text{Cartan\_RIGHT}(1, 4, 2) = -2\cos(\theta)^2\,aa\,r^2\sin(\theta)$$

$$\text{Cartan\_RIGHT}(2, 4, 1) = -\frac{1}{2}\frac{\sin(\theta)\,(-4\,aa\,r^3 - 5\,bb\,aa\,r^2 + 4\,aa\,r^3\cos(\theta)^2 + 5\,bb\,aa\,r^2\cos(\theta)^2 - bb\,aa0)}{(r+bb)\,r^2}$$

$$\text{Cartan\_RIGHT}(2, 4, 2) = -2\cos(\theta)\,aa\,r\,(\cos(\theta) - 1)\,(\cos(\theta) + 1)$$

$$\text{Cartan\_RIGHT}(4, 4, 1) = -\frac{1}{2}\frac{bb}{(r+bb)\,r}$$

---------- Non Zero Christoffel Connection Coefficients {Gamma} for the perturbations ----------

$\text{Christoffel\_Gamma2}(1, 1, 1) = 2\,(\cos(\theta)^6\,aa\,r^6\,ee\,cc^2 - 3\cos(\theta)^4\,aa\,r^6\,ee\,cc^2 - \cos(\theta)^4\,r^4\,aa0\,ee\,cc^2$

$\quad - r^4\cos(\theta)^4\,aa - 2\cos(\theta)^4\,aa\,r^4\,ee\,cc\,cc0 + 4\cos(\theta)^2\,aa\,r^4\,ee\,cc\,cc0 + \cos(\theta)^2\,aa\,r^2\,ee\,cc0^2$

$\quad + 2\cos(\theta)^2\,r^2\,aa0\,ee\,cc\,cc0 + 3\cos(\theta)^2\,aa\,r^6\,ee\,cc^2 + 2\,aa\,r^4\cos(\theta)^2 + 2\cos(\theta)^2\,r^4\,aa0\,ee\,cc^2 + r^2\cos(\theta)^2\,aa0$

$\quad - ee\,cc0^2\,aa0 - ee\,cc^2\,aa0\,r^4 - aa\,r^4 - ee\,cc^2\,r^6\,aa - 2\,aa\,r^4\,ee\,cc\,cc0 - r^2\,aa0 - 2\,r^2\,ee\,cc\,cc0\,aa0 - aa\,ee\,cc0^2\,r^2)$

$\quad aa\cos(\theta)^2\,/\,((r+bb)\,ee)$

$\text{Christoffel\_Gamma2}(1, 1, 2) = -\cos(\theta)\sin(\theta)\,aa\,r\,(r^2\,aa0 + aa\,r^4 + ee\,cc^2\,r^6\,aa + ee\,cc^2\,aa0\,r^4 + 2\,aa\,r^4\,ee\,cc\,cc0$

$\quad + ee\,cc0^2\,aa0 + 2\,r^2\,ee\,cc\,cc0\,aa0 + aa\,ee\,cc0^2\,r^2 + 2\cos(\theta)^4\,r^2\,aa0 - 2\cos(\theta)^6\,aa\,r^4 - 2\cos(\theta)^6\,r^4\,aa0\,ee\,cc^2$

$\quad + 2\cos(\theta)^8\,aa\,r^6\,ee\,cc^2 + 4\cos(\theta)^4\,r^2\,aa0\,ee\,cc\,cc0 + 2\cos(\theta)^4\,aa\,r^2\,ee\,cc0^2 - 4\cos(\theta)^6\,aa\,r^4\,ee\,cc\,cc0$

$\quad - 6\cos(\theta)^2\,r^2\,aa0\,ee\,cc\,cc0 - 5\cos(\theta)^2\,aa\,r^6\,ee\,cc^2 - 4\cos(\theta)^2\,r^4\,aa0\,ee\,cc^2 - 3\,r^2\cos(\theta)^2\,aa0$

$\quad - 7\cos(\theta)^6\,aa\,r^6\,ee\,cc^2 + 9\cos(\theta)^4\,aa\,r^6\,ee\,cc^2 + 5\cos(\theta)^4\,r^4\,aa0\,ee\,cc^2 + 10\cos(\theta)^4\,aa\,r^4\,ee\,cc\,cc0$

$\quad - 8\cos(\theta)^2\,aa\,r^4\,ee\,cc\,cc0 - 3\cos(\theta)^2\,aa\,r^2\,ee\,cc0^2 - 4\,aa\,r^4\cos(\theta)^2 + 5\,r^4\cos(\theta)^4\,aa - 2\cos(\theta)^2\,aa0\,ee\,cc0^2)\,\Big/\,($

$\quad ee\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2))$

$\text{Christoffel\_Gamma2}(1, 1, 3) = -\dfrac{1}{4}\cos(\theta)\,(5\,ee\,aa\,r^8\,cc^3\,bb + 5\,aa0\,r^2\,cc0\,bb + 5\,aa\,r^4\,cc0\,bb + ee\,aa\,r^2\,cc0^3\,bb$

$$+ 8\, ee\, aa0\, cc^2\, r^5\, cc0 + 4\, aa\, r^5\, cc0 + 7\, ee\, aa\, r^4\, cc\, cc0^2\, bb + 4\, ee\, aa0\, cc\, r^3\, cc0^2 + aa0\, r^4\, cc\, bb + 4\, ee\, aa\, r^9\, cc^3$$

$$+ 11\, ee\, aa0\, cc^2\, r^4\, cc0\, bb + 4\, ee\, aa\, r^5\, cc\, cc0^2 + aa\, r^6\, cc\, bb + 11\, ee\, aa\, r^6\, cc^2\, cc0\, bb + 7\, ee\, aa0\, cc\, r^2\, cc0^2\, bb$$

$$+ ee\, aa0\, cc0^3\, bb + 5\, ee\, aa0\, cc^3\, r^6\, bb + 8\, ee\, aa\, r^7\, cc^2\, cc0 + 4\, aa0\, r^3\, cc0 + 4\, ee\, aa0\, cc^3\, r^7 - 8\, \cos(\theta)^6\, ee\, aa\, r^7\, cc^2\, cc0$$

$$- 11\, \cos(\theta)^6\, ee\, aa\, r^6\, cc^2\, cc0\, bb + 7\, \cos(\theta)^4\, ee\, aa\, r^4\, cc\, cc0^2\, bb + 4\, \cos(\theta)^4\, ee\, aa\, r^5\, cc\, cc0^2 + 5\, \cos(\theta)^4\, aa\, r^4\, cc0\, bb$$

$$- 14\, \cos(\theta)^2\, ee\, aa\, r^4\, cc\, cc0^2\, bb - 20\, \cos(\theta)^6\, ee\, aa\, r^8\, cc^3\, bb - 16\, \cos(\theta)^6\, ee\, aa\, r^9\, cc^3 + 4\, \cos(\theta)^8\, ee\, aa\, r^9\, cc^3$$

$$- 12\, \cos(\theta)^2\, ee\, aa0\, cc^3\, r^7 - 4\, \cos(\theta)^2\, ee\, aa0\, cc\, r^3\, cc0^2 + 8\, \cos(\theta)^4\, ee\, aa0\, cc^2\, r^5\, cc0 + \cos(\theta)^4\, aa0\, r^4\, cc\, bb$$

$$- 4\, \cos(\theta)^2\, aa0\, r^3\, cc0 - 15\, \cos(\theta)^2\, ee\, aa0\, cc^3\, r^6\, bb + 15\, \cos(\theta)^4\, ee\, aa0\, cc^3\, r^6\, bb - \cos(\theta)^2\, ee\, aa\, r^2\, cc0^3\, bb$$

$$+ 3\, \cos(\theta)^4\, aa\, r^6\, cc\, bb - \cos(\theta)^6\, aa\, r^6\, cc\, bb + 30\, \cos(\theta)^4\, ee\, aa\, r^8\, cc^3\, bb - 16\, \cos(\theta)^2\, ee\, aa\, r^9\, cc^3$$

$$- 7\, \cos(\theta)^2\, ee\, aa0\, cc\, r^2\, cc0^2\, bb - 5\, \cos(\theta)^6\, ee\, aa0\, cc^3\, r^6\, bb - 4\, \cos(\theta)^6\, ee\, aa0\, cc^3\, r^7$$

$$- 22\, \cos(\theta)^2\, ee\, aa0\, cc^2\, r^4\, cc0\, bb - 20\, \cos(\theta)^2\, ee\, aa\, r^8\, cc^3\, bb + 12\, \cos(\theta)^4\, ee\, aa0\, cc^3\, r^7 - 2\, \cos(\theta)^2\, aa0\, r^4\, cc\, bb$$

$$- 5\, \cos(\theta)^2\, aa0\, r^2\, cc0\, bb + 5\, \cos(\theta)^8\, ee\, aa\, r^8\, cc^3\, bb + 24\, \cos(\theta)^4\, ee\, aa\, r^7\, cc^2\, cc0 + 33\, \cos(\theta)^4\, ee\, aa\, r^6\, cc^2\, cc0\, bb$$

$$+ 24\, \cos(\theta)^4\, ee\, aa\, r^9\, cc^3 - 33\, \cos(\theta)^2\, ee\, aa\, r^6\, cc^2\, cc0\, bb - 16\, \cos(\theta)^2\, ee\, aa0\, cc^2\, r^5\, cc0 - 10\, \cos(\theta)^2\, aa\, r^4\, cc0\, bb$$

$$- 3\, \cos(\theta)^2\, aa\, r^6\, cc\, bb - 24\, \cos(\theta)^2\, ee\, aa\, r^7\, cc^2\, cc0 - 8\, \cos(\theta)^2\, ee\, aa\, r^5\, cc\, cc0^2 + 11\, \cos(\theta)^4\, ee\, aa0\, cc^2\, r^4\, cc0\, bb$$

$$- 8\, \cos(\theta)^2\, aa\, r^5\, cc0 + 4\, \cos(\theta)^4\, aa\, r^5\, cc0 \Big) \Big/ \left(r^3\, (-bb + bb\, \cos(\theta)^2 - r + r\, \cos(\theta)^2) \sqrt{\frac{r + bb}{r}} \right)$$

$$\text{Christoffel\_Gamma2}(1, 1, 4) = \frac{1}{4} \cos(\theta)\, (-4\, aa^3\, r^9 - 12\, \cos(\theta)^4\, aa0^2\, ee\, cc^2\, r^7\, aa + 16\, \cos(\theta)^2\, aa^3\, r^9$$

$$- 24\, \cos(\theta)^4\, aa^3\, r^9 + 16\, \cos(\theta)^6\, aa^3\, r^9 - 4\, \cos(\theta)^8\, aa^3\, r^9 - 8\, aa0\, r^7\, aa^2 + 4\, aa0\, ee^2\, cc0\, cc\, r^3 + 4\, aa0\, ee^2\, cc^2\, r^5$$

$$+ aa0\, ee^2\, cc0^2\, bb + aa\, r^2\, ee^2\, cc0^2\, bb - 4\, aa0^2\, r^5\, aa - 8\, aa^3\, r^9\, ee\, cc\, cc0 - 4\, aa0^2\, ee\, cc0^2\, aa\, r^3 - 4\, aa^3\, r^{11}\, ee\, cc^2$$

$$+ 6\, aa\, r^4\, ee^2\, cc\, cc0\, bb + 4\, aa\, r^5\, ee^2\, cc0\, cc + 4\, aa\, r^7\, ee^2\, cc^2 + 2\, aa0\, r^2\, ee\, bb - 8\, aa0^2\, ee\, cc\, r^5\, cc0\, aa + 2\, aa\, r^4\, ee\, bb$$

$$+ 5\, aa\, r^6\, ee^2\, cc^2\, bb - 4\, aa^3\, r^7\, ee\, cc0^2 + 6\, aa0\, ee^2\, cc\, r^2\, cc0\, bb - 4\, aa0^2\, ee\, cc^2\, r^7\, aa - 8\, aa0\, ee\, cc0^2\, aa^2\, r^5$$

$$- 8\, aa0\, ee\, cc^2\, r^9\, aa^2 + 5\, aa0\, ee^2\, cc^2\, r^4\, bb - 16\, aa0\, ee\, cc\, r^7\, cc0\, aa^2 - 8\, \cos(\theta)^2\, aa0\, ee^2\, cc^2\, r^5$$

$$+ 5\, \cos(\theta)^4\, aa0\, ee^2\, cc^2\, r^4\, bb + 16\, \cos(\theta)^2\, aa0^2\, ee\, cc\, r^5\, cc0\, aa + 4\, \cos(\theta)^2\, aa0^2\, ee\, cc0^2\, aa\, r^3$$

$$- 2\, \cos(\theta)^2\, aa0\, r^2\, ee\, bb + 16\, \cos(\theta)^6\, aa0\, ee\, cc\, r^7\, cc0\, aa^2 + 12\, \cos(\theta)^4\, aa\, r^7\, ee^2\, cc^2 + 12\, \cos(\theta)^2\, aa0^2\, ee\, cc^2\, r^7\, aa$$

$$+ 4\, \cos(\theta)^6\, aa^3\, r^7\, ee\, cc0^2 - 5\, \cos(\theta)^6\, aa\, r^6\, ee^2\, cc^2\, bb + 2\, \cos(\theta)^4\, aa\, r^4\, ee\, bb - 4\, \cos(\theta)^6\, aa\, r^7\, ee^2\, cc^2$$

$$- 8\, \cos(\theta)^4\, aa^2\, r^5\, ee\, cc0^2\, aa0 - 48\, \cos(\theta)^4\, aa^2\, r^9\, ee\, cc^2\, aa0 - 8\, \cos(\theta)^4\, aa0^2\, ee\, cc\, r^5\, cc0\, aa$$

$$- 8\, \cos(\theta)^8\, aa0\, ee\, cc^2\, r^9\, aa^2 - 8\, \cos(\theta)^2\, aa\, r^5\, ee^2\, cc0\, cc - 48\, \cos(\theta)^4\, aa^3\, r^9\, ee\, cc\, cc0 - 12\, \cos(\theta)^4\, aa^3\, r^7\, ee\, cc0^2$$

$$+ 6\, \cos(\theta)^4\, aa\, r^4\, ee^2\, cc\, cc0\, bb - 40\, \cos(\theta)^4\, aa^3\, r^{11}\, ee\, cc^2 + 4\, \cos(\theta)^4\, aa\, r^5\, ee^2\, cc0\, cc + 32\, \cos(\theta)^6\, aa^3\, r^9\, ee\, cc\, cc0$$

$$- 8\, \cos(\theta)^8\, aa^3\, r^9\, ee\, cc\, cc0 + 40\, \cos(\theta)^6\, aa^3\, r^{11}\, ee\, cc^2 - 24\, \cos(\theta)^4\, aa^2\, r^7\, aa0 + 24\, \cos(\theta)^2\, aa0\, r^7\, aa^2$$

$$+ 32\, \cos(\theta)^2\, aa^3\, r^9\, ee\, cc\, cc0 + 4\, \cos(\theta)^6\, aa0^2\, ee\, cc^2\, r^7\, aa + 12\, \cos(\theta)^2\, aa^3\, r^7\, ee\, cc0^2 - 12\, \cos(\theta)^2\, aa\, r^7\, ee^2\, cc^2$$

$$+ 20\, \cos(\theta)^2\, aa^3\, r^{11}\, ee\, cc^2 - 12\, \cos(\theta)^2\, aa\, r^4\, ee^2\, cc\, cc0\, bb - 20\, \cos(\theta)^8\, aa^3\, r^{11}\, ee\, cc^2 + 32\, \cos(\theta)^6\, aa^2\, r^9\, ee\, cc^2\, aa0$$

$$+ 32\, \cos(\theta)^2\, aa^2\, r^9\, ee\, cc^2\, aa0 - 4\, \cos(\theta)^2\, aa\, r^4\, ee\, bb - 6\, \cos(\theta)^2\, aa0\, ee^2\, cc\, r^2\, cc0\, bb + 15\, \cos(\theta)^4\, aa\, r^6\, ee^2\, cc^2\, bb$$

$$- 15\, \cos(\theta)^2\, aa\, r^6\, ee^2\, cc^2\, bb - \cos(\theta)^2\, aa\, r^2\, ee^2\, cc0^2\, bb + 16\, \cos(\theta)^2\, aa^2\, r^5\, ee\, cc0^2\, aa0$$

$$+ 48\, \cos(\theta)^2\, aa^2\, r^7\, ee\, cc\, cc0\, aa0 - 10\, \cos(\theta)^2\, aa0\, ee^2\, cc^2\, r^4\, bb + 8\, \cos(\theta)^2\, aa0^2\, r^5\, aa + 8\, \cos(\theta)^6\, aa0\, r^7\, aa^2$$

$$- 4\, \cos(\theta)^4\, aa0^2\, r^5\, aa + 4\, \cos(\theta)^4\, aa0\, ee^2\, cc^2\, r^5 - 4\, \cos(\theta)^2\, aa0\, ee^2\, cc0\, r^3\, cc + 4\, \cos(\theta)^{10}\, aa^3\, r^{11}\, ee\, cc^2$$

$$- 48\, \cos(\theta)^4\, aa^2\, r^7\, ee\, cc\, cc0\, aa0 \Big) \Big/ \left(ee\, r^3\, (-bb + bb\, \cos(\theta)^2 - r + r\, \cos(\theta)^2) \right)$$

$$\text{Christoffel\_Gamma2}(1, 2, 1) = - \cos(\theta)\, \sin(\theta)\, aa\, r\, (r^2\, aa0 + aa\, r^4 + ee\, cc^2\, r^6\, aa + ee\, cc^2\, aa0\, r^4 + 2\, aa\, r^4\, ee\, cc\, cc0$$

$$+ ee\, cc0^2\, aa0 + 2\, r^2\, ee\, cc\, cc0\, aa0 + aa\, ee\, cc0^2\, r^2 + 2\, \cos(\theta)^4\, r^2\, aa0 - 2\, \cos(\theta)^6\, aa\, r^4 - 2\, \cos(\theta)^6\, r^4\, aa0\, ee\, cc^2$$

$$+ 2\, \cos(\theta)^8\, aa\, r^6\, ee\, cc^2 + 4\, \cos(\theta)^4\, r^2\, aa0\, ee\, cc\, cc0 + 2\, \cos(\theta)^4\, aa\, r^2\, ee\, cc0^2 - 4\, \cos(\theta)^6\, aa\, r^4\, ee\, cc\, cc0$$

$$- 6\, \cos(\theta)^2\, r^2\, aa0\, ee\, cc\, cc0 - 5\, \cos(\theta)^2\, aa\, r^6\, ee\, cc^2 - 4\, \cos(\theta)^2\, r^4\, aa0\, ee\, cc^2 - 3\, r^2\, \cos(\theta)^2\, aa0$$

$$- 7\, \cos(\theta)^6\, aa\, r^6\, ee\, cc^2 + 9\, \cos(\theta)^4\, aa\, r^6\, ee\, cc^2 + 5\, \cos(\theta)^4\, r^4\, aa0\, ee\, cc^2 + 10\, \cos(\theta)^4\, aa\, r^4\, ee\, cc\, cc0$$

$$- 8\, \cos(\theta)^2\, aa\, r^4\, ee\, cc\, cc0 - 3\, \cos(\theta)^2\, aa\, r^2\, ee\, cc0^2 - 4\, aa\, r^4\, \cos(\theta)^2 + 5\, r^4\, \cos(\theta)^4\, aa - 2\, \cos(\theta)^2\, aa0\, ee\, cc0^2 \Big) \Big/ \Big($$

$$ee\,(-bb+bb\cos(\theta)^2-r+r\cos(\theta)^2))$$

$\text{Christoffel\_Gamma2}(1,2,2)=-(2\,ee\,cc^2\,r^7\cos(\theta)^8\,aa^2-4\,ee\,cc\,r^5\cos(\theta)^6\,cc0\,aa^2-2\,r^5\cos(\theta)^6\,aa^2$

$\quad-6\,ee\,cc^2\,r^7\cos(\theta)^6\,aa^2-2\,ee\,cc^2\,r^5\cos(\theta)^6\,aa\,aa0+4\,aa^2\,r^5\cos(\theta)^4+2\,ee\,cc0^2\,aa^2\,r^3\cos(\theta)^4$

$\quad+8\,ee\,cc\,r^5\,cc0\,aa^2\cos(\theta)^4+4\,ee\,cc\,r^3\cos(\theta)^4\,cc0\,aa\,aa0+4\,ee\,cc^2\,r^5\cos(\theta)^4\,aa\,aa0+6\,ee\,cc^2\,r^7\,aa^2\cos(\theta)^4$

$\quad+2\,r^3\cos(\theta)^4\,aa\,aa0-2\,ee\,cc0^2\,aa^2\,r^3\cos(\theta)^2-2\,ee\,cc^2\,r^5\,aa\cos(\theta)^2\,aa0-2\,ee\,cc^2\,r^7\,aa^2\cos(\theta)^2$

$\quad-4\,ee\,cc\,r^3\,cc0\,aa\cos(\theta)^2\,aa0-2\,ee\,cc0^2\,aa\,r\cos(\theta)^2\,aa0-2\,aa^2\,r^5\cos(\theta)^2-4\,ee\,cc\,r^5\,cc0\,aa^2\cos(\theta)^2$

$\quad-2\,aa\,r^3\cos(\theta)^2\,aa0+ee\,r+ee\,bb)\,r\,/\,((r+bb)\,ee)$

$\text{Christoffel\_Gamma2}(1,2,3)=\cos(\theta)^2\sqrt{\dfrac{r+bb}{r}}\,r\sin(\theta)\,(-ee\,r^6\,aa\,cc^3+3\cos(\theta)^2\,ee\,r^6\,aa\,cc^3$

$\quad-2\,ee\,r^4\,aa\,cc^2\,cc0-3\cos(\theta)^4\,ee\,r^6\,aa\,cc^3+4\cos(\theta)^2\,ee\,r^4\,aa\,cc^2\,cc0-r^2\,aa\,cc0-ee\,r^2\,aa\,cc\,cc0^2$

$\quad+\cos(\theta)^6\,ee\,r^6\,aa\,cc^3-2\cos(\theta)^4\,ee\,r^4\,aa\,cc^2\,cc0+r^2\cos(\theta)^2\,aa\,cc0+\cos(\theta)^2\,ee\,r^2\,aa\,cc\,cc0^2-ee\,r^4\,aa0\,cc^3$

$\quad+2\cos(\theta)^2\,ee\,r^4\,aa0\,cc^3-2\,ee\,r^2\,aa0\,cc^2\,cc0-\cos(\theta)^4\,ee\,r^4\,aa0\,cc^3+2\cos(\theta)^2\,ee\,r^2\,aa0\,cc^2\,cc0-aa0\,cc0$

$\quad-ee\,aa0\,cc\,cc0^2)\,\Big/\,(-bb+bb\cos(\theta)^2-r+r\cos(\theta)^2)$

$\text{Christoffel\_Gamma2}(1,2,4)=-\sin(\theta)\,(3\,aa^3\,r^7\cos(\theta)^6+2\,aa^2\,r^5\cos(\theta)^6\,aa0+aa\,r^3\cos(\theta)^2\,aa0^2$

$\quad-3\,aa^3\,r^7\cos(\theta)^4-4\,aa^2\,r^5\cos(\theta)^4\,aa0+aa^3\,r^7\cos(\theta)^2-aa\,r^3\cos(\theta)^4\,aa0^2+2\,aa^2\,r^5\cos(\theta)^2\,aa0$

$\quad+aa\,r^3\cos(\theta)^2\,ee-aa\,r^2\,ee\,bb-aa^3\,r^7\cos(\theta)^8+4\,aa^2\,r^5\,ee\,cc\,cc0\cos(\theta)^2\,aa0-\cos(\theta)^6\,ee^2\,cc^2\,aa\,r^5$

$\quad+\cos(\theta)^4\,ee^2\,cc^2\,r^3\,aa0-\cos(\theta)^2\,ee^2\,cc^2\,r^3\,aa0+\cos(\theta)^4\,ee^2\,cc\,aa\,r^2\,bb\,cc0-\cos(\theta)^2\,ee^2\,cc\,bb\,aa\,r^2\,cc0$

$\quad-2\,aa^2\,r^3\cos(\theta)^4\,ee\,cc0^2\,aa0+2\cos(\theta)^4\,ee^2\,cc^2\,aa\,r^5-2\,aa\,r^3\cos(\theta)^4\,ee\,cc\,cc0\,aa0^2+aa\,r^5\cos(\theta)^6\,ee\,cc^2\,aa0^2$

$\quad-2\,aa^3\,r^7\cos(\theta)^8\,ee\,cc\,cc0-\cos(\theta)^2\,ee^2\,cc^2\,aa\,r^5+aa\,r\cos(\theta)^2\,ee\,cc0^2\,aa0^2+aa^3\,r^5\cos(\theta)^6\,ee\,cc0^2$

$\quad+4\,aa^2\,r^5\cos(\theta)^6\,ee\,cc\,cc0\,aa0+aa^3\,r^9\cos(\theta)^{10}\,ee\,cc^2-aa\,r^3\,ee-4\,aa^3\,r^9\,ee\,cc^2\cos(\theta)^4$

$\quad+6\,aa^2\,r^7\,ee\,cc^2\cos(\theta)^6\,aa0-2\,aa^3\,r^5\,ee\,cc0^2\cos(\theta)^4+aa^3\,r^9\,ee\,cc^2\cos(\theta)^2+2\,aa^2\,r^3\,ee\,cc0^2\cos(\theta)^2\,aa0$

$\quad+\cos(\theta)^4\,ee^2\,cc^2\,bb\,aa0\,r^2-\cos(\theta)^2\,ee^2\,cc^2\,bb\,aa0\,r^2-\cos(\theta)^2\,ee^2\,cc\,r\,aa0\,cc0-\cos(\theta)^6\,ee^2\,cc^2\,aa\,r^4\,bb$

$\quad+\cos(\theta)^4\,ee^2\,cc\,aa\,r^3\,cc0+2\cos(\theta)^4\,ee^2\,cc^2\,bb\,aa\,r^4+2\,aa\,r^3\,ee\,cc\cos(\theta)^2\,cc0\,aa0^2-\cos(\theta)^2\,ee^2\,cc^2\,bb\,aa\,r^4$

$\quad-8\,aa^2\,r^5\,ee\,cc\cos(\theta)^4\,cc0\,aa0-\cos(\theta)^2\,ee^2\,cc\,aa\,r^3\,cc0+6\,aa^3\,r^9\,ee\,cc^2\cos(\theta)^6-2\,aa^2\,r^7\cos(\theta)^8\,ee\,cc^2\,aa0$

$\quad-6\,aa^2\,r^7\,ee\,cc^2\cos(\theta)^4\,aa0+aa^3\,r^5\,ee\,cc0^2\cos(\theta)^2+aa\,r^2\cos(\theta)^2\,ee\,bb-6\,aa^3\,r^7\,ee\,cc\,cc0\cos(\theta)^4$

$\quad-4\,aa^3\,r^9\,ee\,cc^2\cos(\theta)^8+2\,aa^3\,r^7\,ee\,cc\,cc0\cos(\theta)^2+aa\,r^5\,ee\,cc^2\cos(\theta)^2\,aa0^2-\cos(\theta)^2\,ee^2\,cc\,bb\,aa0\,cc0$

$\quad+6\,aa^3\,r^7\,ee\,cc\cos(\theta)^6\,cc0-2\,aa\,r^5\,ee\,cc^2\cos(\theta)^4\,aa0^2+2\,aa^2\,r^7\,ee\,cc^2\cos(\theta)^2\,aa0)\,\Big/\,(ee$

$(-bb+bb\cos(\theta)^2-r+r\cos(\theta)^2))$

$\text{Christoffel\_Gamma2}(1,3,1)=-\dfrac{1}{4}\cos(\theta)\,(5\,ee\,aa\,r^8\,cc^3\,bb+5\,aa0\,r^2\,cc0\,bb+5\,aa\,r^4\,cc0\,bb+ee\,aa\,r^2\,cc0^3\,bb$

$\quad+8\,ee\,aa0\,cc^2\,r^5\,cc0+4\,aa\,r^5\,cc0+7\,ee\,aa\,r^4\,cc\,cc0^2\,bb+4\,ee\,aa0\,cc\,r^3\,cc0^2+aa0\,r^4\,cc\,bb+4\,ee\,aa\,r^9\,cc^3$

$\quad+11\,ee\,aa0\,cc^2\,r^4\,cc0\,bb+4\,ee\,aa\,r^5\,cc\,cc0^2+aa\,r^6\,cc\,bb+11\,ee\,aa\,r^6\,cc^2\,cc0\,bb+7\,ee\,aa0\,cc\,r^2\,cc0^2\,bb$

$\quad+ee\,aa0\,cc0^3\,bb+5\,ee\,aa0\,cc^3\,r^6\,bb+8\,ee\,aa\,r^7\,cc^2\,cc0+4\,aa0\,r^3\,cc0+4\,ee\,aa0\,cc^3\,r^7-8\cos(\theta)^6\,ee\,aa\,r^7\,cc^2\,cc0$

$\quad-11\cos(\theta)^6\,ee\,aa\,r^6\,cc^2\,cc0\,bb+7\cos(\theta)^4\,ee\,aa\,r^4\,cc\,cc0^2\,bb+4\cos(\theta)^4\,ee\,aa\,r^5\,cc\,cc0^2+5\cos(\theta)^4\,aa\,r^4\,cc0\,bb$

$\quad-14\cos(\theta)^2\,ee\,aa\,r^4\,cc\,cc0^2\,bb-20\cos(\theta)^6\,ee\,aa\,r^8\,cc^3\,bb-16\cos(\theta)^6\,ee\,aa\,r^9\,cc^3+4\cos(\theta)^8\,ee\,aa\,r^9\,cc^3$

$\quad-12\cos(\theta)^2\,ee\,aa0\,cc^3\,r^7-4\cos(\theta)^2\,ee\,aa0\,cc\,r^3\,cc0^2+8\cos(\theta)^4\,ee\,aa0\,cc^2\,r^5\,cc0+\cos(\theta)^4\,aa0\,r^4\,cc\,bb$

$\quad-4\cos(\theta)^2\,aa0\,r^3\,cc0-15\cos(\theta)^2\,ee\,aa0\,cc^3\,r^6\,bb+15\cos(\theta)^4\,ee\,aa0\,cc^3\,r^6\,bb-\cos(\theta)^2\,ee\,aa\,r^2\,cc0^3\,bb$

$\quad+3\cos(\theta)^4\,aa\,r^6\,cc\,bb-\cos(\theta)^6\,aa\,r^6\,cc\,bb+30\cos(\theta)^4\,ee\,aa\,r^8\,cc^3\,bb-16\cos(\theta)^2\,ee\,aa\,r^9\,cc^3$

$\quad-7\cos(\theta)^2\,ee\,aa0\,cc\,r^2\,cc0^2\,bb-5\cos(\theta)^6\,ee\,aa0\,cc^3\,r^6\,bb-4\cos(\theta)^6\,ee\,aa0\,cc^3\,r^7$

$\quad-22\cos(\theta)^2\,ee\,aa0\,cc^2\,r^4\,cc0\,bb-20\cos(\theta)^2\,ee\,aa\,r^8\,cc^3\,bb+12\cos(\theta)^4\,ee\,aa0\,cc^3\,r^7-2\cos(\theta)^2\,aa0\,r^4\,cc\,bb$

$\quad-5\cos(\theta)^2\,aa0\,r^2\,cc0\,bb+5\cos(\theta)^8\,ee\,aa\,r^8\,cc^3\,bb+24\cos(\theta)^4\,ee\,aa\,r^7\,cc^2\,cc0+33\cos(\theta)^4\,ee\,aa\,r^6\,cc^2\,cc0\,bb$

$\quad+24\cos(\theta)^4\,ee\,aa\,r^9\,cc^3-33\cos(\theta)^2\,ee\,aa\,r^6\,cc^2\,cc0\,bb-16\cos(\theta)^2\,ee\,aa0\,cc^2\,r^5\,cc0-10\cos(\theta)^2\,aa\,r^4\,cc0\,bb$

$$- 3 \cos(\theta)^2 \, aa \, r^6 \, cc \, bb - 24 \cos(\theta)^2 \, ee \, aa \, r^7 \, cc^2 \, cc0 - 8 \cos(\theta)^2 \, ee \, aa \, r^5 \, cc \, cc0^2 + 11 \cos(\theta)^4 \, ee \, aa0 \, cc^2 \, r^4 \, cc0 \, bb$$

$$- 8 \cos(\theta)^2 \, aa \, r^5 \, cc0 + 4 \cos(\theta)^4 \, aa \, r^5 \, cc0 \Big) \Big/ \left(r^3 \, (-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2) \sqrt{\frac{r+bb}{r}} \right)$$

$$\text{Christoffel\_Gamma2}(1, 3, 2) = \cos(\theta)^2 \sqrt{\frac{r+bb}{r}} \, r \sin(\theta) \, (-ee \, r^6 \, aa \, cc^3 + 3 \cos(\theta)^2 \, ee \, r^6 \, aa \, cc^3$$

$$- 2 \, ee \, r^4 \, aa \, cc^2 \, cc0 - 3 \cos(\theta)^4 \, ee \, r^6 \, aa \, cc^3 + 4 \cos(\theta)^2 \, ee \, r^4 \, aa \, cc^2 \, cc0 - r^2 \, aa \, cc0 - ee \, r^2 \, aa \, cc \, cc0^2$$

$$+ \cos(\theta)^6 \, ee \, r^6 \, aa \, cc^3 - 2 \cos(\theta)^4 \, ee \, r^4 \, aa \, cc^2 \, cc0 + r^2 \cos(\theta)^2 \, aa \, cc0 + \cos(\theta)^2 \, ee \, r^2 \, aa \, cc \, cc0^2 - ee \, r^4 \, aa0 \, cc^3$$

$$+ 2 \cos(\theta)^2 \, ee \, r^4 \, aa0 \, cc^3 - 2 \, ee \, r^2 \, aa0 \, cc^2 \, cc0 - \cos(\theta)^4 \, ee \, r^4 \, aa0 \, cc^3 + 2 \cos(\theta)^2 \, ee \, r^2 \, aa0 \, cc^2 \, cc0 - aa0 \, cc0$$

$$- ee \, aa0 \, cc \, cc0^2 \Big) \Big/ (-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2)$$

$$\text{Christoffel\_Gamma2}(1, 3, 3) =$$

$$-r + r \cos(\theta)^2 - 2 \, ee \, cc^2 \, r^3 + 4 \, ee \, cc^2 \, r^3 \cos(\theta)^2 - 2 \, ee \, cc \, r \, cc0 - 2 \, ee \, cc^2 \, r^3 \cos(\theta)^4 + 2 \, ee \, cc \, r \cos(\theta)^2 \, cc0$$

$$\text{Christoffel\_Gamma2}(1, 3, 4) = -\frac{1}{4} \, (-3 \, ee \, cc^2 \, r^8 \cos(\theta)^{10} \, aa^2 \, cc0 \, bb - 2 \, ee \, cc^3 \, r^8 \cos(\theta)^{10} \, aa \, aa0 \, bb$$

$$- 2 \, r^4 \cos(\theta)^6 \, aa \, aa0 \, cc0 \, bb + 2 \, ee \, cc0^3 \, aa \, r^2 \cos(\theta)^4 \, aa0 \, bb + ee \, cc^3 \, r^6 \cos(\theta)^8 \, aa0^2 \, bb + ee \, cc^3 \, r^{10} \cos(\theta)^{12} \, aa^2 \, bb$$

$$- r^8 \cos(\theta)^{10} \, aa^2 \, cc \, bb - r^4 \cos(\theta)^6 \, aa0^2 \, cc \, bb + 6 \, ee \, cc^2 \, r^6 \cos(\theta)^8 \, aa \, aa0 \, cc0 \, bb + r^2 \cos(\theta)^4 \, aa0^2 \, cc0 \, bb$$

$$+ 3 \, ee \, cc0^2 \, aa0^2 \, cc \, r^2 \cos(\theta)^4 \, bb + 3 \, ee \, cc0^2 \, aa^2 \, r^6 \cos(\theta)^8 \, cc \, bb - 6 \, ee \, cc0^2 \, aa \, r^4 \cos(\theta)^6 \, aa0 \, cc \, bb$$

$$+ r^6 \cos(\theta)^8 \, aa^2 \, cc0 \, bb - ee \, cc0^3 \, aa0^2 \, bb \cos(\theta)^2 + 2 \, r^6 \cos(\theta)^8 \, aa \, aa0 \, cc \, bb - ee \, cc0^3 \, aa^2 \, r^4 \cos(\theta)^6 \, bb$$

$$- 3 \, ee \, cc^2 \, r^4 \cos(\theta)^6 \, cc0 \, aa0^2 \, bb - 3 \, ee \, cc0^2 \, aa^2 \, r^6 \, cc \cos(\theta)^2 \, bb + r \cos(\theta)^2 \, ee \, bb^2 \, cc0 + 3 \, ee \, cc^3 \, r^6 \cos(\theta)^4 \, aa0^2 \, bb$$

$$+ 10 \, ee \, cc^3 \, r^{10} \cos(\theta)^8 \, aa^2 \, bb - 5 \, ee \, cc^3 \, r^{10} \cos(\theta)^{10} \, aa^2 \, bb - 12 \, ee \, cc^3 \, r^8 \cos(\theta)^6 \, aa \, aa0 \, bb$$

$$+ 12 \, ee \, cc^2 \, r^8 \cos(\theta)^8 \, aa^2 \, cc0 \, bb + 8 \, ee \, cc^3 \, r^8 \cos(\theta)^8 \, aa \, aa0 \, bb - 2 \, ee \, cc0^3 \, aa \, r^2 \cos(\theta)^2 \, aa0 \, bb$$

$$+ 6 \, ee \, cc^2 \, r^4 \cos(\theta)^4 \, cc0 \, aa0^2 \, bb + r^2 \cos(\theta)^2 \, ee \, cc0 \, bb - 6 \, ee \, cc0^2 \, aa \, r^4 \, aa0 \, cc \cos(\theta)^2 \, bb - r^2 \cos(\theta)^2 \, aa0^2 \, cc0 \, bb$$

$$+ 2 \, r^4 \cos(\theta)^4 \, aa0^2 \, cc \, bb + 4 \, r^4 \cos(\theta)^4 \, aa \, aa0 \, cc0 \, bb - 6 \, r^6 \cos(\theta)^6 \, aa \, aa0 \, cc \, bb + 3 \, r^3 \cos(\theta)^4 \, ee \, bb^2 \, cc$$

$$- 2 \, r^4 \, aa \cos(\theta)^2 \, aa0 \, cc0 \, bb + 8 \, ee \, cc^3 \, r^8 \, aa \cos(\theta)^4 \, aa0 \, bb - 3 \, ee \, cc^3 \, r^6 \cos(\theta)^6 \, aa0^2 \, bb$$

$$+ 9 \, ee \, cc \, r^6 \, cc0^2 \, aa^2 \cos(\theta)^4 \, bb + 2 \, ee \, cc0^3 \, aa^2 \, r^4 \cos(\theta)^4 \, bb - ee \, cc0^3 \, aa^2 \, r^4 \cos(\theta)^2 \, bb + 5 \, ee \, cc^3 \, r^{10} \, aa^2 \cos(\theta)^4 \, bb$$

$$- 9 \, ee \, cc0^2 \, aa^2 \, r^6 \cos(\theta)^6 \, cc \, bb - 10 \, ee \, cc^3 \, r^{10} \cos(\theta)^6 \, aa^2 \, bb - 18 \, ee \, cc^2 \, r^6 \cos(\theta)^6 \, aa \, aa0 \, cc0 \, bb$$

$$+ 12 \, ee \, cc0^2 \, aa \, r^4 \cos(\theta)^4 \, aa0 \, cc \, bb + 12 \, ee \, cc^2 \, r^8 \, cc0 \, aa^2 \cos(\theta)^4 \, bb - 18 \, ee \, cc^2 \, r^8 \, cc0 \, aa^2 \cos(\theta)^6 \, bb$$

$$+ 18 \, ee \, cc^2 \, r^6 \, cc0 \, aa \cos(\theta)^4 \, aa0 \, bb + 4 \, r^5 \cos(\theta)^4 \, ee \, cc + 4 \, r^8 \cos(\theta)^8 \, aa^2 \, cc \, bb - ee \, cc^3 \, r^{10} \, aa^2 \cos(\theta)^2 \, bb$$

$$- r^6 \, aa^2 \cos(\theta)^2 \, cc0 \, bb - r^4 \, aa0^2 \, cc \cos(\theta)^2 \, bb + 4 \, r^8 \, aa^2 \cos(\theta)^4 \, cc \, bb - r^8 \, aa^2 \cos(\theta)^2 \, cc \, bb$$

$$+ 3 \, r^6 \, aa^2 \cos(\theta)^4 \, cc0 \, bb - 6 \, r^8 \, aa^2 \cos(\theta)^6 \, cc \, bb + 4 \, r^5 \, ee \, cc + 7 \, r^4 \cos(\theta)^4 \, ee \, cc \, bb - 14 \, r^4 \cos(\theta)^2 \, ee \, cc \, bb$$

$$- 3 \, r^6 \cos(\theta)^6 \, aa^2 \, cc0 \, bb - r \, ee \, bb^2 \, cc0 + 3 \, r^3 \, ee \, bb^2 \, cc - 2 \, ee \, cc^3 \, r^8 \, aa \, aa0 \cos(\theta)^2 \, bb$$

$$- 3 \, ee \, cc0^2 \, aa0^2 \, cc \, r^2 \cos(\theta)^2 \, bb - 3 \, ee \, cc^2 \, r^4 \, cc0 \, aa0^2 \cos(\theta)^2 \, bb + 6 \, r^6 \cos(\theta)^4 \, aa \, aa0 \, cc \, bb$$

$$- 2 \, r^6 \, aa \, aa0 \, cc \cos(\theta)^2 \, bb - 3 \, ee \, cc^2 \, r^8 \, cc0 \, aa^2 \cos(\theta)^2 \, bb - 6 \, r^3 \, ee \, bb^2 \, cc \cos(\theta)^2 - r^2 \, ee \, cc0 \, bb + 7 \, r^4 \, ee \, cc \, bb$$

$$- ee \, cc^3 \, r^6 \, aa0^2 \cos(\theta)^2 \, bb - 6 \, ee \, cc^2 \, r^6 \, cc0 \, aa \, aa0 \cos(\theta)^2 \, bb - 8 \, r^5 \cos(\theta)^2 \, ee \, cc \Big) \Big/ \left(\sqrt{\frac{r+bb}{r}} \, r^3 \right.$$

$$\left. (-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2) \right)$$

$$\text{Christoffel\_Gamma2}(1, 4, 1) = \frac{1}{4} \cos(\theta) \, (-4 \, aa^3 \, r^9 - 12 \cos(\theta)^4 \, aa0^2 \, ee \, cc^2 \, r^7 \, aa + 16 \cos(\theta)^2 \, aa^3 \, r^9$$

$$- 24 \cos(\theta)^4 \, aa^3 \, r^9 + 16 \cos(\theta)^6 \, aa^3 \, r^9 - 4 \cos(\theta)^8 \, aa^3 \, r^9 - 8 \, aa0 \, r^7 \, aa^2 + 4 \, aa0 \, ee^2 \, cc0 \, cc \, r^3 + 4 \, aa0 \, ee^2 \, cc^2 \, r^5$$

$$+ aa0 \, ee^2 \, cc0^2 \, bb + aa \, r^2 \, ee^2 \, cc0^2 \, bb - 4 \, aa0^2 \, r^5 \, aa - 8 \, aa^3 \, r^9 \, ee \, cc \, cc0 - 4 \, aa0^2 \, ee \, cc0^2 \, aa \, r^3 - 4 \, aa^3 \, r^{11} \, ee \, cc^2$$

$$+ 6 \, aa \, r^4 \, ee^2 \, cc \, cc0 \, bb + 4 \, aa \, r^5 \, ee^2 \, cc0 \, cc + 4 \, aa \, r^7 \, ee^2 \, cc^2 + 2 \, aa0 \, r^2 \, ee \, bb - 8 \, aa0^2 \, ee \, cc \, r^5 \, cc0 \, aa + 2 \, aa \, r^4 \, ee \, bb$$

$$+ 5 \, aa \, r^6 \, ee^2 \, cc^2 \, bb - 4 \, aa^3 \, r^7 \, ee \, cc0^2 + 6 \, aa0 \, ee^2 \, cc \, r^2 \, cc0 \, bb - 4 \, aa0^2 \, ee \, cc^2 \, r^7 \, aa - 8 \, aa0 \, ee \, cc0^2 \, aa^2 \, r^5$$

$$- 8\ aa0\ ee\ cc^2\ r^9\ aa^2 + 5\ aa0\ ee^2\ cc^2\ r^4\ bb - 16\ aa0\ ee\ cc\ r^7\ cc0\ aa^2 - 8\ \cos(\theta)^2\ aa0\ ee^2\ cc^2\ r^5$$

$$+ 5\ \cos(\theta)^4\ aa0\ ee^2\ cc^2\ r^4\ bb + 16\ \cos(\theta)^2\ aa0^2\ ee\ cc\ r^5\ cc0\ aa + 4\ \cos(\theta)^2\ aa0^2\ ee\ cc0^2\ aa\ r^3$$

$$- 2\ \cos(\theta)^2\ aa0\ r^2\ ee\ bb + 16\ \cos(\theta)^6\ aa0\ ee\ cc\ r^7\ cc0\ aa^2 + 12\ \cos(\theta)^4\ aa\ r^7\ ee^2\ cc^2 + 12\ \cos(\theta)^2\ aa0^2\ ee\ cc^2\ r^7\ aa$$

$$+ 4\ \cos(\theta)^6\ aa^3\ r^7\ ee\ cc0^2 - 5\ \cos(\theta)^6\ aa\ r^6\ ee^2\ cc^2\ bb + 2\ \cos(\theta)^4\ aa\ r^4\ ee\ bb - 4\ \cos(\theta)^6\ aa\ r^7\ ee^2\ cc^2$$

$$- 8\ \cos(\theta)^4\ aa^2\ r^5\ ee\ cc0^2\ aa0 - 48\ \cos(\theta)^4\ aa^2\ r^9\ ee\ cc^2\ aa0 - 8\ \cos(\theta)^4\ aa0^2\ ee\ cc\ r^5\ cc0\ aa$$

$$- 8\ \cos(\theta)^8\ aa0\ ee\ cc^2\ r^9\ aa^2 - 8\ \cos(\theta)^2\ aa\ r^5\ ee^2\ cc0\ cc - 48\ \cos(\theta)^4\ aa^3\ r^9\ ee\ cc\ cc0 - 12\ \cos(\theta)^4\ aa^3\ r^7\ ee\ cc0^2$$

$$+ 6\ \cos(\theta)^4\ aa\ r^4\ ee^2\ cc\ cc0\ bb - 40\ \cos(\theta)^4\ aa^3\ r^{11}\ ee\ cc^2 + 4\ \cos(\theta)^4\ aa\ r^5\ ee^2\ cc0\ cc + 32\ \cos(\theta)^6\ aa^3\ r^9\ ee\ cc\ cc0$$

$$- 8\ \cos(\theta)^8\ aa^3\ r^9\ ee\ cc\ cc0 + 40\ \cos(\theta)^6\ aa^3\ r^{11}\ ee\ cc^2 - 24\ \cos(\theta)^4\ aa^2\ r^7\ aa0 + 24\ \cos(\theta)^2\ aa0\ r^7\ aa^2$$

$$+ 32\ \cos(\theta)^2\ aa^3\ r^9\ ee\ cc\ cc0 + 4\ \cos(\theta)^6\ aa0^2\ ee\ cc^2\ r^7\ aa + 12\ \cos(\theta)^2\ aa^3\ r^7\ ee\ cc0^2 - 12\ \cos(\theta)^2\ aa\ r^7\ ee^2\ cc^2$$

$$+ 20\ \cos(\theta)^2\ aa^3\ r^{11}\ ee\ cc^2 - 12\ \cos(\theta)^2\ aa\ r^4\ ee^2\ cc\ cc0\ bb - 20\ \cos(\theta)^8\ aa^3\ r^{11}\ ee\ cc^2 + 32\ \cos(\theta)^6\ aa^2\ r^9\ ee\ cc^2\ aa0$$

$$+ 32\ \cos(\theta)^2\ aa^2\ r^9\ ee\ cc^2\ aa0 - 4\ \cos(\theta)^2\ aa\ r^4\ ee\ bb - 6\ \cos(\theta)^2\ aa0\ ee^2\ cc\ r^2\ cc0\ bb + 15\ \cos(\theta)^4\ aa\ r^6\ ee^2\ cc^2\ bb$$

$$- 15\ \cos(\theta)^2\ aa\ r^6\ ee^2\ cc^2\ bb - \cos(\theta)^2\ aa\ r^2\ ee^2\ cc0^2\ bb + 16\ \cos(\theta)^2\ aa^2\ r^5\ ee\ cc0^2\ aa0$$

$$+ 48\ \cos(\theta)^2\ aa^2\ r^7\ ee\ cc\ cc0\ aa0 - 10\ \cos(\theta)^2\ aa0\ ee^2\ cc^2\ r^4\ bb + 8\ \cos(\theta)^2\ aa0^2\ r^5\ aa + 8\ \cos(\theta)^6\ aa0\ r^7\ aa^2$$

$$- 4\ \cos(\theta)^4\ aa0^2\ r^5\ aa + 4\ \cos(\theta)^4\ aa0\ ee^2\ cc^2\ r^5 - 4\ \cos(\theta)^2\ aa0\ ee^2\ cc0\ r^3\ cc + 4\ \cos(\theta)^{10}\ aa^3\ r^{11}\ ee\ cc^2$$

$$- 48\ \cos(\theta)^4\ aa^2\ r^7\ ee\ cc\ cc0\ aa0\Big) \Big/ (ee\ r^3\ (-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2))$$

Christoffel_Gamma2(1, 4, 2) $= -\sin(\theta)\,\big(3\ aa^3\ r^7\ \cos(\theta)^6 + 2\ aa^2\ r^5\ \cos(\theta)^6\ aa0 + aa\ r^3\ \cos(\theta)^2\ aa0^2$

$$- 3\ aa^3\ r^7\ \cos(\theta)^4 - 4\ aa^2\ r^5\ \cos(\theta)^4\ aa0 + aa^3\ r^7\ \cos(\theta)^2 - aa\ r^3\ \cos(\theta)^4\ aa0^2 + 2\ aa^2\ r^5\ \cos(\theta)^2\ aa0$$

$$+ aa\ r^3\ \cos(\theta)^2\ ee - aa\ r^2\ ee\ bb - aa^3\ r^7\ \cos(\theta)^8 + 4\ aa^2\ r^5\ ee\ cc\ cc0\ \cos(\theta)^2\ aa0 - \cos(\theta)^6\ ee^2\ cc^2\ aa\ r^5$$

$$+ \cos(\theta)^4\ ee^2\ cc^2\ r^3\ aa0 - \cos(\theta)^2\ ee^2\ cc^2\ r^3\ aa0 + \cos(\theta)^4\ ee^2\ cc\ aa\ r^2\ bb\ cc0 - \cos(\theta)^2\ ee^2\ cc\ bb\ aa\ r^2\ cc0$$

$$- 2\ aa^2\ r^3\ \cos(\theta)^4\ ee\ cc0^2\ aa0 + 2\ \cos(\theta)^4\ ee^2\ cc^2\ aa\ r^5 - 2\ aa\ r^3\ \cos(\theta)^4\ ee\ cc\ cc0\ aa0^2 + aa\ r^5\ \cos(\theta)^6\ ee\ cc^2\ aa0^2$$

$$- 2\ aa^3\ r^7\ \cos(\theta)^8\ ee\ cc\ cc0 - \cos(\theta)^2\ ee^2\ cc^2\ aa\ r^5 + aa\ r\ \cos(\theta)^2\ ee\ cc0^2\ aa0^2 + aa^3\ r^5\ \cos(\theta)^6\ ee\ cc0^2$$

$$+ 4\ aa^2\ r^5\ \cos(\theta)^6\ ee\ cc\ cc0\ aa0 + aa^3\ r^9\ \cos(\theta)^{10}\ ee\ cc^2 - aa\ r^3\ ee - 4\ aa^3\ r^9\ ee\ cc^2\ \cos(\theta)^4$$

$$+ 6\ aa^2\ r^7\ ee\ cc^2\ \cos(\theta)^6\ aa0 - 2\ aa^3\ r^5\ ee\ cc0^2\ \cos(\theta)^4 + aa^3\ r^9\ ee\ cc^2\ \cos(\theta)^2 + 2\ aa^2\ r^3\ ee\ cc0^2\ \cos(\theta)^2\ aa0$$

$$+ \cos(\theta)^4\ ee^2\ cc^2\ bb\ aa0\ r^2 - \cos(\theta)^2\ ee^2\ cc^2\ bb\ aa0\ r^2 - \cos(\theta)^2\ ee^2\ cc\ r\ aa0\ cc0 - \cos(\theta)^6\ ee^2\ cc^2\ aa\ r^4\ bb$$

$$+ \cos(\theta)^4\ ee^2\ cc\ aa\ r^3\ cc0 + 2\ \cos(\theta)^4\ ee^2\ cc^2\ bb\ aa\ r^4 + 2\ aa\ r^3\ ee\ cc\ \cos(\theta)^2\ cc0\ aa0^2 - \cos(\theta)^2\ ee^2\ cc^2\ bb\ aa\ r^4$$

$$- 8\ aa^2\ r^5\ ee\ cc\ \cos(\theta)^4\ cc0\ aa0 - \cos(\theta)^2\ ee^2\ cc\ aa\ r^3\ cc0 + 6\ aa^3\ r^9\ ee\ cc^2\ \cos(\theta)^6 - 2\ aa^2\ r^7\ \cos(\theta)^8\ ee\ cc^2\ aa0$$

$$- 6\ aa^2\ r^7\ ee\ cc^2\ \cos(\theta)^4\ aa0 + aa^3\ r^5\ ee\ cc0^2\ \cos(\theta)^2 + aa\ r^2\ \cos(\theta)^2\ ee\ bb - 6\ aa^3\ r^7\ ee\ cc\ cc0\ \cos(\theta)^4$$

$$- 4\ aa^3\ r^9\ ee\ cc^2\ \cos(\theta)^8 + 2\ aa^3\ r^7\ ee\ cc\ cc0\ \cos(\theta)^2 + aa\ r^5\ ee\ cc^2\ \cos(\theta)^2\ aa0^2 - \cos(\theta)^2\ ee^2\ cc\ bb\ aa0\ cc0$$

$$+ 6\ aa^3\ r^7\ ee\ cc\ \cos(\theta)^6\ cc0 - 2\ aa\ r^5\ ee\ cc^2\ \cos(\theta)^4\ aa0^2 + 2\ aa^2\ r^7\ ee\ cc^2\ \cos(\theta)^2\ aa0\big) \Big/ (ee$$
$$(-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2))$$

Christoffel_Gamma2(1, 4, 3) $= -\dfrac{1}{4}\,(-3\ ee\ cc^2\ r^8\ \cos(\theta)^{10}\ aa^2\ cc0\ bb - 2\ ee\ cc^3\ r^8\ \cos(\theta)^{10}\ aa\ aa0\ bb$

$$- 2\ r^4\ \cos(\theta)^6\ aa\ aa0\ cc0\ bb + 2\ ee\ cc0^3\ aa\ r^2\ \cos(\theta)^4\ aa0\ bb + ee\ cc^3\ r^6\ \cos(\theta)^8\ aa0^2\ bb + ee\ cc^3\ r^{10}\ \cos(\theta)^{12}\ aa^2\ bb$$

$$- r^8\ \cos(\theta)^{10}\ aa^2\ cc\ bb - r^4\ \cos(\theta)^6\ aa0^2\ cc\ bb + 6\ ee\ cc^2\ r^6\ \cos(\theta)^8\ aa\ aa0\ cc0\ bb + r^2\ \cos(\theta)^4\ aa0^2\ cc0\ bb$$

$$+ 3\ ee\ cc0^2\ aa0^2\ cc\ r^2\ \cos(\theta)^4\ bb + 3\ ee\ cc0^2\ aa^2\ r^6\ \cos(\theta)^8\ cc\ bb - 6\ ee\ cc0^2\ aa\ r^4\ \cos(\theta)^6\ aa0\ cc\ bb$$

$$+ r^6\ \cos(\theta)^8\ aa^2\ cc0\ bb - ee\ cc0^3\ aa0^2\ bb\ \cos(\theta)^2 + 2\ r^6\ \cos(\theta)^8\ aa\ aa0\ cc\ bb - ee\ cc0^3\ aa^2\ r^4\ \cos(\theta)^6\ bb$$

$$- 3\ ee\ cc^2\ r^4\ \cos(\theta)^6\ cc0\ aa0^2\ bb - 3\ ee\ cc0^2\ aa^2\ r^6\ cc\ \cos(\theta)^2\ bb + r\ \cos(\theta)^2\ ee\ bb^2\ cc0 + 3\ ee\ cc^3\ r^6\ \cos(\theta)^4\ aa0^2\ bb$$

$$+ 10\ ee\ cc^3\ r^{10}\ \cos(\theta)^8\ aa^2\ bb - 5\ ee\ cc^3\ r^{10}\ \cos(\theta)^{10}\ aa^2\ bb - 12\ ee\ cc^3\ r^8\ \cos(\theta)^6\ aa\ aa0\ bb$$

$$+ 12\ ee\ cc^2\ r^8\ \cos(\theta)^8\ aa^2\ cc0\ bb + 8\ ee\ cc^3\ r^8\ \cos(\theta)^8\ aa\ aa0\ bb - 2\ ee\ cc0^3\ aa\ r^2\ \cos(\theta)^2\ aa0\ bb$$

$$+ 6\ ee\ cc^2\ r^4\ \cos(\theta)^4\ cc0\ aa0^2\ bb + r^2\ \cos(\theta)^2\ ee\ cc0\ bb - 6\ ee\ cc0^2\ aa\ r^4\ aa0\ cc\ \cos(\theta)^2\ bb - r^2\ \cos(\theta)^2\ aa0^2\ cc0\ bb$$

$$+ 2\ r^4\ \cos(\theta)^4\ aa0^2\ cc\ bb + 4\ r^4\ \cos(\theta)^4\ aa\ aa0\ cc0\ bb - 6\ r^6\ \cos(\theta)^6\ aa\ aa0\ cc\ bb + 3\ r^3\ \cos(\theta)^4\ ee\ bb^2\ cc$$

$$- 2\ r^4\ aa\ \cos(\theta)^2\ aa0\ cc0\ bb + 8\ ee\ cc^3\ r^8\ aa\ \cos(\theta)^4\ aa0\ bb - 3\ ee\ cc^3\ r^6\ \cos(\theta)^6\ aa0^2\ bb$$

$+\ 9\ ee\ cc\ r^6\ cc0^2\ aa^2\ \cos(\theta)^4\ bb + 2\ ee\ cc0^3\ aa^2\ r^4\ \cos(\theta)^4\ bb - ee\ cc0^3\ aa^2\ r^4\ \cos(\theta)^2\ bb + 5\ ee\ cc^3\ r^{10}\ aa^2\ \cos(\theta)^4\ bb$

$-\ 9\ ee\ cc0^2\ aa^2\ r^6\ \cos(\theta)^6\ cc\ bb - 10\ ee\ cc^3\ r^{10}\ \cos(\theta)^6\ aa^2\ bb - 18\ ee\ cc^2\ r^6\ \cos(\theta)^6\ aa\ aa0\ cc0\ bb$

$+\ 12\ ee\ cc0^2\ aa\ r^4\ \cos(\theta)^4\ aa0\ cc\ bb + 12\ ee\ cc^2\ r^8\ cc0\ aa^2\ \cos(\theta)^4\ bb - 18\ ee\ cc^2\ r^8\ cc0\ aa^2\ \cos(\theta)^6\ bb$

$+\ 18\ ee\ cc^2\ r^6\ cc0\ aa\ \cos(\theta)^4\ aa0\ bb + 4\ r^5\ \cos(\theta)^4\ ee\ cc + 4\ r^8\ \cos(\theta)^8\ aa^2\ cc\ bb - ee\ cc^3\ r^{10}\ aa^2\ \cos(\theta)^2\ bb$

$-\ r^6\ aa^2\ \cos(\theta)^2\ cc0\ bb - r^4\ aa0^2\ cc\ \cos(\theta)^2\ bb + 4\ r^8\ aa^2\ \cos(\theta)^4\ cc\ bb - r^8\ aa^2\ \cos(\theta)^2\ cc\ bb$

$+\ 3\ r^6\ aa^2\ \cos(\theta)^4\ cc0\ bb - 6\ r^8\ aa^2\ \cos(\theta)^6\ cc\ bb + 4\ r^5\ ee\ cc + 7\ r^4\ \cos(\theta)^4\ ee\ cc\ bb - 14\ r^4\ \cos(\theta)^2\ ee\ cc\ bb$

$-\ 3\ r^6\ \cos(\theta)^6\ aa^2\ cc0\ bb - r\ ee\ bb^2\ cc0 + 3\ r^3\ ee\ bb^2\ cc - 2\ ee\ cc^3\ r^8\ aa\ aa0\ \cos(\theta)^2\ bb$

$-\ 3\ ee\ cc0^2\ aa0^2\ cc\ r^2\ \cos(\theta)^2\ bb - 3\ ee\ cc^2\ r^4\ cc0\ aa0^2\ \cos(\theta)^2\ bb + 6\ r^6\ \cos(\theta)^4\ aa\ aa0\ cc\ bb$

$-\ 2\ r^6\ aa\ aa0\ cc\ \cos(\theta)^2\ bb - 3\ ee\ cc^2\ r^8\ cc0\ aa^2\ \cos(\theta)^2\ bb - 6\ r^3\ ee\ bb^2\ cc\ \cos(\theta)^2 - r^2\ ee\ cc0\ bb + 7\ r^4\ ee\ cc\ bb$

$-\ ee\ cc^3\ r^6\ aa0^2\ \cos(\theta)^2\ bb - 6\ ee\ cc^2\ r^6\ cc0\ aa\ aa0\ \cos(\theta)^2\ bb - 8\ r^5\ \cos(\theta)^2\ ee\ cc\ )\ \Big/\ \Big(\sqrt{\dfrac{r+bb}{r}}\ r^3$

$(-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2\ )\Big)$

Christoffel_Gamma2$(1,4,4) = -\dfrac{1}{2}\ (-4\ r^4\ bb\ aa\ aa0 - 4\ r^7\ aa^2 - 4\ r^5\ aa\ aa0 + r\ ee\ bb^2 + r^2\ ee\ bb$

$+\ 10\ r^4\ bb\ aa\ \cos(\theta)^2\ aa0 - 15\ r^6\ bb\ aa^2\ \cos(\theta)^4 + r^2\ \cos(\theta)^2\ aa0^2\ bb - 4\ r^5\ \cos(\theta)^4\ aa\ aa0$

$+\ ee\ cc0^2\ aa^2\ r^4\ \cos(\theta)^2\ bb + 6\ ee\ cc^2\ r^6\ \cos(\theta)^6\ aa\ aa0\ bb + 2\ ee\ cc\ r^2\ \cos(\theta)^2\ cc0\ aa0^2\ bb$

$-\ 2\ ee\ cc^2\ r^4\ \cos(\theta)^4\ aa0^2\ bb - 6\ ee\ cc^2\ r^6\ \cos(\theta)^4\ aa\ aa0\ bb + ee\ cc^2\ r^8\ aa^2\ \cos(\theta)^2\ bb + 7\ r^6\ \cos(\theta)^6\ aa^2\ bb$

$-\ 8\ ee\ cc\ r^4\ \cos(\theta)^4\ cc0\ aa\ aa0\ bb + ee\ cc^2\ r^4\ \cos(\theta)^2\ aa0^2\ bb + 4\ ee\ cc\ r^4\ \cos(\theta)^6\ cc0\ aa\ aa0\ bb$

$+\ 6\ ee\ cc^2\ r^8\ \cos(\theta)^6\ aa^2\ bb - 8\ r^4\ \cos(\theta)^4\ aa\ aa0\ bb + 2\ ee\ cc^2\ r^6\ aa\ \cos(\theta)^2\ aa0\ bb + ee\ cc0^2\ aa^2\ r^4\ \cos(\theta)^6\ bb$

$-\ 2\ ee\ cc^2\ r^6\ \cos(\theta)^8\ aa\ aa0\ bb - 2\ ee\ cc0^2\ aa\ r^2\ \cos(\theta)^4\ aa0\ bb + ee\ cc^2\ r^8\ \cos(\theta)^{10}\ aa^2\ bb - r^6\ \cos(\theta)^8\ aa^2\ bb$

$-\ 2\ ee\ cc\ r^2\ \cos(\theta)^4\ cc0\ aa0^2\ bb - 2\ ee\ cc\ r^6\ \cos(\theta)^8\ cc0\ aa^2\ bb + 2\ ee\ cc0^2\ aa\ r^2\ \cos(\theta)^2\ aa0\ bb$

$+\ 2\ ee\ cc\ r^6\ cc0\ aa^2\ \cos(\theta)^2\ bb + 13\ r^6\ bb\ aa^2\ \cos(\theta)^2 + 8\ r^5\ \cos(\theta)^2\ aa\ aa0 + 2\ r^4\ \cos(\theta)^6\ aa\ aa0\ bb$

$-\ r^2\ \cos(\theta)^4\ aa0^2\ bb - r^2\ \cos(\theta)^2\ ee\ bb - r\ \cos(\theta)^2\ ee\ bb^2 + 4\ r^7\ \cos(\theta)^6\ aa^2 - 4\ ee\ cc^2\ r^8\ \cos(\theta)^8\ aa^2\ bb$

$-\ 2\ ee\ cc0^2\ aa^2\ r^4\ \cos(\theta)^4\ bb - 12\ r^7\ \cos(\theta)^4\ aa^2 + 6\ ee\ cc\ r^6\ \cos(\theta)^6\ cc0\ aa^2\ bb + 4\ ee\ cc\ r^4\ cc0\ aa\ \cos(\theta)^2\ aa0\ bb$

$-\ 4\ ee\ cc^2\ r^8\ aa^2\ \cos(\theta)^4\ bb + ee\ cc^2\ r^4\ \cos(\theta)^6\ aa0^2\ bb + ee\ cc0^2\ aa0^2\ bb\ \cos(\theta)^2 - 6\ ee\ cc\ r^6\ cc0\ aa^2\ \cos(\theta)^4\ bb$

$-\ 4\ r^6\ bb\ aa^2 + 12\ r^7\ \cos(\theta)^2\ aa^2\ )\ \Big/\ (r^3\ (-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2\ ))$

Christoffel_Gamma2$(2,1,1) = -2\ (\cos(\theta)^6\ aa\ r^6\ ee\ cc^2 - 3\ \cos(\theta)^4\ aa\ r^6\ ee\ cc^2 - \cos(\theta)^4\ r^4\ aa0\ ee\ cc^2$

$-\ r^4\ \cos(\theta)^4\ aa - 2\ \cos(\theta)^4\ aa\ r^4\ ee\ cc\ cc0 + 4\ \cos(\theta)^2\ aa\ r^4\ ee\ cc\ cc0 + \cos(\theta)^2\ aa\ r^2\ ee\ cc0^2$

$+\ 2\ \cos(\theta)^2\ r^2\ aa0\ ee\ cc\ cc0 + 3\ \cos(\theta)^2\ aa\ r^6\ ee\ cc^2 + 2\ aa\ r^4\ \cos(\theta)^2 + 2\ \cos(\theta)^2\ r^4\ aa0\ ee\ cc^2 + r^2\ \cos(\theta)^2\ aa0$

$-\ ee\ cc0^2\ aa0 - ee\ cc^2\ aa0\ r^4 - aa\ r^4 - ee\ cc^2\ r^6\ aa - 2\ aa\ r^4\ ee\ cc\ cc0 - r^2\ aa0 - 2\ r^2\ ee\ cc\ cc0\ aa0 - aa\ ee\ cc0^2\ r^2\ )$

$\sin(\theta)\ \cos(\theta)\ aa\ /\ ((r+bb)\ ee\ r)$

Christoffel_Gamma2$(2,1,2) = -\ (aa^2\ r^5 - ee\ r - ee\ bb - 3\ aa\ r^3\ \cos(\theta)^2\ aa0 - 2\ r^5\ \cos(\theta)^6\ aa^2$

$+\ 2\ r^3\ \cos(\theta)^4\ aa\ aa0 + ee\ cc^2\ r^5\ aa\ aa0 + 2\ ee\ cc\ r^5\ cc0\ aa^2 + 2\ ee\ cc\ r^3\ cc0\ aa\ aa0 + ee\ cc0^2\ aa\ r\ aa0 + ee\ cc^2\ r^7\ aa^2$

$+\ ee\ cc0^2\ aa^2\ r^3 - 6\ ee\ cc\ r^3\ cc0\ aa\ \cos(\theta)^2\ aa0 - 2\ ee\ cc0^2\ aa\ r\ \cos(\theta)^2\ aa0 - 8\ ee\ cc\ r^5\ cc0\ aa^2\ \cos(\theta)^2$

$+\ 2\ ee\ cc^2\ r^7\ \cos(\theta)^8\ aa^2 - 4\ ee\ cc\ r^5\ \cos(\theta)^6\ cc0\ aa^2 - 7\ ee\ cc^2\ r^7\ \cos(\theta)^6\ aa^2 - 2\ ee\ cc^2\ r^5\ \cos(\theta)^6\ aa\ aa0$

$+\ 2\ ee\ cc0^2\ aa^2\ r^3\ \cos(\theta)^4 + 10\ ee\ cc\ r^5\ cc0\ aa^2\ \cos(\theta)^4 + 4\ ee\ cc\ r^3\ \cos(\theta)^4\ cc0\ aa\ aa0 + 5\ ee\ cc^2\ r^5\ \cos(\theta)^4\ aa\ aa0$

$+\ 9\ ee\ cc^2\ r^7\ aa^2\ \cos(\theta)^4 - 3\ ee\ cc0^2\ aa^2\ r^3\ \cos(\theta)^2 - 5\ ee\ cc^2\ r^7\ aa^2\ \cos(\theta)^2 - 4\ ee\ cc^2\ r^5\ aa\ \cos(\theta)^2\ aa0$

$+\ 5\ aa^2\ r^5\ \cos(\theta)^4 - 4\ aa^2\ r^5\ \cos(\theta)^2 + aa\ r^3\ aa0)\ /\ (r\ (r+bb)\ ee)$

Christoffel_Gamma2$(2,1,3) = \dfrac{1}{4}\ \sin(\theta)\ (5\ ee\ aa\ r^8\ cc^3\ bb + 5\ aa0\ r^2\ cc0\ bb + 5\ aa\ r^4\ cc0\ bb + ee\ aa\ r^2\ cc0^3\ bb$

$+\ 8\ ee\ aa0\ cc^2\ r^5\ cc0 + 4\ aa\ r^5\ cc0 + 7\ ee\ aa\ r^4\ cc\ cc0^2\ bb + 4\ ee\ aa0\ cc\ r^3\ cc0^2 + aa0\ r^4\ cc\ bb + 4\ ee\ aa\ r^9\ cc^3$

$+ 11\ ee\ aa0\ cc^2\ r^4\ cc0\ bb + 4\ ee\ aa\ r^5\ cc\ cc0^2 + aa\ r^6\ cc\ bb + 11\ ee\ aa\ r^6\ cc^2\ cc0\ bb + 7\ ee\ aa0\ cc\ r^2\ cc0^2\ bb$

$+ ee\ aa0\ cc0^3\ bb + 5\ ee\ aa0\ cc^3\ r^6\ bb + 8\ ee\ aa\ r^7\ cc^2\ cc0 + 4\ aa0\ r^3\ cc0 + 4\ ee\ aa0\ cc^3\ r^7 - 8\ \cos(\theta)^6\ ee\ aa\ r^7\ cc^2\ cc0$

$- 11\ \cos(\theta)^6\ ee\ aa\ r^6\ cc^2\ cc0\ bb + 7\ \cos(\theta)^4\ ee\ aa\ r^4\ cc\ cc0^2\ bb + 4\ \cos(\theta)^4\ ee\ aa\ r^5\ cc\ cc0^2 + 5\ \cos(\theta)^4\ aa\ r^4\ cc0\ bb$

$- 14\ \cos(\theta)^2\ ee\ aa\ r^4\ cc\ cc0^2\ bb - 20\ \cos(\theta)^6\ ee\ aa\ r^8\ cc^3\ bb - 16\ \cos(\theta)^6\ ee\ aa\ r^9\ cc^3 + 4\ \cos(\theta)^8\ ee\ aa\ r^9\ cc^3$

$- 12\ \cos(\theta)^2\ ee\ aa0\ cc^3\ r^7 - 4\ \cos(\theta)^2\ ee\ aa0\ cc\ r^3\ cc0^2 + 8\ \cos(\theta)^4\ ee\ aa0\ cc^2\ r^5\ cc0 + \cos(\theta)^4\ aa0\ r^4\ cc\ bb$

$- 4\ \cos(\theta)^2\ aa0\ r^3\ cc0 - 15\ \cos(\theta)^2\ ee\ aa0\ cc^3\ r^6\ bb + 15\ \cos(\theta)^4\ ee\ aa0\ cc^3\ r^6\ bb - \cos(\theta)^2\ ee\ aa\ r^2\ cc0^3\ bb$

$+ 3\ \cos(\theta)^4\ aa\ r^6\ cc\ bb - \cos(\theta)^6\ aa\ r^6\ cc\ bb + 30\ \cos(\theta)^4\ ee\ aa\ r^8\ cc^3\ bb - 16\ \cos(\theta)^2\ ee\ aa\ r^9\ cc^3$

$- 7\ \cos(\theta)^2\ ee\ aa0\ cc\ r^2\ cc0^2\ bb - 5\ \cos(\theta)^6\ ee\ aa0\ cc^3\ r^6\ bb - 4\ \cos(\theta)^6\ ee\ aa0\ cc^3\ r^7$

$- 22\ \cos(\theta)^2\ ee\ aa0\ cc^2\ r^4\ cc0\ bb - 20\ \cos(\theta)^2\ ee\ aa\ r^8\ cc^3\ bb + 12\ \cos(\theta)^4\ ee\ aa0\ cc^3\ r^7 - 2\ \cos(\theta)^2\ aa0\ r^4\ cc\ bb$

$- 5\ \cos(\theta)^2\ aa0\ r^2\ cc0\ bb + 5\ \cos(\theta)^8\ ee\ aa\ r^8\ cc^3\ bb + 24\ \cos(\theta)^4\ ee\ aa\ r^7\ cc^2\ cc0 + 33\ \cos(\theta)^4\ ee\ aa\ r^6\ cc^2\ cc0\ bb$

$+ 24\ \cos(\theta)^4\ ee\ aa\ r^9\ cc^3 - 33\ \cos(\theta)^2\ ee\ aa\ r^6\ cc^2\ cc0\ bb - 16\ \cos(\theta)^2\ ee\ aa0\ cc^2\ r^5\ cc0 - 10\ \cos(\theta)^2\ aa\ r^4\ cc0\ bb$

$- 3\ \cos(\theta)^2\ aa\ r^6\ cc\ bb - 24\ \cos(\theta)^2\ ee\ aa\ r^7\ cc^2\ cc0 - 8\ \cos(\theta)^2\ ee\ aa\ r^5\ cc\ cc0^2 + 11\ \cos(\theta)^4\ ee\ aa0\ cc^2\ r^4\ cc0\ bb$

$- 8\ \cos(\theta)^2\ aa\ r^5\ cc0 + 4\ \cos(\theta)^4\ aa\ r^5\ cc0 \Big) \Big/ \left(r^4\ (-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2) \sqrt{\dfrac{r+bb}{r}} \right)$

$\text{Christoffel\_Gamma2}(2, 1, 4) = -\dfrac{1}{4} \sin(\theta)\ (-4\ aa^3\ r^9 - 12\ \cos(\theta)^4\ aa0^2\ ee\ cc^2\ r^7\ aa + 16\ \cos(\theta)^2\ aa^3\ r^9$

$- 24\ \cos(\theta)^4\ aa^3\ r^9 + 16\ \cos(\theta)^6\ aa^3\ r^9 - 4\ \cos(\theta)^8\ aa^3\ r^9 + 4\ aa\ r^5\ ee - 4\ aa\ r^5\ \cos(\theta)^2\ ee - 8\ aa0\ r^7\ aa^2$

$+ 4\ aa0\ ee^2\ cc0\ cc\ r^3 + 4\ aa0\ ee^2\ cc^2\ r^5 + aa0\ ee^2\ cc0^2\ bb + aa\ r^2\ ee^2\ cc0^2\ bb - 4\ aa0^2\ r^5\ aa - 8\ aa^3\ r^9\ ee\ cc\ cc0$

$- 4\ aa0^2\ ee\ cc0^2\ aa\ r^3 - 4\ aa^3\ r^{11}\ ee\ cc^2 + 6\ aa\ r^4\ ee^2\ cc\ cc0\ bb + 4\ aa\ r^5\ ee^2\ cc0\ cc + 4\ aa\ r^7\ ee^2\ cc^2 + 2\ aa0\ r^2\ ee\ bb$

$- 8\ aa0^2\ ee\ cc\ r^5\ cc0\ aa + 6\ aa\ r^4\ ee\ bb + 5\ aa\ r^6\ ee^2\ cc^2\ bb - 4\ aa^3\ r^7\ ee\ cc0^2 + 6\ aa0\ ee^2\ cc\ r^2\ cc0\ bb$

$- 4\ aa0^2\ ee\ cc^2\ r^7\ aa - 8\ aa0\ ee\ cc0^2\ aa^2\ r^5 - 8\ aa0\ ee\ cc^2\ r^9\ aa^2 + 5\ aa0\ ee^2\ cc^2\ r^4\ bb - 16\ aa0\ ee\ cc\ r^7\ cc0\ aa^2$

$- 8\ \cos(\theta)^2\ aa0\ ee^2\ cc^2\ r^5 + 5\ \cos(\theta)^4\ aa0\ ee^2\ cc^2\ r^4\ bb + 16\ \cos(\theta)^2\ aa0^2\ ee\ cc\ r^5\ cc0\ aa$

$+ 4\ \cos(\theta)^2\ aa0^2\ ee\ cc0^2\ aa\ r^3 - 2\ \cos(\theta)^2\ aa0\ r^2\ ee\ bb + 16\ \cos(\theta)^6\ aa0\ ee\ cc\ r^7\ cc0\ aa^2 + 12\ \cos(\theta)^4\ aa\ r^7\ ee^2\ cc^2$

$+ 12\ \cos(\theta)^2\ aa0^2\ ee\ cc^2\ r^7\ aa + 4\ \cos(\theta)^6\ aa^3\ r^7\ ee\ cc0^2 - 5\ \cos(\theta)^6\ aa\ r^6\ ee^2\ cc^2\ bb + 2\ \cos(\theta)^4\ aa\ r^4\ ee\ bb$

$- 4\ \cos(\theta)^6\ aa\ r^7\ ee^2\ cc^2 - 8\ \cos(\theta)^4\ aa^2\ r^5\ ee\ cc0^2\ aa0 - 48\ \cos(\theta)^4\ aa^2\ r^9\ ee\ cc^2\ aa0$

$- 8\ \cos(\theta)^4\ aa0^2\ ee\ cc\ r^5\ cc0\ aa - 8\ \cos(\theta)^8\ aa0\ ee\ cc^2\ r^9\ aa^2 - 8\ \cos(\theta)^2\ aa\ r^5\ ee^2\ cc0\ cc$

$- 48\ \cos(\theta)^4\ aa^3\ r^9\ ee\ cc\ cc0 - 12\ \cos(\theta)^4\ aa^3\ r^7\ ee\ cc0^2 + 6\ \cos(\theta)^4\ aa\ r^4\ ee^2\ cc\ cc0\ bb - 40\ \cos(\theta)^4\ aa^3\ r^{11}\ ee\ cc^2$

$+ 4\ \cos(\theta)^4\ aa\ r^5\ ee^2\ cc0\ cc + 32\ \cos(\theta)^6\ aa^3\ r^9\ ee\ cc\ cc0 - 8\ \cos(\theta)^8\ aa^3\ r^9\ ee\ cc\ cc0 + 40\ \cos(\theta)^6\ aa^3\ r^{11}\ ee\ cc^2$

$- 24\ \cos(\theta)^4\ aa^2\ r^7\ aa0 + 24\ \cos(\theta)^2\ aa0\ r^7\ aa^2 + 32\ \cos(\theta)^2\ aa^3\ r^9\ ee\ cc\ cc0 + 4\ \cos(\theta)^6\ aa0^2\ ee\ cc^2\ r^7\ aa$

$+ 12\ \cos(\theta)^2\ aa^3\ r^7\ ee\ cc0^2 - 12\ \cos(\theta)^2\ aa\ r^7\ ee^2\ cc^2 + 20\ \cos(\theta)^2\ aa^3\ r^{11}\ ee\ cc^2 - 12\ \cos(\theta)^2\ aa\ r^4\ ee^2\ cc\ cc0\ bb$

$- 20\ \cos(\theta)^8\ aa^3\ r^{11}\ ee\ cc^2 + 32\ \cos(\theta)^6\ aa^2\ r^9\ ee\ cc^2\ aa0 + 32\ \cos(\theta)^2\ aa^2\ r^9\ ee\ cc^2\ aa0 - 8\ \cos(\theta)^2\ aa\ r^4\ ee\ bb$

$- 6\ \cos(\theta)^2\ aa0\ ee^2\ cc\ r^2\ cc0\ bb + 15\ \cos(\theta)^4\ aa\ r^6\ ee^2\ cc^2\ bb - 15\ \cos(\theta)^2\ aa\ r^6\ ee^2\ cc^2\ bb$

$- \cos(\theta)^2\ aa\ r^2\ ee^2\ cc0^2\ bb + 16\ \cos(\theta)^2\ aa^2\ r^5\ ee\ cc0^2\ aa0 + 48\ \cos(\theta)^2\ aa^2\ r^7\ ee\ cc\ cc0\ aa0$

$- 10\ \cos(\theta)^2\ aa0\ ee^2\ cc^2\ r^4\ bb + 8\ \cos(\theta)^2\ aa0^2\ r^5\ aa + 8\ \cos(\theta)^6\ aa0\ r^7\ aa^2 - 4\ \cos(\theta)^4\ aa0^2\ r^5\ aa$

$+ 4\ \cos(\theta)^4\ aa0\ ee^2\ cc^2\ r^5 - 4\ \cos(\theta)^2\ aa0\ ee^2\ cc0\ r^3\ cc + 4\ \cos(\theta)^{10}\ aa^3\ r^{11}\ ee\ cc^2 - 48\ \cos(\theta)^4\ aa^2\ r^7\ ee\ cc\ cc0\ aa0)$

$\Big/ (r^4\ ee\ (-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2))$

$\text{Christoffel\_Gamma2}(2, 2, 1) = -\ (aa^2\ r^5 - ee\ r - ee\ bb - 3\ aa\ r^3\ \cos(\theta)^2\ aa0 - 2\ r^5\ \cos(\theta)^6\ aa^2$

$+ 2\ r^3\ \cos(\theta)^4\ aa\ aa0 + ee\ cc^2\ r^5\ aa\ aa0 + 2\ ee\ cc\ r^5\ cc0\ aa^2 + 2\ ee\ cc\ r^3\ cc0\ aa\ aa0 + ee\ cc0^2\ aa\ r\ aa0 + ee\ cc^2\ r^7\ aa^2$

$+ ee\ cc0^2\ aa^2\ r^3 - 6\ ee\ cc\ r^3\ cc0\ aa\ \cos(\theta)^2\ aa0 - 2\ ee\ cc0^2\ aa\ r\ \cos(\theta)^2\ aa0 - 8\ ee\ cc\ r^5\ cc0\ aa^2\ \cos(\theta)^2$

$+ 2\ ee\ cc^2\ r^7\ \cos(\theta)^8\ aa^2 - 4\ ee\ cc\ r^5\ \cos(\theta)^6\ cc0\ aa^2 - 7\ ee\ cc^2\ r^7\ \cos(\theta)^6\ aa^2 - 2\ ee\ cc^2\ r^5\ \cos(\theta)^6\ aa\ aa0$

$+ 2\ ee\ cc0^2\ aa^2\ r^3\ \cos(\theta)^4 + 10\ ee\ cc\ r^5\ cc0\ aa^2\ \cos(\theta)^4 + 4\ ee\ cc\ r^3\ \cos(\theta)^4\ cc0\ aa\ aa0 + 5\ ee\ cc^2\ r^5\ \cos(\theta)^4\ aa\ aa0$

$+ 9\ ee\ cc^2\ r^7\ aa^2\ \cos(\theta)^4 - 3\ ee\ cc0^2\ aa^2\ r^3\ \cos(\theta)^2 - 5\ ee\ cc^2\ r^7\ aa^2\ \cos(\theta)^2 - 4\ ee\ cc^2\ r^5\ aa\ \cos(\theta)^2\ aa0$

$+ 5\ aa^2\ r^5 \cos(\theta)^4 - 4\ aa^2\ r^5 \cos(\theta)^2 + aa\ r^3\ aa0\ ) / (\ r\ (r + bb)\ ee\ )$

Christoffel_Gamma2(2, 2, 2) $= 2\ (\cos(\theta)^6\ aa\ r^6\ ee\ cc^2 - 3 \cos(\theta)^4\ aa\ r^6\ ee\ cc^2 - \cos(\theta)^4\ r^4\ aa0\ ee\ cc^2$

$- r^4 \cos(\theta)^4\ aa - 2 \cos(\theta)^4\ aa\ r^4\ ee\ cc\ cc0 + 4 \cos(\theta)^2\ aa\ r^4\ ee\ cc\ cc0 + \cos(\theta)^2\ aa\ r^2\ ee\ cc0^2$

$+ 2 \cos(\theta)^2\ r^2\ aa0\ ee\ cc\ cc0 + 3 \cos(\theta)^2\ aa\ r^6\ ee\ cc^2 + 2\ aa\ r^4 \cos(\theta)^2 + 2 \cos(\theta)^2\ r^4\ aa0\ ee\ cc^2 + r^2 \cos(\theta)^2\ aa0$

$- ee\ cc0^2\ aa0 - ee\ cc^2\ aa0\ r^4 - aa\ r^4 - ee\ cc^2\ r^6\ aa - 2\ aa\ r^4\ ee\ cc\ cc0 - r^2\ aa0 - 2\ r^2\ ee\ cc\ cc0\ aa0 - aa\ ee\ cc0^2\ r^2\ )$

$aa\ r \sin(\theta) \cos(\theta) / ((r + bb)\ ee\ )$

Christoffel_Gamma2(2, 2, 3) $= (-ee\ r^6\ aa\ cc^3 + 3 \cos(\theta)^2\ ee\ r^6\ aa\ cc^3 - 2\ ee\ r^4\ aa\ cc^2\ cc0 - 3 \cos(\theta)^4\ ee\ r^6\ aa\ cc^3$

$+ 4 \cos(\theta)^2\ ee\ r^4\ aa\ cc^2\ cc0 - r^2\ aa\ cc0 - ee\ r^2\ aa\ cc\ cc0^2 + \cos(\theta)^6\ ee\ r^6\ aa\ cc^3 - 2 \cos(\theta)^4\ ee\ r^4\ aa\ cc^2\ cc0$

$+ r^2 \cos(\theta)^2\ aa\ cc0 + \cos(\theta)^2\ ee\ r^2\ aa\ cc\ cc0^2 - ee\ r^4\ aa0\ cc^3 + 2 \cos(\theta)^2\ ee\ r^4\ aa0\ cc^3 - 2\ ee\ r^2\ aa0\ cc^2\ cc0$

$- \cos(\theta)^4\ ee\ r^4\ aa0\ cc^3 + 2 \cos(\theta)^2\ ee\ r^2\ aa0\ cc^2\ cc0 - aa0\ cc0 - ee\ aa0\ cc\ cc0^2\ ) \cos(\theta) \sqrt{\dfrac{r + bb}{r}} / (r + bb)$

Christoffel_Gamma2(2, 2, 4) $= - (-aa^3\ r^7 \cos(\theta)^6 - aa\ r^3 \cos(\theta)^2\ aa0^2 + 3\ aa^3\ r^7 \cos(\theta)^4 + 2\ aa^2\ r^5 \cos(\theta)^4\ aa0$

$- 3\ aa^3\ r^7 \cos(\theta)^2 - 4\ aa^2\ r^5 \cos(\theta)^2\ aa0 - 8\ aa^2\ r^5\ ee\ cc\ cc0 \cos(\theta)^2\ aa0 + \cos(\theta)^2\ ee^2\ cc^2\ r^3\ aa0$

$+ \cos(\theta)^2\ ee^2\ cc\ bb\ aa\ r^2\ cc0 - \cos(\theta)^4\ ee^2\ cc^2\ aa\ r^5 + 2 \cos(\theta)^2\ ee^2\ cc^2\ aa\ r^5 + 6\ aa^3\ r^9\ ee\ cc^2 \cos(\theta)^4$

$- 2\ aa^2\ r^7\ ee\ cc^2 \cos(\theta)^6\ aa0 + aa^3\ r^5\ ee\ cc0^2 \cos(\theta)^4 - 4\ aa^3\ r^9\ ee\ cc^2 \cos(\theta)^2 - 2\ aa^2\ r^3\ ee\ cc0^2 \cos(\theta)^2\ aa0$

$+ \cos(\theta)^2\ ee^2\ cc^2\ bb\ aa0\ r^2 - \cos(\theta)^4\ ee^2\ cc^2\ bb\ aa\ r^4 - 2\ aa\ r^3\ ee\ cc \cos(\theta)^2\ cc0\ aa0^2 + 2 \cos(\theta)^2\ ee^2\ cc^2\ bb\ aa\ r^4$

$+ 4\ aa^2\ r^5\ ee\ cc \cos(\theta)^4\ cc0\ aa0 + \cos(\theta)^2\ ee^2\ cc\ aa\ r^3\ cc0 - 4\ aa^3\ r^9\ ee\ cc^2 \cos(\theta)^6 + aa^3\ r^7$

$+ 6\ aa^2\ r^7\ ee\ cc^2 \cos(\theta)^4\ aa0 - 2\ aa^3\ r^5\ ee\ cc0^2 \cos(\theta)^2 + 6\ aa^3\ r^7\ ee\ cc\ cc0 \cos(\theta)^4 + aa^3\ r^9\ ee\ cc^2 \cos(\theta)^8$

$- 6\ aa^3\ r^7\ ee\ cc\ cc0 \cos(\theta)^2 - 2\ aa\ r^5\ ee\ cc^2 \cos(\theta)^2\ aa0^2 - 2\ aa^3\ r^7\ ee\ cc \cos(\theta)^6\ cc0 + aa\ r^5\ ee\ cc^2 \cos(\theta)^4\ aa0^2$

$- 6\ aa^2\ r^7\ ee\ cc^2 \cos(\theta)^2\ aa0 + 4\ aa^2\ r^5\ ee\ cc\ cc0\ aa0 - aa0\ ee^2\ cc^2\ bb\ r^2 - aa\ r^3\ ee^2\ cc\ cc0\ bb + 2\ aa0^2\ ee\ cc\ r^3\ cc0\ aa$

$+ 2\ aa0\ ee\ cc^2\ r^7\ aa^2 + 2\ aa0\ ee\ cc0^2\ aa^2\ r^3 - aa\ r^5\ ee^2\ cc^2 - aa0\ ee^2\ cc\ r\ cc0 - aa0\ ee^2\ cc\ cc0\ bb + 2\ aa0\ aa^2\ r^5$

$+ aa^3\ r^9\ ee\ cc^2 - aa\ r^2\ ee^2\ cc\ cc0\ bb - aa\ r^4\ ee^2\ cc^2\ bb + 2\ aa^3\ r^7\ ee\ cc\ cc0 + aa0^2\ ee\ cc^2\ r^5\ aa + aa^3\ r^5\ ee\ cc0^2$

$- aa0\ ee^2\ cc^2\ r^3 + aa0^2\ ee\ cc0^2\ aa\ r + aa0^2\ aa\ r^3\ ) \cos(\theta) / ((r + bb)\ ee\ r\ )$

Christoffel_Gamma2(2, 3, 1) $= \dfrac{1}{4} \sin(\theta)\ (5\ ee\ aa\ r^8\ cc^3\ bb + 5\ aa0\ r^2\ cc0\ bb + 5\ aa\ r^4\ cc0\ bb + ee\ aa\ r^2\ cc0^3\ bb$

$+ 8\ ee\ aa0\ cc^2\ r^5\ cc0 + 4\ aa\ r^5\ cc0 + 7\ ee\ aa\ r^4\ cc\ cc0^2\ bb + 4\ ee\ aa0\ cc\ r^3\ cc0^2 + aa0\ r^4\ cc\ bb + 4\ ee\ aa\ r^9\ cc^3$

$+ 11\ ee\ aa0\ cc^2\ r^4\ cc0\ bb + 4\ ee\ aa\ r^5\ cc\ cc0^2 + aa\ r^6\ cc\ bb + 11\ ee\ aa\ r^6\ cc^2\ cc0\ bb + 7\ ee\ aa0\ cc\ r^2\ cc0^2\ bb$

$+ ee\ aa0\ cc0^3\ bb + 5\ ee\ aa0\ cc^3\ r^6\ bb + 8\ ee\ aa\ r^7\ cc^2\ cc0 + 4\ aa0\ r^3\ cc0 + 4\ ee\ aa0\ cc^3\ r^7 - 8 \cos(\theta)^6\ ee\ aa\ r^7\ cc^2\ cc0$

$- 11 \cos(\theta)^6\ ee\ aa\ r^6\ cc^2\ cc0\ bb + 7 \cos(\theta)^4\ ee\ aa\ r^4\ cc\ cc0^2\ bb + 4 \cos(\theta)^4\ ee\ aa\ r^5\ cc\ cc0^2 + 5 \cos(\theta)^4\ aa\ r^4\ cc0\ bb$

$- 14 \cos(\theta)^2\ ee\ aa\ r^4\ cc\ cc0^2\ bb - 20 \cos(\theta)^6\ ee\ aa\ r^8\ cc^3\ bb - 16 \cos(\theta)^6\ ee\ aa\ r^9\ cc^3 + 4 \cos(\theta)^8\ ee\ aa\ r^9\ cc^3$

$- 12 \cos(\theta)^2\ ee\ aa0\ cc^3\ r^7 - 4 \cos(\theta)^2\ ee\ aa0\ cc\ r^3\ cc0^2 + 8 \cos(\theta)^4\ ee\ aa0\ cc^2\ r^5\ cc0 + \cos(\theta)^4\ aa0\ r^4\ cc\ bb$

$- 4 \cos(\theta)^2\ aa0\ r^3\ cc0 - 15 \cos(\theta)^2\ ee\ aa0\ cc^3\ r^6\ bb + 15 \cos(\theta)^4\ ee\ aa0\ cc^3\ r^6\ bb - \cos(\theta)^2\ ee\ aa\ r^2\ cc0^3\ bb$

$+ 3 \cos(\theta)^4\ aa\ r^6\ cc\ bb - \cos(\theta)^6\ aa\ r^6\ cc\ bb + 30 \cos(\theta)^4\ ee\ aa\ r^8\ cc^3\ bb - 16 \cos(\theta)^2\ ee\ aa\ r^9\ cc^3$

$- 7 \cos(\theta)^2\ ee\ aa0\ cc\ r^2\ cc0^2\ bb - 5 \cos(\theta)^6\ ee\ aa0\ cc^3\ r^6\ bb - 4 \cos(\theta)^6\ ee\ aa0\ cc^3\ r^7$

$- 22 \cos(\theta)^2\ ee\ aa0\ cc^2\ r^4\ cc0\ bb - 20 \cos(\theta)^2\ ee\ aa\ r^8\ cc^3\ bb + 12 \cos(\theta)^4\ ee\ aa0\ cc^3\ r^7 - 2 \cos(\theta)^2\ aa0\ r^4\ cc\ bb$

$- 5 \cos(\theta)^2\ aa0\ r^2\ cc0\ bb + 5 \cos(\theta)^8\ ee\ aa\ r^8\ cc^3\ bb + 24 \cos(\theta)^4\ ee\ aa\ r^7\ cc^2\ cc0 + 33 \cos(\theta)^4\ ee\ aa\ r^6\ cc^2\ cc0\ bb$

$+ 24 \cos(\theta)^4\ ee\ aa\ r^9\ cc^3 - 33 \cos(\theta)^2\ ee\ aa\ r^6\ cc^2\ cc0\ bb - 16 \cos(\theta)^2\ ee\ aa0\ cc^2\ r^5\ cc0 - 10 \cos(\theta)^2\ aa\ r^4\ cc0\ bb$

$- 3 \cos(\theta)^2\ aa\ r^6\ cc\ bb - 24 \cos(\theta)^2\ ee\ aa\ r^7\ cc^2\ cc0 - 8 \cos(\theta)^2\ ee\ aa\ r^5\ cc\ cc0^2 + 11 \cos(\theta)^4\ ee\ aa0\ cc^2\ r^4\ cc0\ bb$

$- 8 \cos(\theta)^2\ aa\ r^5\ cc0 + 4 \cos(\theta)^4\ aa\ r^5\ cc0) \Bigg/ \left(r^4\ (-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2) \sqrt{\dfrac{r + bb}{r}} \right)$

Christoffel_Gamma2(2, 3, 2) $= (-ee\ r^6\ aa\ cc^3 + 3 \cos(\theta)^2\ ee\ r^6\ aa\ cc^3 - 2\ ee\ r^4\ aa\ cc^2\ cc0 - 3 \cos(\theta)^4\ ee\ r^6\ aa\ cc^3$

$+ 4 \cos(\theta)^2\ ee\ r^4\ aa\ cc^2\ cc0 - r^2\ aa\ cc0 - ee\ r^2\ aa\ cc\ cc0^2 + \cos(\theta)^6\ ee\ r^6\ aa\ cc^3 - 2 \cos(\theta)^4\ ee\ r^4\ aa\ cc^2\ cc0$

$+ r^2 \cos(\theta)^2\, aa\, cc0 + \cos(\theta)^2\, ee\, r^2\, aa\, cc\, cc0^2 - ee\, r^4\, aa0\, cc^3 + 2\cos(\theta)^2\, ee\, r^4\, aa0\, cc^3 - 2\, ee\, r^2\, aa0\, cc^2\, cc0$

$- \cos(\theta)^4\, ee\, r^4\, aa0\, cc^3 + 2\cos(\theta)^2\, ee\, r^2\, aa0\, cc^2\, cc0 - aa0\, cc0 - ee\, aa0\, cc\, cc0^2)\cos(\theta)\sqrt{\dfrac{r+bb}{r}}\, / \, (r+bb)$

$$\text{Christoffel\_Gamma2}(2,3,3) = (-1 - 2\, ee\, cc^2\, r^2 + 2\, ee\, cc^2\, r^2\cos(\theta)^2 - 2\, ee\, cc\, cc0)\sin(\theta)\cos(\theta)$$

$\text{Christoffel\_Gamma2}(2,3,4) = \dfrac{1}{4}\cos(\theta)\sin(\theta)\,(-2\, ee\, cc0^3\, aa\, r^2\, aa0\, bb - ee\, cc0^3\, aa0^2\, bb - ee\, cc^3\, r^{10}\, aa^2\, bb$

$- ee\, cc0^3\, aa^2\, r^4\, bb - ee\, cc^3\, r^6\, aa0^2\, bb - 3\, ee\, cc^2\, r^4\, aa0^2\, cc0\, bb - 3\, ee\, cc\, r^2\, cc0^2\, aa0^2\, bb - 2\, r^4\, aa\, aa0\, cc0\, bb$

$- 3\, ee\, cc^2\, r^8\, cc0\, aa^2\, bb - 6\, ee\, cc^2\, r^6\, aa\, aa0\, cc0\, bb - 2\, r^6\, aa\, aa0\, cc\, bb - r^6\, aa^2\, cc0\, bb - 2\, ee\, cc^3\, r^8\, aa\, aa0\, bb$

$- r^4\, aa0^2\, cc\, bb - r^2\, aa0^2\, cc0\, bb - r^8\, aa^2\, cc\, bb + 9\, ee\, cc0^2\, aa^2\, r^6\, cc\cos(\theta)^2\, bb - 3\, ee\, cc^3\, r^6\cos(\theta)^4\, aa0^2\, bb$

$- 5\, ee\, cc^3\, r^{10}\cos(\theta)^8\, aa^2\, bb + ee\, cc^3\, r^{10}\cos(\theta)^{10}\, aa^2\, bb + 8\, ee\, cc^3\, r^8\cos(\theta)^6\, aa\, aa0\, bb$

$- 3\, ee\, cc^2\, r^8\cos(\theta)^8\, aa^2\, cc0\, bb - 2\, ee\, cc^3\, r^8\cos(\theta)^8\, aa\, aa0\, bb + 2\, ee\, cc0^3\, aa\, r^2\cos(\theta)^2\, aa0\, bb$

$- 3\, ee\, cc^2\, r^4\cos(\theta)^4\, cc0\, aa0^2\, bb + 12\, ee\, cc0^2\, aa\, r^4\, aa0\, cc\cos(\theta)^2\, bb + r^2\cos(\theta)^2\, aa0^2\, cc0\, bb$

$- r^4\cos(\theta)^4\, aa0^2\, cc\, bb - 2\, r^4\cos(\theta)^4\, aa\, aa0\, cc0\, bb + 2\, r^6\cos(\theta)^6\, aa\, aa0\, cc\, bb + 4\, r^4\, aa\cos(\theta)^2\, aa0\, cc0\, bb$

$- 12\, ee\, cc^3\, r^8\, aa\cos(\theta)^4\, aa0\, bb + ee\, cc^3\, r^6\cos(\theta)^6\, aa0^2\, bb - 9\, ee\, cc\, r^6\, cc0^2\, aa^2\cos(\theta)^4\, bb$

$- ee\, cc0^3\, aa^2\, r^4\cos(\theta)^4\, bb + 2\, ee\, cc0^3\, aa^2\, r^4\cos(\theta)^2\, bb - 10\, ee\, cc^3\, r^{10}\, aa^2\cos(\theta)^4\, bb$

$+ 3\, ee\, cc0^2\, aa^2\, r^6\cos(\theta)^6\, cc\, bb + 10\, ee\, cc^3\, r^{10}\cos(\theta)^6\, aa^2\, bb + 6\, ee\, cc^2\, r^6\cos(\theta)^6\, aa\, aa0\, cc0\, bb$

$- 6\, ee\, cc0^2\, aa\, r^4\cos(\theta)^4\, aa0\, cc\, bb - 18\, ee\, cc^2\, r^8\, cc0\, aa^2\cos(\theta)^4\, bb + 12\, ee\, cc^2\, r^8\, cc0\, aa^2\cos(\theta)^6\, bb$

$- 18\, ee\, cc^2\, r^6\, cc0\, aa\cos(\theta)^4\, aa0\, bb - r^8\cos(\theta)^8\, aa^2\, cc\, bb + 5\, ee\, cc^3\, r^{10}\, aa^2\cos(\theta)^2\, bb + 3\, r^6\, aa^2\cos(\theta)^2\, cc0\, bb$

$+ 2\, r^4\, aa0^2\, cc\cos(\theta)^2\, bb - 6\, r^8\, aa^2\cos(\theta)^4\, cc\, bb + 4\, r^8\, aa^2\cos(\theta)^2\, cc\, bb - 3\, r^6\, aa^2\cos(\theta)^4\, cc0\, bb$

$+ 4\, r^8\, aa^2\cos(\theta)^6\, cc\, bb - 4\, r^5\, ee\, cc + 8\, r^4\cos(\theta)^2\, ee\, cc\, bb + r^6\cos(\theta)^6\, aa^2\, cc0\, bb - 4\, r^3\, ee\, bb^2\, cc$

$- 3\, ee\, cc\, r^6\, cc0^2\, aa^2\, bb - 6\, ee\, cc\, r^4\, cc0^2\, aa\, aa0\, bb + 8\, ee\, cc^3\, r^8\, aa\, aa0\cos(\theta)^2\, bb + 3\, ee\, cc0^2\, aa0^2\, cc\, r^2\cos(\theta)^2\, bb$

$+ 6\, ee\, cc^2\, r^4\, cc0\, aa0^2\cos(\theta)^2\, bb - 6\, r^6\cos(\theta)^4\, aa\, aa0\, cc\, bb + 6\, r^6\, aa\, aa0\, cc\cos(\theta)^2\, bb$

$+ 12\, ee\, cc^2\, r^8\, cc0\, aa^2\cos(\theta)^2\, bb + 4\, r^3\, ee\, bb^2\, cc\cos(\theta)^2 - 8\, r^4\, ee\, cc\, bb + 3\, ee\, cc^3\, r^6\, aa0^2\cos(\theta)^2\, bb$

$+ 18\, ee\, cc^2\, r^6\, cc0\, aa\, aa0\cos(\theta)^2\, bb + 4\, r^5\cos(\theta)^2\, ee\, cc)\, \Big/ \left(r^4(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)\sqrt{\dfrac{r+bb}{r}}\right)$

$\text{Christoffel\_Gamma2}(2,4,1) = -\dfrac{1}{4}\sin(\theta)\,(-4\, aa^3\, r^9 - 12\cos(\theta)^4\, aa0^2\, ee\, cc^2\, r^7\, aa + 16\cos(\theta)^2\, aa^3\, r^9$

$- 24\cos(\theta)^4\, aa^3\, r^9 + 16\cos(\theta)^6\, aa^3\, r^9 - 4\cos(\theta)^8\, aa^3\, r^9 + 4\, aa\, r^5\, ee - 4\, aa\, r^5\cos(\theta)^2\, ee - 8\, aa0\, r^7\, aa^2$

$+ 4\, aa0\, ee^2\, cc0\, cc\, r^3 + 4\, aa0\, ee^2\, cc^2\, r^5 + aa0\, ee^2\, cc0^2\, bb + aa\, r^2\, ee^2\, cc0^2\, bb - 4\, aa0^2\, r^5\, aa - 8\, aa^3\, r^9\, ee\, cc\, cc0$

$- 4\, aa0^2\, ee\, cc0^2\, aa\, r^3 - 4\, aa^3\, r^{11}\, ee\, cc^2 + 6\, aa\, r^4\, ee^2\, cc\, cc0\, bb + 4\, aa\, r^5\, ee^2\, cc0\, cc + 4\, aa\, r^7\, ee^2\, cc^2 + 2\, aa0\, r^2\, ee\, bb$

$- 8\, aa0^2\, ee\, cc\, r^5\, cc0\, aa + 6\, aa\, r^4\, ee\, bb + 5\, aa\, r^6\, ee^2\, cc^2\, bb - 4\, aa^3\, r^7\, ee\, cc0^2 + 6\, aa0\, ee^2\, cc\, r^2\, cc0\, bb$

$- 4\, aa0^2\, ee\, cc^2\, r^7\, aa - 8\, aa0\, ee\, cc0^2\, aa^2\, r^5 - 8\, aa0\, ee\, cc^2\, r^9\, aa^2 + 5\, aa0\, ee^2\, cc^2\, r^4\, bb - 16\, aa0\, ee\, cc\, r^7\, cc0\, aa^2$

$- 8\cos(\theta)^2\, aa0\, ee^2\, cc^2\, r^5 + 5\cos(\theta)^4\, aa0\, ee^2\, cc^2\, r^4\, bb + 16\cos(\theta)^2\, aa0^2\, ee\, cc\, r^5\, cc0\, aa$

$+ 4\cos(\theta)^2\, aa0^2\, ee\, cc0^2\, aa\, r^3 - 2\cos(\theta)^2\, aa0\, r^2\, ee\, bb + 16\cos(\theta)^6\, aa0\, ee\, cc\, r^7\, cc0\, aa^2 + 12\cos(\theta)^4\, aa\, r^7\, ee^2\, cc^2$

$+ 12\cos(\theta)^2\, aa0^2\, ee\, cc^2\, r^7\, aa + 4\cos(\theta)^6\, aa^3\, r^7\, ee\, cc0^2 - 5\cos(\theta)^6\, aa\, r^6\, ee^2\, cc^2\, bb + 2\cos(\theta)^4\, aa\, r^4\, ee\, bb$

$- 4\cos(\theta)^6\, aa\, r^7\, ee^2\, cc^2 - 8\cos(\theta)^4\, aa^2\, r^5\, ee\, cc0^2\, aa0 - 48\cos(\theta)^4\, aa^2\, r^9\, ee\, cc^2\, aa0$

$- 8\cos(\theta)^4\, aa0^2\, ee\, cc\, r^5\, cc0\, aa - 8\cos(\theta)^8\, aa0\, ee\, cc^2\, r^9\, aa^2 - 8\cos(\theta)^2\, aa\, r^5\, ee^2\, cc0\, cc$

$- 48\cos(\theta)^4\, aa^3\, r^9\, ee\, cc\, cc0 - 12\cos(\theta)^4\, aa^3\, r^7\, ee\, cc0^2 + 6\cos(\theta)^4\, aa\, r^4\, ee^2\, cc\, cc0\, bb - 40\cos(\theta)^4\, aa^3\, r^{11}\, ee\, cc^2$

$+ 4\cos(\theta)^4\, aa\, r^5\, ee^2\, cc0\, cc + 32\cos(\theta)^6\, aa^3\, r^9\, ee\, cc\, cc0 - 8\cos(\theta)^8\, aa^3\, r^9\, ee\, cc\, cc0 + 40\cos(\theta)^6\, aa^3\, r^{11}\, ee\, cc^2$

$- 24\cos(\theta)^4\, aa^2\, r^7\, aa0 + 24\cos(\theta)^2\, aa0\, r^7\, aa^2 + 32\cos(\theta)^2\, aa^3\, r^9\, ee\, cc\, cc0 + 4\cos(\theta)^6\, aa0^2\, ee\, cc^2\, r^7\, aa$

$+ 12\cos(\theta)^2\, aa^3\, r^7\, ee\, cc0^2 - 12\cos(\theta)^2\, aa\, r^7\, ee^2\, cc^2 + 20\cos(\theta)^2\, aa^3\, r^{11}\, ee\, cc^2 - 12\cos(\theta)^2\, aa\, r^4\, ee^2\, cc\, cc0\, bb$

$- 20\cos(\theta)^8\, aa^3\, r^{11}\, ee\, cc^2 + 32\cos(\theta)^6\, aa^2\, r^9\, ee\, cc^2\, aa0 + 32\cos(\theta)^2\, aa^2\, r^9\, ee\, cc^2\, aa0 - 8\cos(\theta)^2\, aa\, r^4\, ee\, bb$

$$- 6 \cos(\theta)^2 \, aa0 \, ee^2 \, cc \, r^2 \, cc0 \, bb + 15 \cos(\theta)^4 \, aa \, r^6 \, ee^2 \, cc^2 \, bb - 15 \cos(\theta)^2 \, aa \, r^6 \, ee^2 \, cc^2 \, bb$$

$$- \cos(\theta)^2 \, aa \, r^2 \, ee^2 \, cc0^2 \, bb + 16 \cos(\theta)^2 \, aa^2 \, r^5 \, ee \, cc0^2 \, aa0 + 48 \cos(\theta)^2 \, aa^2 \, r^7 \, ee \, cc \, cc0 \, aa0$$

$$- 10 \cos(\theta)^2 \, aa0 \, ee^2 \, cc^2 \, r^4 \, bb + 8 \cos(\theta)^2 \, aa0^2 \, r^5 \, aa + 8 \cos(\theta)^6 \, aa0 \, r^7 \, aa^2 - 4 \cos(\theta)^4 \, aa0^2 \, r^5 \, aa$$

$$+ 4 \cos(\theta)^4 \, aa0 \, ee^2 \, cc^2 \, r^5 - 4 \cos(\theta)^2 \, aa0 \, ee^2 \, cc0 \, r^3 \, cc + 4 \cos(\theta)^{10} \, aa^3 \, r^{11} \, ee \, cc^2 - 48 \cos(\theta)^4 \, aa^2 \, r^7 \, ee \, cc \, cc0 \, aa0)$$

$$\Big/ \, (r^4 \, ee \, (-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2))$$

Christoffel_Gamma2$(2, 4, 2) = - (-aa^3 \, r^7 \cos(\theta)^6 - aa \, r^3 \cos(\theta)^2 \, aa0^2 + 3 \, aa^3 \, r^7 \cos(\theta)^4 + 2 \, aa^2 \, r^5 \cos(\theta)^4 \, aa0$

$$- 3 \, aa^3 \, r^7 \cos(\theta)^2 - 4 \, aa^2 \, r^5 \cos(\theta)^2 \, aa0 - 8 \, aa^2 \, r^5 \, ee \, cc \, cc0 \cos(\theta)^2 \, aa0 + \cos(\theta)^2 \, ee^2 \, cc^2 \, r^3 \, aa0$$

$$+ \cos(\theta)^2 \, ee^2 \, cc \, bb \, aa \, r^2 \, cc0 - \cos(\theta)^4 \, ee^2 \, cc^2 \, aa \, r^5 + 2 \cos(\theta)^2 \, ee^2 \, cc^2 \, aa \, r^5 + 6 \, aa^3 \, r^9 \, ee \, cc^2 \cos(\theta)^4$$

$$- 2 \, aa^2 \, r^7 \, ee \, cc^2 \cos(\theta)^6 \, aa0 + aa^3 \, r^5 \, ee \, cc0^2 \cos(\theta)^4 - 4 \, aa^3 \, r^9 \, ee \, cc^2 \cos(\theta)^2 - 2 \, aa^2 \, r^3 \, ee \, cc0^2 \cos(\theta)^2 \, aa0$$

$$+ \cos(\theta)^2 \, ee^2 \, cc^2 \, bb \, aa0 \, r^2 - \cos(\theta)^4 \, ee^2 \, cc^2 \, bb \, aa \, r^4 - 2 \, aa \, r^3 \, ee \, cc \cos(\theta)^2 \, cc0 \, aa0^2 + 2 \cos(\theta)^2 \, ee^2 \, cc^2 \, bb \, aa \, r^4$$

$$+ 4 \, aa^2 \, r^5 \, ee \, cc \cos(\theta)^4 \, cc0 \, aa0 + \cos(\theta)^2 \, ee^2 \, cc \, aa \, r^3 \, cc0 - 4 \, aa^3 \, r^9 \, ee \, cc^2 \cos(\theta)^6 + aa^3 \, r^7$$

$$+ 6 \, aa^2 \, r^7 \, ee \, cc^2 \cos(\theta)^4 \, aa0 - 2 \, aa^3 \, r^5 \, ee \, cc0^2 \cos(\theta)^2 + 6 \, aa^3 \, r^7 \, ee \, cc \, cc0 \cos(\theta)^4 + aa^3 \, r^9 \, ee \, cc^2 \cos(\theta)^8$$

$$- 6 \, aa^3 \, r^7 \, ee \, cc \, cc0 \cos(\theta)^2 - 2 \, aa \, r^5 \, ee \, cc^2 \cos(\theta)^2 \, aa0^2 - 2 \, aa^3 \, r^7 \, ee \, cc \cos(\theta)^6 \, cc0 + aa \, r^5 \, ee \, cc^2 \cos(\theta)^4 \, aa0^2$$

$$- 6 \, aa^2 \, r^7 \, ee \, cc^2 \cos(\theta)^2 \, aa0 + 4 \, aa^2 \, r^5 \, ee \, cc \, cc0 \, aa0 - aa0 \, ee^2 \, cc^2 \, bb \, r^2 - aa \, r^3 \, ee^2 \, cc \, cc0 + 2 \, aa0^2 \, ee \, cc \, r^3 \, cc0 \, aa$$

$$+ 2 \, aa0 \, ee \, cc^2 \, r^7 \, aa^2 + 2 \, aa0 \, ee \, cc0^2 \, aa^2 \, r^3 - aa \, r^5 \, ee^2 \, cc^2 - aa0 \, ee^2 \, cc \, r \, cc0 - aa0 \, ee^2 \, cc \, cc0 \, bb + 2 \, aa0 \, aa^2 \, r^5$$

$$+ aa^3 \, r^9 \, ee \, cc^2 - aa \, r^2 \, ee^2 \, cc \, cc0 \, bb - aa \, r^4 \, ee^2 \, cc^2 \, bb + 2 \, aa^3 \, r^7 \, ee \, cc \, cc0 + aa0^2 \, ee \, cc^2 \, r^5 \, aa + aa^3 \, r^5 \, ee \, cc0^2$$

$$- aa0 \, ee^2 \, cc^2 \, r^3 + aa0^2 \, ee \, cc0^2 \, aa \, r + aa0^2 \, aa \, r^3) \cos(\theta) \, / \, ((r + bb) \, ee \, r)$$

Christoffel_Gamma2$(2, 4, 3) = \dfrac{1}{4} \cos(\theta) \sin(\theta) \, (-2 \, ee \, cc0^3 \, aa \, r^2 \, aa0 \, bb - ee \, cc0^3 \, aa0^2 \, bb - ee \, cc^3 \, r^{10} \, aa^2 \, bb$

$$- ee \, cc0^3 \, aa^2 \, r^4 \, bb - ee \, cc^3 \, r^6 \, aa0^2 \, bb - 3 \, ee \, cc^2 \, r^4 \, aa0^2 \, cc0 \, bb - 3 \, ee \, cc \, r^2 \, cc0^2 \, aa0^2 \, bb - 2 \, r^4 \, aa \, aa0 \, cc0 \, bb$$

$$- 3 \, ee \, cc^2 \, r^8 \, cc0 \, aa^2 \, bb - 6 \, ee \, cc^2 \, r^6 \, aa \, aa0 \, cc0 \, bb - 2 \, r^6 \, aa \, aa0 \, cc \, bb - r^6 \, aa^2 \, cc0 \, bb - 2 \, ee \, cc^3 \, r^8 \, aa \, aa0 \, bb$$

$$- r^4 \, aa0^2 \, cc \, bb - r^2 \, aa0^2 \, cc0 \, bb - r^8 \, aa^2 \, cc \, bb + 9 \, ee \, cc0^2 \, aa^2 \, r^6 \, cc \cos(\theta)^2 \, bb - 3 \, ee \, cc^3 \, r^6 \cos(\theta)^4 \, aa0^2 \, bb$$

$$- 5 \, ee \, cc^3 \, r^{10} \cos(\theta)^8 \, aa^2 \, bb + ee \, cc^3 \, r^{10} \cos(\theta)^{10} \, aa^2 \, bb + 8 \, ee \, cc^3 \, r^8 \cos(\theta)^6 \, aa \, aa0 \, bb$$

$$- 3 \, ee \, cc^2 \, r^8 \cos(\theta)^8 \, aa^2 \, cc0 \, bb - 2 \, ee \, cc^3 \, r^8 \cos(\theta)^8 \, aa \, aa0 \, bb + 2 \, ee \, cc0^3 \, aa \, r^2 \cos(\theta)^2 \, aa0 \, bb$$

$$- 3 \, ee \, cc^2 \, r^4 \cos(\theta)^4 \, cc0 \, aa0^2 \, bb + 12 \, ee \, cc0^2 \, aa \, r^4 \, aa0 \, cc \cos(\theta)^2 \, bb + r^2 \cos(\theta)^2 \, aa0^2 \, cc0 \, bb$$

$$- r^4 \cos(\theta)^4 \, aa0^2 \, cc \, bb - 2 \, r^4 \cos(\theta)^4 \, aa \, aa0 \, cc0 \, bb + 2 \, r^6 \cos(\theta)^6 \, aa \, aa0 \, cc \, bb + 4 \, r^4 \, aa \cos(\theta)^2 \, aa0 \, cc0 \, bb$$

$$- 12 \, ee \, cc^3 \, r^8 \, aa \cos(\theta)^4 \, aa0 \, bb + ee \, cc^3 \, r^6 \cos(\theta)^6 \, aa0^2 \, bb - 9 \, ee \, cc \, r^6 \, cc0^2 \, aa^2 \cos(\theta)^4 \, bb$$

$$- ee \, cc0^3 \, aa^2 \, r^4 \cos(\theta)^4 \, bb + 2 \, ee \, cc0^3 \, aa^2 \, r^4 \cos(\theta)^2 \, bb - 10 \, ee \, cc^3 \, r^{10} \, aa^2 \cos(\theta)^4 \, bb$$

$$+ 3 \, ee \, cc0^2 \, aa^2 \, r^6 \cos(\theta)^6 \, cc \, bb + 10 \, ee \, cc^3 \, r^{10} \cos(\theta)^6 \, aa^2 \, bb + 6 \, ee \, cc^2 \, r^6 \cos(\theta)^6 \, aa \, aa0 \, cc0 \, bb$$

$$- 6 \, ee \, cc0^2 \, aa \, r^4 \cos(\theta)^4 \, aa0 \, cc \, bb - 18 \, ee \, cc^2 \, r^8 \, cc0 \, aa^2 \cos(\theta)^4 \, bb + 12 \, ee \, cc^2 \, r^8 \, cc0 \, aa^2 \cos(\theta)^6 \, bb$$

$$- 18 \, ee \, cc^2 \, r^6 \, cc0 \, aa \cos(\theta)^4 \, aa0 \, bb - r^8 \cos(\theta)^8 \, aa^2 \, cc \, bb + 5 \, ee \, cc^3 \, r^{10} \, aa^2 \cos(\theta)^2 \, bb + 3 \, r^6 \, aa^2 \cos(\theta)^2 \, cc0 \, bb$$

$$+ 2 \, r^4 \, aa0^2 \, cc \cos(\theta)^2 \, bb - 6 \, r^8 \, aa^2 \cos(\theta)^4 \, cc \, bb + 4 \, r^8 \, aa^2 \cos(\theta)^2 \, cc \, bb - 3 \, r^6 \, aa^2 \cos(\theta)^4 \, cc0 \, bb$$

$$+ 4 \, r^8 \, aa^2 \cos(\theta)^6 \, cc \, bb - 4 \, r^5 \, ee \, cc + 8 \, r^4 \cos(\theta)^2 \, ee \, cc \, bb + r^6 \cos(\theta)^6 \, aa^2 \, cc0 \, bb - 4 \, r^3 \, ee \, bb^2 \, cc$$

$$- 3 \, ee \, cc \, r^6 \, cc0^2 \, aa^2 \, bb - 6 \, ee \, cc \, r^4 \, cc0^2 \, aa \, aa0 \, bb + 8 \, ee \, cc^3 \, r^8 \, aa \, aa0 \cos(\theta)^2 \, bb + 3 \, ee \, cc0^2 \, aa0^2 \, cc \, r^2 \cos(\theta)^2 \, bb$$

$$+ 6 \, ee \, cc^2 \, r^4 \, cc0 \, aa0^2 \cos(\theta)^2 \, bb - 6 \, r^6 \cos(\theta)^4 \, aa \, aa0 \, cc \, bb + 6 \, r^6 \, aa \, aa0 \, cc \cos(\theta)^2 \, bb$$

$$+ 12 \, ee \, cc^2 \, r^8 \, cc0 \, aa^2 \cos(\theta)^2 \, bb + 4 \, r^3 \, ee \, bb^2 \, cc \cos(\theta)^2 - 8 \, r^4 \, ee \, cc \, bb + 3 \, ee \, cc^3 \, r^6 \, aa0^2 \cos(\theta)^2 \, bb$$

$$+ 18 \, ee \, cc^2 \, r^6 \, cc0 \, aa \, aa0 \cos(\theta)^2 \, bb + 4 \, r^5 \cos(\theta)^2 \, ee \, cc) \, \Big/ \, \left(r^4 \, (-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2) \sqrt{\dfrac{r + bb}{r}} \right)$$

Christoffel_Gamma2$(2, 4, 4) = \dfrac{1}{2} \cos(\theta) \sin(\theta) \, (6 \, r^4 \, bb \, aa \, aa0 + 4 \, r^7 \, aa^2 + 4 \, r^5 \, aa \, aa0 - 8 \, r^4 \, bb \, aa \cos(\theta)^2 \, aa0$

$$+ 7 \, r^6 \, bb \, aa^2 \cos(\theta)^4 - r^2 \cos(\theta)^2 \, aa0^2 \, bb - 2 \, ee \, cc0^2 \, aa^2 \, r^4 \cos(\theta)^2 \, bb - 2 \, ee \, cc^2 \, r^6 \cos(\theta)^6 \, aa \, aa0 \, bb$$

$$- 2 \, ee \, cc \, r^2 \cos(\theta)^2 \, cc0 \, aa0^2 \, bb + ee \, cc^2 \, r^4 \cos(\theta)^4 \, aa0^2 \, bb + 6 \, ee \, cc^2 \, r^6 \cos(\theta)^4 \, aa \, aa0 \, bb$$

$- 4\ ee\ cc^2\ r^8\ aa^2\ \cos(\theta)^2\ bb - r^6\ \cos(\theta)^6\ aa^2\ bb + 4\ ee\ cc\ r^4\ \cos(\theta)^4\ cc0\ aa\ aa0\ bb - 2\ ee\ cc^2\ r^4\ \cos(\theta)^2\ aa0^2\ bb$

$- 4\ ee\ cc^2\ r^8\ \cos(\theta)^6\ aa^2\ bb + 2\ r^4\ \cos(\theta)^4\ aa\ aa0\ bb - 6\ ee\ cc^2\ r^6\ aa\ \cos(\theta)^2\ aa0\ bb - 2\ ee\ cc0^2\ aa\ r^2\ \cos(\theta)^2\ aa0\ bb$

$- 6\ ee\ cc\ r^6\ cc0\ aa^2\ \cos(\theta)^2\ bb - 11\ r^6\ bb\ aa^2\ \cos(\theta)^2 - 4\ r^5\ \cos(\theta)^2\ aa\ aa0 + ee\ cc^2\ r^8\ \cos(\theta)^8\ aa^2\ bb$

$+ ee\ cc0^2\ aa^2\ r^4\ \cos(\theta)^4\ bb + 4\ r^7\ \cos(\theta)^4\ aa^2 - 2\ ee\ cc\ r^6\ \cos(\theta)^6\ cc0\ aa^2\ bb - 8\ ee\ cc\ r^4\ cc0\ aa\ \cos(\theta)^2\ aa0\ bb$

$+ 6\ ee\ cc^2\ r^8\ aa^2\ \cos(\theta)^4\ bb + 6\ ee\ cc\ r^6\ cc0\ aa^2\ \cos(\theta)^4\ bb + 5\ r^6\ bb\ aa^2 - 8\ r^7\ \cos(\theta)^2\ aa^2 + ee\ aa0^2\ cc0^2\ bb$

$+ aa0^2\ r^2\ bb + ee\ aa^2\ r^8\ cc^2\ bb + ee\ aa0^2\ cc^2\ r^4\ bb + ee\ aa^2\ r^4\ cc0^2\ bb + 2\ ee\ aa0\ aa\ r^2\ cc0^2\ bb + 2\ ee\ aa0^2\ cc\ r^2\ cc0\ bb$

$+ 2\ ee\ aa\ r^6\ aa0\ cc^2\ bb + 4\ ee\ aa\ r^4\ aa0\ cc\ cc0\ bb + 2\ ee\ aa^2\ r^6\ cc\ cc0\ bb) \Big/ (r^4\ (-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2))$

$$\text{Christoffel\_Gamma2}(3, 1, 1) = 2\ \frac{(-cc\ r^2 + cc\ r^2\ \cos(\theta)^2 - cc0)\ \sqrt{\dfrac{r + bb}{r}}\ \cos(\theta)\ aa}{r + bb}$$

$\text{Christoffel\_Gamma2}(3, 1, 2) =$

$$-\frac{\sqrt{\dfrac{r + bb}{r}}\ \sin(\theta)\ aa\ r\ (cc\ r^2 - 3\ cc\ r^2\ \cos(\theta)^2 + 2\ r^2\ cc\ \cos(\theta)^4 + cc0 - 2\ cc0\ \cos(\theta)^2)}{-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2}$$

$\text{Christoffel\_Gamma2}(3, 1, 3) = -\dfrac{1}{4}\ (4\ r^2\ bb - 4\ r^3\ \cos(\theta)^2 + 4\ r^5\ ee\ cc^2\ \cos(\theta)^4 - 8\ r^5\ ee\ cc^2\ \cos(\theta)^2$

$- 4\ r^2\ \cos(\theta)^2\ bb + 5\ r^4\ ee\ bb\ cc^2\ \cos(\theta)^4 - 10\ r^4\ ee\ bb\ cc^2\ \cos(\theta)^2 - 4\ r^3\ ee\ cc\ \cos(\theta)^2\ cc0$

$- 6\ r^2\ ee\ bb\ cc\ \cos(\theta)^2\ cc0 + 4\ r^3 + 4\ r^5\ ee\ cc^2 + 6\ r^2\ ee\ bb\ cc\ cc0 + 4\ r^3\ ee\ cc\ cc0 + 5\ r^4\ ee\ bb\ cc^2 + ee\ cc0^2\ bb) \Big/ (r^3$

$(-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2))$

$\text{Christoffel\_Gamma2}(3, 1, 4) = \dfrac{1}{4}\ (-5\ bb\ ee\ cc\ r^2\ \cos(\theta)^2 + bb\ ee\ cc0 + 5\ ee\ cc\ r^2\ bb - 4\ aa^2\ r^5\ \cos(\theta)^4\ cc0$

$- 4\ ee\ r^3\ cc\ \cos(\theta)^2 + 4\ aa\ r^3\ \cos(\theta)^2\ aa0\ cc0 - 4\ aa\ r^5\ \cos(\theta)^4\ aa0\ cc + 4\ aa^2\ r^7\ \cos(\theta)^6\ cc - 12\ aa^2\ r^7\ \cos(\theta)^4\ cc$

$+ 8\ aa^2\ r^5\ \cos(\theta)^2\ cc0 - 4\ aa\ r^3\ aa0\ cc0 - 4\ aa\ r^5\ aa0\ cc - 4\ aa^2\ r^5\ cc0 + 4\ ee\ r^3\ cc - 4\ aa^2\ r^7\ cc$

$+ 8\ aa\ r^5\ aa0\ cc\ \cos(\theta)^2 + 12\ aa^2\ r^7\ cc\ \cos(\theta)^2) \Big/ \left(\sqrt{\dfrac{r + bb}{r}}\ (-1 + \cos(\theta)^2)\ r^4\right)$

$\text{Christoffel\_Gamma2}(3, 2, 1) =$

$$-\frac{\sqrt{\dfrac{r + bb}{r}}\ \sin(\theta)\ aa\ r\ (cc\ r^2 - 3\ cc\ r^2\ \cos(\theta)^2 + 2\ r^2\ cc\ \cos(\theta)^4 + cc0 - 2\ cc0\ \cos(\theta)^2)}{-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2}$$

$$\text{Christoffel\_Gamma2}(3, 2, 2) = -2\ \frac{aa\ r^2\ (-cc\ r^2 + cc\ r^2\ \cos(\theta)^2 - cc0)\ \sqrt{\dfrac{r + bb}{r}}\ \cos(\theta)}{r + bb}$$

$$\text{Christoffel\_Gamma2}(3, 2, 3) = \frac{(-1 - ee\ cc^2\ r^2 - ee\ cc\ cc0 + ee\ cc^2\ r^2\ \cos(\theta)^2)\ \sin(\theta)\ \cos(\theta)}{-1 + \cos(\theta)^2}$$

$\text{Christoffel\_Gamma2}(3, 2, 4) = -\sqrt{\dfrac{r + bb}{r}}\ \cos(\theta)\ \sin(\theta)\ (-2\ aa^2\ r^5\ cc\ \cos(\theta)^2 - ee\ bb\ cc - aa^2\ r^3\ cc0\ \cos(\theta)^2$

$+ aa^2\ r^5\ cc\ \cos(\theta)^4 - ee\ cc\ r + cc\ r^5\ aa^2 + aa\ r\ aa0\ cc0 + aa\ r^3\ aa0\ cc - aa\ r^3\ cc\ \cos(\theta)^2\ aa0 + aa^2\ r^3\ cc0) \Big/ ($

$-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2)$

$\text{Christoffel\_Gamma2}(3, 3, 1) = -\dfrac{1}{4}\ (4\ r^2\ bb - 4\ r^3\ \cos(\theta)^2 + 4\ r^5\ ee\ cc^2\ \cos(\theta)^4 - 8\ r^5\ ee\ cc^2\ \cos(\theta)^2$

$- 4\ r^2\ \cos(\theta)^2\ bb + 5\ r^4\ ee\ bb\ cc^2\ \cos(\theta)^4 - 10\ r^4\ ee\ bb\ cc^2\ \cos(\theta)^2 - 4\ r^3\ ee\ cc\ \cos(\theta)^2\ cc0$

$- 6\ r^2\ ee\ bb\ cc\ \cos(\theta)^2\ cc0 + 4\ r^3 + 4\ r^5\ ee\ cc^2 + 6\ r^2\ ee\ bb\ cc\ cc0 + 4\ r^3\ ee\ cc\ cc0 + 5\ r^4\ ee\ bb\ cc^2 + ee\ cc0^2\ bb) \Big/ (r^3$

$$(-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2))$$

$$\text{Christoffel\_Gamma2}(3, 3, 2) = \frac{(-1 - ee\, cc^2\, r^2 - ee\, cc\, cc0 + ee\, cc^2\, r^2 \cos(\theta)^2) \sin(\theta) \cos(\theta)}{-1 + \cos(\theta)^2}$$

$\text{Christoffel\_Gamma2}(3, 3, 4) = -\frac{1}{4} \cos(\theta)\, ee\, bb\, (\cos(\theta)^6\, aa\, r^6\, cc^2 - 2 \cos(\theta)^4\, aa\, r^4\, cc0\, cc - \cos(\theta)^4\, aa0\, cc^2\, r^4$

$+ 2 \cos(\theta)^2\, aa0\, cc^2\, r^4 + \cos(\theta)^2\, aa\, r^2\, cc0^2 + 4 \cos(\theta)^2\, aa\, r^4\, cc0\, cc + 2 \cos(\theta)^2\, aa0\, cc0\, cc\, r^2 - 3 \cos(\theta)^4\, aa\, r^6\, cc^2$

$+ 3 \cos(\theta)^2\, aa\, r^6\, cc^2 - aa0\, cc^2\, r^4 - 2\, aa0\, cc0\, cc\, r^2 - aa0\, cc0^2 - 2\, aa\, r^4\, cc\, cc0 - aa\, r^6\, cc^2 - aa\, r^2\, cc0^2) \big/ (r^3$

$(-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2))$

$\text{Christoffel\_Gamma2}(3, 4, 1) = \frac{1}{4}(-5\, bb\, ee\, cc\, r^2 \cos(\theta)^2 + bb\, ee\, cc0 + 5\, ee\, cc\, r^2\, bb - 4\, aa^2\, r^5 \cos(\theta)^4\, cc0$

$- 4\, ee\, r^3\, cc \cos(\theta)^2 + 4\, aa\, r^3 \cos(\theta)^2\, aa0\, cc0 - 4\, aa\, r^5 \cos(\theta)^4\, aa0\, cc + 4\, aa^2\, r^7 \cos(\theta)^6\, cc - 12\, aa^2\, r^7 \cos(\theta)^4\, cc$

$+ 8\, aa^2\, r^5 \cos(\theta)^2\, cc0 - 4\, aa\, r^3\, aa0\, cc0 - 4\, aa\, r^5\, aa0\, cc - 4\, aa^2\, r^5\, cc0 + 4\, ee\, r^3\, cc - 4\, aa^2\, r^7\, cc$

$+ 8\, aa\, r^5\, aa0\, cc \cos(\theta)^2 + 12\, aa^2\, r^7\, cc \cos(\theta)^2) \Big/ \left(\sqrt{\frac{r + bb}{r}}\, (-1 + \cos(\theta)^2)\, r^4 \right)$

$\text{Christoffel\_Gamma2}(3, 4, 2) = -\sqrt{\frac{r + bb}{r}}\, \cos(\theta) \sin(\theta)\, (-2\, aa^2\, r^5\, cc \cos(\theta)^2 - ee\, bb\, cc - aa^2\, r^3\, cc0 \cos(\theta)^2$

$+ aa^2\, r^5\, cc \cos(\theta)^4 - ee\, cc\, r + cc\, r^5\, aa^2 + aa\, r\, aa0\, cc0 + aa\, r^3\, aa0\, cc - aa\, r^3\, cc \cos(\theta)^2\, aa0 + aa^2\, r^3\, cc0) \big/ ($

$-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2)$

$\text{Christoffel\_Gamma2}(3, 4, 3) = -\frac{1}{4} \cos(\theta)\, ee\, bb\, (\cos(\theta)^6\, aa\, r^6\, cc^2 - 2 \cos(\theta)^4\, aa\, r^4\, cc0\, cc - \cos(\theta)^4\, aa0\, cc^2\, r^4$

$+ 2 \cos(\theta)^2\, aa0\, cc^2\, r^4 + \cos(\theta)^2\, aa\, r^2\, cc0^2 + 4 \cos(\theta)^2\, aa\, r^4\, cc0\, cc + 2 \cos(\theta)^2\, aa0\, cc0\, cc\, r^2 - 3 \cos(\theta)^4\, aa\, r^6\, cc^2$

$+ 3 \cos(\theta)^2\, aa\, r^6\, cc^2 - aa0\, cc^2\, r^4 - 2\, aa0\, cc0\, cc\, r^2 - aa0\, cc0^2 - 2\, aa\, r^4\, cc\, cc0 - aa\, r^6\, cc^2 - aa\, r^2\, cc0^2) \big/ (r^3$

$(-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2))$

$\text{Christoffel\_Gamma2}(3, 4, 4) = -\frac{1}{2} \cos(\theta) \sqrt{\frac{r + bb}{r}}\, ee\, bb\, (aa0\, cc0 + r^2\, aa0\, cc + r^2\, aa\, cc0 + r^4\, aa\, cc$

$- r^2 \cos(\theta)^2\, aa\, cc0 + r^4 \cos(\theta)^4\, aa\, cc - 2\, r^4 \cos(\theta)^2\, aa\, cc - r^2 \cos(\theta)^2\, aa0\, cc) \big/ (r^3$

$(-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2))$

$\text{Christoffel\_Gamma2}(4, 1, 1) = -2\, aa \cos(\theta)$

$(r^2 - r^2 \cos(\theta)^2 + ee\, cc^2\, r^4 - 2\, ee\, cc^2\, r^4 \cos(\theta)^2 + 2\, ee\, cc\, r^2\, cc0 + ee\, cc^2\, r^4 \cos(\theta)^4 - 2\, ee\, cc\, r^2 \cos(\theta)^2\, cc0 + ee\, cc0^2)$

$\big/ ((r + bb)\, ee)$

$\text{Christoffel\_Gamma2}(4, 1, 2) = \sin(\theta)\, aa\, r\, (-r^2 + 3\, r^2 \cos(\theta)^2 - 2\, r^2 \cos(\theta)^4 - ee\, cc^2\, r^4 + 4\, ee\, cc^2\, r^4 \cos(\theta)^2$

$- 5\, ee\, cc^2\, r^4 \cos(\theta)^4 - 2\, ee\, cc\, r^2\, cc0 + 6\, ee\, cc\, r^2 \cos(\theta)^2\, cc0 + 2\, r^4\, ee\, cc^2 \cos(\theta)^6 - 4\, r^2\, ee\, cc \cos(\theta)^4\, cc0 - ee\, cc0^2$

$+ 2\, ee\, cc0^2 \cos(\theta)^2) \big/ (ee\, (-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2))$

$\text{Christoffel\_Gamma2}(4, 1, 3) = \frac{1}{4}(16\, ee\, cc^2\, r^5 \cos(\theta)^2\, cc0 + 4\, ee\, cc\, r^3 \cos(\theta)^2\, cc0^2 - 8\, ee\, cc^2\, r^5 \cos(\theta)^4\, cc0$

$+ 12\, ee\, cc^3\, r^7 \cos(\theta)^2 - 12\, ee\, cc^3\, r^7 \cos(\theta)^4 - r^4 \cos(\theta)^4\, cc\, bb + 2\, r^4\, cc \cos(\theta)^2\, bb - 15\, ee\, cc^3\, r^6 \cos(\theta)^4\, bb$

$+ 15\, ee\, cc^3\, r^6 \cos(\theta)^2\, bb + 22\, ee\, cc^2\, r^4 \cos(\theta)^2\, cc0\, bb + 5\, ee\, cc^3\, r^6 \cos(\theta)^6\, bb + 7\, ee\, cc\, r^2 \cos(\theta)^2\, cc0^2\, bb$

$- 11\, ee\, cc^2\, r^4 \cos(\theta)^4\, cc0\, bb - 4\, ee\, cc^3\, r^7 - 8\, ee\, cc^2\, r^5\, cc0 - ee\, cc0^3\, bb - 5\, r^2\, cc0\, bb - 4\, ee\, cc\, r^3\, cc0^2 - r^4\, cc\, bb$

$- 4\, r^3\, cc0 - 7\, ee\, cc\, r^2\, cc0^2\, bb - 11\, ee\, cc^2\, r^4\, cc0\, bb + 4\, ee\, cc^3\, r^7 \cos(\theta)^6 - 5\, ee\, cc^3\, r^6\, bb + 5\, r^2 \cos(\theta)^2\, cc0\, bb$

$+ 4\, r^3 \cos(\theta)^2\, cc0) \Big/ \left(r^3\, (-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2) \sqrt{\frac{r + bb}{r}} \right)$

$\text{Christoffel\_Gamma2}(4, 1, 4) = -\dfrac{1}{4}\big(4\,r^7\,aa^2 + 4\,r^5\,aa\,aa0 - 2\,r^2\,ee\,bb + 8\,ee^2\,cc^2\,r^5\cos(\theta)^2 + 4\,r^5\cos(\theta)^4\,aa\,aa0$

$\quad - 8\,r^5\cos(\theta)^2\,aa\,aa0 + 2\,r^2\cos(\theta)^2\,ee\,bb - 4\,r^7\cos(\theta)^6\,aa^2 + 12\,r^7\cos(\theta)^4\,aa^2 - 12\,r^7\cos(\theta)^2\,aa^2$

$\quad + 4\,ee\,cc0^2\,aa\,r^3\,aa0 + 8\,ee\,cc\,r^7\,cc0\,aa^2 + 8\,ee\,cc\,r^5\,cc0\,aa\,aa0 + 4\,ee\,cc^2\,r^7\,aa\,aa0 + 4\,ee\,cc^2\,r^9\,aa^2$

$\quad + 4\,ee\,cc0^2\,aa^2\,r^5 - 6\,ee^2\,cc\,r^2\,cc0\,bb - 5\,ee^2\,cc^2\,r^4\,bb - ee^2\,cc0^2\,bb - 4\,ee^2\,cc^2\,r^5\cos(\theta)^4 + 4\,ee^2\,cc\,r^3\cos(\theta)^2\,cc0$

$\quad - 4\,ee\,cc0^2\,aa\,r^3\cos(\theta)^2\,aa0 + 4\,ee\,cc0^2\,aa^2\,r^5\cos(\theta)^4 - 8\,ee\,cc\,r^7\cos(\theta)^6\,cc0\,aa^2 + 8\,ee\,cc\,r^5\cos(\theta)^4\,cc0\,aa\,aa0$

$\quad - 8\,ee\,cc0^2\,aa^2\,r^5\cos(\theta)^2 + 4\,ee\,cc^2\,r^9\cos(\theta)^8\,aa^2 - 4\,ee\,cc^2\,r^7\cos(\theta)^6\,aa\,aa0 - 16\,ee\,cc^2\,r^9\,aa^2\cos(\theta)^2$

$\quad - 24\,ee\,cc\,r^7\,cc0\,aa^2\cos(\theta)^2 + 12\,ee\,cc^2\,r^7\cos(\theta)^4\,aa\,aa0 + 24\,ee\,cc\,r^7\,cc0\,aa^2\cos(\theta)^4$

$\quad - 16\,ee\,cc\,r^5\,cc0\,aa\cos(\theta)^2\,aa0 + 24\,ee\,cc^2\,r^9\,aa^2\cos(\theta)^4 - 12\,ee\,cc^2\,r^7\,aa\cos(\theta)^2\,aa0 + 10\,ee^2\,cc^2\,r^4\cos(\theta)^2\,bb$

$\quad - 5\,ee^2\,cc^2\,r^4\cos(\theta)^4\,bb + 6\,ee^2\,cc\,r^2\cos(\theta)^2\,cc0\,bb - 16\,ee\,cc^2\,r^9\cos(\theta)^6\,aa^2 - 4\,ee^2\,cc\,r^3\,cc0 - 4\,ee^2\,cc^2\,r^5\big)\Big/\big(ee$

$\quad r^3\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)\big)$

$\text{Christoffel\_Gamma2}(4, 2, 1) = \sin(\theta)\,aa\,r\,(-r^2 + 3\,r^2\cos(\theta)^2 - 2\,r^2\cos(\theta)^4 - ee\,cc^2\,r^4 + 4\,ee\,cc^2\,r^4\cos(\theta)^2$

$\quad - 5\,ee\,cc^2\,r^4\cos(\theta)^4 - 2\,ee\,cc\,r^2\,cc0 + 6\,ee\,cc\,r^2\cos(\theta)^2\,cc0 + 2\,r^4\,ee\,cc^2\cos(\theta)^6 - 4\,r^2\,ee\,cc\cos(\theta)^4\,cc0 - ee\,cc0^2$

$\quad + 2\,ee\,cc0^2\cos(\theta)^2)\Big/\big(ee\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)\big)$

$\text{Christoffel\_Gamma2}(4, 2, 2) = 2\,aa\,r^2$

$\quad (r^2 - r^2\cos(\theta)^2 + ee\,cc^2\,r^4 - 2\,ee\,cc^2\,r^4\cos(\theta)^2 + 2\,ee\,cc\,r^2\,cc0 + ee\,cc^2\,r^4\cos(\theta)^4 - 2\,ee\,cc\,r^2\cos(\theta)^2\,cc0 + ee\,cc0^2)$

$\quad \cos(\theta)\,/\,((r + bb)\,ee)$

$\text{Christoffel\_Gamma2}(4, 2, 3) = -\sqrt{\dfrac{r + bb}{r}}\,r\cos(\theta)\sin(\theta)$

$\quad (cc^3\,r^4\,ee - 2\,cc^3\,r^4\,ee\cos(\theta)^2 + 2\,cc^2\,r^2\,ee\,cc0 + cc^3\,r^4\cos(\theta)^4\,ee - 2\,cc^2\,r^2\cos(\theta)^2\,ee\,cc0 + cc0 + ee\,cc\,cc0^2)\Big/\big($

$\quad -bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2\big)$

$\text{Christoffel\_Gamma2}(4, 2, 4) = \cos(\theta)\sin(\theta)\,(-aa^2\,r^5 + aa\,r^3\cos(\theta)^2\,aa0 - ee^2\,cc^2\,bb\,r^2\cos(\theta)^2 - ee\,cc^2\,r^5\,aa\,aa0$

$\quad - 2\,ee\,cc\,r^5\,cc0\,aa^2 - 2\,ee\,cc\,r^3\,cc0\,aa\,aa0 - ee\,cc0^2\,aa\,r\,aa0 - ee\,cc^2\,r^7\,aa^2 - ee\,cc0^2\,aa^2\,r^3$

$\quad + 2\,ee\,cc\,r^3\,cc0\,aa\cos(\theta)^2\,aa0 + 4\,ee\,cc\,r^5\,cc0\,aa^2\cos(\theta)^2 + ee\,cc^2\,r^7\cos(\theta)^6\,aa^2 - 2\,ee\,cc\,r^5\,cc0\,aa^2\cos(\theta)^4$

$\quad - ee\,cc^2\,r^5\cos(\theta)^4\,aa\,aa0 - 3\,ee\,cc^2\,r^7\,aa^2\cos(\theta)^4 + ee\,cc0^2\,aa^2\,r^3\cos(\theta)^2 + 3\,ee\,cc^2\,r^7\,aa^2\cos(\theta)^2$

$\quad + 2\,ee\,cc^2\,r^5\,aa\cos(\theta)^2\,aa0 - aa^2\,r^5\cos(\theta)^4 + 2\,aa^2\,r^5\cos(\theta)^2 - aa\,r^3\,aa0 - ee^2\,cc^2\,r^3\cos(\theta)^2 + ee^2\,cc^2\,r^3$

$\quad + ee^2\,cc\,r\,cc0 + ee^2\,cc^2\,bb\,r^2 + ee^2\,cc\,cc0\,bb)\Big/\big(ee\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)\big)$

$\text{Christoffel\_Gamma2}(4, 3, 1) = \dfrac{1}{4}\big(16\,ee\,cc^2\,r^5\cos(\theta)^2\,cc0 + 4\,ee\,cc\,r^3\cos(\theta)^2\,cc0^2 - 8\,ee\,cc^2\,r^5\cos(\theta)^4\,cc0$

$\quad + 12\,ee\,cc^3\,r^7\cos(\theta)^2 - 12\,ee\,cc^3\,r^7\cos(\theta)^4 - r^4\cos(\theta)^4\,cc\,bb + 2\,r^4\,cc\cos(\theta)^2\,bb - 15\,ee\,cc^3\,r^6\cos(\theta)^4\,bb$

$\quad + 15\,ee\,cc^3\,r^6\cos(\theta)^2\,bb + 22\,ee\,cc^2\,r^4\cos(\theta)^2\,cc0\,bb + 5\,ee\,cc^3\,r^6\cos(\theta)^6\,bb + 7\,ee\,cc\,r^2\cos(\theta)^2\,cc0^2\,bb$

$\quad - 11\,ee\,cc^2\,r^4\cos(\theta)^4\,cc0\,bb - 4\,ee\,cc^3\,r^7 - 8\,ee\,cc^2\,r^5\,cc0 - ee\,cc0^3\,bb - 5\,r^2\,cc0\,bb - 4\,ee\,cc\,r^3\,cc0^2 - r^4\,cc\,bb$

$\quad - 4\,r^3\,cc0 - 7\,ee\,cc\,r^2\,cc0^2\,bb - 11\,ee\,cc^2\,r^4\,cc0\,bb + 4\,ee\,cc^3\,r^7\cos(\theta)^6 - 5\,ee\,cc^3\,r^6\,bb + 5\,r^2\cos(\theta)^2\,cc0\,bb$

$\quad + 4\,r^3\cos(\theta)^2\,cc0\big)\Big/\left(r^3\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)\sqrt{\dfrac{r + bb}{r}}\right)$

$\text{Christoffel\_Gamma2}(4, 3, 2) = -\sqrt{\dfrac{r + bb}{r}}\,r\cos(\theta)\sin(\theta)$

$\quad (cc^3\,r^4\,ee - 2\,cc^3\,r^4\,ee\cos(\theta)^2 + 2\,cc^2\,r^2\,ee\,cc0 + cc^3\,r^4\cos(\theta)^4\,ee - 2\,cc^2\,r^2\cos(\theta)^2\,ee\,cc0 + cc0 + ee\,cc\,cc0^2)\Big/\big($

$\quad -bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2\big)$

$\text{Christoffel\_Gamma2}(4, 3, 4) = \dfrac{1}{4}\cos(\theta)\,bb\,(-4\cos(\theta)^6\,ee\,aa\,r^8\,cc^3 + ee\,aa0\,cc0^3 + aa\,r^6\,cc + aa0\,r^4\,cc + aa\,r^4\,cc0$

$\quad + aa0\,r^2\,cc0 + ee\,aa0\,cc^3\,r^6 + 3\,ee\,aa0\,cc\,r^2\,cc0^2 + 3\,ee\,aa\,r^6\,cc^2\,cc0 + 3\,ee\,aa0\,cc^2\,r^4\,cc0 + ee\,aa\,r^2\,cc0^3$

$+\ ee\ aa\ r^8\ cc^3 + 3\ ee\ aa\ r^4\ cc\ cc0^2 + \cos(\theta)^4\ aa0\ r^4\ cc + \cos(\theta)^4\ aa\ r^4\ cc0 + 3\cos(\theta)^4\ aa\ r^6\ cc - 3\cos(\theta)^2\ aa\ r^6\ cc$

$-\ 2\cos(\theta)^2\ aa0\ r^4\ cc - \cos(\theta)^6\ aa\ r^6\ cc - \cos(\theta)^2\ aa0\ r^2\ cc0 - 2\cos(\theta)^2\ aa\ r^4\ cc0 - 3\cos(\theta)^2\ ee\ aa0\ cc^3\ r^6$

$-\cos(\theta)^2\ ee\ aa\ r^2\ cc0^3 - 4\cos(\theta)^2\ ee\ aa\ r^8\ cc^3 + 6\cos(\theta)^4\ ee\ aa\ r^8\ cc^3 - 6\cos(\theta)^2\ ee\ aa0\ cc^2\ r^4\ cc0$

$-\cos(\theta)^6\ ee\ aa0\ cc^3\ r^6 - 3\cos(\theta)^2\ ee\ aa0\ cc\ r^2\ cc0^2 + 3\cos(\theta)^4\ ee\ aa0\ cc^3\ r^6 - 6\cos(\theta)^2\ ee\ aa\ r^4\ cc\ cc0^2$

$+\ 3\cos(\theta)^4\ ee\ aa0\ cc^2\ r^4\ cc0 - 9\cos(\theta)^2\ ee\ aa\ r^6\ cc^2\ cc0 + \cos(\theta)^8\ ee\ aa\ r^8\ cc^3 + 9\cos(\theta)^4\ ee\ aa\ r^6\ cc^2\ cc0$

$\left.+\ 3\cos(\theta)^4\ ee\ aa\ r^4\ cc\ cc0^2 - 3\cos(\theta)^6\ ee\ aa\ r^6\ cc^2\ cc0\right)\left/\left(r^3\left(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2\right)\sqrt{\dfrac{r+bb}{r}}\right)\right.$

Christoffel_Gamma2$(4, 4, 1) = -\dfrac{1}{4}\left(4\ r^7\ aa^2 + 4\ r^5\ aa\ aa0 - 2\ r^2\ ee\ bb + 8\ ee^2\ cc^2\ r^5\cos(\theta)^2 + 4\ r^5\cos(\theta)^4\ aa\ aa0\right.$

$-\ 8\ r^5\cos(\theta)^2\ aa\ aa0 + 2\ r^2\cos(\theta)^2\ ee\ bb - 4\ r^7\cos(\theta)^6\ aa^2 + 12\ r^7\cos(\theta)^4\ aa^2 - 12\ r^7\cos(\theta)^2\ aa^2$

$+\ 4\ ee\ cc0^2\ aa\ r^3\ aa0 + 8\ ee\ cc\ r^7\ cc0\ aa^2 + 8\ ee\ cc\ r^5\ cc0\ aa\ aa0 + 4\ ee\ cc^2\ r^7\ aa\ aa0 + 4\ ee\ cc^2\ r^9\ aa^2$

$+\ 4\ ee\ cc0^2\ aa^2\ r^5 - 6\ ee^2\ cc\ r^2\ cc0\ bb - 5\ ee^2\ cc^2\ r^4\ bb - ee^2\ cc0^2\ bb - 4\ ee^2\ cc^2\ r^5\cos(\theta)^4 + 4\ ee^2\ cc\ r^3\cos(\theta)^2\ cc0$

$-\ 4\ ee\ cc0^2\ aa\ r^3\cos(\theta)^2\ aa0 + 4\ ee\ cc0^2\ aa^2\ r^5\cos(\theta)^4 - 8\ ee\ cc\ r^7\cos(\theta)^6\ cc0\ aa^2 + 8\ ee\ cc\ r^5\cos(\theta)^4\ cc0\ aa\ aa0$

$-\ 8\ ee\ cc0^2\ aa^2\ r^5\cos(\theta)^2 + 4\ ee\ cc^2\ r^9\cos(\theta)^8\ aa^2 - 4\ ee\ cc^2\ r^7\cos(\theta)^6\ aa\ aa0 - 16\ ee\ cc^2\ r^9\ aa^2\cos(\theta)^2$

$-\ 24\ ee\ cc\ r^7\ cc0\ aa^2\cos(\theta)^2 + 12\ ee\ cc^2\ r^7\cos(\theta)^4\ aa\ aa0 + 24\ ee\ cc\ r^7\ cc0\ aa^2\cos(\theta)^4$

$-\ 16\ ee\ cc\ r^5\ cc0\ aa\cos(\theta)^2\ aa0 + 24\ ee\ cc^2\ r^9\ aa^2\cos(\theta)^4 - 12\ ee\ cc^2\ r^7\ aa\cos(\theta)^2\ aa0 + 10\ ee^2\ cc^2\ r^4\cos(\theta)^2\ bb$

$\left.-\ 5\ ee^2\ cc^2\ r^4\cos(\theta)^4\ bb + 6\ ee^2\ cc\ r^2\cos(\theta)^2\ cc0\ bb - 16\ ee\ cc^2\ r^9\cos(\theta)^6\ aa^2 - 4\ ee^2\ cc\ r^3\ cc0 - 4\ ee^2\ cc^2\ r^5\right)\left/\left(ee\right.\right.$

$\left.r^3\left(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2\right)\right)$

Christoffel_Gamma2$(4, 4, 2) = \cos(\theta)\sin(\theta)\left(-aa^2\ r^5 + aa\ r^3\cos(\theta)^2\ aa0 - ee^2\ cc^2\ bb\ r^2\cos(\theta)^2 - ee\ cc^2\ r^5\ aa\ aa0\right.$

$-\ 2\ ee\ cc\ r^5\ cc0\ aa^2 - 2\ ee\ cc\ r^3\ cc0\ aa\ aa0 - ee\ cc0^2\ aa\ r\ aa0 - ee\ cc^2\ r^7\ aa^2 - ee\ cc0^2\ aa^2\ r^3$

$+\ 2\ ee\ cc\ r^3\ cc0\ aa\cos(\theta)^2\ aa0 + 4\ ee\ cc\ r^5\ cc0\ aa^2\cos(\theta)^2 + ee\ cc^2\ r^7\cos(\theta)^6\ aa^2 - 2\ ee\ cc\ r^5\ cc0\ aa^2\cos(\theta)^4$

$-\ ee\ cc^2\ r^5\cos(\theta)^4\ aa\ aa0 - 3\ ee\ cc^2\ r^7\ aa^2\cos(\theta)^4 + ee\ cc0^2\ aa^2\ r^3\cos(\theta)^2 + 3\ ee\ cc^2\ r^7\ aa^2\cos(\theta)^2$

$+\ 2\ ee\ cc^2\ r^5\ aa\cos(\theta)^2\ aa0 - aa^2\ r^5\cos(\theta)^4 + 2\ aa^2\ r^5\cos(\theta)^2 - aa\ r^3\ aa0 - ee^2\ cc^2\ r^3\cos(\theta)^2 + ee^2\ cc^2\ r^3$

$\left.+\ ee^2\ cc\ r\ cc0 + ee^2\ cc^2\ bb\ r^2 + ee^2\ cc\ cc0\ bb\right)\left/\left(ee\left(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2\right)\right)\right.$

Christoffel_Gamma2$(4, 4, 3) = \dfrac{1}{4}\cos(\theta)\ bb\left(-4\cos(\theta)^6\ ee\ aa\ r^8\ cc^3 + ee\ aa0\ cc0^3 + aa\ r^6\ cc + aa0\ r^4\ cc + aa\ r^4\ cc0\right.$

$+\ aa0\ r^2\ cc0 + ee\ aa0\ cc^3\ r^6 + 3\ ee\ aa0\ cc\ r^2\ cc0^2 + 3\ ee\ aa\ r^6\ cc^2\ cc0 + 3\ ee\ aa0\ cc^2\ r^4\ cc0 + ee\ aa\ r^2\ cc0^3$

$+\ ee\ aa\ r^8\ cc^3 + 3\ ee\ aa\ r^4\ cc\ cc0^2 + \cos(\theta)^4\ aa0\ r^4\ cc + \cos(\theta)^4\ aa\ r^4\ cc0 + 3\cos(\theta)^4\ aa\ r^6\ cc - 3\cos(\theta)^2\ aa\ r^6\ cc$

$-\ 2\cos(\theta)^2\ aa0\ r^4\ cc - \cos(\theta)^6\ aa\ r^6\ cc - \cos(\theta)^2\ aa0\ r^2\ cc0 - 2\cos(\theta)^2\ aa\ r^4\ cc0 - 3\cos(\theta)^2\ ee\ aa0\ cc^3\ r^6$

$-\cos(\theta)^2\ ee\ aa\ r^2\ cc0^3 - 4\cos(\theta)^2\ ee\ aa\ r^8\ cc^3 + 6\cos(\theta)^4\ ee\ aa\ r^8\ cc^3 - 6\cos(\theta)^2\ ee\ aa0\ cc^2\ r^4\ cc0$

$-\cos(\theta)^6\ ee\ aa0\ cc^3\ r^6 - 3\cos(\theta)^2\ ee\ aa0\ cc\ r^2\ cc0^2 + 3\cos(\theta)^4\ ee\ aa0\ cc^3\ r^6 - 6\cos(\theta)^2\ ee\ aa\ r^4\ cc\ cc0^2$

$+\ 3\cos(\theta)^4\ ee\ aa0\ cc^2\ r^4\ cc0 - 9\cos(\theta)^2\ ee\ aa\ r^6\ cc^2\ cc0 + \cos(\theta)^8\ ee\ aa\ r^8\ cc^3 + 9\cos(\theta)^4\ ee\ aa\ r^6\ cc^2\ cc0$

$\left.+\ 3\cos(\theta)^4\ ee\ aa\ r^4\ cc\ cc0^2 - 3\cos(\theta)^6\ ee\ aa\ r^6\ cc^2\ cc0\right)\left/\left(r^3\left(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2\right)\sqrt{\dfrac{r+bb}{r}}\right)\right.$

Christoffel_Gamma2$(4, 4, 4) = \dfrac{1}{2}\cos(\theta)\ bb\left(\cos(\theta)^6\ aa\ r^6\ ee\ cc^2 - 3\cos(\theta)^4\ aa\ r^6\ ee\ cc^2 - \cos(\theta)^4\ r^4\ aa0\ ee\ cc^2\right.$

$-\ r^4\cos(\theta)^4\ aa - 2\cos(\theta)^4\ aa\ r^4\ ee\ cc\ cc0 + 4\cos(\theta)^2\ aa\ r^4\ ee\ cc\ cc0 + \cos(\theta)^2\ aa\ r^2\ ee\ cc0^2$

$+\ 2\cos(\theta)^2\ r^2\ aa0\ ee\ cc\ cc0 + 3\cos(\theta)^2\ aa\ r^6\ ee\ cc^2 + 2\ aa\ r^4\cos(\theta)^2 + 2\cos(\theta)^2\ r^4\ aa0\ ee\ cc^2 + r^2\cos(\theta)^2\ aa0$

$-\ ee\ cc0^2\ aa0 - ee\ cc^2\ aa0\ r^4 - aa\ r^4 - ee\ cc^2\ r^6\ aa - 2\ aa\ r^4\ ee\ cc\ cc0 - r^2\ aa0 - 2\ r^2\ ee\ cc\ cc0\ aa0 - aa\ ee\ cc0^2\ r^2\right)$

$\left/\left(r^3\left(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2\right)\right)\right.$

---------- Non Zero Residue Connection Coefficients {T}= {C} - {Gamma} for the given perturbations ----------

$T(1,1,1) = -2\,(\cos(\theta)^6\,aa\,r^6\,ee\,cc^2 - 3\,\cos(\theta)^4\,aa\,r^6\,ee\,cc^2 - \cos(\theta)^4\,r^4\,aa0\,ee\,cc^2 - r^4\,\cos(\theta)^4\,aa$

$\quad - 2\,\cos(\theta)^4\,aa\,r^4\,ee\,cc\,cc0 + 4\,\cos(\theta)^2\,aa\,r^4\,ee\,cc\,cc0 + \cos(\theta)^2\,aa\,r^2\,ee\,cc0^2 + 2\,\cos(\theta)^2\,r^2\,aa0\,ee\,cc\,cc0$

$\quad + 3\,\cos(\theta)^2\,aa\,r^6\,ee\,cc^2 + 2\,aa\,r^4\,\cos(\theta)^2 + 2\,\cos(\theta)^2\,r^4\,aa0\,ee\,cc^2 + r^2\,\cos(\theta)^2\,aa0 - ee\,cc0^2\,aa0 - ee\,cc^2\,aa0\,r^4$

$\quad - aa\,r^4 - ee\,cc^2\,r^6\,aa - 2\,aa\,r^4\,ee\,cc\,cc0 - r^2\,aa0 - 2\,r^2\,ee\,cc\,cc0\,aa0 - aa\,ee\,cc0^2\,r^2)\,aa\,\cos(\theta)^2\,/\,((r+bb)\,ee)$

$T(1,1,2) = \cos(\theta)\,\sin(\theta)\,aa\,r\,(r^2\,aa0 + aa\,r^4 + ee\,cc^2\,r^6\,aa + ee\,cc^2\,aa0\,r^4 + 2\,aa\,r^4\,ee\,cc\,cc0 + ee\,cc0^2\,aa0$

$\quad + 2\,r^2\,ee\,cc\,cc0\,aa0 + aa\,ee\,cc0^2\,r^2 + 2\,\cos(\theta)^4\,r^2\,aa0 - 2\,\cos(\theta)^6\,aa\,r^4 - 2\,\cos(\theta)^6\,r^4\,aa0\,ee\,cc^2$

$\quad + 2\,\cos(\theta)^8\,aa\,r^6\,ee\,cc^2 + 4\,\cos(\theta)^4\,r^2\,aa0\,ee\,cc\,cc0 + 2\,\cos(\theta)^4\,aa\,r^2\,ee\,cc0^2 - 4\,\cos(\theta)^6\,aa\,r^4\,ee\,cc\,cc0$

$\quad - 6\,\cos(\theta)^2\,r^2\,aa0\,ee\,cc\,cc0 - 5\,\cos(\theta)^2\,aa\,r^6\,ee\,cc^2 - 4\,\cos(\theta)^2\,r^4\,aa0\,ee\,cc^2 - 3\,r^2\,\cos(\theta)^2\,aa0$

$\quad - 7\,\cos(\theta)^6\,aa\,r^6\,ee\,cc^2 + 9\,\cos(\theta)^4\,aa\,r^6\,ee\,cc^2 + 5\,\cos(\theta)^4\,r^4\,aa0\,ee\,cc^2 + 10\,\cos(\theta)^4\,aa\,r^4\,ee\,cc\,cc0$

$\quad - 8\,\cos(\theta)^2\,aa\,r^4\,ee\,cc\,cc0 - 3\,\cos(\theta)^2\,aa\,r^2\,ee\,cc0^2 - 4\,aa\,r^4\,\cos(\theta)^2 + 5\,r^4\,\cos(\theta)^4\,aa - 2\,\cos(\theta)^2\,aa0\,ee\,cc0^2)\,\big/\,($

$\quad ee\,(-bb + bb\,\cos(\theta)^2 - r + r\,\cos(\theta)^2))$

$T(1,1,3) = \dfrac{1}{4}\cos(\theta)\,(-4\,r^5\,aa0\,cc - 4\,r^7\,aa\,cc - 4\,r^5\,\cos(\theta)^4\,aa0\,cc + 8\,r^5\,aa0\,cc\,\cos(\theta)^2 - 12\,r^7\,\cos(\theta)^4\,aa\,cc$

$\quad + 4\,r^7\,\cos(\theta)^6\,aa\,cc + 5\,ee\,aa\,r^8\,cc^3\,bb + aa0\,r^2\,cc0\,bb + aa\,r^4\,cc0\,bb + ee\,aa\,r^2\,cc0^3\,bb + 8\,ee\,aa0\,cc^2\,r^5\,cc0$

$\quad + 7\,ee\,aa\,r^4\,cc\,cc0^2\,bb + 4\,ee\,aa0\,cc\,r^3\,cc0^2 - 3\,aa0\,r^4\,cc\,bb + 4\,ee\,aa\,r^9\,cc^3 + 11\,ee\,aa0\,cc^2\,r^4\,cc0\,bb$

$\quad + 4\,ee\,aa\,r^5\,cc\,cc0^2 - 3\,aa\,r^6\,cc\,bb + 11\,ee\,aa\,r^6\,cc^2\,cc0\,bb + 7\,ee\,aa0\,cc\,r^2\,cc0^2\,bb + ee\,aa0\,cc0^3\,bb$

$\quad + 5\,ee\,aa0\,cc^3\,r^6\,bb + 8\,ee\,aa\,r^7\,cc^2\,cc0 + 4\,ee\,aa0\,cc^3\,r^7 - 8\,\cos(\theta)^6\,ee\,aa\,r^7\,cc^2\,cc0 - 11\,\cos(\theta)^6\,ee\,aa\,r^6\,cc^2\,cc0\,bb$

$\quad + 7\,\cos(\theta)^4\,ee\,aa\,r^4\,cc\,cc0^2\,bb + 4\,\cos(\theta)^4\,ee\,aa\,r^5\,cc\,cc0^2 + \cos(\theta)^4\,aa\,r^4\,cc0\,bb - 14\,\cos(\theta)^2\,ee\,aa\,r^4\,cc\,cc0^2\,bb$

$\quad - 20\,\cos(\theta)^6\,ee\,aa\,r^8\,cc^3\,bb - 16\,\cos(\theta)^6\,ee\,aa\,r^9\,cc^3 + 4\,\cos(\theta)^8\,ee\,aa\,r^9\,cc^3 - 12\,\cos(\theta)^2\,ee\,aa0\,cc^3\,r^7$

$\quad - 4\,\cos(\theta)^2\,ee\,aa0\,cc\,r^3\,cc0^2 + 8\,\cos(\theta)^4\,ee\,aa0\,cc^2\,r^5\,cc0 - 3\,\cos(\theta)^4\,aa0\,r^4\,cc\,bb - 15\,\cos(\theta)^2\,ee\,aa0\,cc^3\,r^6\,bb$

$\quad + 15\,\cos(\theta)^4\,ee\,aa0\,cc^3\,r^6\,bb - \cos(\theta)^2\,ee\,aa\,r^2\,cc0^3\,bb - 9\,\cos(\theta)^4\,aa\,r^6\,cc\,bb + 3\,\cos(\theta)^6\,aa\,r^6\,cc\,bb$

$\quad + 30\,\cos(\theta)^4\,ee\,aa\,r^8\,cc^3\,bb - 16\,\cos(\theta)^2\,ee\,aa\,r^9\,cc^3 - 7\,\cos(\theta)^2\,ee\,aa0\,cc\,r^2\,cc0^2\,bb - 5\,\cos(\theta)^6\,ee\,aa0\,cc^3\,r^6\,bb$

$\quad - 4\,\cos(\theta)^6\,ee\,aa0\,cc^3\,r^7 - 22\,\cos(\theta)^2\,ee\,aa0\,cc^2\,r^4\,cc0\,bb - 20\,\cos(\theta)^2\,ee\,aa\,r^8\,cc^3\,bb + 12\,\cos(\theta)^4\,ee\,aa0\,cc^3\,r^7$

$\quad + 6\,\cos(\theta)^2\,aa0\,r^4\,cc\,bb - \cos(\theta)^2\,aa0\,r^2\,cc0\,bb + 5\,\cos(\theta)^8\,ee\,aa\,r^8\,cc^3\,bb + 24\,\cos(\theta)^4\,ee\,aa\,r^7\,cc^2\,cc0$

$\quad + 33\,\cos(\theta)^4\,ee\,aa\,r^6\,cc^2\,cc0\,bb + 24\,\cos(\theta)^4\,ee\,aa\,r^9\,cc^3 - 33\,\cos(\theta)^2\,ee\,aa\,r^6\,cc^2\,cc0\,bb$

$\quad - 16\,\cos(\theta)^2\,ee\,aa0\,cc^2\,r^5\,cc0 - 2\,\cos(\theta)^2\,aa\,r^4\,cc0\,bb + 9\,\cos(\theta)^2\,aa\,r^6\,cc\,bb - 24\,\cos(\theta)^2\,ee\,aa\,r^7\,cc^2\,cc0$

$\quad - 8\,\cos(\theta)^2\,ee\,aa\,r^5\,cc\,cc0^2 + 11\,\cos(\theta)^4\,ee\,aa0\,cc^2\,r^4\,cc0\,bb + 12\,r^7\,aa\,cc\,\cos(\theta)^2)\,\Big/\,\Big(r^3$

$\quad (-bb + bb\,\cos(\theta)^2 - r + r\,\cos(\theta)^2)\,\sqrt{\dfrac{r+bb}{r}}\,\Big)$

$T(1,1,4) = -\dfrac{1}{4}\cos(\theta)\,(-4\,aa^3\,r^9 - 12\,\cos(\theta)^4\,aa0^2\,ee\,cc^2\,r^7\,aa + 16\,\cos(\theta)^2\,aa^3\,r^9 - 24\,\cos(\theta)^4\,aa^3\,r^9$

$\quad + 16\,\cos(\theta)^6\,aa^3\,r^9 - 4\,\cos(\theta)^8\,aa^3\,r^9 - 8\,aa0\,r^7\,aa^2 + 4\,aa0\,ee^2\,cc0\,cc\,r^3 + 4\,aa0\,ee^2\,cc^2\,r^5 + aa0\,ee^2\,cc0^2\,bb$

$\quad + aa\,r^2\,ee^2\,cc0^2\,bb - 4\,aa0^2\,r^5\,aa - 8\,aa^3\,r^9\,ee\,cc\,cc0 - 4\,aa0^2\,ee\,cc0^2\,aa\,r^3 - 4\,aa^3\,r^{11}\,ee\,cc^2 + 6\,aa\,r^4\,ee^2\,cc\,cc0\,bb$

$\quad + 4\,aa\,r^5\,ee^2\,cc0\,cc + 4\,aa\,r^7\,ee^2\,cc^2 + 2\,aa0\,r^2\,ee\,bb - 8\,aa0^2\,ee\,cc\,r^5\,cc0\,aa + 2\,aa\,r^4\,ee\,bb + 5\,aa\,r^6\,ee^2\,cc^2\,bb$

$\quad - 4\,aa^3\,r^7\,ee\,cc0^2 + 6\,aa0\,ee^2\,cc\,r^2\,cc0\,bb - 4\,aa0^2\,ee\,cc^2\,r^7\,aa - 8\,aa0\,ee\,cc0^2\,aa^2\,r^5 - 8\,aa0\,ee\,cc^2\,r^9\,aa^2$

$\quad + 5\,aa0\,ee^2\,cc^2\,r^4\,bb - 16\,aa0\,ee\,cc\,r^7\,cc0\,aa^2 - 8\,\cos(\theta)^2\,aa0\,ee^2\,cc^2\,r^5 + 5\,\cos(\theta)^4\,aa0\,ee^2\,cc^2\,r^4\,bb$

$\quad + 16\,\cos(\theta)^2\,aa0^2\,ee\,cc\,r^5\,cc0\,aa + 4\,\cos(\theta)^2\,aa0^2\,ee\,cc0^2\,aa\,r^3 - 2\,\cos(\theta)^2\,aa0\,r^2\,ee\,bb$

$\quad + 16\,\cos(\theta)^6\,aa0\,ee\,cc\,r^7\,cc0\,aa^2 + 12\,\cos(\theta)^4\,aa\,r^7\,ee^2\,cc^2 + 12\,\cos(\theta)^2\,aa0^2\,ee\,cc^2\,r^7\,aa + 4\,\cos(\theta)^6\,aa^3\,r^7\,ee\,cc0^2$

$\quad - 5\,\cos(\theta)^6\,aa\,r^6\,ee^2\,cc^2\,bb + 2\,\cos(\theta)^4\,aa\,r^4\,ee\,bb - 4\,\cos(\theta)^6\,aa\,r^7\,ee^2\,cc^2 - 8\,\cos(\theta)^4\,aa^2\,r^5\,ee\,cc0^2\,aa0$

$\quad - 48\,\cos(\theta)^4\,aa^2\,r^9\,ee\,cc^2\,aa0 - 8\,\cos(\theta)^4\,aa0^2\,ee\,cc\,r^5\,cc0\,aa - 8\,\cos(\theta)^8\,aa0\,ee\,cc^2\,r^9\,aa^2$

$$-8\cos(\theta)^2\,aa\,r^5\,ee^2\,cc0\,cc-48\cos(\theta)^4\,aa^3\,r^9\,ee\,cc\,cc0-12\cos(\theta)^4\,aa^3\,r^7\,ee\,cc0^2+6\cos(\theta)^4\,aa\,r^4\,ee^2\,cc\,cc0\,bb$$
$$-40\cos(\theta)^4\,aa^3\,r^{11}\,ee\,cc^2+4\cos(\theta)^4\,aa\,r^5\,ee^2\,cc0\,cc+32\cos(\theta)^6\,aa^3\,r^9\,ee\,cc\,cc0-8\cos(\theta)^8\,aa^3\,r^9\,ee\,cc\,cc0$$
$$+40\cos(\theta)^6\,aa^3\,r^{11}\,ee\,cc^2-24\cos(\theta)^4\,aa^2\,r^7\,aa0+24\cos(\theta)^2\,aa0\,r^7\,aa^2+32\cos(\theta)^2\,aa^3\,r^9\,ee\,cc\,cc0$$
$$+4\cos(\theta)^6\,aa0^2\,ee\,cc^2\,r^7\,aa+12\cos(\theta)^2\,aa^3\,r^7\,ee\,cc0^2-12\cos(\theta)^2\,aa\,r^7\,ee^2\,cc^2+20\cos(\theta)^2\,aa^3\,r^{11}\,ee\,cc^2$$
$$-12\cos(\theta)^2\,aa\,r^4\,ee^2\,cc\,cc0\,bb-20\cos(\theta)^8\,aa^3\,r^{11}\,ee\,cc^2+32\cos(\theta)^6\,aa^2\,r^9\,ee\,cc^2\,aa0$$
$$+32\cos(\theta)^2\,aa^2\,r^9\,ee\,cc^2\,aa0-4\cos(\theta)^2\,aa\,r^4\,ee\,bb-6\cos(\theta)^2\,aa0\,ee^2\,cc\,r^2\,cc0\,bb+15\cos(\theta)^4\,aa\,r^6\,ee^2\,cc^2\,bb$$
$$-15\cos(\theta)^2\,aa\,r^6\,ee^2\,cc^2\,bb-\cos(\theta)^2\,aa\,r^2\,ee^2\,cc0^2\,bb+16\cos(\theta)^2\,aa^2\,r^5\,ee\,cc0^2\,aa0$$
$$+48\cos(\theta)^2\,aa^2\,r^7\,ee\,cc\,cc0\,aa0-10\cos(\theta)^2\,aa0\,ee^2\,cc^2\,r^4\,bb+8\cos(\theta)^2\,aa0^2\,r^5\,aa+8\cos(\theta)^6\,aa0\,r^7\,aa^2$$
$$-4\cos(\theta)^4\,aa0^2\,r^5\,aa+4\cos(\theta)^4\,aa0\,ee^2\,cc^2\,r^5-4\cos(\theta)^2\,aa0\,ee^2\,cc0\,r^3\,cc+4\cos(\theta)^{10}\,aa^3\,r^{11}\,ee\,cc^2$$
$$-48\cos(\theta)^4\,aa^2\,r^7\,ee\,cc\,cc0\,aa0\Big)\Big/\big(ee\,r^3\,(-bb+bb\cos(\theta)^2-r+r\cos(\theta)^2)\big)$$

$$T(1,2,1)=\cos(\theta)\sin(\theta)\,aa\,r\,(r^2\,aa0+aa\,r^4+ee\,cc^2\,r^6\,aa+ee\,cc^2\,aa0\,r^4+2\,aa\,r^4\,ee\,cc\,cc0+ee\,cc0^2\,aa0$$
$$+2\,r^2\,ee\,cc\,cc0\,aa0+aa\,ee\,cc0^2\,r^2+2\cos(\theta)^4\,r^2\,aa0-2\cos(\theta)^6\,aa\,r^4-2\cos(\theta)^6\,r^4\,aa0\,ee\,cc^2$$
$$+2\cos(\theta)^8\,aa\,r^6\,ee\,cc^2+4\cos(\theta)^4\,r^2\,aa0\,ee\,cc\,cc0+2\cos(\theta)^4\,aa\,r^2\,ee\,cc0^2-4\cos(\theta)^6\,aa\,r^4\,ee\,cc\,cc0$$
$$-6\cos(\theta)^2\,r^2\,aa0\,ee\,cc\,cc0-5\cos(\theta)^2\,aa\,r^6\,ee\,cc^2-4\cos(\theta)^2\,r^4\,aa0\,ee\,cc^2-3\,r^2\cos(\theta)^2\,aa0$$
$$-7\cos(\theta)^6\,aa\,r^6\,ee\,cc^2+9\cos(\theta)^4\,aa\,r^6\,ee\,cc^2+5\cos(\theta)^4\,r^4\,aa0\,ee\,cc^2+10\cos(\theta)^4\,aa\,r^4\,ee\,cc\,cc0$$
$$-8\cos(\theta)^2\,aa\,r^4\,ee\,cc\,cc0-3\cos(\theta)^2\,aa\,r^2\,ee\,cc0^2-4\,aa\,r^4\cos(\theta)^2+5\,r^4\cos(\theta)^4\,aa-2\cos(\theta)^2\,aa0\,ee\,cc0^2)\Big/\big($$
$$ee\,(-bb+bb\cos(\theta)^2-r+r\cos(\theta)^2)\big)$$

$$T(1,2,2)=2\,r^2\cos(\theta)^2\,aa\,(\cos(\theta)^6\,aa\,r^6\,ee\,cc^2-3\cos(\theta)^4\,aa\,r^6\,ee\,cc^2-\cos(\theta)^4\,r^4\,aa0\,ee\,cc^2-r^4\cos(\theta)^4\,aa$$
$$-2\cos(\theta)^4\,aa\,r^4\,ee\,cc\,cc0+4\cos(\theta)^2\,aa\,r^4\,ee\,cc\,cc0+\cos(\theta)^2\,aa\,r^2\,ee\,cc0^2+2\cos(\theta)^2\,r^2\,aa0\,ee\,cc\,cc0$$
$$+3\cos(\theta)^2\,aa\,r^6\,ee\,cc^2+2\,aa\,r^4\cos(\theta)^2+2\cos(\theta)^2\,r^4\,aa0\,ee\,cc^2+r^2\cos(\theta)^2\,aa0-ee\,cc0^2\,aa0-ee\,cc^2\,aa0\,r^4$$
$$-aa\,r^4-ee\,cc^2\,r^6\,aa-2\,aa\,r^4\,ee\,cc\,cc0-r^2\,aa0-2\,r^2\,ee\,cc\,cc0\,aa0-aa\,ee\,cc0^2\,r^2)/((r+bb)\,ee)$$

$$T(1,2,3)=\cos(\theta)^2\,(\cos(\theta)^6\,aa\,r^6\,ee\,cc^2+r^4\cos(\theta)^4\,aa-3\cos(\theta)^4\,aa\,r^6\,ee\,cc^2-2\cos(\theta)^4\,aa\,r^4\,ee\,cc\,cc0$$
$$-\cos(\theta)^4\,r^4\,aa0\,ee\,cc^2+3\cos(\theta)^2\,aa\,r^6\,ee\,cc^2+2\cos(\theta)^2\,r^4\,aa0\,ee\,cc^2-r^2\cos(\theta)^2\,aa0+4\cos(\theta)^2\,aa\,r^4\,ee\,cc\,cc0$$
$$+2\cos(\theta)^2\,r^2\,aa0\,ee\,cc\,cc0-2\,aa\,r^4\cos(\theta)^2+\cos(\theta)^2\,aa\,r^2\,ee\,cc0^2+aa\,r^4-ee\,cc0^2\,aa0-2\,aa\,r^4\,ee\,cc\,cc0$$
$$-2\,r^2\,ee\,cc\,cc0\,aa0-ee\,cc^2\,aa0\,r^4-aa\,ee\,cc0^2\,r^2+r^2\,aa0-ee\,cc^2\,r^6\,aa)\,cc\,\Bigg/\left(\sin(\theta)\sqrt{\dfrac{r+bb}{r}}\right)$$

$$T(1,2,4)=\sin(\theta)\,(3\,aa^3\,r^7\cos(\theta)^6+2\,aa^2\,r^5\cos(\theta)^6\,aa0+aa\,r^3\cos(\theta)^2\,aa0^2-3\,aa^3\,r^7\cos(\theta)^4$$
$$-4\,aa^2\,r^5\cos(\theta)^4\,aa0+aa^3\,r^7\cos(\theta)^2-aa\,r^3\cos(\theta)^4\,aa0^2+2\,aa^2\,r^5\cos(\theta)^2\,aa0+aa\,r^3\cos(\theta)^2\,ee-aa\,r^2\,ee\,bb$$
$$-aa^3\,r^7\cos(\theta)^8+4\,aa^2\,r^5\,ee\,cc\,cc0\cos(\theta)^2\,aa0-\cos(\theta)^6\,ee^2\,cc^2\,aa\,r^5+\cos(\theta)^4\,ee^2\,cc^2\,r^3\,aa0$$
$$-\cos(\theta)^2\,ee^2\,cc^2\,r^3\,aa0+\cos(\theta)^4\,ee^2\,cc\,aa\,r^2\,bb\,cc0-\cos(\theta)^2\,ee^2\,cc\,bb\,aa\,r^2\,cc0-2\,aa^2\,r^3\cos(\theta)^4\,ee\,cc0^2\,aa0$$
$$+2\cos(\theta)^4\,ee^2\,cc^2\,aa\,r^5-2\,aa\,r^3\cos(\theta)^4\,ee\,cc\,cc0\,aa0^2+aa\,r^5\cos(\theta)^6\,ee\,cc^2\,aa0^2-2\,aa^3\,r^7\cos(\theta)^8\,ee\,cc\,cc0$$
$$-\cos(\theta)^2\,ee^2\,cc^2\,aa\,r^5+aa\,r\cos(\theta)^2\,ee\,cc0^2\,aa0^2+aa^3\,r^5\cos(\theta)^6\,ee\,cc0^2+4\,aa^2\,r^5\cos(\theta)^6\,ee\,cc\,cc0\,aa0$$
$$+aa^3\,r^9\cos(\theta)^{10}\,ee\,cc^2-aa\,r^3\,ee-4\,aa^3\,r^9\,ee\,cc^2\cos(\theta)^4+6\,aa^2\,r^7\,ee\,cc^2\cos(\theta)^6\,aa0-2\,aa^3\,r^5\,ee\,cc0^2\cos(\theta)^4$$
$$+aa^3\,r^9\,ee\,cc^2\cos(\theta)^2+2\,aa^2\,r^3\,ee\,cc0^2\cos(\theta)^2\,aa0+\cos(\theta)^4\,ee^2\,cc^2\,bb\,aa0\,r^2-\cos(\theta)^2\,ee^2\,cc^2\,bb\,aa0\,r^2$$
$$-\cos(\theta)^2\,ee^2\,cc\,r\,aa0\,cc0-\cos(\theta)^6\,ee^2\,cc^2\,aa\,r^4\,bb+\cos(\theta)^4\,ee^2\,cc\,aa\,r^3\,cc0+2\cos(\theta)^4\,ee^2\,cc^2\,bb\,aa\,r^4$$
$$+2\,aa\,r^3\,ee\,cc\cos(\theta)^2\,cc0\,aa0^2-\cos(\theta)^2\,ee^2\,cc^2\,bb\,aa\,r^4-8\,aa^2\,r^5\,ee\,cc\cos(\theta)^4\,cc0\,aa0-\cos(\theta)^2\,ee^2\,cc\,aa\,r^3\,cc0$$
$$+6\,aa^3\,r^9\,ee\,cc^2\cos(\theta)^6-2\,aa^2\,r^7\cos(\theta)^8\,ee\,cc^2\,aa0-6\,aa^2\,r^7\,ee\,cc^2\cos(\theta)^4\,aa0+aa^3\,r^5\,ee\,cc0^2\cos(\theta)^2$$
$$+aa\,r^2\cos(\theta)^2\,ee\,bb-6\,aa^3\,r^7\,ee\,cc\,cc0\cos(\theta)^4-4\,aa^3\,r^9\,ee\,cc^2\cos(\theta)^8+2\,aa^3\,r^7\,ee\,cc\,cc0\cos(\theta)^2$$
$$+aa\,r^5\,ee\,cc^2\cos(\theta)^2\,aa0^2-\cos(\theta)^2\,ee^2\,cc\,bb\,aa0\,cc0+6\,aa^3\,r^7\,ee\,cc\cos(\theta)^6\,cc0-2\,aa\,r^5\,ee\,cc^2\cos(\theta)^4\,aa0^2$$
$$+2\,aa^2\,r^7\,ee\,cc^2\cos(\theta)^2\,aa0\Big)\Big/\big(ee\,(-bb+bb\cos(\theta)^2-r+r\cos(\theta)^2)\big)$$

$$T(1,3,1)=\frac{1}{4}\cos(\theta)\,(4\,r^5\,aa0\,cc+4\,r^7\,aa\,cc+4\,r^5\cos(\theta)^4\,aa0\,cc-8\,r^5\,aa0\,cc\cos(\theta)^2+12\,r^7\cos(\theta)^4\,aa\,cc$$

$$- 4 \, r^7 \cos(\theta)^6 \, aa \, cc + 5 \, ee \, aa \, r^8 \, cc^3 \, bb + aa0 \, r^2 \, cc0 \, bb + aa \, r^4 \, cc0 \, bb + ee \, aa \, r^2 \, cc0^3 \, bb + 8 \, ee \, aa0 \, cc^2 \, r^5 \, cc0$$

$$+ 7 \, ee \, aa \, r^4 \, cc \, cc0^2 \, bb + 4 \, ee \, aa0 \, cc \, r^3 \, cc0^2 + 5 \, aa0 \, r^4 \, cc \, bb + 4 \, ee \, aa \, r^9 \, cc^3 + 11 \, ee \, aa0 \, cc^2 \, r^4 \, cc0 \, bb$$

$$+ 4 \, ee \, aa \, r^5 \, cc \, cc0^2 + 5 \, aa \, r^6 \, cc \, bb + 11 \, ee \, aa \, r^6 \, cc^2 \, cc0 \, bb + 7 \, ee \, aa0 \, cc \, r^2 \, cc0^2 \, bb + ee \, aa0 \, cc0^3 \, bb$$

$$+ 5 \, ee \, aa0 \, cc^3 \, r^6 \, bb + 8 \, ee \, aa \, r^7 \, cc^2 \, cc0 + 4 \, ee \, aa0 \, cc^3 \, r^7 - 8 \cos(\theta)^6 \, ee \, aa \, r^7 \, cc^2 \, cc0 - 11 \cos(\theta)^6 \, ee \, aa \, r^6 \, cc^2 \, cc0 \, bb$$

$$+ 7 \cos(\theta)^4 \, ee \, aa \, r^4 \, cc \, cc0^2 \, bb + 4 \cos(\theta)^4 \, ee \, aa \, r^5 \, cc \, cc0^2 + \cos(\theta)^4 \, aa \, r^4 \, cc0 \, bb - 14 \cos(\theta)^2 \, ee \, aa \, r^4 \, cc \, cc0^2 \, bb$$

$$- 20 \cos(\theta)^6 \, ee \, aa \, r^8 \, cc^3 \, bb - 16 \cos(\theta)^6 \, ee \, aa \, r^9 \, cc^3 + 4 \cos(\theta)^8 \, ee \, aa \, r^9 \, cc^3 - 12 \cos(\theta)^2 \, ee \, aa0 \, cc^3 \, r^7$$

$$- 4 \cos(\theta)^2 \, ee \, aa0 \, cc \, r^3 \, cc0^2 + 8 \cos(\theta)^4 \, ee \, aa0 \, cc^2 \, r^5 \, cc0 + 5 \cos(\theta)^4 \, aa0 \, r^4 \, cc \, bb - 15 \cos(\theta)^2 \, ee \, aa0 \, cc^3 \, r^6 \, bb$$

$$+ 15 \cos(\theta)^4 \, ee \, aa0 \, cc^3 \, r^6 \, bb - \cos(\theta)^2 \, ee \, aa \, r^2 \, cc0^3 \, bb + 15 \cos(\theta)^4 \, aa \, r^6 \, cc \, bb - 5 \cos(\theta)^6 \, aa \, r^6 \, cc \, bb$$

$$+ 30 \cos(\theta)^4 \, ee \, aa \, r^8 \, cc^3 \, bb - 16 \cos(\theta)^2 \, ee \, aa \, r^9 \, cc^3 - 7 \cos(\theta)^2 \, ee \, aa0 \, cc \, r^2 \, cc0^2 \, bb - 5 \cos(\theta)^6 \, ee \, aa0 \, cc^3 \, r^6 \, bb$$

$$- 4 \cos(\theta)^6 \, ee \, aa0 \, cc^3 \, r^7 - 22 \cos(\theta)^2 \, ee \, aa0 \, cc^2 \, r^4 \, cc0 \, bb - 20 \cos(\theta)^2 \, ee \, aa \, r^8 \, cc^3 \, bb + 12 \cos(\theta)^4 \, ee \, aa0 \, cc^3 \, r^7$$

$$- 10 \cos(\theta)^2 \, aa0 \, r^4 \, cc \, bb - \cos(\theta)^2 \, aa0 \, r^2 \, cc0 \, bb + 5 \cos(\theta)^8 \, ee \, aa \, r^8 \, cc^3 \, bb + 24 \cos(\theta)^4 \, ee \, aa \, r^7 \, cc^2 \, cc0$$

$$+ 33 \cos(\theta)^4 \, ee \, aa \, r^6 \, cc^2 \, cc0 \, bb + 24 \cos(\theta)^4 \, ee \, aa \, r^9 \, cc^3 - 33 \cos(\theta)^2 \, ee \, aa \, r^6 \, cc^2 \, cc0 \, bb$$

$$- 16 \cos(\theta)^2 \, ee \, aa0 \, cc^2 \, r^5 \, cc0 - 2 \cos(\theta)^2 \, aa \, r^4 \, cc0 \, bb - 15 \cos(\theta)^2 \, aa \, r^6 \, cc \, bb - 24 \cos(\theta)^2 \, ee \, aa \, r^7 \, cc^2 \, cc0$$

$$- 8 \cos(\theta)^2 \, ee \, aa \, r^5 \, cc \, cc0^2 + 11 \cos(\theta)^4 \, ee \, aa0 \, cc^2 \, r^4 \, cc0 \, bb - 12 \, r^7 \, aa \, cc \cos(\theta)^2 \Bigg) \Bigg/ \Bigg(r^3$$

$$(-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2) \sqrt{\frac{r + bb}{r}} \Bigg)$$

$$T(1,3,2) = \cos(\theta)^2 \, (\cos(\theta)^6 \, aa \, r^6 \, ee \, cc^2 - 3 \cos(\theta)^4 \, aa \, r^6 \, ee \, cc^2 - \cos(\theta)^4 \, r^4 \, aa0 \, ee \, cc^2 - r^4 \cos(\theta)^4 \, aa$$

$$- 2 \cos(\theta)^4 \, aa \, r^4 \, ee \, cc \, cc0 + 4 \cos(\theta)^2 \, aa \, r^4 \, ee \, cc \, cc0 + \cos(\theta)^2 \, aa \, r^2 \, ee \, cc0^2 + 2 \cos(\theta)^2 \, r^2 \, aa0 \, ee \, cc \, cc0$$

$$+ 3 \cos(\theta)^2 \, aa \, r^6 \, ee \, cc^2 + 2 \, aa \, r^4 \cos(\theta)^2 + 2 \cos(\theta)^2 \, r^4 \, aa0 \, ee \, cc^2 + r^2 \cos(\theta)^2 \, aa0 - ee \, cc0^2 \, aa0 - ee \, cc^2 \, aa0 \, r^4$$

$$- aa \, r^4 - ee \, cc^2 \, r^6 \, aa - 2 \, aa \, r^4 \, ee \, cc \, cc0 - r^2 \, aa0 - 2 \, r^2 \, ee \, cc \, cc0 \, aa0 - aa \, ee \, cc0^2 \, r^2) \, cc \Bigg/ \Bigg(\sin(\theta) \sqrt{\frac{r + bb}{r}} \Bigg)$$

$$T(1,3,3) = 2 \, ee \, cc^2 \, r^3 \cos(\theta)^4 - 2 \, ee \, cc \, r \cos(\theta)^2 \, cc0 - 4 \, ee \, cc^2 \, r^3 \cos(\theta)^2 + 2 \, ee \, cc \, r \, cc0 + 2 \, ee \, cc^2 \, r^3$$

$$T(1,3,4) = \frac{1}{4} (-3 \, ee \, cc^2 \, r^8 \cos(\theta)^{10} \, aa^2 \, cc0 \, bb - 2 \, ee \, cc^3 \, r^8 \cos(\theta)^{10} \, aa \, aa0 \, bb - 2 \, r^4 \cos(\theta)^6 \, aa \, aa0 \, cc0 \, bb$$

$$+ 2 \, ee \, cc0^3 \, aa \, r^2 \cos(\theta)^4 \, aa0 \, bb + ee \, cc^3 \, r^6 \cos(\theta)^8 \, aa0^2 \, bb + ee \, cc^3 \, r^{10} \cos(\theta)^{12} \, aa^2 \, bb - r^8 \cos(\theta)^{10} \, aa^2 \, cc \, bb$$

$$- r^4 \cos(\theta)^6 \, aa0^2 \, cc \, bb + 6 \, ee \, cc^2 \, r^6 \cos(\theta)^8 \, aa \, aa0 \, cc0 \, bb + r^2 \cos(\theta)^4 \, aa0^2 \, cc0 \, bb + 3 \, ee \, cc0^2 \, aa0^2 \, cc \, r^2 \cos(\theta)^4 \, bb$$

$$+ 3 \, ee \, cc0^2 \, aa^2 \, r^6 \cos(\theta)^8 \, cc \, bb - 6 \, ee \, cc0^2 \, aa \, r^4 \cos(\theta)^6 \, aa0 \, cc \, bb + r^6 \cos(\theta)^8 \, aa^2 \, cc0 \, bb - ee \, cc0^3 \, aa0^2 \, bb \cos(\theta)^2$$

$$+ 2 \, r^6 \cos(\theta)^8 \, aa \, aa0 \, cc \, bb - ee \, cc0^3 \, aa^2 \, r^4 \cos(\theta)^6 \, bb - 3 \, ee \, cc^2 \, r^4 \cos(\theta)^6 \, cc0 \, aa0^2 \, bb$$

$$- 3 \, ee \, cc0^2 \, aa^2 \, r^6 \, cc \cos(\theta)^2 \, bb + r \cos(\theta)^2 \, ee \, bb^2 \, cc0 + 3 \, ee \, cc^3 \, r^6 \cos(\theta)^4 \, aa0^2 \, bb + 10 \, ee \, cc^3 \, r^{10} \cos(\theta)^8 \, aa^2 \, bb$$

$$- 5 \, ee \, cc^3 \, r^{10} \cos(\theta)^{10} \, aa^2 \, bb - 12 \, ee \, cc^3 \, r^8 \cos(\theta)^6 \, aa \, aa0 \, bb + 12 \, ee \, cc^2 \, r^8 \cos(\theta)^8 \, aa^2 \, cc0 \, bb$$

$$+ 8 \, ee \, cc^3 \, r^8 \cos(\theta)^8 \, aa \, aa0 \, bb - 2 \, ee \, cc0^3 \, aa \, r^2 \cos(\theta)^2 \, aa0 \, bb + 6 \, ee \, cc^2 \, r^4 \cos(\theta)^4 \, cc0 \, aa0^2 \, bb$$

$$+ r^2 \cos(\theta)^2 \, ee \, cc0 \, bb - 6 \, ee \, cc0^2 \, aa \, r^4 \, aa0 \, cc \cos(\theta)^2 \, bb - r^2 \cos(\theta)^2 \, aa0^2 \, cc0 \, bb + 2 \, r^4 \cos(\theta)^4 \, aa0^2 \, cc \, bb$$

$$+ 4 \, r^4 \cos(\theta)^4 \, aa \, aa0 \, cc0 \, bb - 6 \, r^6 \cos(\theta)^6 \, aa \, aa0 \, cc \, bb + 3 \, r^3 \cos(\theta)^4 \, ee \, bb^2 \, cc - 2 \, r^4 \, aa \cos(\theta)^2 \, aa0 \, cc0 \, bb$$

$$+ 8 \, ee \, cc^3 \, r^8 \, aa \cos(\theta)^4 \, aa0 \, bb - 3 \, ee \, cc^3 \, r^6 \cos(\theta)^6 \, aa0^2 \, bb + 9 \, ee \, cc \, r^6 \, cc0^2 \, aa^2 \cos(\theta)^4 \, bb$$

$$+ 2 \, ee \, cc0^3 \, aa^2 \, r^4 \cos(\theta)^4 \, bb - ee \, cc0^3 \, aa^2 \, r^4 \cos(\theta)^2 \, bb + 5 \, ee \, cc^3 \, r^{10} \, aa^2 \cos(\theta)^4 \, bb - 9 \, ee \, cc0^2 \, aa^2 \, r^6 \cos(\theta)^6 \, cc \, bb$$

$$- 10 \, ee \, cc^3 \, r^{10} \cos(\theta)^6 \, aa^2 \, bb - 18 \, ee \, cc^2 \, r^6 \cos(\theta)^6 \, aa \, aa0 \, cc0 \, bb + 12 \, ee \, cc0^2 \, aa \, r^4 \cos(\theta)^4 \, aa0 \, cc \, bb$$

$$+ 12 \, ee \, cc^2 \, r^8 \, cc0 \, aa^2 \cos(\theta)^4 \, bb - 18 \, ee \, cc^2 \, r^8 \, cc0 \, aa^2 \cos(\theta)^6 \, bb + 18 \, ee \, cc^2 \, r^6 \, cc0 \, aa \cos(\theta)^4 \, aa0 \, bb$$

$$+ 4 \, r^5 \cos(\theta)^4 \, ee \, cc + 4 \, r^8 \cos(\theta)^8 \, aa^2 \, cc \, bb - ee \, cc^3 \, r^{10} \, aa^2 \cos(\theta)^2 \, bb - r^6 \, aa^2 \cos(\theta)^2 \, cc0 \, bb$$

$$- r^4 \, aa0^2 \, cc \cos(\theta)^2 \, bb + 4 \, r^8 \, aa^2 \cos(\theta)^4 \, cc \, bb - r^8 \, aa^2 \cos(\theta)^2 \, cc \, bb + 3 \, r^6 \, aa^2 \cos(\theta)^4 \, cc0 \, bb$$

$$- 6 \, r^8 \, aa^2 \cos(\theta)^6 \, cc \, bb + 4 \, r^5 \, ee \, cc + 7 \, r^4 \cos(\theta)^4 \, ee \, cc \, bb - 14 \, r^4 \cos(\theta)^2 \, ee \, cc \, bb - 3 \, r^6 \cos(\theta)^6 \, aa^2 \, cc0 \, bb$$

$$- r \, ee \, bb^2 \, cc0 + 3 \, r^3 \, ee \, bb^2 \, cc - 2 \, ee \, cc^3 \, r^8 \, aa \, aa0 \cos(\theta)^2 \, bb - 3 \, ee \, cc0^2 \, aa0^2 \, cc \, r^2 \cos(\theta)^2 \, bb$$

$$- 3\, ee\, cc^2\, r^4\, cc0\, aa0^2 \cos(\theta)^2\, bb + 6\, r^6 \cos(\theta)^4\, aa\, aa0\, cc\, bb - 2\, r^6\, aa\, aa0\, cc \cos(\theta)^2\, bb$$

$$- 3\, ee\, cc^2\, r^8\, cc0\, aa^2 \cos(\theta)^2\, bb - 6\, r^3\, ee\, bb^2\, cc \cos(\theta)^2 - r^2\, ee\, cc0\, bb + 7\, r^4\, ee\, cc\, bb - ee\, cc^3\, r^6\, aa0^2 \cos(\theta)^2\, bb$$

$$- 6\, ee\, cc^2\, r^6\, cc0\, aa\, aa0 \cos(\theta)^2\, bb - 8\, r^5 \cos(\theta)^2\, ee\, cc \Big) \Big/ \left(\sqrt{\frac{r + bb}{r}}\, r^3\, (-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2) \right)$$

$T(1, 4, 1) = -\dfrac{1}{4} \big(-4\, aa^3\, r^9 - 12 \cos(\theta)^4\, aa0^2\, ee\, cc^2\, r^7\, aa + 16 \cos(\theta)^2\, aa^3\, r^9 - 24 \cos(\theta)^4\, aa^3\, r^9 + 16 \cos(\theta)^6\, aa^3\, r^9$

$\quad - 4 \cos(\theta)^8\, aa^3\, r^9 - 8\, aa\, r^5\, ee + 16\, aa\, r^5 \cos(\theta)^2\, ee - 8\, aa0\, r^7\, aa^2 + 4\, aa0\, ee^2\, cc0\, cc\, r^3 + 4\, aa0\, ee^2\, cc^2\, r^5$

$\quad + aa0\, ee^2\, cc0^2\, bb + aa\, r^2\, ee^2\, cc0^2\, bb - 4\, aa0^2\, r^5\, aa - 8\, r^5\, ee\, aa \cos(\theta)^4 - 8\, aa^3\, r^9\, ee\, cc\, cc0 - 4\, aa0^2\, ee\, cc0^2\, aa\, r^3$

$\quad - 4\, aa^3\, r^{11}\, ee\, cc^2 + 6\, aa\, r^4\, ee^2\, cc\, cc0\, bb + 4\, aa\, r^5\, ee^2\, cc0\, cc + 4\, aa\, r^7\, ee^2\, cc^2 - 8\, aa0^2\, ee\, cc\, r^5\, cc0\, aa - 8\, aa\, r^4\, ee\, bb$

$\quad + 5\, aa\, r^6\, ee^2\, cc^2\, bb - 4\, aa^3\, r^7\, ee\, cc0^2 + 6\, aa0\, ee^2\, cc\, r^2\, cc0\, bb - 4\, aa0^2\, ee\, cc^2\, r^7\, aa - 8\, aa0\, ee\, cc0^2\, aa^2\, r^5$

$\quad - 8\, aa0\, ee\, cc^2\, r^9\, aa^2 + 5\, aa0\, ee^2\, cc^2\, r^4\, bb - 16\, aa0\, ee\, cc\, r^7\, cc0\, aa^2 - 8 \cos(\theta)^2\, aa0\, ee^2\, cc^2\, r^5$

$\quad + 5 \cos(\theta)^4\, aa0\, ee^2\, cc^2\, r^4\, bb + 16 \cos(\theta)^2\, aa0^2\, ee\, cc\, r^5\, cc0\, aa + 4 \cos(\theta)^2\, aa0^2\, ee\, cc0^2\, aa\, r^3$

$\quad + 16 \cos(\theta)^6\, aa0\, ee\, cc\, r^7\, cc0\, aa^2 + 12 \cos(\theta)^4\, aa\, r^7\, ee^2\, cc^2 + 12 \cos(\theta)^2\, aa0^2\, ee\, cc^2\, r^7\, aa + 4 \cos(\theta)^6\, aa^3\, r^7\, ee\, cc0^2$

$\quad - 5 \cos(\theta)^6\, aa\, r^6\, ee^2\, cc^2\, bb - 8 \cos(\theta)^4\, aa\, r^4\, ee\, bb - 4 \cos(\theta)^6\, aa\, r^7\, ee^2\, cc^2 - 8 \cos(\theta)^4\, aa^2\, r^5\, ee\, cc0^2\, aa0$

$\quad - 48 \cos(\theta)^4\, aa^2\, r^9\, ee\, cc^2\, aa0 - 8 \cos(\theta)^4\, aa0^2\, ee\, cc\, r^5\, cc0\, aa - 8 \cos(\theta)^8\, aa0\, ee\, cc^2\, r^9\, aa^2$

$\quad - 8 \cos(\theta)^2\, aa\, r^5\, ee^2\, cc0\, cc - 48 \cos(\theta)^4\, aa^3\, r^9\, ee\, cc\, cc0 - 12 \cos(\theta)^4\, aa^3\, r^7\, ee\, cc0^2 + 6 \cos(\theta)^4\, aa\, r^4\, ee^2\, cc\, cc0\, bb$

$\quad - 40 \cos(\theta)^4\, aa^3\, r^{11}\, ee\, cc^2 + 4 \cos(\theta)^4\, aa\, r^5\, ee^2\, cc0\, cc + 32 \cos(\theta)^6\, aa^3\, r^9\, ee\, cc\, cc0 - 8 \cos(\theta)^8\, aa^3\, r^9\, ee\, cc\, cc0$

$\quad + 40 \cos(\theta)^6\, aa^3\, r^{11}\, ee\, cc^2 - 24 \cos(\theta)^4\, aa^2\, r^7\, aa0 + 24 \cos(\theta)^2\, aa0\, r^7\, aa^2 + 32 \cos(\theta)^2\, aa^3\, r^9\, ee\, cc\, cc0$

$\quad + 4 \cos(\theta)^6\, aa0^2\, ee\, cc^2\, r^7\, aa + 12 \cos(\theta)^2\, aa^3\, r^7\, ee\, cc0^2 - 12 \cos(\theta)^2\, aa\, r^7\, ee^2\, cc^2 + 20 \cos(\theta)^2\, aa^3\, r^{11}\, ee\, cc^2$

$\quad - 12 \cos(\theta)^2\, aa\, r^4\, ee^2\, cc\, cc0\, bb - 20 \cos(\theta)^8\, aa^3\, r^{11}\, ee\, cc^2 + 32 \cos(\theta)^6\, aa^2\, r^9\, ee\, cc^2\, aa0$

$\quad + 32 \cos(\theta)^2\, aa^2\, r^9\, ee\, cc^2\, aa0 + 16 \cos(\theta)^2\, aa\, r^4\, ee\, bb - 6 \cos(\theta)^2\, aa0\, ee^2\, cc\, r^2\, cc0\, bb + 15 \cos(\theta)^4\, aa\, r^6\, ee^2\, cc^2\, bb$

$\quad - 15 \cos(\theta)^2\, aa\, r^6\, ee^2\, cc^2\, bb - \cos(\theta)^2\, aa\, r^2\, ee^2\, cc0^2\, bb + 16 \cos(\theta)^2\, aa^2\, r^5\, ee\, cc0^2\, aa0$

$\quad + 48 \cos(\theta)^2\, aa^2\, r^7\, ee\, cc\, cc0\, aa0 - 10 \cos(\theta)^2\, aa0\, ee^2\, cc^2\, r^4\, bb + 8 \cos(\theta)^2\, aa0^2\, r^5\, aa + 8 \cos(\theta)^6\, aa0\, r^7\, aa^2$

$\quad - 4 \cos(\theta)^4\, aa0^2\, r^5\, aa + 4 \cos(\theta)^4\, aa0\, ee^2\, cc^2\, r^5 - 4 \cos(\theta)^2\, aa0\, ee^2\, cc0\, r^3\, cc + 4 \cos(\theta)^{10}\, aa^3\, r^{11}\, ee\, cc^2$

$\quad - 48 \cos(\theta)^4\, aa^2\, r^7\, ee\, cc\, cc0\, aa0 \big) \cos(\theta) \big/ (ee\, r^3\, (-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2))$

$T(1, 4, 2) = \sin(\theta) \big(3\, aa^3\, r^7 \cos(\theta)^6 + 2\, aa^2\, r^5 \cos(\theta)^6\, aa0 + aa\, r^3 \cos(\theta)^2\, aa0^2 - 2 \cos(\theta)^4\, aa\, r^2\, ee\, bb$

$\quad - 3\, aa^3\, r^7 \cos(\theta)^4 - 4\, aa^2\, r^5 \cos(\theta)^4\, aa0 + aa^3\, r^7 \cos(\theta)^2 - aa\, r^3 \cos(\theta)^4\, aa0^2 + 2\, aa^2\, r^5 \cos(\theta)^2\, aa0$

$\quad - 2 \cos(\theta)^4\, aa\, r^3\, ee + 3\, aa\, r^3 \cos(\theta)^2\, ee - aa\, r^2\, ee\, bb - aa^3\, r^7 \cos(\theta)^8 + 4\, aa^2\, r^5\, ee\, cc\, cc0 \cos(\theta)^2\, aa0$

$\quad - \cos(\theta)^6\, ee^2\, cc^2\, aa\, r^5 + \cos(\theta)^4\, ee^2\, cc^2\, r^3\, aa0 - \cos(\theta)^2\, ee^2\, cc^2\, r^3\, aa0 + \cos(\theta)^4\, ee^2\, cc\, aa\, r^2\, bb\, cc0$

$\quad - \cos(\theta)^2\, ee^2\, cc\, bb\, aa\, r^2\, cc0 - 2\, aa^2\, r^3 \cos(\theta)^4\, ee\, cc0^2\, aa0 + 2 \cos(\theta)^4\, ee^2\, cc^2\, aa\, r^5 - 2\, aa\, r^3 \cos(\theta)^4\, ee\, cc\, cc0\, aa0^2$

$\quad + aa\, r^5 \cos(\theta)^6\, ee\, cc^2\, aa0^2 - 2\, aa^3\, r^7 \cos(\theta)^8\, ee\, cc\, cc0 - \cos(\theta)^2\, ee^2\, cc^2\, aa\, r^5 + aa\, r \cos(\theta)^2\, ee\, cc0^2\, aa0^2$

$\quad + aa^3\, r^5 \cos(\theta)^6\, ee\, cc0^2 + 4\, aa^2\, r^5 \cos(\theta)^6\, ee\, cc\, cc0\, aa0 + aa^3\, r^9 \cos(\theta)^{10}\, ee\, cc^2 - aa\, r^3\, ee - 4\, aa^3\, r^9\, ee\, cc^2 \cos(\theta)^4$

$\quad + 6\, aa^2\, r^7\, ee\, cc^2 \cos(\theta)^6\, aa0 - 2\, aa^3\, r^5\, ee\, cc0^2 \cos(\theta)^4 + aa^3\, r^9\, ee\, cc^2 \cos(\theta)^2 + 2\, aa^2\, r^3\, ee\, cc0^2 \cos(\theta)^2\, aa0$

$\quad + \cos(\theta)^4\, ee^2\, cc^2\, bb\, aa0\, r^2 - \cos(\theta)^2\, ee^2\, cc^2\, bb\, aa0\, r^2 - \cos(\theta)^2\, ee^2\, cc\, r\, aa0\, cc0 - \cos(\theta)^6\, ee^2\, cc^2\, aa\, r^4\, bb$

$\quad + \cos(\theta)^4\, ee^2\, cc\, aa\, r^3\, cc0 + 2 \cos(\theta)^4\, ee^2\, cc^2\, bb\, aa\, r^4 + 2\, aa\, r^3\, ee\, cc \cos(\theta)^2\, cc0\, aa0^2 - \cos(\theta)^2\, ee^2\, cc^2\, bb\, aa\, r^4$

$\quad - 8\, aa^2\, r^5\, ee\, cc \cos(\theta)^4\, cc0\, aa0 - \cos(\theta)^2\, ee^2\, cc\, aa\, r^3\, cc0 + 6\, aa^3\, r^9\, ee\, cc^2 \cos(\theta)^6 - 2\, aa^2\, r^7 \cos(\theta)^8\, ee\, cc^2\, aa0$

$\quad - 6\, aa^2\, r^7\, ee\, cc^2 \cos(\theta)^4\, aa0 + aa^3\, r^5\, ee\, cc0^2 \cos(\theta)^2 + 3\, aa\, r^2 \cos(\theta)^2\, ee\, bb - 6\, aa^3\, r^7\, ee\, cc\, cc0 \cos(\theta)^4$

$\quad - 4\, aa^3\, r^9\, ee\, cc^2 \cos(\theta)^8 + 2\, aa^3\, r^7\, ee\, cc\, cc0 \cos(\theta)^2 + aa\, r^5\, ee\, cc^2 \cos(\theta)^2\, aa0^2 - \cos(\theta)^2\, ee^2\, cc\, bb\, aa0\, cc0$

$\quad + 6\, aa^3\, r^7\, ee\, cc \cos(\theta)^6\, cc0 - 2\, aa\, r^5\, ee\, cc^2 \cos(\theta)^4\, aa0^2 + 2\, aa^2\, r^7\, ee\, cc^2 \cos(\theta)^2\, aa0 \big) \big/ (ee$

$\quad (-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2))$

$T(1, 4, 3) = \dfrac{1}{4} \big(-3\, ee\, cc^2\, r^8 \cos(\theta)^{10}\, aa^2\, cc0\, bb - 2\, ee\, cc^3\, r^8 \cos(\theta)^{10}\, aa\, aa0\, bb - 2\, r^4 \cos(\theta)^6\, aa\, aa0\, cc0\, bb$

$+ 2\ ee\ cc0^3\ aa\ r^2\ \cos(\theta)^4\ aa0\ bb + ee\ cc^3\ r^6\ \cos(\theta)^8\ aa0^2\ bb + ee\ cc^3\ r^{10}\ \cos(\theta)^{12}\ aa^2\ bb - r^8\ \cos(\theta)^{10}\ aa^2\ cc\ bb$

$- r^4\ \cos(\theta)^6\ aa0^2\ cc\ bb + 6\ ee\ cc^2\ r^6\ \cos(\theta)^8\ aa\ aa0\ cc0\ bb + r^2\ \cos(\theta)^4\ aa0^2\ cc0\ bb + 3\ ee\ cc0^2\ aa0^2\ cc\ r^2\ \cos(\theta)^4\ bb$

$+ 3\ ee\ cc0^2\ aa^2\ r^6\ \cos(\theta)^8\ cc\ bb - 6\ ee\ cc0^2\ aa\ r^4\ \cos(\theta)^6\ aa0\ cc\ bb + r^6\ \cos(\theta)^8\ aa^2\ cc0\ bb - ee\ cc0^3\ aa0^2\ bb\ \cos(\theta)^2$

$+ 2\ r^6\ \cos(\theta)^8\ aa\ aa0\ cc\ bb - ee\ cc0^3\ aa^2\ r^4\ \cos(\theta)^6\ bb - 3\ ee\ cc^2\ r^4\ \cos(\theta)^6\ cc0\ aa0^2\ bb$

$- 3\ ee\ cc0^2\ aa^2\ r^6\ cc\ \cos(\theta)^2\ bb + r\ \cos(\theta)^2\ ee\ bb^2\ cc0 + 3\ ee\ cc^3\ r^6\ \cos(\theta)^4\ aa0^2\ bb + 10\ ee\ cc^3\ r^{10}\ \cos(\theta)^8\ aa^2\ bb$

$- 5\ ee\ cc^3\ r^{10}\ \cos(\theta)^{10}\ aa^2\ bb - 12\ ee\ cc^3\ r^8\ \cos(\theta)^6\ aa\ aa0\ bb + 12\ ee\ cc^2\ r^8\ \cos(\theta)^8\ aa^2\ cc0\ bb$

$+ 8\ ee\ cc^3\ r^8\ \cos(\theta)^8\ aa\ aa0\ bb - 2\ ee\ cc0^3\ aa\ r^2\ \cos(\theta)^2\ aa0\ bb + 6\ ee\ cc^2\ r^4\ \cos(\theta)^4\ cc0\ aa0^2\ bb$

$+ r^2\ \cos(\theta)^2\ ee\ cc0\ bb - 6\ ee\ cc0^2\ aa\ r^4\ aa0\ cc\ \cos(\theta)^2\ bb - r^2\ \cos(\theta)^2\ aa0^2\ cc0\ bb + 2\ r^4\ \cos(\theta)^4\ aa0^2\ cc\ bb$

$+ 4\ r^4\ \cos(\theta)^4\ aa\ aa0\ cc0\ bb - 6\ r^6\ \cos(\theta)^6\ aa\ aa0\ cc\ bb + 3\ r^3\ \cos(\theta)^4\ ee\ bb^2\ cc - 2\ r^4\ aa\ \cos(\theta)^2\ aa0\ cc0\ bb$

$+ 8\ ee\ cc^3\ r^8\ aa\ \cos(\theta)^4\ aa0\ bb - 3\ ee\ cc^3\ r^6\ \cos(\theta)^6\ aa0^2\ bb + 9\ ee\ cc\ r^6\ cc0^2\ aa^2\ \cos(\theta)^4\ bb$

$+ 2\ ee\ cc0^3\ aa^2\ r^4\ \cos(\theta)^4\ bb - ee\ cc0^3\ aa^2\ r^4\ \cos(\theta)^2\ bb + 5\ ee\ cc^3\ r^{10}\ aa^2\ \cos(\theta)^4\ bb - 9\ ee\ cc0^2\ aa^2\ r^6\ \cos(\theta)^6\ cc\ bb$

$- 10\ ee\ cc^3\ r^{10}\ \cos(\theta)^6\ aa^2\ bb - 18\ ee\ cc^2\ r^6\ \cos(\theta)^6\ aa\ aa0\ cc0\ bb + 12\ ee\ cc0^2\ aa\ r^4\ \cos(\theta)^4\ aa0\ cc\ bb$

$+ 12\ ee\ cc^2\ r^8\ cc0\ aa^2\ \cos(\theta)^4\ bb - 18\ ee\ cc^2\ r^8\ cc0\ aa^2\ \cos(\theta)^6\ bb + 18\ ee\ cc^2\ r^6\ cc0\ aa\ \cos(\theta)^4\ aa0\ bb$

$+ 4\ r^5\ \cos(\theta)^4\ ee\ cc + 4\ r^8\ \cos(\theta)^8\ aa^2\ cc\ bb - ee\ cc^3\ r^{10}\ aa^2\ \cos(\theta)^2\ bb - r^6\ aa^2\ \cos(\theta)^2\ cc0\ bb$

$- r^4\ aa0^2\ cc\ \cos(\theta)^2\ bb + 4\ r^8\ aa^2\ \cos(\theta)^4\ cc\ bb - r^8\ aa^2\ \cos(\theta)^2\ cc\ bb + 3\ r^6\ aa^2\ \cos(\theta)^4\ cc0\ bb$

$- 6\ r^8\ aa^2\ \cos(\theta)^6\ cc\ bb + 4\ r^5\ ee\ cc + 7\ r^4\ \cos(\theta)^4\ ee\ cc\ bb - 14\ r^4\ \cos(\theta)^2\ ee\ cc\ bb - 3\ r^6\ \cos(\theta)^6\ aa^2\ cc0\ bb$

$- r\ ee\ bb^2\ cc0 + 3\ r^3\ ee\ bb^2\ cc - 2\ ee\ cc^3\ r^8\ aa\ aa0\ \cos(\theta)^2\ bb - 3\ ee\ cc0^2\ aa0^2\ cc\ r^2\ \cos(\theta)^2\ bb$

$- 3\ ee\ cc^2\ r^4\ cc0\ aa0^2\ \cos(\theta)^2\ bb + 6\ r^6\ \cos(\theta)^4\ aa\ aa0\ cc\ bb - 2\ r^6\ aa\ aa0\ cc\ \cos(\theta)^2\ bb$

$- 3\ ee\ cc^2\ r^8\ cc0\ aa^2\ \cos(\theta)^2\ bb - 6\ r^3\ ee\ bb^2\ cc\ \cos(\theta)^2 - r^2\ ee\ cc0\ bb + 7\ r^4\ ee\ cc\ bb - ee\ cc^3\ r^6\ aa0^2\ \cos(\theta)^2\ bb$

$\left. - 6\ ee\ cc^2\ r^6\ cc0\ aa\ aa0\ \cos(\theta)^2\ bb - 8\ r^5\ \cos(\theta)^2\ ee\ cc\right) \Big/ \left(\sqrt{\dfrac{r+bb}{r}}\ r^3\ (-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2) \right)$

$T(1, 4, 4) = \dfrac{1}{2}\ (-4\ r^4\ bb\ aa\ aa0 - 4\ r^7\ aa^2 - 4\ r^5\ aa\ aa0 + r\ ee\ bb^2 + r^2\ ee\ bb + 10\ r^4\ bb\ aa\ \cos(\theta)^2\ aa0$

$- 15\ r^6\ bb\ aa^2\ \cos(\theta)^4 + r^2\ \cos(\theta)^2\ aa0^2\ bb - 4\ r^5\ \cos(\theta)^4\ aa\ aa0 + ee\ cc0^2\ aa^2\ r^4\ \cos(\theta)^2\ bb$

$+ 6\ ee\ cc^2\ r^6\ \cos(\theta)^6\ aa\ aa0\ bb + 2\ ee\ cc\ r^2\ \cos(\theta)^2\ cc0\ aa0^2\ bb - 2\ ee\ cc^2\ r^4\ \cos(\theta)^4\ aa0^2\ bb$

$- 6\ ee\ cc^2\ r^6\ \cos(\theta)^4\ aa\ aa0\ bb + ee\ cc^2\ r^8\ aa^2\ \cos(\theta)^2\ bb + 7\ r^6\ \cos(\theta)^6\ aa^2\ bb - 8\ ee\ cc\ r^4\ \cos(\theta)^4\ cc0\ aa\ aa0\ bb$

$+ ee\ cc^2\ r^4\ \cos(\theta)^2\ aa0^2\ bb + 4\ ee\ cc\ r^4\ \cos(\theta)^6\ cc0\ aa\ aa0\ bb + 6\ ee\ cc^2\ r^8\ \cos(\theta)^6\ aa^2\ bb - 8\ r^4\ \cos(\theta)^4\ aa\ aa0\ bb$

$+ 2\ ee\ cc^2\ r^6\ aa\ \cos(\theta)^2\ aa0\ bb + ee\ cc0^2\ aa^2\ r^4\ \cos(\theta)^6\ bb - 2\ ee\ cc^2\ r^6\ \cos(\theta)^8\ aa\ aa0\ bb$

$- 2\ ee\ cc0^2\ aa\ r^2\ \cos(\theta)^4\ aa0\ bb + ee\ cc^2\ r^8\ \cos(\theta)^{10}\ aa^2\ bb - r^6\ \cos(\theta)^8\ aa^2\ bb - 2\ ee\ cc\ r^2\ \cos(\theta)^4\ cc0\ aa0^2\ bb$

$- 2\ ee\ cc\ r^6\ \cos(\theta)^8\ cc0\ aa^2\ bb + 2\ ee\ cc0^2\ aa\ r^2\ \cos(\theta)^2\ aa0\ bb + 2\ ee\ cc\ r^6\ cc0\ aa^2\ \cos(\theta)^2\ bb$

$+ 13\ r^6\ bb\ aa^2\ \cos(\theta)^2 + 8\ r^5\ \cos(\theta)^2\ aa\ aa0 + 2\ r^4\ \cos(\theta)^6\ aa\ aa0\ bb - r^2\ \cos(\theta)^4\ aa0^2\ bb - r^2\ \cos(\theta)^2\ ee\ bb$

$- r\ \cos(\theta)^2\ ee\ bb^2 + 4\ r^7\ \cos(\theta)^6\ aa^2 - 4\ ee\ cc^2\ r^8\ \cos(\theta)^8\ aa^2\ bb - 2\ ee\ cc0^2\ aa^2\ r^4\ \cos(\theta)^4\ bb - 12\ r^7\ \cos(\theta)^4\ aa^2$

$+ 6\ ee\ cc\ r^6\ \cos(\theta)^6\ cc0\ aa^2\ bb + 4\ ee\ cc\ r^4\ cc0\ aa\ \cos(\theta)^2\ aa0\ bb - 4\ ee\ cc^2\ r^8\ aa^2\ \cos(\theta)^4\ bb$

$+ ee\ cc^2\ r^4\ \cos(\theta)^6\ aa0^2\ bb + ee\ cc0^2\ aa0^2\ bb\ \cos(\theta)^2 - 6\ ee\ cc\ r^6\ cc0\ aa^2\ \cos(\theta)^4\ bb - 4\ r^6\ bb\ aa^2 + 12\ r^7\ \cos(\theta)^2\ aa^2$

$\Big) \Big/ (r^3\ (-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2))$

$T(2, 1, 1) = 2\ (\cos(\theta)^6\ aa\ r^6\ ee\ cc^2 - 3\ \cos(\theta)^4\ aa\ r^6\ ee\ cc^2 - \cos(\theta)^4\ r^4\ aa0\ ee\ cc^2 - r^4\ \cos(\theta)^4\ aa$

$- 2\ \cos(\theta)^4\ aa\ r^4\ ee\ cc\ cc0 + 4\ \cos(\theta)^2\ aa\ r^4\ ee\ cc\ cc0 + \cos(\theta)^2\ aa\ r^2\ ee\ cc0^2 + 2\ \cos(\theta)^2\ r^2\ aa0\ ee\ cc\ cc0$

$+ 3\ \cos(\theta)^2\ aa\ r^6\ ee\ cc^2 + 2\ aa\ r^4\ \cos(\theta)^2 + 2\ \cos(\theta)^2\ r^4\ aa0\ ee\ cc^2 + r^2\ \cos(\theta)^2\ aa0 - ee\ cc0^2\ aa0 - ee\ cc^2\ aa0\ r^4$

$- aa\ r^4 - ee\ cc^2\ r^6\ aa - 2\ aa\ r^4\ ee\ cc\ cc0 - r^2\ aa0 - 2\ r^2\ ee\ cc\ cc0\ aa0 - aa\ ee\ cc0^2\ r^2)\ \sin(\theta)\ \cos(\theta)\ aa\ /\ ((r+bb)$

$ee\ r)$

$T(2, 1, 2) = aa\ (r^2\ aa0 + aa\ r^4 + ee\ cc^2\ r^6\ aa + ee\ cc^2\ aa0\ r^4 + 2\ aa\ r^4\ ee\ cc\ cc0 + ee\ cc0^2\ aa0 + 2\ r^2\ ee\ cc\ cc0\ aa0$

$+ aa\ ee\ cc0^2\ r^2 + 2\ \cos(\theta)^4\ r^2\ aa0 - 2\ \cos(\theta)^6\ aa\ r^4 - 2\ \cos(\theta)^6\ r^4\ aa0\ ee\ cc^2 + 2\ \cos(\theta)^8\ aa\ r^6\ ee\ cc^2$

$$+ 4\cos(\theta)^4 r^2 aa0\, ee\, cc\, cc0 + 2\cos(\theta)^4 aa\, r^2 ee\, cc0^2 - 4\cos(\theta)^6 aa\, r^4 ee\, cc\, cc0 - 6\cos(\theta)^2 r^2 aa0\, ee\, cc\, cc0$$

$$- 5\cos(\theta)^2 aa\, r^6 ee\, cc^2 - 4\cos(\theta)^2 r^4 aa0\, ee\, cc^2 - 3\, r^2 \cos(\theta)^2 aa0 - 7\cos(\theta)^6 aa\, r^6 ee\, cc^2 + 9\cos(\theta)^4 aa\, r^6 ee\, cc^2$$

$$+ 5\cos(\theta)^4 r^4 aa0\, ee\, cc^2 + 10\cos(\theta)^4 aa\, r^4 ee\, cc\, cc0 - 8\cos(\theta)^2 aa\, r^4 ee\, cc\, cc0 - 3\cos(\theta)^2 aa\, r^2 ee\, cc0^2$$

$$- 4\, aa\, r^4 \cos(\theta)^2 + 5\, r^4 \cos(\theta)^4 aa - 2\cos(\theta)^2 aa0\, ee\, cc0^2) / ((r + bb)\, ee)$$

$$T(2, 1, 3) = -\frac{1}{4}\sin(\theta)\, (-4\, r^5 aa0\, cc - 4\, r^7 aa\, cc - 4\, r^5 \cos(\theta)^4 aa0\, cc + 8\, r^5 aa0\, cc \cos(\theta)^2 - 12\, r^7 \cos(\theta)^4 aa\, cc$$

$$+ 4\, r^7 \cos(\theta)^6 aa\, cc + 5\, ee\, aa\, r^8 cc^3 bb + aa0\, r^2 cc0\, bb + aa\, r^4 cc0\, bb + ee\, aa\, r^2 cc0^3 bb + 8\, ee\, aa0\, cc^2 r^5 cc0$$

$$+ 7\, ee\, aa\, r^4 cc\, cc0^2 bb + 4\, ee\, aa0\, cc\, r^3 cc0^2 - 3\, aa0\, r^4 cc\, bb + 4\, ee\, aa\, r^9 cc^3 + 11\, ee\, aa0\, cc^2 r^4 cc0\, bb$$

$$+ 4\, ee\, aa\, r^5 cc\, cc0^2 - 3\, aa\, r^6 cc\, bb + 11\, ee\, aa\, r^6 cc^2 cc0\, bb + 7\, ee\, aa0\, cc\, r^2 cc0^2 bb + ee\, aa0\, cc0^3 bb$$

$$+ 5\, ee\, aa0\, cc^3 r^6 bb + 8\, ee\, aa\, r^7 cc^2 cc0 + 4\, ee\, aa0\, cc^3 r^7 - 8\cos(\theta)^6 ee\, aa\, r^7 cc^2 cc0 - 11\cos(\theta)^6 ee\, aa\, r^6 cc^2 cc0\, bb$$

$$+ 7\cos(\theta)^4 ee\, aa\, r^4 cc\, cc0^2 bb + 4\cos(\theta)^4 ee\, aa\, r^5 cc\, cc0^2 + \cos(\theta)^4 aa\, r^4 cc0\, bb - 14\cos(\theta)^2 ee\, aa\, r^4 cc\, cc0^2 bb$$

$$- 20\cos(\theta)^6 ee\, aa\, r^8 cc^3 bb - 16\cos(\theta)^6 ee\, aa\, r^9 cc^3 + 4\cos(\theta)^8 ee\, aa\, r^9 cc^3 - 12\cos(\theta)^2 ee\, aa0\, cc^3 r^7$$

$$- 4\cos(\theta)^2 ee\, aa0\, cc\, r^3 cc0^2 + 8\cos(\theta)^4 ee\, aa\, cc^2 r^5 cc0 - 3\cos(\theta)^4 aa0\, r^4 cc\, bb - 15\cos(\theta)^2 ee\, aa0\, cc^3 r^6 bb$$

$$+ 15\cos(\theta)^4 ee\, aa0\, cc^3 r^6 bb - \cos(\theta)^2 ee\, aa\, r^2 cc0^3 bb - 9\cos(\theta)^4 aa\, r^6 cc\, bb + 3\cos(\theta)^6 aa\, r^6 cc\, bb$$

$$+ 30\cos(\theta)^4 ee\, aa\, r^8 cc^3 bb - 16\cos(\theta)^2 ee\, aa\, r^9 cc^3 - 7\cos(\theta)^2 ee\, aa0\, cc\, r^2 cc0^2 bb - 5\cos(\theta)^6 ee\, aa0\, cc^3 r^6 bb$$

$$- 4\cos(\theta)^6 ee\, aa0\, cc^3 r^7 - 22\cos(\theta)^2 ee\, aa\, cc^2 r^4 cc0\, bb - 20\cos(\theta)^2 ee\, aa\, r^8 cc^3 bb + 12\cos(\theta)^4 ee\, aa0\, cc^3 r^7$$

$$+ 6\cos(\theta)^2 aa0\, r^4 cc\, bb - \cos(\theta)^2 aa0\, r^2 cc0\, bb + 5\cos(\theta)^8 ee\, aa\, r^8 cc^3 bb + 24\cos(\theta)^4 ee\, aa\, r^7 cc^2 cc0$$

$$+ 33\cos(\theta)^4 ee\, aa\, r^6 cc^2 cc0\, bb + 24\cos(\theta)^4 ee\, aa\, r^9 cc^3 - 33\cos(\theta)^2 ee\, aa\, r^6 cc^2 cc0\, bb$$

$$- 16\cos(\theta)^2 ee\, aa0\, cc^2 r^5 cc0 - 2\cos(\theta)^2 aa\, r^4 cc0\, bb + 9\cos(\theta)^2 aa\, r^6 cc\, bb - 24\cos(\theta)^2 ee\, aa\, r^7 cc^2 cc0$$

$$- 8\cos(\theta)^2 ee\, aa\, r^5 cc\, cc0^2 + 11\cos(\theta)^4 ee\, aa0\, cc^2 r^4 cc0\, bb + 12\, r^7 aa\, cc \cos(\theta)^2) \Big/ \Big(r^4$$

$$(-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2)\sqrt{\frac{r + bb}{r}}\Big)$$

$$T(2, 1, 4) = \frac{1}{4}\sin(\theta)\, (-4\, aa^3 r^9 - 12\cos(\theta)^4 aa0^2 ee\, cc^2 r^7 aa + 16\cos(\theta)^2 aa^3 r^9 - 24\cos(\theta)^4 aa^3 r^9$$

$$+ 16\cos(\theta)^6 aa^3 r^9 - 4\cos(\theta)^8 aa^3 r^9 + 4\, aa\, r^5 ee - 4\, aa\, r^5 \cos(\theta)^2 ee - 8\, aa0\, r^7 aa^2 + 4\, aa0\, ee^2 cc0\, cc\, r^3$$

$$+ 4\, aa0\, ee^2 cc^2 r^5 + aa0\, ee^2 cc0^2 bb + aa\, r^2 ee^2 cc0^2 bb - 4\, aa0^2 r^5 aa - 8\, aa^3 r^9 ee\, cc\, cc0 - 4\, aa0^2 ee\, cc0^2 aa\, r^3$$

$$- 4\, aa^3 r^{11} ee\, cc^2 + 6\, aa\, r^4 ee^2 cc\, cc0\, bb + 4\, aa\, r^5 ee^2 cc0\, cc + 4\, aa\, r^7 ee^2 cc^2 + 2\, aa0\, r^2 ee\, bb - 8\, aa0^2 ee\, cc\, r^5 cc0\, aa$$

$$+ 6\, aa\, r^4 ee\, bb + 5\, aa\, r^6 ee^2 cc^2 bb - 4\, aa^3 r^7 ee\, cc0^2 + 6\, aa0\, ee^2 cc\, r^2 cc0\, bb - 4\, aa0^2 ee\, cc^2 r^7 aa$$

$$- 8\, aa0\, ee\, cc0^2 aa^2 r^5 - 8\, aa0\, ee\, cc^2 r^9 aa^2 + 5\, aa0\, ee^2 cc^2 r^4 bb - 16\, aa0\, ee\, cc\, r^7 cc0\, aa^2 - 8\cos(\theta)^2 aa0\, ee^2 cc^2 r^5$$

$$+ 5\cos(\theta)^4 aa0\, ee^2 cc^2 r^4 bb + 16\cos(\theta)^2 aa0^2 ee\, cc\, r^5 cc0\, aa + 4\cos(\theta)^2 aa0^2 ee\, cc0^2 aa\, r^3$$

$$- 2\cos(\theta)^2 aa0\, r^2 ee\, bb + 16\cos(\theta)^6 aa0\, ee\, cc\, r^7 cc0\, aa^2 + 12\cos(\theta)^4 aa\, r^7 ee^2 cc^2 + 12\cos(\theta)^2 aa0^2 ee\, cc^2 r^7 aa$$

$$+ 4\cos(\theta)^6 aa^3 r^7 ee\, cc0^2 - 5\cos(\theta)^6 aa\, r^6 ee^2 cc^2 bb + 2\cos(\theta)^4 aa\, r^4 ee\, bb - 4\cos(\theta)^6 aa\, r^7 ee^2 cc^2$$

$$- 8\cos(\theta)^4 aa^2 r^5 ee\, cc0^2 aa0 - 48\cos(\theta)^4 aa^2 r^9 ee\, cc^2 aa0 - 8\cos(\theta)^4 aa0^2 ee\, cc\, r^5 cc0\, aa$$

$$- 8\cos(\theta)^8 aa0\, ee\, cc^2 r^9 aa^2 - 8\cos(\theta)^2 aa\, r^5 ee^2 cc0\, cc - 48\cos(\theta)^4 aa^3 r^9 ee\, cc\, cc0 - 12\cos(\theta)^4 aa^3 r^7 ee\, cc0^2$$

$$+ 6\cos(\theta)^4 aa\, r^4 ee^2 cc\, cc0\, bb - 40\cos(\theta)^4 aa^3 r^{11} ee\, cc^2 + 4\cos(\theta)^4 aa\, r^5 ee^2 cc0\, cc + 32\cos(\theta)^6 aa^3 r^9 ee\, cc\, cc0$$

$$- 8\cos(\theta)^8 aa^3 r^9 ee\, cc\, cc0 + 40\cos(\theta)^6 aa^3 r^{11} ee\, cc^2 - 24\cos(\theta)^4 aa^2 r^7 aa0 + 24\cos(\theta)^2 aa0\, r^7 aa^2$$

$$+ 32\cos(\theta)^2 aa^3 r^9 ee\, cc\, cc0 + 4\cos(\theta)^6 aa0^2 ee\, cc^2 r^7 aa + 12\cos(\theta)^2 aa^3 r^7 ee\, cc0^2 - 12\cos(\theta)^2 aa\, r^7 ee^2 cc^2$$

$$+ 20\cos(\theta)^2 aa^3 r^{11} ee\, cc^2 - 12\cos(\theta)^2 aa\, r^4 ee^2 cc\, cc0\, bb - 20\cos(\theta)^8 aa^3 r^{11} ee\, cc^2 + 32\cos(\theta)^6 aa^2 r^9 ee\, cc^2 aa0$$

$$+ 32\cos(\theta)^2 aa^2 r^9 ee\, cc^2 aa0 - 8\cos(\theta)^2 aa\, r^4 ee\, bb - 6\cos(\theta)^2 aa0\, ee^2 cc\, r^2 cc0\, bb + 15\cos(\theta)^4 aa\, r^6 ee^2 cc^2 bb$$

$$- 15\cos(\theta)^2 aa\, r^6 ee^2 cc^2 bb - \cos(\theta)^2 aa\, r^2 ee^2 cc0^2 bb + 16\cos(\theta)^2 aa^2 r^5 ee\, cc0^2 aa0$$

$$+ 48\cos(\theta)^2 aa^2 r^7 ee\, cc\, cc0\, aa0 - 10\cos(\theta)^2 aa0\, ee^2 cc^2 r^4 bb + 8\cos(\theta)^2 aa0^2 r^5 aa + 8\cos(\theta)^6 aa0\, r^7 aa^2$$

$$- 4\cos(\theta)^4 aa0^2 r^5 aa + 4\cos(\theta)^4 aa0 ee^2 cc^2 r^5 - 4\cos(\theta)^2 aa0 ee^2 cc0 r^3 cc + 4\cos(\theta)^{10} aa^3 r^{11} ee cc^2$$
$$- 48\cos(\theta)^4 aa^2 r^7 ee cc cc0 aa0\Big) \Big/ \big(r^4 ee (-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)\big)$$

$$T(2,2,1) = aa\,(r^2 aa0 + aa\, r^4 + ee\, cc^2 r^6 aa + ee\, cc^2 aa0\, r^4 + 2 aa\, r^4 ee\, cc\, cc0 + ee\, cc0^2 aa0 + 2 r^2 ee\, cc\, cc0\, aa0$$
$$+ aa\, ee\, cc0^2 r^2 + 2\cos(\theta)^4 r^2 aa0 - 2\cos(\theta)^6 aa\, r^4 - 2\cos(\theta)^6 r^4 aa0\, ee\, cc^2 + 2\cos(\theta)^8 aa\, r^6 ee\, cc^2$$
$$+ 4\cos(\theta)^4 r^2 aa0\, ee\, cc\, cc0 + 2\cos(\theta)^4 aa\, r^2 ee\, cc0^2 - 4\cos(\theta)^6 aa\, r^4 ee\, cc\, cc0 - 6\cos(\theta)^2 r^2 aa0\, ee\, cc\, cc0$$
$$- 5\cos(\theta)^2 aa\, r^6 ee\, cc^2 - 4\cos(\theta)^2 r^4 aa0\, ee\, cc^2 - 3 r^2 \cos(\theta)^2 aa0 - 7\cos(\theta)^6 aa\, r^6 ee\, cc^2 + 9\cos(\theta)^4 aa\, r^6 ee\, cc^2$$
$$+ 5\cos(\theta)^4 r^4 aa0\, ee\, cc^2 + 10\cos(\theta)^4 aa\, r^4 ee\, cc\, cc0 - 8\cos(\theta)^2 aa\, r^4 ee\, cc\, cc0 - 3\cos(\theta)^2 aa\, r^2 ee\, cc0^2$$
$$- 4 aa\, r^4 \cos(\theta)^2 + 5 r^4 \cos(\theta)^4 aa - 2\cos(\theta)^2 aa0\, ee\, cc0^2) / ((r+bb)\, ee)$$

$$T(2,2,2) = -2\,(\cos(\theta)^6 aa\, r^6 ee\, cc^2 - 3\cos(\theta)^4 aa\, r^6 ee\, cc^2 - \cos(\theta)^4 r^4 aa0\, ee\, cc^2 - r^4 \cos(\theta)^4 aa$$
$$- 2\cos(\theta)^4 aa\, r^4 ee\, cc\, cc0 + 4\cos(\theta)^2 aa\, r^4 ee\, cc\, cc0 + \cos(\theta)^2 aa\, r^2 ee\, cc0^2 + 2\cos(\theta)^2 r^2 aa0\, ee\, cc\, cc0$$
$$+ 3\cos(\theta)^2 aa\, r^6 ee\, cc^2 + 2 aa\, r^4 \cos(\theta)^2 + 2\cos(\theta)^2 r^4 aa0\, ee\, cc^2 + r^2 \cos(\theta)^2 aa0 - ee\, cc0^2 aa0 - ee\, cc^2 aa0\, r^4$$
$$- aa\, r^4 - ee\, cc^2 r^6 aa - 2 aa\, r^4 ee\, cc\, cc0 - r^2 aa0 - 2 r^2 ee\, cc\, cc0\, aa0 - aa\, ee\, cc0^2 r^2)\, aa\, r \sin(\theta)\cos(\theta) / ((r+bb)$$
$$ee)$$

$$T(2,2,3) = -(\cos(\theta)^6 aa\, r^6 ee\, cc^2 + r^4 \cos(\theta)^4 aa - 3\cos(\theta)^4 aa\, r^6 ee\, cc^2 - 2\cos(\theta)^4 aa\, r^4 ee\, cc\, cc0$$
$$- \cos(\theta)^4 r^4 aa0\, ee\, cc^2 + 3\cos(\theta)^2 aa\, r^6 ee\, cc^2 + 2\cos(\theta)^2 r^4 aa0\, ee\, cc^2 - r^2 \cos(\theta)^2 aa0 + 4\cos(\theta)^2 aa\, r^4 ee\, cc\, cc0$$
$$+ 2\cos(\theta)^2 r^2 aa0\, ee\, cc\, cc0 - 2 aa\, r^4 \cos(\theta)^2 + \cos(\theta)^2 aa\, r^2 ee\, cc0^2 + aa\, r^4 - ee\, cc0^2 aa0 - 2 aa\, r^4 ee\, cc\, cc0$$
$$- 2 r^2 ee\, cc\, cc0\, aa0 - ee\, cc^2 aa0\, r^4 - aa\, ee\, cc0^2 r^2 + r^2 aa0 - ee\, cc^2 r^6 aa)\, cc\cos(\theta) \Big/ \left(r\sqrt{\frac{r+bb}{r}}\right)$$

$$T(2,2,4) = (-aa^3 r^7 \cos(\theta)^6 - aa\, r^3 \cos(\theta)^2 aa0^2 + 3 aa^3 r^7 \cos(\theta)^4 + 2 aa^2 r^5 \cos(\theta)^4 aa0 - 3 aa^3 r^7 \cos(\theta)^2$$
$$- 4 aa^2 r^5 \cos(\theta)^2 aa0 - 8 aa^2 r^5 ee\, cc\, cc0 \cos(\theta)^2 aa0 + \cos(\theta)^2 ee^2 cc^2 r^3 aa0 + \cos(\theta)^2 ee^2 cc\, bb\, aa\, r^2 cc0$$
$$- \cos(\theta)^4 ee^2 cc^2 aa\, r^5 + 2\cos(\theta)^2 ee^2 cc^2 aa\, r^5 + 6 aa^3 r^9 ee\, cc^2 \cos(\theta)^4 - 2 aa^2 r^7 ee\, cc^2 \cos(\theta)^6 aa0$$
$$+ aa^3 r^5 ee\, cc0^2 \cos(\theta)^4 - 4 aa^3 r^9 ee\, cc^2 \cos(\theta)^2 - 2 aa^2 r^3 ee\, cc0^2 \cos(\theta)^2 aa0 + \cos(\theta)^2 ee^2 cc^2 bb\, aa0\, r^2$$
$$- \cos(\theta)^4 ee^2 cc^2 bb\, aa\, r^4 - 2 aa\, r^3 ee\, cc \cos(\theta)^2 cc0\, aa0^2 + 2\cos(\theta)^2 ee^2 cc^2 bb\, aa\, r^4$$
$$+ 4 aa^2 r^5 ee\, cc \cos(\theta)^4 cc0\, aa0 + \cos(\theta)^2 ee^2 cc\, aa\, r^3 cc0 - 4 aa^3 r^9 ee\, cc^2 \cos(\theta)^6 + aa^3 r^7$$
$$+ 6 aa^2 r^7 ee\, cc^2 \cos(\theta)^4 aa0 - 2 aa^3 r^5 ee\, cc0^2 \cos(\theta)^2 + 6 aa^3 r^7 ee\, cc\, cc0 \cos(\theta)^4 + aa^3 r^9 ee\, cc^2 \cos(\theta)^8$$
$$- 6 aa^3 r^7 ee\, cc\, cc0 \cos(\theta)^2 - 2 aa\, r^5 ee\, cc^2 \cos(\theta)^2 aa0^2 - 2 aa^3 r^7 ee\, cc \cos(\theta)^6 cc0 + aa\, r^5 ee\, cc^2 \cos(\theta)^4 aa0^2$$
$$- 6 aa^2 r^7 ee\, cc^2 \cos(\theta)^2 aa0 + 4 aa^2 r^5 ee\, cc\, cc0\, aa0 - aa0\, ee^2 cc^2 bb\, r^2 - aa\, r^3 ee^2 cc\, cc0 + 2 aa0^2 ee\, cc\, r^3 cc0\, aa$$
$$+ 2 aa0\, ee\, cc^2 r^7 aa^2 + 2 aa0\, ee\, cc0^2 aa^2 r^3 - aa\, r^5 ee^2 cc^2 - aa0\, ee^2 cc\, r\, cc0 - aa0\, ee^2 cc\, cc0\, bb + 2 aa0\, aa^2 r^5$$
$$+ aa^3 r^9 ee\, cc^2 - aa\, r^2 ee^2 cc\, cc0\, bb - aa\, r^4 ee^2 cc^2 bb + 2 aa^3 r^7 ee\, cc\, cc0 + aa0^2 ee\, cc^2 r^5 aa + aa^3 r^5 ee\, cc0^2$$
$$- aa0\, ee^2 cc^2 r^3 + aa0^2 ee\, cc0^2 aa\, r + aa0^2 aa\, r^3)\cos(\theta) / ((r+bb)\, ee\, r)$$

$$T(2,3,1) = -\frac{1}{4}\sin(\theta)\,(4 r^5 aa0\, cc + 4 r^7 aa\, cc + 4 r^5 \cos(\theta)^4 aa0\, cc - 8 r^5 aa0\, cc \cos(\theta)^2 + 12 r^7 \cos(\theta)^4 aa\, cc$$
$$- 4 r^7 \cos(\theta)^6 aa\, cc + 5 ee\, aa\, r^8 cc^3 bb + aa0\, r^2 cc0\, bb + aa\, r^4 cc0\, bb + ee\, aa\, r^2 cc0^3 bb + 8 ee\, aa0\, cc^2 r^5 cc0$$
$$+ 7 ee\, aa\, r^4 cc\, cc0^2 bb + 4 ee\, aa0\, cc\, r^3 cc0^2 + 5 aa0\, r^4 cc\, bb + 4 ee\, aa\, r^9 cc^3 + 11 ee\, aa0\, cc^2 r^4 cc0\, bb$$
$$+ 4 ee\, aa\, r^5 cc\, cc0^2 + 5 aa\, r^6 cc\, bb + 11 ee\, aa\, r^6 cc^2 cc0\, bb + 7 ee\, aa0\, cc\, r^2 cc0^2 bb + ee\, aa0\, cc0^3 bb$$
$$+ 5 ee\, aa0\, cc^3 r^6 bb + 8 ee\, aa\, r^7 cc^2 cc0 + 4 ee\, aa0\, cc^3 r^7 - 8\cos(\theta)^6 ee\, aa\, r^7 cc^2 cc0 - 11\cos(\theta)^6 ee\, aa\, r^6 cc^2 cc0\, bb$$
$$+ 7\cos(\theta)^4 ee\, aa\, r^4 cc\, cc0^2 bb + 4\cos(\theta)^4 ee\, aa\, r^5 cc\, cc0^2 + \cos(\theta)^4 aa\, r^4 cc0\, bb - 14\cos(\theta)^2 ee\, aa\, r^4 cc\, cc0^2 bb$$
$$- 20\cos(\theta)^6 ee\, aa\, r^8 cc^3 bb - 16\cos(\theta)^6 ee\, aa\, r^9 cc^3 + 4\cos(\theta)^8 ee\, aa\, r^9 cc^3 - 12\cos(\theta)^2 ee\, aa0\, cc^3 r^7$$
$$- 4\cos(\theta)^2 ee\, aa0\, cc\, r^3 cc0^2 + 8\cos(\theta)^4 ee\, aa0\, cc^2 r^5 cc0 + 5\cos(\theta)^4 aa0\, r^4 cc\, bb - 15\cos(\theta)^2 ee\, aa0\, cc^3 r^6 bb$$
$$+ 15\cos(\theta)^4 ee\, aa0\, cc^3 r^6 bb - \cos(\theta)^2 ee\, aa\, r^2 cc0^3 bb + 15\cos(\theta)^4 aa\, r^6 cc\, bb - 5\cos(\theta)^6 aa\, r^6 cc\, bb$$
$$+ 30\cos(\theta)^4 ee\, aa\, r^8 cc^3 bb - 16\cos(\theta)^2 ee\, aa\, r^9 cc^3 - 7\cos(\theta)^2 ee\, aa0\, cc\, r^2 cc0^2 bb - 5\cos(\theta)^6 ee\, aa0\, cc^3 r^6 bb$$
$$- 4\cos(\theta)^6 ee\, aa0\, cc^3 r^7 - 22\cos(\theta)^2 ee\, aa0\, cc^2 r^4 cc0\, bb - 20\cos(\theta)^2 ee\, aa\, r^8 cc^3 bb + 12\cos(\theta)^4 ee\, aa0\, cc^3 r^7$$

$$- 10 \cos(\theta)^2\, aa0\, r^4\, cc\, bb - \cos(\theta)^2\, aa0\, r^2\, cc0\, bb + 5 \cos(\theta)^8\, ee\, aa\, r^8\, cc^3\, bb + 24 \cos(\theta)^4\, ee\, aa\, r^7\, cc^2\, cc0$$

$$+ 33 \cos(\theta)^4\, ee\, aa\, r^6\, cc^2\, cc0\, bb + 24 \cos(\theta)^4\, ee\, aa\, r^9\, cc^3 - 33 \cos(\theta)^2\, ee\, aa\, r^6\, cc^2\, cc0\, bb$$

$$- 16 \cos(\theta)^2\, ee\, aa0\, cc^2\, r^5\, cc0 - 2 \cos(\theta)^2\, aa\, r^4\, cc0\, bb - 15 \cos(\theta)^2\, aa\, r^6\, cc\, bb - 24 \cos(\theta)^2\, ee\, aa\, r^7\, cc^2\, cc0$$

$$- 8 \cos(\theta)^2\, ee\, aa\, r^5\, cc\, cc0^2 + 11 \cos(\theta)^4\, ee\, aa0\, cc^2\, r^4\, cc0\, bb - 12\, r^7\, aa\, cc\, \cos(\theta)^2\Big) \Big/ \Bigg(r^4$$

$$(-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2)\sqrt{\dfrac{r+bb}{r}}\,\Bigg)$$

$$T(2,3,2) = -\big(\cos(\theta)^6\, aa\, r^6\, ee\, cc^2 - 3 \cos(\theta)^4\, aa\, r^6\, ee\, cc^2 - \cos(\theta)^4\, r^4\, aa0\, ee\, cc^2 - r^4 \cos(\theta)^4\, aa$$

$$- 2 \cos(\theta)^4\, aa\, r^4\, ee\, cc\, cc0 + 4 \cos(\theta)^2\, aa\, r^4\, ee\, cc\, cc0 + \cos(\theta)^2\, aa\, r^2\, ee\, cc0^2 + 2 \cos(\theta)^2\, r^2\, aa0\, ee\, cc\, cc0$$

$$+ 3 \cos(\theta)^2\, aa\, r^6\, ee\, cc^2 + 2\, aa\, r^4 \cos(\theta)^2 + 2 \cos(\theta)^2\, r^4\, aa0\, ee\, cc^2 + r^2 \cos(\theta)^2\, aa0 - ee\, cc0^2\, aa0 - ee\, cc^2\, aa0\, r^4$$

$$- aa\, r^4 - ee\, cc^2\, r^6\, aa - 2\, aa\, r^4\, ee\, cc\, cc0 - r^2\, aa0 - 2\, r^2\, ee\, cc\, cc0\, aa0 - aa\, ee\, cc0^2\, r^2\big)\, cc\, \cos(\theta) \Big/ \Bigg(r \sqrt{\dfrac{r+bb}{r}}\,\Bigg)$$

$$T(2,3,3) = -2 \cos(\theta) \sin(\theta)\, ee\, cc\, (-cc\, r^2 + cc\, r^2 \cos(\theta)^2 - cc0)$$

$$T(2,3,4) = -\dfrac{1}{4} \cos(\theta) \sin(\theta)\, (-2\, ee\, cc0^3\, aa\, r^2\, aa0\, bb - ee\, cc0^3\, aa0^2\, bb - ee\, cc^3\, r^{10}\, aa^2\, bb - ee\, cc0^3\, aa^2\, r^4\, bb$$

$$- ee\, cc^3\, r^6\, aa0^2\, bb - 3\, ee\, cc^2\, r^4\, aa0^2\, cc0\, bb - 3\, ee\, cc\, r^2\, cc0^2\, aa0^2\, bb - 2\, r^4\, aa\, aa0\, cc0\, bb - 3\, ee\, cc^2\, r^8\, cc0\, aa^2\, bb$$

$$- 6\, ee\, cc^2\, r^6\, aa\, aa0\, cc0\, bb - 2\, r^6\, aa\, aa0\, cc\, bb - r^6\, aa^2\, cc0\, bb - 2\, ee\, cc^3\, r^8\, aa\, aa0\, bb - r^4\, aa0^2\, cc\, bb$$

$$- r^2\, aa0^2\, cc0\, bb - r^8\, aa^2\, cc\, bb + 9\, ee\, cc0^2\, aa^2\, r^6\, cc \cos(\theta)^2\, bb - 3\, ee\, cc^3\, r^6 \cos(\theta)^4\, aa0^2\, bb$$

$$- 5\, ee\, cc^3\, r^{10} \cos(\theta)^8\, aa^2\, bb + ee\, cc^3\, r^{10} \cos(\theta)^{10}\, aa^2\, bb + 8\, ee\, cc^3\, r^8 \cos(\theta)^6\, aa\, aa0\, bb$$

$$- 3\, ee\, cc^2\, r^8 \cos(\theta)^8\, aa^2\, cc0\, bb - 2\, ee\, cc^3\, r^8 \cos(\theta)^8\, aa\, aa0\, bb + 2\, ee\, cc0^3\, aa\, r^2 \cos(\theta)^2\, aa0\, bb$$

$$- 3\, ee\, cc^2\, r^4 \cos(\theta)^4\, cc0\, aa0^2\, bb + 12\, ee\, cc0^2\, aa\, r^4\, aa0\, cc \cos(\theta)^2\, bb + r^2 \cos(\theta)^2\, aa0^2\, cc0\, bb$$

$$- r^4 \cos(\theta)^4\, aa0^2\, cc\, bb - 2\, r^4 \cos(\theta)^4\, aa\, aa0\, cc0\, bb + 2\, r^6 \cos(\theta)^6\, aa\, aa0\, cc\, bb + 4\, r^4\, aa \cos(\theta)^2\, aa0\, cc0\, bb$$

$$- 12\, ee\, cc^3\, r^8\, aa \cos(\theta)^4\, aa0\, bb + ee\, cc^3\, r^6 \cos(\theta)^6\, aa0^2\, bb - 9\, ee\, cc\, r^6\, cc0^2\, aa^2 \cos(\theta)^4\, bb$$

$$- ee\, cc0^3\, aa^2\, r^4 \cos(\theta)^4\, bb + 2\, ee\, cc0^3\, aa^2\, r^4 \cos(\theta)^2\, bb - 10\, ee\, cc^3\, r^{10}\, aa^2 \cos(\theta)^4\, bb$$

$$+ 3\, ee\, cc0^2\, aa^2\, r^6 \cos(\theta)^6\, cc\, bb + 10\, ee\, cc^3\, r^{10} \cos(\theta)^6\, aa^2\, bb + 6\, ee\, cc^2\, r^6 \cos(\theta)^6\, aa\, aa0\, cc0\, bb$$

$$- 6\, ee\, cc0^2\, aa\, r^4 \cos(\theta)^4\, aa0\, cc\, bb - 18\, ee\, cc^2\, r^8\, cc0\, aa^2 \cos(\theta)^4\, bb + 12\, ee\, cc^2\, r^8\, cc0\, aa^2 \cos(\theta)^6\, bb$$

$$- 18\, ee\, cc^2\, r^6\, cc0\, aa \cos(\theta)^4\, aa0\, bb - r^8 \cos(\theta)^8\, aa^2\, cc\, bb + 5\, ee\, cc^3\, r^{10}\, aa^2 \cos(\theta)^2\, bb + 3\, r^6\, aa^2 \cos(\theta)^2\, cc0\, bb$$

$$+ 2\, r^4\, aa0^2\, cc \cos(\theta)^2\, bb - 6\, r^8\, aa^2 \cos(\theta)^4\, cc\, bb + 4\, r^8\, aa^2 \cos(\theta)^2\, cc\, bb - 3\, r^6\, aa^2 \cos(\theta)^4\, cc0\, bb$$

$$+ 4\, r^8\, aa^2 \cos(\theta)^6\, cc\, bb - 4\, r^5\, ee\, cc + 8\, r^4 \cos(\theta)^2\, ee\, cc\, bb + r^6 \cos(\theta)^6\, aa^2\, cc0\, bb - 4\, r^3\, ee\, bb^2\, cc$$

$$- 3\, ee\, cc\, r^6\, cc0^2\, aa^2\, bb - 6\, ee\, cc\, r^4\, cc0^2\, aa\, aa0\, bb + 8\, ee\, cc^3\, r^8\, aa\, aa0 \cos(\theta)^2\, bb + 3\, ee\, cc0^2\, aa0^2\, cc\, r^2 \cos(\theta)^2\, bb$$

$$+ 6\, ee\, cc^2\, r^4\, cc0\, aa0^2 \cos(\theta)^2\, bb - 6\, r^6 \cos(\theta)^4\, aa\, aa0\, cc\, bb + 6\, r^6\, aa\, aa0\, cc \cos(\theta)^2\, bb$$

$$+ 12\, ee\, cc^2\, r^8\, cc0\, aa^2 \cos(\theta)^2\, bb + 4\, r^3\, ee\, bb^2\, cc \cos(\theta)^2 - 8\, r^4\, ee\, cc\, bb + 3\, ee\, cc^3\, r^6\, aa0^2 \cos(\theta)^2\, bb$$

$$+ 18\, ee\, cc^2\, r^6\, cc0\, aa\, aa0 \cos(\theta)^2\, bb + 4\, r^5 \cos(\theta)^2\, ee\, cc\,) \Big/ \Bigg(r^4\, (-bb + bb \cos(\theta)^2 - r + r \cos(\theta)^2)\sqrt{\dfrac{r+bb}{r}}\,\Bigg)$$

$$T(2,4,1) = \dfrac{1}{4}\,(-4\, aa^3\, r^9 - 12 \cos(\theta)^4\, aa0^2\, ee\, cc^2\, r^7\, aa + 16 \cos(\theta)^2\, aa^3\, r^9 - 24 \cos(\theta)^4\, aa^3\, r^9 + 16 \cos(\theta)^6\, aa^3\, r^9$$

$$- 4 \cos(\theta)^8\, aa^3\, r^9 - 4\, aa\, r^5\, ee + 12\, aa\, r^5 \cos(\theta)^2\, ee - 8\, aa0\, r^7\, aa^2 + 4\, aa0\, ee^2\, cc0\, cc\, r^3 + 4\, aa0\, ee^2\, cc^2\, r^5$$

$$+ aa0\, ee^2\, cc0^2\, bb + aa\, r^2\, ee^2\, cc0^2\, bb - 4\, aa0^2\, r^5\, aa - 8\, r^5\, ee\, aa \cos(\theta)^4 - 8\, aa^3\, r^9\, ee\, cc\, cc0 - 4\, aa0^2\, ee\, cc0^2\, aa\, r^3$$

$$- 4\, aa^3\, r^{11}\, ee\, cc^2 + 6\, aa\, r^4\, ee^2\, cc\, cc0\, bb + 4\, aa\, r^5\, ee^2\, cc0\, cc + 4\, aa\, r^7\, ee^2\, cc^2 - 8\, aa0^2\, ee\, cc\, r^5\, cc0\, aa - 4\, aa\, r^4\, ee\, bb$$

$$+ 5\, aa\, r^6\, ee^2\, cc^2\, bb - 4\, aa^3\, r^7\, ee\, cc0^2 + 6\, aa0\, ee^2\, cc\, r^2\, cc0\, bb - 4\, aa0^2\, ee\, cc^2\, r^7\, aa - 8\, aa0\, ee\, cc0^2\, aa^2\, r^5$$

$$- 8\, aa0\, ee\, cc^2\, r^9\, aa^2 + 5\, aa0\, ee^2\, cc^2\, r^4\, bb - 16\, aa0\, ee\, cc\, r^7\, cc0\, aa^2 - 8 \cos(\theta)^2\, aa0\, ee^2\, cc^2\, r^5$$

$$+ 5 \cos(\theta)^4\, aa0\, ee^2\, cc^2\, r^4\, bb + 16 \cos(\theta)^2\, aa0^2\, ee\, cc\, r^5\, cc0\, aa + 4 \cos(\theta)^2\, aa0^2\, ee\, cc0^2\, aa\, r^3$$

$+ 16 \cos(\theta)^6\, aa0\, ee\, cc\, r^7\, cc0\, aa^2 + 12 \cos(\theta)^4\, aa\, r^7\, ee^2\, cc^2 + 12 \cos(\theta)^2\, aa0^2\, ee\, cc^2\, r^7\, aa + 4 \cos(\theta)^6\, aa^3\, r^7\, ee\, cc0^2$

$- 5 \cos(\theta)^6\, aa\, r^6\, ee^2\, cc^2\, bb - 8 \cos(\theta)^4\, aa\, r^4\, ee\, bb - 4 \cos(\theta)^6\, aa\, r^7\, ee^2\, cc^2 - 8 \cos(\theta)^4\, aa^2\, r^5\, ee\, cc0^2\, aa0$

$- 48 \cos(\theta)^4\, aa^2\, r^9\, ee\, cc^2\, aa0 - 8 \cos(\theta)^4\, aa0^2\, ee\, cc\, r^5\, cc0\, aa - 8 \cos(\theta)^8\, aa0\, ee\, cc^2\, r^9\, aa^2$

$- 8 \cos(\theta)^2\, aa\, r^5\, ee^2\, cc0\, cc - 48 \cos(\theta)^4\, aa^3\, r^9\, ee\, cc\, cc0 - 12 \cos(\theta)^4\, aa^3\, r^7\, ee\, cc0^2 + 6 \cos(\theta)^4\, aa\, r^4\, ee^2\, cc\, cc0\, bb$

$- 40 \cos(\theta)^4\, aa^3\, r^{11}\, ee\, cc^2 + 4 \cos(\theta)^4\, aa\, r^5\, ee^2\, cc0\, cc + 32 \cos(\theta)^6\, aa^3\, r^9\, ee\, cc\, cc0 - 8 \cos(\theta)^8\, aa^3\, r^9\, ee\, cc\, cc0$

$+ 40 \cos(\theta)^6\, aa^3\, r^{11}\, ee\, cc^2 - 24 \cos(\theta)^4\, aa^2\, r^7\, aa0 + 24 \cos(\theta)^2\, aa0\, r^7\, aa^2 + 32 \cos(\theta)^2\, aa^3\, r^9\, ee\, cc\, cc0$

$+ 4 \cos(\theta)^6\, aa0^2\, ee\, cc^2\, r^7\, aa + 12 \cos(\theta)^2\, aa^3\, r^7\, ee\, cc0^2 - 12 \cos(\theta)^2\, aa\, r^7\, ee^2\, cc^2 + 20 \cos(\theta)^2\, aa^3\, r^{11}\, ee\, cc^2$

$- 12 \cos(\theta)^2\, aa\, r^4\, ee^2\, cc\, cc0\, bb - 20 \cos(\theta)^8\, aa^3\, r^{11}\, ee\, cc^2 + 32 \cos(\theta)^6\, aa^2\, r^9\, ee\, cc^2\, aa0$

$+ 32 \cos(\theta)^2\, aa^2\, r^9\, ee\, cc^2\, aa0 + 12 \cos(\theta)^2\, aa\, r^4\, ee\, bb - 6 \cos(\theta)^2\, aa0\, ee^2\, cc\, r^2\, cc0\, bb + 15 \cos(\theta)^4\, aa\, r^6\, ee^2\, cc^2\, bb$

$- 15 \cos(\theta)^2\, aa\, r^6\, ee^2\, cc^2\, bb - \cos(\theta)^2\, aa\, r^2\, ee^2\, cc0^2\, bb + 16 \cos(\theta)^2\, aa^2\, r^5\, ee\, cc0^2\, aa0$

$+ 48 \cos(\theta)^2\, aa^2\, r^7\, ee\, cc\, cc0\, aa0 - 10 \cos(\theta)^2\, aa0\, ee^2\, cc^2\, r^4\, bb + 8 \cos(\theta)^2\, aa0^2\, r^5\, aa + 8 \cos(\theta)^6\, aa0\, r^7\, aa^2$

$- 4 \cos(\theta)^4\, aa0^2\, r^5\, aa + 4 \cos(\theta)^4\, aa0\, ee^2\, cc^2\, r^5 - 4 \cos(\theta)^2\, aa0\, ee^2\, cc0\, r^3\, cc + 4 \cos(\theta)^{10}\, aa^3\, r^{11}\, ee\, cc^2$

$- 48 \cos(\theta)^4\, aa^2\, r^7\, ee\, cc\, cc0\, aa0)\, \sin(\theta) \Big/ (r^4\, ee\, (-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2))$

$T(2,4,2) = \cos(\theta)\, (-aa^3\, r^7\cos(\theta)^6 - aa\, r^3\cos(\theta)^2\, aa0^2 + 3\, aa^3\, r^7\cos(\theta)^4 + 2\, aa^2\, r^5\cos(\theta)^4\, aa0 - 3\, aa^3\, r^7\cos(\theta)^2$

$- 4\, aa^2\, r^5\cos(\theta)^2\, aa0 - 2\, aa\, r^3\cos(\theta)^2\, ee + 2\, aa\, r^2\, ee\, bb - 8\, aa^2\, r^5\, ee\, cc\, cc0\cos(\theta)^2\, aa0 + \cos(\theta)^2\, ee^2\, cc^2\, r^3\, aa0$

$+ \cos(\theta)^2\, ee^2\, cc\, bb\, aa\, r^2\, cc0 - \cos(\theta)^4\, ee^2\, cc^2\, aa\, r^5 + 2\cos(\theta)^2\, ee^2\, cc^2\, aa\, r^5 + 2\, aa\, r^3\, ee + 6\, aa^3\, r^9\, ee\, cc^2\cos(\theta)^4$

$- 2\, aa^2\, r^7\, ee\, cc^2\cos(\theta)^6\, aa0 + aa^3\, r^5\, ee\, cc0^2\cos(\theta)^4 - 4\, aa^3\, r^9\, ee\, cc^2\cos(\theta)^2 - 2\, aa^2\, r^3\, ee\, cc0^2\cos(\theta)^2\, aa0$

$+ \cos(\theta)^2\, ee^2\, cc^2\, bb\, aa0\, r^2 - \cos(\theta)^4\, ee^2\, cc^2\, bb\, aa\, r^4 - 2\, aa\, r^3\, ee\, cc\cos(\theta)^2\, cc0\, aa0^2 + 2\cos(\theta)^2\, ee^2\, cc^2\, bb\, aa\, r^4$

$+ 4\, aa^2\, r^5\, ee\, cc\cos(\theta)^4\, cc0\, aa0 + \cos(\theta)^2\, ee^2\, cc\, aa\, r^3\, cc0 - 4\, aa^3\, r^9\, ee\, cc^2\cos(\theta)^6 + aa^3\, r^7$

$+ 6\, aa^2\, r^7\, ee\, cc^2\cos(\theta)^4\, aa0 - 2\, aa^3\, r^5\, ee\, cc0^2\cos(\theta)^2 - 2\, aa\, r^2\cos(\theta)^2\, ee\, bb + 6\, aa^3\, r^7\, ee\, cc\, cc0\cos(\theta)^4$

$+ aa^3\, r^9\, ee\, cc^2\cos(\theta)^8 - 6\, aa^3\, r^7\, ee\, cc\, cc0\cos(\theta)^2 - 2\, aa\, r^5\, ee\, cc^2\cos(\theta)^2\, aa0^2 - 2\, aa^3\, r^7\, ee\, cc\cos(\theta)^6\, cc0$

$+ aa\, r^5\, ee\, cc^2\cos(\theta)^4\, aa0^2 - 6\, aa^2\, r^7\, ee\, cc^2\cos(\theta)^2\, aa0 + 4\, aa^2\, r^5\, ee\, cc\, cc0\, aa0 - aa0\, ee^2\, cc^2\, bb\, r^2 - aa\, r^3\, ee^2\, cc\, cc0$

$+ 2\, aa0^2\, ee\, cc\, r^3\, cc0\, aa + 2\, aa0\, ee\, cc^2\, r^7\, aa^2 + 2\, aa0\, ee\, cc0^2\, aa^2\, r^3 - aa\, r^5\, ee^2\, cc^2 - aa0\, ee^2\, cc\, r\, cc0$

$- aa0\, ee^2\, cc\, cc0\, bb + 2\, aa0\, aa^2\, r^5 + aa^3\, r^9\, ee\, cc^2 - aa\, r^2\, ee^2\, cc\, cc0\, bb - aa\, r^4\, ee^2\, cc^2\, bb + 2\, aa^3\, r^7\, ee\, cc\, cc0$

$+ aa0^2\, ee\, cc^2\, r^5\, aa + aa^3\, r^5\, ee\, cc0^2 - aa0\, ee^2\, cc^2\, r^3 + aa0^2\, ee\, cc0^2\, aa\, r + aa0^2\, aa\, r^3) / ((r + bb)\, ee\, r)$

$T(2,4,3) = -\dfrac{1}{4}\cos(\theta)\sin(\theta)\,(-2\, ee\, cc0^3\, aa\, r^2\, aa0\, bb - ee\, cc0^3\, aa0^2\, bb - ee\, cc^3\, r^{10}\, aa^2\, bb - ee\, cc0^3\, aa^2\, r^4\, bb$

$- ee\, cc^3\, r^6\, aa0^2\, bb - 3\, ee\, cc^2\, r^4\, aa0^2\, cc0\, bb - 3\, ee\, cc\, r^2\, cc0^2\, aa0^2\, bb - 2\, r^4\, aa\, aa0\, cc0\, bb - 3\, ee\, cc^2\, r^8\, cc0\, aa^2\, bb$

$- 6\, ee\, cc^2\, r^6\, aa\, aa0\, cc0\, bb - 2\, r^6\, aa\, aa0\, cc\, bb - r^6\, aa^2\, cc0\, bb - 2\, ee\, cc^3\, r^8\, aa\, aa0\, bb - r^4\, aa0^2\, cc\, bb$

$- r^2\, aa0^2\, cc0\, bb - r^8\, aa^2\, cc\, bb + 9\, ee\, cc0^2\, aa^2\, r^6\, cc\cos(\theta)^2\, bb - 3\, ee\, cc^3\, r^6\cos(\theta)^4\, aa0^2\, bb$

$- 5\, ee\, cc^3\, r^{10}\cos(\theta)^8\, aa^2\, bb + ee\, cc^3\, r^{10}\cos(\theta)^{10}\, aa^2\, bb + 8\, ee\, cc^3\, r^8\cos(\theta)^6\, aa\, aa0\, bb$

$- 3\, ee\, cc^2\, r^8\cos(\theta)^8\, aa^2\, cc0\, bb - 2\, ee\, cc^3\, r^8\cos(\theta)^8\, aa\, aa0\, bb + 2\, ee\, cc0^3\, aa\, r^2\cos(\theta)^2\, aa0\, bb$

$- 3\, ee\, cc^2\, r^4\cos(\theta)^4\, cc0\, aa0^2\, bb + 12\, ee\, cc0^2\, aa\, r^4\, aa0\, cc\cos(\theta)^2\, bb + r^2\cos(\theta)^2\, aa0^2\, cc0\, bb$

$- r^4\cos(\theta)^4\, aa0^2\, cc\, bb - 2\, r^4\cos(\theta)^4\, aa\, aa0\, cc0\, bb + 2\, r^6\cos(\theta)^6\, aa\, aa0\, cc\, bb + 4\, r^4\, aa\cos(\theta)^2\, aa0\, cc0\, bb$

$- 12\, ee\, cc^3\, r^8\, aa\cos(\theta)^4\, aa0\, bb + ee\, cc^3\, r^6\cos(\theta)^6\, aa0^2\, bb - 9\, ee\, cc\, r^6\, cc0^2\, aa^2\cos(\theta)^4\, bb$

$- ee\, cc0^3\, aa^2\, r^4\cos(\theta)^4\, bb + 2\, ee\, cc0^3\, aa^2\, r^4\cos(\theta)^2\, bb - 10\, ee\, cc^3\, r^{10}\, aa^2\cos(\theta)^4\, bb$

$+ 3\, ee\, cc0^2\, aa^2\, r^6\cos(\theta)^6\, cc\, bb + 10\, ee\, cc^3\, r^{10}\cos(\theta)^6\, aa^2\, bb + 6\, ee\, cc^2\, r^6\cos(\theta)^6\, aa\, aa0\, cc0\, bb$

$- 6\, ee\, cc0^2\, aa\, r^4\cos(\theta)^4\, aa0\, cc\, bb - 18\, ee\, cc^2\, r^8\, cc0\, aa^2\cos(\theta)^4\, bb + 12\, ee\, cc^2\, r^8\, cc0\, aa^2\cos(\theta)^6\, bb$

$- 18\, ee\, cc^2\, r^6\, cc0\, aa\cos(\theta)^4\, aa0\, bb - r^8\cos(\theta)^8\, aa^2\, cc\, bb + 5\, ee\, cc^3\, r^{10}\, aa^2\cos(\theta)^2\, bb + 3\, r^6\, aa^2\cos(\theta)^2\, cc0\, bb$

$+ 2\, r^4\, aa0^2\, cc\cos(\theta)^2\, bb - 6\, r^8\, aa^2\cos(\theta)^4\, cc\, bb + 4\, r^8\, aa^2\cos(\theta)^2\, cc\, bb - 3\, r^6\, aa^2\cos(\theta)^4\, cc0\, bb$

$+ 4\, r^8\, aa^2\cos(\theta)^6\, cc\, bb - 4\, r^5\, ee\, cc + 8\, r^4\cos(\theta)^2\, ee\, cc\, bb + r^6\cos(\theta)^6\, aa^2\, cc0\, bb - 4\, r^3\, ee\, bb^2\, cc$

$- 3\, ee\, cc\, r^6\, cc0^2\, aa^2\, bb - 6\, ee\, cc\, r^4\, cc0^2\, aa\, aa0\, bb + 8\, ee\, cc^3\, r^8\, aa\, aa0\cos(\theta)^2\, bb + 3\, ee\, cc0^2\, aa0^2\, cc\, r^2\cos(\theta)^2\, bb$

$$+ 6\,ee\,cc^2\,r^4\,cc0\,aa0^2\,\cos(\theta)^2\,bb - 6\,r^6\,\cos(\theta)^4\,aa\,aa0\,cc\,bb + 6\,r^6\,aa\,aa0\,cc\,\cos(\theta)^2\,bb$$

$$+ 12\,ee\,cc^2\,r^8\,cc0\,aa^2\,\cos(\theta)^2\,bb + 4\,r^3\,ee\,bb^2\,cc\,\cos(\theta)^2 - 8\,r^4\,ee\,cc\,bb + 3\,ee\,cc^3\,r^6\,aa0^2\,\cos(\theta)^2\,bb$$

$$+ 18\,ee\,cc^2\,r^6\,cc0\,aa\,aa0\,\cos(\theta)^2\,bb + 4\,r^5\,\cos(\theta)^2\,ee\,cc\Big) \bigg/ \left(r^4\,(-bb + bb\,\cos(\theta)^2 - r + r\,\cos(\theta)^2)\sqrt{\dfrac{r+bb}{r}}\, \right)$$

$T(2,4,4) = -\dfrac{1}{2}\cos(\theta)\sin(\theta)\,(6\,r^4\,bb\,aa\,aa0 + 4\,r^7\,aa^2 + 4\,r^5\,aa\,aa0 - 8\,r^4\,bb\,aa\,\cos(\theta)^2\,aa0 + 7\,r^6\,bb\,aa^2\,\cos(\theta)^4$

$$- r^2\,\cos(\theta)^2\,aa0^2\,bb - 2\,ee\,cc0^2\,aa^2\,r^4\,\cos(\theta)^2\,bb - 2\,ee\,cc^2\,r^6\,\cos(\theta)^6\,aa\,aa0\,bb - 2\,ee\,cc\,r^2\,\cos(\theta)^2\,cc0\,aa0^2\,bb$$

$$+ ee\,cc^2\,r^4\,\cos(\theta)^4\,aa0^2\,bb + 6\,ee\,cc^2\,r^6\,\cos(\theta)^4\,aa\,aa0\,bb - 4\,ee\,cc^2\,r^8\,aa^2\,\cos(\theta)^2\,bb - r^6\,\cos(\theta)^6\,aa^2\,bb$$

$$+ 4\,ee\,cc\,r^4\,\cos(\theta)^4\,cc0\,aa\,aa0\,bb - 2\,ee\,cc^2\,r^4\,\cos(\theta)^2\,aa0^2\,bb - 4\,ee\,cc^2\,r^8\,\cos(\theta)^6\,aa^2\,bb + 2\,r^4\,\cos(\theta)^4\,aa\,aa0\,bb$$

$$- 6\,ee\,cc^2\,r^6\,aa\,\cos(\theta)^2\,aa0\,bb - 2\,ee\,cc0^2\,aa\,r^2\,\cos(\theta)^2\,aa0\,bb - 6\,ee\,cc\,r^6\,cc0\,aa^2\,\cos(\theta)^2\,bb$$

$$- 11\,r^6\,bb\,aa^2\,\cos(\theta)^2 - 4\,r^5\,\cos(\theta)^2\,aa\,aa0 + ee\,cc^2\,r^8\,\cos(\theta)^8\,aa^2\,bb + ee\,cc0^2\,aa^2\,r^4\,\cos(\theta)^4\,bb + 4\,r^7\,\cos(\theta)^4\,aa^2$$

$$- 2\,ee\,cc\,r^6\,\cos(\theta)^6\,cc0\,aa^2\,bb - 8\,ee\,cc\,r^4\,cc0\,aa\,\cos(\theta)^2\,aa0\,bb + 6\,ee\,cc^2\,r^8\,aa^2\,\cos(\theta)^4\,bb$$

$$+ 6\,ee\,cc\,r^6\,cc0\,aa^2\,\cos(\theta)^4\,bb + 5\,r^6\,bb\,aa^2 - 8\,r^7\,\cos(\theta)^2\,aa^2 + ee\,aa0^2\,cc0^2\,bb + aa0^2\,r^2\,bb + ee\,aa^2\,r^8\,cc^2\,bb$$

$$+ ee\,aa0^2\,cc^2\,r^4\,bb + ee\,aa^2\,r^4\,cc0^2\,bb + 2\,ee\,aa0\,aa\,r^2\,cc0^2\,bb + 2\,ee\,aa0^2\,cc\,r^2\,cc0\,bb + 2\,ee\,aa\,r^6\,aa0\,cc^2\,bb$$

$$+ 4\,ee\,aa\,r^4\,aa0\,cc\,cc0\,bb + 2\,ee\,aa^2\,r^6\,cc\,cc0\,bb\Big) \bigg/ \big(r^4\,(-bb + bb\,\cos(\theta)^2 - r + r\,\cos(\theta)^2) \big)$$

$$T(3,1,1) = -2\,\dfrac{(-cc\,r^2 + cc\,r^2\,\cos(\theta)^2 - cc0)\sqrt{\dfrac{r+bb}{r}}\,\cos(\theta)\,aa}{r+bb}$$

$$T(3,1,2) = \dfrac{\sqrt{\dfrac{r+bb}{r}}\,\sin(\theta)\,aa\,r\,(cc\,r^2 - 3\,cc\,r^2\,\cos(\theta)^2 + 2\,r^2\,cc\,\cos(\theta)^4 + cc0 - 2\,cc0\,\cos(\theta)^2)}{-bb + bb\,\cos(\theta)^2 - r + r\,\cos(\theta)^2}$$

$T(3,1,3) = \dfrac{1}{4}\,ee\,(4\,r^5\,cc^2\,\cos(\theta)^4 - 8\,r^5\,cc^2\,\cos(\theta)^2 + 5\,r^4\,bb\,cc^2\,\cos(\theta)^4 - 10\,r^4\,bb\,cc^2\,\cos(\theta)^2 - 4\,r^3\,cc\,\cos(\theta)^2\,cc0$

$$- 6\,r^2\,bb\,cc\,\cos(\theta)^2\,cc0 + 4\,r^5\,cc^2 + 6\,r^2\,bb\,cc\,cc0 + 4\,r^3\,cc\,cc0 + 5\,r^4\,bb\,cc^2 + cc0^2\,bb\Big) \bigg/ \big(r^3$$
$$(-bb + bb\,\cos(\theta)^2 - r + r\,\cos(\theta)^2) \big)$$

$T(3,1,4) = -\dfrac{1}{4}\,(-5\,bb\,ee\,cc\,r^2\,\cos(\theta)^2 + bb\,ee\,cc0 + 5\,ee\,cc\,r^2\,bb - 4\,aa^2\,r^5\,\cos(\theta)^4\,cc0 - 4\,ee\,r^3\,cc\,\cos(\theta)^2$

$$+ 4\,aa\,r^3\,\cos(\theta)^2\,aa0\,cc0 - 4\,aa\,r^5\,\cos(\theta)^4\,aa0\,cc + 4\,aa^2\,r^7\,\cos(\theta)^6\,cc - 12\,aa^2\,r^7\,\cos(\theta)^4\,cc + 8\,aa^2\,r^5\,\cos(\theta)^2\,cc0$$

$$- 4\,aa\,r^3\,aa0\,cc0 - 4\,aa\,r^5\,aa0\,cc - 4\,aa^2\,r^5\,cc0 + 4\,ee\,r^3\,cc - 4\,aa^2\,r^7\,cc + 8\,aa\,r^5\,aa0\,cc\,\cos(\theta)^2$$

$$+ 12\,aa^2\,r^7\,cc\,\cos(\theta)^2\Big) \bigg/ \left(\sqrt{\dfrac{r+bb}{r}}\,(-1 + \cos(\theta)^2)\,r^4 \right)$$

$$T(3,2,1) = \dfrac{\sqrt{\dfrac{r+bb}{r}}\,\sin(\theta)\,aa\,r\,(cc\,r^2 - 3\,cc\,r^2\,\cos(\theta)^2 + 2\,r^2\,cc\,\cos(\theta)^4 + cc0 - 2\,cc0\,\cos(\theta)^2)}{-bb + bb\,\cos(\theta)^2 - r + r\,\cos(\theta)^2}$$

$$T(3,2,2) = 2\,\dfrac{aa\,r^2\,(-cc\,r^2 + cc\,r^2\,\cos(\theta)^2 - cc0)\sqrt{\dfrac{r+bb}{r}}\,\cos(\theta)}{r+bb}$$

$$T(3,2,3) = \dfrac{(-cc\,r^2 + cc\,r^2\,\cos(\theta)^2 - cc0)\,cc\,ee\,\cos(\theta)}{\sin(\theta)}$$

$T(3,2,4) = \sqrt{\dfrac{r+bb}{r}}\,\cos(\theta)\,\sin(\theta)\,(-2\,aa^2\,r^5\,cc\,\cos(\theta)^2 - ee\,bb\,cc - aa^2\,r^3\,cc0\,\cos(\theta)^2 + aa^2\,r^5\,cc\,\cos(\theta)^4$

$$- ee\,cc\,r + cc\,r^5\,aa^2 + aa\,r\,aa0\,cc0 + aa\,r^3\,aa0\,cc - aa\,r^3\,cc\,\cos(\theta)^2\,aa0 + aa^2\,r^3\,cc0\Big) \bigg/ \big($$

$$-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)$$

$$T(3,3,1) = \frac{1}{4}ee\,(4\,r^5\,cc^2\cos(\theta)^4 - 8\,r^5\,cc^2\cos(\theta)^2 + 5\,r^4\,bb\,cc^2\cos(\theta)^4 - 10\,r^4\,bb\,cc^2\cos(\theta)^2 - 4\,r^3\,cc\cos(\theta)^2\,cc0$$

$$- 6\,r^2\,bb\,cc\cos(\theta)^2\,cc0 + 4\,r^5\,cc^2 + 6\,r^2\,bb\,cc\,cc0 + 4\,r^3\,cc\,cc0 + 5\,r^4\,bb\,cc^2 + cc0^2\,bb)\Big/ (r^3$$
$$(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2))$$

$$T(3,3,2) = \frac{(-cc\,r^2 + cc\,r^2\cos(\theta)^2 - cc0)\,cc\,ee\cos(\theta)}{\sin(\theta)}$$

$$T(3,3,4) = \frac{1}{4}\cos(\theta)\,ee\,bb\,(\cos(\theta)^6\,aa\,r^6\,cc^2 - 2\cos(\theta)^4\,aa\,r^4\,cc0\,cc - \cos(\theta)^4\,aa0\,cc^2\,r^4 + 2\cos(\theta)^2\,aa0\,cc^2\,r^4$$

$$+ \cos(\theta)^2\,aa\,r^2\,cc0^2 + 4\cos(\theta)^2\,aa\,r^4\,cc0\,cc + 2\cos(\theta)^2\,aa0\,cc0\,cc\,r^2 - 3\cos(\theta)^4\,aa\,r^6\,cc^2 + 3\cos(\theta)^2\,aa\,r^6\,cc^2$$
$$- aa0\,cc^2\,r^4 - 2\,aa0\,cc0\,cc\,r^2 - aa0\,cc0^2 - 2\,aa\,r^4\,cc\,cc0 - aa\,r^6\,cc^2 - aa\,r^2\,cc0^2)\Big/ (r^3$$
$$(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2))$$

$$T(3,4,1) = -\frac{1}{4}(-5\,bb\,ee\,cc\,r^2\cos(\theta)^2 + bb\,ee\,cc0 + 5\,ee\,cc\,r^2\,bb - 4\,aa^2\,r^5\cos(\theta)^4\,cc0 - 4\,ee\,r^3\,cc\cos(\theta)^2$$

$$+ 4\,aa\,r^3\cos(\theta)^2\,aa0\,cc0 - 4\,aa\,r^5\cos(\theta)^4\,aa0\,cc + 4\,aa^2\,r^7\cos(\theta)^6\,cc - 12\,aa^2\,r^7\cos(\theta)^4\,cc + 8\,aa^2\,r^5\cos(\theta)^2\,cc0$$
$$- 4\,aa\,r^3\,aa0\,cc0 - 4\,aa\,r^5\,aa0\,cc - 4\,aa^2\,r^5\,cc0 + 4\,ee\,r^3\,cc - 4\,aa^2\,r^7\,cc + 8\,aa\,r^5\,aa0\,cc\cos(\theta)^2$$
$$+ 12\,aa^2\,r^7\,cc\cos(\theta)^2)\Big/\left(\sqrt{\frac{r+bb}{r}}\,(-1 + \cos(\theta)^2)\,r^4\right)$$

$$T(3,4,2) = \sqrt{\frac{r+bb}{r}}\cos(\theta)\sin(\theta)\,(-2\,aa^2\,r^5\,cc\cos(\theta)^2 - ee\,bb\,cc - aa^2\,r^3\,cc0\cos(\theta)^2 + aa^2\,r^5\,cc\cos(\theta)^4$$

$$- ee\,cc\,r + cc\,r^5\,aa^2 + aa\,r\,aa0\,cc0 + aa\,r^3\,aa0\,cc - aa\,r^3\,cc\cos(\theta)^2\,aa0 + aa^2\,r^3\,cc0)\Big/ ($$
$$-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)$$

$$T(3,4,3) = \frac{1}{4}\cos(\theta)\,ee\,bb\,(\cos(\theta)^6\,aa\,r^6\,cc^2 - 2\cos(\theta)^4\,aa\,r^4\,cc0\,cc - \cos(\theta)^4\,aa0\,cc^2\,r^4 + 2\cos(\theta)^2\,aa0\,cc^2\,r^4$$

$$+ \cos(\theta)^2\,aa\,r^2\,cc0^2 + 4\cos(\theta)^2\,aa\,r^4\,cc0\,cc + 2\cos(\theta)^2\,aa0\,cc0\,cc\,r^2 - 3\cos(\theta)^4\,aa\,r^6\,cc^2 + 3\cos(\theta)^2\,aa\,r^6\,cc^2$$
$$- aa0\,cc^2\,r^4 - 2\,aa0\,cc0\,cc\,r^2 - aa0\,cc0^2 - 2\,aa\,r^4\,cc\,cc0 - aa\,r^6\,cc^2 - aa\,r^2\,cc0^2)\Big/ (r^3$$
$$(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2))$$

$$T(3,4,4) = \frac{1}{2}\cos(\theta)\sqrt{\frac{r+bb}{r}}\,ee\,bb\,(aa0\,cc0 + r^2\,aa0\,cc + r^2\,aa\,cc0 + r^4\,aa\,cc - r^2\cos(\theta)^2\,aa\,cc0$$

$$+ r^4\cos(\theta)^4\,aa\,cc - 2\,r^4\cos(\theta)^2\,aa\,cc - r^2\cos(\theta)^2\,aa0\,cc)\Big/ (r^3\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2))$$

$$T(4,1,1) = 2\,aa\cos(\theta)$$
$$(r^2 - r^2\cos(\theta)^2 + ee\,cc^2\,r^4 - 2\,ee\,cc^2\,r^4\cos(\theta)^2 + 2\,ee\,cc\,r^2\,cc0 + ee\,cc^2\,r^4\cos(\theta)^4 - 2\,ee\,cc\,r^2\cos(\theta)^2\,cc0 + ee\,cc0^2)$$
$$/\,((r+bb)\,ee)$$

$$T(4,1,2) = -\sin(\theta)\,aa\,r\,(-r^2 + 3\,r^2\cos(\theta)^2 - 2\,r^2\cos(\theta)^4 - ee\,cc^2\,r^4 + 4\,ee\,cc^2\,r^4\cos(\theta)^2 - 5\,ee\,cc^2\,r^4\cos(\theta)^4$$
$$- 2\,ee\,cc\,r^2\,cc0 + 6\,ee\,cc\,r^2\cos(\theta)^2\,cc0 + 2\,r^4\,ee\,cc^2\cos(\theta)^6 - 4\,r^2\,ee\,cc\cos(\theta)^4\,cc0 - ee\,cc0^2 + 2\,ee\,cc0^2\cos(\theta)^2)$$
$$\Big/ (ee\,(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2))$$

$$T(4,1,3) = -\frac{1}{4}(16\,ee\,cc^2\,r^5\cos(\theta)^2\,cc0 + 4\,ee\,cc\,r^3\cos(\theta)^2\,cc0^2 - 8\,ee\,cc^2\,r^5\cos(\theta)^4\,cc0 + 12\,ee\,cc^3\,r^7\cos(\theta)^2$$

$$- 12\,ee\,cc^3\,r^7\cos(\theta)^4 + 3\,r^4\cos(\theta)^4\,cc\,bb - 6\,r^4\,cc\cos(\theta)^2\,bb - 15\,ee\,cc^3\,r^6\cos(\theta)^4\,bb + 15\,ee\,cc^3\,r^6\cos(\theta)^2\,bb$$
$$+ 22\,ee\,cc^2\,r^4\cos(\theta)^2\,cc0\,bb + 5\,ee\,cc^3\,r^6\cos(\theta)^6\,bb + 7\,ee\,cc\,r^2\cos(\theta)^2\,cc0^2\,bb - 11\,ee\,cc^2\,r^4\cos(\theta)^4\,cc0\,bb$$
$$- 4\,ee\,cc^3\,r^7 - 8\,ee\,cc^2\,r^5\,cc0 - ee\,cc0^3\,bb - r^2\,cc0\,bb - 4\,ee\,cc\,r^3\,cc0^2 + 3\,r^4\,cc\,bb + 4\,r^5\cos(\theta)^4\,cc$$
$$- 7\,ee\,cc\,r^2\,cc0^2\,bb - 11\,ee\,cc^2\,r^4\,cc0\,bb + 4\,ee\,cc^3\,r^7\cos(\theta)^6 - 8\,r^5\,cc\cos(\theta)^2 - 5\,ee\,cc^3\,r^6\,bb + r^2\cos(\theta)^2\,cc0\,bb$$

$$+\ 4\ r^5\ cc\ )\ \Big/\ \Big(r^3\ (-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2)\ \sqrt{\dfrac{r+bb}{r}}\ \Big)$$

$T(4, 1, 4) = \dfrac{1}{4}\,(4\ r^7\ aa^2 + 4\ r^5\ aa\ aa0 - 2\ r^2\ ee\ bb + 8\ ee^2\ cc^2\ r^5\ \cos(\theta)^2 + 4\ r^5\ \cos(\theta)^4\ aa\ aa0 - 8\ r^5\ \cos(\theta)^2\ aa\ aa0$

$\quad +\ 2\ r^2\ \cos(\theta)^2\ ee\ bb - 4\ r^7\ \cos(\theta)^6\ aa^2 + 12\ r^7\ \cos(\theta)^4\ aa^2 - 12\ r^7\ \cos(\theta)^2\ aa^2 + 4\ ee\ cc0^2\ aa\ r^3\ aa0$

$\quad +\ 8\ ee\ cc\ r^7\ cc0\ aa^2 + 8\ ee\ cc\ r^5\ cc0\ aa\ aa0 + 4\ ee\ cc^2\ r^7\ aa\ aa0 + 4\ ee\ cc^2\ r^9\ aa^2 + 4\ ee\ cc0^2\ aa^2\ r^5 - 6\ ee^2\ cc\ r^2\ cc0\ bb$

$\quad -\ 5\ ee^2\ cc^2\ r^4\ bb - ee^2\ cc0^2\ bb - 4\ ee^2\ cc^2\ r^5\ \cos(\theta)^4 + 4\ ee^2\ cc\ r^3\ \cos(\theta)^2\ cc0 - 4\ ee\ cc0^2\ aa\ r^3\ \cos(\theta)^2\ aa0$

$\quad +\ 4\ ee\ cc0^2\ aa^2\ r^5\ \cos(\theta)^4 - 8\ ee\ cc\ r^7\ \cos(\theta)^6\ cc0\ aa^2 + 8\ ee\ cc\ r^5\ \cos(\theta)^4\ cc0\ aa\ aa0 - 8\ ee\ cc0^2\ aa^2\ r^5\ \cos(\theta)^2$

$\quad +\ 4\ ee\ cc^2\ r^9\ \cos(\theta)^8\ aa^2 - 4\ ee\ cc^2\ r^7\ \cos(\theta)^6\ aa\ aa0 - 16\ ee\ cc^2\ r^9\ aa^2\ \cos(\theta)^2 - 24\ ee\ cc\ r^7\ cc0\ aa^2\ \cos(\theta)^2$

$\quad +\ 12\ ee\ cc^2\ r^7\ \cos(\theta)^4\ aa\ aa0 + 24\ ee\ cc\ r^7\ cc0\ aa^2\ \cos(\theta)^4 - 16\ ee\ cc\ r^5\ cc0\ aa\ \cos(\theta)^2\ aa0$

$\quad +\ 24\ ee\ cc^2\ r^9\ aa^2\ \cos(\theta)^4 - 12\ ee\ cc^2\ r^7\ aa\ \cos(\theta)^2\ aa0 + 10\ ee^2\ cc^2\ r^4\ \cos(\theta)^2\ bb - 5\ ee^2\ cc^2\ r^4\ \cos(\theta)^4\ bb$

$\quad +\ 6\ ee^2\ cc\ r^2\ \cos(\theta)^2\ cc0\ bb - 16\ ee\ cc^2\ r^9\ \cos(\theta)^6\ aa^2 - 4\ ee^2\ cc\ r^3\ cc0 - 4\ ee^2\ cc^2\ r^5\ )\ \Big/\ (ee\ r^3$

$\quad (-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2))$

$T(4, 2, 1) = -\sin(\theta)\ aa\ r\ (-r^2 + 3\ r^2\ \cos(\theta)^2 - 2\ r^2\ \cos(\theta)^4 - ee\ cc^2\ r^4 + 4\ ee\ cc^2\ r^4\ \cos(\theta)^2 - 5\ ee\ cc^2\ r^4\ \cos(\theta)^4$

$\quad -\ 2\ ee\ cc\ r^2\ cc0 + 6\ ee\ cc\ r^2\ \cos(\theta)^2\ cc0 + 2\ r^4\ ee\ cc^2\ \cos(\theta)^6 - 4\ r^2\ ee\ cc\ \cos(\theta)^4\ cc0 - ee\ cc0^2 + 2\ ee\ cc0^2\ \cos(\theta)^2)$

$\quad \Big/\ (ee\ (-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2))$

$T(4, 2, 2) = -2\ aa\ r^2$

$\quad (r^2 - r^2\ \cos(\theta)^2 + ee\ cc^2\ r^4 - 2\ ee\ cc^2\ r^4\ \cos(\theta)^2 + 2\ ee\ cc\ r^2\ cc0 + ee\ cc^2\ r^4\ \cos(\theta)^4 - 2\ ee\ cc\ r^2\ \cos(\theta)^2\ cc0 + ee\ cc0^2)$

$\quad \cos(\theta) / ((r + bb)\ ee)$

$T(4, 2, 3) = -\cos(\theta)$

$\quad (ee\ cc^2\ r^4\ \cos(\theta)^4 - 2\ ee\ cc^2\ r^4\ \cos(\theta)^2 - 2\ ee\ cc\ r^2\ \cos(\theta)^2\ cc0 + r^2\ \cos(\theta)^2 - r^2 + ee\ cc^2\ r^4 + 2\ ee\ cc\ r^2\ cc0 + ee\ cc0^2)$

$\quad cc\ \Big/\ \Big(\sin(\theta)\ \sqrt{\dfrac{r+bb}{r}}\ \Big)$

$T(4, 2, 4) = -\cos(\theta)\ \sin(\theta)\ (-aa^2\ r^5 + aa\ r^3\ \cos(\theta)^2\ aa0 - ee^2\ cc^2\ bb\ r^2\ \cos(\theta)^2 - ee\ cc^2\ r^5\ aa\ aa0$

$\quad -\ 2\ ee\ cc\ r^5\ cc0\ aa^2 - 2\ ee\ cc\ r^3\ cc0\ aa\ aa0 - ee\ cc0^2\ aa\ r\ aa0 - ee\ cc^2\ r^7\ aa^2 - ee\ cc0^2\ aa^2\ r^3$

$\quad +\ 2\ ee\ cc\ r^3\ cc0\ aa\ \cos(\theta)^2\ aa0 + 4\ ee\ cc\ r^5\ cc0\ aa^2\ \cos(\theta)^2 + ee\ cc^2\ r^7\ \cos(\theta)^6\ aa^2 - 2\ ee\ cc\ r^5\ cc0\ aa^2\ \cos(\theta)^4$

$\quad -\ ee\ cc^2\ r^5\ \cos(\theta)^4\ aa\ aa0 - 3\ ee\ cc^2\ r^7\ aa^2\ \cos(\theta)^4 + ee\ cc0^2\ aa^2\ r^3\ \cos(\theta)^2 + 3\ ee\ cc^2\ r^7\ aa^2\ \cos(\theta)^2$

$\quad +\ 2\ ee\ cc^2\ r^5\ aa\ \cos(\theta)^2\ aa0 - aa^2\ r^5\ \cos(\theta)^4 + 2\ aa^2\ r^5\ \cos(\theta)^2 - aa\ r^3\ aa0 - ee^2\ cc^2\ r^3\ \cos(\theta)^2 + ee^2\ cc^2\ r^3$

$\quad +\ ee^2\ cc\ r\ cc0 + ee^2\ cc^2\ bb\ r^2 + ee^2\ cc\ cc0\ bb\ )\ \Big/\ (ee\ (-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2))$

$T(4, 3, 1) = -\dfrac{1}{4}\,(16\ ee\ cc^2\ r^5\ \cos(\theta)^2\ cc0 + 4\ ee\ cc\ r^3\ \cos(\theta)^2\ cc0^2 - 8\ ee\ cc^2\ r^5\ \cos(\theta)^4\ cc0 + 12\ ee\ cc^3\ r^7\ \cos(\theta)^2$

$\quad -\ 12\ ee\ cc^3\ r^7\ \cos(\theta)^4 - 5\ r^4\ \cos(\theta)^4\ cc\ bb + 10\ r^4\ cc\ \cos(\theta)^2\ bb - 15\ ee\ cc^3\ r^6\ \cos(\theta)^4\ bb + 15\ ee\ cc^3\ r^6\ \cos(\theta)^2\ bb$

$\quad +\ 22\ ee\ cc^2\ r^4\ \cos(\theta)^2\ cc0\ bb + 5\ ee\ cc^3\ r^6\ \cos(\theta)^6\ bb + 7\ ee\ cc\ r^2\ \cos(\theta)^2\ cc0^2\ bb - 11\ ee\ cc^2\ r^4\ \cos(\theta)^4\ cc0\ bb$

$\quad -\ 4\ ee\ cc^3\ r^7 - 8\ ee\ cc^2\ r^5\ cc0 - ee\ cc0^3\ bb - r^2\ cc0\ bb - 4\ ee\ cc\ r^3\ cc0^2 - 5\ r^4\ cc\ bb - 4\ r^5\ \cos(\theta)^4\ cc$

$\quad -\ 7\ ee\ cc\ r^2\ cc0^2\ bb - 11\ ee\ cc^2\ r^4\ cc0\ bb + 4\ ee\ cc^3\ r^7\ \cos(\theta)^6 + 8\ r^5\ cc\ \cos(\theta)^2 - 5\ ee\ cc^3\ r^6\ bb + r^2\ \cos(\theta)^2\ cc0\ bb$

$\quad -\ 4\ r^5\ cc\ )\ \Big/\ \Big(r^3\ (-bb + bb\ \cos(\theta)^2 - r + r\ \cos(\theta)^2)\ \sqrt{\dfrac{r+bb}{r}}\ \Big)$

$T(4, 3, 2) = -\cos(\theta)$

$\quad (r^2 - r^2\ \cos(\theta)^2 + ee\ cc^2\ r^4 - 2\ ee\ cc^2\ r^4\ \cos(\theta)^2 + 2\ ee\ cc\ r^2\ cc0 + ee\ cc^2\ r^4\ \cos(\theta)^4 - 2\ ee\ cc\ r^2\ \cos(\theta)^2\ cc0 + ee\ cc0^2)$

$\quad cc\ \Big/\ \Big(\sin(\theta)\ \sqrt{\dfrac{r+bb}{r}}\ \Big)$

$$T(4, 3, 4) = -\frac{1}{4}\cos(\theta)\, bb\, (-4\cos(\theta)^6\, ee\, aa\, r^8\, cc^3 + ee\, aa0\, cc0^3 + aa\, r^6\, cc + aa0\, r^4\, cc + aa\, r^4\, cc0 + aa0\, r^2\, cc0$$

$$+ ee\, aa0\, cc^3\, r^6 + 3\, ee\, aa0\, cc\, r^2\, cc0^2 + 3\, ee\, aa\, r^6\, cc^2\, cc0 + 3\, ee\, aa0\, cc^2\, r^4\, cc0 + ee\, aa\, r^2\, cc0^3 + ee\, aa\, r^8\, cc^3$$

$$+ 3\, ee\, aa\, r^4\, cc\, cc0^2 + \cos(\theta)^4\, aa0\, r^4\, cc + \cos(\theta)^4\, aa\, r^4\, cc0 + 3\cos(\theta)^4\, aa\, r^6\, cc - 3\cos(\theta)^2\, aa\, r^6\, cc$$

$$- 2\cos(\theta)^2\, aa0\, r^4\, cc - \cos(\theta)^6\, aa\, r^6\, cc - \cos(\theta)^2\, aa0\, r^2\, cc0 - 2\cos(\theta)^2\, aa\, r^4\, cc0 - 3\cos(\theta)^2\, ee\, aa0\, cc^3\, r^6$$

$$- \cos(\theta)^2\, ee\, aa\, r^2\, cc0^3 - 4\cos(\theta)^2\, ee\, aa\, r^8\, cc^3 + 6\cos(\theta)^4\, ee\, aa\, r^8\, cc^3 - 6\cos(\theta)^2\, ee\, aa0\, cc^2\, r^4\, cc0$$

$$- \cos(\theta)^6\, ee\, aa0\, cc^3\, r^6 - 3\cos(\theta)^2\, ee\, aa0\, cc\, r^2\, cc0^2 + 3\cos(\theta)^4\, ee\, aa0\, cc^3\, r^6 - 6\cos(\theta)^2\, ee\, aa\, r^4\, cc\, cc0^2$$

$$+ 3\cos(\theta)^4\, ee\, aa0\, cc^2\, r^4\, cc0 - 9\cos(\theta)^2\, ee\, aa\, r^6\, cc^2\, cc0 + \cos(\theta)^8\, ee\, aa\, r^8\, cc^3 + 9\cos(\theta)^4\, ee\, aa\, r^6\, cc^2\, cc0$$

$$+ 3\cos(\theta)^4\, ee\, aa\, r^4\, cc\, cc0^2 - 3\cos(\theta)^6\, ee\, aa\, r^6\, cc^2\, cc0) \bigg/ \left(r^3\, (-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)\sqrt{\frac{r+bb}{r}}\right)$$

$$T(4, 4, 1) = \frac{1}{4}(4\, r^7\, aa^2 + 4\, r^5\, aa\, aa0 + 8\, ee^2\, cc^2\, r^5\cos(\theta)^2 + 4\, r^5\cos(\theta)^4\, aa\, aa0 - 8\, r^5\cos(\theta)^2\, aa\, aa0$$

$$- 4\, r^7\cos(\theta)^6\, aa^2 + 12\, r^7\cos(\theta)^4\, aa^2 - 12\, r^7\cos(\theta)^2\, aa^2 + 4\, ee\, cc0^2\, aa\, r^3\, aa0 + 8\, ee\, cc\, r^7\, cc0\, aa^2$$

$$+ 8\, ee\, cc\, r^5\, cc0\, aa\, aa0 + 4\, ee\, cc^2\, r^7\, aa\, aa0 + 4\, ee\, cc^2\, r^9\, aa^2 + 4\, ee\, cc0^2\, aa^2\, r^5 - 6\, ee^2\, cc\, r^2\, cc0\, bb - 5\, ee^2\, cc^2\, r^4\, bb$$

$$- ee^2\, cc0^2\, bb - 4\, ee^2\, cc^2\, r^5\cos(\theta)^4 + 4\, ee^2\, cc\, r^3\cos(\theta)^2\, cc0 - 4\, ee\, cc0^2\, aa\, r^3\cos(\theta)^2\, aa0 + 4\, ee\, cc0^2\, aa^2\, r^5\cos(\theta)^4$$

$$- 8\, ee\, cc\, r^7\cos(\theta)^6\, cc0\, aa^2 + 8\, ee\, cc\, r^5\cos(\theta)^4\, cc0\, aa\, aa0 - 8\, ee\, cc0^2\, aa^2\, r^5\cos(\theta)^2 + 4\, ee\, cc^2\, r^9\cos(\theta)^8\, aa^2$$

$$- 4\, ee\, cc^2\, r^7\cos(\theta)^6\, aa\, aa0 - 16\, ee\, cc^2\, r^9\, aa^2\cos(\theta)^2 - 24\, ee\, cc\, r^7\, cc0\, aa^2\cos(\theta)^2 + 12\, ee\, cc^2\, r^7\cos(\theta)^4\, aa\, aa0$$

$$+ 24\, ee\, cc\, r^7\, cc0\, aa^2\cos(\theta)^4 - 16\, ee\, cc\, r^5\, cc0\, aa\cos(\theta)^2\, aa0 + 24\, ee\, cc^2\, r^9\, aa^2\cos(\theta)^4$$

$$- 12\, ee\, cc^2\, r^7\, aa\cos(\theta)^2\, aa0 + 10\, ee^2\, cc^2\, r^4\cos(\theta)^2\, bb - 5\, ee^2\, cc^2\, r^4\cos(\theta)^4\, bb + 6\, ee^2\, cc\, r^2\cos(\theta)^2\, cc0\, bb$$

$$- 16\, ee\, cc^2\, r^9\cos(\theta)^6\, aa^2 - 4\, ee^2\, cc\, r^3\, cc0 - 4\, ee^2\, cc^2\, r^5) \big/ (ee\, r^3\, (-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2))$$

$$T(4, 4, 2) = -\cos(\theta)\sin(\theta)\, (-aa^2\, r^5 + aa\, r^3\cos(\theta)^2\, aa0 - ee^2\, cc^2\, bb\, r^2\cos(\theta)^2 - ee\, cc^2\, r^5\, aa\, aa0$$

$$- 2\, ee\, cc\, r^5\, cc0\, aa^2 - 2\, ee\, cc\, r^3\, cc0\, aa\, aa0 - ee\, cc0^2\, aa\, r\, aa0 - ee\, cc^2\, r^7\, aa^2 - ee\, cc0^2\, aa^2\, r^3$$

$$+ 2\, ee\, cc\, r^3\, cc0\, aa\cos(\theta)^2\, aa0 + 4\, ee\, cc\, r^5\, cc0\, aa^2\cos(\theta)^2 + ee\, cc^2\, r^7\cos(\theta)^6\, aa^2 - 2\, ee\, cc\, r^5\, cc0\, aa^2\cos(\theta)^4$$

$$- ee\, cc^2\, r^5\cos(\theta)^4\, aa\, aa0 - 3\, ee\, cc^2\, r^7\, aa^2\cos(\theta)^4 + ee\, cc0^2\, aa^2\, r^3\cos(\theta)^2 + 3\, ee\, cc^2\, r^7\, aa^2\cos(\theta)^2$$

$$+ 2\, ee\, cc^2\, r^5\, aa\cos(\theta)^2\, aa0 - aa^2\, r^5\cos(\theta)^4 + 2\, aa^2\, r^5\cos(\theta)^2 - aa\, r^3\, aa0 - ee^2\, cc^2\, r^3\cos(\theta)^2 + ee^2\, cc^2\, r^3$$

$$+ ee^2\, cc\, r\, cc0 + ee^2\, cc^2\, bb\, r^2 + ee^2\, cc\, cc0\, bb) \big/ (ee\, (-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2))$$

$$T(4, 4, 3) = -\frac{1}{4}\cos(\theta)\, bb\, (-4\cos(\theta)^6\, ee\, aa\, r^8\, cc^3 + ee\, aa0\, cc0^3 + aa\, r^6\, cc + aa0\, r^4\, cc + aa\, r^4\, cc0 + aa0\, r^2\, cc0$$

$$+ ee\, aa0\, cc^3\, r^6 + 3\, ee\, aa0\, cc\, r^2\, cc0^2 + 3\, ee\, aa\, r^6\, cc^2\, cc0 + 3\, ee\, aa0\, cc^2\, r^4\, cc0 + ee\, aa\, r^2\, cc0^3 + ee\, aa\, r^8\, cc^3$$

$$+ 3\, ee\, aa\, r^4\, cc\, cc0^2 + \cos(\theta)^4\, aa0\, r^4\, cc + \cos(\theta)^4\, aa\, r^4\, cc0 + 3\cos(\theta)^4\, aa\, r^6\, cc - 3\cos(\theta)^2\, aa\, r^6\, cc$$

$$- 2\cos(\theta)^2\, aa0\, r^4\, cc - \cos(\theta)^6\, aa\, r^6\, cc - \cos(\theta)^2\, aa0\, r^2\, cc0 - 2\cos(\theta)^2\, aa\, r^4\, cc0 - 3\cos(\theta)^2\, ee\, aa0\, cc^3\, r^6$$

$$- \cos(\theta)^2\, ee\, aa\, r^2\, cc0^3 - 4\cos(\theta)^2\, ee\, aa\, r^8\, cc^3 + 6\cos(\theta)^4\, ee\, aa\, r^8\, cc^3 - 6\cos(\theta)^2\, ee\, aa0\, cc^2\, r^4\, cc0$$

$$- \cos(\theta)^6\, ee\, aa0\, cc^3\, r^6 - 3\cos(\theta)^2\, ee\, aa0\, cc\, r^2\, cc0^2 + 3\cos(\theta)^4\, ee\, aa0\, cc^3\, r^6 - 6\cos(\theta)^2\, ee\, aa\, r^4\, cc\, cc0^2$$

$$+ 3\cos(\theta)^4\, ee\, aa0\, cc^2\, r^4\, cc0 - 9\cos(\theta)^2\, ee\, aa\, r^6\, cc^2\, cc0 + \cos(\theta)^8\, ee\, aa\, r^8\, cc^3 + 9\cos(\theta)^4\, ee\, aa\, r^6\, cc^2\, cc0$$

$$+ 3\cos(\theta)^4\, ee\, aa\, r^4\, cc\, cc0^2 - 3\cos(\theta)^6\, ee\, aa\, r^6\, cc^2\, cc0) \bigg/ \left(r^3\, (-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2)\sqrt{\frac{r+bb}{r}}\right)$$

$$T(4, 4, 4) = -\frac{1}{2}\cos(\theta)\, bb\, (\cos(\theta)^6\, aa\, r^6\, ee\, cc^2 - 3\cos(\theta)^4\, aa\, r^6\, ee\, cc^2 - \cos(\theta)^4\, r^4\, aa0\, ee\, cc^2 - r^4\cos(\theta)^4\, aa$$

$$- 2\cos(\theta)^4\, aa\, r^4\, ee\, cc\, cc0 + 4\cos(\theta)^2\, aa\, r^4\, ee\, cc\, cc0 + \cos(\theta)^2\, aa\, r^2\, ee\, cc0^2 + 2\cos(\theta)^2\, r^2\, aa0\, ee\, cc\, cc0$$

$$+ 3\cos(\theta)^2\, aa\, r^6\, ee\, cc^2 + 2\, aa\, r^4\cos(\theta)^2 + 2\cos(\theta)^2\, r^4\, aa0\, ee\, cc^2 + r^2\cos(\theta)^2\, aa0 - ee\, cc0^2\, aa0 - ee\, cc^2\, aa0\, r^4$$

$$- aa\, r^4 - ee\, cc^2\, r^6\, aa - 2\, aa\, r^4\, ee\, cc\, cc0 - r^2\, aa0 - 2\, r^2\, ee\, cc\, cc0\, aa0 - aa\, ee\, cc0^2\, r^2) \big/ (r^3$$

$$(-bb + bb\cos(\theta)^2 - r + r\cos(\theta)^2))$$

$$\text{CartanaffineTorsion}(1, 3, 1) = -\frac{\cos(\theta)\, cc\, r\, (-1 + \cos(\theta))\, (\cos(\theta) + 1)\, (-aa\, r^2 + aa\, r^2 \cos(\theta)^2 - aa0)}{\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{CartanaffineTorsion}(1, 4, 1) = \frac{1}{4}\frac{\cos(\theta)\, (-4\, aa\, r^3 - 5\, bb\, aa\, r^2 + 4\, aa\, r^3 \cos(\theta)^2 + 5\, bb\, aa\, r^2 \cos(\theta)^2 - bb\, aa0)}{r\, (r + bb)}$$

$$\text{CartanaffineTorsion}(1, 3, 2) = -\frac{\cos(\theta)^2\, cc\, r^2\, (-1 + \cos(\theta))\, (\cos(\theta) + 1)\, (-aa\, r^2 + aa\, r^2 \cos(\theta)^2 - aa0)}{\sin(\theta)\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{CartanaffineTorsion}(1, 4, 2) = -\cos(\theta)^2\, aa\, r^2 \sin(\theta)$$

$$\text{CartanaffineTorsion}(1, 1, 3) = \frac{\cos(\theta)\, cc\, r\, (-1 + \cos(\theta))\, (\cos(\theta) + 1)\, (-aa\, r^2 + aa\, r^2 \cos(\theta)^2 - aa0)}{\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{CartanaffineTorsion}(1, 2, 3) = \frac{\cos(\theta)^2\, cc\, r^2\, (-1 + \cos(\theta))\, (\cos(\theta) + 1)\, (-aa\, r^2 + aa\, r^2 \cos(\theta)^2 - aa0)}{\sin(\theta)\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{CartanaffineTorsion}(1, 1, 4) = -\frac{1}{4}\frac{\cos(\theta)\, (-4\, aa\, r^3 - 5\, bb\, aa\, r^2 + 4\, aa\, r^3 \cos(\theta)^2 + 5\, bb\, aa\, r^2 \cos(\theta)^2 - bb\, aa0)}{r\, (r + bb)}$$

$$\text{CartanaffineTorsion}(1, 2, 4) = \cos(\theta)^2\, aa\, r^2 \sin(\theta)$$

$$\text{CartanaffineTorsion}(2, 3, 1) = \frac{\sin(\theta)\, cc\, (-1 + \cos(\theta))\, (\cos(\theta) + 1)\, (-aa\, r^2 + aa\, r^2 \cos(\theta)^2 - aa0)}{\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{CartanaffineTorsion}(2, 4, 1) = -\frac{1}{4}\frac{\sin(\theta)\, (-4\, aa\, r^3 - 5\, bb\, aa\, r^2 + 4\, aa\, r^3 \cos(\theta)^2 + 5\, bb\, aa\, r^2 \cos(\theta)^2 - bb\, aa0)}{(r + bb)\, r^2}$$

$$\text{CartanaffineTorsion}(2, 3, 2) = \frac{\cos(\theta)\, cc\, r\, (-1 + \cos(\theta))\, (\cos(\theta) + 1)\, (-aa\, r^2 + aa\, r^2 \cos(\theta)^2 - aa0)}{\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{CartanaffineTorsion}(2, 4, 2) = -\cos(\theta)\, aa\, r\, (-1 + \cos(\theta))\, (\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(2, 1, 3) = -\frac{\sin(\theta)\, cc\, (-1 + \cos(\theta))\, (\cos(\theta) + 1)\, (-aa\, r^2 + aa\, r^2 \cos(\theta)^2 - aa0)}{\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{CartanaffineTorsion}(2, 2, 3) = -\frac{\cos(\theta)\, cc\, r\, (-1 + \cos(\theta))\, (\cos(\theta) + 1)\, (-aa\, r^2 + aa\, r^2 \cos(\theta)^2 - aa0)}{\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{CartanaffineTorsion}(2, 1, 4) = \frac{1}{4}\frac{\sin(\theta)\, (-4\, aa\, r^3 - 5\, bb\, aa\, r^2 + 4\, aa\, r^3 \cos(\theta)^2 + 5\, bb\, aa\, r^2 \cos(\theta)^2 - bb\, aa0)}{(r + bb)\, r^2}$$

$$\text{CartanaffineTorsion}(2, 2, 4) = \cos(\theta)\, aa\, r\, (-1 + \cos(\theta))\, (\cos(\theta) + 1)$$

$$\text{CartanaffineTorsion}(4, 3, 1) = \frac{cc\, r\, (-1 + \cos(\theta))\, (\cos(\theta) + 1)}{\sqrt{\dfrac{r + bb}{r}}}$$

$$\text{CartanaffineTorsion}(4, 4, 1) = -\frac{1}{4}\frac{bb}{(r+bb)\,r}$$

$$\text{CartanaffineTorsion}(4, 3, 2) = \frac{\cos(\theta)\,cc\,r^2\,(-1+\cos(\theta))\,(\cos(\theta)+1)}{\sin(\theta)\,\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{CartanaffineTorsion}(4, 1, 3) = -\frac{cc\,r\,(-1+\cos(\theta))\,(\cos(\theta)+1)}{\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{CartanaffineTorsion}(4, 2, 3) = -\frac{\cos(\theta)\,cc\,r^2\,(-1+\cos(\theta))\,(\cos(\theta)+1)}{\sin(\theta)\,\sqrt{\dfrac{r+bb}{r}}}$$

$$\text{CartanaffineTorsion}(4, 1, 4) = \frac{1}{4}\frac{bb}{(r+bb)\,r}$$

>
>
>
>
>

****

****

>
>
>
>
>

The next items to compute will be the Metric curvature 2-forms from the Christoffel connections.

|