```
> restart:with(plots):with(DEtools):with(linalg):with(difforms):
  with(liesymm):with(plots):setup(x,y,z,t,s):defform(x=0,y=0,z=0,t=0,s=0,a=const,b
  =const,c=const,k=const,mu=const,omega=const,m=const,delta=const):
Warning, new definition for norm
Warning, new definition for trace
Warning, new definition for `&^`
Warning, new definition for close
Warning, new definition for d
Warning, new definition for mixpar
Warning, new definition for translate
Warning, new definition for wdegree
```

diracch.mws

A Dirac Hedge-hog Magnetic Field and SunSpots

R. M. Kiehn, University of Houston

Aug 5, 1998, updated Aug 30,1998

"When is a "monopole" not a monopole?"

An electromagnetic field that has helical structure but ZERO helicity density offers some interesting properties for modeling the magnetic properties of stars like the sun. In particular, the z component of the Lorentz force seems to indicate the possibility that an electromagnetic mechanism might be responsible for the formation of an accretion disk.

****

Preliminary

In 1931 Dirac investigated that which is now called the magnetic monopole. His interest was in a B field that had the form

```
> Bfield:=evalm(m/(2*((x^2+y^2+z^2)^(3/2)))*[x,y,z]):B21:=factor(Bfield[1]);B22:=f
  actor(Bfield[2]);B23:=factor(Bfield[3]);
>
```

$$B21 := \frac{1}{2}\frac{m\,x}{(x^2+y^2+z^2)^{3/2}}$$

$$B22 := \frac{1}{2}\frac{m\,y}{(x^2+y^2+z^2)^{3/2}}$$

$$B23 := \frac{1}{2}\frac{m\,z}{(x^2+y^2+z^2)^{3/2}}$$

The B field is spherically symmetric with a zero divergence everywhere except perhaps at the origin. On a sphere of radius r, the field has the appearance 3-D hedgehog with "bristles" along the radii from the origin.) Dirac and others have been quick to presume the existence of a delta function source at the origin to account for the singularity. However there could be other options of interpretation, namely a topological obstruction..

The literature also demonstrates that this field can be obtained from a vector potential.

The first vector potential formula below is from Gockler & Schucker

VECTOR POTENTIAL 1 Gockler & Schucker

```
> Adirac1:=evalm(m/(2*(x^2+y^2+z^2)^(1/2)*(z-(x^2+y^2+z^2)^(1/2))))*[-y,x,0];
```

$$Adirac1 := \frac{1}{2} \frac{m\,[-y, x, 0]}{\sqrt{x^2+y^2+z^2}\,(z-\sqrt{x^2+y^2+z^2})}$$

Nash and Sen give a similar (but incorrect - and probably a misprint) formula for the vector potential as :

VECTOR POTENTIAL 2 Nash and Sen

```
> Adirac2:=(m*(z-(x^2+y^2+z^2)^(1/2))/(2*(x^2+y^2+z^2)^(1/2)))*[-y,x,0];
```

$$Adirac2 := \frac{1}{2} \frac{m\,(z-\sqrt{x^2+y^2+z^2})\,[-y, x, 0]}{\sqrt{x^2+y^2+z^2}}$$

Both authors then claim that the curl of this vector (curl A) field yields

```
> Bfield:=(m/(2*((x^2+y^2+z^2)^(3/2))))*[x,y,z]);
```

$$Bfield := \frac{1}{2} \frac{m\,[x, y, z]}{(x^2+y^2+z^2)^{3/2}}$$

In each case the direction field is analogous to that of "rigidbody" rotation [-y,x,0] about the z axis, but scaled by a coefficient function that arbitrarily reparameterizes the direction field. Unfortunately the Nash and Sen formula does not work (most likely a misprint), but the Gockler Schucker formula does work. A check is given below using Maple:

```
>
>
> BFIELD1:=curl(Adirac1,[x,y,z]):B11:=factor(BFIELD1[1]);B12:=factor(BFIELD1[2]);B
  13:=factor(subs(x^2+y^2+z^2=r^2,(BFIELD1[3])));
```

The B13 component indeed factors to give the desired format.

$$B11 := \frac{1}{2} \frac{m\,x}{(x^2+y^2+z^2)^{3/2}}$$

$$B12 := \frac{1}{2} \frac{m\,y}{(x^2+y^2+z^2)^{3/2}}$$

$$B13 := \frac{1}{2} \frac{m\,(-x^2 r^2 z + x^2 r^2 \sqrt{r^2} + x^2 (r^2)^{3/2} + 2 r^4 z - 2 (r^2)^{3/2} r^2 - y^2 r^2 z + y^2 r^2 \sqrt{r^2} + y^2 (r^2)^{3/2})}{(r^2)^{3/2} (z-\sqrt{r^2})^2 r^2}$$

The Nash Sen Formula is a mistake:

```
> BFIELD2:=curl(Adirac2,[x,y,z]):B21:=factor(BFIELD2[1]);B22:=factor(BFIELD2[2]);B
  23:=factor(BFIELD2[3]);
```

$$B21 := -\frac{1}{2}\frac{m\,(-z+\sqrt{x^2+y^2+z^2})\,x\,(\sqrt{x^2+y^2+z^2}+z)}{(x^2+y^2+z^2)^{3/2}}$$

$$B22 := -\frac{1}{2}\frac{m\,(-z+\sqrt{x^2+y^2+z^2})\,y\,(\sqrt{x^2+y^2+z^2}+z)}{(x^2+y^2+z^2)^{3/2}}$$

$$B23 := -\frac{1}{2} m\,(3\,x^2\,(x^2+y^2+z^2)^{3/2} - x^4\,z - 2\,x^2\,z\,y^2 - 3\,x^2\,z^3 - x^4\sqrt{x^2+y^2+z^2} - 2\,x^2\sqrt{x^2+y^2+z^2}\,y^2 - x^2\sqrt{x^2+y^2+z^2}\,z^2 - y^4\,z - 3\,y^2\,z^3 - 2\,z^5 + 3\,y^2\,(x^2+y^2+z^2)^{3/2} + 2\,(x^2+y^2+z^2)^{3/2}\,z^2 - y^4\sqrt{x^2+y^2+z^2} - y^2\sqrt{x^2+y^2+z^2}\,z^2) \Big/ (x^2+y^2+z^2)^{5/2}$$

```
>
>
```

****

VECTOR POTENTIAL 3   (rmk)

However consider a formula related to the first expression by means of a renormalization transformation (contact transformation of the first kind), which greatly simplifies the interpretation of the formulas, and gives a sense of cylindrical symmetry to the vector potentials, emphasizing its relationship to the "Kelvin" circulation integral. Define the Kelvin 1-form as:

```
> Kelvin:=evalm([-y,x,0])/(a*x^2+b*y^2);BK:=(curl(evalm(Kelvin),[x,y,z])):curl_Kel
  vin:=evalm([BK[1],BK[2],factor(BK[3])]);
>
```

$$Kelvin := \frac{[-y,\,x,\,0\,]}{a\,x^2+b\,y^2}$$

$$curl\_Kelvin := [\,0,\,0,\,0\,]$$

The Kelvin vector potential has finite circulation = 2pi for any closed path in the z = 0 plane, surrounding the z axis, but has zero curl almost everywhere. (even in the anisotropic case a <> b)

Consider the vector potential of the form

```
> ARMK:=Gamma(x,y,z,t)*Kelvin;
```

$$ARMK := \frac{\Gamma(x,y,z,t)\,[-y,\,x,\,0\,]}{a\,x^2+b\,y^2}$$

Substitute for Gamma:

```
> Gamma(x,y,z,t):=-z*m/(2*(a*x^2+b*y^2+delta*z^2)^(1/2));
```

$$\Gamma(x,y,z,t) := -\frac{1}{2}\frac{z\,m}{\sqrt{a\,x^2+b\,y^2+\delta\,z^2}}$$

to yield:

```
> ARMKH:=evalm(ARMK);
```

$$ARMKH := \left[\frac{1}{2}\frac{z\,m\,y}{\sqrt{a\,x^2+b\,y^2+\delta\,z^2}\,(a\,x^2+b\,y^2)},\,-\frac{1}{2}\frac{z\,m\,x}{\sqrt{a\,x^2+b\,y^2+\delta\,z^2}\,(a\,x^2+b\,y^2)},\,0\right]$$

The kelvin type vector potential above generates the hedgehog B field to within a factor for any anisotropy!

```
> BFIELD3:=curl(ARMKH,[x,y,z]):B31:=factor(BFIELD3[1]);B32:=factor(BFIELD3[2]);B33
  :=factor(BFIELD3[3]);B31/B33;B32/B33;Helicity:=innerprod(ARMKH,BFIELD3);
```

$$B31 := \frac{1}{2}\frac{m\,x}{(a\,x^2+b\,y^2+\delta\,z^2)^{3/2}}$$

$$B32 := \frac{1}{2}\frac{m\,y}{(a\,x^2+b\,y^2+\delta\,z^2)^{3/2}}$$

$$B33 := \frac{1}{2}\frac{z\,m}{(a\,x^2+b\,y^2+\delta\,z^2)^{3/2}}$$

$$\frac{x}{z}$$

$$\frac{y}{z}$$

$$Helicity := 0$$

It is to be noticed that two different vector potentials yield the same Hedgehog B field. There is no Helicity density with either field. The results are valid for any anisotropic constants, a,b,delta.

```
> Kelvin_Helicity:=innerprod(ARMK,BFIELD3);
```

$$Kelvin\_Helicity := 0$$

Since the divergence of B vanishes there is NO MAGNETIC CHARGE !!! (mod the origin)

```
> DIVB:=factor(diverge(BFIELD3,[x,y,z]));
```

$$DIVB := 0$$

Now compute the associated Currents, assuming the Lorentz vacuum constitutive relations, B = muH, D = epsilonE. As there is no E field,

```
>
> Jf3:=curl(BFIELD3/mu,[x,y,z]):J1:=factor(subs(p=2,(Jf3[1])));J2:=factor(subs(p=2
  ,(Jf3[2])));J3:=factor(subs(p=2,(Jf3[3])));J1oblate:=subs(a=1,b=1,J1);J2oblate:=
  subs(a=1,b=1,J2);J3oblate:=subs(a=1,b=1,J3);
```

$$J1 := -\frac{3}{2}\frac{y\,m\,z\,(b-\delta)}{(a\,x^2+b\,y^2+\delta\,z^2)^{5/2}\,\mu}$$

$$J2 := -\frac{3}{2}\frac{z\,x\,m\,(-a+\delta)}{(a\,x^2+b\,y^2+\delta\,z^2)^{5/2}\,\mu}$$

$$J3 := \frac{3}{2}\frac{(-a+b)\,x\,y\,m}{(a\,x^2+b\,y^2+\delta\,z^2)^{5/2}\,\mu}$$

$$J1oblate := -\frac{3}{2}\frac{y\,m\,z\,(1-\delta)}{(x^2+y^2+\delta\,z^2)^{5/2}\,\mu}$$

$$J2oblate := -\frac{3}{2}\frac{z\,x\,m\,(-1+\delta)}{(x^2+y^2+\delta\,z^2)^{5/2}\,\mu}$$

$$J3oblate := 0$$

It is remarkable that if the divisor is isotropic (a=b=delta), then the B Field is **not** associated with a current!!!

It appears that "anisotropy" is the key ingredient of the Hedge Hog field.
In the anisotropic case, the associated current density has a direction field whose equations are similar in form (except for a sign) to the differential system for the the Euler (angular momentum) system of a rotating rigid body. (See R. Hermann's analysis in terms of elliptic functions in "Differential Geometry and the Calculus of variations" AP 1968 p232)

If the anisotropy is oblate, (a=b, 0<delta<1) or prolate(a=b,delta>1) then there is a circulation component to the current density, but J3 is zero for xy symmetry.

The circulation component of the current density changes rotation direction depending on z.

The Current is orthogonal to the B field (The" Magnetic Helicity" = B dot curlB vanishes)

```
> innerprod(Jf3,BFIELD3);
```

$$0$$

The Lorentz force is also highly dependent upon anisotropy. There is no z component of the Lorentz force on the plasma unless there is some anisotropy. The oblate case makes the z = 0 plane an attractor.

```
> LFORCE:=crossprod(Jf3,BFIELD3):LFz:=factor(LFORCE[3]);LFx:=factor(LFORCE[1]);LFy
  :=factor(LFORCE[2]);LFZoblate:=factor(subs(a=1,b=1,LFz));LFXoblate:=factor(subs(
  a=1,b=1,LFx));LFYoblate:=factor(subs(a=1,b=1,LFy));RATIOLFXFY:=LFXoblate/LFYobla
  te;RATIOLFZFX:=LFZoblate/LFXoblate;
```

$$LFz := -\frac{3}{4}\frac{m^2 z\,(b\,y^2 - \delta\,y^2 + a\,x^2 - x^2\,\delta)}{\mu\,(a\,x^2 + b\,y^2 + \delta\,z^2)^4}$$

$$LFx := -\frac{3}{4}\frac{x\,m^2\,(b\,y^2 - a\,y^2 - z^2\,a + \delta\,z^2)}{\mu\,(a\,x^2 + b\,y^2 + \delta\,z^2)^4}$$

$$LFy := \frac{3}{4}\frac{y\,m^2\,(b\,z^2 + b\,x^2 - a\,x^2 - \delta\,z^2)}{\mu\,(a\,x^2 + b\,y^2 + \delta\,z^2)^4}$$

$$LFZoblate := \frac{3}{4}\frac{m^2\,z\,(y^2 + x^2)\,(-1 + \delta)}{\mu\,(x^2 + y^2 + \delta\,z^2)^4}$$

$$LFXoblate := -\frac{3}{4}\frac{x\,m^2\,z^2\,(-1 + \delta)}{\mu\,(x^2 + y^2 + \delta\,z^2)^4}$$

$$LFYoblate := -\frac{3}{4}\frac{y\,m^2\,z^2\,(-1 + \delta)}{\mu\,(x^2 + y^2 + \delta\,z^2)^4}$$

$$RATIOLFXFY := \frac{x}{y}$$

$$RATIOLFZFX := -\frac{y^2 + x^2}{z\,x}$$

The Torsion 3-form is zero, but the Spin 3-form is finite.

```
> Helicity_density:=innerprod(ARMK,BFIELD3);
```

$$Helicity\_density := 0$$

```
> Sp4:=0;
```

$$Sp4 := 0$$

```
> SPIN:=crossprod(ARMK,BFIELD3/mu):sp1:=factor(SPIN[1]);sp2:=factor(SPIN[2]);sp3:=
```

```
factor(SPIN[3]);P1:=factor(diverge(SPIN,[x,y,z]));
```

$$sp1 := -\frac{1}{4}\frac{x\,m^2\,z^2}{\mu\,(a\,x^2+b\,y^2)\,(a\,x^2+b\,y^2+\delta\,z^2)^2}$$

$$sp2 := -\frac{1}{4}\frac{y\,m^2\,z^2}{\mu\,(a\,x^2+b\,y^2)\,(a\,x^2+b\,y^2+\delta\,z^2)^2}$$

$$sp3 := \frac{1}{4}\frac{(y^2+x^2)\,m^2\,z}{\mu\,(a\,x^2+b\,y^2)\,(a\,x^2+b\,y^2+\delta\,z^2)^2}$$

$$P1 := \frac{1}{4}\frac{m^2\,(b\,y^4+4\,z^2\,y^2\,b+b\,y^2\,x^2+y^2\,a\,x^2-3\,y^2\,\delta\,z^2+4\,z^2\,x^2\,a+x^4\,a-3\,x^2\,\delta\,z^2)}{\mu\,(a\,x^2+b\,y^2)\,(a\,x^2+b\,y^2+\delta\,z^2)^3}$$

The remarkable result is there is a component of the Lorentz force that forms an accretion disc if the system is at least oblate!! 0<delta<1 implies the LORENTZ force is expanding radially away from the z axis and contracting to the z =0 plane. The Spin vector is proportional to the Lorentz force (in the sense of a radiation reaction). The first Poincare invariant is not zero, meaning the 3-form of Spin is not conserved.

****

Now use exterior differential forms to evaluate the charge current 3-form, the Torsion 3-form and the Spin 3-form for a time dependent potential that generates a HedgeHog B field. The time dependent part will produce an E field.

****

THE EXAMPLE ELECTROMAGNETIC FIELD for Rotating Plasma.

will have zero magnetic helicity density, but finite spin density.

In the first example the vector potential will be modified to admit small oscillations along the z axis. At first no Coulomb term will be examined (A4=0). Then the effect of some simple Coulomb additions will be considered.

****

First consider the case of xy cylindrical symmetry.:

```
> dR:=[d(x),d(y),d(z),d(t)]:
```

USE VECTOR POTENTIAL FORMAT 3 modified for oscillations:

```
> A:=evalm(((1+beta*cos(omega*t))*z/(2*k*(x^2+y^2+delta*z^2)^(1/2)*(x^2+y^2)))*([-y,x,0]));A4:=0*beta*z*cos(omega*t);
```

$$A := \left[-\frac{1}{2}\frac{(1+\beta\cos(\omega\,t))\,z\,y}{k\,\sqrt{x^2+y^2+\delta\,z^2}\,(y^2+x^2)},\ \frac{1}{2}\frac{(1+\beta\cos(\omega\,t))\,z\,x}{k\,\sqrt{x^2+y^2+\delta\,z^2}\,(y^2+x^2)},\ 0\right]$$

$$A4 := 0$$

```
>
> Action:=evalm(A)[1]*d(x)+evalm(A)[2]*d(y)+A[3]*d(z)+A4*d(t);
>
```

$$Action := -\frac{1}{2}\frac{(1+\beta\cos(\omega t))\,z\,y\,\mathrm{d}(x)}{k\sqrt{x^2+y^2+\delta z^2}\,(y^2+x^2)} + \frac{1}{2}\frac{(1+\beta\cos(\omega t))\,z\,x\,\mathrm{d}(y)}{k\sqrt{x^2+y^2+\delta z^2}\,(y^2+x^2)}$$

```
>
> F:=wcollect(factor(d(Action)));
```

$$F := \frac{1}{2}\frac{(x^3+x^3\beta\cos(\omega t)+x\,y^2+x\,\beta\cos(\omega t)\,y^2)\,((\mathrm{d}(z))\,\&\wedge\,(\mathrm{d}(y)))}{k\,\%1^{3/2}\,(y^2+x^2)}$$
$$+\frac{1}{2}\frac{(-\beta\sin(\omega t)\,\omega\,z\,x^3-\beta\sin(\omega t)\,\omega\,z\,x\,y^2-\beta\sin(\omega t)\,\omega\,z^3\,x\,\delta)\,((\mathrm{d}(t))\,\&\wedge\,(\mathrm{d}(y)))}{k\,\%1^{3/2}\,(y^2+x^2)}$$
$$+\frac{1}{2}\frac{(-y^3\beta\cos(\omega t)-x^2\,y\,\beta\cos(\omega t)-y\,x^2-y^3)\,((\mathrm{d}(z))\,\&\wedge\,(\mathrm{d}(x)))}{k\,\%1^{3/2}\,(y^2+x^2)}$$
$$+\frac{1}{2}\frac{(\beta\sin(\omega t)\,\omega\,z\,y\,x^2+\beta\sin(\omega t)\,\omega\,z^3\,y\,\delta+\beta\sin(\omega t)\,\omega\,z\,y^3)\,((\mathrm{d}(t))\,\&\wedge\,(\mathrm{d}(x)))}{k\,\%1^{3/2}\,(y^2+x^2)}$$
$$+\frac{1}{2}\frac{(z\,y^2+z\,\beta\cos(\omega t)\,y^2+x^2\,z\,\beta\cos(\omega t)+x^2\,z)\,((\mathrm{d}(y))\,\&\wedge\,(\mathrm{d}(x)))}{k\,\%1^{3/2}\,(y^2+x^2)}$$
$$\%1 := x^2+y^2+\delta z^2$$

F is the electromagnetic 2-form in covariant language for all diffeomorphisms. It is gauge invariant with respect to all closed 1-form additions to the 1-form of Action.

The three components of the Vector potential are:

```
> A:=[evalm(A)[1],evalm(A)[2],evalm(A)[3]];
```

$$A := \left[-\frac{1}{2}\frac{(1+\beta\cos(\omega t))\,z\,y}{k\sqrt{x^2+y^2+\delta z^2}\,(y^2+x^2)},\ \frac{1}{2}\frac{(1+\beta\cos(\omega t))\,z\,x}{k\sqrt{x^2+y^2+\delta z^2}\,(y^2+x^2)},\ 0\right]$$

The three components of the Magnetic field are:

```
> B:=(curl(A,[x,y,z])):B1:=factor(B[1]);B2:=factor(B[2]);B3:=factor(B[3]);DIVB:=diverge(B,[x,y,z]);CURLB:=curl(B,[x,y,z]):CURLB1:=factor(CURLB[1]);CURLB2:=factor(CURLB[2]);CURLB3:=factor(CURLB[3]);
```

$$B1 := -\frac{1}{2}\frac{(1+\beta\cos(\omega t))\,x}{(x^2+y^2+\delta z^2)^{3/2}\,k}$$
$$B2 := -\frac{1}{2}\frac{(1+\beta\cos(\omega t))\,y}{(x^2+y^2+\delta z^2)^{3/2}\,k}$$
$$B3 := -\frac{1}{2}\frac{(1+\beta\cos(\omega t))\,z}{(x^2+y^2+\delta z^2)^{3/2}\,k}$$
$$DIVB := 0$$
$$CURLB1 := -\frac{3}{2}\frac{y\,z\,(1+\beta\cos(\omega t))\,(-1+\delta)}{(x^2+y^2+\delta z^2)^{5/2}\,k}$$
$$CURLB2 := \frac{3}{2}\frac{x\,z\,(1+\beta\cos(\omega t))\,(-1+\delta)}{(x^2+y^2+\delta z^2)^{5/2}\,k}$$
$$CURLB3 := 0$$

Note that the magnetic field is radially harmonic, and oscillates in time, first pointing in towards the origin and the outwards, with the frequency omega. (An oscillating hedgehog B field) There are no amperian currents of the type curlH if the oblateness factor delta = 1.. There will be currents due to the displacement currents even if there is no oblateness.

The three components of the Electric Field are (neglecting any Coulomb potential):

```
> E:=[-diff(A4,x)-diff(A[1],t),-diff(A4,y)-diff(A[2],t),-diff(A4,z)-diff(A[3],t)]:
  E1:=factor(E[1]);E2:=factor(E[2]);E3:=factor(E[3]);E1/A[1];E2/A[2];rho:=epsilon*
  diverge(E,[x,y,z]);
```

$$E1 := -\frac{1}{2}\frac{\beta\sin(\omega t)\,\omega z y}{k\sqrt{x^2+y^2+\delta z^2}\,(y^2+x^2)}$$

$$E2 := \frac{1}{2}\frac{\beta\sin(\omega t)\,\omega z x}{k\sqrt{x^2+y^2+\delta z^2}\,(y^2+x^2)}$$

$$E3 := 0$$

$$\frac{\beta\sin(\omega t)\,\omega}{1+\beta\cos(\omega t)}$$

$$\frac{\beta\sin(\omega t)\,\omega}{1+\beta\cos(\omega t)}$$

$$\rho := 0$$

The x,y components of the E field are proportional to the Vector potential., and as will be shown below both A and E are proportional to the Current, J, when the system is isotropic
Topological Parity 4 form (Second Poncare invariant) can be computed and it vanishes.

```
> EdotB:=factor(innerprod(E,B));EdotE:=innerprod(E,E):BdotB:=innerprod(B,B):ExH:=c
  rossprod(E,B/mu):Poynt1:=factor(ExH[1]);Poynt2:=factor(ExH[2]);Poynt3:=factor(Ex
  H[3]);RATIOPWRZX:=Poynt3/Poynt1; MagnPoynt:=factor(innerprod(ExH,ExH));
```

$$EdotB := 0$$

$$Poynt1 := -\frac{1}{4}\frac{(1+\beta\cos(\omega t))\,x z^2\,\omega\sin(\omega t)\,\beta}{\mu\,(y^2+x^2)\,(x^2+y^2+\delta z^2)^2\,k^2}$$

$$Poynt2 := -\frac{1}{4}\frac{(1+\beta\cos(\omega t))\,y z^2\,\omega\sin(\omega t)\,\beta}{\mu\,(y^2+x^2)\,(x^2+y^2+\delta z^2)^2\,k^2}$$

$$Poynt3 := \frac{1}{4}\frac{\beta\sin(\omega t)\,\omega z\,(1+\beta\cos(\omega t))}{\mu\,(x^2+y^2+\delta z^2)^2\,k^2}$$

$$RATIOPWRZX := -\frac{y^2+x^2}{z x}$$

$$MagnPoynt := \frac{1}{16}\frac{(x^2+y^2+z^2)\,\beta^2\sin(\omega t)^2\,\omega^2 z^2\,(1+\beta\cos(\omega t))^2}{k^4\,(y^2+x^2)\,\mu^2\,(x^2+y^2+\delta z^2)^4}$$

The Poynting vector density (E x H) is finite. The radiation is outbound in the z direction and inbound in the x,y directions!

The spatial part of the Torsion current can be computed directly to yield

```
> ExA:=crossprod(E,A);Bphi:=[B1*A4,B2*A4,B3*A4];
```

$$ExA := [0, 0, 0]$$

$$Bphi := [0, 0, 0]$$

```
>
```

The Full Torsion current vanishes !! There is NO HELICITY DENSITY OR TORSION CURRENT


```
> ExA:=crossprod(E,A);Bphi:=[B1*A4,B2*A4,B3*A4];
```

$$ExA := [0, 0, 0]$$
$$Bphi := [0, 0, 0]$$

```
> TORS:=evalm(ExA+A4*B);
```

$$TORS := [0, 0, 0]$$

```
> AdotB:=factor(inner(A,B));
```

$$AdotB := 0$$

THE HELICITY density IS ZERO !!!!!

```
> TORSION:=[factor(TORS[1]),factor(TORS[2]),factor(TORS[3]),AdotB];
```

$$TORSION := [0, 0, 0, 0]$$

The Torsion 3-form vanishes identically when beta = zero.

THE TORISON VECTOR vanishes identically !!!!!

P2 is the second poincare measure, and equals the Divergence of the Torsion current.

```
> P2:=factor(diverge(TORSION,[x,y,z,t]));
```

$$P2 := 0$$

```
>
```

Now compute the induced charge current assuming the constitutive properties of the Lorentz vacuum.

```
> J:= evalm(curl(B,[x,y,z])/mu-epsilon*[diff(E1,t),diff(E2,t),diff(E3,t)]):
> J1:=factor(J[1]);J2:=factor(J[2]);J3:=factor(J[3]);rho:=factor(diverge(E,[x,y,z]
  ));
> RATIO_J1toA1:=J1/A[1];RATIO_J2toA2:=J2/A[2];RATIO_J1toE1:=J1/E[1];RATIO_J2toE2:=
  J2/E[2];RATIO_J1toJ2:=factor(J1/J2);
```

$$J1 := \frac{1}{2}(3\,x^2 + 3\,x^2\,\beta\cos(\omega\,t) + 3\,\beta\cos(\omega\,t)\,y^2 - 3\,x^2\,\delta - 3\,\delta\,y^2 - 3\,\beta\cos(\omega\,t)\,\delta\,x^2 - 3\,\beta\cos(\omega\,t)\,\delta\,y^2 + 3\,y^2$$
$$+ \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,x^4 + 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^2\,x^2 + 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,x^2\,\delta\,z^2 + \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^4$$
$$+ 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^2\,\delta\,z^2 + \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,\delta^2\,z^4)\,z\,y \Big/ ((y^2 + x^2)\,(x^2 + y^2 + \delta\,z^2)^{5/2}\,k\,\mu)$$

$$J2 := -\frac{1}{2}(3\,x^2 + 3\,x^2\,\beta\cos(\omega\,t) + 3\,\beta\cos(\omega\,t)\,y^2 - 3\,x^2\,\delta - 3\,\delta\,y^2 - 3\,\beta\cos(\omega\,t)\,\delta\,x^2 - 3\,\beta\cos(\omega\,t)\,\delta\,y^2 + 3\,y^2$$
$$+ \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,x^4 + 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^2\,x^2 + 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,x^2\,\delta\,z^2 + \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^4$$
$$+ 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^2\,\delta\,z^2 + \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,\delta^2\,z^4)\,z\,x \Big/ ((y^2 + x^2)\,(x^2 + y^2 + \delta\,z^2)^{5/2}\,k\,\mu)$$

$$J3 := 0$$
$$\rho := 0$$

$$RATIO\_J1toA1 := -(3\,x^2 + 3\,x^2\,\beta\cos(\omega\,t) + 3\,\beta\cos(\omega\,t)\,y^2 - 3\,x^2\,\delta - 3\,\delta\,y^2 - 3\,\beta\cos(\omega\,t)\,\delta\,x^2 - 3\,\beta\cos(\omega\,t)\,\delta\,y^2$$
$$+ 3\,y^2 + \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,x^4 + 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^2\,x^2 + 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,x^2\,\delta\,z^2 + \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^4$$
$$+ 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^2\,\delta\,z^2 + \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,\delta^2\,z^4) \Big/ ((x^2 + y^2 + \delta\,z^2)^2\,\mu\,(1 + \beta\cos(\omega\,t)))$$

$$RATIO\_J2toA2 := -(3\,x^2 + 3\,x^2\,\beta\cos(\omega\,t) + 3\,\beta\cos(\omega\,t)\,y^2 - 3\,x^2\,\delta - 3\,\delta\,y^2 - 3\,\beta\cos(\omega\,t)\,\delta\,x^2 - 3\,\beta\cos(\omega\,t)\,\delta\,y^2$$
$$+ 3\,y^2 + \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,x^4 + 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^2\,x^2 + 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,x^2\,\delta\,z^2 + \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^4$$
$$+ 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^2\,\delta\,z^2 + \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,\delta^2\,z^4) \Big/ ((x^2 + y^2 + \delta\,z^2)^2\,\mu\,(1 + \beta\cos(\omega\,t)))$$

$$RATIO\_J1toE1 := -(3\,x^2 + 3\,x^2\,\beta\cos(\omega\,t) + 3\,\beta\cos(\omega\,t)\,y^2 - 3\,x^2\,\delta - 3\,\delta\,y^2 - 3\,\beta\cos(\omega\,t)\,\delta\,x^2 - 3\,\beta\cos(\omega\,t)\,\delta\,y^2$$
$$+ 3\,y^2 + \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,x^4 + 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^2\,x^2 + 2\,\varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,x^2\,\delta\,z^2 + \varepsilon\,\beta\cos(\omega\,t)\,\omega^2\,\mu\,y^4$$

$$+ 2\,\varepsilon\,\beta\cos(\omega t)\,\omega^2\,\mu\,y^2\,\delta z^2 + \varepsilon\,\beta\cos(\omega t)\,\omega^2\,\mu\,\delta^2 z^4) \Big/ ((x^2+y^2+\delta z^2)^2\,\mu\,\beta\sin(\omega t)\,\omega)$$

$$RATIO\_J2toE2 := -(3x^2 + 3x^2\beta\cos(\omega t) + 3\beta\cos(\omega t)\,y^2 - 3x^2\delta - 3\delta y^2 - 3\beta\cos(\omega t)\,\delta x^2 - 3\beta\cos(\omega t)\,\delta y^2 + 3y^2 + \varepsilon\beta\cos(\omega t)\,\omega^2\mu x^4 + 2\varepsilon\beta\cos(\omega t)\,\omega^2\mu y^2x^2 + 2\varepsilon\beta\cos(\omega t)\,\omega^2\mu x^2\delta z^2 + \varepsilon\beta\cos(\omega t)\,\omega^2\mu y^4 + 2\varepsilon\beta\cos(\omega t)\,\omega^2\mu y^2\delta z^2 + \varepsilon\beta\cos(\omega t)\,\omega^2\mu\delta^2 z^4) \Big/ ((x^2+y^2+\delta z^2)^2\,\mu\,\beta\sin(\omega t)\,\omega)$$

$$RATIO\_J1toJ2 := -\frac{y}{x}$$

Note that the currents are entirely of the displacement type if delta = 1 (isotropic case)

Now compute the Lorentz force = rho E + J x B. note that if there is no rho, the plasma force is just due to the J x B term:

```
> JXB:=crossprod(J,B):JxB_Z:=factor(JXB[3]);JxB_X:=factor(JXB[1]);JxB_Y:=factor(JXB[2]);MagHEL:=innerprod(J,B);RatioLFXY:=JxB_X/JxB_Y;RatioLFZX:=JxB_Z/JxB_X;
```

$$JxB\_Z := -\frac{1}{4}z\,(3x^2 + 3x^2\beta\cos(\omega t) + 3\beta\cos(\omega t)\,y^2 - 3x^2\delta - 3\delta y^2 - 3\beta\cos(\omega t)\,\delta x^2 - 3\beta\cos(\omega t)\,\delta y^2 + 3y^2 + \varepsilon\beta\cos(\omega t)\,\omega^2\mu x^4 + 2\varepsilon\beta\cos(\omega t)\,\omega^2\mu y^2x^2 + 2\varepsilon\beta\cos(\omega t)\,\omega^2\mu x^2\delta z^2 + \varepsilon\beta\cos(\omega t)\,\omega^2\mu y^4 + 2\varepsilon\beta\cos(\omega t)\,\omega^2\mu y^2\delta z^2 + \varepsilon\beta\cos(\omega t)\,\omega^2\mu\delta^2 z^4)\,(1+\beta\cos(\omega t)) \Big/ (\mu k^2 (x^2+y^2+\delta z^2)^4)$$

$$JxB\_X := \frac{1}{4}(1+\beta\cos(\omega t))\,(3x^2 + 3x^2\beta\cos(\omega t) + 3\beta\cos(\omega t)\,y^2 - 3x^2\delta - 3\delta y^2 - 3\beta\cos(\omega t)\,\delta x^2 - 3\beta\cos(\omega t)\,\delta y^2 + 3y^2 + \varepsilon\beta\cos(\omega t)\,\omega^2\mu x^4 + 2\varepsilon\beta\cos(\omega t)\,\omega^2\mu y^2x^2 + 2\varepsilon\beta\cos(\omega t)\,\omega^2\mu x^2\delta z^2 + \varepsilon\beta\cos(\omega t)\,\omega^2\mu y^4 + 2\varepsilon\beta\cos(\omega t)\,\omega^2\mu y^2\delta z^2 + \varepsilon\beta\cos(\omega t)\,\omega^2\mu\delta^2 z^4)\,x\,z^2 \Big/ ((y^2+x^2)\,(x^2+y^2+\delta z^2)^4\,k^2\mu)$$

$$JxB\_Y := \frac{1}{4}(1+\beta\cos(\omega t))\,(3x^2 + 3x^2\beta\cos(\omega t) + 3\beta\cos(\omega t)\,y^2 - 3x^2\delta - 3\delta y^2 - 3\beta\cos(\omega t)\,\delta x^2 - 3\beta\cos(\omega t)\,\delta y^2 + 3y^2 + \varepsilon\beta\cos(\omega t)\,\omega^2\mu x^4 + 2\varepsilon\beta\cos(\omega t)\,\omega^2\mu y^2x^2 + 2\varepsilon\beta\cos(\omega t)\,\omega^2\mu x^2\delta z^2 + \varepsilon\beta\cos(\omega t)\,\omega^2\mu y^4 + 2\varepsilon\beta\cos(\omega t)\,\omega^2\mu y^2\delta z^2 + \varepsilon\beta\cos(\omega t)\,\omega^2\mu\delta^2 z^4)\,y\,z^2 \Big/ ((y^2+x^2)\,(x^2+y^2+\delta z^2)^4\,k^2\mu)$$

$$MagHEL := 0$$

$$RatioLFXY := \frac{x}{y}$$

$$RatioLFZX := -\frac{y^2+x^2}{z\,x}$$

Note that the ratios of the Lorentz force components in the time dependent case are essentially the same as for the static case given above. Each component of the Lorentz Force has the same scaling function.

>
>

It is remarkable that the z component of the Lorentz Force, JxB_Z, is a restoring force such as to acrete the plasma to the Z=0 plane. This is the first time I have seen a mechanism for forming an accretion disc.

The net charge densities are zero, but there is a SPIRAL current that oscillates with time at the frequency omega. The current is proportional to the Vector potential !
Like a London Supercurrent

```
> MagHelicity3:=factor(innerprod(B,J));
```

$$MagHelicity3 := 0$$

Magnetic Energy B.H compared to interaction energy A.J.

```
> MagE3:=factor(subs(k=1,beta=1,epsilon=1,mu=1,omega=1,innerprod(B,B)/mu));ElecE3:
  =factor(subs(k=1,beta=1,epsilon=1,mu=1,omega=1,epsilon*innerprod(E,E)));
```

$$MagE3 := \frac{1}{4}\frac{\cos(t)^2\,(x^2+y^2+z^2)}{(x^2+y^2+\delta\,z^2)^3}$$

$$ElecE3 := \frac{1}{4}\frac{\sin(t)^2\,z^2}{(x^2+y^2)\,(x^2+y^2+\delta\,z^2)}$$

```
> IntE3:=factor(subs(k=1,beta=1,epsilon=1,mu=1,omega=1,innerprod(A,J)));
```

$$IntE3 := -\frac{1}{4}\frac{(3\,y^2+3\,x^2-3\,\delta\,x^2-3\,\delta\,y^2+x^4+2\,x^2\,y^2+2\,x^2\,\delta\,z^2+y^4+2\,y^2\,\delta\,z^2+\delta^2\,z^4)\,z^2\cos(t)^2}{(x^2+y^2)\,(x^2+y^2+\delta\,z^2)^3}$$

Next compute the Spin 3-form

```
> SSPIN:=evalm(crossprod(A,B/mu)+A4*epsilon*E):S4:=innerprod(A,epsilon*E);
>
```

$$S4 := \frac{1}{4}\frac{\cos(\omega\,t)\,z^2\,\varepsilon\,\sin(\omega\,t)\,\omega}{(x^2+y^2)\,(x^2+y^2+\delta\,z^2)\,k^2}$$

```
> SPIN:=[factor(SSPIN[1]),factor(SSPIN[2]),factor(SSPIN[3]),S4];
```

$$SPIN := \left[-\frac{1}{4}\frac{x\,z^2\cos(\omega\,t)^2}{\mu\,(x^2+y^2)\,\%1^2\,k^2},\ -\frac{1}{4}\frac{y\,z^2\cos(\omega\,t)^2}{\mu\,(x^2+y^2)\,\%1^2\,k^2},\ \frac{1}{4}\frac{\cos(\omega\,t)^2\,z}{\mu\,\%1^2\,k^2},\ \frac{1}{4}\frac{\cos(\omega\,t)\,z^2\,\varepsilon\,\sin(\omega\,t)\,\omega}{(x^2+y^2)\,\%1\,k^2}\right]$$

$$\%1 := x^2+y^2+\delta\,z^2$$

The Spin 3-form does not vanish, and moreover the first Poincare measure (equal to the divergence of the spin 3-form) does not vanish!

THE SPIN 3-FORM DOES NOT VANISH and DOES NOT HAVE ZERO DIVERGENCE !!!!!

```
> P1:=factor(simplify(factor(subs(diverge(SPIN,[x,y,z,t]))),trig));
```

$$P1 := \frac{1}{4}(4\,x^2\,z^2\cos(\omega\,t)^2 - z^6\,\varepsilon\,\omega^2\,\mu\,\delta^2 - y^4\,z^2\,\varepsilon\,\omega^2\,\mu - 2\,y^2\,z^2\,\varepsilon\,\omega^2\,\mu\,x^2 - 2\,y^2\,z^4\,\varepsilon\,\omega^2\,\mu\,\delta - z^2\,\varepsilon\,\omega^2\,\mu\,x^4$$
$$- 2\,z^4\,\varepsilon\,\omega^2\,\mu\,x^2\,\delta + 2\cos(\omega\,t)^2\,z^6\,\varepsilon\,\omega^2\,\mu\,\delta^2 + 2\,y^4\cos(\omega\,t)^2\,z^2\,\varepsilon\,\omega^2\,\mu + 4\,y^2\cos(\omega\,t)^2\,z^2\,\varepsilon\,\omega^2\,\mu\,x^2 - 3\,y^2\cos(\omega\,t)^2\,z^2\,\delta$$
$$+ 4\,y^2\cos(\omega\,t)^2\,z^4\,\varepsilon\,\omega^2\,\mu\,\delta + 2\cos(\omega\,t)^2\,z^2\,\varepsilon\,\omega^2\,\mu\,x^4 - 3\cos(\omega\,t)^2\,x^2\,\delta\,z^2 + 4\cos(\omega\,t)^2\,z^4\,\varepsilon\,\omega^2\,\mu\,x^2\,\delta + y^4\cos(\omega\,t)^2$$
$$+ \cos(\omega\,t)^2\,x^4 + 2\,y^2\cos(\omega\,t)^2\,x^2 + 4\,y^2\,z^2\cos(\omega\,t)^2)\Big/\left(\mu\,k^2\,(x^2+y^2)\,(x^2+y^2+\delta\,z^2)^3\right)$$

```
>
>
>
```

It is remarkable that the Spin density (A.D) oscillates at twice the frequency of the Driving Curent. Could it be that omega = 22 years to mimic the field reversal of the SUN, and the double frequency of 11 year period is the alternating sunspot cycles. ??? Can a small prolate oscillation be the cause of sunspots?

THIS WORK was STIMULATED BY THE CHAPMAN MAGNETIC HELICITY Conference at BOULDER CO, July 28, 1998

NOW EXAMINE THE EFFECT OF A COULOMB POTENTIAL
No OBLATENESS:

```
> A:=evalm((cos(omega*t)*z/(2*k*(x^2+y^2+z^2)^(1/2)*(x^2+y^2)))*([-y,x,0]));A4:=beta*z*sin(omega*t);
```

$$A := \left[-\frac{1}{2}\frac{\cos(\omega t)\, z\, y}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)}, \frac{1}{2}\frac{\cos(\omega t)\, z\, x}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)}, 0 \right]$$

$$A4 := \beta\, z \sin(\omega t)$$

```
>
> Action:=evalm(A)[1]*d(x)+evalm(A)[2]*d(y)+A[3]*d(z)+A4*d(t);
>
```

$$Action := -\frac{1}{2}\frac{\cos(\omega t)\, z\, y\, \mathrm{d}(x)}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)} + \frac{1}{2}\frac{\cos(\omega t)\, z\, x\, \mathrm{d}(y)}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)} + \beta\, z \sin(\omega t)\, \mathrm{d}(t)$$

```
>
> F:=wcollect(factor(d(Action)));
```

$$F := \frac{1}{2}\frac{(2\, x^2 \beta \sin(\omega t)\, k\, (x^2+y^2+z^2)^{3/2} + 2\, \beta \sin(\omega t)\, k\, (x^2+y^2+z^2)^{3/2}\, y^2)\, ((\mathrm{d}(z)) \,\&\wedge\, (\mathrm{d}(t)))}{k\, (x^2+y^2+z^2)^{3/2}\, (y^2+x^2)}$$
$$+ \frac{1}{2}\frac{(\cos(\omega t)\, x^3 + x \cos(\omega t)\, y^2)\, ((\mathrm{d}(z)) \,\&\wedge\, (\mathrm{d}(y)))}{k\, (x^2+y^2+z^2)^{3/2}\, (y^2+x^2)}$$
$$+ \frac{1}{2}\frac{(-\sin(\omega t)\, \omega\, z\, x^3 - \sin(\omega t)\, \omega\, z\, x\, y^2 - \sin(\omega t)\, \omega\, z^3\, x)\, ((\mathrm{d}(t)) \,\&\wedge\, (\mathrm{d}(y)))}{k\, (x^2+y^2+z^2)^{3/2}\, (y^2+x^2)}$$
$$+ \frac{1}{2}\frac{(-x^2 \cos(\omega t)\, y - \cos(\omega t)\, y^3)\, ((\mathrm{d}(z)) \,\&\wedge\, (\mathrm{d}(x)))}{k\, (x^2+y^2+z^2)^{3/2}\, (y^2+x^2)}$$
$$+ \frac{1}{2}\frac{(\sin(\omega t)\, \omega\, z\, y\, x^2 + \sin(\omega t)\, \omega\, z\, y^3 + \sin(\omega t)\, \omega\, z^3\, y)\, ((\mathrm{d}(t)) \,\&\wedge\, (\mathrm{d}(x)))}{k\, (x^2+y^2+z^2)^{3/2}\, (y^2+x^2)}$$
$$+ \frac{1}{2}\frac{(\cos(\omega t)\, z\, x^2 + \cos(\omega t)\, z\, y^2)\, ((\mathrm{d}(y)) \,\&\wedge\, (\mathrm{d}(x)))}{k\, (x^2+y^2+z^2)^{3/2}\, (y^2+x^2)}$$

F is the electromagnetic 2-form in covariant language for all diffeomorphisms. It is gauge invariant with respect to all closed 1-form additions to the 1-form of Action.

The three components of the Vector potential are:

```
> A:=[evalm(A)[1],evalm(A)[2],evalm(A)[3]];
```

$$A := \left[-\frac{1}{2}\frac{\cos(\omega t)\, z\, y}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)}, \frac{1}{2}\frac{\cos(\omega t)\, z\, x}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)}, 0 \right]$$

The three components of the Magnetic field are:

```
> B:=(curl(A,[x,y,z])):B1:=factor(B[1]);B2:=factor(B[2]);B3:=factor(B[3]);DIVB:=diverge(B,[x,y,z]);CURLB:=curl(B,[x,y,z]):CURLB1:=factor(CURLB[1]);CURLB2:=factor(CURLB[2]);CURLB3:=factor(CURLB[3]);
```

$$B1 := -\frac{1}{2}\frac{\cos(\omega t)\, x}{(x^2+y^2+z^2)^{3/2}\, k}$$

$$B2 := -\frac{1}{2}\frac{\cos(\omega t)\, y}{(x^2+y^2+z^2)^{3/2}\, k}$$

$$B3 := -\frac{1}{2}\frac{\cos(\omega t)\, z}{(x^2+y^2+z^2)^{3/2}\, k}$$

$$DIVB := 0$$

$$CURLB1 := 0$$

$$CURLB2 := 0$$

$$CURLB3 := 0$$

Note that the magnetic field is radially harmonic, and oscillates in time, first pointing in towards the origin and the outwards, with the frequency omega. (An oscillating hedgehog B field) The B field does not depend upon the Coulomb Potential terms. NO Amperian currents exist unless the system is anisotropic (oblate).

The three components of the Electric Field are (now including the Coulomb potential):

```
> E:=[-diff(A4,x)-diff(A[1],t),-diff(A4,y)-diff(A[2],t),-diff(A4,z)-diff(A[3],t)]:
  E1:=factor(E[1]);E2:=factor(E[2]);E3:=factor(E[3]);E1/A[1];E2/A[2];
```

$$E1 := -\frac{1}{2}\frac{\sin(\omega t)\,\omega\, z\, y}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)}$$

$$E2 := \frac{1}{2}\frac{\sin(\omega t)\,\omega\, z\, x}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)}$$

$$E3 := -\beta \sin(\omega t)$$

$$\frac{\sin(\omega t)\,\omega}{\cos(\omega t)}$$

$$\frac{\sin(\omega t)\,\omega}{\cos(\omega t)}$$

The x,y components of the E field are proportional to the Vector potential., and as will be shown below both A and E are proportional to the Current, J, when beta is zero. Note that there is now a component of the E field in the direction of the Z axis.

Topological Parity 4 form (Second Poncare invariant) can be computed and it vanishes when beta is zero, but not for finite beta!

```
> EdotB:=factor(innerprod(E,B));EdotE:=innerprod(E,E):BdotB:=innerprod(B,B):ExB:=c
  rossprod(E,B):Poynt1c=factor(ExB[1]/mu);Poynt2c=factor(ExB[2]/mu);Poynt3c=factor
  (ExB[3]/mu);
```

$$EdotB := \frac{1}{2}\frac{z\cos(\omega t)\sin(\omega t)\,\beta}{(x^2+y^2+z^2)^{3/2}\, k}$$

$$Poynt1c = \frac{1}{4}\frac{(-\omega z^2 x + 2\beta y k\sqrt{x^2+y^2+z^2}\, z^2 - 2\beta y k (x^2+y^2+z^2)^{3/2})\sin(\omega t)\cos(\omega t)}{\mu (y^2+x^2)(x^2+y^2+z^2)^2 k^2}$$

$$Poynt2c = -\frac{1}{4}\frac{(-2 x\beta k (x^2+y^2+z^2)^{3/2} + 2 x \beta k\sqrt{x^2+y^2+z^2}\, z^2 + \omega z^2 y)\sin(\omega t)\cos(\omega t)}{\mu (y^2+x^2)(x^2+y^2+z^2)^2 k^2}$$

$$Poynt3c = \frac{1}{4}\frac{\cos(\omega t)\, z\, \omega \sin(\omega t)}{k^2 (x^2+y^2+z^2)^2 \mu}$$

The Poynting vector density (E x H) is finite.

The spatial part of the Torsion current can be computed directly to yield

```
> ExA:=crossprod(E,A);Bphi:=[B1*A4,B2*A4,B3*A4];
```

$$ExA := \left[\frac{1}{2}\frac{\beta \sin(\omega t)\cos(\omega t)\, z\, x}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)}, \frac{1}{2}\frac{\beta \sin(\omega t)\cos(\omega t)\, z\, y}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)}, 0\right]$$

$$Bphi := \left[-\frac{1}{2}\frac{\cos(\omega t)\, x\, \beta\, z \sin(\omega t)}{(x^2+y^2+z^2)^{3/2} k}, -\frac{1}{2}\frac{\cos(\omega t)\, y\, \beta\, z \sin(\omega t)}{(x^2+y^2+z^2)^{3/2} k}, -\frac{1}{2}\frac{\cos(\omega t)\, z^2 \beta \sin(\omega t)}{(x^2+y^2+z^2)^{3/2} k}\right]$$

```
>
```

The Full Torsion current does not vanish unless beta=0. !! But even though there is a TORSION CURRENT, the HELICITY DENSITY VANISHES.!!!


```
> ExA:=crossprod(E,A);Bphi:=[B1*A4,B2*A4,B3*A4];
```

$$ExA := \left[\frac{1}{2}\frac{\beta \sin(\omega t)\cos(\omega t)\, z\, x}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)}, \frac{1}{2}\frac{\beta \sin(\omega t)\cos(\omega t)\, z\, y}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)}, 0\right]$$

$$Bphi := \left[-\frac{1}{2}\frac{\cos(\omega t)\, x\, \beta\, z \sin(\omega t)}{(x^2+y^2+z^2)^{3/2} k}, -\frac{1}{2}\frac{\cos(\omega t)\, y\, \beta\, z \sin(\omega t)}{(x^2+y^2+z^2)^{3/2} k}, -\frac{1}{2}\frac{\cos(\omega t)\, z^2 \beta \sin(\omega t)}{(x^2+y^2+z^2)^{3/2} k}\right]$$


```
> TORS:=evalm(ExA+A4*B):
>
> AdotB:=factor(inner(A,B));
```

$$AdotB := 0$$

THE HELICITY density IS ZERO !!!!!

```
> TORSION:=[factor(TORS[1]),factor(TORS[2]),factor(TORS[3]),AdotB];
```

$$TORSION := \left[\frac{1}{2}\frac{\beta \sin(\omega t)\cos(\omega t)\, z^3 x}{k (x^2+y^2+z^2)^{3/2}(y^2+x^2)}, \frac{1}{2}\frac{\beta \sin(\omega t)\cos(\omega t)\, z^3 y}{k (x^2+y^2+z^2)^{3/2}(y^2+x^2)}, -\frac{1}{2}\frac{\cos(\omega t)\, z^2 \beta \sin(\omega t)}{(x^2+y^2+z^2)^{3/2} k}, 0\right]$$

P2 is the second poincare measure, and equals the Divergence of the Torsion current.

```
> P2:=factor(diverge(TORSION,[x,y,z,t]));
```

$$P2 := -\frac{z \cos(\omega t)\sin(\omega t)\,\beta}{(x^2+y^2+z^2)^{3/2} k}$$

```
>
```

Now compute the induced charge current assuming the constitutive properties of the Lorentz vacuum. There are no Amperian currents unless the system is oblate, but there are displacement currents.

```
> J:= evalm(curl(B,[x,y,z])/mu-epsilon*[diff(E1,t),diff(E2,t),diff(E3,t)]):
> J1:=factor(J[1]);J2:=factor(J[2]);J3:=factor(J[3]);rho:=factor(diverge(E,[x,y,z]
  ));
> RATIO_J1toA1:=J1/A[1];RATIO_J2toA2:=J2/A[2];RATIO_J3toE3:=J3/E[3];RATIO_J1toE1:=
```

```
J1/E[1];RATIO_J2toE2:=J2/E[2];
```

$$J1 := \frac{1}{2}\frac{\varepsilon \cos(\omega t)\,\omega^2 z\, y}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)}$$

$$J2 := -\frac{1}{2}\frac{\varepsilon \cos(\omega t)\,\omega^2 z\, x}{k\sqrt{x^2+y^2+z^2}\,(y^2+x^2)}$$

$$J3 := \varepsilon\,\beta \cos(\omega t)\,\omega$$

$$\rho := 0$$

$$RATIO\_J1toA1 := -\varepsilon\,\omega^2$$

$$RATIO\_J2toA2 := -\varepsilon\,\omega^2$$

$$RATIO\_J3toE3 := -\frac{\varepsilon \cos(\omega t)\,\omega}{\sin(\omega t)}$$

$$RATIO\_J1toE1 := -\frac{\varepsilon \cos(\omega t)\,\omega}{\sin(\omega t)}$$

$$RATIO\_J2toE2 := -\frac{\varepsilon \cos(\omega t)\,\omega}{\sin(\omega t)}$$

Now compute the Lorentz force = rho E + J x B. note that if there is no rho, the plasma force is just due to the J x B term: Note further that the time dependent Coluomb term induces a current along the z axis.

The E field has the Ohmic property that it is proportional to the current density.

```
> JXB:=crossprod(J,B):JxB_Z:=factor(JXB[3]);JxB_X:=factor(JXB[1]);JxB_Y:=factor(JXB[2]);
```

$$JxB\_Z := -\frac{1}{4}\frac{\cos(\omega t)^2 z\,\varepsilon\,\omega^2}{k^2 (x^2+y^2+z^2)^2}$$

$$JxB\_X := -\frac{1}{4}\varepsilon\,\omega\,(-x^5 z^2 \omega - 2 x^3 z^2 \omega y^2 - 2 x^3 z^4 \omega - x z^2 \omega y^4 - 2 x z^4 \omega y^2 - x z^6 \omega - 2 \beta y k (x^2+y^2+z^2)^{7/2} + 2 \beta y k (x^2+y^2+z^2)^{5/2} z^2) \cos(\omega t)^2 \Big/ ((y^2+x^2)(x^2+y^2+z^2)^4 k^2)$$

$$JxB\_Y := \frac{1}{4}\varepsilon\,\omega\,(-2 x \beta k (x^2+y^2+z^2)^{7/2} + 2 x \beta k (x^2+y^2+z^2)^{5/2} z^2 + z^2 y \omega x^4 + 2 z^2 y^3 \omega x^2 + 2 z^4 y \omega x^2 + z^2 y^5 \omega + 2 z^4 y^3 \omega + z^6 y \omega) \cos(\omega t)^2 \Big/ ((y^2+x^2)(x^2+y^2+z^2)^4 k^2)$$

It is remarkable that even with the time dependent Coulomb potential (and no anisotropy) the z component of the Lorentz Force, JxB_Z, is still a restoring force such as to acrete the plasma to the Z=0 plane. This robust feature is due to the HedgeHog quality of the B field.

The net charge densities are zero, but there is a SPIRAL current that oscillates with time at the frequency omega. The current is proportional to the Vector potential ! Like a London super current.!

****

Next compute the Spin 3-form

```
> SSPIN:=evalm(crossprod(A,B/mu)+A4*epsilon*E):S4:=innerprod(A,epsilon*E);
>
```

$$S4 := \frac{1}{4}\,\frac{\cos(\omega t)\, z^2\, \varepsilon \sin(\omega t)\, \omega}{(y^2+x^2)(x^2+y^2+z^2)\, k^2}$$

```
> SPIN:=[factor(SSPIN[1]),factor(SSPIN[2]),factor(SSPIN[3]),S4];
```

$$SPIN := \left[-\frac{1}{4}\,\frac{(\cos(\omega t)^2 x + 2\,\beta \sin(\omega t)^2\, \varepsilon\, \omega\, y\, k\, (x^2+y^2+z^2)^{3/2}\, \mu)\, z^2}{\mu\, (y^2+x^2)(x^2+y^2+z^2)^2\, k^2}, \right.$$

$$\frac{1}{4}\,\frac{(2\,\beta \sin(\omega t)^2\, \varepsilon\, \omega\, x\, k\, (x^2+y^2+z^2)^{3/2}\, \mu - \cos(\omega t)^2\, y)\, z^2}{\mu\, (y^2+x^2)(x^2+y^2+z^2)^2\, k^2}, -\frac{1}{4} z\, (4\,\beta^2 \sin(\omega t)^2\, \varepsilon\, k^2\, \mu\, x^4$$

$$+ 8\, x^2 \beta^2 \sin(\omega t)^2\, \varepsilon\, k^2\, \mu\, y^2 + 8\, x^2 \beta^2 \sin(\omega t)^2\, \varepsilon\, k^2\, \mu\, z^2 + 8\,\beta^2 \sin(\omega t)^2\, \varepsilon\, k^2\, \mu\, z^2 y^2 + 4\,\beta^2 \sin(\omega t)^2\, \varepsilon\, k^2\, \mu\, z^4$$

$$\left. - \cos(\omega t)^2 + 4\,\beta^2 \sin(\omega t)^2\, \varepsilon\, k^2\, \mu\, y^4) \Big/ (k^2 (x^2+y^2+z^2)^2\, \mu), \frac{1}{4}\,\frac{\cos(\omega t)\, z^2\, \varepsilon \sin(\omega t)\, \omega}{(y^2+x^2)(x^2+y^2+z^2)\, k^2} \right]$$

The Spin 3-form does not vanish, and moreover the first Poincare measure (equal to the divergence of the spin 3-form) does not vanish!

THE SPIN 3-FORM DOES NOT VANISH and DOES NOT HAVE ZERO DIVERGENCE !!!!!

```
> P1:=factor(simplify(factor(subs(diverge(SPIN,[x,y,z,t]))),trig));
```

$$P1 := \frac{1}{4}(2\, x^2 \cos(\omega t)^2 z^2\, \varepsilon\, \omega^2\, \mu + 2 \cos(\omega t)^2 z^2\, \varepsilon\, \omega^2\, \mu\, y^2 + 2 \cos(\omega t)^2 z^4\, \varepsilon\, \omega^2\, \mu + \cos(\omega t)^2 x^2 + \cos(\omega t)^2 y^2$$

$$- x^2 \omega^2 z^2\, \varepsilon\, \mu - \omega^2 z^4\, \varepsilon\, \mu - 4\,\beta^2 \varepsilon\, k^2 \mu\, x^2 z^4 + 4\,\beta^2 \varepsilon\, k^2 \mu\, x^2 z^4 \cos(\omega t)^2 + 4\,\beta^2 \varepsilon\, k^2 \mu\, y^2 z^4 \cos(\omega t)^2 - 4\,\beta^2 \varepsilon\, k^2 \mu\, y^2 z^4$$

$$- 8\,\beta^2 \varepsilon\, k^2 \mu\, y^4 z^2 + 8\,\beta^2 \varepsilon\, k^2 \mu\, y^4 z^2 \cos(\omega t)^2 + 16\,\beta^2 \varepsilon\, k^2 \mu\, x^2 z^2 y^2 \cos(\omega t)^2 - 16\,\beta^2 \varepsilon\, k^2 \mu\, x^2 z^2 y^2 - \omega^2 z^2\, \varepsilon\, \mu\, y^2$$

$$- 12\,\beta^2 \varepsilon\, k^2 \mu\, x^2 y^4 + 12\,\beta^2 \varepsilon\, k^2 \mu\, x^2 y^4 \cos(\omega t)^2 - 4\,\beta^2 \varepsilon\, k^2 \mu\, y^6 + 4\,\beta^2 \varepsilon\, k^2 \mu\, y^6 \cos(\omega t)^2 - 8\,\beta^2 \varepsilon\, k^2 \mu\, x^4 z^2$$

$$+ 8\,\beta^2 \varepsilon\, k^2 \mu\, x^4 z^2 \cos(\omega t)^2 - 12\,\beta^2 \varepsilon\, k^2 \mu\, x^4 y^2 + 12\,\beta^2 \varepsilon\, k^2 \mu\, x^4 y^2 \cos(\omega t)^2 - 4\,\beta^2 \varepsilon\, k^2 \mu\, x^6 + 4\,\beta^2 \varepsilon\, k^2 \mu\, x^6 \cos(\omega t)^2$$

$$) \Big/ (\mu\, k^2 (y^2+x^2)(x^2+y^2+z^2)^2)$$

```
>
>
```