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[ Magnetic field
> B:=curl(A,[x,y,z]);
B := \left[ \left( \frac{\partial}{\partial y} Az(x, y, z, t) \right) - \left( \frac{\partial}{\partial z} Ay(x, y, z, t) \right), \left( \frac{\partial}{\partial z} Ax(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} Az(x, y, z, t) \right), \left( \frac{\partial}{\partial x} Ay(x, y, z, t) \right) - \left( \frac{\partial}{\partial y} Ax(x, y, z, t) \right) \right]
[ Electric Field
> E:=[-diff(A4,x)-diff(A[1],t),-diff(A4,y)-diff(A[2],t),-diff(A4,z)-diff(A[3],t)];
E := \left[ -\left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ax(x, y, z, t) \right), -\left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ay(x, y, z, t) \right), -\left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Az(x, y, z, t) \right) \right]
[ Parity (Second Poncare invariant)
> EdotB:=inner(E,B);
EdotB := \left( -\left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ax(x, y, z, t) \right) \right) \left( \left( \frac{\partial}{\partial y} Az(x, y, z, t) \right) - \left( \frac{\partial}{\partial z} Ay(x, y, z, t) \right) \right) + \left( -\left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ay(x, y, z, t) \right) \right) \left( \left( \frac{\partial}{\partial z} Ax(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} Az(x, y, z, t) \right) \right) + \left( -\left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Az(x, y, z, t) \right) \right) \left( \left( \frac{\partial}{\partial x} Ay(x, y, z, t) \right) - \left( \frac{\partial}{\partial y} Ax(x, y, z, t) \right) \right)
[ The Torsion current.
> ExA:=crossprod(E,A);Bphi:=[B[1]*phi,B[2]*phi,B[3]*phi];
ExA := \left[ \left( -\left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ay(x, y, z, t) \right) \right) Az(x, y, z, t) - \left( -\left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Az(x, y, z, t) \right) \right) Ay(x, y, z, t), \left( -\left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Az(x, y, z, t) \right) \right) Ax(x, y, z, t) - \left( -\left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ax(x, y, z, t) \right) \right) Az(x, y, z, t), \left( -\left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ax(x, y, z, t) \right) \right) Ay(x, y, z, t) - \left( -\left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ay(x, y, z, t) \right) \right) Ax(x, y, z, t) \right]
Bphi := \left[ \left( \left( \frac{\partial}{\partial y} Az(x, y, z, t) \right) - \left( \frac{\partial}{\partial z} Ay(x, y, z, t) \right) \right) \phi, \left( \left( \frac{\partial}{\partial z} Ax(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} Az(x, y, z, t) \right) \right) \phi, \left( \left( \frac{\partial}{\partial x} Ay(x, y, z, t) \right) - \left( \frac{\partial}{\partial y} Ax(x, y, z, t) \right) \right) \phi \right]
> TORS:=evalm(ExA+A4*B);
TORS := \left[ \left( -\left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ay(x, y, z, t) \right) \right) Az(x, y, z, t) - \left( -\left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Az(x, y, z, t) \right) \right) Ay(x, y, z, t) + \phi(x, y, z, t) \left( \left( \frac{\partial}{\partial y} Az(x, y, z, t) \right) - \left( \frac{\partial}{\partial z} Ay(x, y, z, t) \right) \right), \left( -\left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Az(x, y, z, t) \right) \right) Ax(x, y, z, t) - \left( -\left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ax(x, y, z, t) \right) \right) Az(x, y, z, t) + \phi(x, y, z, t) \left( \left( \frac{\partial}{\partial z} Ax(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} Az(x, y, z, t) \right) \right), \left( -\left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ay(x, y, z, t) \right) \right) Ax(x, y, z, t) - \left( -\left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ay(x, y, z, t) \right) \right) Ax(x, y, z, t) + \phi(x, y, z, t) \left( \left( \frac{\partial}{\partial x} Ay(x, y, z, t) \right) - \left( \frac{\partial}{\partial y} Ax(x, y, z, t) \right) \right) \right]

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$$\left[ -\left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} \text{Ay}(x, y, z, t) \right) \right] \text{Ax}(x, y, z, t) + \phi(x, y, z, t) \left[ \left( \frac{\partial}{\partial x} \text{Ay}(x, y, z, t) \right) - \left( \frac{\partial}{\partial y} \text{Ax}(x, y, z, t) \right) \right]$$

> **AdotB:=inner(A,B);**

$$\begin{aligned} \text{AdotB} := & \text{Ax}(x, y, z, t) \left[ \left( \frac{\partial}{\partial y} \text{Az}(x, y, z, t) \right) - \left( \frac{\partial}{\partial z} \text{Ay}(x, y, z, t) \right) \right] \\ & + \text{Ay}(x, y, z, t) \left[ \left( \frac{\partial}{\partial z} \text{Ax}(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} \text{Az}(x, y, z, t) \right) \right] \\ & + \text{Az}(x, y, z, t) \left[ \left( \frac{\partial}{\partial x} \text{Ay}(x, y, z, t) \right) - \left( \frac{\partial}{\partial y} \text{Ax}(x, y, z, t) \right) \right] \end{aligned}$$

> **TORSION:=[TORS[1],TORS[2],TORS[3],AdotB];**

$$\begin{aligned} \text{TORSION} := & \left[ \left( -\left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} \text{Ay}(x, y, z, t) \right) \right) \text{Az}(x, y, z, t) \right. \\ & - \left. \left( -\left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} \text{Az}(x, y, z, t) \right) \right) \text{Ay}(x, y, z, t) + \phi(x, y, z, t) \left[ \left( \frac{\partial}{\partial y} \text{Az}(x, y, z, t) \right) - \left( \frac{\partial}{\partial z} \text{Ay}(x, y, z, t) \right) \right] \right. \\ & \left. \left( -\left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} \text{Az}(x, y, z, t) \right) \right) \text{Ax}(x, y, z, t) - \left( -\left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} \text{Ax}(x, y, z, t) \right) \right) \text{Az}(x, y, z, t) \right. \\ & + \phi(x, y, z, t) \left[ \left( \frac{\partial}{\partial z} \text{Ax}(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} \text{Az}(x, y, z, t) \right) \right] \left. \left( -\left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} \text{Ax}(x, y, z, t) \right) \right) \text{Ay}(x, y, z, t) \right. \\ & - \left. \left( -\left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} \text{Ay}(x, y, z, t) \right) \right) \text{Ax}(x, y, z, t) + \phi(x, y, z, t) \left[ \left( \frac{\partial}{\partial x} \text{Ay}(x, y, z, t) \right) - \left( \frac{\partial}{\partial y} \text{Ax}(x, y, z, t) \right) \right] \right. \\ & \text{Ax}(x, y, z, t) \left[ \left( \frac{\partial}{\partial y} \text{Az}(x, y, z, t) \right) - \left( \frac{\partial}{\partial z} \text{Ay}(x, y, z, t) \right) \right] + \text{Ay}(x, y, z, t) \left[ \left( \frac{\partial}{\partial z} \text{Ax}(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} \text{Az}(x, y, z, t) \right) \right] \\ & \left. + \text{Az}(x, y, z, t) \left[ \left( \frac{\partial}{\partial x} \text{Ay}(x, y, z, t) \right) - \left( \frac{\partial}{\partial y} \text{Ax}(x, y, z, t) \right) \right] \right] \end{aligned}$$

[ Divergence of the Torsion current.

> **DIVT:=factor(diverge(TORSION,[x,y,z,t]));**

$$\begin{aligned} \text{DIVT} := & 2 \left( \frac{\partial}{\partial z} \text{Ax}(x, y, z, t) \right) \left( \frac{\partial}{\partial t} \text{Ay}(x, y, z, t) \right) - 2 \left( \frac{\partial}{\partial y} \text{Ax}(x, y, z, t) \right) \left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) \\ & - 2 \left( \frac{\partial}{\partial y} \text{Ax}(x, y, z, t) \right) \left( \frac{\partial}{\partial t} \text{Az}(x, y, z, t) \right) + 2 \left( \frac{\partial}{\partial y} \text{Az}(x, y, z, t) \right) \left( \frac{\partial}{\partial t} \text{Ax}(x, y, z, t) \right) \\ & + 2 \left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) \left( \frac{\partial}{\partial z} \text{Ax}(x, y, z, t) \right) - 2 \left( \frac{\partial}{\partial z} \text{Ay}(x, y, z, t) \right) \left( \frac{\partial}{\partial t} \text{Ax}(x, y, z, t) \right) \\ & - 2 \left( \frac{\partial}{\partial x} \text{Az}(x, y, z, t) \right) \left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) - 2 \left( \frac{\partial}{\partial x} \text{Az}(x, y, z, t) \right) \left( \frac{\partial}{\partial t} \text{Ay}(x, y, z, t) \right) \\ & + 2 \left( \frac{\partial}{\partial x} \text{Ay}(x, y, z, t) \right) \left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) + 2 \left( \frac{\partial}{\partial x} \text{Ay}(x, y, z, t) \right) \left( \frac{\partial}{\partial t} \text{Az}(x, y, z, t) \right) \\ & + 2 \left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) \left( \frac{\partial}{\partial y} \text{Az}(x, y, z, t) \right) - 2 \left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) \left( \frac{\partial}{\partial z} \text{Ay}(x, y, z, t) \right) \end{aligned}$$

[ An evolutionary dynamical system.

> **VV:=[U(x,y,z,t),V(x,y,z,t),W(x,y,z,t)];**

$$\text{VV} := [\text{U}(x, y, z, t), \text{V}(x, y, z, t), \text{W}(x, y, z, t)]$$

[ The Lorentz force.

> **FL:=evalm(E+crossprod(VV,B));P:=innerprod(VV,E);**

$$\begin{aligned}
FL := & \left[ -\left(\frac{\partial}{\partial x} \phi(x, y, z, t)\right) - \left(\frac{\partial}{\partial t} Ax(x, y, z, t)\right) + V(x, y, z, t) \left( \left(\frac{\partial}{\partial x} Ay(x, y, z, t)\right) - \left(\frac{\partial}{\partial y} Ax(x, y, z, t)\right) \right) \right. \\
& - W(x, y, z, t) \left( \left(\frac{\partial}{\partial z} Ax(x, y, z, t)\right) - \left(\frac{\partial}{\partial x} Az(x, y, z, t)\right) \right) - \left(\frac{\partial}{\partial y} \phi(x, y, z, t)\right) - \left(\frac{\partial}{\partial t} Ay(x, y, z, t)\right) \\
& + W(x, y, z, t) \left( \left(\frac{\partial}{\partial y} Az(x, y, z, t)\right) - \left(\frac{\partial}{\partial z} Ay(x, y, z, t)\right) \right) - U(x, y, z, t) \left( \left(\frac{\partial}{\partial x} Ay(x, y, z, t)\right) - \left(\frac{\partial}{\partial y} Ax(x, y, z, t)\right) \right) \\
& - \left(\frac{\partial}{\partial z} \phi(x, y, z, t)\right) - \left(\frac{\partial}{\partial t} Az(x, y, z, t)\right) + U(x, y, z, t) \left( \left(\frac{\partial}{\partial z} Ax(x, y, z, t)\right) - \left(\frac{\partial}{\partial x} Az(x, y, z, t)\right) \right) \\
& \left. - V(x, y, z, t) \left( \left(\frac{\partial}{\partial y} Az(x, y, z, t)\right) - \left(\frac{\partial}{\partial z} Ay(x, y, z, t)\right) \right) \right] \\
P := & -U(x, y, z, t) \left(\frac{\partial}{\partial x} \phi(x, y, z, t)\right) - U(x, y, z, t) \left(\frac{\partial}{\partial t} Ax(x, y, z, t)\right) - V(x, y, z, t) \left(\frac{\partial}{\partial y} \phi(x, y, z, t)\right) \\
& - V(x, y, z, t) \left(\frac{\partial}{\partial t} Ay(x, y, z, t)\right) - W(x, y, z, t) \left(\frac{\partial}{\partial z} \phi(x, y, z, t)\right) - W(x, y, z, t) \left(\frac{\partial}{\partial t} Az(x, y, z, t)\right)
\end{aligned}$$

The virtual work 1-form

> **Work := FL[1]\*d(x)+FL[2]\*d(y)+FL[3]\*d(z)-P\*d(t);**

$$\begin{aligned}
Work := & \left( -\left(\frac{\partial}{\partial x} \phi(x, y, z, t)\right) - \left(\frac{\partial}{\partial t} Ax(x, y, z, t)\right) + V(x, y, z, t) \left( \left(\frac{\partial}{\partial x} Ay(x, y, z, t)\right) - \left(\frac{\partial}{\partial y} Ax(x, y, z, t)\right) \right) \right. \\
& - W(x, y, z, t) \left( \left(\frac{\partial}{\partial z} Ax(x, y, z, t)\right) - \left(\frac{\partial}{\partial x} Az(x, y, z, t)\right) \right) \Big) d(x) + \left( -\left(\frac{\partial}{\partial y} \phi(x, y, z, t)\right) - \left(\frac{\partial}{\partial t} Ay(x, y, z, t)\right) \right. \\
& + W(x, y, z, t) \left( \left(\frac{\partial}{\partial y} Az(x, y, z, t)\right) - \left(\frac{\partial}{\partial z} Ay(x, y, z, t)\right) \right) - U(x, y, z, t) \left( \left(\frac{\partial}{\partial x} Ay(x, y, z, t)\right) - \left(\frac{\partial}{\partial y} Ax(x, y, z, t)\right) \right) \Big) \\
& d(y) + \left( -\left(\frac{\partial}{\partial z} \phi(x, y, z, t)\right) - \left(\frac{\partial}{\partial t} Az(x, y, z, t)\right) + U(x, y, z, t) \left( \left(\frac{\partial}{\partial z} Ax(x, y, z, t)\right) - \left(\frac{\partial}{\partial x} Az(x, y, z, t)\right) \right) \right. \\
& \left. - V(x, y, z, t) \left( \left(\frac{\partial}{\partial y} Az(x, y, z, t)\right) - \left(\frac{\partial}{\partial z} Ay(x, y, z, t)\right) \right) \right) d(z) - \left( -U(x, y, z, t) \left(\frac{\partial}{\partial x} \phi(x, y, z, t)\right) \right. \\
& - U(x, y, z, t) \left(\frac{\partial}{\partial t} Ax(x, y, z, t)\right) - V(x, y, z, t) \left(\frac{\partial}{\partial y} \phi(x, y, z, t)\right) - V(x, y, z, t) \left(\frac{\partial}{\partial t} Ay(x, y, z, t)\right) \\
& \left. - W(x, y, z, t) \left(\frac{\partial}{\partial z} \phi(x, y, z, t)\right) - W(x, y, z, t) \left(\frac{\partial}{\partial t} Az(x, y, z, t)\right) \right) d(t)
\end{aligned}$$

> **Closure := wcollect(d(Work));**

$$\begin{aligned}
Closure := & \left( -\left(\frac{\partial}{\partial y} V(x, y, z, t)\right) \left(\frac{\partial}{\partial x} Ay(x, y, z, t)\right) + \left(\frac{\partial}{\partial y} V(x, y, z, t)\right) \left(\frac{\partial}{\partial y} Ax(x, y, z, t)\right) + \left(\frac{\partial^2}{\partial y \partial t} Ax(x, y, z, t)\right) \right. \\
& + \left(\frac{\partial}{\partial y} W(x, y, z, t)\right) \left(\frac{\partial}{\partial z} Ax(x, y, z, t)\right) - \left(\frac{\partial}{\partial y} W(x, y, z, t)\right) \left(\frac{\partial}{\partial x} Az(x, y, z, t)\right) - V(x, y, z, t) \left(\frac{\partial^2}{\partial y \partial x} Ay(x, y, z, t)\right) \\
& + V(x, y, z, t) \left(\frac{\partial^2}{\partial y^2} Ax(x, y, z, t)\right) + W(x, y, z, t) \left(\frac{\partial^2}{\partial z \partial y} Ax(x, y, z, t)\right) + \left(\frac{\partial}{\partial x} W(x, y, z, t)\right) \left(\frac{\partial}{\partial y} Az(x, y, z, t)\right) \\
& - \left(\frac{\partial}{\partial x} W(x, y, z, t)\right) \left(\frac{\partial}{\partial z} Ay(x, y, z, t)\right) - W(x, y, z, t) \left(\frac{\partial^2}{\partial z \partial x} Ay(x, y, z, t)\right) - \left(\frac{\partial^2}{\partial x \partial t} Ay(x, y, z, t)\right) \\
& \left. - \left(\frac{\partial}{\partial x} U(x, y, z, t)\right) \left(\frac{\partial}{\partial x} Ay(x, y, z, t)\right) + \left(\frac{\partial}{\partial x} U(x, y, z, t)\right) \left(\frac{\partial}{\partial y} Ax(x, y, z, t)\right) - U(x, y, z, t) \left(\frac{\partial^2}{\partial x^2} Ay(x, y, z, t)\right) \right)
\end{aligned}$$



$$\begin{aligned}
& - \left( \frac{\partial}{\partial x} U(x, y, z, t) \right) \left( \frac{\partial}{\partial x} Az(x, y, z, t) \right) + U(x, y, z, t) \left( \frac{\partial^2}{\partial z \partial x} Ax(x, y, z, t) \right) - U(x, y, z, t) \left( \frac{\partial^2}{\partial x^2} Az(x, y, z, t) \right) \\
& - \left( \frac{\partial^2}{\partial x \partial t} Az(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} V(x, y, z, t) \right) \left( \frac{\partial}{\partial y} Az(x, y, z, t) \right) + \left( \frac{\partial}{\partial x} V(x, y, z, t) \right) \left( \frac{\partial}{\partial z} Ay(x, y, z, t) \right) \\
& - V(x, y, z, t) \left( \frac{\partial^2}{\partial y \partial x} Az(x, y, z, t) \right) - \left( \frac{\partial}{\partial z} V(x, y, z, t) \right) \left( \frac{\partial}{\partial x} Ay(x, y, z, t) \right) + \left( \frac{\partial}{\partial z} V(x, y, z, t) \right) \left( \frac{\partial}{\partial y} Ax(x, y, z, t) \right) \\
& + \left( \frac{\partial}{\partial z} W(x, y, z, t) \right) \left( \frac{\partial}{\partial z} Ax(x, y, z, t) \right) - \left( \frac{\partial}{\partial z} W(x, y, z, t) \right) \left( \frac{\partial}{\partial x} Az(x, y, z, t) \right) + V(x, y, z, t) \left( \frac{\partial^2}{\partial z \partial y} Ax(x, y, z, t) \right) \\
& + \left( \frac{\partial^2}{\partial z \partial t} Ax(x, y, z, t) \right) + W(x, y, z, t) \left( \frac{\partial^2}{\partial z^2} Ax(x, y, z, t) \right) - W(x, y, z, t) \left( \frac{\partial^2}{\partial z \partial x} Az(x, y, z, t) \right) \\
& ((d(x)) \&^{\wedge}(d(z))) + \left( - \left( \frac{\partial}{\partial t} V(x, y, z, t) \right) \left( \frac{\partial}{\partial x} Ay(x, y, z, t) \right) + \left( \frac{\partial}{\partial t} V(x, y, z, t) \right) \left( \frac{\partial}{\partial y} Ax(x, y, z, t) \right) \right) \\
& + \left( \frac{\partial^2}{\partial x \partial t} \phi(x, y, z, t) \right) + \left( \frac{\partial}{\partial t} W(x, y, z, t) \right) \left( \frac{\partial}{\partial z} Ax(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} W(x, y, z, t) \right) \left( \frac{\partial}{\partial x} Az(x, y, z, t) \right) \\
& + V(x, y, z, t) \left( \frac{\partial^2}{\partial y \partial t} Ax(x, y, z, t) \right) + \left( \frac{\partial^2}{\partial t^2} Ax(x, y, z, t) \right) + W(x, y, z, t) \left( \frac{\partial^2}{\partial z \partial t} Ax(x, y, z, t) \right) \\
& + V(x, y, z, t) \left( \frac{\partial^2}{\partial y \partial x} \phi(x, y, z, t) \right) + \left( \frac{\partial}{\partial x} V(x, y, z, t) \right) \left( \frac{\partial}{\partial t} Ay(x, y, z, t) \right) + \left( \frac{\partial}{\partial x} U(x, y, z, t) \right) \left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) \\
& + U(x, y, z, t) \left( \frac{\partial^2}{\partial x^2} \phi(x, y, z, t) \right) + \left( \frac{\partial}{\partial x} W(x, y, z, t) \right) \left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) + \left( \frac{\partial}{\partial x} U(x, y, z, t) \right) \left( \frac{\partial}{\partial t} Ax(x, y, z, t) \right) \\
& + W(x, y, z, t) \left( \frac{\partial^2}{\partial z \partial x} \phi(x, y, z, t) \right) + U(x, y, z, t) \left( \frac{\partial^2}{\partial x \partial t} Ax(x, y, z, t) \right) + \left( \frac{\partial}{\partial x} W(x, y, z, t) \right) \left( \frac{\partial}{\partial t} Az(x, y, z, t) \right) \\
& + \left( \frac{\partial}{\partial x} V(x, y, z, t) \right) \left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) ((d(x)) \&^{\wedge}(d(t)))
\end{aligned}$$

[ The Lagrange (internal or free energy)

> **Lag:=innerprod(VV,A)-A4;**

$$Lag := U(x, y, z, t) Ax(x, y, z, t) + V(x, y, z, t) Ay(x, y, z, t) + W(x, y, z, t) Az(x, y, z, t) - \phi(x, y, z, t)$$

> **wcollect(d(Lag));**

$$\begin{aligned}
& \left( U(x, y, z, t) \left( \frac{\partial}{\partial x} Ax(x, y, z, t) \right) + \left( \frac{\partial}{\partial x} V(x, y, z, t) \right) Ay(x, y, z, t) + V(x, y, z, t) \left( \frac{\partial}{\partial x} Ay(x, y, z, t) \right) \right) \\
& - \left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) + \left( \frac{\partial}{\partial x} W(x, y, z, t) \right) Az(x, y, z, t) + \left( \frac{\partial}{\partial x} U(x, y, z, t) \right) Ax(x, y, z, t) \\
& + W(x, y, z, t) \left( \frac{\partial}{\partial x} Az(x, y, z, t) \right) d(x) + \left( U(x, y, z, t) \left( \frac{\partial}{\partial y} Ax(x, y, z, t) \right) + \left( \frac{\partial}{\partial y} U(x, y, z, t) \right) Ax(x, y, z, t) \right) \\
& + V(x, y, z, t) \left( \frac{\partial}{\partial y} Ay(x, y, z, t) \right) + \left( \frac{\partial}{\partial y} V(x, y, z, t) \right) Ay(x, y, z, t) + \left( \frac{\partial}{\partial y} W(x, y, z, t) \right) Az(x, y, z, t) \\
& - \left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) + W(x, y, z, t) \left( \frac{\partial}{\partial y} Az(x, y, z, t) \right) d(y) + \left( V(x, y, z, t) \left( \frac{\partial}{\partial z} Ay(x, y, z, t) \right) \right) \\
& + \left( \frac{\partial}{\partial z} U(x, y, z, t) \right) Ax(x, y, z, t) - \left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) + W(x, y, z, t) \left( \frac{\partial}{\partial z} Az(x, y, z, t) \right) \\
& + U(x, y, z, t) \left( \frac{\partial}{\partial z} Ax(x, y, z, t) \right) + \left( \frac{\partial}{\partial z} V(x, y, z, t) \right) Ay(x, y, z, t) + \left( \frac{\partial}{\partial z} W(x, y, z, t) \right) Az(x, y, z, t) d(z) + \left(
\end{aligned}$$

$$\left( \frac{\partial}{\partial t} U(x, y, z, t) \right) Ax(x, y, z, t) + U(x, y, z, t) \left( \frac{\partial}{\partial t} Ax(x, y, z, t) \right) + V(x, y, z, t) \left( \frac{\partial}{\partial t} Ay(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} \phi(x, y, z, t) \right) \\ + \left( \frac{\partial}{\partial t} V(x, y, z, t) \right) Ay(x, y, z, t) + W(x, y, z, t) \left( \frac{\partial}{\partial t} Az(x, y, z, t) \right) + \left( \frac{\partial}{\partial t} W(x, y, z, t) \right) Az(x, y, z, t) \right) d(t)$$

> **Theta := wcollect(Work+d(Lag)) ;**

$$\Theta := \left( U(x, y, z, t) \left( \frac{\partial}{\partial x} Ax(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ax(x, y, z, t) \right) \right) \\ + V(x, y, z, t) \left( \left( \frac{\partial}{\partial x} Ay(x, y, z, t) \right) - \left( \frac{\partial}{\partial y} Ax(x, y, z, t) \right) \right) - W(x, y, z, t) \left( \left( \frac{\partial}{\partial z} Ax(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} Az(x, y, z, t) \right) \right) \\ + \left( \frac{\partial}{\partial x} V(x, y, z, t) \right) Ay(x, y, z, t) + V(x, y, z, t) \left( \frac{\partial}{\partial x} Ay(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) \\ + \left( \frac{\partial}{\partial x} W(x, y, z, t) \right) Az(x, y, z, t) + \left( \frac{\partial}{\partial x} U(x, y, z, t) \right) Ax(x, y, z, t) + W(x, y, z, t) \left( \frac{\partial}{\partial x} Az(x, y, z, t) \right) \right) d(x) + \left( \right. \\ \left. - \left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ay(x, y, z, t) \right) + W(x, y, z, t) \left( \left( \frac{\partial}{\partial y} Az(x, y, z, t) \right) - \left( \frac{\partial}{\partial z} Ay(x, y, z, t) \right) \right) \right) \\ - U(x, y, z, t) \left( \left( \frac{\partial}{\partial x} Ay(x, y, z, t) \right) - \left( \frac{\partial}{\partial y} Ax(x, y, z, t) \right) \right) + \left( \frac{\partial}{\partial y} V(x, y, z, t) \right) Ay(x, y, z, t) - \left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) \\ + U(x, y, z, t) \left( \frac{\partial}{\partial y} Ax(x, y, z, t) \right) + \left( \frac{\partial}{\partial y} U(x, y, z, t) \right) Ax(x, y, z, t) + V(x, y, z, t) \left( \frac{\partial}{\partial y} Ay(x, y, z, t) \right) \\ + \left( \frac{\partial}{\partial y} W(x, y, z, t) \right) Az(x, y, z, t) + W(x, y, z, t) \left( \frac{\partial}{\partial y} Az(x, y, z, t) \right) \right) d(y) + \left( V(x, y, z, t) \left( \frac{\partial}{\partial z} Ay(x, y, z, t) \right) \right) \\ + \left( \frac{\partial}{\partial z} V(x, y, z, t) \right) Ay(x, y, z, t) + U(x, y, z, t) \left( \frac{\partial}{\partial z} Ax(x, y, z, t) \right) - \left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) \\ - \left( \frac{\partial}{\partial t} Az(x, y, z, t) \right) + U(x, y, z, t) \left( \left( \frac{\partial}{\partial z} Ax(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} Az(x, y, z, t) \right) \right) \\ - V(x, y, z, t) \left( \left( \frac{\partial}{\partial y} Az(x, y, z, t) \right) - \left( \frac{\partial}{\partial z} Ay(x, y, z, t) \right) \right) + \left( \frac{\partial}{\partial z} W(x, y, z, t) \right) Az(x, y, z, t) \\ + \left( \frac{\partial}{\partial z} U(x, y, z, t) \right) Ax(x, y, z, t) + W(x, y, z, t) \left( \frac{\partial}{\partial z} Az(x, y, z, t) \right) \right) d(z) + \left( \left( \frac{\partial}{\partial t} U(x, y, z, t) \right) Ax(x, y, z, t) \right) \\ + W(x, y, z, t) \left( \frac{\partial}{\partial t} Az(x, y, z, t) \right) + V(x, y, z, t) \left( \frac{\partial}{\partial t} Ay(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} \phi(x, y, z, t) \right) \\ + U(x, y, z, t) \left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) + U(x, y, z, t) \left( \frac{\partial}{\partial t} Ax(x, y, z, t) \right) + V(x, y, z, t) \left( \frac{\partial}{\partial y} \phi(x, y, z, t) \right) \\ + V(x, y, z, t) \left( \frac{\partial}{\partial t} Ay(x, y, z, t) \right) + W(x, y, z, t) \left( \frac{\partial}{\partial z} \phi(x, y, z, t) \right) + W(x, y, z, t) \left( \frac{\partial}{\partial t} Az(x, y, z, t) \right) \\ + \left( \frac{\partial}{\partial t} V(x, y, z, t) \right) Ay(x, y, z, t) + \left( \frac{\partial}{\partial t} W(x, y, z, t) \right) Az(x, y, z, t) + U(x, y, z, t) \left( \frac{\partial}{\partial t} Ax(x, y, z, t) \right) \right) d(t)$$

> **q := getcoeff(Theta&^d(y)&^d(z)&^d(t)) ;**

$$q := U(x, y, z, t) \left( \frac{\partial}{\partial x} Ax(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) - \left( \frac{\partial}{\partial t} Ax(x, y, z, t) \right) + V(x, y, z, t) \left( \frac{\partial}{\partial x} Ay(x, y, z, t) \right) \\ - V(x, y, z, t) \left( \frac{\partial}{\partial y} Ax(x, y, z, t) \right) - W(x, y, z, t) \left( \frac{\partial}{\partial z} Ax(x, y, z, t) \right) + W(x, y, z, t) \left( \frac{\partial}{\partial x} Az(x, y, z, t) \right)$$

$$\begin{aligned}
 & + \left( \frac{\partial}{\partial x} V(x, y, z, t) \right) Ay(x, y, z, t) + V(x, y, z, t) \left( \frac{\partial}{\partial x} Ay(x, y, z, t) \right) - \left( \frac{\partial}{\partial x} \phi(x, y, z, t) \right) \\
 & + \left( \frac{\partial}{\partial x} W(x, y, z, t) \right) Az(x, y, z, t) + \left( \frac{\partial}{\partial x} U(x, y, z, t) \right) Ax(x, y, z, t) + W(x, y, z, t) \left( \frac{\partial}{\partial x} Az(x, y, z, t) \right)
 \end{aligned}$$

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