

```
[ > restart:with(plots):
```

Differential Equation routine with plots.

Uses arclength as parameter (direction field of unit tangent vector).

snhopf2de.mws

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```
[ > Omega:=20;F:=4;
```

```
[ > OPTS:=(numpoints=500,axes=BOXED,scaling=CONSTRAINED,orientation=[5  
9,59],shading=Z,labels=['x','y','z'],titlefont=[TIMES,BOLD,12]):
```

```
Omega := 20
```

```
F := 4
```

### SADDLE NODE HOPF equations

The vector field (below) satisfies the Navier-Stokes equations in a rotating frame of reference.

It is of interest to find the direction field for the flow lines. The initial conditions are such that the flow starts at a point near the z axis. Two cases are considered.  $z(0) = -1$  and  $z(0) = -4$ .

The first case generates a reentrant flow whose z direction changes sign inside a bubble. The second case generates a non-reentrant flow which extends from the initial conditions to  $z = \text{plus infinity}$  and circulates about the bubble.

If the value of F is negative, no bubble occurs.

```
[ > Vx:=-z(t)*x(t)-Omega*y(t);
```

```
Vx := -z(t) x(t) - 20 y(t)
```

```
[ > Vy:=-z(t)*y(t)+Omega*x(t);
```

```
Vy := -z(t) y(t) + 20 x(t)
```

```
[ > Vz:=- (F-z(t)^2-(x(t)^2+y(t)^2));
```

```
Vz := -4 + z(t)^2 + x(t)^2 + y(t)^2
```

```
[ > Sc:=1/(Vx*Vx+Vy*Vy+Vz*Vz)^(1/2):
```

Scale by magnitude of vector field

```
[ > DEx:= diff(x(t),t)=Vx*Sc:
```

```
[ > DEy:= diff(y(t),t)=Vy*Sc:
```

```
[ > DEz:= diff(z(t),t)=Vz*Sc:
```

```
[ > fcns := {x(t),y(t),z(t)}:
```

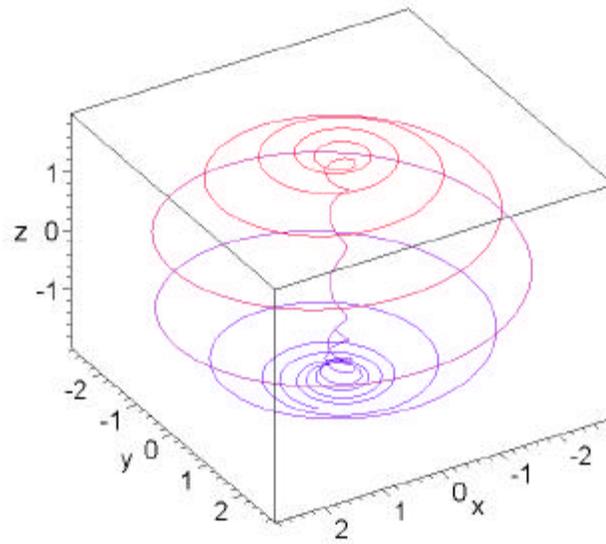
Set up initial conditions, solve the ODE's and plot.

```
[ > INITS:={x(0)=0.1,y(0)=.1,z(0)=-1}:
```

```
[ > p:=dsolve({DEx,DEy,DEz} union INITS,fcns,type=numeric):
```

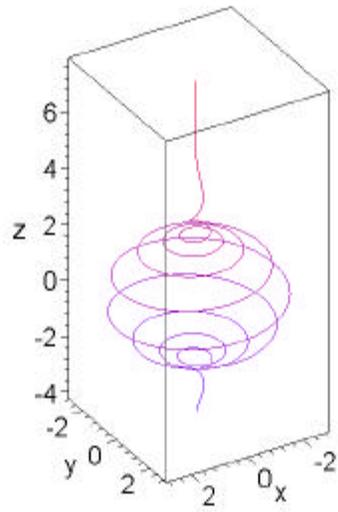
```
[ > odeplot(p, [x(t),y(t),z(t)], -0..100, OPTS,title='INSIDE_BUBBLE');
```

## INSIDE\_BUBBLE



```
[ >  
[ > INITS:={x(0)=0.1,y(0)=.1,z(0)=-4}:  
[ > p:=dsolve({DEx,DEy,DEz} union INITS,fcns,type=numeric):  
[ >  
[ > odeplot(p, [x(t),y(t),z(t)], -0..89, OPTS,title='OUTSIDE_BUBBLE');
```

# OUTSIDE\_BUBBLE



[ >