```
> restart:
```

faraday.mws (R.M.Kiehn, example created 9/1/97)
Using MAPLE as a symbolic math calculator to compute
Maxell-Faraday formulas from an arbitrary 1-form of Action on 4D space time.
The space-time is not constrained by any metric, group, or connection.
The program allows the computation of the vector part of the torsion current,
R. M. Kiehn,et.al.Phys Rev A, 43 , (1991) p. 5665
or
http://www.uh.edu/~rkiehn/pdf/maxwell.pdf
(which, if it exists, is the equivalent to Poincare formulation of the B3 concept given by Evans in
"Helicity and the Electromagnetic Field" Eq 55
www.europa.com/~rsc/physics/B3/evans

```
>  with(liesymm):with(linalg):with(plots):
Warning, new definition for close
Warning, new definition for norm
Warning, new definition for trace
```

```
> setup(x,y,z,t);
```

$$[x, y, z, t]$$

The independent variables are assumed as {x,y,z,t}
Forms are intitialized below:

```
> defform(x=0,y=0,z=0,t=0,a=const,b=const,c=const,k=const,mu=const,m=const,alpha=c
  onst);
```

$$\mathrm{defform}(x = 0, y = 0, z = 0, t = 0, a = const, b = const, c = const, k = const, \mu = const, m = const, \alpha = const)$$

The differential position vector on the domain:

```
> dR:=[d(x),d(y),d(z),d(t)];
```

$$dR := [\mathrm{d}(x), \mathrm{d}(y), \mathrm{d}(z), \mathrm{d}(t)]$$

Specify the four functions that are the covariant components of the Action 1-form.

```
> A1:=Ax(x,y,z,t);A2:=Ay(x,y,z,t);A2:=Ay(x,y,z,t);A4:=phi(x,y,z,t);
>
```

$$A1 := \mathrm{Ax}(x, y, z, t)$$

$$A2 := \mathrm{Ay}(x, y, z, t)$$

$$A2 := \mathrm{Ay}(x, y, z, t)$$

$$A4 := \phi(x, y, z, t)$$

Skip the next line for abstract formulas, otherwise insert explicit functional forms

```
> A1:=x^3+x*y;A2:=x^3;A3:=z;A4:=cos(t)*x;
```

$$A1 := x^3 + x\,y$$

$$A2 := x^3$$

$$A3 := z$$

$$A4 := \cos(t)\,x$$

```
>
```

```
> Action:=A1*d(x)+A2*d(y)+A3*d(z)-A4*d(t);
```

$$Action := (x^3 + x\,y)\,\mathrm{d}(x) + x^3\,\mathrm{d}(y) + z\,\mathrm{d}(z) - \cos(t)\,x\,\mathrm{d}(t)$$

```
> F:=wcollect(d(Action));
```

$$F := \cos(t)\,((\mathrm{d}(t))\ \&\hat{}\ (\mathrm{d}(x))) + (-x + 3\,x^2)\,((\mathrm{d}(x))\ \&\hat{}\ (\mathrm{d}(y)))$$

Vector potential Av

```
> Av:=[A1,A2,A3];
```

$$Av := [x^3 + x\,y,\ x^3,\ z]$$

Magnetic field vector Bv

```
> Bv:=curl(Av,[x,y,z]);
```

$$Bv := [0,\ 0,\ -x + 3\,x^2]$$

Electric Field

```
> E:=[-diff(A4,x)-diff(Av[1],t),-diff(A4,y)-diff(Av[2],t),-diff(A4,z)-diff(Av[3],t
)];
```

$$E := [-\cos(t),\ 0,\ 0]$$

The E and B fields satisfy Maxwell's equations, curlE+partialdB/dt = 0 and div B = 0

Parity (Second Poncare invariant)

```
> EdotB:=factor(simplify(inner(E,Bv)));
```

$$EdotB := 0$$

The Torsion current.

```
> ExAv:=crossprod(E,Av);Bphi:=[Bv[1]*phi,Bv[2]*phi,Bv[3]*phi];
```

$$ExAv := [0,\ \cos(t)\,z,\ -\cos(t)\,x^3]$$

$$Bphi := [0,\ 0,\ (-x + 3\,x^2)\,\phi]$$

The vector part of the Torsion current is the Poincare form of B3

```
> TORSB3:=evalm(ExAv+A4*Bv);
```

$$TORSB3 := [0,\ \cos(t)\,z,\ -\cos(t)\,x^3 + \cos(t)\,x\,(-x + 3\,x^2)]$$

```
> AdotB:=inner(Av,Bv);
```

$$AdotB := z\,(-x + 3\,x^2)$$

The 4 vector of Torsion current

```
> TORSION:=[TORSB3[1],TORSB3[2],TORSB3[3],AdotB];
```

$$TORSION := [0,\ \cos(t)\,z,\ -\cos(t)\,x^3 + \cos(t)\,x\,(-x + 3\,x^2),\ z\,(-x + 3\,x^2)]$$

Divergence of the Torsion current. If the E.B Poincare invariant is zero then the Torsion current satisfies a conservation law. 4divT = 0.

```
> DIVT:=factor(diverge(TORSION,[x,y,z,t]));
```

$$DIVT := 0$$

An evolutionary dynamical system.

```
> VV:=[U(x,y,z,t),V(x,y,z,t),W(x,y,z,t)];
```

$$VV := [\mathrm{U}(x, y, z, t),\ \mathrm{V}(x, y, z, t),\ \mathrm{W}(x, y, z, t)]$$

The Lorentx force.

```
> FL:=evalm(E+crossprod(VV,Bv));P:=innerprod(VV,E);
```

$$FL := [-\cos(t) + \mathrm{V}(x, y, z, t)\,(-x + 3\,x^2),\ -\mathrm{U}(x, y, z, t)\,(-x + 3\,x^2),\ 0]$$

$$P := -\mathrm{U}(x, y, z, t)\,\cos(t)$$

The virtual work 1-form

```
> Work:=FL[1]*d(x)+FL[2]*d(y)+FL[3]*d(z)-P*d(t);
```

$$Work := (-\cos(t) + \mathrm{V}(x, y, z, t)\,(-x + 3\,x^2))\,\mathrm{d}(x) - \mathrm{U}(x, y, z, t)\,(-x + 3\,x^2)\,\mathrm{d}(y) + \mathrm{U}(x, y, z, t)\,\cos(t)\,\mathrm{d}(t)$$

If the exterior differential of the Work 1-form vanishes, then the motion induced by the dynamical system satisfies the Helmholtz condition (and vorticity is conserved.) In space time this is implies that the evolution is a symplectomorphism. A subset of symplecto-morphisms are conservative Hamiltonian systems.

```
> Helmholtz:=wcollect(d(Work));
```

$$Helmholtz := \left(\sin(t) - \left(\frac{\partial}{\partial t}\mathrm{V}(x,y,z,t)\right)x + 3\left(\frac{\partial}{\partial t}\mathrm{V}(x,y,z,t)\right)x^2 - \left(\frac{\partial}{\partial x}\mathrm{U}(x,y,z,t)\right)\cos(t)\right)((\mathrm{d}(t))\ \&\hat{}\ (\mathrm{d}(x))) + \left(\left(\frac{\partial}{\partial x}\mathrm{U}(x,y,z,t)\right)x - 3\left(\frac{\partial}{\partial x}\mathrm{U}(x,y,z,t)\right)x^2 + \mathrm{U}(x,y,z,t) - 6\,\mathrm{U}(x,y,z,t)\,x - \left(\frac{\partial}{\partial y}\mathrm{V}(x,y,z,t)\right)x\,(-1+3\,x)\right)((\mathrm{d}(x))\ \&\hat{}\ (\mathrm{d}(y))) + \left(-\left(\frac{\partial}{\partial y}\mathrm{U}(x,y,z,t)\right)\cos(t) - \left(\frac{\partial}{\partial t}\mathrm{U}(x,y,z,t)\right)x\,(-1+3\,x)\right)((\mathrm{d}(t))\ \&\hat{}\ (\mathrm{d}(y))) + \left(\frac{\partial}{\partial z}\mathrm{V}(x,y,z,t)\right)x\,(-1+3\,x)\,((\mathrm{d}(z))\ \&\hat{}\ (\mathrm{d}(x))) + \left(\frac{\partial}{\partial z}\mathrm{U}(x,y,z,t)\right)\cos(t)\,((\mathrm{d}(z))\ \&\hat{}\ (\mathrm{d}(t))) - \left(\frac{\partial}{\partial z}\mathrm{U}(x,y,z,t)\right)x\,(-1+3\,x)\,((\mathrm{d}(z))\ \&\hat{}\ (\mathrm{d}(y)))$$

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Each algebraic factor must vanish it the Helmholtz conservation law is to be true. The implication is that there are six partial differential equations that must be satisfied for the evolution to be thermodynamically reversible. Such a constraint is always true if EdotB vanishes, for then there exists a UNIQUE hamiltonian vector field of conservative evolution on the resultant contact manifold of topological dimension 3.

Note that the Poincare formulation of B3 is as a covariant tensor field of rank 3, which is covariant with respect to ALL diffeomorphisms, Lorentz, Galilean, or any map with an inverse and a differentiable Jacobian inverse.

. If E.B=0, the conservation law is valid in all frames of reference that can be mapped c1 onto the original formulation. In this case, the conservation law is retrodictive even though the topology of the range space is Not the same as the topology of the target space.

1. NOTE THAT the Poincare completely covariant formulation of the Topological Torsion tensor is NOT Gauge invariant. Any closed 1-form can be added to the Action and the MAXWELL FIELD EQUATIONS are always GAUGE INVARIANT, but the Topological Torsion Tensor depends upon the Closed, but not Exact contributions (Harmonic, not gradient contributions) to the vector potentials. It depends on the induced Cartan Topology!

2. NOTE that the Poincare completely covariant formulation of the Topological Torsion tensor is not dependent upon the existence of complex or a non-abelian set of vector potentials.

As the B3 formalism of Evans seems to require a Non-Abelian vector valued set of potentials, then such a set of constraints implies that additional structure has been imposed upon the electromagnetic domain, other that that given by the topological considerations above.

There is no doubt that longitudinal B fields can be constructed from set of vector potentials

The fundamental question is that of their evolution. Are they truely waves in the simple sense?

I think not. The solutions that lead to waves (propagating discontinuities) with longitudinal components must be of a quaternion nature, and here I agree with Evans that such fields must belong to a non-abelian set.

see Kiehn. et. al. Phys Rev A, 43 , (1991) p. 5665

These waves may be projectively equivalent to the vacuum state, but it is hard for me to believe that they are Lorentz equivalent to the vacuum state. The Lorentz transformation preserves isotropy and homogeneity, which need not be part of the projective group.

See V. Fock "Space Time and Gravitation" 1932 appendix

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[>