Conservative, Dissipative and Irreversible Processes

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Details and Examples at

http://www.uh.edu/~rkiehn/carfre12.htm

You are invited to visit Cartan's Corner
http://www.uh.edu/~rkiehn

Abstract: When a mechanical or hydrodynamic system, with n degrees of freedom, is modeled in terms of the Cartan-Hilbert-Lagrange differential 1-form of Action, including anholonomic differential constraints, the resulting topological domain is a 2n+2 dimensional non-compact symplectic manifold. Imposition of the constraints of canonical momentum reduce the manifold to the familiar 2n+1 dimensional space of states. On this 2n+1 domain there exists a unique vector field that leaves the Action integral stationary. A unique extremal vector field, similar to that which describes reversible conservative processes on the space of states, does not exist on the 2n+2 symplectic domain. However, there does exist a unique Torsion vector field which does not leave the Action integral stationary, but only conformally invariant. Processes represented by the unique Torsion field are irreversible in a thermodynamic sense. In addition to supporting non-canonical momentum, the symplectic domain must support anholonomic differential fluctuations, $d\mathbf{v}-\mathbf{a}dt \neq 0$ in the velocity and/or in position, $d\mathbf{x}-\mathbf{v}dt \neq 0$. The implication is that such dissipative evolution can not be described kinematically in terms of a single parameter group. Such symplectic systems admit a non-zero temperature gradient and/or a non-zero pressure gradient with a zero point energy. These domains can act as a source of magnetic dynamo action in a plasma, producing an acceleration mechanism in electromagnetic domains where $\mathbf{E} \bullet \mathbf{B} \neq 0$. Anholonomic differential fluctuations lead to the dissipative Navier-Stokes equations. Using Cartan's magic formula, it is possible to deduce a simple non-statistical test to see if a given evolutionary processes is irreversible in a thermodynamic sense. For example, conformal processes in the direction of the Torsion vector produce a heat 1-form, Q, which is non-integrable and does not admit an integrating factor. Therefore such process are irreversible. The symplectic manifold does admit vector fields that leave the Action integral stationary, but such fields are not unique and can lead to a hierarchy of stationary states. The subset of stationary symplectomorphisms are reversible in the thermodynamic sense described above. As the key feature of the turbulent state admitted to by everyone is irreversibility, turbulence cannot be represented by a symplectomorphism. Arguments are presented that turbulence cannot occur on a domain of less than four variables.

Some Observations:

1. **Non-isolated thermodynamic systems often decay irreversibly into self-organized coherent states.**

   (Creation of coherent structures in a turbulent flow)

2. **Dynamical systems generated by vector fields exhibit attractors.**

3. **Dissipative Hamiltonian processes are not thermodynamically irreversible.**

   (A damped harmonic oscillator does not appear to be dissipative in a shrinking coordinate system.)

Objective of this talk

SHOW HOW THESE OBSERVATIONS CAN BE EXPLAINED

(without statistical camouflage)

In Terms Of the Concept of

TOPOLOGICAL TORSION

on a symplectic domain of 2n+2 dimensions

producing a dynamical definition
of thermodynamically irreversible processes
that indicates that

TURBULENCE
and
OTHER IRREVERSIBLE PROCESSES
OCCUR ONLY ON SPACES OF PFAFF TOPOLOGICAL DIMENSION > 3

TOOLS:

1. Ultimate Domain of definition ≡ {x,y,z,t}
2. Physical System ≡ 1-form of Action, A
3. A Process ≡ A Vector Field, V

INTUITIVELY:

Irreversible process in 2n+2 will decay to "stationary" states of measure zero.
(i.e.; 2n+1 domains)

The stationary state is a deformable Topologically Coherent Structure.

MATHEMATICALLY:

The physical system (that is the 1-form of Action A) defines a 4D symplectic volume element (dA^dA) with a measure (density) ρ.

The topologically coherent structure

$$\oiiint_{boundary} A\wedge dA$$

is a "stationary state" (evolutionary invariant) on the 3 dimensional sets of measure zero (ρ=0).

Concepts associated with
IRREVERSIBLE EVOLUTION

Decays To

Turbulent State ⇨ Coherent State

Pfaff Dimension 4 ⇨ Pfaff Dimension 3

Symplectic Manifold ⇨ Contact Manifold

Differential Anholonomic Fluctuations

$\Delta x = dx - vdt \neq 0 \quad \Rightarrow \quad \Delta x = dx - vdt = 0$

$\Delta v = dv - adt \neq 0 \quad \Rightarrow \quad \Delta v = dv - adt = 0$

Irreversibility implies topological evolution on a domain of Pfaff dimension 4 $\quad dA \wedge dA \neq 0$

On this symplectic domain there exists a 3-form of

TOPOLOGICAL TORSION $\quad A \wedge dA \neq 0$

1. **Torsion Defects**
2. **Non-Uniqueness, Envelopes, Regression**
3. **Non-Riemannian Geometries, Projectivized Finsler Spaces.**

Fundamental Assumptions:

1. **Physical systems can be described adequately by a Lagrangian with suitable non-holonomic constraints.**

Implies the existence of a 1-form of Action, $A = L(x,v,t)dt + p(dx-vdt)$

2. **The fundamental equation of evolution is given by Cartan's Magic Formula.**

$$\mathcal{L}_{(V)}A = i(V)dA + d(i(V)A) = Q$$

3. **The correspondence between Cartan's magic Formula and the First Law of Thermodynamics is to be taken literally.**

$$\mathcal{L}_{(V)}A = W + d(U) = Q$$

4. **A process is irreversible in the thermodynamic sense iif the 1-form of heat, Q, does not admit an integrating factor.**

$$Q \wedge dQ <> 0$$

Non-Holonomic Differential Fluctuations

Consider a 1-form of Action for a fluid system constrained with non-holonomic differential fluctuations, $\Delta x = dx - Vdt \neq 0$.

$$A = L(x,v,t)dt + p \bullet (dx - V\, dt)$$

If all of the variables (including the Lagrange multipliers, p) are independent, the domain of definition is 10 dimensional, (x, v, p; t}.

For the 10 dimensional velocity vector V = {v, a, f; 1}, the virtual work 1-form becomes :

$$W = i(V)dA =$$
$$(f - \partial L/\partial x)\{dx\text{-}vdt\} + (p - \partial L/\partial v)\{dv\text{-}adt\}.$$

But on a symplectic manifold the virtual work 1-form, W, cannot vanish. Therefore the non-holonomic differential fluctuations of either position, $\Delta x = \{dx\text{-}vdt\}$, and/or velocity, $\Delta v = \{dv\text{-}adt\}$ cannot vanish, and neither can their coefficients.

The non-holonomic differential fluctuations $\Delta x = \{dx\text{-}vdt\}$, and/or velocity, $\Delta v = \{dv\text{-}adt\}$ become the 4D differential topology generalizations of Langevin noise.

The theory of non-holonomic fluctuations
leads to a symplectic manifold of
topological dimension 2n+2.

Evolution in the Direction of the
Unique Topological Torsion Vector
on the 2n+2 D Symplectic Manifold
induces

Exponential (irreversible) Decay

to sets of measure zero on the symplectic manifold: the inertial manifold attractors.

The attractors are defects in 2n+2,
but form a 2n+1

Contact Manifold
with a unique extremal field.

The non-holonomic fluctuations vanish on the Contact Manifold.

Fundamental Theorems:

The components of the 3-form A^dA define the Torsion Current, T

$$i(T)(dx\wedge dt\wedge dz\wedge dt) = A\wedge dA$$

Theorem 1.

An evolutionary vector field V

(on the symplectic manifold $dA\wedge dA \neq 0$)

in the direction of the Torsion vector is irreversible in the thermodynamic sense.

Theorem 2.

The three dimensional integral,

$$\oiiint_{boundary}\{T_x dy\wedge dz\wedge dt ... - T_t dx\wedge dy\wedge dz\}$$

(integration defined on the sets of $dA\wedge dA \approx Div_4 T \Rightarrow 0$)

Is a Relative Integral Evolutionary Invariant

Corollary:

Topological defects = $\oiiint_{cycle}\{T_x dy\wedge dz\wedge dt ... - T_t dx\wedge dy\wedge dz\}$

come in pairs, with rational ratios.

TOPOLOGICAL CLASSIFICATIONS
of
PHYSICAL SYSTEMS

The Pfaff Sequence and Pfaff Dimension

Given a 1-form of Action, and its closure dA, compute algebraic terms until you get a zero.

{A, dA, A^dA, dA^dA....}

Number of terms, n = Pfaff dimension

(or class of the 1-form)

If n=2m+1, A defines a Contact manifold.
If n = 2n+2, A defines a Symplectic manifold.
(The symplectic manifold is not compact without boundary!)

On Pfaff dimension n = 4 space,

Topological Torsion ≡ A^dA

equivalent to a 4 dimensional current.

Given A, the Torsion Current is uniquely determined on the symplectic manifold.

Gauge independent !

THE TORSION VECTOR

The Action defines the Torsion Vector and its Divergence. Non-zero divergence is required for the existence of a symplectic 4D manifold.

$$di(T)dx\wedge dy\wedge dz\wedge dt = dA\wedge dA$$

or

$$(Div_4 T)dx\wedge dy\wedge dz\wedge dt = \rho(x,y,z,t)\ dx\wedge dy\wedge dz\wedge dt$$

The Singular Set: $Div_4 T = \rho(x,y,z,t) \Rightarrow 0$

defines a set of measure zero on 4D space time.

(The attractor or inertial manifold)

Topological (Pfaff) Dimension becomes 2n+1 = 3

Properties:

1. **Torsion Vector is Orthogonal to the Action.**

2. **Torsion vector generates exponential decay of momenta.**

3. **Conformal invariance:** $L_{(T)}A = \Gamma\ A$

Many uses of CARTAN's Magic Formula:

$$\mathcal{L}_{(V)}A = i(V)dA + d(i(V)A) = Q$$

1. Describes topological evolution

2. Describes "stationary" paths if Q = 0

3. Is equivalent to the First Law of Thermodynamics.

$$\mathcal{L}_{(V)}A = W + d(U) = Q$$

$$W = i(V)dA \equiv \quad \text{1-form of virtual Work}$$

Axiom T: A thermodynamic process is irreversible when the Heat 1-form Q does not admit an integrating factor.

Criteria for existence of Integrating Factor
Frobenius Condition ≡ $Q \wedge dQ = 0$.

Therefore

$$L_{(V)}A \wedge L_{(V)}dA = Q \wedge dQ \neq 0$$

implies

Process V is irreversible

on Physical System represented by A the 1-form of Action.

Processes in the direction of the Torsion Current are irreversible.

$$L_{(T)}A \wedge L_{(T)}dA = \Gamma^2 A \wedge dA \neq 0$$

on the symplectic manifold.

Classification of Processes
based on concept of Virtual Work, W

REVERSIBLE PROCESSES:

Hamiltonian

$$W = i(V)dA = 0 \quad \text{Extremals}$$

Unique on 2n+1 Contact Manifold
Does not exist on 2n+2 Symplectic Manifold.
Conservative system with a domain constant

$$W = i(V)dA = d\Theta \quad \text{Bernoulli-Casimirs}$$

Non-unique on 2n+2 symplectic manifold.
Bernoulli function is an evolutionary but not a domain constant.
Process can be dissipative, but not thermodynamically irreversible.

Symplectic

$$dW = d(i(V)dA) = 0 \quad \text{Helmholtz Symplectic}$$

Gauge dependent. Note: Work 1-form is closed – forces without curl component)

FOR ALL SUCH PROCESSES

$$L_{(V)}A \wedge L_{(V)}dA = Q \wedge dQ = 0$$

Hence, they are

thermodynamically reversible.

(even though apparently dissipative in certain coordinate systems)

IRREVERSIBLE PROCESSES:

$$W = i(V)dA = \Gamma A \qquad \text{Conformal}$$

(Intuitive assumption of rmk 1974 article)

But from the definition of the Torsion Current,

$$L(T)A = \Gamma A , \qquad i(T)A = 0$$

or $$i(T)dA = \Gamma A$$

translated to engineering terms:

$$dv/dt \approx -\sigma v..$$

a result which implies

Exponential decay to set of measure zero.

In general

$$L_{(T)}A \wedge L_{(T)}dA = Q\wedge dQ = \Gamma^2 A\wedge dA \neq 0$$

$\therefore$ Evolution along T is irreversible !!!

THERMODYNAMIC IRREVERSIBILITY

EXAMPLES

Example 1 MAXWELL THEORY

Action: $A = A_x dx + A_y dy + A_z dz + \varphi dt \Leftarrow pdq - Hdt$

$dA = B_z dx \wedge dy ... E_x dx \wedge dt ...$

with $B = \text{curl } A$, $E = -\text{grad } \phi - \partial A/\partial t$

$ddA \Rightarrow 0$ (yields Maxwell's equations)

$\text{Curl } E + \partial B/\partial t = 0$

$\text{div } B = 0$

$A \wedge dA \Rightarrow (E \times A + B\varphi, A \bullet B)$ --- the Torsion Vector

$dA \wedge dA = -2(E \bullet B)\, dx \wedge dy \wedge dz \wedge dt$

$\text{Div}_4\, T = -2(E \bullet B)$

When $(E \bullet B) \neq 0$, the domain is 4D symplectic !

E field has component parallel to B field.

Example 2. Action with NAVIER STOKES Constraints

Action $A = v_x dx + v_y dy + v_z dz - (v \bullet v/2 + \phi)dt \Leftarrow pdq - Hdt$

Classification of various flows via Virtual Work

$$W = i(V)dA = (\partial v/\partial t + \mathrm{grad}(v \bullet v/2 + \phi) - v \times \mathrm{curl}\, v) \bullet dr$$

(spatial part = the convective derivative !)

$$+ ([(\partial v/\partial t + \mathrm{grad}(v \bullet v/2 + \phi)] \bullet v)dt$$

(the timelike part = the dissipative power)

$W \Rightarrow 0$ (Extremal Case gives Euler fluid equations)

$W \Rightarrow \mathrm{grad}(P/\rho)$ (Symplectic Case gives Barotropic fluid)

Bernoulli-Casimir function is (P/ρ)

Enthalpy evolutionary invariant is $(P/\rho + v \bullet v/2 + \phi)$

$W \Rightarrow \mathrm{grad}(P/\rho) + \nu \nabla^2 v$ (Navier Stokes constraints)

special case (dissipative but reversible when dW = 0)

The Torsion Vector becomes:

$$T \approx \{(v \bullet \mathrm{curl} v)v - (v \bullet v/2)\mathrm{curl} v - \nu\, \mathrm{curl}\, \mathrm{curl} v;\ v \bullet \mathrm{curl} v\}$$

for a Navier - Stokes system (note helicity term $v \bullet \mathrm{curl} v$)

When $\{\mathrm{curl}\, v \bullet \mathrm{curl}\, \mathrm{curl}\, v\} \neq 0$
domain is 4D symplectic (!)
and the flow v is irreversible.

Signature of Symplectic manifold would be given by defect lines of vorticity in the shape of twisted helices.

(Recently observed by Kuibin, et. al.)

EXAMPLE 3. The sliding ball.