

```

> restart: with (linalg):with(liesymm):with(diffforms):with(plots):
> setup(x,y,z,t):deform(x=0,y=0,z=0,u=0,f=0,t=0,Vx=0,Vy=0,Vz=0,D1=0,D2=0,D3=0,Ax=
  0,Ay=0,Az=0,C=0,Phi=0,a=const,b=const,c=const,q=const,Lx=0,Ly=0,Lz=0);
Warning, the protected names norm and trace have been redefined and unprotected

Warning, the protected name close has been redefined and unprotected

Warning, the names &^, d and wdegree have been redefined

Warning, the name changecoords has been redefined

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# Implicit Surfaces for discriminant of Pfaff D

## 4 Jacobian

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OLD NAME implinor.mws

NEW NAME impldisc.mws

1. The Cayley - Hamilton Polynomial is used to formulate the zero set of a universal phase function, Theta, which is calculated from the Jacobian Matrix of the given 1-form of Action, of Pfaff dimension 4, used to encode a specific physical system.

$$\Theta := p^4 - M p^3 + G p^2 - A p + K$$

The variable p represents the eigen values of the Jacobian matrix in terms of the similarity invariant functions, M,G,A,K, of the Jacobian polynomial.

\*

The 4 coefficient functions of the Action 1-form (in 4D) may be used to represent a normal field to a hypersurface, and when appropriately scaled by dividing each term by the square root of the sum of squares of the coefficients (the Gauss map) the Jacobian matrix of the "unit" normal field becomes equivalent to the shape matrix of differential geometry. The functions M, G, A become the similarity curvature invariants. The Gauss map causes the fourth order similarity value to be zero. The Gauss map reduces the problem to those domains where the space is of Pfaff dimension 3. It is apparent that the use of geometrical curvature concepts prohibits systems of Pfaff dimension 4.

The divergence of the renormalized normal field vanishes! The 4 volume element is zero. The Gauss map scaling implies that the 4D Topological Torsion vector can be non-zero, but if it is non-zero, it has zero divergence, and evolutionary processes in the direction of the Topological Torsion vector are not thermodynamically reversible.

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The

## Discriminant for Gibbs function

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> G4D:=q^4-M*q^3+G*q^2-A*q+K;Phi:=discrim(G4D,q);Gibbs:=factor(subs(K=0,Phi));Minn
:=factor(subs(M=0,Phi));

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$$G4D := q^4 - M q^3 + G q^2 - A q + K$$

$$\Phi := 18 M^3 A G K - 4 M^3 A^3 - 128 G^2 K^2 + 16 G^4 K - 4 G^3 A^2 - 27 M^4 K^2 + 256 K^3 - 27 A^4 - 80 M A G^2 K - 4 G^3 M^2 K + G^2 M^2 A^2 + 144 G M^2 K^2 + 18 M A^3 G - 192 M A K^2 - 6 M^2 A^2 K + 144 K A^2 G$$

$$Gibbs := -A^2 (4 A M^3 + 4 G^3 + 27 A^2 - M^2 G^2 - 18 M G A)$$

$$Minn := -128 G^2 K^2 + 16 G^4 K - 4 G^3 A^2 + 256 K^3 - 27 A^4 + 144 K A^2 G$$

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> AA:=solve(Gibbs,A);AP:=factor(AA[3]);AM:=factor(AA[4]);

```

$$AA := 0, 0, -\frac{2}{27} M^3 + \frac{1}{3} M G + \frac{2}{27} \sqrt{M^6 - 9 M^4 G + 27 M^2 G^2 - 27 G^3},$$

$$-\frac{2}{27}M^3 + \frac{1}{3}MG - \frac{2}{27}\sqrt{M^6 - 9M^4G + 27M^2G^2 - 27G^3}$$

$$AP := -\frac{2}{27}M^3 + \frac{1}{3}MG + \frac{2}{27}\sqrt{-(3G - M^2)^3}$$

$$AM := -\frac{2}{27}M^3 + \frac{1}{3}MG - \frac{2}{27}\sqrt{-(3G - M^2)^3}$$

> factor(AP\*AM);

$$\frac{1}{729}(-2M^3 + 9MG + 2\sqrt{-(3G - M^2)^3})(-2M^3 + 9MG - 2\sqrt{-(3G - M^2)^3})$$

>

>

>

> imag:=factor(27\*M^2\*G^2-9\*M^4\*G+M^6-27\*G^3);

>

$$imag := -(3G - M^2)^3$$

> G4DMM:=collect(factor(subs(q=y+M/4,G4D)),y);

$$G4DMM := y^4 + \left(G - \frac{3}{8}M^2\right)y^2 + \left(\frac{1}{2}MG - \frac{1}{8}M^3 - A\right)y - \frac{1}{4}MA + K + \frac{1}{16}GM^2 - \frac{3}{256}M^4$$

> factor((1/2\*M\*G-1/8\*M^3-A)\*y-1/4\*M\*A+K+1/16\*G\*M^2-3/256\*M^4);

$$\frac{1}{2}GyM - \frac{1}{8}yM^3 - Ay - \frac{1}{4}MA + K + \frac{1}{16}GM^2 - \frac{3}{256}M^4$$

> G3D:=factor((G4D-K)/q);G3DMM:=collect(subs(q=x+M/3,factor(G4D-K)/q),x);GR3Dq:=diff(G3D,q);GR3Dqq:=diff(GR3Dq,q);

$$G3D := q^3 - Mq^2 + Gq - A$$

$$G3DMM := x^3 + \left(-\frac{1}{3}M^2 + G\right)x - \frac{2}{27}M^3 - A + \frac{1}{3}MG$$

$$GR3Dq := 3q^2 - 2Mq + G$$

$$GR3Dqq := 6q - 2M$$

> MM:=solve(GR3Dqq,M);GG:=solve(subs(M=3\*q,GR3Dq),G);

$$MM := 3q$$

$$GG := 3q^2$$

> AA:=solve(subs(G=GG,M=0,G3D),A);

> R:=[-MM,-GG,-AA];

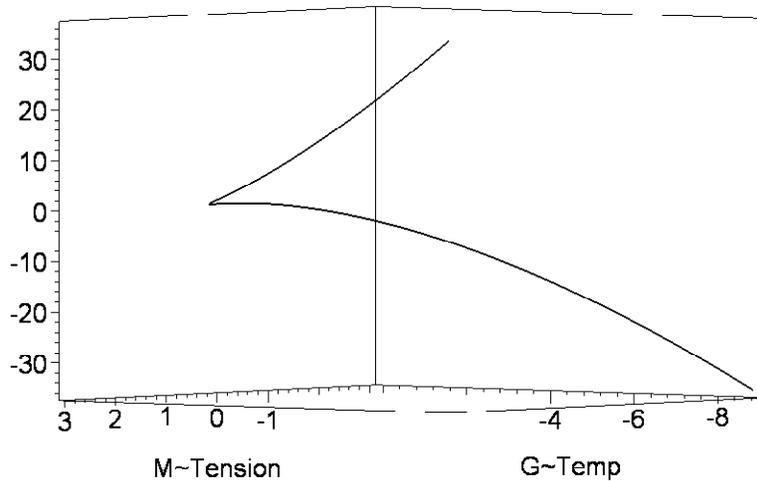
>

$$AA := 4q^3$$

$$R := [-3q, -3q^2, -4q^3]$$

> spacecurve(R/3,q=-3..3,orientation=[-90,90],labels=["M~Tension","G~Temp","A~Pressure"],title=`HYSTERETIC`,axes=normal,thickness =2,color = black);

# HYSTERETIC



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> G4D:=q^4+(x)*q^2-y*q;Phi:=discrim(G4D,q);collect(Phi,y);subs(z=0,Phi);
```

$$G4D := q^4 + x q^2 - y q$$

$$\Phi := -y^2 (4 x^3 + 27 y^2)$$

$$-27 y^4 - 4 x^3 y^2$$

$$-y^2 (4 x^3 + 27 y^2)$$

```
> G4D:=q^4+(x)*q^2-y*q+z;Phi:=discrim(G4D,q);collect(Phi,y);subs(z=0,Phi);
```

$$G4D := q^4 + q^2 x - y q + z$$

$$\Phi := -4 x^3 y^2 - 27 y^4 + 16 x^4 z - 128 x^2 z^2 + 144 z y^2 x + 256 z^3$$

$$-27 y^4 + (-4 x^3 + 144 x z) y^2 + 256 z^3 + 16 x^4 z - 128 x^2 z^2$$

$$-4 x^3 y^2 - 27 y^4$$

```
> DISCGG:=collect(simplify(factor(discrim(G4D,q))),z);DISCGGx:=diff(DISCGG,z);A:=factor(coeff(DISCGG,z^2));B:=factor(coeff(DISCGG,z^1));C:=factor(DISCGG-256*z^3-A*z^2-B*z);Binodal:=factor(subs(y=0,DISCGG));
```

$$DISCGG := 256 z^3 - 128 x^2 z^2 + (16 x^4 + 144 y^2 x) z - 4 x^3 y^2 - 27 y^4$$

$$DISCGGx := 768 z^2 - 256 z x^2 + 16 x^4 + 144 y^2 x$$

$$A := -128 x^2$$

$$B := 16 x (x^3 + 9 y^2)$$

$$C := -y^2 (4 x^3 + 27 y^2)$$

$$Binodal := 16 z (-4 z + x^2)^2$$

```
> DiscGG0:=subs(x=-0,256*z^3+A*z^2+B*z+C)/16;DiscGGm1:=subs(x=-1,256*z^3+A*z^2+B*z+C)/16;DiscGGp1:=subs(x=2,256*z^3+A*z^2+B*z+C)/16;
```

$$DiscGG0 := 16 z^3 - \frac{27}{16} y^4$$

$$DiscGGm1 := 16 z^3 - 8 z^2 - (-1 + 9 y^2) z - \frac{1}{16} y^2 (-4 + 27 y^2)$$

$$DiscGGp1 := 16 z^3 - 32 z^2 + 2 (8 + 9 y^2) z - \frac{1}{16} y^2 (32 + 27 y^2)$$

```
[ >
[ >
[ > Phi:=DiscGG;
[ > NN:=eval([diff(Phi,x),diff(Phi,y),diff(Phi,z)]);
[ > magn:=simplify((NN[1]^2+NN[2]^2+NN[3]^2)^(1/2));
[ >
[ > unitnormal:=(evalm(NN/magn));NN1:=factor(unitnormal[1]);
[ Compute the Jacobian matrix of the unit normal field:
[ > JACNN:=jacobian(unitnormal,[x,y,z]);
[ > DETJAC:=det(JACNN);
[ > MEAN_CURVATURE:=factor(simplify(trace(JACNN)/2));
[ > ADJOINTJACNN:=factor(adj(JACNN));
[ >
[ >
[ > GAUSS_CURVATURE:=simplify(trace(ADJOINTJACNN));
```

Besides the similarity invariants above there are three other invariants that are useful for describing the 3x3 Jacobian matrix. These are called the Brand invariants and are discussed in L. Brand's book on Vector and Tensor Analysis, Wiley 1962, page 173.

These three other invariants are related to the anti-symmetric parts of the Jacobian matrix ( a dyadic in the Brand-Gibbs notation). For a 3x3 matrix the anti-symmetric matrix elements are related to the "curl" of the unitnormal field, which will be defined herein as the VORTICITY.

The adjoint matrix can also have antisymmetric parts. In 3 dimensions these antisymmetric parts form another "curl" type vector field. This second curl-type vector field can be constructed by multiplying the curl of the unit normal field by the Jacobian matrix itself. This second anti-symmetric construction will be defined as the "BRAND" vector.

In Hydrodynamics, the "inner" product of the VORTICITY with itself is called the "enstrophy" of the flow. The innerproduct of the VORTICITY and the BRAND vector has been associated with "vortex stretching. The innerproduct of the BRAND vector with itself has never been defined.

Yet all three are invariant scalars under similarity transformations.

The unit normal field can have vorticity, but no helicity if generated from a implicit function. If the unit normal field is generated from a 1-form then the system may not be integrable and helicity will be non-zero.

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[ >
[ >
[ For the Implicit sphere, the vorticity vanishes.
[ > VORTICITY:=evalm(curl(unitnormal,[x,y,z]));
[ > BRAND:=innerprod(JACNN,VORTICITY);
[ > HELICITY:=simplify(innerprod(unitnormal,VORTICITY));
[ >
[ > MAGNUS_FORCE:=(crossprod(unitnormal,VORTICITY));
[ > # Brandt invariants
[ > enstrophy:=simplify(innerprod(VORTICITY,VORTICITY));
[ >
[ > stretch:=simplify(innerprod(BRAND,VORTICITY));
[ > brand_invariant:=simplify(innerprod(BRAND,BRAND));
[ >
```