Coslab Conference - Bilbao, Spain - June 10, 2003

FALACO SOLITONS

Cosmic Strings in a Swimming Pool

A Topological Perspective of Cosmology

http://www22.pair.com/csdc/pdf/cosmos.pdf

R. M. Kiehn (Emeritus) Physics Dept., Un. Houston

http://www.cartan.pair.com

Presentation in Two Parts

1 Falaco Solitons:

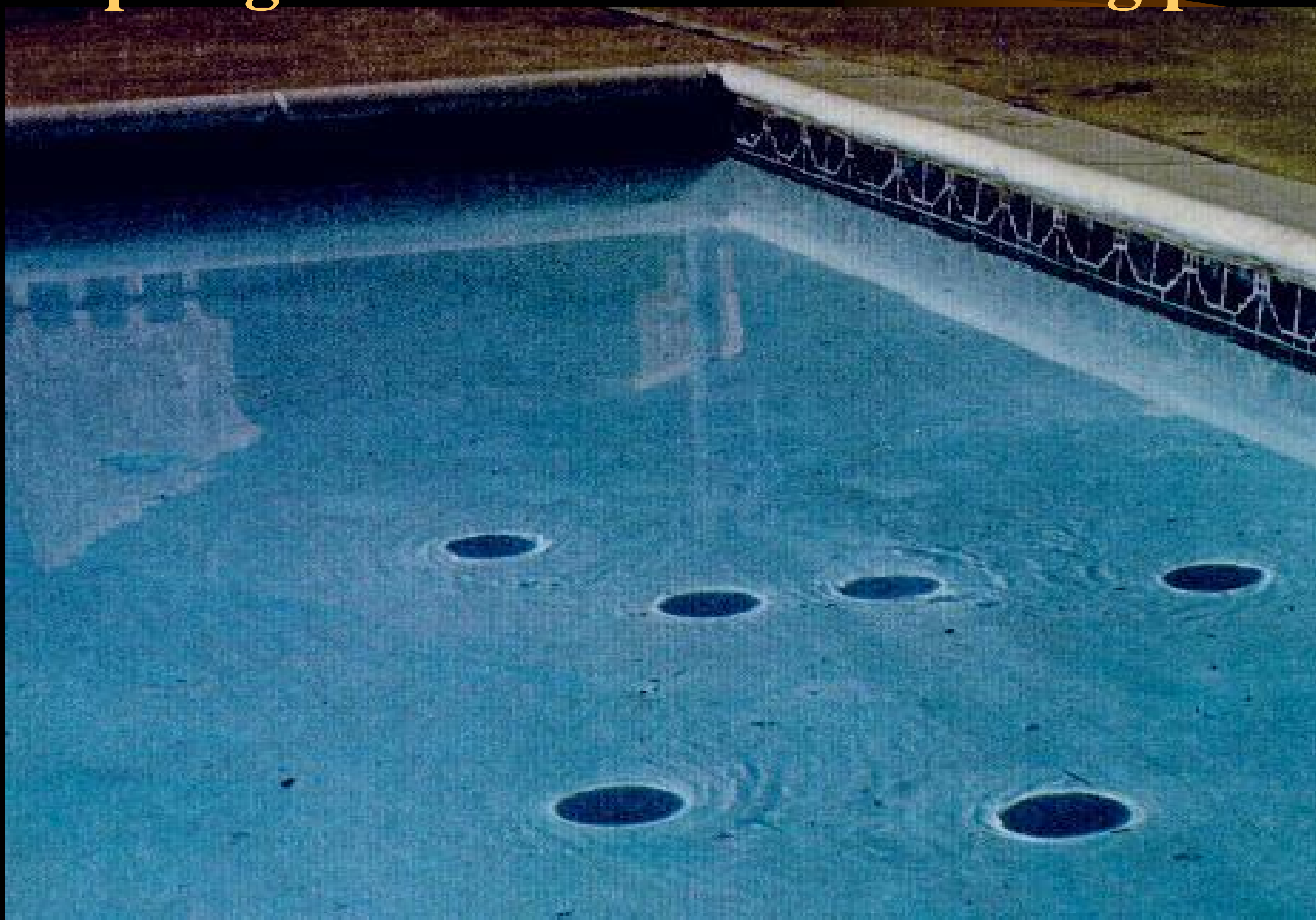
Topological defects in a swimming pool.

2 Cosmology **from a topological perspective**

Space-time as a turbulent non-equilibrium dilute gas near its critical point, of Pfaff topological dimension 4. Stars and galaxies as topological defects, of Pfaff dimension 3.

Part 1. Falaco Solitons 1986

Topological Defects in a swimming pool



History of Falaco Solitons

- 1986 visit to Rio de Janeiro and the mountain side house of my MIT roomate, Jose Haraldo H. Falcao.

- On the mountain side above the beach

History of Falaco Solitons

- 1986 visit to Rio de Janeiro at the house of my MIT roomate, Jose Haraldo H. Falcao.

- To the swimming pool

FALACO SOLITONS

Topological Defects in a swimming pool

Optical Properties of Falaco Solitons

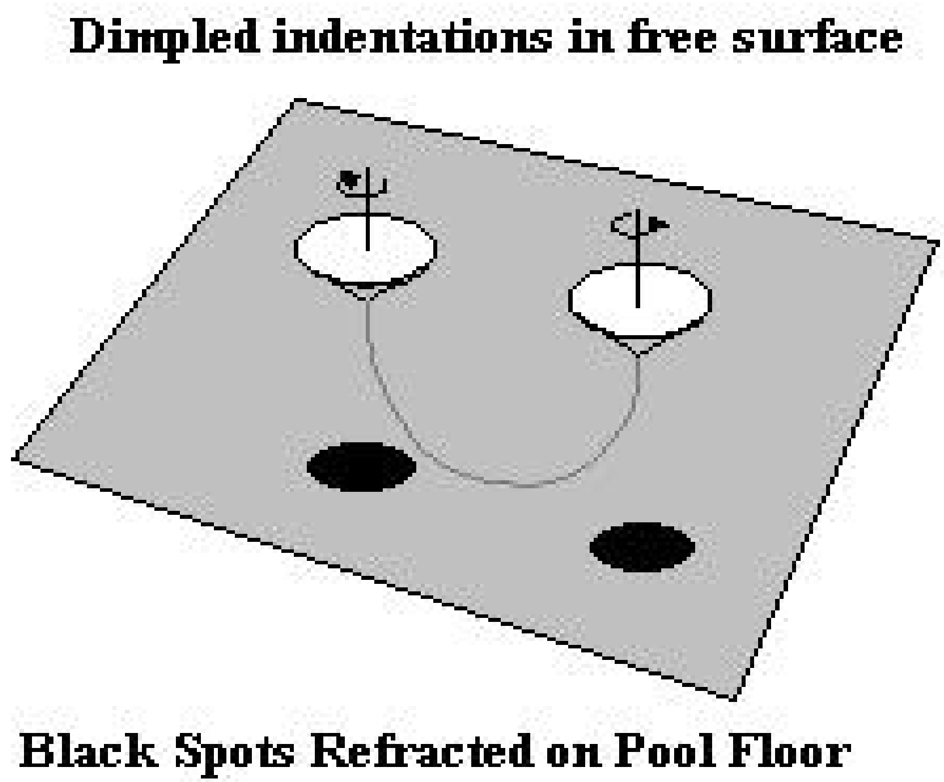


Figure 2. The Falaco Soliton

Optical Properties of Falaco Solitons

- Snell Refraction from a rotational vertex.

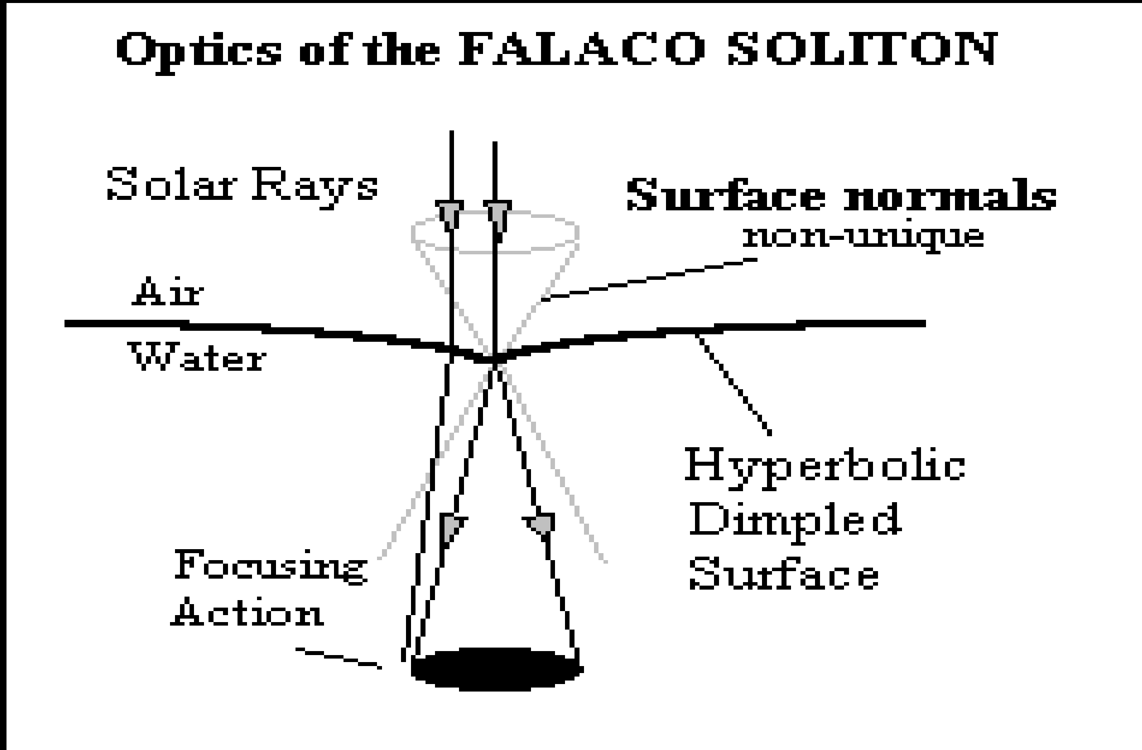
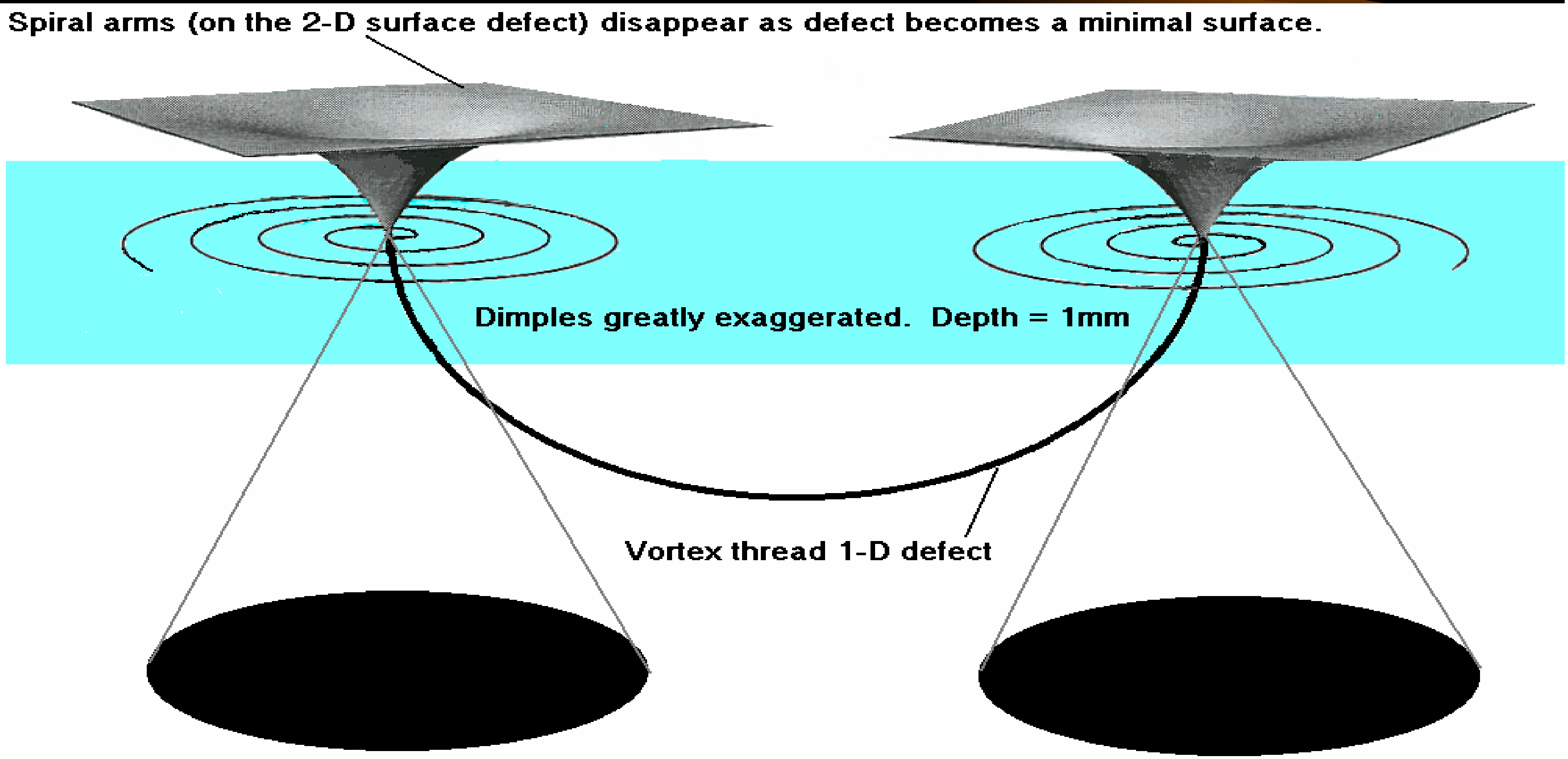


Figure 3. The Snell Black Disk

CGL theory applied to Falaco Solitons



Black Holes by Snell Refraction from Minimal Surface

Falaco Solitons

Adapted from O. Tornkvist and E. Schroeder, PRL, 78, 10 1997 p.1980

Topology of Falaco Solitons

- String (vortex) defect related to GPL theory.
- Spiral arms obvious during formation phase indicate end caps are related to CGL theory.
- Stability and confinement of 2D topological defect end caps produced by 1D defect string.

Topology of Falaco Solitons

- The topological defects and CGL analysis do not depend upon a Low Temperature !!!

- The Falaco Soliton is a long lived state far from equilibrium (Pfaff Dim=3), produced in a dissipative media by irreversible processes, (Pfaff Dim=4).

Motivation from Falaco Solitons

- **Falaco Solitons** are universal topological defects that can be created easily at a macroscopic level - without low temperature.
- **Falaco Solitons** mimic microscopic quarks with a confinement problem.
- **Falaco Solitons** mimic cosmological strings on a space-time manifold.

Part 2: Topological Cosmology

Basic Ideas from Lev Landau 1958

- **A low density van der Waals gas near its critical point exhibits large fluctuations in density.**
- **The density fluctuations are correlated and indicate a force of attractive interaction equivalent to the law of Newtonian gravity.**

Topological Cosmology

Conjectures RMK 1965

- **The Universe is a dilute gas or plasma near its critical point.**
- **The large fluctuations in density form topological defects of Pfaff dimension 3, called stars and galaxies.**
- **The density defects attract as $1/r^2$.**

Topological Cosmology

From Topological Thermodynamics RMK 2003

- The Universe is a dilute TURBULENT gas or plasma, NOT in EQUILIBRIUM due to expansion, of PFAFF TOPOLOGICAL DIMENSION 4, near its critical point.
- The large fluctuations in density are the result of Topological defects of Pfaff Topological dimension 3 being created by topological evolution in the background of Pfaff dimension 4. The topological condensation defects, are called stars and galaxies.
- The density defects attract as $1/r^2$.

Topological Cosmology

From Topological Thermodynamics RMK 2003

- The thermodynamics of a non-equilibrium, turbulent, dissipative system of Pfaff Topological Dimension 4, explains,
- the granularity of the night sky,
- the $1/r^2$ gravitation attraction,
- the expansion of the Universe,

creating stars and galaxies by formation of topological defects as “stationary” states of Pfaff topological dimension 3, far from equilibrium (which requires Pfaff dimension 2).

Topological Cosmology

Conjectures 2003

- Are the thin spiral arm galaxies formed on discontinuity layers, as are the endcaps of the Falaco solitons?

- Are the spiral arm galaxies M31 and the Milky Way confined by a singular, globally stabilizing thread connecting the dimpled vertex points of the galactic cores - as is observed in the Falaco Solitons?

Topological Cosmology

Conjectures 2003

Are

Cosmic Strings

the equivalent to

Falaco Solitons

in a

Swimming Pool?

Topological Thermodynamics

- **Open Thermodynamic System:**
  - **Pfaff Topological Dimension 4**
- **Closed Thermodynamic System**
  - **Pfaff Topological Dimension 3**
- **Isolated-Equilibrium Thermodynamic System**
  - **Pfaff Topological Dimension 2**

(Pfaff Topological Dimension > 2 implies non-equilibrium)

Decay to Topological Defects in 4D

Continuous Topological Evolution can describe the irreversible evolution on an

"Open" symplectic domain of Pfaff dimension 4, with evolutionary orbits being irreversibly attracted to a

"Closed" contact domain of Pfaff dimension 3, with topological defects (stationary states), and a possible ultimate decay to the

"Isolated-Equilibrium" domain of Pfaff dimension 2 or less (integrable Caratheodory surface).

Topological Thermodynamics

- **Consider ANY vector field V with a Jacobian matrix of rank (topological dimension) 4.**
- **The Cayley-Hamilton theorem creates a characteristic polynomial of 4th degree describing a non-equilibrium open system:**

A Universal Thermodynamic Phase Function

$$\Theta = \Psi^4 - M\,\Psi^3 + G\,\Psi^2 - A\,\Psi + K \Rightarrow 0$$

Topological Thermodynamics

- The vector field V could be the generator of a exterior differential 1-form of Action, A, representing the topological properties of a physical system.
- The vector field V could be the generator of thermodynamic process.
- The vector field V could be the generator of a dynamical system.

Topological Thermodynamics

- The **Universal Topological Phase Function** generated by a non-singular Jacobian matrix is holomorphic in the complex variable Ψ.
- **THEN, from a theorem of Sophus Lie:**

- The Phase Function Θ creates

Conjugate Minimal Surfaces in 4D.

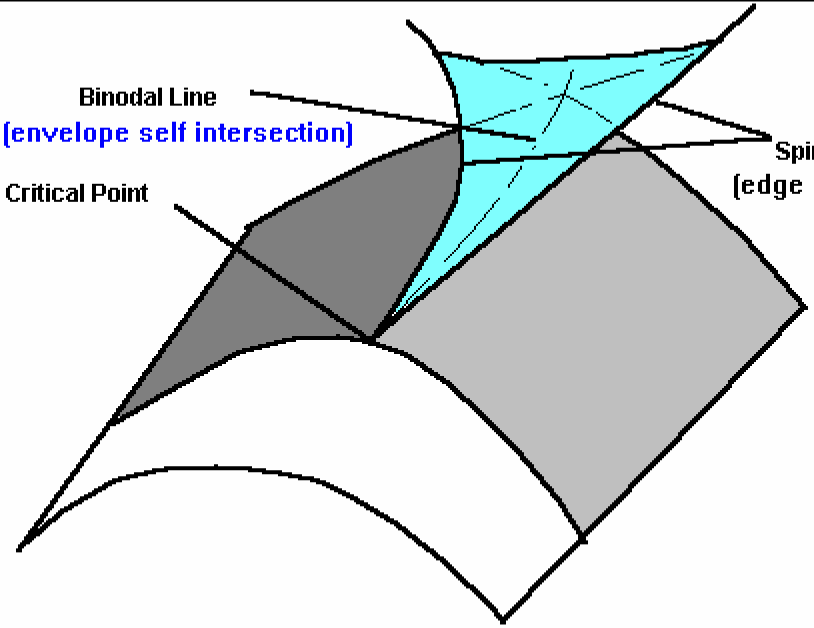
Topological Thermodynamics

- The **UNIVERSAL TOPOLOGICAL PHASE** function supports an envelope, which when constrained to a minimal surface (**M = 0**), generates the Swallow-Tail bifurcation set.

- The UNIVERSAL minimal surface ENVELOPE is homeomorphic to the

Gibbs Surface of a van der Waals gas

Topological Thermodynamics



A Universal Thermodynamic Swallowtail

The Gibbs surface of a Van der Waals gas

Topological Thermodynamics

- The determinant of the Jacobian matrix is given by the similarity coefficient K.

$$K = \Psi^4 - M\,\Psi^3 + G\,\Psi^2 - A\,\Psi$$

- Every determinant can be related to the divergence of a current. Hence

$$\mathrm{div}J + \partial\rho/\partial t = \Psi(\Psi^3 - M\,\Psi^2 + G\,\Psi - A)$$

Universal Landau Ginsburg format

Topological Thermodynamics

- If K = 0, the Jacobian matrix is singular.
  - The Pfaff topological dimension $\Rightarrow$ 3.

$$K = \Psi(\Psi^3 - M\Psi^2 + G\Psi - A) \Rightarrow 0$$

- The singular UNIVERSAL Phase function becomes homeomorphic to the cubic equation of state for a van der Waals gas!

COSMOLOGICAL SUMMARY 1

- **Vector fields of Pfaff Topological Dimension 4**

encode a non-equilibrium

van der Waals Gas

- **If the Universe is of Topological Dimension 4, it should**

mimic a non-equilibrium

COSMOLOGICAL SUMMARY 2

- **The thermodynamics of a non-equilibrium, turbulent, dissipative system of Pfaff Topological Dimension 4, explains,**
  - **the granularity of the night sky,**
  - **the $1/r^2$ gravitational attraction,**
  - **the expansion of the Universe,**

  creating stars and galaxies by formation of topological defects as “stationary” states of Pfaff topological dimension 3, far from equilibrium (which requires Pfaff dimension 2).

COSMOLOGICAL note 1

- It is important to realize that the topological method does not depend upon metric, connection, scales, equilibrium statistics, gauge symmetries, or quantum hypotheses. Yet a cosmology based on a non-equilibrium turbulent dissipative thermodynamic system of topological dimension 4, near its critical point, does give a reason for :
  - the granularity of the night sky,
  - the $1/r^2$ gravitational attraction,
  - the expansion of the Universe.

COSMOLOGICAL note 2

WARNING

- **The failure of a symmetric METRIC approach**

A symmetric METRIC matrix has no complex eigenvalues. Such matrices can not represent a non-equilibrium thermodynamic system.

Part 3: Topological Evolution

Basic Ideas of Topology Evolution

- Properties that are independent from size and shape are topological properties. Such objects are deformation invariants.
- Topological evolution can take place continuously or discontinuously.
- Cutting is a discontinuous process. Pasting is a continuous process.

Simple Topological Examples

- A condensed fluid is a thermodynamic phase with a connected components.
- A vapor is a thermodynamic phase with disconnected components.
- A change of phase implies a change of topology. Condensation is continuous. Vaporization is discontinuous.

Emphasis on

Continuous Topological Evolution

- Continuous Evolution can change Topology.
  - An arrow of time and thermodynamic irreversibility require **Topological Change.**
- **Exterior Differential Forms,** unlike tensors, are functionally well behaved - with respect to those C1 maps which are neither diffeomorpisms nor homeomorpisms.
  - Hence Cartan's methods can be used to describe **Continuous Topological Evolution.**

Objectives of CTE

(Continuous Topological Evolution)

- **Establish the long sought for connection between** **Irreversible Thermodynamics and Dynamical Systems -- without Statistics!**
- **Demonstrate the connection between** **Thermodynamic Irreversibility and Topological (Pfaff) Dimension 4.**

Topological Thermodynamics

- **Topological Structure of Physical Systems** is encoded in an Action differential 1-form **A**.
- **Physical Processes** can be defined in terms of contravariant vector direction fields, **V.**
- **Continuous Topological Evolution** is encoded by Cartan's magic formula :

$$L_{(V)}A = i(V)dA + di(V)A$$

($L_{(V)}A$ is the Lie differential with respect to the direction field V acting on the 1-form A)

Theorems of CTE

- **Topological evolution** is a necessary condition for both time asymmetry and thermodynamic irreversibility
- **A unique extremal direction field** which represents a conservative reversible Hamiltonian process always exists on subspaces of topological dimension **2n+1**.
- **A unique torsional direction field** which represents a thermodynamically irreversible process always exists on subspaces of even topological dimension **2n+2**.

Cartan's Magic Formula

Define the exterior differential forms:

Work = W= i(V)dA, Energy = U= i(V)A, Heat = Q.

Then Cartan's Magic Formula of CTE,

$$L_{(V)}A = i(V)dA + di(V)A = Q,$$

becomes the First Law of Thermodynamics,

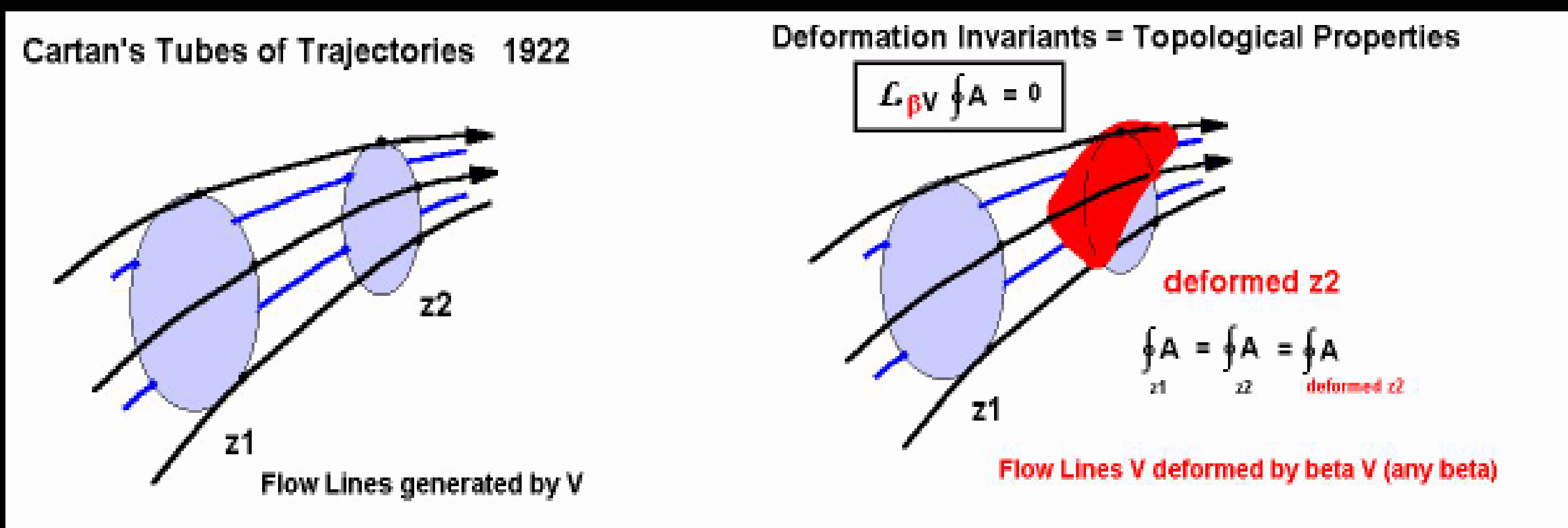
$$L_{(V)}A = W + dU = Q,$$

connecting Dynamical Systems and Thermodynamics.

($L_{(V)}A$ is the Lie differential with respect to the direction field V acting on the 1-form A)

Cartan's Magic Formula

Cartan used an integral version of his Lie derivative formula - in terms of hydrodynamic flow along a tube of trajectories - to prove that all Hamiltonian processes preserve the closed integrals of Action, and conversely.



Thermodynamic Processes page 1

- **Classical Thermodynamics**:

  A Process acting on a Physical System that creates a 1-form of Heat Q, is an irreversible process unless Q admits an integrating factor.

- **Frobenius Theorem**:

  An integrating factor exists iff Q^dQ = 0.
  (Pfaff dimension of Q < 3.)

Thermodynamic Processes page 2

For a Process **V** acting on a Physical System represented by a 1-form of Action, **A**:

A Dynamical Test for a Reversible Process is

$$L_{(V)}A\wedge L_{(V)}\,dA = Q\wedge dQ = 0.$$

A Dynamical Test for an Irreversible Process is

$$L_{(V)}A\wedge L_{(V)}\,dA = Q\wedge dQ \neq 0.$$

(Q^$dQ = 0$ implies a Pfaff dimension of <3; Q^$dQ \neq 0$ implies a Pfaff dimension of 3 or more.)

Thermodynamic Processes page 3

All classic Hamiltonian, Symplectic, Bernoulli and Stokes Processes, satisfy the Helmholtz-Poincare constraint ("conservation of vorticity").

$$L_{(V)}dA = dQ = 0.$$

and are therefore

Thermodynamically Reversible.

as

$$L_{(V)}A \wedge L_{(V)}\, dA = Q \wedge dQ = 0.$$

Pfaff Topological Dimension

A physical system represented by a 1-form of Action, A, has a minimum number of functions required for its topological definition. This number,

PTD = Pfaff Topological Dimension,

is equal or less than the geometrical dimension N of the domain of support. The PTD is also equal to the number of non-zero terms in the

Pfaff Sequence = {A,dA,A^dA,dA^dA…}.

Subspaces of lesser PTD form coherent topological structures, defects, or thermodynamic phases.

Pfaff Topological Dimension

A physical system for which the 1-forms of Action, A, of Work, W, and Heat, Q, are all of PTD = 4 defines a dissipative, non-equilibrium turbulent system.

The Jacobian matrix of A has a characteristic polynomial with 4 non-zero roots. If the system evolves to regions of PTD = 3, then the Jacobian matrix has a characteristic cubic polynomial with three non-zero roots. The similarity invariants of the polynomial can be related to the curvatures of a hyper surface whose normal field is proportional to the components of A.

The cubic polynomial defines a universal

van der Waals gas.

Pfaff Topological Dimension

Hence all thermodynamic systems of PTD = 4 have topological defect structures of PTD = 3, which are universally related topologically to a van der Waals gas.

The similarity invariants are the linear Mean Curvature, the quadratic Gauss curvature and the cubic Adjoint curvature.

The critical point occurs when M=0, G=0, A=0, K=0

The Spinodal line when G=0, M=0

The Binodal line occurs when

Cosmological Conclusion

The universe can be represented by a dilute dissipative non equilibrium turbulent system of PTD = 4, with topological defects of PTD = 3.

PTD = 3 implies a universal van der Waals gas. The similarity invariants of the cubic polynomial are adjusted to represent the neighborhood of the critical point.

The topological defects are the stars representing extreme density fluctuations attracted by a Newtonian force law.

Contact Manifolds, n = 2k+1 .

On subspaces of Pfaff dimension $n = 2k + 1 \leq m$, called contact manifolds, the Principle of Least Action implies that the evolution obeys the Helmholtz-Poincare constraint ("conservation of vorticity")

$$L_{(V)}dA = dQ = 0.$$

and all such evolutionary processes are therefore

Thermodynamically Reversible.

A unique direction field, **V**, completely determined by the topological features of the Action, A, of odd Pfaff dimension, such that $W = i(V)dA=0$ is called the **Extremal Field,** and if $U = i(V)A = 1$, a **Reeb** field.

Symplectic Manifolds n = 2k+2.

On subspaces of Pfaff dimension $n = 2k + 2 \leq m$, called symplectic manifolds, extremal fields do not exist. However, a unique direction field T can be defined in terms of the topological features of the physical system, A:

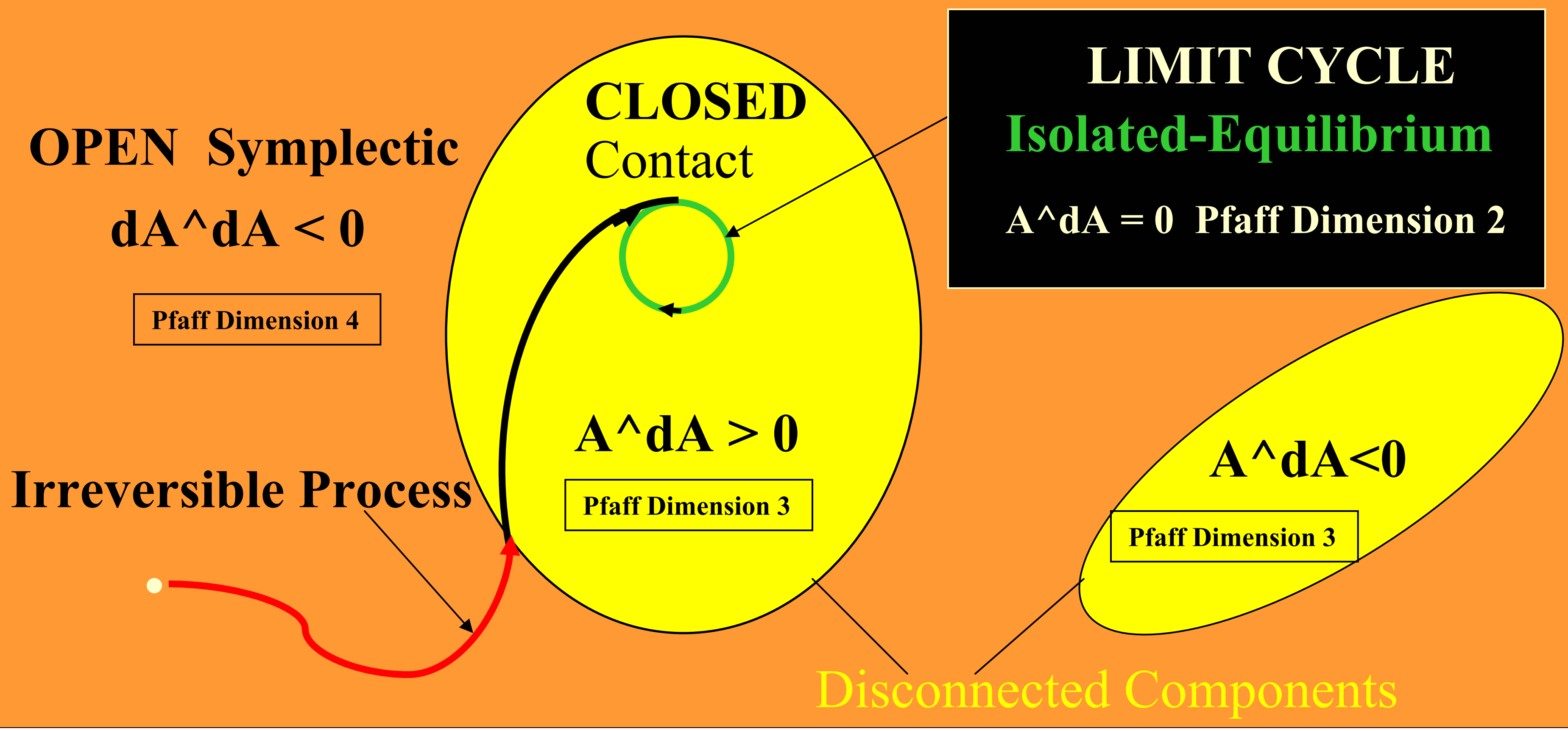
$$i(T)\{dx \wedge dy \wedge dz \wedge dt\} = A \wedge dA.$$

Processes in the direction of the Torsion Vector, T, are Thermodynamically Irreversible, as

$$L_{(T)} A \wedge L_{(T)} dA = Q \wedge dQ = \Gamma^2 A \wedge dA \neq 0.$$

Irreversible Decay on a Symplectic Manifold (PTD=4) to a Contact Manifold (PTD=3) of disconnected components, then possibly to an Isolated-Equilibrium (PTD = 2) State.

Turbulent Non-Equilibrium Pfaff dimension 4



Connectivity and the Arrow of time

Regions where PTD ≤ 2 generate a connected topology;

PTD ≥ 3 generate a disconnected topology.

Continuous Processes can represent the evolution from disconnected topology (≥ 3) to a connected topology (≤ 2) .

Continuous Processes can NOT represent the evolution from a connected topology (≤ 2) to a disconnected topology (≥ 3) .

(An Arrow of Time.)

You can describe the decay of turbulence continuously, but NOT the creation of turbulence.

Topological Fluctuations 4D

Recall: The kinematic assumption is a Topological Constraint:

$$\Delta \mathbf{x} = d\mathbf{x} - \mathbf{v}dt \Rightarrow \mathbf{0}.$$

Define a transverse **topological fluctuation in position** as

$$\Delta \mathbf{x} = d\mathbf{x} - \mathbf{v}dt \neq \mathbf{0}. \qquad \textbf{(\sim Pressure)}$$

Define a transverse **topological fluctuation in velocity** as

$$\Delta \mathbf{v} = d\mathbf{v} - \mathbf{A}dt \neq \mathbf{0}. \qquad \textbf{(\sim Temperature)}$$

On a variety {P,v,x,t} define the 1-form of Action as:

$$A = L(v,x,t)\,dt + P\,(dx - vdt) = P\,dx + H\,dt \Rightarrow$$

$$A = L(v,x,t)\,dt + P\,\Delta x$$

Topological Fluctuations 4D

Define the topological fluctuation in momenta as:

$$\Delta p = dP - (\partial L/\partial x)\, dt - f\, \Delta x\ .$$

Then compute the elements of the Pfaff sequence:

Action: $$A = L(v,x,t)\, dt + P\, \Delta x$$

Vorticity: $$dA = (\partial L/\partial v - P)\, \Delta v \wedge dt + \Delta p \wedge \Delta x$$

Torsion: $$A \wedge dA = L\, \Delta p \wedge \Delta x \wedge dt - P(\partial L/\partial v - P)\, \Delta v \wedge \Delta x \wedge dt$$

Parity: $$dA \wedge dA = -2\, (\partial L/\partial v - P)\, \Delta p \wedge \Delta v \wedge \Delta x \wedge dt$$

Topological Fluctuations 4D

The Topological constraint of Canonical Momenta, $P = \partial L/\partial v$ reduces the Pfaff dimension from 2n+2 to 2n+1.

- Theorem 1. There exists a unique extremal direction field on the 2n+1 Contact manifold, a Hamiltonian conservative representation, such that the Virtual Work 1-form is zero.
- Theorem 2 (analogue to Heisenberg), The existence of Topological Vorticity and Topological Torsion require that the product of fluctuations in momenta and fluctuations in position is NOT zero.

$$\Delta p \hat{} \Delta x \neq 0$$

Topological Fluctuations 4D

Consider the Work 1-form on a 2n+2 manifold, $(\partial L/\partial v\text{-}P) \neq \mathbf{0}$.

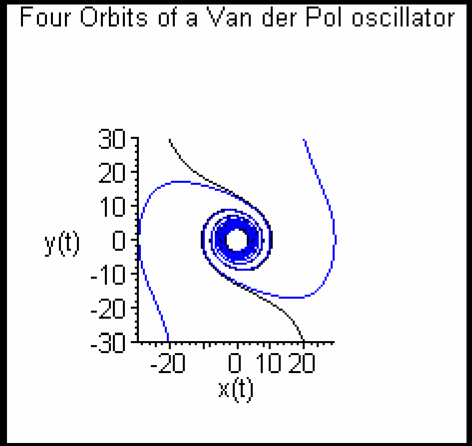
$$\mathbf{W = i(V)dA = (\partial L/\partial v\text{-}P)\,\Delta v + \Delta p \hat{}\Delta x}$$

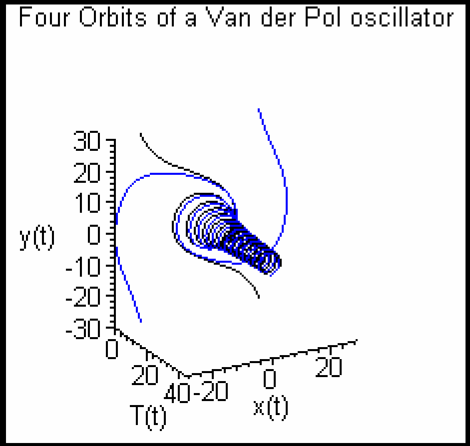
Processes on a 2n+2 symplectic manifold require $W \neq \mathbf{0}$.

- To be a symplectic manifold requires that the first term in the expression for work, W, is not zero. The momenta cannot be canonical, and the Velocity fluctuations must be non-zero. This implies the existence of a non-zero temperature, and leads to the analogue of the Planck concept of a zero point energy on the symplectic 2n+2 topological manifold.

Van der Pol Example

Perhaps the most famous limit cycle is that of the Van der Pol Oscillator. The evolutionary decay from arbitrary initial conditions {x,y,t} to a unique attracting limit cycle {f(x,y),t} is an obvious example of continuous topological change.





Lagrangian Example page 1

A Cartan-Hilbert 1-form of Action, A, for a physical system can be written as

$$A = L(t,x,v,p)dt + p \bullet (dx - v\, dt)$$

The k+1 base variables are {t, x}.

The 2k "fiber" variables are {v,p}.

The Lagrange function $L(t,x,v,p)$ is a function of the 3k+1 variables, (t,x,v,p) .

Lagrangian Example page 2

However, the maximum Pfaff topological dimension is 2k+2 and the top Pfaffian on the symplectic manifold is

$$(dA)^{k+1} = (k+1)!\{\partial L/\partial v - p)\bullet dv\}\wedge\Omega_p\wedge\Omega_q\wedge dt$$

$$\Omega_p = dp_1\wedge..\wedge dp_n \qquad \Omega_q = dq^1\wedge..\wedge dq^n$$

Note that the "symplectic momenta" are not canonically defined:

$$p - \partial L/\partial v \neq 0.$$

Lagrangian Example page 3

Evolution starts on the 2k+2 symplectic manifold with orbits being attracted to 2k+1 domains where the momenta become canonical: $p - \partial L/\partial v \Rightarrow 0$.

- Topological evolution can either continue to reduce the Pfaff topological dimension, or
- the process on the Contact 2k+1 manifold can become "extremal", and the topological change stops.

The resulting contact manifold becomes a "stationary" non-dissipating Hamiltonian state,

"Far from Equilibrium".

The Sliding - Rolling Ball

Consider a bowling ball with initial translational and rotational energy, thrown to the floor of the bowling alley.

Initially the ball skids or slips on a 2k + 2 symplectic manifold irreversibly reducing its energy and angular momentum via “friction” forces.

From arbitrary initial conditions, the evolution is attracted to a 2k + 1 contact manifold, where the ball rolls without slipping, and the anholonomic constraint vanishes.

$$dx - \lambda \, d\Theta = 0$$

The Sliding - Rolling Ball page 2

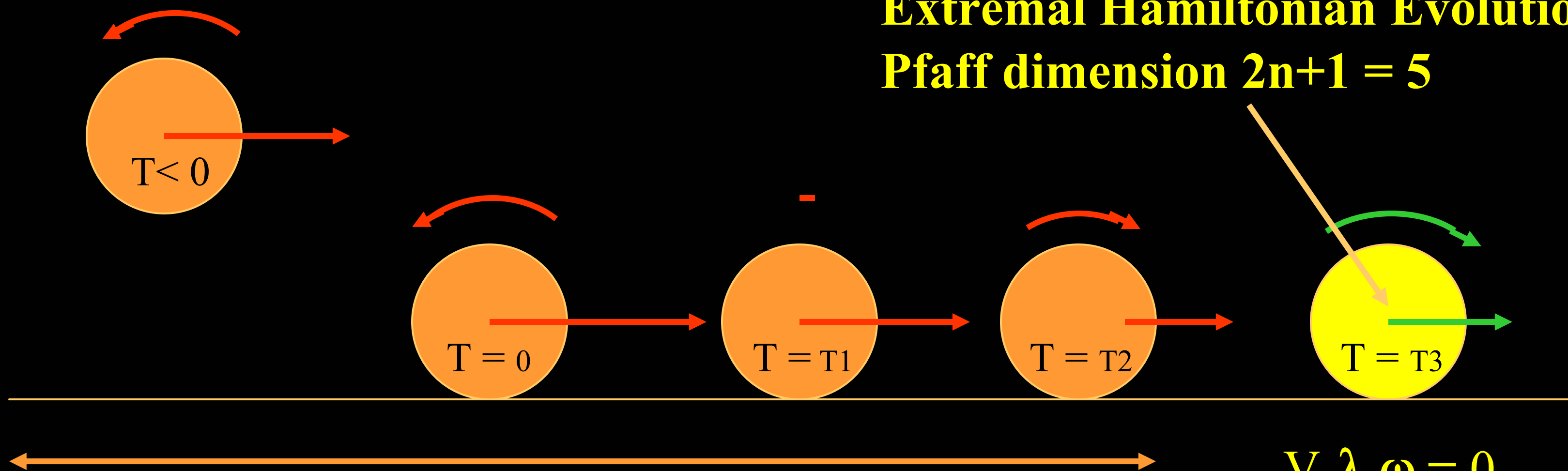
The subsequent motion, neglecting air resistance, continues in a Hamiltonian manner without change of Kinetic Energy or Angular Momentum.

The 1-form of Action can be written as:

$$A = L(t,\mathbf{x}, \Theta,v,\omega)dt + \ldots + \mathbf{s} \bullet (d\mathbf{x} - \lambda\, d\Theta)$$

The Sliding - Rolling Ball page 3

Extremal Hamiltonian Evolution
Pfaff dimension 2n+1 = 5



$V-\lambda\ \omega = 0$

Irreversible Evolution - Pfaff dimension 2n+2 = 6

$V-\lambda\ \omega > 0$	$V-\lambda\ \omega > 0$	$V-\lambda\ \omega > 0$
$\lambda\ \omega < 0$	$\lambda\ \omega = 0$	$\lambda\ \omega > 0$

Note how friction changes Angular Momentum

Summary

- Without Topological Evolution, there is no Arrow of Time and no Thermodynamic Irreversibility.
- Physical Systems of Pfaff dimension 4 generate a unique continuous evolutionary process which is thermodynamically irreversible.
- Cartan's Magic formula combines continuous topological evolution and thermodynamics

Entropy production rate

Construct the 3-form A^F = A^dA

Topological Torsion vector T

A ^ dA = i[T, h](dx^dy^dz^dt)

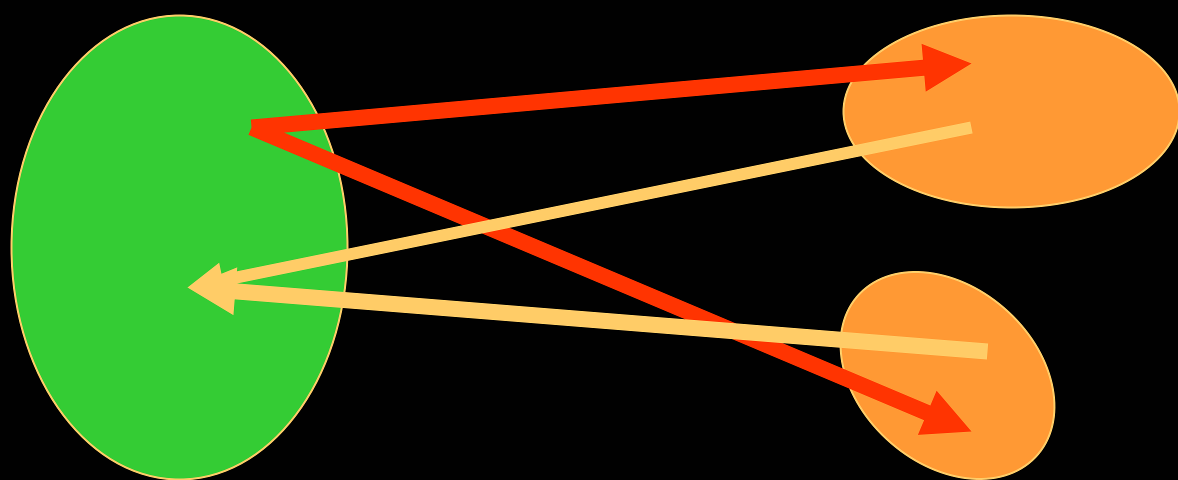
with the 4 component Torsion Direction Field

Γ^2 ≈ Entropy Production Rate

≈(1/2 divergence of T) ^{2}

Arrow of Time and Turbulence

Creation of Turbulence is a Discontinuous Process



Decay of Turbulence is a Continuous Process

Streamline Flow

Connected Topology $D_{pfaff} \leq \mathbf{2}$

Turbulent Flow

Disconnected Topology $D_{pfaff} > \mathbf{3}$

Electromagnetic Example page 1

Use the 4D electromagnetic 1-form of Action:

$$A = \mathbf{A}(x,y,z,t) \bullet d\mathbf{r} - \phi(x,y,z,t)dt.$$

Define: $\mathbf{E} = -\text{curl}\, \mathbf{A} - \text{grad}\, \phi(x,y,z,t)$, $\mathbf{B} = \text{curl}\, \mathbf{A}$

Construct the 2-form F = dA,

$$F = dA = B_z\, dx\wedge dy \ldots - E_z\, dz\wedge dt \ldots$$

Then the 3-form $dF = ddA = 0$, and generates the Maxwell-Faraday PDE's.

Electromagnetic Example page 2

Construct the 3-form A^F = A^dA

Topological Torsion: **$A \wedge dA = i[T, h](dx \wedge dy \wedge dz \wedge dt)$**

with the 4 component Torsion Direction Field

$$T = [T, h] = [\, E \times A + \phi B,\ A \bullet B].$$

Then,

$$L_{(T)} A = (E \bullet B)\, A$$

(The 4th component **$A \bullet B$** is often defined as the Helicity density)

Electromagnetic Example page 3

Construct the 4-form F^F = dA^dA :

Topological Parity: **dA ^ dA = -2(E • B) dx^dy^dz^dt).**

On regions where Γ = (E • B) $\neq$ 0,

- **the Pfaff dimension is 4,**
- **evolution in the direction of the Torsion vector is thermodynamically irreversible.**

$$L_{(T)} A\wedge L_{(T)} dA = Q\wedge dQ = (-E \bullet B)^2 A\wedge dA \neq 0.$$

(The divergence of the Torsion vector is equal to -2(**E • B.**)

Electromagnetic Example page 4

On regions where (E • B) = 0,

- **the Pfaff dimension is 3,**
- **and the evolution can proceed in the direction of the extremal field, which is reversible. (The Torsion vector on the 4D base space has zero divergence).**

The closed integrals of the 3-form A^dA are deformation (topological) invariants for all processes V on domains of Pfaff dimension 3

(The values of the closed integrals are "topologically quantized")!

Darboux Format 4D

Consider a map Φ from **{x,y,z,t}** $\Rightarrow$ **{P,H,Q,T}** and a 1-form of Action on the target space in Darboux Format,

$$\mathbf{A = PdQ + HdT.}$$

By Functional Substitution, pull back the target to the domain

$$\mathbf{A = A(x,y,z,t) \bullet dr - \phi(x,y,z,t)dt \Leftarrow A = PdQ + HdT.}$$

Then the vector and scalar potentials of the previous electromagnetic example are well defined functions of $\{P_{(x,y,z,t)}, H_{(x,y,z,t)}, Q_{(x,y,z,t)}, T_{(x,y,z,t)}\}$ and their differentials.

Result: All of the topological features of the Darboux representation now have an electromagnetic interpretation.

Darboux Format 4D

To reduce the algebraic complexity, constrain the map such that "time" on the final state is equal to "time" on the intial state: **T= t.** Then the important topological properties of the Darboux format in electromagnetic interpretation are:

$$\mathbf{E} = (\partial P/\partial t\ \nabla Q - \partial Q/\partial t\ \nabla P) - \nabla H, \quad \mathbf{B} = \nabla P \times \nabla Q$$

Frenet Torsion ⇒ 0: $\mathbf{A} \bullet \mathbf{B} = 0$

Topological Torsion: $\mathbf{T} = - P\ (\partial Q/\partial t\ \nabla P - \nabla H) \times \nabla Q$

Topological Parity : $\mathbf{E} \bullet \mathbf{B} = \nabla H \bullet \nabla P \times \nabla Q$