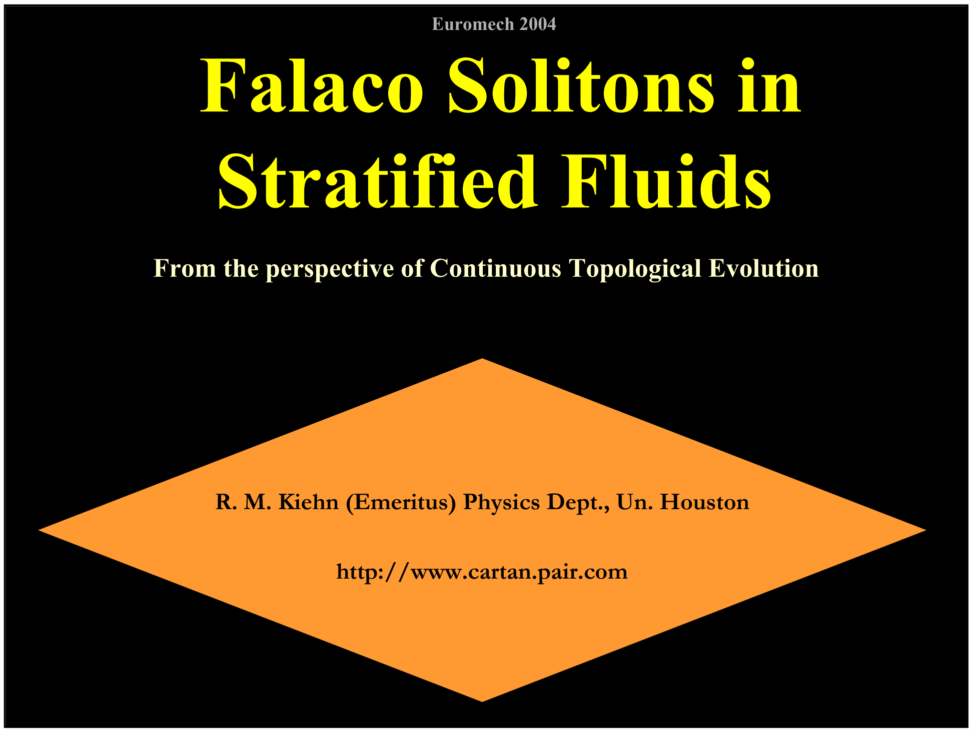


Before I explain the features of Falaco Solitons I believe that is appropriate that I describe 3 visual experiences that have made an extraordinary impact on, and have stimulated my interests in science, over the last 50 years.

The first photo was taken when I was working with the Los Alamos nuclear testing group, J-10, conducting atmospheric nuclear explosions in the Nevada desert. The time was 1957. The thing that startled me was that amongst all that non-equilibrium, turbulence, and irreversible processes that existed in the mushroom cloud, there was created an ionized ring of obvious topological coherence, a coherent structure far from equilibrium that persisted for a relatively long lifetime.

Three Stimulating Events #2

Long Lived Wake

1962

http://www.airtoair.net photo by Paul Bowan

The second stimulating event occurred in 1962 when I was flying with the strategic air command over the Pacific conducting measurements of hydrogen bomb explosions. When I saw the wakes generated by aircraft flying out cloud banks, again I was startled by the creation of a topological coherent structure, with a relatively long persistent lifetime, which was formed in an obviously non-equilibrium diffuse dissipative medium.

The third stimulating event occurred in Rio de Janeiro in 1986. It is these long lived coherent topological structures in a swimming pool, defined as Falaco Solitons, that I will discuss in more detail today.

What is the Common Thread ?

They all are artifacts of :

Continuous Topological Evolution

- **Coherent Topological Structures**
- **as Long lived States far from Equilibrium.**
- **Created by Irreversible processes.**

All of these stimulating photos have a common thread. They appear to be artifacts of Continuous Topological Evolution in a dissipative 4D domain of space-time producing long-lived topologically coherent, deformable, structures.

History of Falaco Solitons

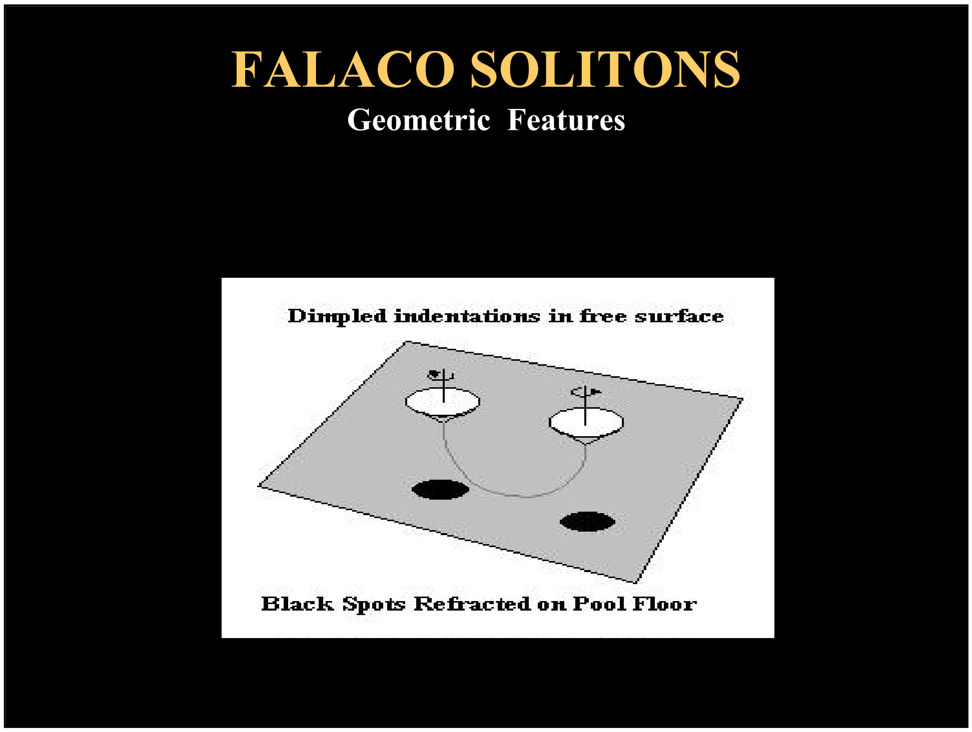
- 1986 visit to Rio de Janeiro and the mountain side house of my MIT roommate, Jose Haraldo H. Falcao.

- On the mountain side above the beach

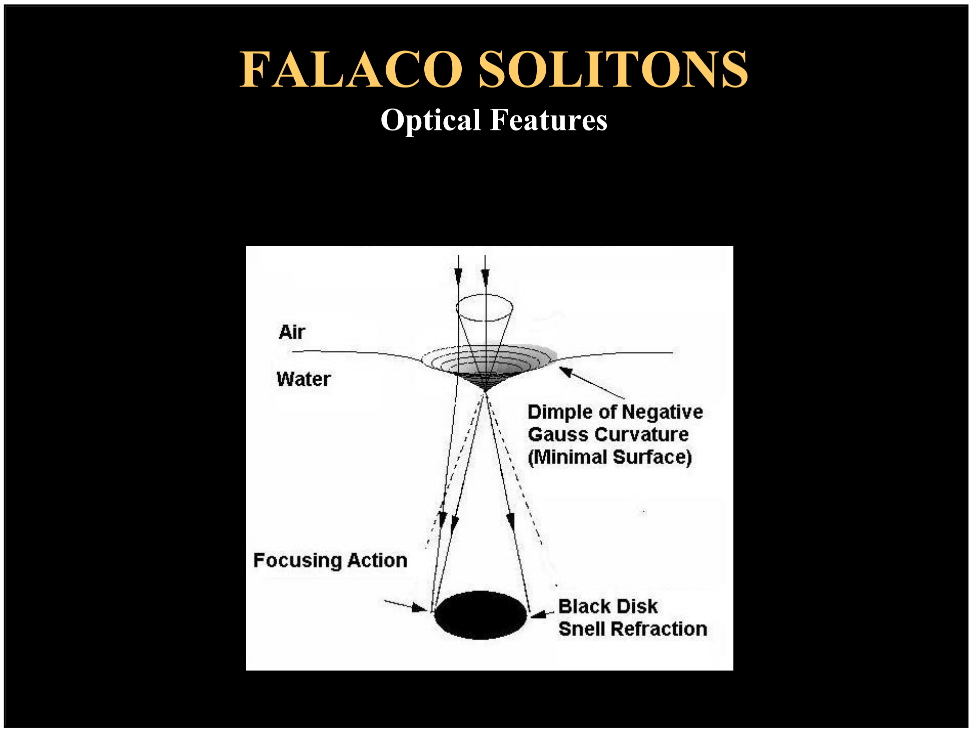
Now to Falaco Solitons. In 1986, I went to Rio de Janeiro to visit in my old MIT roommate, Jose Haraldo H. Falcao. He had married into wealth and had built a fabulous house (that his wife designed) hanging on the mountain side and overlooking Sao Coronado beach south of Rio.

The morning after my arrival, I went downstairs to the open air game room and took a dip in his pristine white marble pool flooded with bright Brazilian sunshine. After a few minutes I got out of the pool and was met by two servant girls offering terry cloth robes, coffee, and croissants. I drank my coffee and turned back to the pool some 5 minutes after my exit. When I looked at the pool I saw a pair of very dark Black Circular Spots about 15 cm in diameter, with bright contrasting rings, more or less gliding slowly across the pool floor.

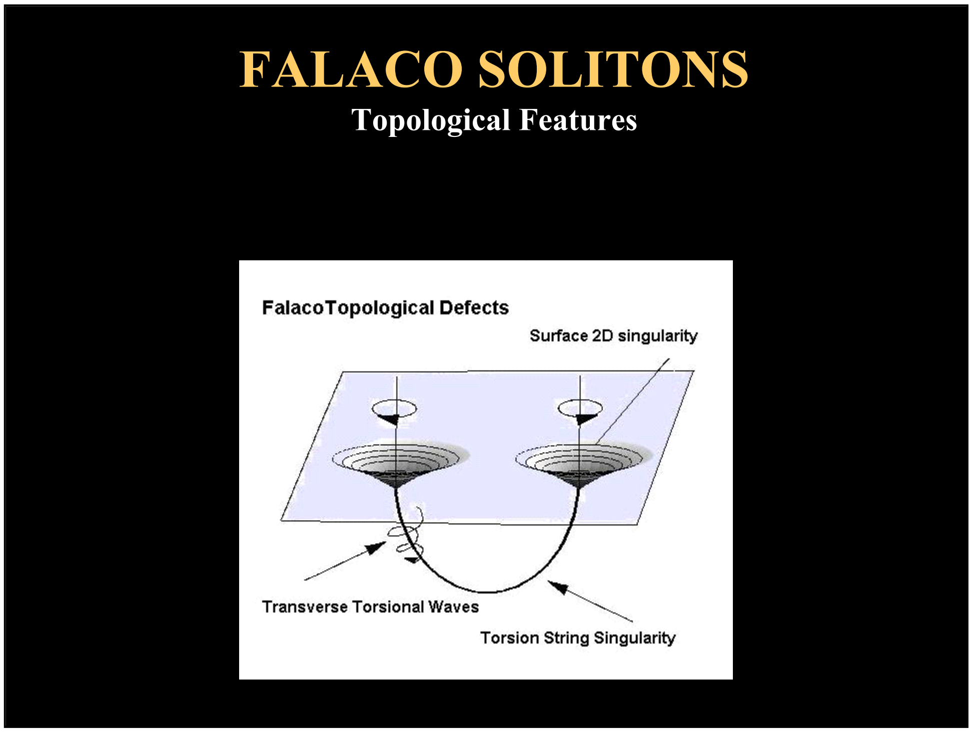
I jumped into the pool, and immediately the black spots disappeared. My first encounter of the third kind and I blew it. I climbed out of the pool again, and then I saw what had happened. My hips (somehow) induced what I now believe to be a Kelvin – Helmholtz instability interior to fluid, which with in 10 to 15 seconds decayed to produce a pair of circular, rotational, inverted dimples in the pool surface.



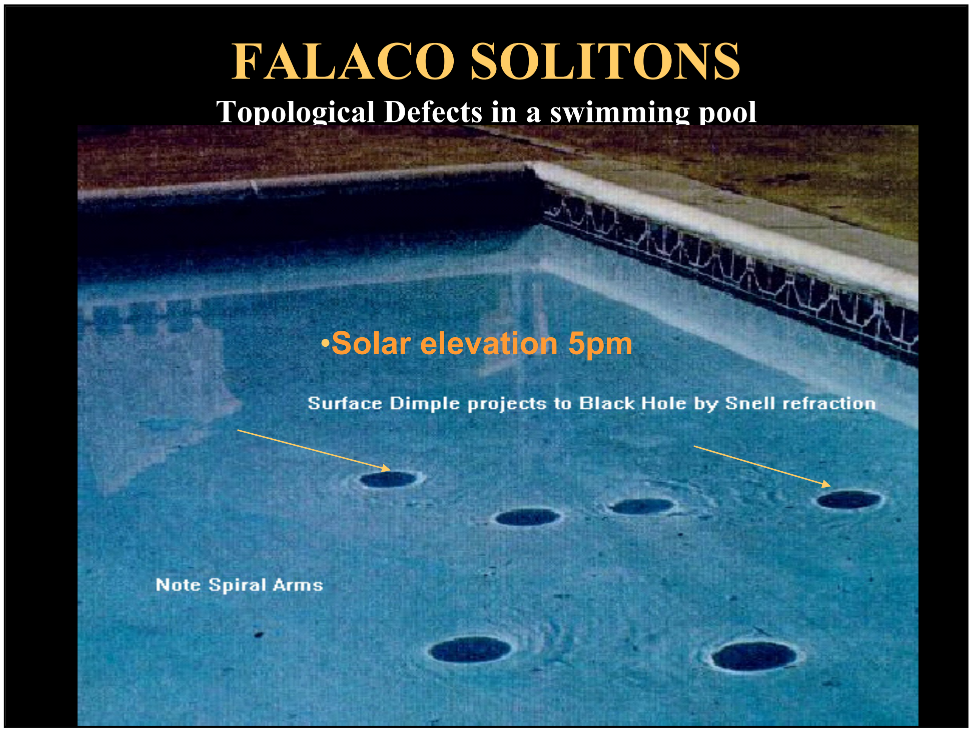
The rotational dimples seemed to be compact and were about 15 cm in diameter, and formed a depression perhaps a millimeter or two deep, as if one had poked a sharp pencil into a rubber sheet.



On the spot I figured out that Snell Refraction from a surface of negative Gauss curvature would give the optical effect, and explain both the "black holes" and the optical halo. The black disks were circular, even though the angle of incidence was not 90 degrees. This observation implied that the refracting surface had to be a Minimal Surface of negative Gauss curvature. What was not explained at the time was the long life (many minutes in a still pool) of these coherent topological structures.



Later experiments with dye drops injected near a dimple vertex made it apparent that there was also a 1D topological defect, in the form of a circular thread, or arc, that connected the vertices of the 2D topological defects. The dye drops would execute torsional wave motion around the guiding center of the thread which apparently connected the vertices of the dimpled minimal surfaces of rotation. The 1D thread appears to behave as an elastic string with a tension that globally stabilizes or confines the two unstable 2D surfaces. If the thread was "cut", the 2D endcaps would not diffuse away, but would disappear almost immediately.



In the Photo displayed, the black refracted circular discs are clearly visible even though the solar elevation was at the time about 30 degrees above the horizon. Note the contrast distortions that identify the pair of surface topological defects associated with each Black Disc.

I now have come to the conclusion that these Falaco Solitons were long-lived, topological coherent structures, and consist of a pair of 2D topological defect surfaces connected by a 1D topological defect string. These non-equilibrium structures give credence to the dynamical concept that I call Topological Torsion.

Topological Torsion

What is it?

A topological idea applied to the transition to turbulence in 1977 (NASA-AMES)

Presented at the 1989 Cambridge Conference on Topological Fluid Mechanics

“but effectively ignored”

Examples presented at the Permb conference 1990

Two Useful Thermodynamic Methods

The concept of Topological Torsion was introduced in a NASA AMES report in 1977, relating a topological description of the differences between streamline flow and turbulence. Later (1989) I introduced the concept at the Cambridge conference on Topological Fluid Dynamics (attended by a number of people at this meeting), where again the concept of Topological Torsion was presented, but more or less ignored. In 1990 at the Permb conference, I extended the ideas on topological coherent structures and Topological Torsion with a number of examples exhibiting phase transitions.

It became apparent that

New Theoretical Foundations

were needed to describe

Non Equilibrium systems

and

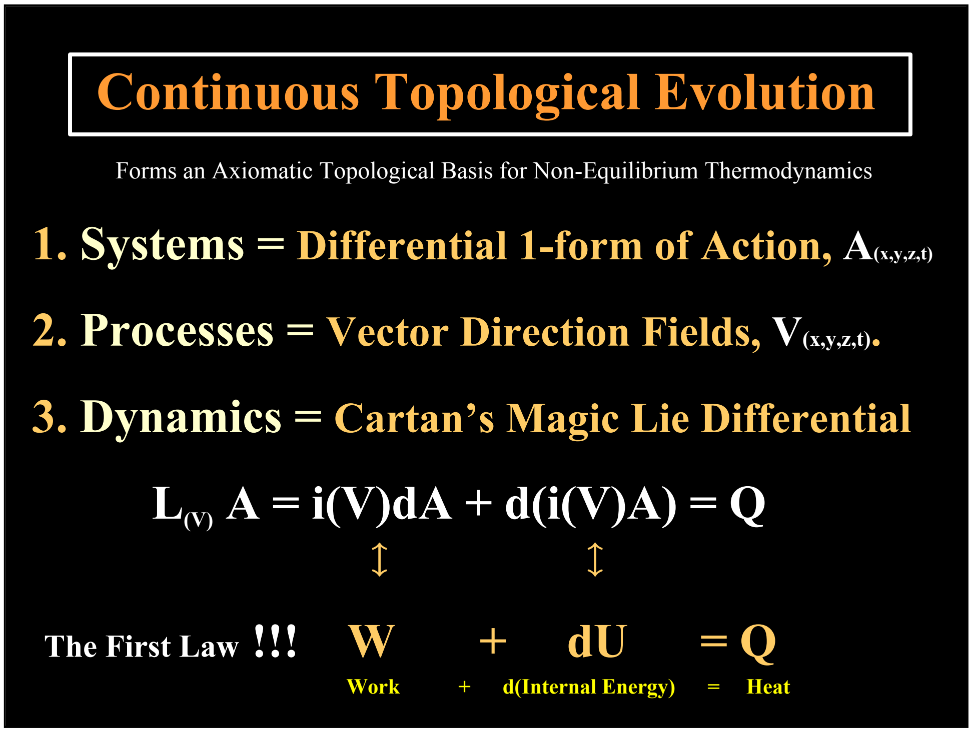
Continuous Irreversible Processes,

which require

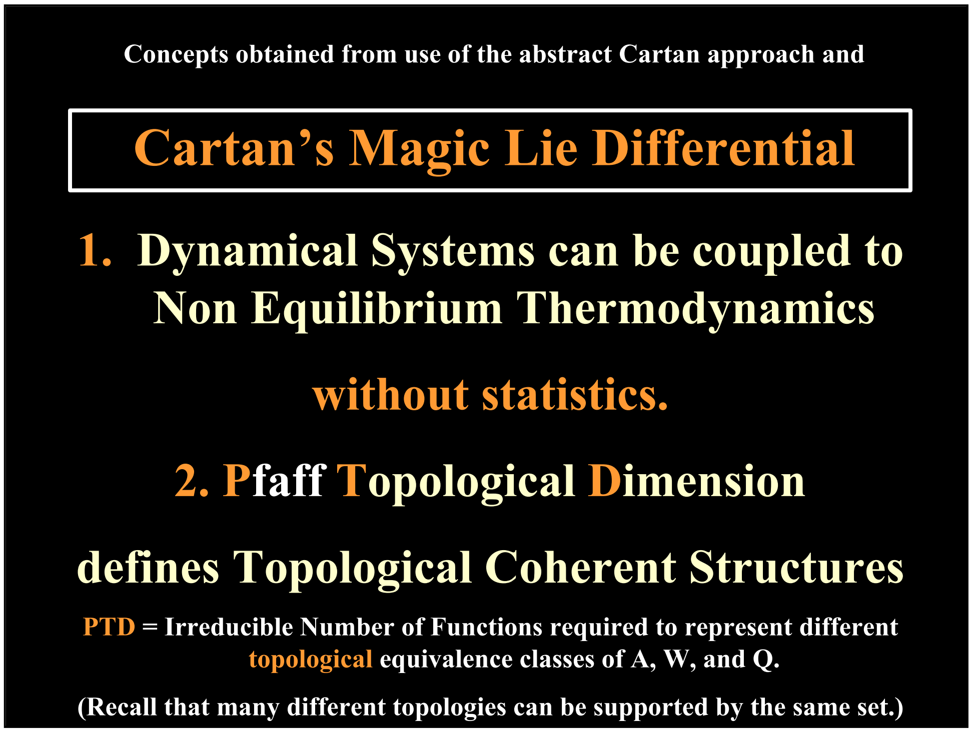
Topological (not geometrical) Evolution.

The method selected was to use Cartan's Methods of Exterior Differential Topology to encode Continuous Topological Evolution. The basic idea is that an exterior differential 1-form induces a topology on a variety of independent variables.

Over the years I have challenged a number of hydrodynamicists and string theory physicists to help me solve this real world effect, but without response. It became apparent that New Theoretical Foundations were needed. I knew early on (1977) that irreversible processes must involve topological change, and that classical tensor analysis, constrained by diffeomorphisms which by definition preserve topology, was inadequate. Also it had been determined that any exterior differential 1-form induced a topology on a variety of independent variables (Santa Barbara 1990).



In the era 1992 to 1998 it became evident that non-equilibrium thermodynamics and irreversible processes could be understood in terms of Continuous Topological Evolution, and that Cartan's theory of exterior differential forms was the mathematics of choice. The basic axioms were: Physical Systems can be encoded by 1-form of Action. Processes are encoded by vector fields. Dynamics are encoded by Cartan's Magic formula for the Lie differential. Cartan's Magic formula was, abstractly, the cohomological expression for the First Law of Thermodynamics. The topological dimension of isolated-equilibrium systems was 2 or less (Caratheodory), while the topological dimension of non-equilibrium systems is 3 or more.



With these topological axioms, a marriage can be made between Non Equilibrium Thermodynamics, Irreversible Processes, and Dynamical Systems. The marriage does not require Statistics, or geometric constraints of metric or connection. Coherent Topological structures could be defined in terms of the Pfaff Topological dimensions of the 1-forms of Action, Work and Heat. (Recall that the same domain of sets can support several different topologies.) The Pfaff topological dimension is equal to the minimum number of functions required to encode the topological properties of the system. It is easily evaluated in terms of the Pfaff sequence, { A, dA, A^dA, dA^dA..}

3. Topological Torsion vector, T4(x,y,z,t)

An exterior differential 3-form in 4D Space-Time,
uniquely determined by the components of the system Action 1-form
i(T4)dx^dy^dz^dt = A^dA,
and NOT equivalent to the unique Extremal field of Hamiltonian Dynamics,
and the Calculus of Variations.

The 4 coefficients of the 3-form A^dA are equivalent to
a 4 component vector "Vortex field", T4,
dominated by, but not identical to, vorticity,
and with a 4th component equal to helicity
(in hydrodynamics).

T4, must be non-zero in all Non-Equilibrium systems: hence PTD(A) = 3 or 4.
All 1-forms with PTD(A) > 2 induce a Disconnected Topology, with non-unique solutions.

Must have non-zero 4 divergence (= dissipation coefficient)
if the Process in the direction of T4
is thermodynamically irreversible: hence PTD(A) = 4.

The 4D vector direction field, T4, is uniquely defined by the coefficient functions of the 3-form of Topological Torsion, which in term is generated by the 1-form A that encodes the 4D physical system. From a hydrodynamic point of view, T4 is dominated by (but not exactly equal to) vorticity, and the 4th component is the Helicity, <V ||curl V>. The universal topological dissipation coefficient is equal to the 4divergence of T4 in space time.

4. Continuous Irreversible Evolution

occurs in the direction of T4

when the dissipation coefficient, 4div(T4) <> 0.

5. Dissipative Processes with 4div(T4) <> 0

can cause the 4D symplectic manifold with PTD(A) = 4
to evolve to a 3D contact manifold of PTD(A) = 3, with 4div(T4) = 0,
creating a long-lived, topologically coherent, state far from equilibrium.

Further evolution in the direction of T4, now with 4div(T4) = 0,

is reversible, non dissipative and Hamiltonian extremal.

If the topological dissipation coefficient is not zero, then the evolution on the 4D **symplectic** manifold in the direction of T4 is thermodynamically Irreversible (PTD(Q) > 2). However, continuous topological evolution can evolve to domains where the dissipation coefficient vanishes. Such domains are 3D **contact** manifolds (or topological defects in the 4D symplectic manifold), and continued evolution in the direction of T4 on the contact manifold is Hamiltonian, reversible and not dissipative. The 3D contact manifold may have many disconnected compact components, which form long-lived topological coherent structures far from equilibrium, created by irreversible processes on a symplectic manifold. For purposes of this meeting these compact structures are the “vortices” in the background “turbulent” field.

Objectives

Use Cartan's universal methods of

Differential Topology and Thermodynamics

to find solutions describing the features of

FALACO SOLITONS

as long lived coherent states far from equilibrium.

Coda: Demonstrate that the non-equilibrium thermodynamic solutions are among those that satisfy the

Navier-Stokes Equations.

The objective is to use these methods of differential topology which encode non-equilibrium thermodynamic systems to find evolutionary solutions that replicate the observable features of the Falaco Solitons. Examples will demonstrate that some of these solutions are also solutions to the Navier-Stokes equations for swirling flow. The fundamental creation process is essentially a projection from a 4D **symplectic** manifold to a 3D **contact** manifold. Therefore, I now want to present three Mathematical Projections from 4D to 3D (which are not diffeomorphic) that have features useful to the analysis of the thermodynamics of non-equilibrium systems and Falaco Solitons.

Three Topological Projections #1

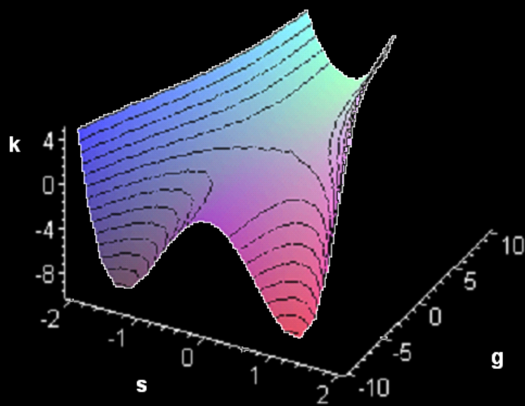
The Hopf map and Topological Torsion

Pfaff Topological dimension 4

http://hopf.math.purdue.edu/pub/hopf.html created by Ken Shoemake

The first example is the Hopf map which is a topological projection from a 4D space (XYZT) to a 3D space (xyz). The Hopf map admits an adjoint 1-form A, of Pfaff topological dimension 4, which therefore can be associated with irreversible processes defined by a Topological Torsion vector.

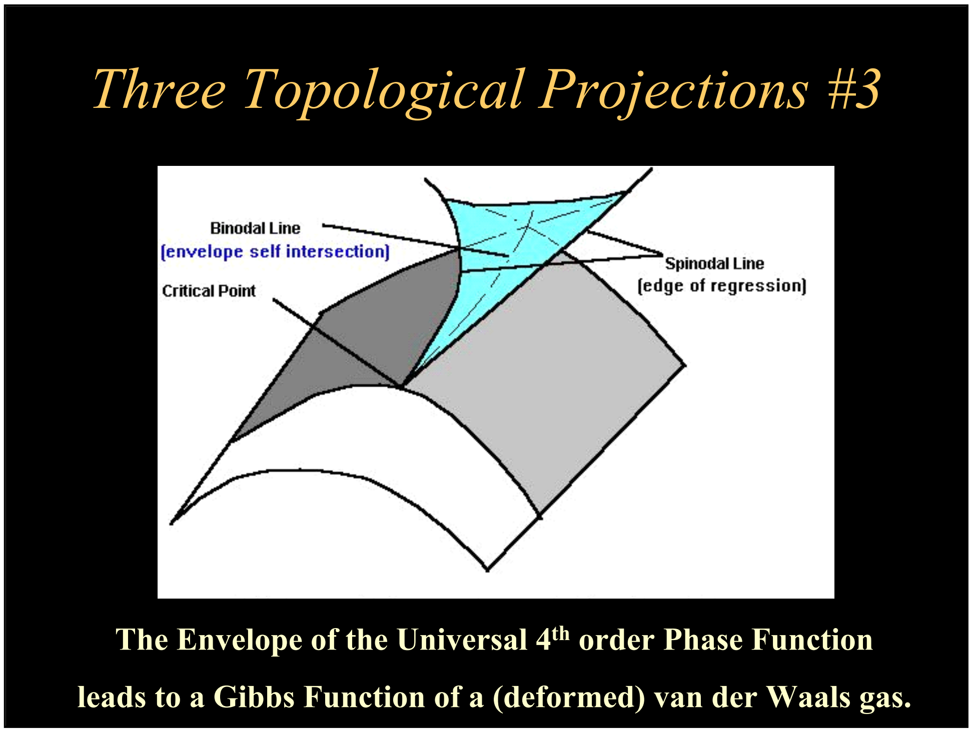
Three Topological Projections #2



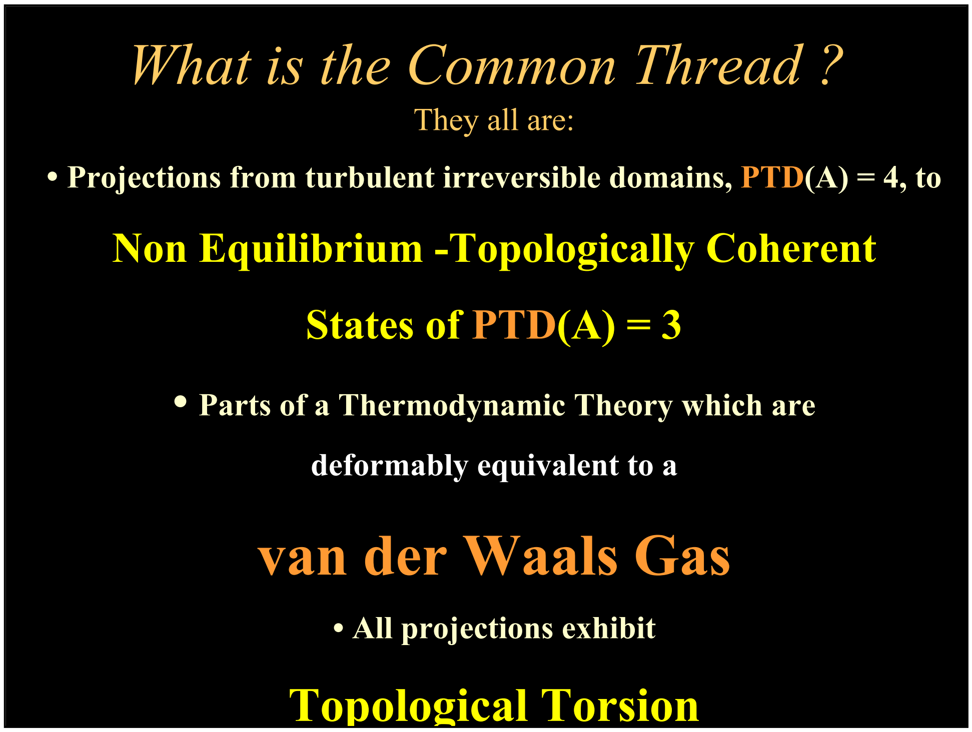
Universal 4^{th} order Thermodynamic Phase Function

is a van der Waals gas with a Higgs potential

The second projection 4D to 3D is deduced from the Jacobian matrix of an arbitrary 1-form of Pfaff Topological Dimension 4. The Characteristic Polynomial of the Cayley – Hamilton theorem implies that there exists a family of 4th order implicit hypersurfaces, defined herein as the universal Thermodynamic Phase function. The Phase function may be written in terms of similarity invariants (as coordinates in a 4D space) constructed from the eigenfunctions of Jacobian matrix at a point. The fourth order similarity invariant admits a solution hypersurface which has the features of a quartic Higgs function below a critical point, and a quadratic function above the critical point. Evolutionary trajectories on the phase hypersurface admit zero sets and bifurcations that universally define the Binodal and Spinodal lines of a (deformed) van der Waals gas. The similarity invariants capture more that the strain coefficients as the can be composed of complex eigenvalues, while the symmetric strain coefficients imply that the Jacobian eigen values are real.



The family of Universal Phase function hypersurfaces also admits an envelope which is the Swallow-Tail image of a universal Gibbs function for a van der Waals gas. It can be shown that Topological Torsion can not be zero for envelope systems. The basic result is that the idea of a deformable van der Waals gas is a universal topological result of a continuous system on 4D space time.



The common thread between all of these mathematic projections from 4D to 3D is that they all generate states of Pfaff Topological Dimension 3 from states of Pfaff topological dimension 4. All have features of a (deformed) van der Waals gas. All admit topological structures with Topological Torsion.

Two practical Topological Thermodynamic techniques are based on

The Pfaff Topological Dimension

and its equivalence classes

The Thermodynamic Phase Function

And the Similarity Invariants of the Jacobian Matrix, [J(A)]

The two practical, topological, techniques for analyzing problems are based on the Pfaff topological dimension of the thermodynamic 1-forms A, W, and Q, and the similarity invariants of the Jacobian matrix of 1-form of Action that encodes the physical system.

Method 1. **Pfaff Topological Dimension**

Compute Pfaff Sequence from the 1 form A(x,y,z,t) that encodes the system:

{A, dA, A^dA, dA^dA}

Find topological defects, defined as subspaces of

dA^dA = 0, or 4DivT_4 = 0

in domains

dA^dA < > 0, or 4DivT_4 < > 0

The 4D Dissipation coefficient = 4Div T_4

The Pfaff topological dimension is easily evaluated in terms of the number of non-zero elements in the Pfaff sequence (Cartan) for a specific 1-form.

Method 2. Characteristic Polynomial Method

For a 1-form of Action A(x,y,z,t) with PTD(A) = 4

(a turbulent non equilibrium domain)

Compute Jacobian matrix of A and its

Cayley-Hamilton Characteristic Polynomial, defined as the

Universal Phase Function,

$$\Theta = \rho^4 - X_M \rho^3 + Y_G \rho^2 - Z_A \rho + T_K = 0$$

Compute Similarity-Curvature invariants.

$$X_M \quad Y_G \quad Z_A \quad T_K$$

Compare with Experiment and the van der Waals gas

The similarity invariants are constructed from the Cayley Hamilton theorem to form a Universal Phase Function.

Example Solutions demonstrate

1. Falaco type solutions satisfy the Navier-Stokes equations for a swirling fluid.

2. Solutions are related to Langford's concept of a tertiary Saddle – Node Hopf bifurcation.

3. Visual Minimal Surface pairs and "string" effects are replicated.

4. Topological Torsion is chiral sensitive, Similarity-Curvature invariants are not.

Example solutions can be extracted from Langford's tertiary bifurcation equations, such as those representing a saddle-node Hopf bifurcation. Solutions near the critical point mimic the Falaco Solitons, with minimal surface 2D singularities and 1D connecting threads. It is important to remark that during the course of analysis it was determined that the Pfaffian methods were chirally sensitive, while the similarity invariants (hence strain concepts) were not.

Details (An exact Solution to the Navier Stokes equations)

Modified Langford SNHopf dynamical system

$$dx/dt = V_x = x\,(G+Cz) \pm \Omega\, y$$

$$dy/dt = V_y = y\,(G+Cz) \pm \Omega\, x$$

$$dz/dt = V_z = A + P\sinh(\alpha z) + D(x^2+y^2)$$

Define Action 1-form (Projective dual)

$$A = V_x\,dx + V_y\,dy + V_z\,dz - (V \bullet V)dt$$

To demonstrate the details, a modification of the Saddle Node Hopf tertiary bifurcation mode is used to construct the dissipation coefficient and the similarity coefficients. This dynamical system can be shown to satisfy the Navier – Stokes equations in a rotating frame

Local Stability Theorem

for Curvature Similarity Invariants

If all eigenvalues of Jacobian have no positive real parts. Then

Linear	$X_M \leq 0$	**= 0 for Critical Point**
Quadratic	$Y_G \geq 0,$	**= 0 for Critical Point**
Cubic	$Z_A \leq 0,$	**= 0 for Critical Point**
Quartic	$T_K \geq 0.$	**= 0 for Critical Point**

Local stability theory indicates that eigenvalues of the Jacobian matrix can have no real parts. All similarity invariants are real, even though the eigenvalues can be complex. In all cases, a local stability constraint implies that the odd similarity invariants are zero or negative, while the even similarity invariants are zero or positive.

Local Stability - Hopf map

A Projection 4D to 3D

All eigenvalues of Jacobian matrix are pure imaginary.

$X_M = 0$

$Y_G \geq 0,$ = 0 for Critical Bifurcation Point

$Z_A = 0,$

$T_K \geq 0.$ = 0 for Critical Bifurcation Point

For the Hopf map, the eigenvalues are pure imaginary plus or minus pairs. The odd similarity invariants are zero, and the even similarity invariants are positive. At the critical point of the Phase function, all similarity invariants vanish. The Hopf map is locally stable.

Global Stability - Falaco Solitons

One zero eigen value, one negative real = b, two complex conjugates with (possible) positive real part, $\sigma \pm i\,\Omega$.

$$X_M = b + 2\sigma \leq 0$$

$$Y_G = (\sigma^2 + \Omega^2 + 2b\sigma) < 0 \quad \text{is possible} \quad ,$$

$$Z_A = b\,(\sigma^2 + \Omega^2) \leq 0,$$

$$T_K = 0. \qquad = 0 \text{ for Critical Bifurcation Point}$$

For Falaco Solitons, one eigenvalue is zero as Tk = 0, one eigenvalue is assumed to be negative, b<0, to make Za=0, and the complex pair of eigen values is presumed to have a real component greater than zero. Hence the Falaco Soliton is not locally stable.

Global Stability - Falaco Solitons

Choose $b < 0$, solve for possible expansion,

positive magnitude of $\sigma = |b|/2$.

Substitute in Y_G

Obtain Falaco stability condition on rotation rate

$$\Omega^2 - \tfrac{3}{4} b^2 = 0$$

However, global stability can be achieved by assuming values such that Xm = 0 and yet Yg < 0. The constraint gives an equation relating the rotation swirl rate of the fluid and the stabilizing string tension coefficient, b.

Similarity-Curvature Invariants

Linear $X_M = 2(G+Cz)+\alpha P\cosh(\alpha z)$

Quadratic $Y_G = +\,\Omega^2 - 2CD(x^2+y^2)+(G+Cz)^2 + 2(G+Cz)P\alpha\cosh(\alpha z)$

Cubic $Z_A = (+\Omega^2 + (G+Cz)^2)P\alpha\cosh(\alpha z) -2CD(G+Cz)(x^2+y^2)$

Quartic $T_K = 0$

The explicit values for the similarity coefficients can be determined

When the 1D string tension balances the 2D rotation expansion, then

Global Stability - Falaco Solitons

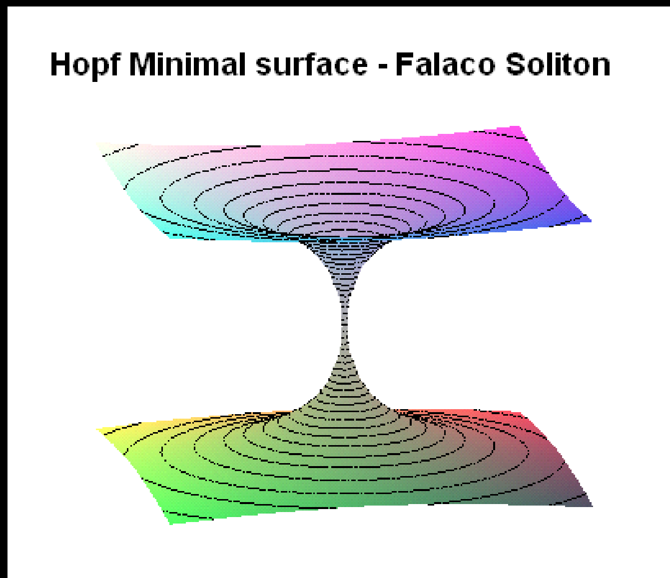
Critical point at $Y_G = 0$

implies

$$(x^2+y^2) = +\alpha^2 P^2 (\cosh(\alpha z))^2/(3|DC|)$$

The critical point satisfies the formula given,

$$(x^2+y^2) = +\alpha^2\ P^2\ (\cosh(\alpha z))^2/(3|DC|)$$



And the globally stabilized hypersurface is a minimal surface which approximates the minimal surface and connecting thread features of the experimental Falaco Solitons

Important Lesson from Falaco Solitons

Topologically Coherent, Non Equilibrium

4D "Vortex" Structures

can be related to

Conjugate "Chiral" Minimal Surfaces

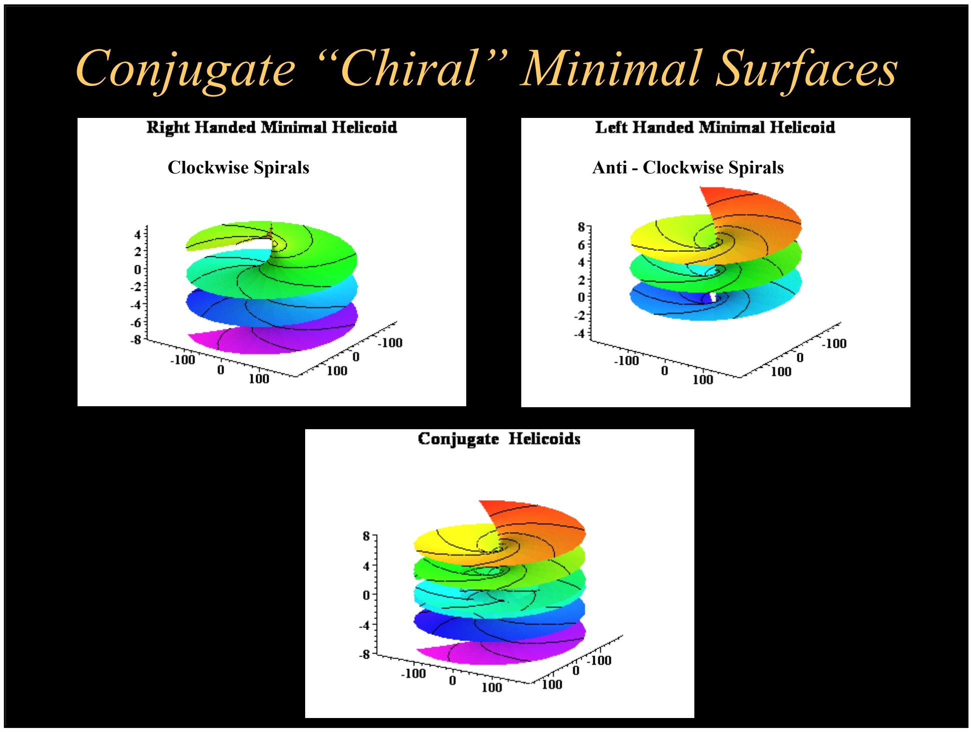
Superposition of R and L handed

with To-Fro Expansion Contraction

Think directional and rotational symmetry breaking

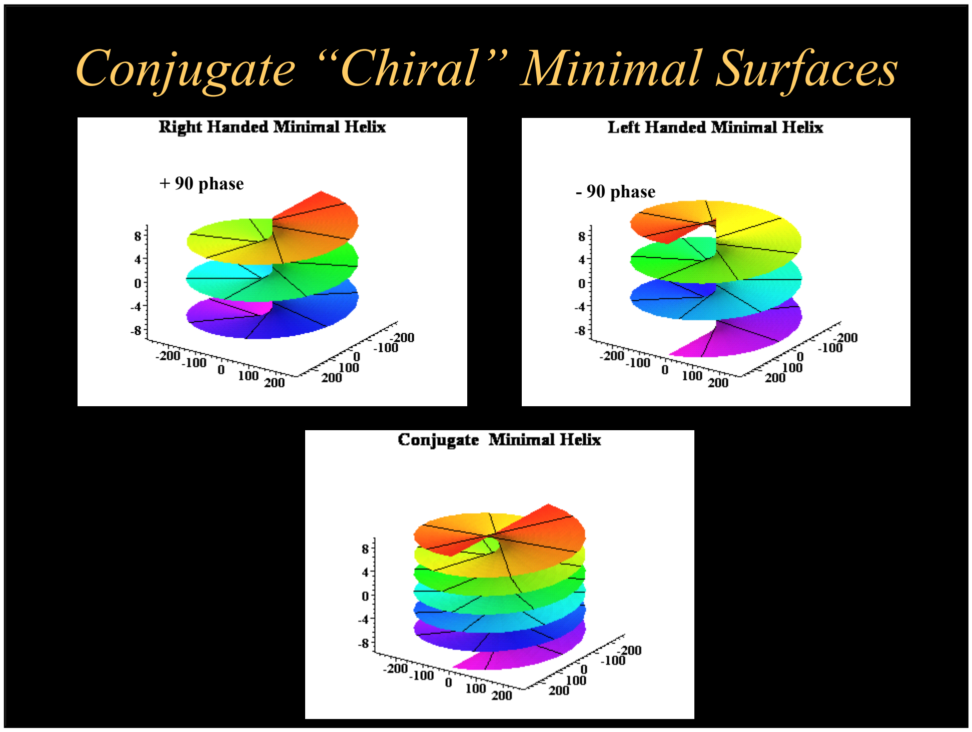
See Phys. Rev A 43, (1991) p. 5165 for applications to EM signals

Important lessons can be learned from these solutions, that indicate that the minimal surfaces in 4D are double valued and chirally sensitive. It is well known (Sophus Lie) that all holomorphic functions in 4D generate complex conjugate pairs of minimal surfaces. The Universal Phase function is an example. Such things are related to topological torsion which has a fixed point of rotation or expansion contraction. (Note that Affine Torsion is not the same as there is no fixed point.)

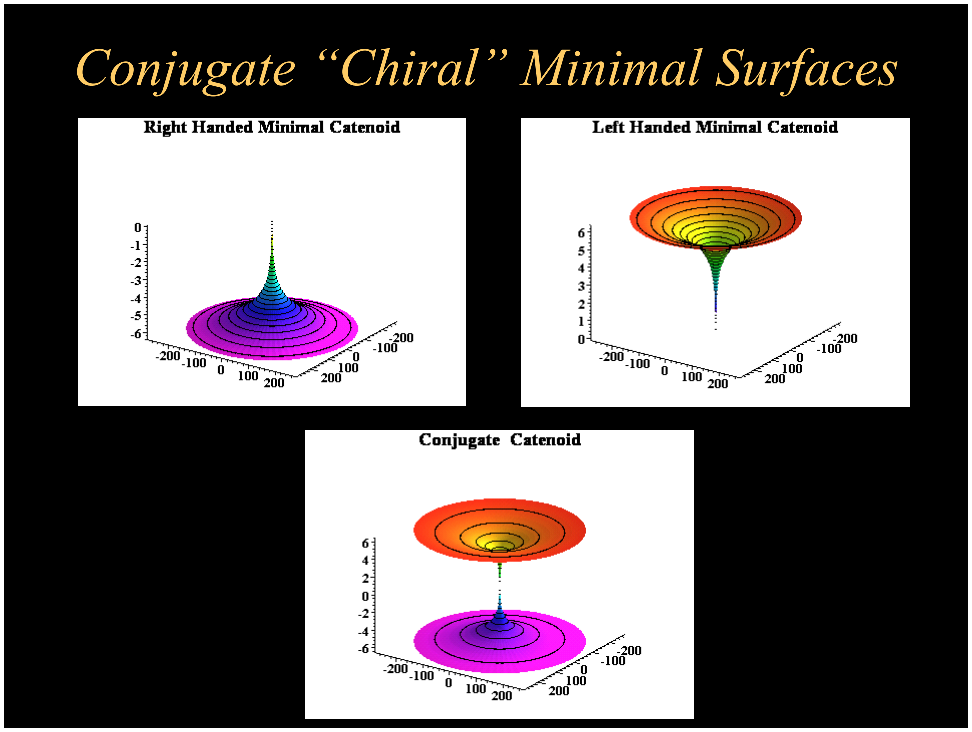


I feel that it is important to present some visual images of these (little appreciated) chiral sensitivities associated with Minimal surfaces. I became interested in this phenomena in 1990 relative to propagating electromagnetic discontinuities.

Conjugate Helicoids



Conjugate Helices



Conjugate Cateniods

One reason that stimulated me to attend this meeting was the chance to discuss with OKULOV how his idea of

Bifurcation of Helical Symmetries

With

Distinct Chiral Dynamics

could be related, perhaps, to Thermodynamic Phase bifurcations to

FALACO SOLITONS.

I saw that V. Okulov, an expert in matters of chiral hydrodynamic bifurcations, is present at this conference and I looked forward to talking to him again. The last time we met was at the IUTAM conference in Denmark, 1997, where I announced that Turbulence in solutions to the Navier-Stokes equations must be associated with topological defect threads of ***helical*** vorticity. The next day Pavel Kuibin and Valery Okulov presented experimental evidence of such structures in a turbulent open flow.

Now some Interesting Conjectures

Application to BOSE CONDENSATES

For All 4th Order Universal Phase Functions,

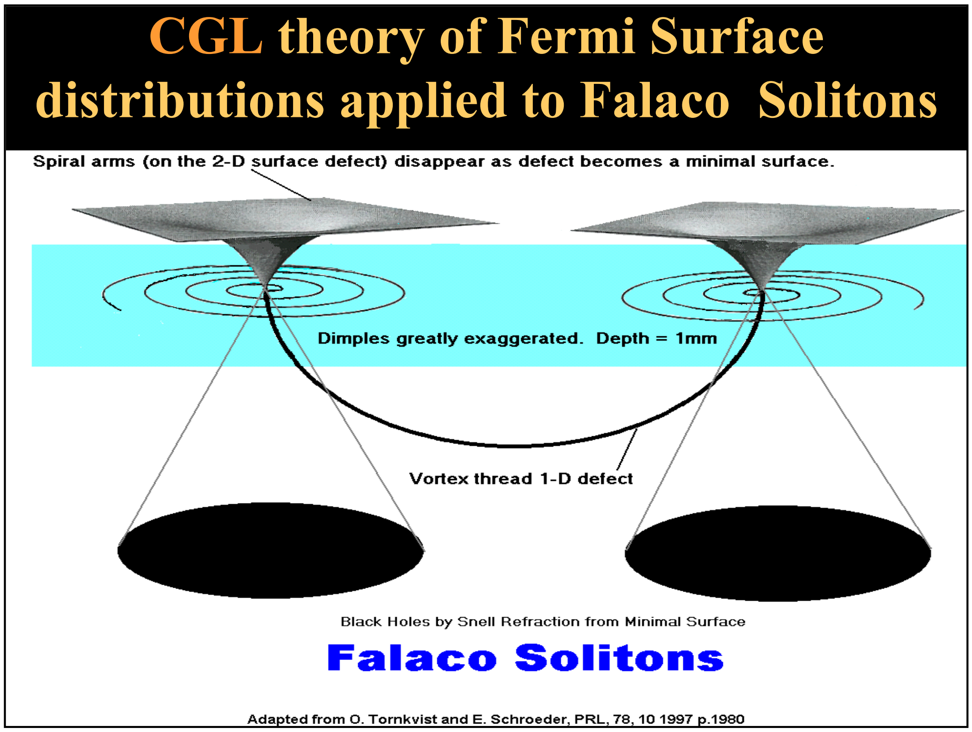
$$\Theta = \Psi^{4} - X_{M}\Psi^{3} + Y_{G}\Psi^{2} - Z_{A}\Psi + T_{K} = 0$$

the determinant of the Jacobian matrix is given by the similarity coefficient T_K. Every determinant can be related to the divergence of a current. Hence

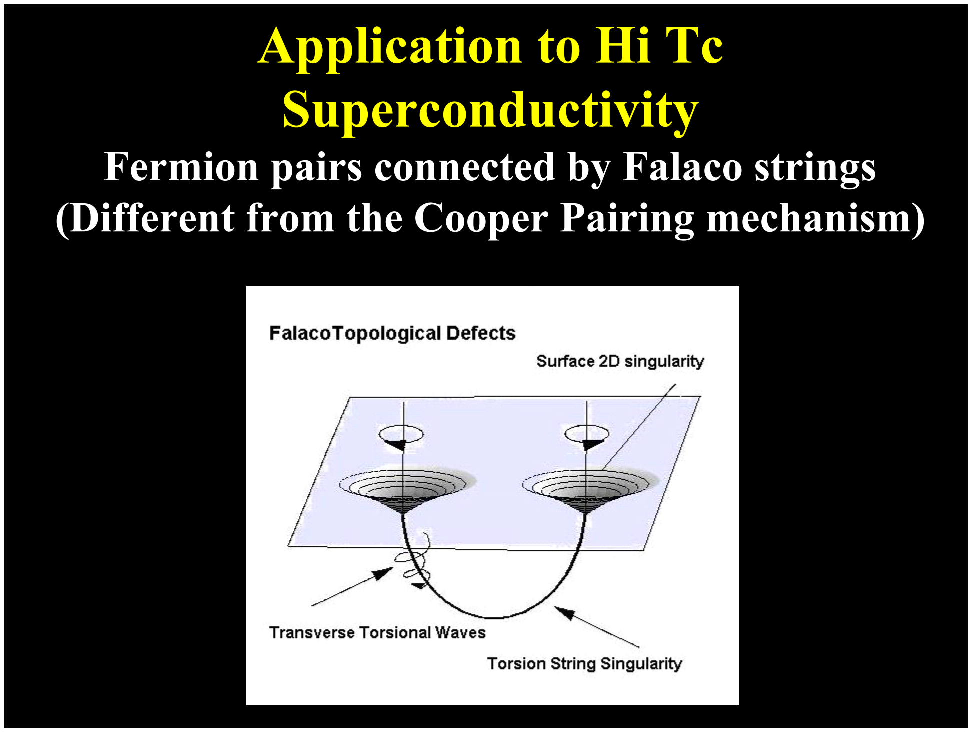
$$\mathrm{div}J + \partial\rho/\partial t = \Psi(\Psi^{3} - X_{M}\Psi^{2} + Y_{G}\Psi - Z_{A})$$

A Complex Ginsburg Landau format

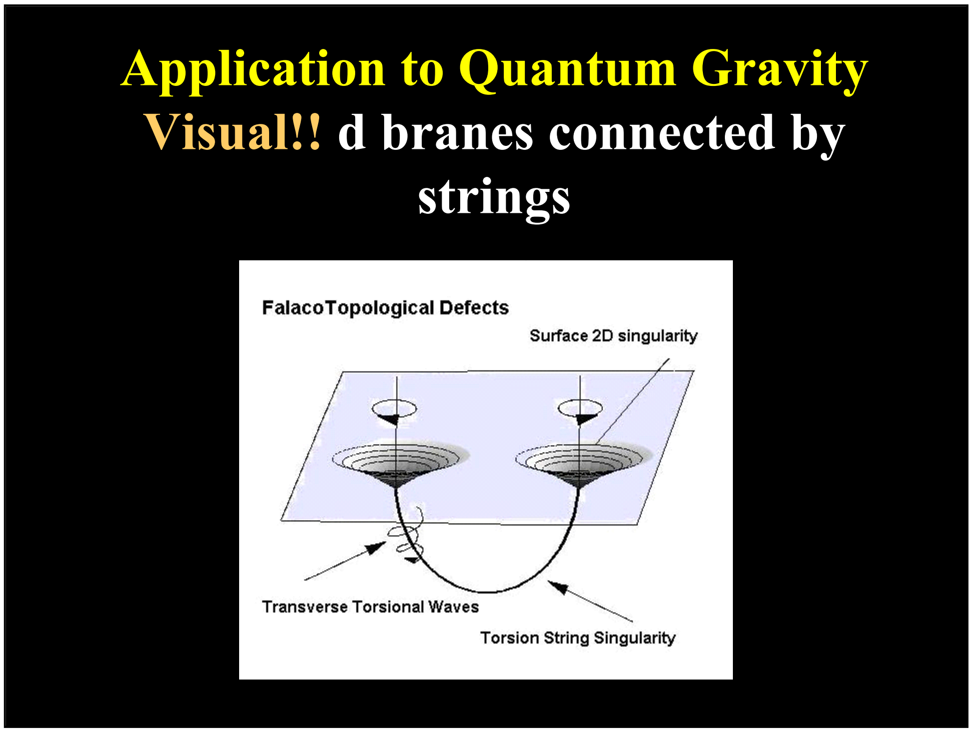
Applications The universal Phase function permits a derivation of the Ginsburg Landau formalism



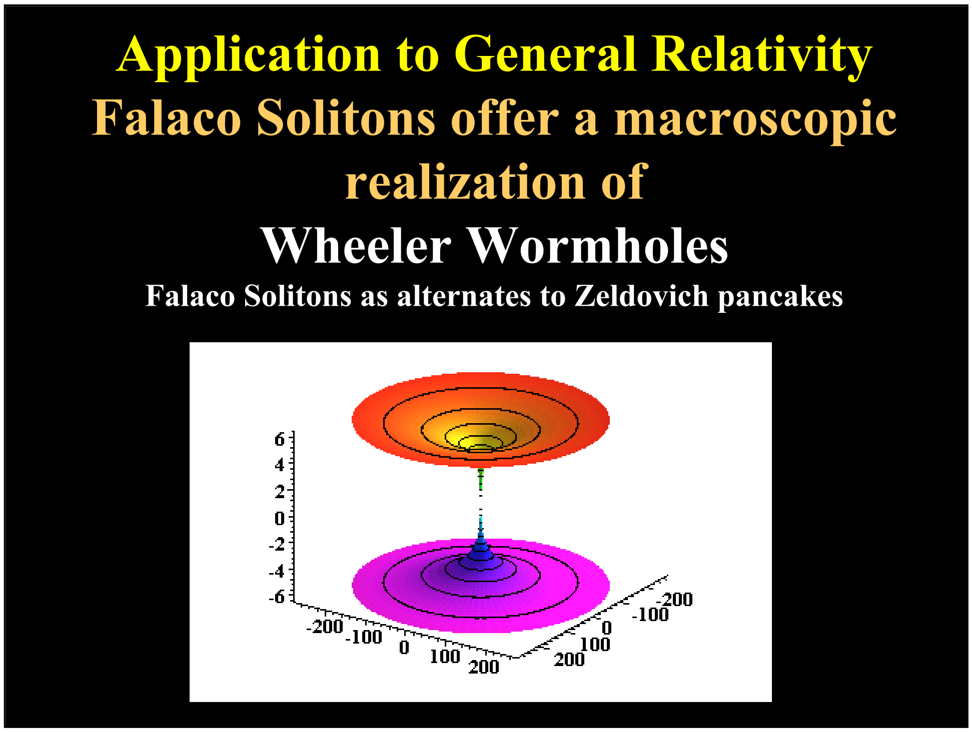
Compare to CGL theory presented by Tornquist and Schroeder. The dimples are from their publication. The Snell projections and the connecting thread are my additions to their diagram. The conclusion is that Falaco Solitons are evident structures in mesoscopic quantum systems, such as superfluids and condensates.



Applications The Falaco pair gives another method for combining Fermion pairs into Bosons.



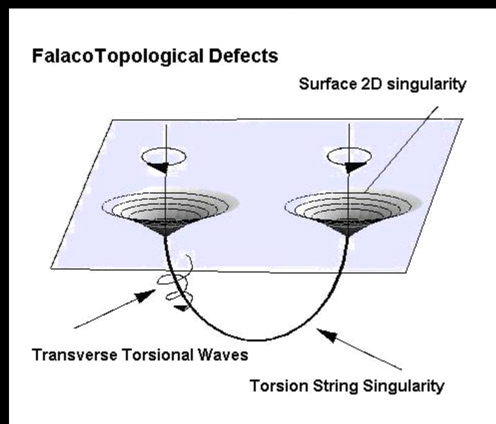
Applications The Falaco Solitons are macroscopic realizations of d branes with connecting threads.



Applications The Falaco Solitons give macroscopic realizations of Wheeler wormholes.

Application to Elementary Particles

Falaco Solitons as Macroscopic Quarks on the ends of a confinement string.



Applications The Falaco Solitons give a macroscopic realization of quarks on the end of a string. If an attempt is made to isolate one quark, they both disappear. When the Falaco thread is severed, the endcaps disappear rapidly and non-diffusively. The Falaco thread is a confinement string.

Application to Astrophysics

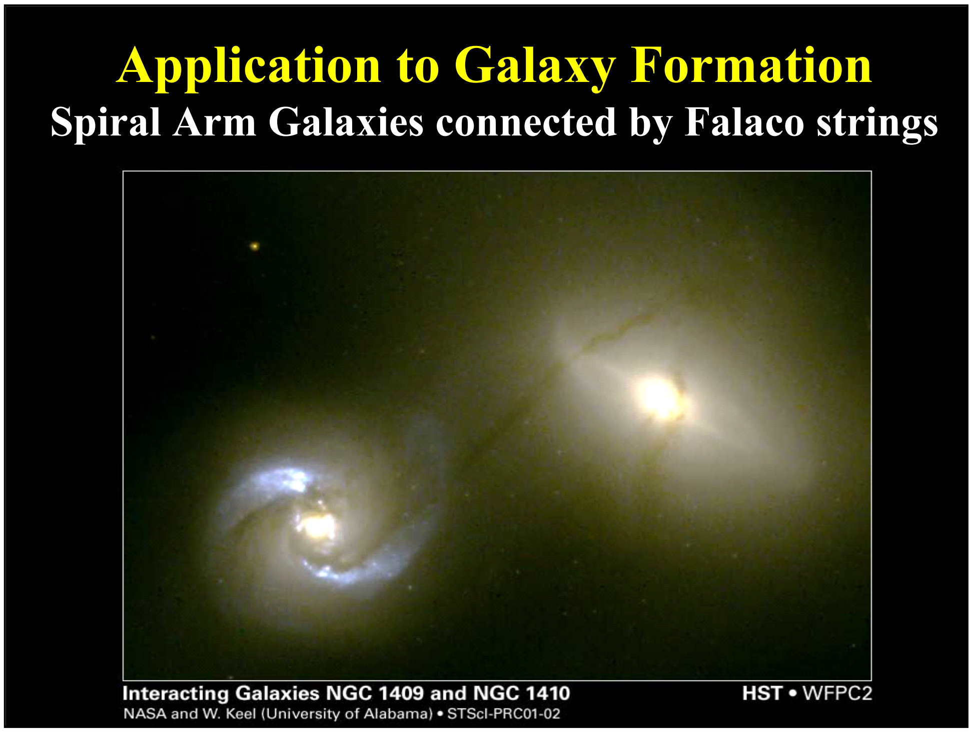
The Universe as a turbulent, dilute, universal (topological) van der Waals gas near its critical point.

Stars and galaxies as long lived topologically coherent structures, far from equilibrium, caused by density fluctuations near critical point.

Correlations of density fluctuations cause $1/r^2$ attraction of defects (Lev Landau).

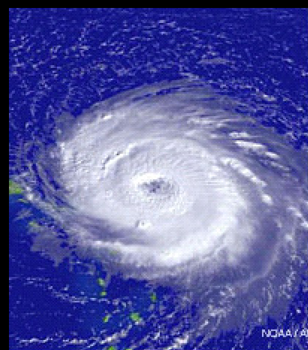
Thermodynamic non-metrical explanations for dark matter, dark energy and negative pressure.

Applications If it assumed from the properties of the universal Phase function, that the universe may be considered as a van der Waals gas near its critical point, then a number of recent astrophysics conjectures have more palatable explanations.



Applications The Formation of Spiral Arms during the formative stages of the Falaco Soliton evolving irreversibly in a turbulent media to a long lived state far from equilibrium leads to the suggestion that spiral arm galaxies may be connect by cosmic strings. The recent Hubble above photograph is very suggestive.

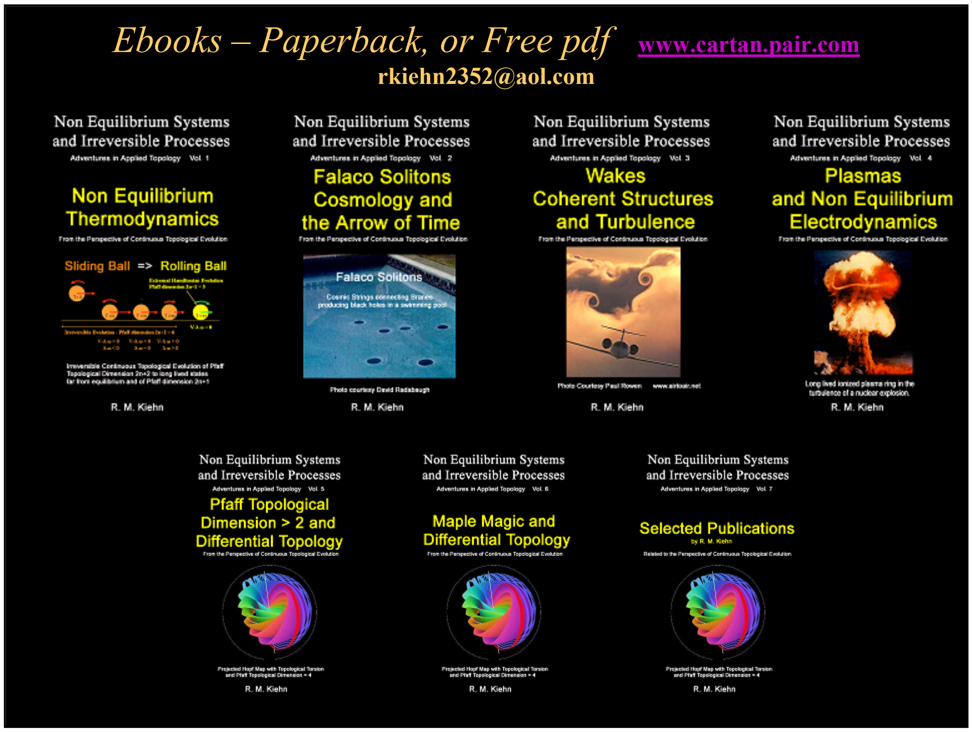
Hurricane Frances



Falaco Soliton in early stages of Formation?

Cubic compatification?

Applications The recent work of Crowdy (reported at this conference) suggests that compact realizations of topological Falaco Solitons between layered discontinuities (the ocean and the Tropopause) may lead to a better understanding of hurricanes. After hearing Crowdy's lecture on circulation and vorticity in the complex plane, I was able that night to cast his ideas in the topological language of differential forms, such that the compactification ideas could an be generalized in terms of exterior differential forms to 3 and 4 dimensions.



If you want unbound copies of the monographs - free in downloadable pdf format - send me an email and I will send you a clickable URL to retrieve a copy (5 to 8 megabytes each. 1 and 7now now complete. 2 id 90% complete and should be available by October 10, 3 by November 1, 4 by December 1. 5,6 before January 1.) rkiehn2352@aol.com

Topological Torsion gone berserk

Note Spiral Arms and Inverse Fractal Dimples

In France they eat such things !!! -- a cross between broccoli and cauliflower