Torsion-Helicity and Spin as Topological Coherent Structures in Plasmas

Emphasizing the Topological difference
between Torsion-Helicity and Spin.
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R. M. Kiehn
University of Houston
Physics Department
http://www.un.edu/~rkiehn/car/carhomep.htm
Full Article as a pdf download at
http://www.un.edu/~rkiehn/pdf/chapman.pdf
(with example solutions to Maxwell's Equations)

Electromagnetism $\cong$ Two Topological Species

Existence of Potentials $\{\mathbf{A}, \phi\}$ implies

the domain of support for

Field Intensities $\mathbf{F}(\mathbf{E}, \mathbf{B})$

$$\mathbf{A} \Rightarrow \mathbf{F} = d\mathbf{A}$$

NOT COMPACT without BOUNDARY

(Open or Compact with boundary.)

Conserved Charge-Current density $\{\mathbf{J}, \rho\}$

implies the existence

Field Excitations $\{\mathbf{D}, \mathbf{H}\}$

$$\mathbf{G} \Rightarrow \mathbf{J} = d\mathbf{G}$$

the domain of support for $\mathbf{G}(\mathbf{D}, \mathbf{H})$

can be compact without boundary.

Two Species of

EM Coherent Structures

Definition: A coherent structure is a deformable integrable domain with the same observable topological features; i.e., **An Evolutionary Deformation Invariant**:

Example 1: $\iiiint_{closed} \textbf{Poincare2}\ d\Omega_4$

Where Poincare2 :=

$$2\{\mathbf{E} \circ \mathbf{B}\}$$

Example 2: $\iiiint_{closed} \textbf{Poincare1}\ d\Omega_4$

Where Poincare1 :=

$$(\mathbf{B} \circ \mathbf{H} - \mathbf{D} \circ \mathbf{E}) - (\mathbf{A} \circ \mathbf{J} - \rho\phi)$$

Two Topological Currents:

Torsion-Helicity and Spin

Four component 3rd rank tensors
or Topological 3-forms:

The Spin current:

$$\boldsymbol{Spin_4} = \mathbf{A} \times \mathbf{H} + \phi\mathbf{D}, \mathbf{A} \circ \mathbf{D}$$

$$\mathbf{Spin}_{3-form} = \mathbf{A}\wedge\mathbf{G}$$

The Torsion (or Helicity) Current:

$$\boldsymbol{Torsion_4} = \mathbf{E} \times \mathbf{A} + \phi\mathbf{B}, \mathbf{A} \circ \mathbf{B}$$

$$\mathbf{Torsion}_{3-form} = \mathbf{A}\wedge\mathbf{F}$$

Note: Magnetic Helicity is only the 4th component of a third rank tensor field.

Conservation Laws for

Torsion and Spin

When the 4-Divergences of the Spin and Torsion currents vanish, the closed integrals of the 3-forms form evolutionary coherent structures.

$$\mathbf{4}\boldsymbol{Div}\,\mathbf{S_4} \equiv \boldsymbol{d}(\boldsymbol{A}\wedge\boldsymbol{G}) = \boldsymbol{Poincare}\mathbf{1}$$

$$\mathbf{4}\boldsymbol{Div}\,\mathbf{T_4} \equiv \boldsymbol{d}(\boldsymbol{A}\wedge\boldsymbol{F}) = \boldsymbol{Poincare}\mathbf{2}$$

For definitions of the Poincare invariants, see page 3

In particular, when $\mathbf{E.B} = \mathbf{0}$

$$\iiint_{closed}(\mathbf{E}\times\mathbf{A}+\phi\mathbf{B})_x dy\wedge dz\wedge dt + \ldots\mathbf{A}\circ\mathbf{B}dx\wedge dy\wedge dz$$

is the Topological Helicity Invariant, not

$$\iiint\mathbf{A}\circ\mathbf{B}dx\wedge dy\wedge dz$$

The Source of Torsion-Helicity change (formation of Links and Knots) is the non-zero divergence of the Torsion Helicity current: A^F.

1. A necessary condition for the evolutionary change of the Torsion-Helicity integral (the number of Links and Knots) is that the Second Poincare invariant does not vanish, $\mathbf{E} \circ \mathbf{B} \neq 0$.
2. When $\mathbf{E} \circ \mathbf{B} = 0$, the ratios of the Torsion-Helicity closed integrals are rational. (Topological counters)
3. The value of the second Poincare invariant is insensitive to gauge additions, but not to renormalization.
4. There exist an infinite number of integrating (renormalization) factors for the 3-form A^F. (Example: divide A by any Holder Norm homogeneous of degree 1)

Evolutionary Processes

Define an evolutionary process **V** in terms of a 4-vector field on space time.Evolution of differential forms obey

Cartan's Magic Formula

$$\boldsymbol{L}(\boldsymbol{V})\boldsymbol{A} = \boldsymbol{i}(\boldsymbol{V})\boldsymbol{dA} + \boldsymbol{d}(\boldsymbol{i}(\boldsymbol{V})\boldsymbol{A}$$

$$= \boldsymbol{W} + \boldsymbol{dU} = \boldsymbol{Q}$$

Define a Plasma process as a process that preserves the number of charges.

$$\boldsymbol{L}_{(\boldsymbol{V})}\boldsymbol{G} = \boldsymbol{0} \supset \boldsymbol{J} = \rho\boldsymbol{V}$$

The Ideal Plasma Processes.

Define an Ideal Plasma process as a process that preserves the number of charges and is Force Free (the Virtual Work vanishes).

1. $\mathbf{L}_{(\rho\mathbf{V})}\mathbf{G} = \mathbf{0} \supset \mathbf{J} = \rho\mathbf{V}$.
2. $\mathbf{W} = \mathbf{i}(\rho\mathbf{V})\mathbf{dA} = \mathbf{0} \supset \mathbf{E} + \mathbf{V} \times \mathbf{B} = \mathbf{0}$.

Zero Virtual Work implies

$$\mathbf{E} \circ \mathbf{B} = \mathbf{0}$$

Hence the closed integrals of Torsion-Helicity current are topological invariants of such *Extremal* processes.

Semi-Ideal Plasma Processes.

Define a Semi-Ideal Plasma process as a process that preserves the number of charges and for which the Virtual Work is exact. (Symplectic, NOT extremal)

1. $\mathbf{L}_{(\rho\mathbf{V})}\mathbf{G} = \mathbf{0} \supset \mathbf{J} = \rho\mathbf{V}$.
2. $\mathbf{W} = \mathbf{i}(\rho\mathbf{V})\mathbf{dA} = \mathbf{d}\Theta \supset \mathbf{E} + \mathbf{V} \times \mathbf{B} = \nabla\Theta$.

For the Virtual Work to be exact

$$\mathbf{E} \circ \mathbf{B} = \mathbf{B} \circ \mathbf{grad}\Theta$$

and

$$\mathbf{E} \circ \mathbf{V} = \mathbf{V} \circ \mathbf{grad}\Theta$$

(Compare to Hornig and Schindler)

If $\mathbf{B} \circ \mathbf{grad}\Theta \neq \mathbf{0}$, then $\mathbf{E} \circ \mathbf{B} \neq \mathbf{0}$ and it is possible to induce topological change in the Torsion-Helicity integral, and dissipation.

Grad T ~B induces a dynamo action.

Suppose that the Bernoulli function is

$$\Theta = kT$$

and the temperature gradient has a component in the direction of the **B** field. Such a temperature gradient would be consistent with the production of an Ohmic current flow, and an **E** field parallel to the **B** field.

Conjecture:

Large temperature gradients along the **B** field lines can act as a source of stellar plasma jets in rotating, magnetic neutron stars. (A dynamo effect in a dissipative media).

IRREVERSIBLE PROCESSES

From classical thermodyamics, if the Heat, **Q**, does not admit an integrating factor the process is irreversible.

$$Irreversible \supset \mathbf{Q}\wedge d\mathbf{Q} \neq 0$$

Theorem 1:

All symplectic processes are reversible.

Theorem 2:

Processes in the direction of the Torsion Current are Irreversible.

Theorem 3:

Plasma processes in the direction of the Torsion vector, leave both the Torsion current and the Spin current ”frozen in”.

The Torsion Current is Unique

The Torsion Current direction field is uniquely defined by the 1-form that defines the Action, for systems of Even Pfaff dimension. This unique Conformal field for even Pfaff dimension is the dual of the Hamiltonian Extremal field in the odd Pfaff dimension case.

1. $\mathbf{L}_{(\mathbf{T})}\mathbf{A} = \Gamma\mathbf{A}$
2. $\mathbf{U} = \mathbf{i}(\mathbf{T})\mathbf{A} = \mathbf{0}$
3. $\Gamma \simeq Div_4\mathbf{T}_4$

For processes in the direction of the Torsion field, the fields decay in a conformal manner, even in the dissipative irreversible regime. The decay can asymptotically approach a long lived ”steady state”.