AMS-ABQ Oct 17, 2004

Non Equilibrium Thermodynamic Systems and Irreversible Processes

From the perspective of Continuous Topological Evolution

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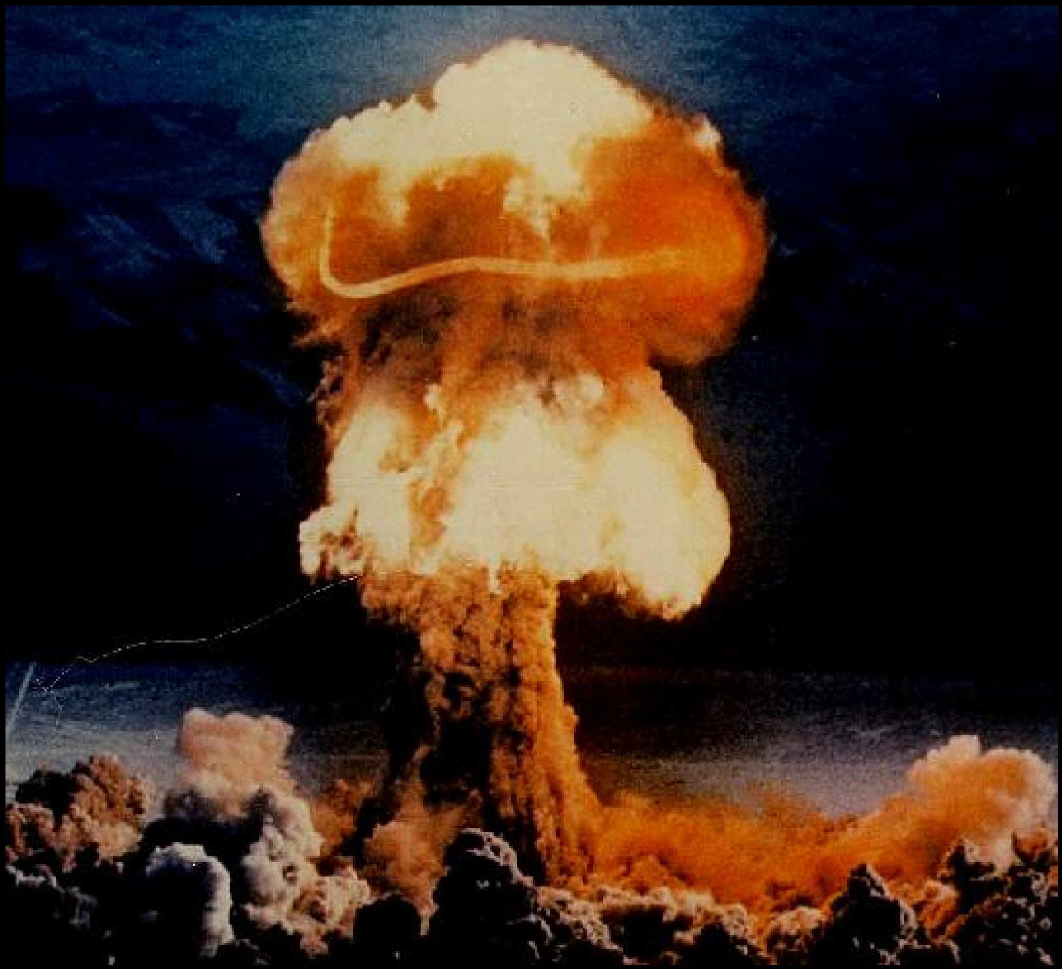
Any mathematician knows it is impossible to understand an elementary course in thermodynamics .... V. Arnold 1990

It is always emphasized that thermodynamics is concerned with reversible processes and equilibrium states, and that it can have nothing to do with irreversible processes or systems out of equilibrium ......Bridgman 1941

... No one knows what entropy really is, so in a debate (if you use the term entropy) you will always have an advantage .... Von Neumann (1971)

Three Stimulating Events #1

Long Lived Ionized Ring



1957

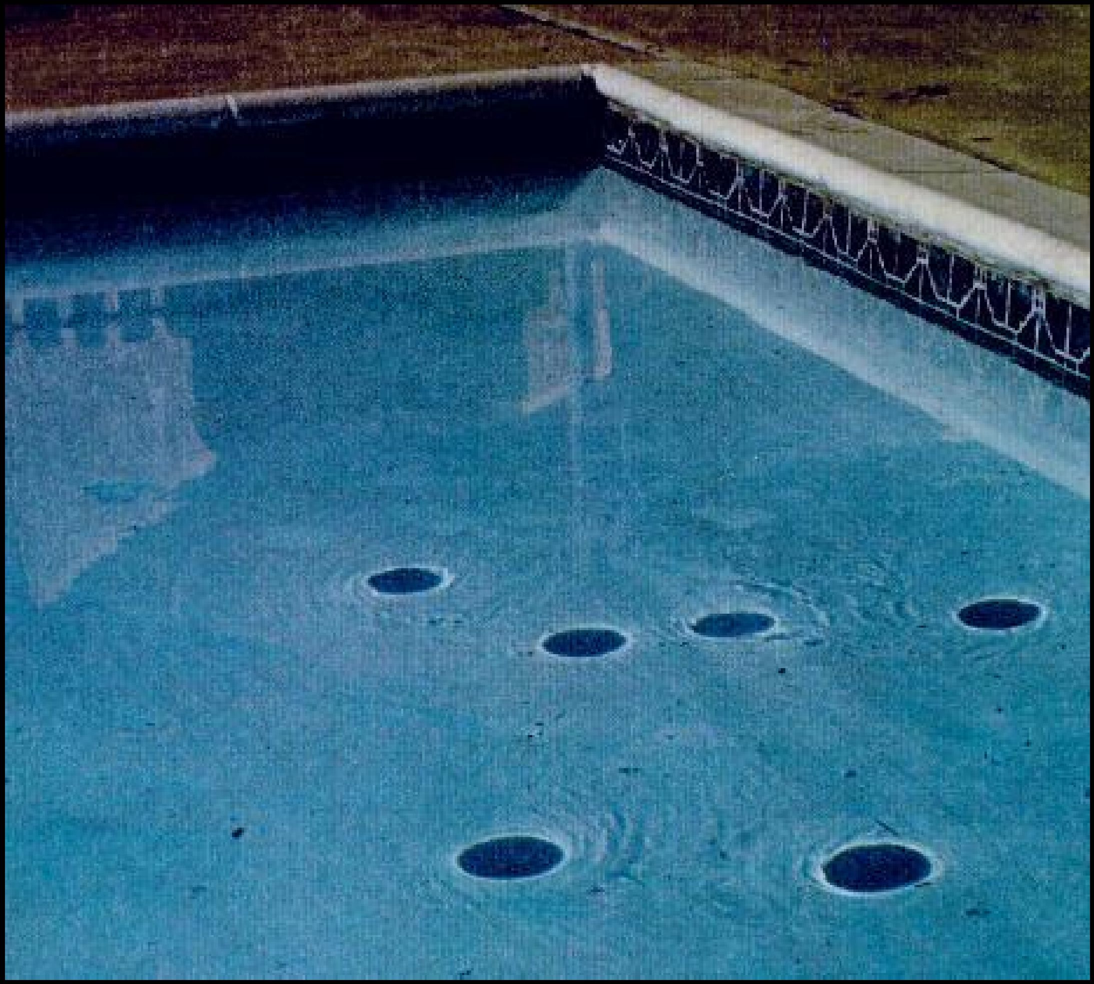
Three Stimulating Events #2

Long Lived Wake

1962

http://www.airtoair.net **photo by Paul Bowan**

Three Stimulating Events #3
Long Lived Falaco Solitons



1986

What is the Common Thread ?

They all are artifacts of :

Continuous Topological Evolution

creating

- **Coherent Topological Structures**
- **as Long lived States far from Equilibrium**
- **by means of Irreversible processes.**

I contend that these

Universal Dynamical Topological Defects,

or deformation invariants independent from size and shape, forming

Topologically Coherent Structures

of Pfaff Topological dimension 3 or more, all exhibit non-zero

Topological Torsion

A topological artifact of thermodynamic non-equilibrium

During the period 1965-1992 it became apparent that New Theoretical Foundations were needed to describe

Non Equilibrium systems

and

Continuous Irreversible Processes,

which require

Topological (not geometrical) Evolution.

The method selected was to use Cartan's Methods of Exterior Differential Topology to encode Continuous Topological Evolution.

Objectives of CTE

(Continuous Topological Evolution)

- Establish the long sought for connection between **Irreversible Thermodynamics** and **Dynamical Systems** -- without Statistics!
- Demonstrate the connection between **Thermodynamic Irreversibility** and **Topological (Pfaff) Dimension 4**.
- Demonstrate the universality of a 4 dimensional **van der Waals** gas.

Simple Ideas of CTE

Topological evolution can take place continuously or discontinuously. From a simplistic viewpoint:

Cutting is a discontinuous process.

Pasting is a continuous process.

Evolutionary invariants that are independent from size and shape (deformation invariants such as number of parts, holes, orientation, Pfaff Topological Dimension) are physically measurable, useful, properties of applied topology.

Any 1-form A induces a Topology on the domain.

(The induced topology may not be a connected topology)

Mathematical Objects in CTE

Exterior Differential Forms,

unlike tensors, are functionally well behaved - with respect to those C1 maps which are neither diffeomorphisms nor homeomorphisms.

Cartan's methods of Exterior Differential Systems,

can be used to describe

Continuous Topological Evolution.

(Hamiltonian methods can not!)

Theorems of CTE

- **Topological evolution (change)** is a necessary condition for both time asymmetry (the Arrow of Time) and Thermodynamic Irreversibility.
- **A unique Extremal direction field** which represents a conservative reversible Hamiltonian process always exists on subspaces of topological dimension 2n+1.
- **A unique Torsional direction field** which represents a thermodynamically irreversible process always exists on subspaces of even topological dimension 2n+2.

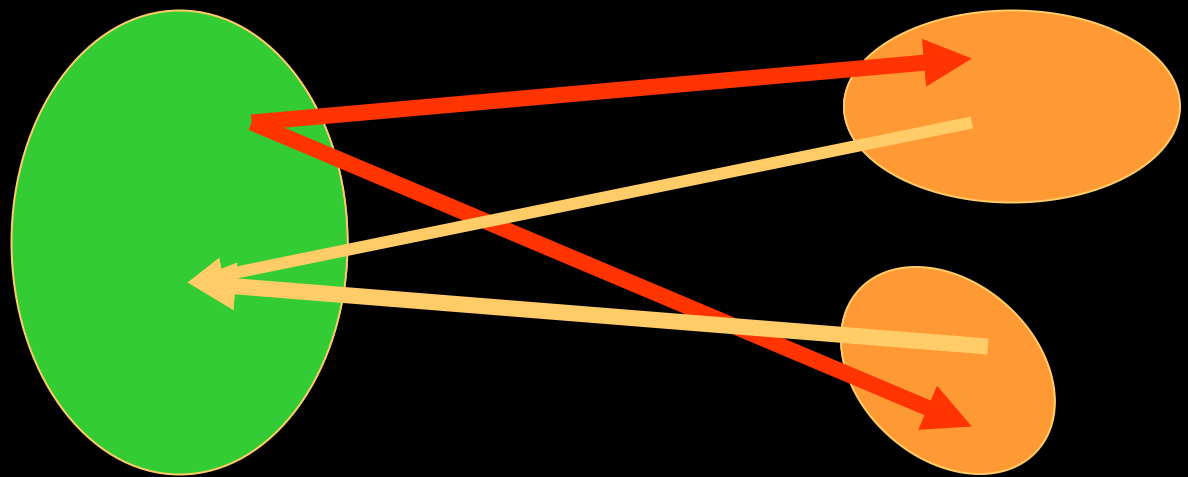
Theorems of CTE

- The Creation of Turbulence (and non equilibrium systems) is represented by a discontinuous process.
- The Decay of Turbulence (and the ultimate decay to equilibrium) is represented by a continuous process.
- Digital methods that match slope and value can NOT be used to describe the creation of turbulence, but can be used to describe the decay of turbulence.
- Irreversible turbulence is an artifact of 4D, hence time dependent 2D turbulence is a myth.

Creation of Turbulence is a

Discontinuous Process

And can NOT be studied with digital techniques that match slope and value



Decay of Turbulence is a

Continuous Process

Which can be studied with digital techniques that match slope and value

Streamline Flow

Connected Topology PTD ≤ **2**

Turbulent Flow

Disconnected Topology PTD > **3**

Encoding Non Equilibrium physical systems and Processes for use with CTE

Systems = Differential 1-form of Action, $A_{k(x,y,z,t)}$

Processes = Vector Direction Fields, $V^{k}_{(x,y,z,t)}$.

Dynamics = Cartan's Magic Lie Differential

$$L_{(V)}\,A = i(V)dA + d(i(V)A) = Q$$

$$\updownarrow \qquad\qquad \updownarrow$$

The First Law !!! $\quad W \quad + \quad dU \quad = Q$

Work + d(Internal Energy) = Heat

Two practical Topological Thermodynamic techniques are based on

The Pfaff Topological Dimension

generated from the 1-forms of

Action, A, Work, W, and Heat, Q.

The Thermodynamic Phase Function

and the Similarity Invariants of the Jacobian Matrix, [J(A)]

Part 1: Utilization of the idea of

Pfaff Topological Dimension

and the equivalence classes generated from

the 1-form of Action, A,

the 1-form of Work, W.

and the 1-form of Heat, Q.

Pfaff Topological Dimension

A physical system represented by a 1-form of Action, A, has a minimum number of functions required for its topological definition. This number,

PTD = Pfaff Topological Dimension,

is equal or less than the geometrical dimension N of the domain of support. The PTD is also equal to the number of non-zero terms in the

Pfaff Sequence = {A,dA,A^dA,dA^dA…}.

Subspaces of lesser PTD form coherent topological structures, defects, or thermodynamic phases.

PTD(A)=4 and Topological Torsion

Compute Pfaff Sequence from the 1 form A(x,y,z,t) that encodes the physical system:

$$\{A, dA, A\wedge dA, dA\wedge dA\}$$

On a 4D symplectic manifold generated by a 1-form of Action, A, with PTD(A)=4, there exists a unique direction field, T4, such that the 3-form of Topological Torsion becomes:

$$A\wedge dA = i(T4)dx\wedge dy\wedge dz\wedge dt$$

The same idea works in 2n+2 Dimensions

Compute the 4-form

$$dA\hat{}dA = \Gamma\,\Omega = (4DivT4)\Omega.$$

Where Ω = dx^dy^dz^dt

The Topological Dissipation coefficient = Γ

Find topological defects, where PTD(A)=3, Γ = 0,

defined as subspaces of PTD(A)=4, which are

Topologically coherent states of PTD(A)=3

with zero topological dissipation, Γ = 0

(Hence of long lifetime, yet far from equilibrium, PTD(A)=3)

Topological Torsion, A^dA=i(T4)Ω

What is it?   An artifact of non-equilibrium!!

A topological idea applied to the transition to turbulence

(1977 NASA-AMES, 1989 Cambridge Conference on Topological Fluid Mechanics)

Streamline flow: A^dA=0

Turbulent flow: A^dA <>0

A statement related to

Frobenius Unique Integrability.

Equilibrium Systems: A^dA=0

Non Equilibrium systems: A^dA <>0

Concepts obtained from use of the abstract Cartan approach and

Cartan's Magic Lie Differential

1. **Dynamical Systems can be coupled to Non Equilibrium Thermodynamics**

without statistics.

2. **Pfaff Topological Dimension defines Topological Coherent Defect Structures**

PTD = Irreducible Number of Functions required to represent different topological equivalence classes of A, W, and Q.

(Recall that many different topologies can be supported by the same set.)

3. Thermodynamic Systems can be put into topological PTD(A) equivalence classes

Open: The Pfaff topological dimension of the Action 1-form, A, is

PTD(A) = 4

Closed: The Pfaff topological dimension of the Action 1-form, A, is

PTD(A) = 3

Isolated-Equilibrium The topological dimension of the Action 1-form, A, is

PTD(A) = 2 or less

(Pfaff Topological Dimension > 2 implies non-equilibrium)

4. Reversible Processes can be put into Topological PTD(W) Equivalence Classes

Extremal: PTD(W) = 0

Hamiltonian: PTD(W) = 1, W exact

Stokes: PTD(W) = 1, W closed

Helmholtz: PTD(W) = 2

5. Irreversible Processes can be put into Topological PTD(Q) Equivalence Classes

Reversible : PTD(Q) < 3

Irreversible : PTD(Q) > 2

Navier - Stokes: PTD(Q) > 2

Q^dQ < > 0 implies that Q is not integrable

Classic Definition:

A process is irreversible if the Heat 1-form, Q, does not admit an integrating factor.

6. Evolution in the direction of T4, the Topological Torsion vector, is Irreversible

on the symplectic manifold, dA^dA < > 0, PTD(A) =4

$$L(T4)A = \Gamma A$$

$$\Gamma = 4\mathrm{Div}\,(T4) = \mathrm{coeff}\,\{dA\wedge dA\}$$

$$Q\wedge dQ = L(T4)A\wedge L(T4)dA$$
$$= \Gamma^2 A\wedge dA <> 0$$

Γ = topological dissipation coefficient

7. Evolution in the direction of T4, the Topological Torsion vector, is Reversible

on the Contact submanifold, A^dA < > 0, dA^dA=0, PTD(A) =3

$$L(T4)A = \Gamma\, A \Rightarrow 0$$

$$\text{as } \Gamma = \text{coeff}\,\{dA\wedge dA\} \Rightarrow 0$$

$$Q\wedge dQ = \Gamma^2 A\wedge dA \Rightarrow 0$$

On the contact manifold of Pfaff topological dimension 3,

the process T4 becomes Hamiltonian Characteristic and reversible .

Hence irreversibility is an artifact of Pfaff Topological dimension = 4.

Once created, the long lived coherent defect structure (an "excited" topologically stationary state) on the Contact Submanifold domain, evolves in the direction of the (now) divergence free Hamiltonian process, T4. The System is far from equilibrium as PTD(A) = 3.

Decay to Topological Defects in 4D

Continuous Topological Evolution can describe the irreversible evolution on an

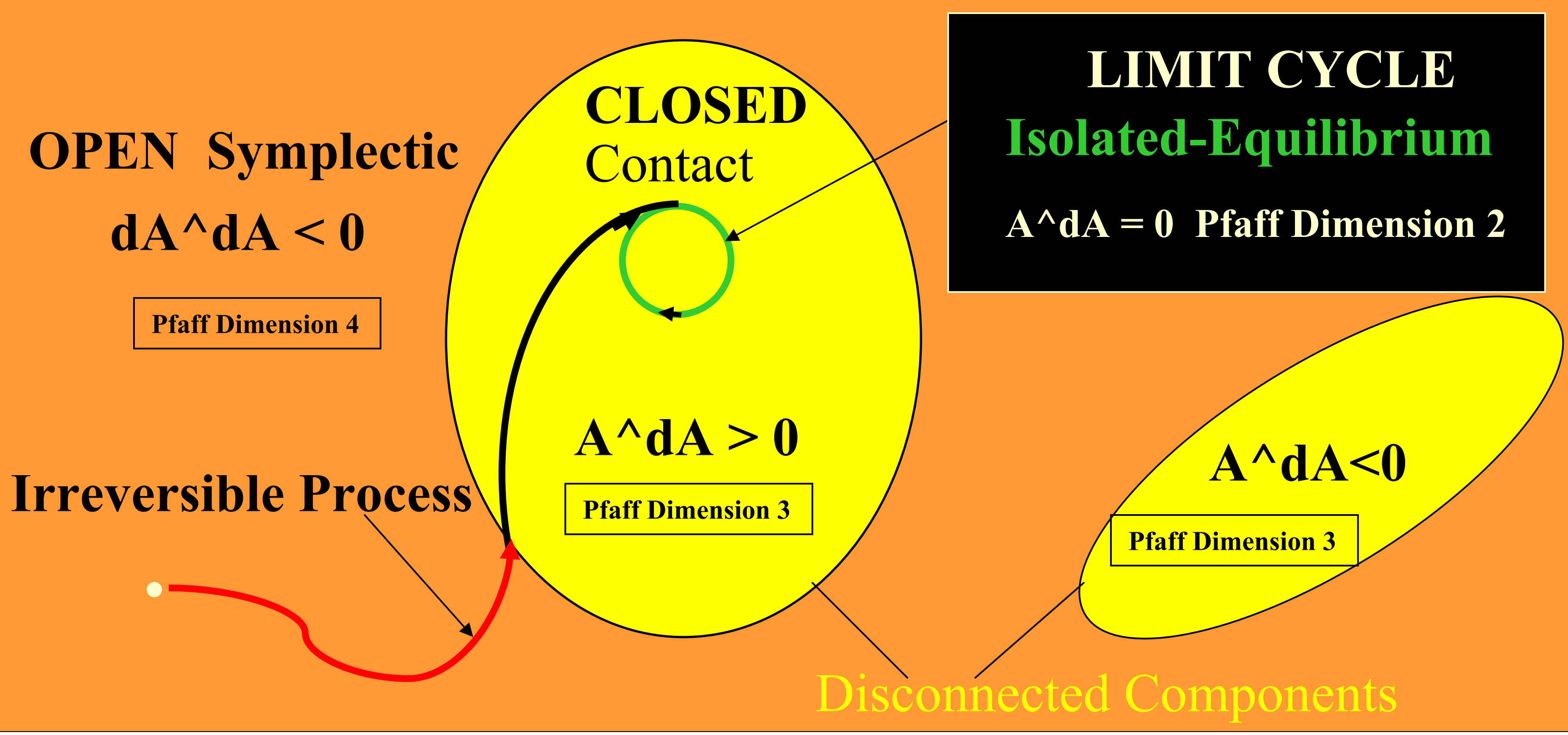
"Open" symplectic domain of Pfaff dimension 4, with evolutionary orbits being irreversibly attracted to a

"Closed" contact domain of Pfaff dimension 3, with topological defects (stationary states and coherent structures), and a possible ultimate decay to the

"Isolated-Equilibrium" domain of Pfaff dimension 2 or less (integrable Caratheodory surface).

Irreversible Decay on a Symplectic Manifold (PTD=4) to a Contact Manifold (PTD=3) of disconnected components, then possibly to an Isolated-Equilibrium (PTD = 2) State.

Turbulent Non-Equilibrium Pfaff dimension 4



Part 2: Utilization of the idea of a Universal Thermodynamic Phase Function

and the Similarity Invariants of the Jacobian Matrix, [J(A)]

The universal van der Waals gas

The Characteristic Polynomial Method

For a 1-form of Action A(x,y,z,t) with PTD(A) = 4

(a turbulent non equilibrium domain)

Compute Jacobian matrix of A and its

Cayley-Hamilton Characteristic Polynomial, defined as the

Universal Phase Function,

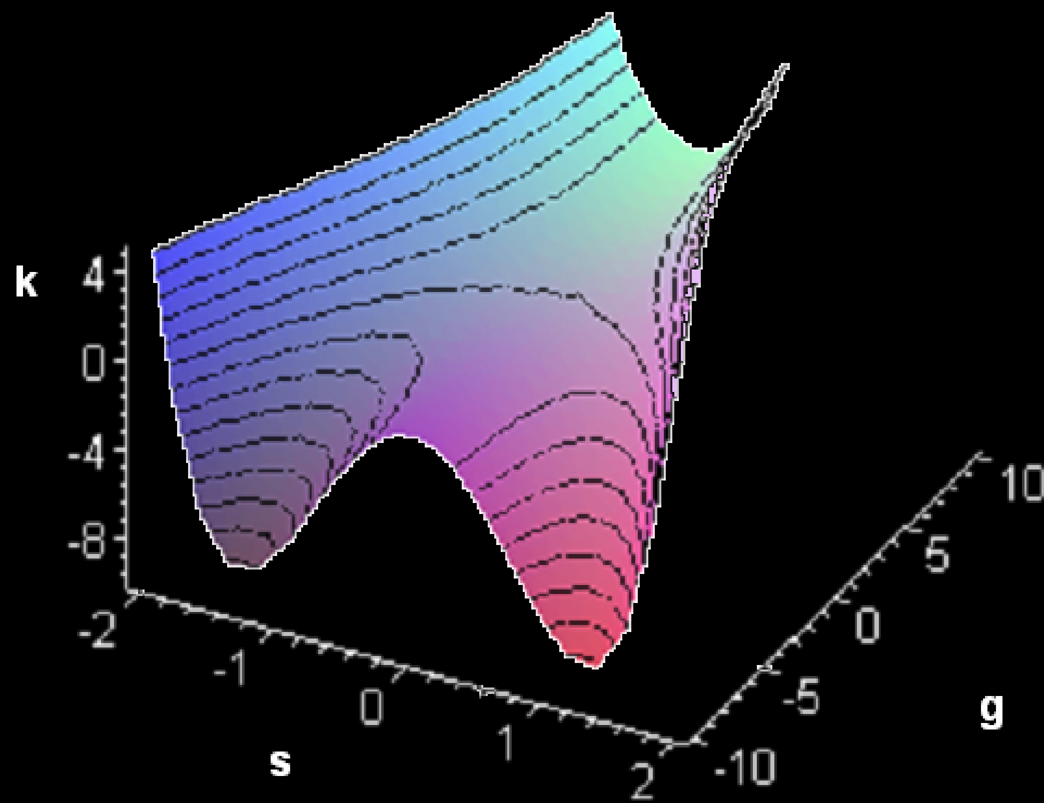
$$\Theta = \rho^4 - X_M \rho^3 + Y_G \rho^2 - Z_A \rho + T_K = 0$$

Compute Similarity-Curvature invariants.

$$X_M \quad Y_G \quad Z_A \quad T_K$$

Solve for Tk to generate a Universal Higgs surface.

Universal Topological Projections



Universal 4^{th} order Thermodynamic Phase Function

is a van der Waals gas with a Higgs potential

$\partial\Theta/\partial\rho \Rightarrow 0$ **generates a pitchfork bifurcation of the "pressure" and defines the Binodal line of the universal van der Waals gas.**

$\partial^2\Theta/\partial\rho^2 \Rightarrow 0$ **generates an edge of regression and the Spinodal line of the universal van der Waals gas.**

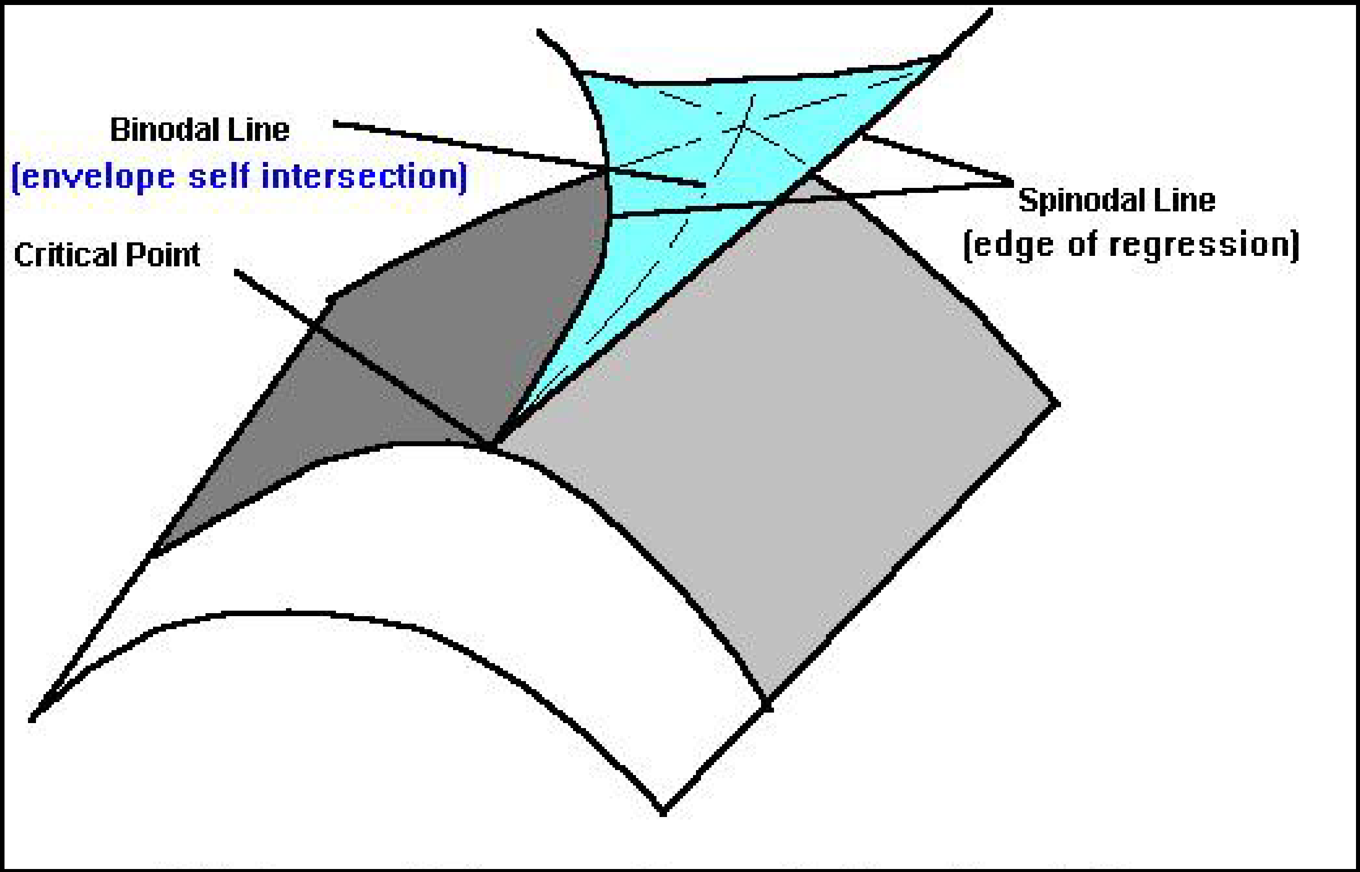
Topological Thermodynamics

- The UNIVERSAL TOPOLOGICAL PHASE function supports an envelope, which, when constrained to a minimal surface ($X_M = 0$), generates the Swallow-Tail bifurcation set.

- The UNIVERSAL minimal surface ENVELOPE is homeomorphic to the

Gibbs Surface of a van der Waals gas

Universal Topological Projections



The Envelope of the Universal 4th order Phase Function leads to a Gibbs Function of a (deformed) van der Waals gas.

Topological Thermodynamics

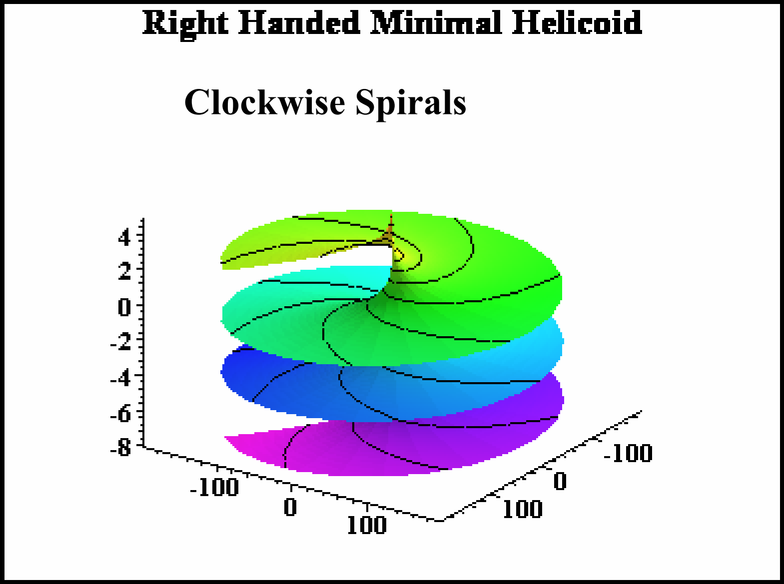
- The Universal Topological Phase Function, Θ generated by a non-singular Jacobian matrix is holomorphic in the complex variable ρ.

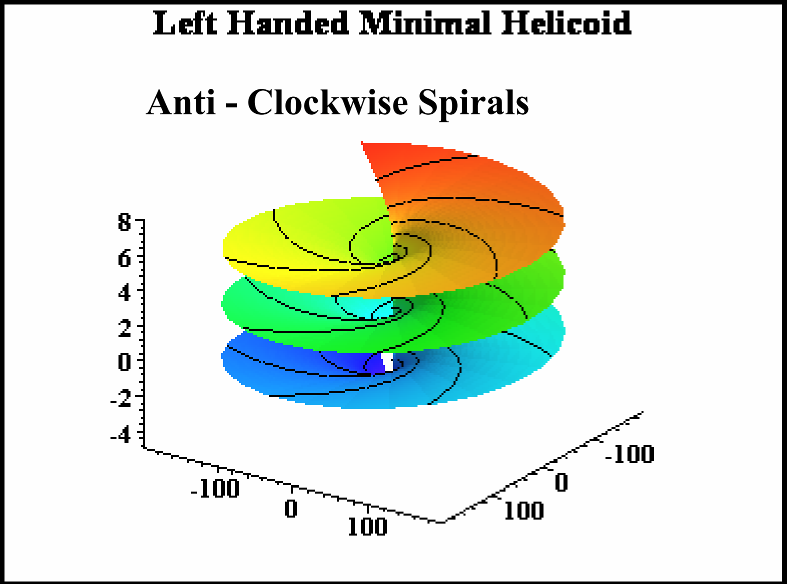
- THEN, from a theorem of Sophus Lie:

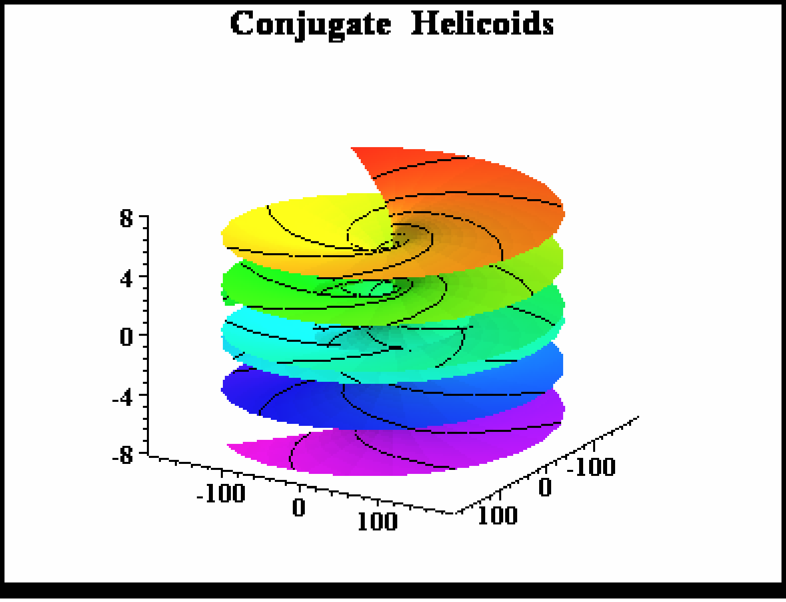
- The Phase Function Θ creates

Conjugate Minimal Surfaces in 4D.

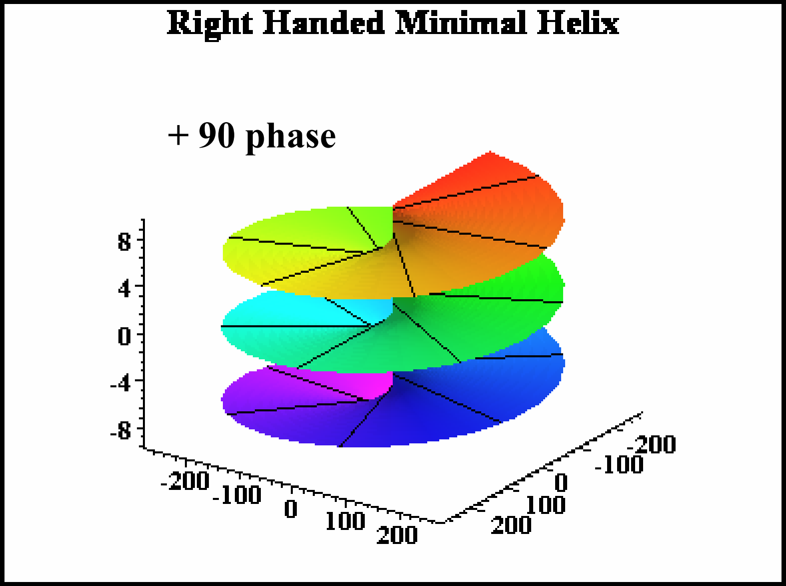
Conjugate “Chiral” Minimal Surfaces

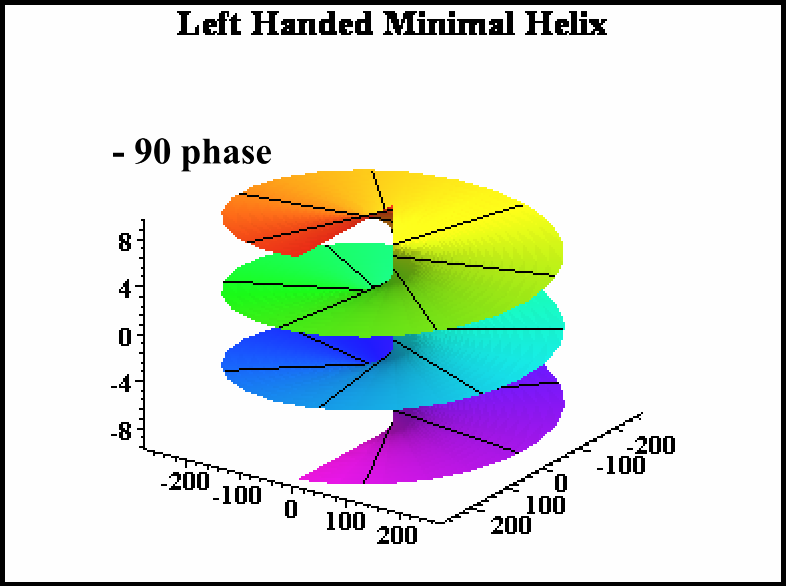


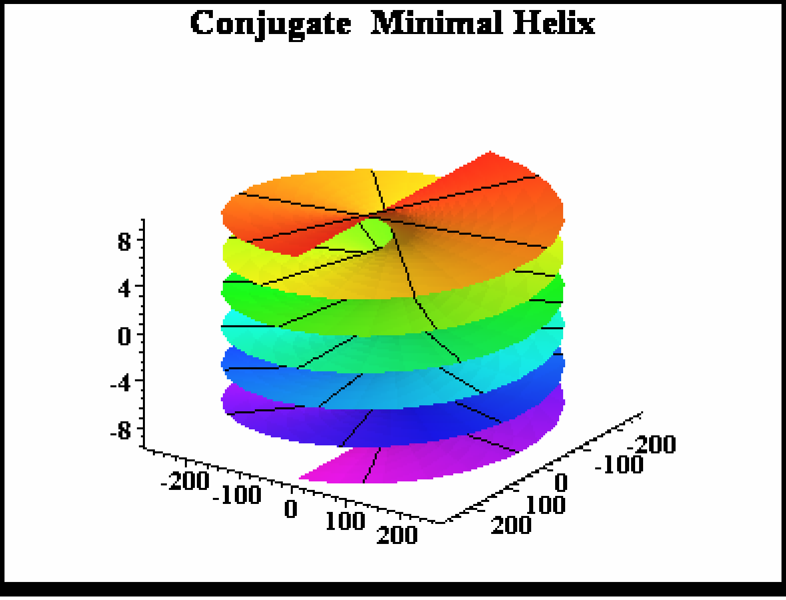




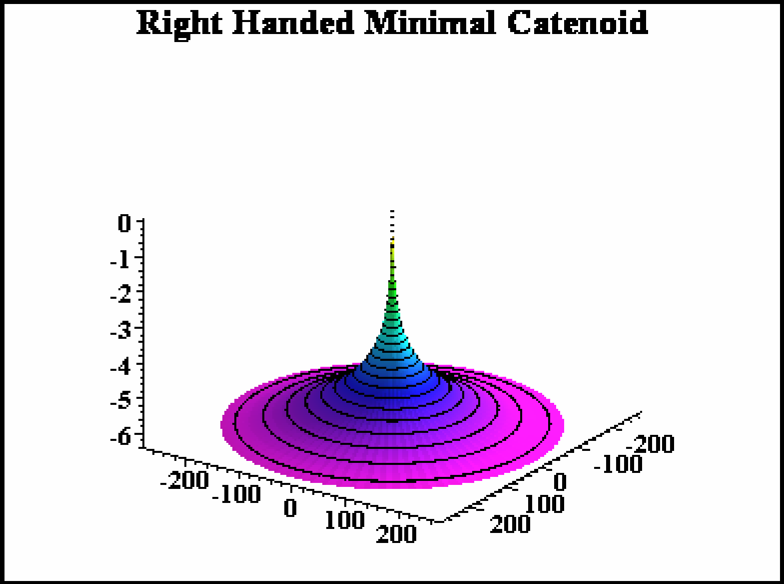
Conjugate “Chiral” Minimal Surfaces

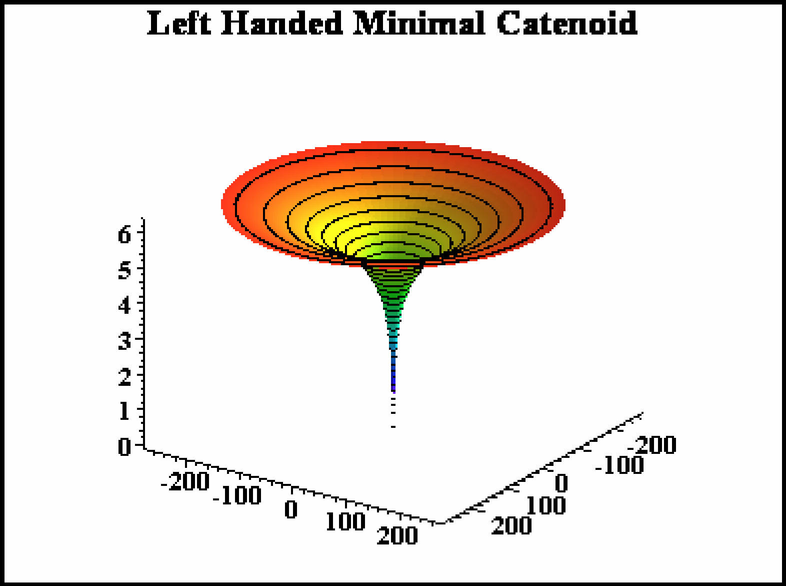


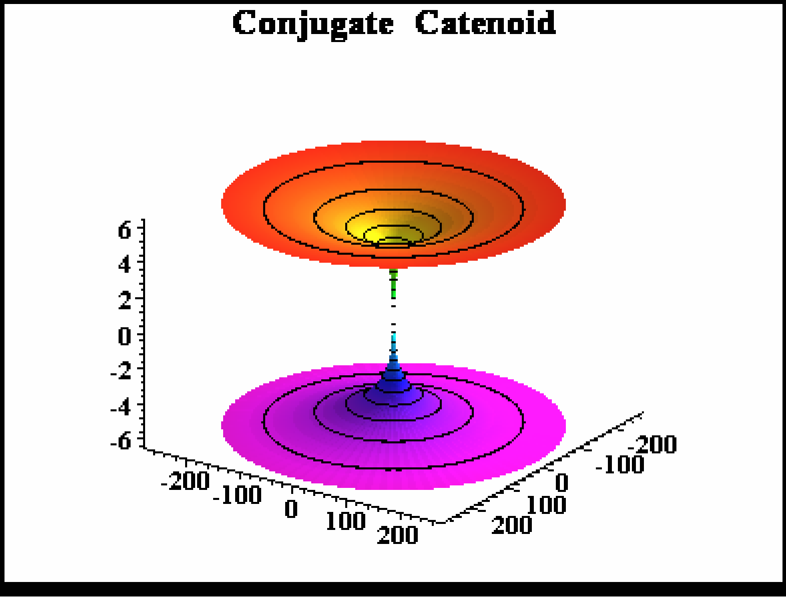




Conjugate "Chiral" Minimal Surfaces







Similarity Invariants are

NOT Chiral Sensitive.

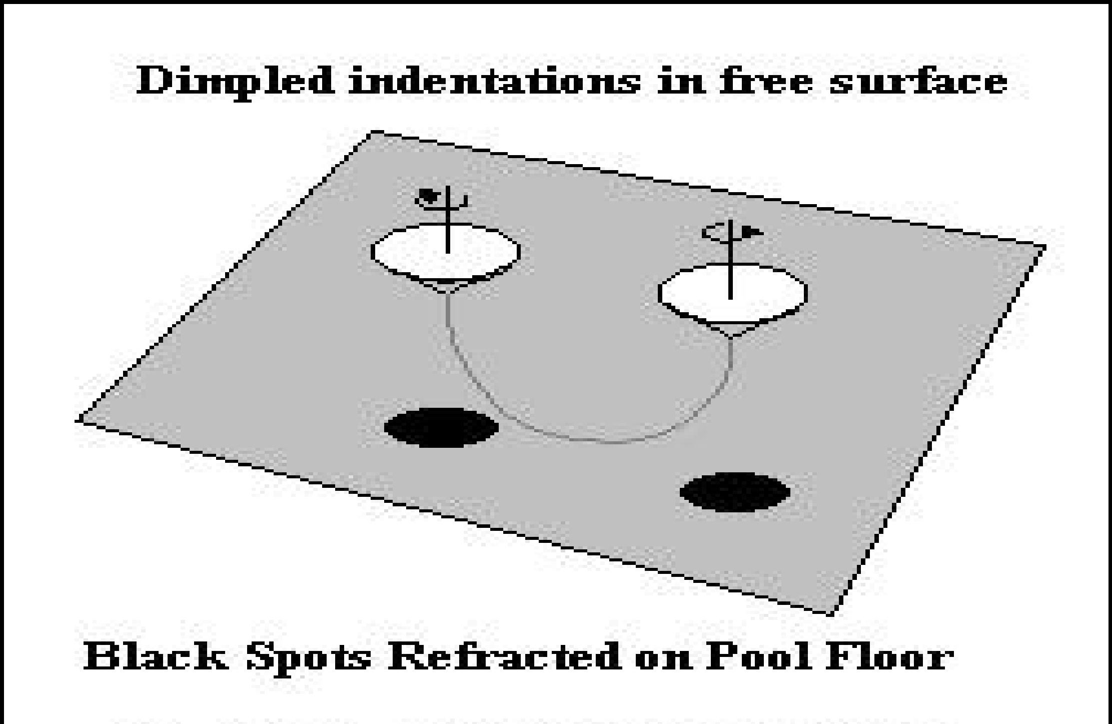
Topological properties such as Pfaff Topological Dimension can be Chiral Sensitive.

FALACO SOLITONS

Cosmic Strings and Black Holes in a swimming pool

FALACO SOLITONS

Geometric Features



FALACO SOLITONS

Geometric Features

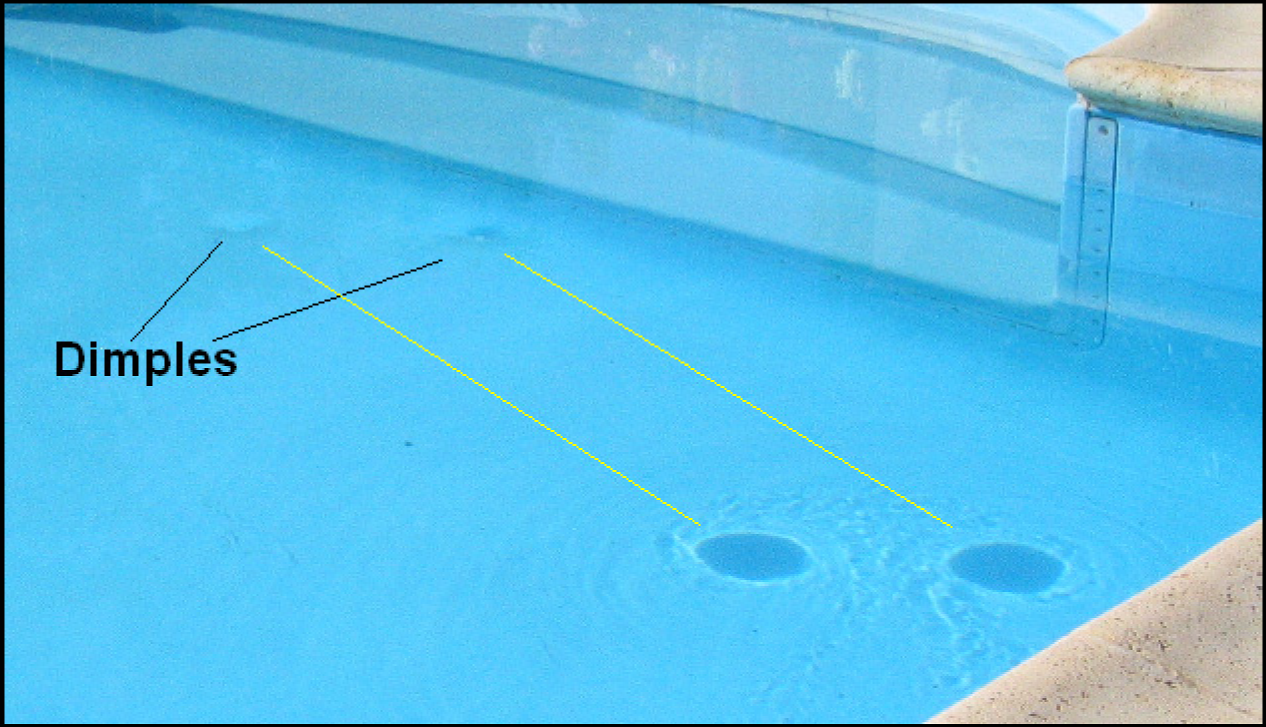
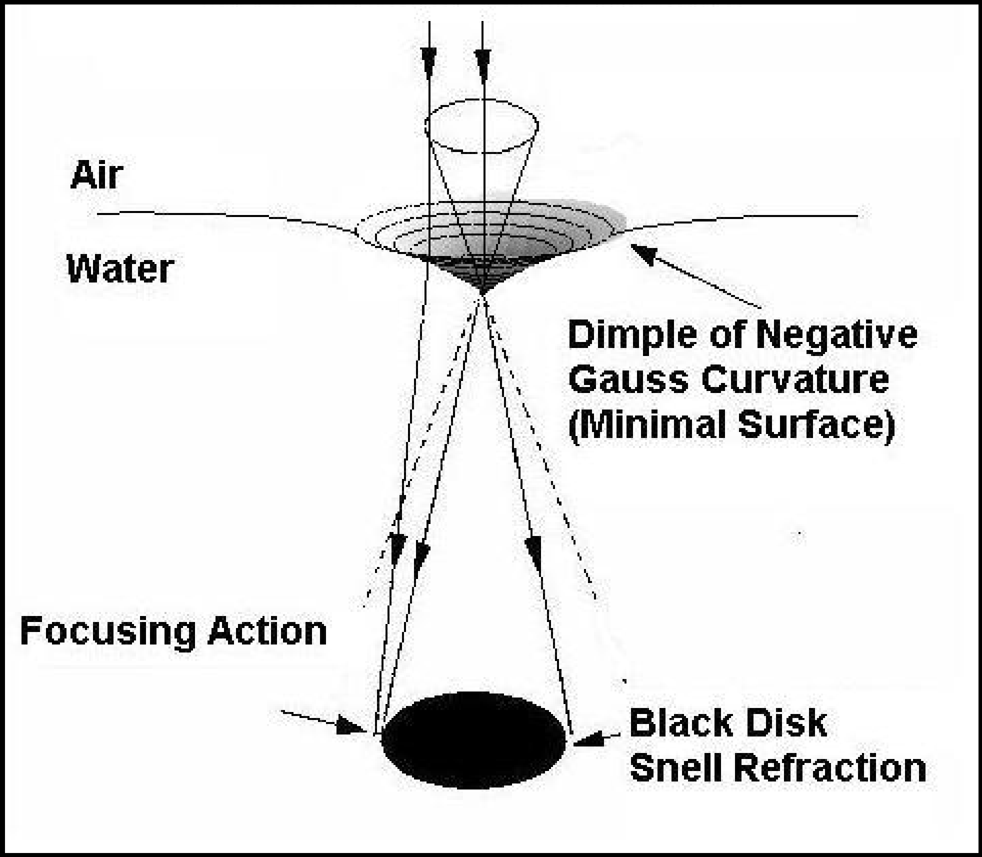


Photo by Best Boy, David Radabaugh Sept 28, 2004

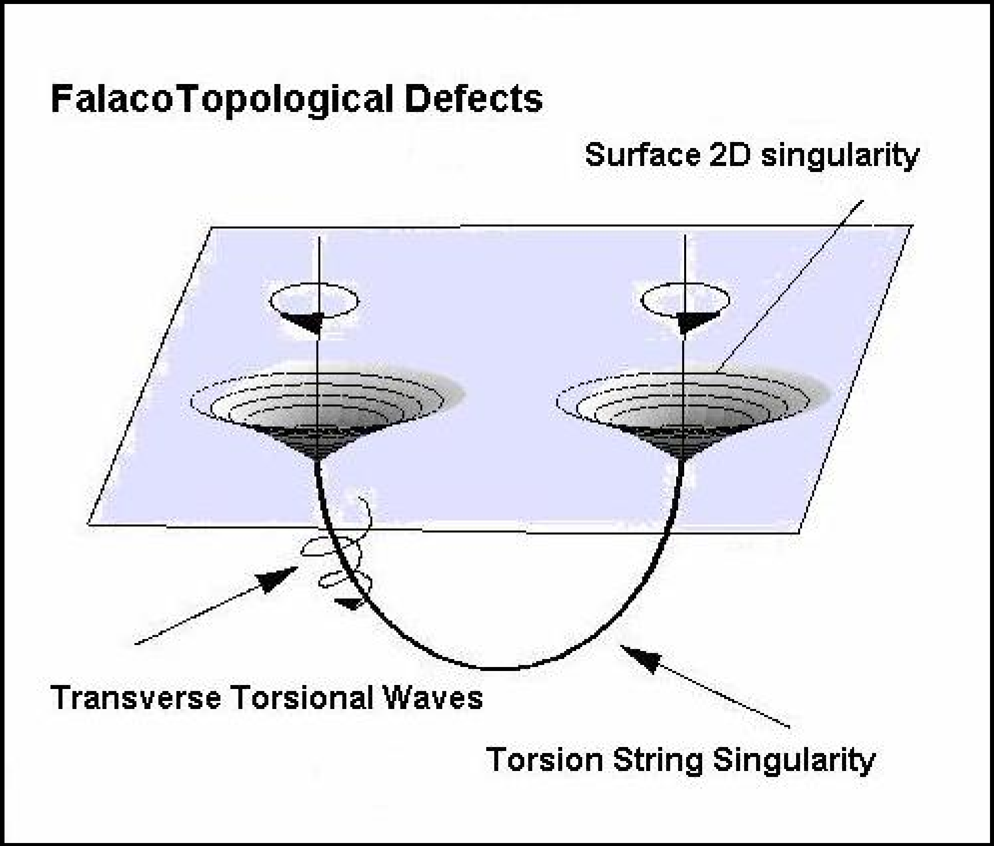
FALACO SOLITONS

Optical Features



FALACO SOLITONS

Topological Features



FALACO SOLITONS

Topological Defects in a swimming pool

I contend that Falaco Solitons are

Universal Dynamical Topological Defects,

or deformation invariants independent from size and shape, forming

Topologically Coherent Structures

of Pfaff Topological dimension 3 or more, at all scales,
and exhibit non-zero

Topological Torsion

Therefore they are non-equilibrium systems
of Pfaff topological dimension 3.

Now some Interesting Conjectures

Application to BOSE CONDENSATES

For All 4th Order Universal Phase Functions,

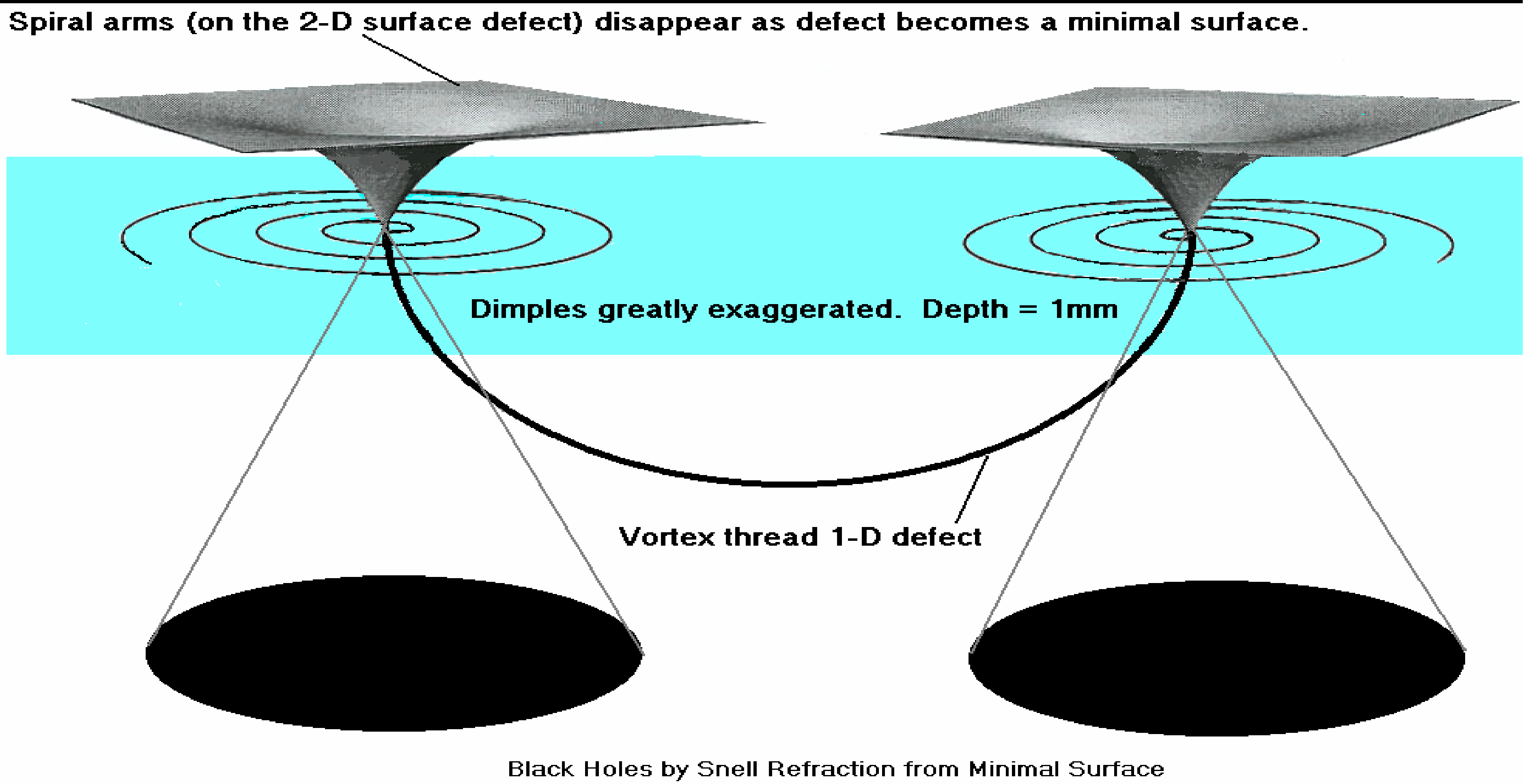
$$\Theta = \Psi^{4} - X_M \Psi^{3} + Y_G \Psi^{2} - Z_A \Psi + T_K = 0$$

the determinant of the Jacobian matrix is given by the similarity coefficient T_K.
Every determinant can be related to the divergence of a current. Hence

$$\mathrm{div}J + \partial\rho/\partial t = \Psi(\Psi^{3} - X_M \Psi^{2} + Y_G \Psi - Z_A)$$

A Complex Ginsburg Landau format

CGL theory of Fermi Surface distributions applied to Falaco Solitons

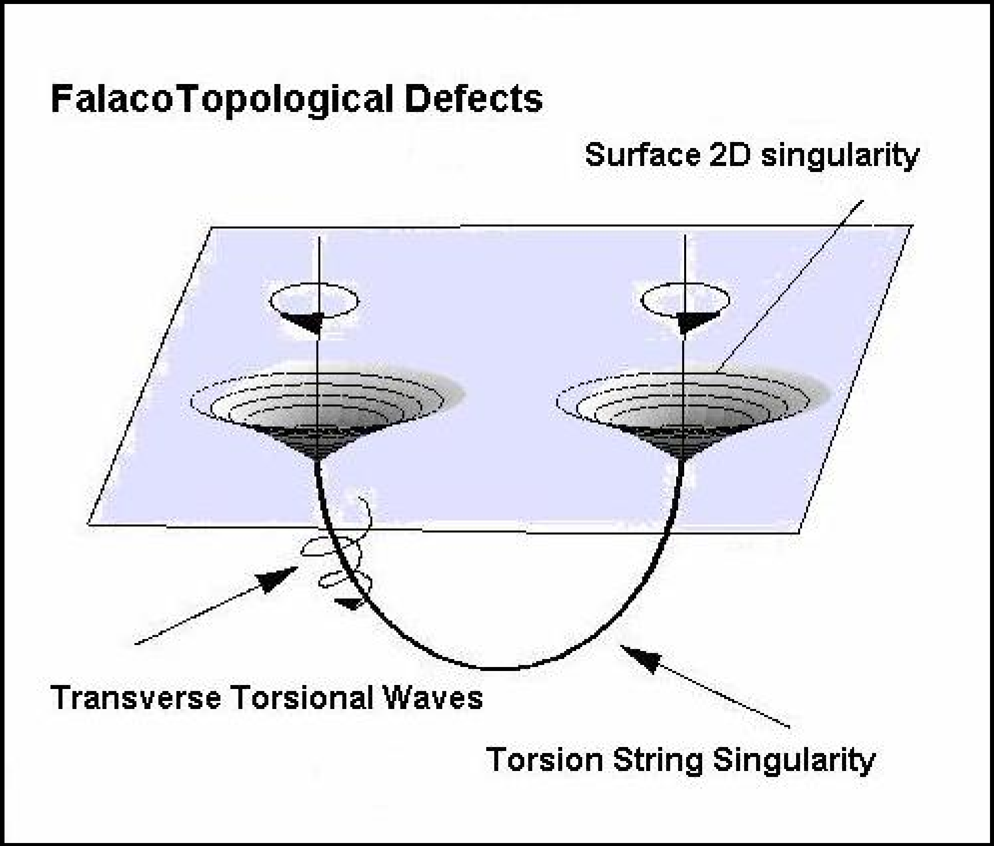


Falaco Solitons

Adapted from O. Tornkvist and E. Schroeder, PRL, 78, 10 1997 p.1980

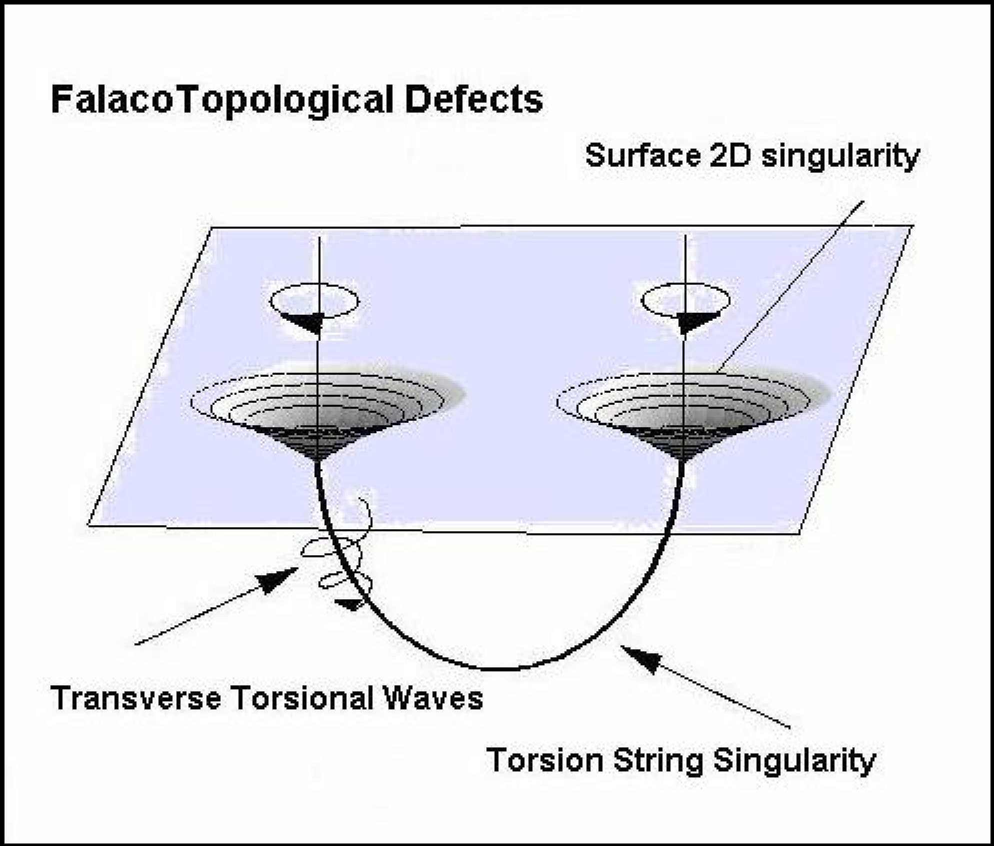
Application to Quantum Gravity

Visual!! 2d branes connected by strings



Application to Elementary Particles

Falaco Solitons as Macroscopic Quarks on the ends of a confinement string.

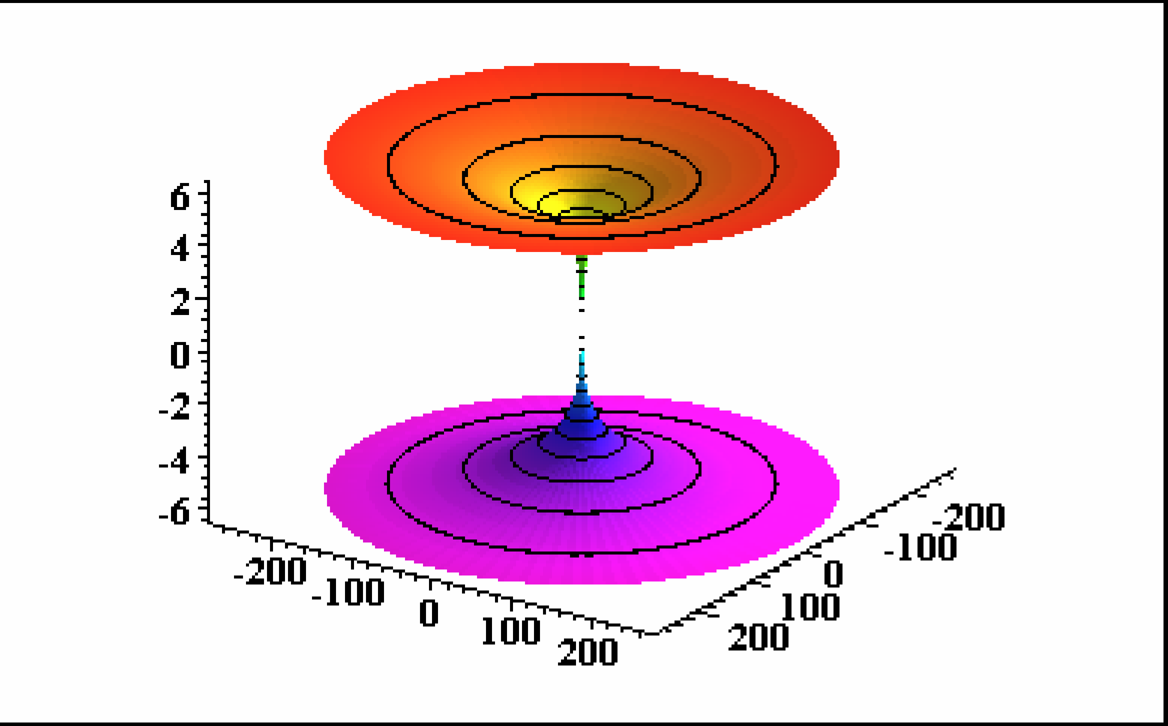


Application to General Relativity

Falaco Solitons offer a macroscopic realization of

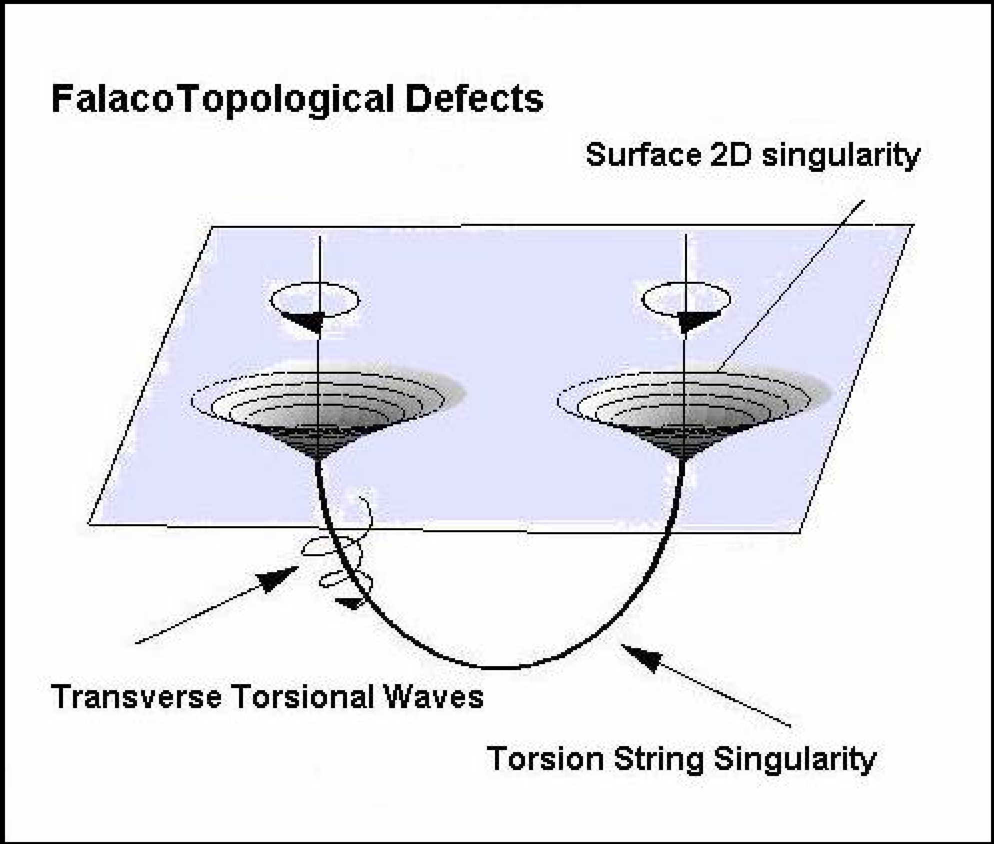
Wheeler Wormholes

Falaco Solitons as alternates to Zeldovich pancakes



Application to Hi Tc Superconductivity

Fermion pairs connected by Falaco strings (Different from the Cooper Pairing mechanism)



Application to Astrophysics

The Universe as a turbulent, dilute, universal (topological) van der Waals gas near its critical point.

Stars and galaxies as long lived topologically coherent structures, far from equilibrium, caused by density fluctuations near critical point.

Correlations of density fluctuations cause $1/r^2$ attraction of defects (Lev Landau).

Thermodynamic non-metrical explanations for dark matter, dark energy and negative pressure.

Application to Galaxy Formation

Spiral Arm Galaxies connected by Falaco strings

Interacting Galaxies NGC 1409 and NGC 1410 **HST • WFPC2**

NASA and W. Keel (University of Alabama) • STScI-PRC01-02

Hurricane Frances

Falaco Soliton in early stages of Formation?

Cubic compatification?

Topological Torsion gone berserk

Note Spiral Arms and Inverse Fractal Dimples

In France they eat such things !!! -- a cross between broccoli and cauliflower

Non Equilibrium Thermodynamics

and

Contact (PTD(A) = 2k+1 > 2)

vs.

Symplectic (PTD(A) = 2k+2 > 2)

manifolds.

Contact Manifolds, n = 2k+1 .

On a Contact Manifold there exists a unique Extremal process V_E (the null eigenvector of dA).

Such processes are Hamiltonian and reversible.

The evolution obeys the Helmholtz-Poincare constraint ("conservation of vorticity")

$$L_{(V_E)}dA = dQ = 0.$$

and all such evolutionary processes are therefore

Thermodynamically Reversible.

If $U = i(V_E)A = 1$, then V_E is called a Reeb field.

Symplectic Manifolds $n = 2k+2$.

On symplectic manifolds extremal fields do not exist.

However, a unique direction field T can be defined in terms of the topological features of the physical system, A:

$$i(T)\{dx\wedge dy\wedge dz\wedge dt\}=A\wedge dA.$$

Processes in the direction of the Torsion Vector, T, are Thermodynamically Irreversible, as

$$L_{(T)} A\wedge L_{(T)} dA = Q\wedge dQ = \Gamma^2 A\wedge dA \neq 0.$$

Lagrangian Example page 1

A Cartan-Hilbert 1-form of Action, A, for a physical system can be written as

$$A = L(t,x,v,p)dt + p \bullet (dx - v\,dt)$$

The k+1 base variables are {t, x}.

The 2k "fiber" variables are {v,p}.

The Lagrange function L(t,x,v,p) is a function of the 3k+1 variables, (t,x,v,p) .

Lagrangian Example page 2

However, direct computation shows that the maximum Pfaff topological dimension is 2k+2,

and the top Pfaffian on the

symplectic manifold is equal to

$$(dA)^{k+1} = (k+1)!\{\partial L/\partial v - p)\bullet dv\} \wedge \Omega_p \wedge \Omega_q \wedge dt$$

$$\Omega_p = dp_1 \wedge .. \wedge dp_n \qquad \Omega_q = dq^1 \wedge .. \wedge dq^n$$

Note that the “symplectic momenta” are NOT canonically defined: $p - \partial L/\partial v \neq 0.$

Lagrangian Example page 3

Evolution starts on the 2k+2 symplectic manifold with orbits being attracted to 2k+1 domains where the momenta become canonical: $p - \partial L/\partial v \Rightarrow 0$.

- Topological evolution can either continue to reduce the Pfaff topological dimension, or
- the process on the Contact 2k+1 manifold can become "extremal", and the topological change stops.

The resulting contact manifold becomes a "stationary" non-dissipating Hamiltonian state, $n > 2$.

"Far from Equilibrium".

Topological Fluctuations 4D

Recall: The kinematic assumption is a Topological Constraint:

$$\Delta x = dx - vdt \Rightarrow 0.$$

The Pfaff Topological Dimension of $\Delta x < 3$

Define a transverse topological fluctuation in position as

$$\Delta x = dx - vdt \neq 0. \qquad (\sim\text{Pressure})$$

The Pfaff Topological Dimension of $\Delta x > 2$

Define a transverse topological fluctuation in velocity as

$$\Delta v = dv - Adt \neq 0. \qquad (\sim\text{Temperature})$$

The Pfaff Topological Dimension of $\Delta v > 2$

On a variety {P,v,x,t} encode a 1-form of Action as:

$$A = L(v,x,t)\, dt + P\,(dx - vdt) = P\, dx + H\, dt \Rightarrow$$

$$A = L(v,x,t)\, dt + P\, \Delta x$$

Topological Fluctuations 4D

Define the topological fluctuation in momenta as:

$$\Delta p = dP - (\partial L/\partial x)\, dt - f\, \Delta x\ .$$

Then compute the elements of the Pfaff sequence:

Action: $A = L(v,x,t)\, dt + P\, \Delta x$

Vorticity: $dA = (\partial L/\partial v - P)\, \Delta v \wedge dt + \Delta p \wedge \Delta x$

Torsion: $A \wedge dA = L\, \Delta p \wedge \Delta x \wedge dt - P(\partial L/\partial v - P)\, \Delta v \wedge \Delta x \wedge dt$

Parity: $dA \wedge dA = -\, 2\, (\partial L/\partial v - P)\, \Delta p \wedge \Delta v \wedge \Delta x \wedge dt$

Topological Fluctuations 4D

The Topological constraint of Canonical Momenta, $P = \partial L/\partial v$ reduces the Pfaff dimension from 2n+2 to 2n+1.

- **Theorem 1**. There exists a unique extremal direction field on the 2n+1 Contact manifold, a Hamiltonian conservative representation, such that the Virtual Work 1-form is zero.
- **Theorem 2 (analogue to Heisenberg)**, The existence of Topological Vorticity and Topological Torsion require that the product of fluctuations in momenta and fluctuations in position is NOT zero.

$$\Delta p \wedge \Delta x \neq 0$$

Topological Fluctuations 4D

Consider the Work 1-form on a 2n+2 manifold, $(\partial L/\partial v - P) \neq 0$.

$$\text{Work} = i(V)dA = (\partial L/\partial v - P)\,\Delta v + \Delta p\hat{}\Delta x$$

Processes on a 2n+2 symplectic manifold require $W \neq 0$.

- **To be a symplectic manifold requires that the first term in the expression for work, W, is not zero. The momenta cannot be canonical, and the Velocity fluctuations must be non-zero. This implies the existence of a non-zero temperature, and leads to the analogue of the Planck concept of a zero point energy on the symplectic 2n+2 topological manifold.**

Electromagnetic Example page 1

Use the 4D electromagnetic 1-form of Action:

$$A = A(x,y,z,t) \bullet dr - \phi(x,y,z,t)dt.$$

Define: $E = -\text{curl } A - \text{grad } \phi(x,y,z,t)$, $B = \text{curl } A$

Construct the 2-form F = dA,

$$F = dA = B_z\, dx \wedge dy \ldots - E_z\, dz \wedge dt \ldots$$

Then the 3-form dF = ddA = 0, and generates the Maxwell-Faraday PDE's.

Electromagnetic Example page 2

Construct the 3-form A^F = A^dA

Topological Torsion: **A ^ dA = i[T, h](dx^dy^dz^dt)**

with the 4 component Torsion Direction Field

T = [T, h] = - [E x A + ϕB, A • B].

(The 4th component **A • B** is often defined as the Helicity density)

Then, $L_{(T)}$ A => (E • B) A = Γ A

Γ = topological dissipation coefficient

(Bulk Viscosity due to expansion and rotation, not affine shears)

Electromagnetic Example page 3

Construct the 4-form F^F = dA^dA :

Topological Parity: **dA ^ dA = -2(E • B) dx^dy^dz^dt).**

On regions where Γ = (E • B) ≠ 0,

- **the Pfaff dimension is 4,**
- **evolution in the direction of the Torsion vector T4 is**

thermodynamically irreversible.

$$L_{(T)} A\wedge L_{(T)} dA = Q\wedge dQ = (-E \bullet B)^2 A\wedge dA \neq 0.$$

(The divergence of the Topological Torsion vector **T4** is equal to -2(**E • B.**)

Electromagnetic Example page 4

On regions where (E • B) = 0,

- the Pfaff dimension is 3,
- and the evolution can proceed in the direction of the extremal field, which is reversible. (The Torsion vector on the 4D base space has zero divergence).

The closed integrals of the 3-form A^dA are deformation (topological) invariants for all processes V on domains of Pfaff dimension 3

(The values of the closed integrals are "topologically quantized")!

Darboux Format 4D

Consider a map Φ from $\{x,y,z,t\} \Rightarrow \{P,H,Q,T\}$ and a 1-form of Action on the target space in Darboux Format,

$$A = PdQ + HdT.$$

By Functional Substitution, pull back the target to the domain

$$A = \mathbf{A}(x,y,z,t) \bullet d\mathbf{r} - \phi(x,y,z,t)dt \Leftarrow A = PdQ + HdT.$$

Then the vector and scalar potentials of the previous electromagnetic example are well defined functions of $\{P_{(x,y,z,t)}, H_{(x,y,z,t)}, Q_{(x,y,z,t)}, T_{(x,y,z,t)}\}$ and their differentials.

Result: All of the topological features of the Darboux representation now have an electromagnetic interpretation.

Darboux Format 4D

To reduce the algebraic complexity, constrain the map such that "time" on the final state is equal to "time" on the initial state: **T= t.** Then the important topological properties of the Darboux format in electromagnetic interpretation are:

$$\mathbf{E} = (\partial P/\partial t\ \nabla Q - \partial Q/\partial t\ \nabla P) - \nabla H, \quad \mathbf{B} = \nabla P \times \nabla Q$$

Frenet Torsion $\Rightarrow$ 0: $\mathbf{A} \bullet \mathbf{B} = 0$

Topological Torsion: $\mathbf{T} = -P\ (\partial Q/\partial t\ \nabla P - \nabla H) \times \nabla Q$

Topological Parity : $\mathbf{E} \bullet \mathbf{B} = \nabla H \bullet \nabla P \times \nabla Q$

Entropy production rate

Construct the 3-form A^F = A^dA

Topological Torsion vector T= [T, h]

A ^ dA = i[T, h](dx^dy^dz^dt)

with the 4 component Torsion Direction Field

$\Gamma^2 \approx$ Entropy Production Rate

$\approx$(1/2 divergence of T) ^{2}

The Sliding - Rolling Ball page 1

Consider a bowling ball with initial translational and rotational energy, thrown to the floor of the bowling alley.

Initially the ball skids or slips on a 2k + 2 symplectic manifold irreversibly reducing its energy and angular momentum via "friction" forces.

From arbitrary initial conditions, the evolution is attracted to a 2k + 1 contact manifold, where the ball rolls without slipping, and the anholonomic constraint vanishes.

$$dx - \lambda \, d\Theta = 0$$

The Sliding - Rolling Ball page 2

The subsequent motion, neglecting air resistance, continues in a Hamiltonian manner without change of Kinetic Energy or Angular Momentum.

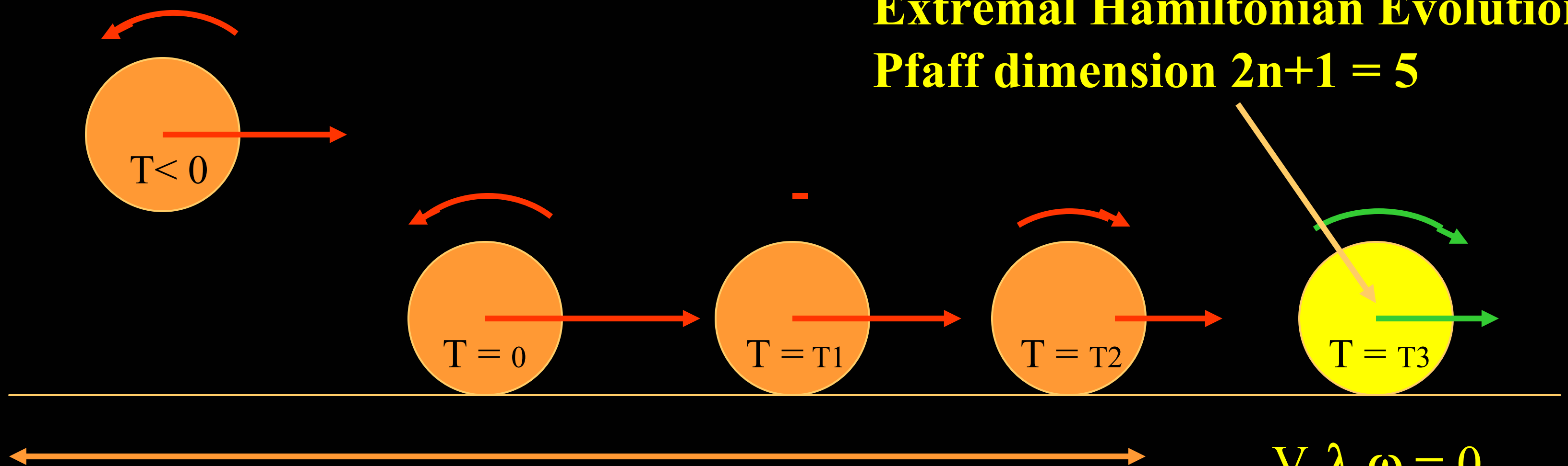
The 1-form of Action can be written as:

$$A = L(t,\mathbf{x}, \Theta,v,\omega)dt + \ldots + \mathbf{s} \bullet (d\mathbf{x} - \lambda\, d\Theta)$$

$$\mathrm{PTD}(A) = 6$$

The Sliding - Rolling Ball page 3

Extremal Hamiltonian Evolution
Pfaff dimension 2n+1 = 5



$V\text{-}\lambda\ \omega = 0$

Irreversible Evolution - Pfaff dimension 2n+2 = 6

$V\text{-}\lambda\ \omega > 0$	$V\text{-}\lambda\ \omega > 0$	$V\text{-}\lambda\ \omega > 0$
$\lambda\ \omega < 0$	$\lambda\ \omega = 0$	$\lambda\ \omega > 0$

Note how friction changes Angular Momentum

Summary

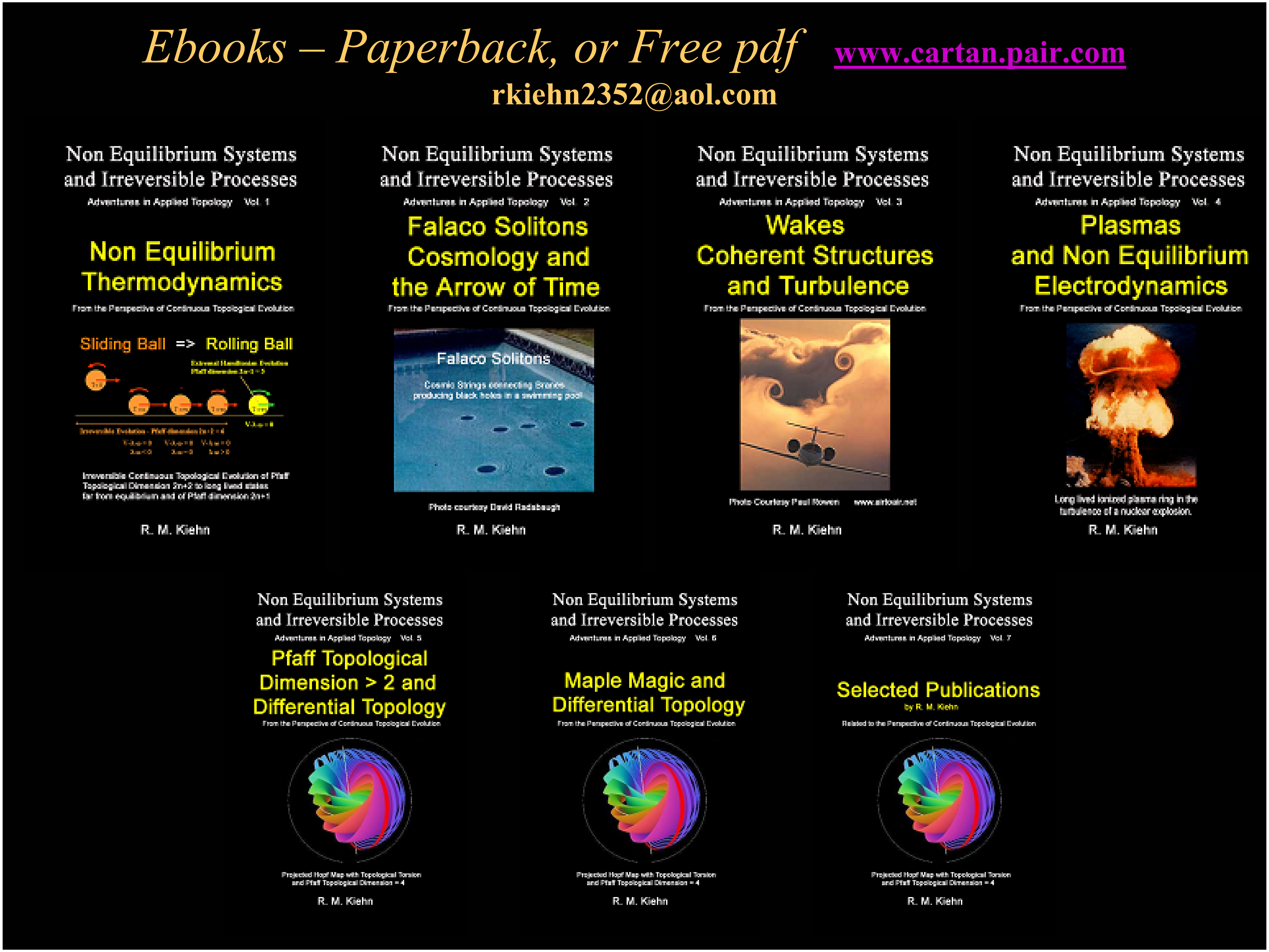
- Without Topological Evolution, there is no Arrow of Time and no Thermodynamic Irreversibility.
- Physical Systems of Pfaff Topological dimension 4 admit a unique continuous thermodynamically irreversible evolutionary process, which can lead to long lived topologically coherent excited states of Pfaff Topological Dimension 3, far from equilibrium. These states appear as topological defects, and can be embedded in turbulent dissipative media.
- Cartan's Magic formula combines the dynamics of continuous topological evolution and the First Law of Thermodynamics, without statistics.

Summary

- Physical systems of Pfaff Topological Dimension greater than 2 are NOT equilibrium systems, and consist of a disconnected topology.
- From the point of view of Continuous Topological Evolution, Thermodynamics, Hydrodynamics, and Maxwell – Faraday Electrodynamics are equivalent.
- The divergence of the Topological Torsion Vector determines a Topological Dissipation coefficient.

Summary

- Topological Fluctuations (of Pfaff Topological Dimension) can cause ultimate decay of states far from equilibrium.
- Recognition of Chirality Symmetry Breaking can lead to many new practical applications.
- The methods of continuous topological evolution could be applied to any synergetic multi-component non-equilibrium system. Examples include Economic, Political, and Biological systems.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Non Equilibrium Systems and Irreversible Processes
Adventures in Applied Topology Vol. 1
Non Equilibrium Thermodynamics
From the Perspective of Continuous Topological Evolution
Sliding Ball => Rolling Ball
Irreversible Continuous Topological Evolution of Pfaff Topological Dimension 2n+2 to long lived states far from equilibrium and of Pfaff dimension 2n+1
R. M. Kiehn
Non Equilibrium Systems and Irreversible Processes
Adventures in Applied Topology Vol. 2
Falaco Solitons Cosmology and the Arrow of Time
From the Perspective of Continuous Topological Evolution
Falaco Solitons
Cosmic Strings connecting Branes producing black holes in a swimming pool
Photo courtesy David Radabaugh
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Long lived ionized plasma ring in the turbulence of a nuclear explosion.
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Pfaff Topological Dimension > 2 and Differential Topology
From the Perspective of Continuous Topological Evolution
Projected Hopf Map with Topological Torsion and Pfaff Topological Dimension = 4
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Maple Magic and Differential Topology
From the Perspective of Continuous Topological Evolution
Projected Hopf Map with Topological Torsion and Pfaff Topological Dimension = 4
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Selected Publications
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Projected Hopf Map with Topological Torsion and Pfaff Topological Dimension = 4
R. M. Kiehn

END AMS – ABQ Presentation

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[redacted]

Thermodynamic Processes page 1

- **Classical Thermodynamics**:

  A Process acting on a Physical System that creates a 1-form of Heat Q, is an irreversible process unless Q admits an integrating factor.

- **Frobenius Theorem**:

  An integrating factor exists iff $Q \wedge dQ = 0$.
  (Pfaff dimension of $Q < 3$.)

History of Falaco Solitons

- **1986 visit to Rio de Janeiro and the mountain side house of my MIT roommate, Jose Haraldo H. Falcao.**

- **On the mountain side above the beach**

History of Falaco Solitons

- Out door living - RC, game room, breakfast room and attached swimming pool of a Brazilian Mansion

- To the swimming pool

Three Topological Projections #1

The Hopf map and Topological Torsion

Pfaff Topological dimension 4

http://hopf.math.purdue.edu/pub/hopf.html created by Ken Shoemake

Example Solutions demonstrate

1. Falaco type solutions satisfy the Navier-Stokes equations for a swirling fluid.

2. Solutions are related to Langford's concept of a tertiary Saddle – Node Hopf bifurcation.

3. Visual Minimal Surface pairs and "string" effects are replicated.

4. Topological Torsion is chiral sensitive, Similarity-Curvature invariants are not.

Details (An exact Solution to the Navier Stokes equations)

Modified Langford SNHopf dynamical system

$$dx/dt = V_x = x\,(G+Cz) \pm \Omega\, y$$

$$dy/dt = V_y = y\,(G+Cz) \pm \Omega\, x$$

$$dz/dt = V_z = A + P\sinh(\alpha z) + D(x^2+y^2)$$

Define Action 1-form (Projective dual)

$$A = V_x\,dx + V_y\,dy + V_z\,dz - (V \bullet V)dt$$

Local Stability Theorem

for Curvature Similarity Invariants

If all eigenvalues of Jacobian have no positive real parts. Then

Linear $X_M \leq 0$ **= 0 for Critical Point**

Quadratic $Y_G \geq 0$, **= 0 for Critical Point**

Cubic $Z_A \leq 0$, **= 0 for Critical Point**

Quartic $T_K \geq 0$. **= 0 for Critical Point**

Local Stability - Hopf map

A Projection 4D to 3D

All eigenvalues of Jacobian matrix are pure imaginary.

$X_M = 0$

$Y_G \geq 0,$ **= 0 for Critical Bifurcation Point**

$Z_A = 0,$

$T_K \geq 0.$ **= 0 for Critical Bifurcation Point**

Global Stability - Falaco Solitons

One zero eigen value, one negative real = b, two complex conjugates with (possible) positive real part, $\sigma \pm i\,\Omega$.

$$\mathbf{X_M = b + 2\,\sigma \leq 0}$$

$\mathbf{Y_G = (\sigma^2 + \Omega^2 + 2b\,\sigma) < 0}$ is possible,

$$\mathbf{Z_A = b\,(\sigma^2 + \Omega^2) \leq 0,}$$

$\mathbf{T_K = 0.}$ **= 0 for Critical Bifurcation Point**

Global Stability - Falaco Solitons

Choose $b < 0$, solve for possible expansion,

positive magnitude of $\sigma = |b|/2$.

Substitute in Y_G

Obtain Falaco stability condition on rotation rate

$$\Omega^2 - \tfrac{3}{4} b^2 \leq 0$$

Similarity-Curvature Invariants

Linear $X_M = 2(G+Cz)+\alpha P\cosh(\alpha z)$

Quadratic $Y_G = +\,\Omega^2 - 2CD(x^2+y^2)+(G+Cz)^2 + 2(G+Cz)P\alpha\cosh(\alpha z)$

Cubic $Z_A = (+\Omega^2 + (G+Cz)^2)P\alpha\cosh(\alpha z) - 2CD(G+Cz)(x^2+y^2)$

Quartic $T_K = 0$

Global Stability - Falaco Solitons

Mean Curvature => 0, Gauss curvature < 0

Leads to a real Minimal Surface

String Tension b

balances

Rotation Expansion

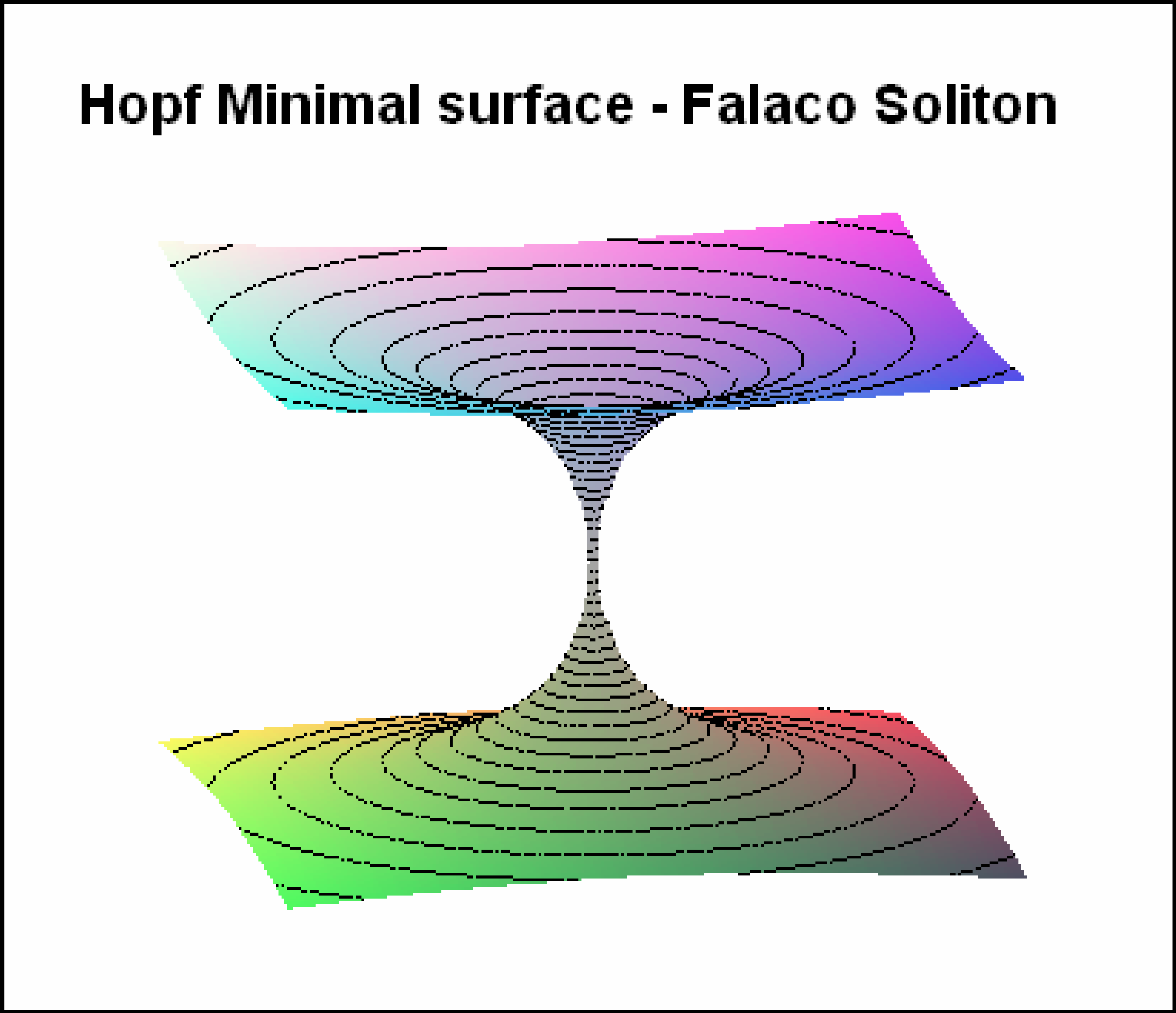
Global Stability - Falaco Solitons

Critical point at $Y_G = 0$

implies

$$(x^2+y^2) = +\alpha^2 P^2 (\cosh(\alpha z))^2/(3|DC|)$$

$$(x^2+y^2) = +\alpha^2 P^2 (\cosh(\alpha z))^2/(3|DC|)$$



Important Lesson from Falaco Solitons

Topologically Coherent, Non Equilibrium

4D "Vortex" Structures

can be related to

Conjugate "Chiral" Minimal Surfaces

Superposition of R and L handed

with To-Fro Expansion Contraction

Think directional and rotational symmetry breaking

See Phys. Rev A 43, (1991) p. 5165 for applications to EM signals

One reason that stimulated me to attend this meeting was the chance to discuss with OKULOV how his idea of

Bifurcation of Helical Symmetries

With

Distinct Chiral Dynamics

could be related, perhaps, to Thermodynamic Phase bifurcations to

FALACO SOLITONS.

Objectives

Use Cartan's universal methods of

Differential Topology and Thermodynamics

to find solutions describing the features of

FALACO SOLITONS

as long lived coherent states far from equilibrium.

Coda: Demonstrate that the non-equilibrium thermodynamic solutions are among those that satisfy the

Navier-Stokes Equations.

[redacted]

Cartan's Magic Formula

Define the exterior differential forms:

Work = W= i(V)dA, Energy = U= i(V)A, Heat = Q.

Then Cartan's Magic Formula of CTE,

$$L_{(V)}A = i(V)dA + di(V)A = Q,$$

becomes the First Law of Thermodynamics,

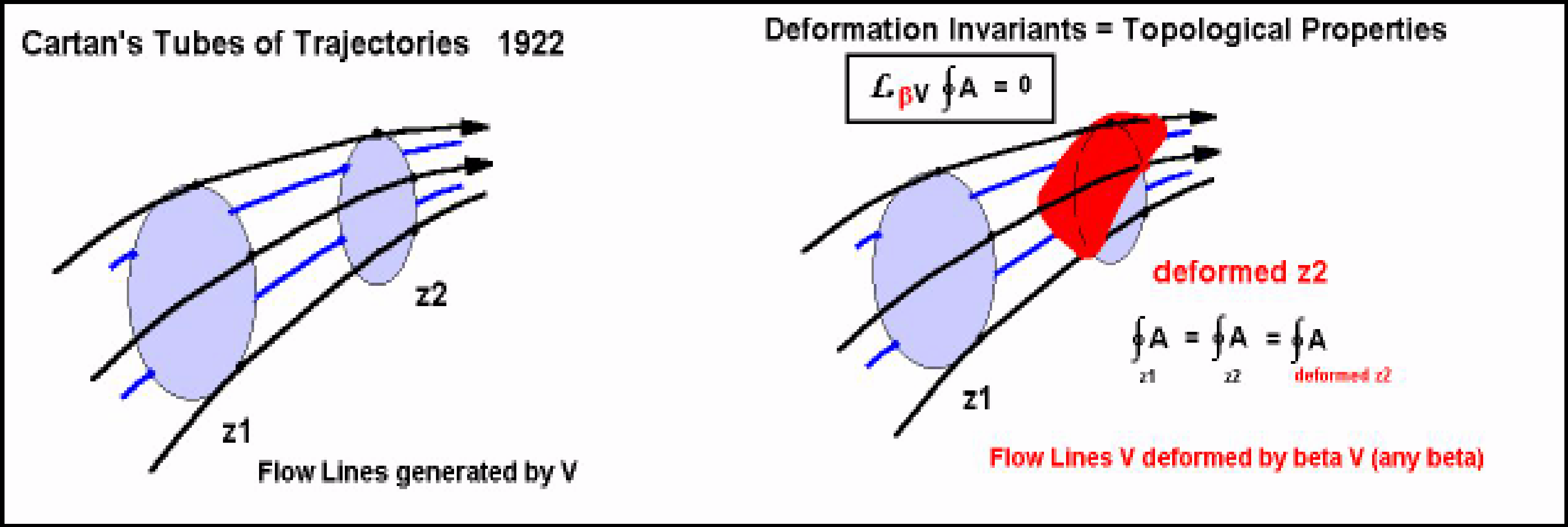
$$L_{(V)}A = W + dU = Q,$$

connecting Dynamical Systems and Thermodynamics.

($L_{(V)}A$ is the Lie differential with respect to the direction field V acting on the 1-form A)

Cartan's Magic Formula

Cartan used an integral version of his Lie derivative formula - in terms of hydrodynamic flow along a tube of trajectories - to prove that all Hamiltonian processes preserve the closed integrals of Action, and conversely.



Thermodynamic Processes page 2

For a Process V acting on a Physical System represented by a 1-form of Action, A:

A Dynamical Test for a Reversible Process is

$$L_{(V)}A \wedge L_{(V)}\, dA = Q \wedge dQ = 0.$$

A Dynamical Test for an Irreversible Process is

$$L_{(V)}A \wedge L_{(V)}\, dA = Q \wedge dQ \neq 0.$$

($Q \wedge dQ = 0$ implies a Pfaff dimension of <3; $Q \wedge dQ \neq 0$ implies a Pfaff dimension of 3 or more.)

Thermodynamic Processes page 3

All classic Hamiltonian, Symplectic, Bernoulli and Stokes Processes, satisfy the Helmholtz-Poincare constraint ("conservation of vorticity").

$$L_{(V)}dA = dQ = 0.$$

and are therefore

Thermodynamically Reversible.

as

$$L_{(V)}A \hat{}\ L_{(V)}\, dA = Q \hat{}\, dQ = 0.$$

Pfaff Topological Dimension

A physical system for which the 1-forms of Action, **A**, of Work, **W**, and Heat, **Q**, are all of PTD = 4 defines a dissipative, non-equilibrium turbulent system.

The Jacobian matrix of **A** has a characteristic polynomial with 4 non-zero roots. If the system evolves to regions of PTD = 3, then the Jacobian matrix has a characteristic cubic polynomial with three non-zero roots. The similarity invariants of the polynomial can be related to the curvatures of a hyper surface whose normal field is proportional to the components of **A**.

The cubic polynomial defines a universal

Pfaff Topological Dimension

Hence all thermodynamic systems of PTD = 4 have topological defect structures of PTD = 3, which are universally related topologically to a van der Waals gas.

The similarity invariants are related to the linear Mean Curvature, the quadratic Gauss curvature and the cubic Adjoint curvature.

The critical point occurs when M=0, G=0, A=0, K=0

The Spinodal line when G=0, M=0

The Binodal line occurs when

Cosmological Conclusion

The universe can be represented by a dilute dissipative non equilibrium turbulent system of PTD = 4, with topological defects of PTD = 3.

PTD = 3 implies a universal van der Waals gas. The similarity invariants of the cubic polynomial are adjusted to represent the neighborhood of the critical point.

The topological defects are the stars representing extreme density fluctuations attracted by a Newtonian force law.

Van der Pol Example

Perhaps the most famous limit cycle is that of the Van der Pol Oscillator. The evolutionary decay from arbitrary initial conditions {x,y,t} to a unique attracting limit cycle {f(x,y),t} is an obvious example of continuous topological change.