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COPENHAGEN'S SINGLE SYSTEM PREMISE PREVENTS A UNIFIED VIEW OF INTEGER AND FRACTIONAL QUANTUM HALL EFFECT

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Summary: This essay presents conclusive evidence of the impermissibility of Copenhagen's single system interpretation of the Schroedinger process. The latter needs to be viewed as a tool exclusively describing phase and orientation randomized ensembles and is not to be used for isolated single systems. Asymptotic closeness of single system and ensemble behavior and the rare nature of true single system manifestations have prevented a definitive identification of this Copenhagen deficiency over the past three quarter century. Quantum uncertainty so becomes a basic trade mark of phase and orientation disordered ensembles. The ensuing void of usable single system tools opens a new inquiry for tools without statistical connotations. Three, in part already known, period integrals here identified as flux, charge and action counters emerge as diffeo-4 invariant tools fully compatible with the demands of the general theory of relativity. The discovery of the quantum Hall effect has been instrumental in forcing a distinction between ensemble disorder as in the normal Hall effect versus ensemble order in the plateau states. Since the order of the latter permits a view of the plateau states as a macro- or meso-scopic single system, the period integral description applies, yielding a straightforward unified description of integer and fractional quantum Hall effects.

Key words: Copenhagen, integer, fractional, Quantum Hall effect, period-residue integrals

Introduction

In contemporary physics fundamental quantum problems, not ab initio processed with Schroedinger-Copenhagen methods, tend to be viewed as possible frauds that are better ignored. Yet, this trend, of one recipe fits all, is risky, because here it concerns a procedure, whose limitations have been shrouded in the Copenhagen clouds of nonclassical. mystique. The risks of such single recipe processing was brought home to this author in the course of an almost two decades attempt at discussing a simple pre-1925 unified approach to the integer and fractional quantum Hall effects (QHE) (section IV of this paper).

The initial naive expectation that such simplification might be a welcome contribution turned out a major miscalculation. Perhaps I had expected either a reluctant approval or sophisticated Copenhagen-based arguments refuting my pre-Copenhagen procedures. Yet, no such thing has been forthcoming, instead a more than ominous silence settled over the realm of QHE unification. While at first disappointed and frustrated by a total absence of response, an awareness dawned that Copenhagen arguments did not measure up to this job. Its traditions were due for a reassessment. After some soul searching surprising tidbits of physics history could be uncovered. Here is first a summary account of those findings, which should have been the focus of much earlier concern.

It is argued here that the nonclassical conceptualization of quantum mechanics has been precipitated by a rather silent yet faulty assumption underlying Copenhagen views. For peace of mind, let it be known that this conceptual deficiency can be corrected without unduly affecting standard tools invoked when working with everyday physical or technical problems. So, except for examples of special illustrating power in sections III and IV, no mathematical equations need to be displayed to get the principal message across. This gives disciplines in adjacent realms of epistemics a glimpse of a long standing conceptual conflict within the realms of physics. In the ensuing process of readjustment, a major psychological hurdle is the policy of Copenhagen protagonists to consider their nonclassical rationale as a unique reflection of reality, which they *believe* to be immune from being affected by classical arguments.

This presentation is organized in toastmaster fashion,* meaning those interested in the main theme can start out reading the introduction and conclusion. The central sections sketch more esoteric arguments and technical examples that have contributed to a strong suspicion that the principal claim expressed in the title is solid. The bonus is an opening of perspectives where none were before. Subject to change are not disciplines but their validity domains. They need a redefining, in some cases a shrinking of applicability, for others an extension. The relation between theories of quanta and theories of relativity benefits from this conceptual reorganization. The so-called incompatibility of quantum theory and the theory of general relativity is a *nonproblem*, caused by comparing wrong subdivisions of theory. The essentials germane to such a comparison are found to be perfectly compatible.

The older quantum theory of Planck and Einstein, which later culminated in the Bohr-Sommerfeld integral condition, has been traditionally regarded as an approximation of what is now (mistakenly) believed to be the more exact Schroedinger process. The Wentzel-Kramers-Brillouin (WKB) methodology capitalizes on this asymptotic closeness for obtaining approximate solutions of Schroedinger's equation and so acquires an added physical perspective.

During the early euphoria after the 1925 quantum revolution, the WKB rationale was predicated by *a silent assumption* that the Bohr-Sommerfeld integral method and the Schroedinger equation were believed to be addressing one and the same physical situation. A dramatic break with the past ensues from questioning this unproven identity of purpose. The Copenhagen interpretation so appears as a byproduct of this inadmissible identification. Its view of Schroedinger's equation as an instrument describing single quantum systems was totally unsubstantiated. This silent assumption is the culprit that has unleashed the avalanche of nonclassical propositions, which are then needed to accommodate the consequences of an unproven assumption.

With the help of quite elementary arguments based on dimensional analysis and transformation theory, this discussion aims at verifying the existence of a global set of nearly ideal single system quantum tools, fully compatible with the requirements of the general theory of relativity. Ironically it is, in many ways, an already existing *global* super-structure of *local* Maxwell theory. It consists of a set of residue integrals counting flux, charge and action quanta. The Bohr-Sommerfeld integral emerges as a special reduction of one of these residue integrals. The *metric-independent features* of the general invariance of these residue integrals is an essential aspect of their applicability and is characteristic of this conceptual reorganization.

This global superstructure of Maxwell theory is naturally compatible with the invariance requirements of the general theory of relativity and can address itself to single systems without invoking a priori notions of statistics. So if we were to believe Copenhagen claims we would now have two candidates for single system description. Yet, only one fully complies with the exigencies of the general theory of relativity. In this manner, we have sort of backed up into the earlier conclusion that Schroedinger's equation is not a single system tool. Its probability connotation identifies it as a plurality tool. It describes either a collective of identical real single systems or a collective of abstract manifestation of one single system. Copenhagen unfortunately opted for the much less realistic and less promising ensemble of abstract manifestation of one single system, yet not quite ruling out an actual physical ensemble. A brief review of history is now in order to see how this state of indecision came about.

From the beginning, the Copenhagen probability interpretation, had been woven around this silent single system thesis. In the contemporary text book literature the single system idea is injected as a near-foregone conclusion. At the time, nobody seemed to question this single system idea, except perhaps Slater [1], one of the Copenhageners who may have hesitantly expressed second thoughts about this matter to Bohr. The young Slater was probably no match for Bohr's persuasiveness. Perhaps unbeknown to Bohr and Slater, there was at that time earlier work by Planck [2] in which he showed how a phase averaging of an ensemble of harmonic oscillators requires a zero-point energy to maintain the ensemble in a state of random phase. Schroedinger [3] mentions how Planck's *zero-point* average agrees with his own but seems thrown off by two versions of Planck's radiation theory. The source here

* First tell them what you are going to tell them, then you tell them and then you tell them what you told them.

available of ref. [2] has a 1912 preface by Planck; presumably it is his second version. In it, Planck introduces his ensemble view of zero-point energy. Had Schroedinger been more intensely confronted with Planck's ensemble option, the ensuing seventy years of nonclassical pursuit of a single system illusion might not have been, and perhaps, the Copenhageners might have adopted another course of action.

Jammer[4] extensively reviews how real ensemble alternatives of wave mechanics were vividly pursued in the Thirties as well as later. The names of Popper, Kemble, Groenewold, Collins, Blokhintsev and Ballentine are mentioned in this context. Yet, it is not obvious whether these ensemble protagonists were well aware of Planck's ensemble-based introduction of zero-point energy, because the Copenhagen-type nonclassical probability kept also dominating the scene in ensemble views of the Schroedinger methodology. It shows errors are harder to remove, once they have become anchored and hidden in a universally accepted procedure.

The preliminary conclusion of this intermezzo reveals how the Schroedinger equation and the Bohr-Sommerfeld integral and its companions are addressing different physical situations. Their mathematical asymptotics reflects a realistic physical aspect. So, having the Schroedinger equation as an ensemble instrument and Bohr-Sommerfeld *et al* as the natural single system tool emerges as a better assignment of their respective physical potentials.

This new role for the Bohr-Sommerfeld integral reestablishes a primary function for it as well as for its cyclic companions. By the same token, Schroedinger's equation retains full honors even if its earlier services were rendered under a misleading label. However, it has to be taken off its Copenhagen single system pedestal. Against this background of changes, the relation between quanta and relativity can be reexamined leading to successful reconciliation. For many years this adaptation had been viewed as a hopeless undertaking. A faulty premise about Schroedinger's object of description had been blocking the view. Now in retrospect, it can be said that past attempts at blindly imposing general covariance on a combination of fundamental law and ensemble information cannot be regarded as an act with a well defined physical objective.

Since this conclusion strikes at the heart of a traditional feature of existing quantum views, it can be expected to cause considerable opposition. This opposition can be expected to harden in the light of *three quarter century of nonclassical conceptualization, which has conditioned Copenhagen supporters to assume a frame of mind that does not accept classical arguments as a valid means to refute a Copenhagen edifice put together by nonclassical ingredients.*

The ensuing predicament is a confrontation between two alien worlds, constructed as two distinct and foreign logical systems. They don't quite have a common cross-section permitting operations in their common realm. To safeguard its position, the nonclassical world has been using a technique of unduly undercutting the reality of the classical world. Classical reality, it was said, is mistakenly accepted as reality, it merely is a probability appearance.

This nonclassical game of usurping its sine qua non position for describing contemporary physics can only be invalidated by disproving its claim of uniqueness. In retrospect, Planck's counter example [2] serves that purpose. It proves that the body of nonclassical conceptualization has been due to a single system's inability to act as a universe of discourse for a classical statistics.

I. Physical Reference Systems and the Uniqueness of their Units

A primary requirement for quantitative descriptions of physics is a basic agreement on measure references. Amazingly, the foursome of length, time, mass and electric charge [$\boldsymbol{\ell, t, m, q}$] suffices. This sequence of symbols sort of illustrates the historic evolution of man's awareness of nature. Length is a first concept in man's exploration of space, the duration of processes taking place gives a concept of time, so the duo [$\boldsymbol{\ell, t}$] gives us a means of getting around in space and time. Newton specified the concept of mass [$\boldsymbol{m}$] with its aspects of inertia and gravity. Electric charge [$\boldsymbol{q}$], initially received an *irrational* measure in terms of [$\boldsymbol{\ell, t, m}$], was later given independent status.

Ever since Faraday established the laws of electrolytic deposit there had been an overriding suspicion that nature provided a fundamental electric unit, known as the elementary electric charge [e]. This knowledge was further substantiated by Millikan's famous oil drop experiment. Compared with the irrational charge reference in terms of [ℓ, t, m], an independent [q] was added as having a more fundamental connotation. The Coulomb as independent unit for [q] can now be defined as an exact multiple of the elementary charge [e] .

There is no unique unit of mass. There are electron- proton- and neutron- mass units, their ratios though don't resemble rational fractions. This defies the existence of a unique universal measure of [m] similar to that of [q].

In the foursome [ℓ, t, m, q] only the new-comer [q] has that special property of universal uniqueness, which does not apply to the other three. This raises questions, whether nature provides other natural units that have comparable universality and uniqueness as [e]. A quantity that comes to mind is action, for which Planck has established the existence of a very unique unit known as Planck's unit of action [h]. Since action has dimension [h] = [$m\ \ell^2\ t^{-1}$], it is permissible to adopt a fundamental reference system in which the [m] in [ℓ, t, m, q] is replaced by [h]. This gives a new reference foursome, distinguished by two frame related metric units [ℓ, t] and two physical references sharing the special property of being countable in terms of unique natural units [h, e]. Let this new reference system, for which practical units can be adapted according to the internationally agreed SI convention, be referred to as the action-charge (h,e) reference [ℓ, t, h, e].

This action-charge reference has some conspicuous advantages over the traditional [ℓ, t, m, q] system. Unlike the old reference in which m retains metric connotations, in the (h,e) reference system the physical and geometric references are now completely separated. *This separation is underlined by what is called the general relativistic invariance of the units* [h] *and* [e] *as domain-scalars of space-time*. In the standard SI system q is that sort of a scalar, m is not, because it is the component of a four-vector.

The fundamental significance of the (h,e) reference is strongly emphasized by experimentation on macro- and mesoscopic quantum systems. The Josephson effect gives very accurate data for the quantum flux unit h/e. The quantum Hall effect gives accurate data for the Hall impedance h/e^2 . Together they give measurements of fundamental constants approaching 9 decimal places.

Flux and Hall impedance are global system properties of a more primitive or elementary nature than spectral data requiring more structural knowledge of the global mechanisms producing those spectra. Despite sophisticated corrections of a QED nature, fundamental constants obtained from spectral data lack a reproducibility comparable to the Josephson-Hall effect data. Compare hereto compiled data extracted from Cohen-Dumond tables on p.135 of ref.6.

The ratio h/e^2 though has a basic role in the study of spectral fine structure phenomena. Spectral observations on distant galaxies confirm its apparent constancy throughout the visible universe. This fact, all by itself, lends strong global support for adopting an action-charge (h,e) reference system that makes the independent nature of the metric [ℓ, t] basis and physical [h, e] basis an explicit feature of general theory.

The (h,e) reference system has a critical role in establishing basic transformation features of tensor fields in general theory [5], which in turn is helpful in identifying physical relations that are independent of metric specifications. If, at this point, such metric-independence sounds too esoteric, keep in mind: *a counting* of identical natural physical [h, e] units should not depend on choices of units of length or time [ℓ, t].

II. Metric and Premetric Aspects of Spacetime

From the times of early Greek mathematics until the present there has always been a vivid awareness that geometry is a discipline that can be mostly pursued independent from physical specifics. Mathematicians have always claimed geometry as their domain and, sometimes fortunately for geometry, they have succeeded in doing so without undue interference of physicists. The dimensional situation in physics, depicted in the previous section, delineates in detail how some parts of physics depend on metric-geometry notions, whereas other parts are topological in nature, *i.e.*, independent of metric specifications. In the latter part the quanta h and e hold a crucial position as domain-scalars. Using current mathematical terminology quantization assumes here a rather exclusive **global** feature!

The notions *global* versus *local* have emerged in mathematics earlier this century. These concepts are not normally explicitly defined, not even in mathematical encyclopedia. It means standard dictionary definitions suffice. Their recent introduction has accompanied an era of topological development, which has as yet not affected physics to the same degree as it has affected mathematics. Typical physical concepts in the global realm are: *orientability* of manifolds and the objects therein (*e.g.*, *enantiomorphism*), the distinction between *exact* and *closed* differential forms, the *odd-even* or *pair-impair* distinction between differential forms, their integrals and their periods or residues. Last but not least *spinors* and *spinor formalisms* are a local artifact that have their roots in the orientability of micro objects.

Yet, even today 99% of physics' conceptualization, predicated by Euclidian premises and its vehicle of vector analysis, obliterates most of the just listed distinctions. The theory of general relativity postulates spacetime to be Riemannian, which gives it locally Euclidian properties. So, not surprisingly, much Euclidian simplifications carried over also into the general theory.

Euclidean and Riemannian geometry both are metric geometries, which means there is a metric tensor of reference. Euclidian geometry permits frames of reference in which the metric tensor is constant, Riemannian geometry does not! In fact, the general theory of relativity relates gravity to intrinsic changes in the metric. Geometry is here further encroaching on physical territory and so becomes a joint responsibility of mathematics and physics.

Since in much of physics the metric can be taken as constant, mathematical procedures common in physics traditionally choose frames of reference that make the metric invisible. Even in the general theory the metric is still used to recover some of that good old Euclidean simplicity. For instance, the metric tensor is used to reduce the transformation manifestations of physical fields; it is called *the process of pulling tensor indices up and down.* However, in view of the gravitational implications of the metric, those operations obscure the physical nature of those tensors. All of which raises the question whether tensorial physical fields have a preferred intrinsic transformation feature unblemished by the obscure metric operations of raising and lowering indices. Modern so-called *anholonomic* renditions of the general theory of relativity perpetuate the old desire of making the metric invisible, which compounds or rather sacrifices the possibility of recognizing metric independence.

To answer questions whether or not physical statements are possible independent of the metric structure in which physical events take place, it does not suffice to make the metric invisible. In witness of the metric relation to gravity, the question is whether there is a part of physics that remains unaffected by gravity. Some people have indeed explored this territory.

As an aftermath of the general theory of relativity, in the early Twenties some workers in the borderline field of physics and mathematics discovered a number of physical relations within the realm of the general theory of relativity, which could be rendered in a completely metric-independent manner. It means, it is possible to give these laws a mathematical formulation that merely calls on the spacetime manifold properties, whereas any reference to the spacetime metric tensor field for length and time references is allowed to drop out of the picture. This feature persists under arbitrary (diffeo-4) changes of spacetime coordinates.

The initial response by physicists to these rather mathematical inquiries was one of puzzlement and mild disbelief. An attempt at a down to earth assessment suggests that everything in physics invokes measurements of length and time. What does "metric-free" mean, in fact, how could anything in physics be metric-free?

The interest in these mathematical observations soon began to wane after the first amazement had worn off. Now, at the end of this century, almost all physicists are either unaware of these matters, or long ago they dismissed them as peculiarities of mere mathematical concern.

Even if there was at the time a growing perceptiveness of topology related matters, the metric-free physical relations of the early Twenties were, with few exceptions, all cast in the form of metric-independent differential statements that have strictly local implications. *The purely mathematical interest, however, becomes a matter of relevant physical interest as soon as the local interest is extended to a realm of global concern.* That is where the topology comes in. From a physical point of view that is most easily done by looking at some of the familiar integral laws of physics.

In quite introductory physics courses, it is shown that Gauss' law of electrostatics really counts the number of net elementary charges residing inside a closed surface. This statement is said to remain true even if the enclosure is deformed, as long as no charges cross the enclosing surface. *Since Gauss' law statement really counts the net number of charges inside an enclosure, this counting will also have to be independent of arbitrary changes in the frame of reference and its metric specifications*.

The standard formulations of electromagnetic theory, even the most advanced presentations amongst them, don't make the here cited frame- and metric independence a matter of obvious mathematical perspicacity. The standard mathematical vehicle of vector analysis, commonly used in physics, is exclusively metric-based. It makes the option of separating out metric-free features either very cumbersome or practically impossible.

Even so, the reader may still ask: why all this unrelenting insistence on the pursuit of metric-free options? To answer this question, be reminded that absence of metric makes non-metric qualities stand out more clearly. Topological structure is a prime example of a nonmetric quality. More important though: *metric-free law statements have validity in macro- and in micro-domains, because the metric is the one and only reference of what is physically small or large.* Hence a pursuit of metric-free options opens the doors to topological explorations in macro- and micro-domains. Last but not least, since the metric is known as an exclusive agent of gravity action, *metric-free relations have a position in physics that holds independent of gravity*.

When seen in this light, it seems worthwhile to repair shortcomings in contemporary mathematical presentations of physics if they stand in the way of a more discerning view of these matters. The explicit discovery of metric-free statements of physical laws first emerged from the use of tensorial descriptions, mostly because in some cases the metric-based process of covariant differentiation would reduce to ordinary differentiation. Since tensor methods are mostly local in nature, the topological implication of metric-free physics points towards the global connotations of integral statements.

In answer to these topological needs, mathematics has singled out a technique specifically meant for dealing with certain metric-free integration aspects of tensor analysis. It has become known as the method of differential forms. It lifts the metric-free global aspect out of the realm of appropriately general invariant tensor methods.

Forms were first introduced by Cartan and later developed by de Rham for purposes of topology. The integral theorems of Stokes and Gauss encountered in vector analysis and their generalizations reemerge in differential forms as metric-independent theorems The so called de Rham theorems relate to the category of residue integrals of which Gauss' integral of electrostatics is an early classical example. So, adopting differential forms we get reacquainted with all sorts of items with which there was already an earlier partial familiarity, yet they now appear in a new more enhanced context that permits a more discerning assessment. The flux integral, perhaps first introduced as a period integral by Fritz London, has more recently become known as the integral of Aharonov-Bohm. There is a 3-dimensional action integral that is a product integral of this 1-dimensional flux integral and a 2-dimensional Ampère-Gauss integral.* If the latter yields a point charge residue, the 3-dimensional integral reduces to a 1-dimensional period integral that has long been known as the Bohr-Sommerfeld integral of the early quantum theory.

A trend is emerging in physics favoring methods of differential forms as a future tool for purposes of physics. Yet, contemporary physics has remained largely uninformed about the pre-metric discoveries of the early Twenties. Hence no clear distinction emerges between metric-free and metric dependent forms. As a result, forms in physics are introduced in somewhat ad hoc manner, not taking advantage of this chosen opportunity to readdress the physical issues associated with pre-metric physics: e.g., macro- as well as micro-topological structure invoking the invariants of action ***h*** and charge ***e***.

These options have been either ignored or denied for so long, because a continued use of the traditional dimensional reference system [***ℓ, t, m, q***] detracts from a topologically more discerning view of physical structure. **The Action Charge revised dimensional reference system [*ℓ, t, h, e*], by contrast, serves as a reminder how to identify and home in on countable quantities.** To create meaningful order in a realm of tradition where too many distinctions have been lumped together under the motto of **one tool serves all needs**, one has to take advantage of all reminders to disantangle the mesh.

III. Schroedinger's Recipe as a Derivation

Here is first a brief reminder of Schroedinger's recipe, after which an added rationale is given of how this recipe can be elevated to the status of a derivation from first principles.

Schroedinger starts with the Hamilton-Jacobi equation of mechanics. Let S be the action function, p,q the generalized momenta and coordinates of the system under consideration and H its hamiltonian, the equation is normally stated in the form

$$H(p,q,t)=-\$S/\$t.$$

Schroedinger started out with a conserved one-particle system of mass m in a central potential field V depending on radius r. In probing the origin and underlying physical principles of the process, it is well advised to focus on the same example chosen by Schroedinger. If E is the constant system energy, the H-J equation assumes the explicit form.

$$\frac{1}{2m}\left[\left(\frac{\$S}{\$x}\right)^2+\left(\frac{\$S}{\$y}\right)^2+\left(\frac{\$S}{\$z}\right)^2\right]+V=E \qquad 1$$

Following de Broglie, Schroedinger then changes the dependent variable S

$$S=\sqrt{-1}\ ¡\ \ln ×\ , \qquad 2$$

and obtains an accordingly modified Hamilton-Jacobi equation

$$-\frac{¡^2}{2m}\left[\left(\frac{\$×}{\$x}\right)^2+\left(\frac{\$×}{\$y}\right)^2+\left(\frac{\$×}{\$z}\right)^2\right]+V×^2=E×^2. \qquad 3$$

He then optimizes this functional with the help of the Euler-Lagrangean derivative and so obtains his wave equation in the familiar form:

$$-\frac{¡^2}{2m}\left[\frac{\$^2×}{\$x^2}+\frac{\$^2×}{\$y^2}+\frac{\$^2×}{\$z^2}\right]+V×=E×. \qquad 4$$

Solutions of this equation are sought for *eigenvalues* E that permit × to be square integrable and single valued.

For V a Coulomb potential, Schroedinger showed that such solutions can be obtained if E assumes discrete values, which are identical to the Bohr states of hydrogen-like atoms. Yet, the corresponding

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* This integral, of course, includes the Maxwell displacement term, so it could be denoted as the AGM integral. Similarly recalling London's first suggestion in the Thirties of flux quantization, the Aharonov-Bohm integral might be denoted as the LAB integral. The product integral is due to R M Kiehn, compare ref.6 for more detailed explanations.

discrete angular momentum states came out to be L=$\sqrt{n(n+1)}$ ¡, whereas the Bohr angular momentum states are L=n¡. This difference in angular momentum was right from the start viewed as fundamental. The spectral analysis of more complex situations with spin orbit coupling seemed to support the new L expression.

An inspection of the Feynman Lectures [7] shows how L= $\sqrt{n(n+1)}$ ¡ can be calculated as an ensemble average of arbitrarily oriented angular momenta L=n¡. This calculation seems to have surfaced earlier in the Russian literature [8]. Neither text, though, asked questions whether or not this coincidence might affect the Copenhagen interpretation of the Schroedinger equation.

Another remarkable difference between Schroedinger results and the older quantum mechanics shows up if the potential function V is taken to be that of a harmonic oscillator. Landau [9] has shown that cyclotron orbits are in this category. This example is of current interest, because of its relation to the quantum Hall effect, treated in the next section. In that case there is no direction averaging, because in a planar configuration all cyclotron orbits have the same orientation. Solving the Schroedinger equations yields energy states E=(n+$\frac{1}{2}$)¡ø, in which ø=(e/m)B is the cyclotron frequency of a particle of charge e and mass m in a magnetic field B.

It is not too well known that Schroedinger's spectrum E= (n+$\frac{1}{2}$)¡ø had already been anticipated by Planck [2] in 1913. Schroedinger was aware of Planck's work and makes the appropriate reference. He mentions the coincidence of results as a point for future investigation. There is no evidence, however, that he followed up on his intention. Heisenberg, who came up with the same result, was apparently unaware of Planck's earlier conclusion and so is the bulk of the contemporary text book literature on quantum mechanics.

So, how did Planck obtain his result? He considered an ensemble of identical harmonic oscillators of arbitrary mutual phase and showed how a process of phase averaging yields E= (n+$\frac{1}{2}$)¡ø. Hence even for n=0, a residual zero-point energy remains to retain the state of phase disorder. Yet Planck views the latter as half of the oscillators being in the state n=0, the other half in the energy state n=1. In other words, **single oscillators are not equipped with an energy residue** E=$\frac{1}{2}$ ¡ø. That idea was Copenhagen's addition; an unnecessary assumption, solely precipitated by an unproven single system premise.

In the light of these combined experiences relating to orientation and phase averaging, the question arises whether the Schroedinger equation itself cannot be directly viewed as an averaging procedure. In fact, the eigenvalue process has long been known as an optimization process. The Schroedinger recipe indeed opens up a striking ensemble perspective of optimum probability.

Consider hereto the H-J Eq.1, its general solution invokes a number of integration constants and every set of values of those constants represents a system of given orientation and phase. Hence the set of all possible values these constants can assume represents an (infinite) ensemble of possible single system solutions; it is the solution manifold of the H-J Eq.1.

The change of dependent variable S‘× given by Eq.2 transforms the solution manifold S into a corresponding solution collective × such as appears in the functional Eq.3. The latter is then *extremized* by a variational process that leads from Eq.3 to Eq.4: the Schroedinger equation. Since the functional Eq.3 is the equivalent of the H-J solution manifold, Schroedinger's equation optimizes the ensemble represented by that manifold. The solutions of Eq.4 subject to the boundary conditions of square integrability and single valuedness of × can now be seen as ensemble specifications for the solution manifold. The single valuedness secures Bohr-Sommerfeld type quantization and the square integrability is a necessary condition for a probability connotations of × such as established by Born. So, Eq.4 establishes a most probable ensemble state!

The option of viewing the Schroedinger process as a procedure derivable from a collective of single system situations, which are all quantized according to the older quantum theory, has discomforting consequences for Copenhagen's interpretation of quantum mechanics. They can be briefly listed as follows:

1 The Schroedinger equation describes phase and orientation randomized ensembles of single systems.

2 This ensemble view of the Schroedinger equation reinstates a measure of exactness of the Bohr-Sommerfeld theory of single systems. Hence precise orbits no longer are forbidden items.

3 Ensemble specifications completely obviate Copenhagen's attempt at a nonclassical imaging of single systems with fuzzy orbits obeying a statistics whose universe of discourse had been blocked from view by Eq.2 and then secured in hiding by Copenhagen's premature single system premise.

4 The zero-point energy is now no longer a single system attribute, but instead becomes an ensemble attribute necessary to maintain phase and orientation disorder.

While the four listed points are devastating for Copenhagen's single system premise, it constitutes in no way a conclusive encompassing procedure. It is necessary to keep in mind that these conclusions have been extracted from a one particle system. Many particle systems have been a major hurdle in astronomy and in atomic theory. For the latter Pauli's exclusion principle and in some way Fermi-Dirac and Bose-Einstein statistics have been major steps on the road to further understanding. These fermion-boson and exclusion aspects don't really come out of the Schroedinger process.

There is a qualitative understanding of the spectra of many electron atomic systems, but nowhere the precision of Hydrogen related spectra. In some recent literature hybrid possibilities are being explored by combining pre and post 1925 methods (see p42 ref.6). Since the now well established single system versus ensemble distinction has been found to correspond to the pre- and post 1925 methodology, such added insights may cast further light on the use of hybrid techniques. Copenhagen's fuzzy orbits can now be regarded as so well enough defined to make orbital topology a new challenge. Yet, mindful how earlier premature actions had adverse effects, it is well to go step by step.

IV. The Quantum Hall Effect

Two-dimensional conducting structures exposed to a magnetic field B, perpendicular to its surface, exhibit a transverse (Hall) voltage V_H measured in a direction perpendicular to B and the two-dimensional forward current I_F. The ratio of the two

$$\frac{V_H}{I_F} = Z_H \qquad 5$$

is known as the Hall impedance. It has the physical dimension of a resistance and is frequently referred to as such. However, since Z_H is not dissipative the term impedance seems more appropriate.

Looking first at the normal Hall effect, let the charge carriers making up I_F have an average drift velocity v. Assuming v and B constant across the width w of the sample, one obtains for $V_H = vBw$. If σ is the electron density per unit area, the forward current $I_F = \sigma vw$. The ratio of the two gives Eq.6

$$Z_H = \frac{B}{\sigma} \;, \qquad 6$$

No sample dimensions appear in Eq.6, neither do the magnitudes of I_F nor V_H for that matter. In other words the drift velocity v can approach zero and Eq.6 still remains valid. It means Eq.6 describes the physical state of a local neighborhood of the sample. For the quantum state this local neighborhood assumes a more global character and pervades the whole sample.

In some semi-conductor devices it is possible to change the magnitude of σ by charge carrier injection with the help of an interlayer injection voltage V_i. MOSFET devices, which stands *for metal oxide semi-conductor field effect transistor*, lend themselves to such experimentation. The quantum Hall effect was discovered on exactly such a device [10]. A normal test run of Z_H with respect to V_i gives a smooth hyperbolic graph for Z_H as a function of V_i, the latter being proportional to σ. However, at extremely low temperatures and very high B, the hyperbolic graph develops narrow horizontal sections, the so-called "plateau" states, for which Z_H remains constant in small intervals of V_i. Solid state consideration, calling on Landau's wave equation analysis of cyclotron states,* have led to a conclusion [10] that the measured impedance states could be described by the simple, yet very fundamental formula:

$$Z_H = \frac{1}{i}\,\frac{h}{e^2}\ ;\ i= \text{an integer.} \qquad (7)$$

The observations were so reproducible and precise that it was immediately suggested as a measurement procedure in conjunction with Josephson effect observations measuring h/e, thus leading to e and h measurements yielding up to 9 decimal places.

Varying B instead of σ gives a linear graph. Experiments have yielded in addition to the integer i states also additional plateau states for which i appeared to be a rational fraction [11]. This noninteger situation is referred to as the *fractional quantum Hall effect*. A Schroedinger-based analysis of the latter effect has led to convictions indicating that integer and fractional effects are fundamentally very different physical manifestations. This conviction has led to a tentative conclusion that the fractional effect might be revealing a possible equivalent of fractional charge. The 1998 Nobel prize for physics has sort of underlined exactly such conclusions.

However, if the critique of the Copenhagen interpretation given in the previous sections is relevant and correct, one should not be using a statistical tool on the highly ordered situation confronted in the quantum Hall effect. Instead the techniques of period (residue) integration, such as available in the Aharonov-Bohm and Ampère-Gauss integrals, now assume a primary status as physical laws. So, Landau's Schroedinger analysis of cyclotron states, suggesting a phase random ensemble as object of analysis, should be replaced by a single system procedure. Here is a brief intermezzo of how to use those "new" global tools for a single system that is a multi-electron cyclotron state.

The electric field E_c seen by an electron circulating at angular rate ø and radius r in a magnetic field B is E_c = ørB, integrating from zero to r gives for the orbital potential V_c=ør^{2}B/2. The period of circulation T=2¹/ø and multiplying the two gives the cyclic time component of the Aharonov-Bohm period integral, which equals the magnetic flux F encircled by the orbit:

$$§V_c dt = V_c T = {}^1r^2B = F.$$

In an external field, F is a multiple n of h/e (compare ref.6. chapterVI). Hence

$$V_c = n¡ø/e.$$

This condition fixes the energy quantization in the following manner. Let se be the orbiting electron charge, its energy in the orbital potential is now E= seV_c, which equals the orbital kinetic energy, since B=øm/e

$$E = seV_c = \tfrac{1}{2}\ seør^2B = \tfrac{1}{2}\ sm(ør)^2\,.$$

The energy quantization compatible with flux quanta h/e is thus found to be the Landau energy per system **without the zero-point energy:**

$$E = sn¡ø,\ n = \text{integer and } ¡=h/2¹.$$

All of which shows no more worries about zero-point energy and conceivable flux residues thereof: end of this cyclotron intermezzo.

The mathematical advantage of using residue integrals in physics is even enhanced with respect to mathematics, because nature gives us standard residues that are multiples of fundamental quanta constructed from h and e. There is no solving of difficult integrals, instead physical judgment is needed whether or not given situations justify an application of the integral laws that have now been given primary status.

In the normal state of the sample the current consists of a disordered variety of cyclotron orbits, not all in the same quantum state and of arbitrary phase. The phase randomness must be expected to reveal the presence of an ensemble-based zero-point energy in the sense of Planck.

Now visualize the order changes taking place when the whole sample goes into a (single) quantum state. All cyclotron states assume the same radius and they approach a global mesoscopic state of order

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* Please note how the zero-point energy of the Landau state for n=0 leads to a predicament whether a lowest state still links an associated flux residue? The predicament is avoided by regarding zero-point energy as an ensemble attribute. Keep in mind, zero-point energy as a single system attribute also leads to the vacuum infinities of QED.

by circulating in phase synchronicity. A lattice of such identical cyclotron orbitals is slowly drifting through the sample matrix. This lattice is acting as if it were a meso- or macroscopic single system devoid of Planck's zero-point energy. Such a meso-scopic system has less of a chance of interacting with the sample matrix and might thus be indicating a drop in resistivity. Experiments indeed show a superconductive I_F throughout the plateau intervals.

The lattice uniformity invites the acceptance of one cyclotron system as representative of the global entity. Multiplying enumerator and denominator of Eq.6 both by the cyclotron orbital area S big enough to include the charge at the cyclotron boundary (or a multiple thereof) gives as

flux through the orbital area $F = BS$ 8

and as total orbital charge $Q = \sigma S$ 9

Substitution in Eq.6 gives for the Hall impedance $Z_H = F/Q$ 10

The earlier brief intermezzo of the single system analysis of the cyclotron state found F a multiple n of the flux quantum h/e and Q a multiple s of the electron charge circulating in the cyclotron orbit. Substitution in Eq.10 yields the simple result

$$Z_H = \frac{n}{s}\,\frac{h}{e^2} \qquad 11$$

which contains Eq.7 as special case, covering integer and fractional effects both.** More formally it can be said: **Z_H equals the ratio of the period integral of Aharonov-Bohm over the Ampère-Gauss period integral.**

A further point needs to be made about the number s of electrons accumulating in cyclotron orbits. Such accumulation suggests boson formation by electron pairing, in accordance with the principle of pair-forming in super conductors. So. s is even. Comparison of Eqs.7 and 11 gives:

$$i = s/n. \qquad 12$$

Since s is even, factor two reductions in the ratio of Eq.12 lead to a prevalence of odd denominator ratios reported in the established Quantum Hall literature.

Two critical ideas guide this compelling (non-Schroedinger) approach to the quantum Hall effect:

First, the general nature of this highly ordered physical situation is such that one may speak of a single system rather than an ensemble of phase random single systems bringing about the plateau states.

Second, the plateau impedance is independent of the drift velocity v of I_F. This aspect is already manifest in the normal Hall effect, as displayed in Eq.6, and is shared by the quantum effects.

Hence for a vanishing forward current v‘0, normal states and plateau states describe a local state of the sample, which in view of the magnetic field and low temperature is bound to reveal charge carriers in cyclotron states. The only conceivable distinction between these two-dimensional normal states and plateau states can only be a matter of whether or not cyclotron charge carriers are orbiting out of phase (normal) or in phase (plateau). In the latter case there is global order, all cyclotron orbits are in phase, have the same radius (same quantum state), and may assumed to be orderly positioned in a two-dimensional lattice to maintain the phase lock. The global order, when viewed in the perpective of Eq.6, makes it clear that one cyclotron state suffices to obtain the result for the whole lattice of identical cyclotron states acting as one single system.

Eq.11 expresses a perception of the quantum Hall effect(s) as a case of independent flux and charge quantization, not easily incorporated in a Schroedinger description. Eq.11 was first published in Phys. Lett.94A**,343(1983). Its derivation as a period integral ratio drew no response whatsoever. A letter by Kiehn in Physics Today, April 1987, p.122 (which triggered the appearance of a preceding letter on p,120) inquiring about this absence of response to the joined description offered by Eq.11. Still no response! So, undeterred the following articles kept hammering the same topic: Phys. Lett. **A125**, 225(1987); Physics Essays **2**,55(1989) and in *A Two-Tier Quantum Mechanics;* (*period integrals for single systems and Schroedinger process for ensembles*); p.219 of the Proceedings of a 1992 meeting at Columbia Un. on **The Interpretation of Quantum Theory: Where Do We Stand?** (Published by the Enciclopedia Italiana, Roma,1994) ed. Luigi Accardi. Finally *Quantum Reprogramming* (Kluwer, Boston-Dordrecht, 1995), which is ref.6. None of these made the grade as objects of consideration in the established quantum Hall literature.

Following the dictates of *point one* for a single system that is not part of a phase random ensemble, it is necessary to call on the period integrals as the appropriate tools for assessing the physical situation. Eq.11 is then the unavoidable result of that assessment.

Unfortunately all the currently approved approaches to the integer and fractional quantum Hall effects have been using Schroedinger techniques, which lead to unavoidable predicaments involving zero-point energies in the plateaus. This methodology is faced with a compromise between a physical situation, seemingly free of any statistical implications, and a Schroedinger- based tool with an inherent statistical connotation. Obtaining the observed and well established experimental end results requires, in some way or another, the not so classical act of ridding the end result of conceivable manifestation of that presumed nonclassical statistics.

Finally it should be mentioned that there are spectacular scaling features in the transitions between plateau and normal transitions, which have been extensively reviewed by Huckestein [12]. Preliminary investigations have shown that a needed build-up of zero-point energy in the transiton from plateau to normal state presents a very natural mechanism making these transitions physically more transparent.

Conclusion

In retrospect, this little quantum reprogramming exercise leaves all of physics' major tools from Maxwell to Schroedinger-Dirac fully intact. It is the very reason why this plea for conceptual change can in principle be made without writing down many mathematical equations. The major casualty in this rearrangement of basics is the conceptual picture of the Copenhagen interpretation and its realm of tortured nonclassical propositions.

Accepting Born's probability identification of Schroedinger's × function as basically correct, it was Copenhagen's single system premise that left Born's statistics without a realistic universe of discourse as a suitable physical home. That is how the idea of a nonclassical statistics was born. Henceforth, a nonclassical statistics be defined as one lacking a universe of discourse.

Nonclassical protagonists have gone so far as to claim that classical equivalents of nonclassical don't exist. They used to say that such questions reveal a basic incapability of understanding modern physics. Such dictates, however, resemble a familiar policy of people who don't like to be contradicted. Here, the position is taken that classical counterparts of presumably nonclassical entities have been cited to exist. In 1913, nobody less than Max Planck [2] gave an example, which, in retrospect, contradicts the nonclassical statistical propositions that were going to get hold of physics for the rest of the century. If continuing their views, Copenhageners will be faced with the daunting task of proving that classical counterparts to their nonclassical propositions can't exist!

The Copenhagen view cannot be rescued by a Gibbs-type ensemble of *conceivable* statistical manifestations of one and the same single system. This proposition would endow every harmonic oscillator with an ever-present zero-point energy, leading to the notorious QED infinities of vacuum.

Knowing that the Gibbs ensemble was meant as an abstract substitute for an actual ensemble, such vacuum infinities can be avoided by restricting the Schroedinger as well as the Dirac equations to real ensembles of identical systems that are taken to be random in phase and orientation. Most spectroscopic samples meet that condition. Hence, spectroscopists may not have to worry too much about this restriction, moreover most, but not all, spectral differences between asymptotic situations are not all that conspicuous!

Since a conceptual reorganization of contemporary physics, as here indicated, restricts Schroedinger applicability to ensembles, the upshot of doing so leaves single systems without an appropriate tool of analysis. So, a complementary path to reorganization had to start out at the opposite end: *i.e.*, the single system. It becomes a matter of exploring whether or not the old Bohr-Sommerfeld condition (by its very nature a single system tool) has brothers and sisters. This search led to the Aharonov-Bohm and Ampère-Gauss integrals, which could be reunited as siblings, that had been separated at birth. Their typical single system connotations enhance insight into flux, charge and Hall impedance quantization, as well as some QED phenomena [6].

The methodology of establishing a basis of general features for universal single system tools, as used here, is based on dimensional analysis and general spacetime theory of transformation. Their metric-independence holds a key role in macro- and micro-applicability, thus finally establishing a sound physical objective for a number of curious and puzzling mathematical investigations of the Twenties and the Thirties. Their diffeo-4 invariance amply secures compatibility with requirements of the general theory of relativity.

It is this existence of a set of useful single system tools that poses a major problem for a continued support of Copenhagen's single system premise and its aftermath of nonclassical conceptualization. Their metric-independent general invariance naturally invites a description of the general invariant quantum Hall impedance as expressed by Eq.11. In no way can the Schroedinger methodology, with its inherent ensemble connotation, compete for a single system approach. This point of view had already been indirectly reached by some people in the Thirties. The reason why this early attempt did not get off the ground, at that time, was the obscuring influence of that early persistent Copenhagen myth of an all pervasive, a priori, nonclassical statistics.

The counter examples disproving this nonclassical statistics deal with a statistics of mutual phase and orientation of the systems in an ensemble. The universe of discourse of this statistics is identified through the free constants of integration provided by the solution manifold of the H-J equation. *Schroedinger's exponential transformation of dependent variables injects the ensemble information from S into × and in this process an easily visualizable contact with the universe of discourse was lost.* Explicit proof shows how phase and orientation averaging of an ensemble of systems retrieves Schroedinger results. Imposing single-valuedness of × is a provable Bohr-Sommerfeld equivalent. The variational processing of × in the H-J functional Eq.3 parallels Planck's equilibrium optimizing of phase [2] and the Feynman-Kompaneyets [7,8] averaging of orientation of systems in an ensemble.

All of this hints at a reality that makes the Schroedinger equation a derived secondary law of nature with the practical consequence of restricting its applicability to ensembles. By the same token, the Aharonov-Bohm, the Ampère-Gauss and the Bohr-Sommerfeld integrals are elevated to primary quantum laws with a greatly enhanced realm of applicability in macro- and micro-domains. From these primary laws the Schroedinger equation can be derived. In fact, the ensemble picture is clearly invited by a (quantum restricted) solution manifold of the H-J equation, *Schroedinger's recipe for obtaining his wave equation so automatically turns into a derivation from primary laws.*

These interlocking conclusions now stand firm enough to realize that there is no avoiding of the trauma caused by putting an end to near-three quarter century of nonclassical paradigms. The ensemble option also deeply affects the later studies about completeness and Bell's theorem, because they all reason from a perspective of the questionable single system premise. A reassessment of these studies is unavoidable. Many new questions emerge about how this affects adjacent territories of physics. They don't all have to be resolved at once to further substantiate the here cited conclusions or to plan future action to map out new courses that need to be laid out.

The perhaps most puzzling question in this course of investigation may well be a psychological one: how could it have happened that the option between single system and ensemble was so single-handedly, and really without argument, decided in favor of the single system? In 1934 Popper reopened the ensemble issue. Yet, eight years after the initiation of the Schroedinger process, the nonclassical conceptualization had already a sufficiently strong foothold in physics circles preventing a more complete reversal of policy. Amidst extensive criticism about other matters, Popper's ensemble interlude was oddly approved by Einstein. These matters are discussed in some detail in ref.6.

Schroedinger's reference to Planck's 1913 ensemble-based introduction of zero-point energy, was so very close to a favoring of the ensemble over the single system that it is rather a mystery how Schroedinger himself, and everybody else, started favoring the single system. After 1925 there seems to have been a collective euphoria favoring a "final theory" by celebrating an equation of uncertain origin, that was believed to have near-unlimited potential.

There is some ground for suspicion that Schroedinger perhaps never got around to completely reading and weighing the Planck references. It raises questions to what extent the point ever came up in subsequent encounters with other physicists What exactly did Schroedinger and Bohr talk about during their not so agreeable meeting in Copenhagen? One wonders, did Schroedinger ever talk to Slater about his concerns about emerging Copenhagen views?

It might pay off to know a little more about the communication gaps of that era in the hope of learning to avoid conceivable future pitfalls. The idea that physics had its priorities the wrong way around for so long may seem outrageous. Yet it is true, and by virtue of this fact, it is another feather on the hat of Max Planck. Despite serious efforts on my part, **I have not understood why the here reported and still easily accessible work by Planck has not more deeply affected later developments.**

To better understand the present and what the future holds in stock, it may be well advised to first reassess the past. For this purpose, a truly reliable historical inquiry is called for to clarify these matters with the benefit of an improved insight. In the meantime, contemporary theory does well to reconcile itself with the idea that the unparalleled success of Schroedinger's process was really a byproduct of profiting from an asymptotic physical closeness of single system- and (dilute) ensemble behavior. This led to an unwarranted elevation of Schroedinger's equation too close to the edge of its potential to deliver.

The asymptotic closeness of ensemble and single system results was even made less conspicuous by the fact that the vast majority of experimental observation assessed by the Schroedinger process were ensemble based. True single system observations were either nonexistent or much too rare to conspicuously exhibit the limitations of the Schroedinger process as a single system tool. Here we see how Copenhagen's magic extrapolation of the Schroedinger process could last for so long. The quantum Hall effect is really one of those exceptional cases forcing an observer to make the ensemble versus single system distinction: *i.e.,* phase disordered versus phase ordered ensemble states, in which the latter assumes all qualities of an honest to goodness single system.

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