```
> restart:with(plots):
```

gencat.mws

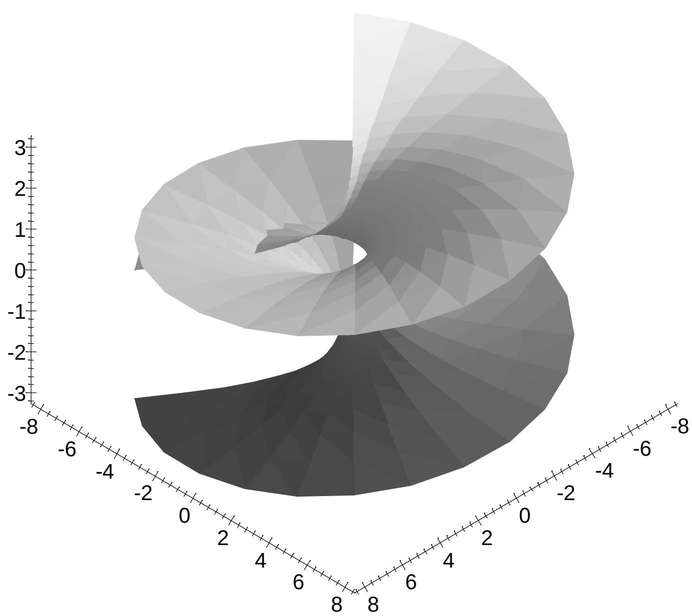
The HELI-CATENOID as a parametric surface of intersecting twisted minimal surfaces. In the limits (a=0 b=1) to (a=1,b=0), the minimal surface changes from a catenoid of revolution to a helicoid.

First, a picture:

```
> R:=[a*sinh(v)*cos(u)-b*cosh(v)*sin(u),a*sinh(v)*sin(u)+b*cosh(v)*cos(u),a*u+b*v]
;
>
```

$$R := [a\sinh(v)\cos(u) - b\cosh(v)\sin(u),\ a\sinh(v)\sin(u) + b\cosh(v)\cos(u),\ a\,u + b\,v]$$

```
> a:=.5:b:=.5:
> plot3d(R(u,v),u=-Pi..Pi,v=-Pi..Pi,shading=zgreyscale,lightmodel=light4,axes=fram
ed,style=PATCHNOGRID);
```



Now restart the computation of the gauss and mean curvature using Cartan's Repere Mobile

```
> restart:with(linalg):
Warning, new definition for norm
Warning, new definition for trace

> with(difforms):
> with(liesymm): setup(u,v):
Warning, new definition for `&^`
Warning, new definition for close
Warning, new definition for d
Warning, new definition for mixpar
Warning, new definition for wdegree
```

The Position Vector in R3 parametrized with (u,v). The example is for a Monge Surface z=g(u,v)

```
> RR:=[a*sinh(v)*cos(u)-b*cosh(v)*sin(u),a*sinh(v)*sin(u)+b*cosh(v)*cos(u),a*u+b*v
```

```
];
```

$$RR := [a \sinh(v)\cos(u) - b\cosh(v)\sin(u),\ a\sinh(v)\sin(u) + b\cosh(v)\cos(u),\ a\,u + b\,v]$$

```
> XX:=RR[1]:YY:=RR[2]:ZZ:=RR[3]:
> Yu:=diff(RR,u);
```

$$Yu := [-a\sinh(v)\sin(u) - b\cosh(v)\cos(u),\ a\sinh(v)\cos(u) - b\cosh(v)\sin(u),\ a]$$

```
> Yv:=diff(RR,v);
```

$$Yv := [a\cosh(v)\cos(u) - b\sinh(v)\sin(u),\ a\cosh(v)\sin(u) + b\sinh(v)\cos(u),\ b]$$

```
> NNU:=crossprod(Yu,Yv);
```

$$NNU := [(a\sinh(v)\cos(u) - b\cosh(v)\sin(u))\,b - a\,(a\cosh(v)\sin(u) + b\sinh(v)\cos(u)),\ a\,(a\cosh(v)\cos(u) - b\sinh(v)\sin(u)) - (-a\sinh(v)\sin(u) - b\cosh(v)\cos(u))\,b,\ (-a\sinh(v)\sin(u) - b\cosh(v)\cos(u))\,(a\cosh(v)\sin(u) + b\sinh(v)\cos(u)) - (a\sinh(v)\cos(u) - b\cosh(v)\sin(u))\,(a\cosh(v)\cos(u) - b\sinh(v)\sin(u))]$$

Scale the adjoint normal field here by rho

```
> rho:=innerprod(NNU,NNU)^(1/2);
```

$$\rho := (b^4\cosh(v)^2\sin(u)^2 + 2\,b^2\cosh(v)^2\sin(u)^2 a^2 + a^4\cosh(v)^2\sin(u)^2 + a^4\cosh(v)^2\cos(u)^2 + 2\,a^2\cosh(v)^2\cos(u)^2 b^2 + b^4\cosh(v)^2\cos(u)^2 + a^4\sinh(v)^2\sin(u)^4\cosh(v)^2 + 4\,a^2\sinh(v)^2\sin(u)^2\cosh(v)^2 b^2\cos(u)^2 + 2\,a^4\sinh(v)^2\sin(u)^2\cosh(v)^2\cos(u)^2 + 2\,a^2\sinh(v)^2\sin(u)^4\cosh(v)^2 b^2 + b^4\cosh(v)^2\cos(u)^4\sinh(v)^2 + 2\,b^2\cosh(v)^2\cos(u)^4\sinh(v)^2 a^2 + 2\,b^4\cosh(v)^2\cos(u)^2\sinh(v)^2\sin(u)^2 + a^4\sinh(v)^2\cos(u)^4\cosh(v)^2 + b^4\cosh(v)^2\sin(u)^4\sinh(v)^2)^{1/2}$$

```
> #rho:=1;
>
```

This vector (surface normal) NNU can be computed from the Adjoint Matrix operation on the two tangent vectors Yu and Yv. The basis frame utilizes this surface normal with arbitrary scaling

```
> NN:=([factor(NNU[1]),factor(NNU[2]),simplify(factor(NNU[3]))]);
```

$$NN := [-\cosh(v)\sin(u)\,(b^2 + a^2),\ \cosh(v)\cos(u)\,(b^2 + a^2),\ -a^2\sinh(v)\cosh(v) - b^2\cosh(v)\sinh(v)]$$

```
> FF:=array([[Yu[1],Yv[1],NN[1]/rho],[Yu[2],Yv[2],NN[2]/rho],[Yu[3],Yv[3],NN[3]/rh
  o]  ] );
```

$$FF := \begin{bmatrix} -a\sinh(v)\sin(u) - b\cosh(v)\cos(u) & a\cosh(v)\cos(u) - b\sinh(v)\sin(u) & -\dfrac{\cosh(v)\sin(u)\,(b^2+a^2)}{\sqrt{\%1}} \\ a\sinh(v)\cos(u) - b\cosh(v)\sin(u) & a\cosh(v)\sin(u) + b\sinh(v)\cos(u) & \dfrac{\cosh(v)\cos(u)\,(b^2+a^2)}{\sqrt{\%1}} \\ a & b & \dfrac{-a^2\sinh(v)\cosh(v) - b^2\cosh(v)\sinh(v)}{\sqrt{\%1}} \end{bmatrix}$$

$$\%1 := b^4\cosh(v)^2\sin(u)^2 + 2\,b^2\cosh(v)^2\sin(u)^2 a^2 + a^4\cosh(v)^2\sin(u)^2 + a^4\cosh(v)^2\cos(u)^2 + 2\,a^2\cosh(v)^2\cos(u)^2 b^2 + b^4\cosh(v)^2\cos(u)^2 + a^4\sinh(v)^2\sin(u)^4\cosh(v)^2 + 4\,a^2\sinh(v)^2\sin(u)^2\cosh(v)^2 b^2\cos(u)^2 + 2\,a^4\sinh(v)^2\sin(u)^2\cosh(v)^2\cos(u)^2 + 2\,a^2\sinh(v)^2\sin(u)^4\cosh(v)^2 b^2 + b^4\cosh(v)^2\cos(u)^4\sinh(v)^2 + 2\,b^2\cosh(v)^2\cos(u)^4\sinh(v)^2 a^2 + 2\,b^4\cosh(v)^2\cos(u)^2\sinh(v)^2\sin(u)^2 + a^4\sinh(v)^2\cos(u)^4\cosh(v)^2 + b^4\cosh(v)^2\sin(u)^4\sinh(v)^2$$

The Repere Mobile or FRAME MATRIX, FF. note that the frame matrix is not orthonormal.!!

```
> detFF:=simplify((det(FF)));
```

$$detFF := \sqrt{\cosh(v)^4\,(b^2 + a^2)^2}$$

For the Monge casel the deteminant is non-zero globally, hence an inverse always exists.

```
> FFINVD:=evalm(FF^(-1));
```

$FFINVD :=$

$$\left[-\frac{\sinh(v)\,a\cosh(v)\sin(u)+b\sinh(v)^2\cos(u)+\cos(u)\,b}{\cosh(v)\,\%1},\right.$$
$$\left.\frac{\sinh(v)\,a\cosh(v)\cos(u)-b\sinh(v)^2\sin(u)-\sin(u)\,b}{\cosh(v)\,\%1},\frac{a}{a^2\sinh(v)^2+b^2\sinh(v)^2+b^2+a^2}\right]$$
$$\left[\frac{a\sinh(v)^2\cos(u)-\sinh(v)\,b\cosh(v)\sin(u)+\cos(u)\,a}{\cosh(v)\,\%1},\right.$$
$$\left.\frac{a\sinh(v)^2\sin(u)+\sinh(v)\,b\cosh(v)\cos(u)+\sin(u)\,a}{\cosh(v)\,\%1},\frac{b}{a^2\sinh(v)^2+b^2\sinh(v)^2+b^2+a^2}\right]$$
$$\left[-\frac{\sqrt{\%2}\,\sin(u)}{\cosh(v)\,\%1},\frac{\sqrt{\%2}\,\cos(u)}{\cosh(v)\,\%1},-\frac{\sinh(v)\sqrt{\%2}}{\cosh(v)\,(a^2\sinh(v)^2+b^2\sinh(v)^2+b^2+a^2)}\right]$$
$$\%1 := a^2\sinh(v)^2\sin(u)^2+b^2\cos(u)^2\sinh(v)^2+b^2\cos(u)^2+a^2\sinh(v)^2\cos(u)^2+b^2\sin(u)^2\sinh(v)^2+b^2\sin(u)^2+a^2\sin(u)^2+a^2\cos(u)^2$$
$$\%2 := b^4\cosh(v)^2\sin(u)^2+2\,b^2\cosh(v)^2\sin(u)^2a^2+a^4\cosh(v)^2\sin(u)^2+a^4\cosh(v)^2\cos(u)^2+2\,a^2\cosh(v)^2\cos(u)^2b^2+b^4\cosh(v)^2\cos(u)^2+a^4\sinh(v)^2\sin(u)^4\cosh(v)^2+4\,a^2\sinh(v)^2\sin(u)^2\cosh(v)^2b^2\cos(u)^2+2\,a^4\sinh(v)^2\sin(u)^2\cosh(v)^2\cos(u)^2+2\,a^2\sinh(v)^2\sin(u)^4\cosh(v)^2b^2+b^4\cosh(v)^2\cos(u)^4\sinh(v)^2+2\,b^2\cosh(v)^2\cos(u)^4\sinh(v)^2a^2+2\,b^4\cosh(v)^2\cos(u)^2\sinh(v)^2\sin(u)^2+a^4\sinh(v)^2\cos(u)^4\cosh(v)^2+b^4\cosh(v)^2\sin(u)^4\sinh(v)^2$$

The 1-form components of the differential position vector with respect to the Basis Frame, F.

```
> dR:=innerprod(FFINVD,[d(XX),d(YY),d(ZZ)]);
```

$$dR := [\,\mathrm{d}(u), \mathrm{d}(v), 0\,]$$

```
> sigma1:=(wcollect(dR[1]));
```

$$\sigma1 := \mathrm{d}(u)$$

```
> sigma2:=wcollect(dR[2]);
```

$$\sigma2 := \mathrm{d}(v)$$

Note that sigma1 is du and sigma2 is dv for a parametric Monge surfaces!!

```
> omega:=(wcollect(dR[3]));
```

$$\omega := 0$$

Note that this term vanishes for a parametric Monge surface, hence parametric Monge surfaces exhibit no TORSION!! of the Affine type (that is there is no translational shear defects!)

```
>
>
```

Compute the Cartan Matrix of connection forms from C=[F(inverse)] times d[F]

```
> dFF:=array([[d(FF[1,1]),d(FF[1,2]),d(FF[1,3])],[d(FF[2,1]),d(FF[2,2]),d(FF[2,3])],[d(FF[3,1]),d(FF[3,2]),d(FF[3,3])]]);
```

$dFF :=$

$$\left[-a\cosh(v)\,\mathrm{d}(v)\sin(u)-a\sinh(v)\cos(u)\,\mathrm{d}(u)-b\sinh(v)\,\mathrm{d}(v)\cos(u)+b\cosh(v)\sin(u)\,\mathrm{d}(u),\right.$$
$$a\sinh(v)\,\mathrm{d}(v)\cos(u)-a\cosh(v)\sin(u)\,\mathrm{d}(u)-b\cosh(v)\,\mathrm{d}(v)\sin(u)-b\sinh(v)\cos(u)\,\mathrm{d}(u),$$
$$\left.-\frac{\sinh(v)\,\mathrm{d}(v)\sin(u)\,(b^2+a^2)}{\sqrt{\%2}}-\frac{\cosh(v)\cos(u)\,\mathrm{d}(u)\,(b^2+a^2)}{\sqrt{\%2}}+\frac{1}{2}\frac{\cosh(v)\sin(u)\,(b^2+a^2)\,\%1\,\mathrm{d}(v)}{\%2^{3/2}}\right]$$
$$\left[a\cosh(v)\,\mathrm{d}(v)\cos(u)-a\sinh(v)\sin(u)\,\mathrm{d}(u)-b\sinh(v)\,\mathrm{d}(v)\sin(u)-b\cosh(v)\cos(u)\,\mathrm{d}(u),\right.$$
$$a\sinh(v)\,\mathrm{d}(v)\sin(u)+a\cosh(v)\cos(u)\,\mathrm{d}(u)+b\cosh(v)\,\mathrm{d}(v)\cos(u)-b\sinh(v)\sin(u)\,\mathrm{d}(u),$$

$$\frac{\sinh(v)\,d(v)\cos(u)(b^2+a^2)}{\sqrt{\%2}}-\frac{\cosh(v)\sin(u)\,d(u)(b^2+a^2)}{\sqrt{\%2}}-\frac{1}{2}\frac{\cosh(v)\cos(u)(b^2+a^2)\,\%1\,d(v)}{\%2^{3/2}}\Bigg]$$

$$\Bigg[0\,,0\,,\frac{-a^2\cosh(v)^2 d(v)-a^2\sinh(v)^2 d(v)-b^2\sinh(v)^2 d(v)-b^2\cosh(v)^2 d(v)}{\sqrt{\%2}}$$

$$-\frac{1}{2}\frac{(-a^2\sinh(v)\cosh(v)-b^2\cosh(v)\sinh(v))\,\%1\,d(v)}{\%2^{3/2}}\Bigg]$$

$\%1 := 2\,b^4\cosh(v)\sin(u)^2\sinh(v)+4\,b^2\cosh(v)\sin(u)^2 a^2\sinh(v)+2\,a^4\cosh(v)\sin(u)^2\sinh(v)$
$+2\,a^4\cosh(v)\cos(u)^2\sinh(v)+4\,a^2\cosh(v)\cos(u)^2 b^2\sinh(v)+2\,b^4\cosh(v)\cos(u)^2\sinh(v)$
$+2\,a^4\sinh(v)\sin(u)^4\cosh(v)^3+2\,a^4\sinh(v)^3\sin(u)^4\cosh(v)+8\,a^2\sinh(v)\sin(u)^2\cosh(v)^3 b^2\cos(u)^2$
$+8\,a^2\sinh(v)^3\sin(u)^2\cosh(v)\,b^2\cos(u)^2+4\,a^4\sinh(v)\sin(u)^2\cosh(v)^3\cos(u)^2$
$+4\,a^4\sinh(v)^3\sin(u)^2\cosh(v)\cos(u)^2+4\,a^2\sinh(v)\sin(u)^4\cosh(v)^3 b^2+4\,a^2\sinh(v)^3\sin(u)^4\cosh(v)\,b^2$
$+2\,b^4\cosh(v)\cos(u)^4\sinh(v)^3+2\,b^4\cosh(v)^3\cos(u)^4\sinh(v)+4\,b^2\cosh(v)\cos(u)^4\sinh(v)^3 a^2$
$+4\,b^2\cosh(v)^3\cos(u)^4\sinh(v)\,a^2+4\,b^4\cosh(v)\cos(u)^2\sinh(v)^3\sin(u)^2+4\,b^4\cosh(v)^3\cos(u)^2\sinh(v)\sin(u)^2$
$+2\,a^4\sinh(v)\cos(u)^4\cosh(v)^3+2\,a^4\sinh(v)^3\cos(u)^4\cosh(v)+2\,b^4\cosh(v)\sin(u)^4\sinh(v)^3$
$+2\,b^4\cosh(v)^3\sin(u)^4\sinh(v)$

$\%2 := b^4\cosh(v)^2\sin(u)^2+2\,b^2\cosh(v)^2\sin(u)^2 a^2+a^4\cosh(v)^2\sin(u)^2+a^4\cosh(v)^2\cos(u)^2$
$+2\,a^2\cosh(v)^2\cos(u)^2 b^2+b^4\cosh(v)^2\cos(u)^2+a^4\sinh(v)^2\sin(u)^4\cosh(v)^2$
$+4\,a^2\sinh(v)^2\sin(u)^2\cosh(v)^2 b^2\cos(u)^2+2\,a^4\sinh(v)^2\sin(u)^2\cosh(v)^2\cos(u)^2$
$+2\,a^2\sinh(v)^2\sin(u)^4\cosh(v)^2 b^2+b^4\cosh(v)^2\cos(u)^4\sinh(v)^2+2\,b^2\cosh(v)^2\cos(u)^4\sinh(v)^2 a^2$
$+2\,b^4\cosh(v)^2\cos(u)^2\sinh(v)^2\sin(u)^2+a^4\sinh(v)^2\cos(u)^4\cosh(v)^2+b^4\cosh(v)^2\sin(u)^4\sinh(v)^2$

```
> cartan:=evalm(FFINVD&*dFF);
```

cartan :=

$$\Bigg[-\frac{\%5\,\%4}{\cosh(v)\,\%2}+\frac{\%3\,\%1}{\cosh(v)\,\%2}\,,-\frac{\%5\,\%7}{\cosh(v)\,\%2}+\frac{\%3\,\%6}{\cosh(v)\,\%2}\,,$$

$$-\frac{\%5\,\%12}{\cosh(v)\,\%2}+\frac{\%3\,\%11}{\cosh(v)\,\%2}+\frac{a\,\%10}{a^2\sinh(v)^2+b^2\sinh(v)^2+b^2+a^2}\Bigg]$$

$$\Bigg[\frac{\%14\,\%4}{\cosh(v)\,\%2}+\frac{\%13\,\%1}{\cosh(v)\,\%2}\,,\frac{\%14\,\%7}{\cosh(v)\,\%2}+\frac{\%13\,\%6}{\cosh(v)\,\%2}\,,$$

$$\frac{\%14\,\%12}{\cosh(v)\,\%2}+\frac{\%13\,\%11}{\cosh(v)\,\%2}+\frac{b\,\%10}{a^2\sinh(v)^2+b^2\sinh(v)^2+b^2+a^2}\Bigg]$$

$$\Bigg[-\frac{\sqrt{\%9}\,\sin(u)\,\%4}{\cosh(v)\,\%2}+\frac{\sqrt{\%9}\,\cos(u)\,\%1}{\cosh(v)\,\%2}\,,-\frac{\sqrt{\%9}\,\sin(u)\,\%7}{\cosh(v)\,\%2}+\frac{\sqrt{\%9}\,\cos(u)\,\%6}{\cosh(v)\,\%2}\,,$$

$$-\frac{\sqrt{\%9}\,\sin(u)\,\%12}{\cosh(v)\,\%2}+\frac{\sqrt{\%9}\,\cos(u)\,\%11}{\cosh(v)\,\%2}-\frac{\sinh(v)\sqrt{\%9}\,\%10}{\cosh(v)(a^2\sinh(v)^2+b^2\sinh(v)^2+b^2+a^2)}\Bigg]$$

$\%1 := a\cosh(v)\,d(v)\cos(u)-a\sinh(v)\sin(u)\,d(u)-b\sinh(v)\,d(v)\sin(u)-b\cosh(v)\cos(u)\,d(u)$
$\%2 := a^2\sinh(v)^2\sin(u)^2+b^2\cos(u)^2\sinh(v)^2+b^2\cos(u)^2+a^2\sinh(v)^2\cos(u)^2+b^2\sin(u)^2\sinh(v)^2+b^2\sin(u)^2$
$+a^2\sin(u)^2+a^2\cos(u)^2$
$\%3 := \sinh(v)\,a\cosh(v)\cos(u)-b\sinh(v)^2\sin(u)-\sin(u)\,b$
$\%4 := -a\cosh(v)\,d(v)\sin(u)-a\sinh(v)\cos(u)\,d(u)-b\sinh(v)\,d(v)\cos(u)+b\cosh(v)\sin(u)\,d(u)$
$\%5 := \sinh(v)\,a\cosh(v)\sin(u)+b\sinh(v)^2\cos(u)+\cos(u)\,b$
$\%6 := a\sinh(v)\,d(v)\sin(u)+a\cosh(v)\cos(u)\,d(u)+b\cosh(v)\,d(v)\cos(u)-b\sinh(v)\sin(u)\,d(u)$

$\%7 := a \sinh(v)\, d(v) \cos(u) - a \cosh(v) \sin(u)\, d(u) - b \cosh(v)\, d(v) \sin(u) - b \sinh(v) \cos(u)\, d(u)$

$\%8 := 2\, b^4 \cosh(v) \sin(u)^2 \sinh(v) + 4\, b^2 \cosh(v) \sin(u)^2 a^2 \sinh(v) + 2\, a^4 \cosh(v) \sin(u)^2 \sinh(v)$
$+ 2\, a^4 \cosh(v) \cos(u)^2 \sinh(v) + 4\, a^2 \cosh(v) \cos(u)^2 b^2 \sinh(v) + 2\, b^4 \cosh(v) \cos(u)^2 \sinh(v)$
$+ 2\, a^4 \sinh(v) \sin(u)^4 \cosh(v)^3 + 2\, a^4 \sinh(v)^3 \sin(u)^4 \cosh(v) + 8\, a^2 \sinh(v) \sin(u)^2 \cosh(v)^3 b^2 \cos(u)^2$
$+ 8\, a^2 \sinh(v)^3 \sin(u)^2 \cosh(v)\, b^2 \cos(u)^2 + 4\, a^4 \sinh(v) \sin(u)^2 \cosh(v)^3 \cos(u)^2$
$+ 4\, a^4 \sinh(v)^3 \sin(u)^2 \cosh(v) \cos(u)^2 + 4\, a^2 \sinh(v) \sin(u)^4 \cosh(v)^3 b^2 + 4\, a^2 \sinh(v)^3 \sin(u)^4 \cosh(v)\, b^2$
$+ 2\, b^4 \cosh(v) \cos(u)^4 \sinh(v)^3 + 2\, b^4 \cosh(v)^3 \cos(u)^4 \sinh(v) + 4\, b^2 \cosh(v) \cos(u)^4 \sinh(v)^3 a^2$
$+ 4\, b^2 \cosh(v)^3 \cos(u)^4 \sinh(v)\, a^2 + 4\, b^4 \cosh(v) \cos(u)^2 \sinh(v)^3 \sin(u)^2 + 4\, b^4 \cosh(v)^3 \cos(u)^2 \sinh(v) \sin(u)^2$
$+ 2\, a^4 \sinh(v) \cos(u)^4 \cosh(v)^3 + 2\, a^4 \sinh(v)^3 \cos(u)^4 \cosh(v) + 2\, b^4 \cosh(v) \sin(u)^4 \sinh(v)^3$
$+ 2\, b^4 \cosh(v)^3 \sin(u)^4 \sinh(v)$

$\%9 := b^4 \cosh(v)^2 \sin(u)^2 + 2\, b^2 \cosh(v)^2 \sin(u)^2 a^2 + a^4 \cosh(v)^2 \sin(u)^2 + a^4 \cosh(v)^2 \cos(u)^2$
$+ 2\, a^2 \cosh(v)^2 \cos(u)^2 b^2 + b^4 \cosh(v)^2 \cos(u)^2 + a^4 \sinh(v)^2 \sin(u)^4 \cosh(v)^2$
$+ 4\, a^2 \sinh(v)^2 \sin(u)^2 \cosh(v)^2 b^2 \cos(u)^2 + 2\, a^4 \sinh(v)^2 \sin(u)^2 \cosh(v)^2 \cos(u)^2$
$+ 2\, a^2 \sinh(v)^2 \sin(u)^4 \cosh(v)^2 b^2 + b^4 \cosh(v)^2 \cos(u)^4 \sinh(v)^2 + 2\, b^2 \cosh(v)^2 \cos(u)^4 \sinh(v)^2 a^2$
$+ 2\, b^4 \cosh(v)^2 \cos(u)^2 \sinh(v)^2 \sin(u)^2 + a^4 \sinh(v)^2 \cos(u)^4 \cosh(v)^2 + b^4 \cosh(v)^2 \sin(u)^4 \sinh(v)^2$

$$\%10 := \frac{-a^2 \cosh(v)^2 d(v) - a^2 \sinh(v)^2 d(v) - b^2 \sinh(v)^2 d(v) - b^2 \cosh(v)^2 d(v)}{\sqrt{\%9}}$$
$$- \frac{1}{2} \frac{(-a^2 \sinh(v) \cosh(v) - b^2 \cosh(v) \sinh(v))\, \%8\, d(v)}{\%9^{3/2}}$$

$$\%11 := \frac{\sinh(v)\, d(v) \cos(u)\, (b^2 + a^2)}{\sqrt{\%9}} - \frac{\cosh(v) \sin(u)\, d(u)\, (b^2 + a^2)}{\sqrt{\%9}} - \frac{1}{2} \frac{\cosh(v) \cos(u)\, (b^2 + a^2)\, \%8\, d(v)}{\%9^{3/2}}$$

$$\%12 := -\frac{\sinh(v)\, d(v) \sin(u)\, (b^2 + a^2)}{\sqrt{\%9}} - \frac{\cosh(v) \cos(u)\, d(u)\, (b^2 + a^2)}{\sqrt{\%9}} + \frac{1}{2} \frac{\cosh(v) \sin(u)\, (b^2 + a^2)\, \%8\, d(v)}{\%9^{3/2}}$$

$\%13 := a \sinh(v)^2 \sin(u) + \sinh(v)\, b \cosh(v) \cos(u) + \sin(u)\, a$

$\%14 := a \sinh(v)^2 \cos(u) - \sinh(v)\, b \cosh(v) \sin(u) + \cos(u)\, a$

The interior connection coefficients (can be Christoffel symbols on the parameter space

```
> Gamma11:=(wcollect(cartan[1,1]));
```

$$\Gamma 11 := \left(-\frac{(\sinh(v)\, a \cosh(v) \sin(u) + b \sinh(v)^2 \cos(u) + \cos(u)\, b)\, (-a \sinh(v) \cos(u) + b \cosh(v) \sin(u))}{\cosh(v)\, \%1} \right.$$
$$\left. + \frac{(\sinh(v)\, a \cosh(v) \cos(u) - b \sinh(v)^2 \sin(u) - \sin(u)\, b)\, (-a \sinh(v) \sin(u) - b \cosh(v) \cos(u))}{\cosh(v)\, \%1} \right) d(u) + \left(\right.$$
$$-\frac{(\sinh(v)\, a \cosh(v) \sin(u) + b \sinh(v)^2 \cos(u) + \cos(u)\, b)\, (-a \cosh(v) \sin(u) - b \sinh(v) \cos(u))}{\cosh(v)\, \%1}$$
$$\left. + \frac{(\sinh(v)\, a \cosh(v) \cos(u) - b \sinh(v)^2 \sin(u) - \sin(u)\, b)\, (a \cosh(v) \cos(u) - b \sinh(v) \sin(u))}{\cosh(v)\, \%1} \right) d(v)$$

$\%1 := a^2 \sinh(v)^2 \sin(u)^2 + b^2 \cos(u)^2 \sinh(v)^2 + b^2 \cos(u)^2 + a^2 \sinh(v)^2 \cos(u)^2 + b^2 \sin(u)^2 \sinh(v)^2 + b^2 \sin(u)^2$
$+ a^2 \sin(u)^2 + a^2 \cos(u)^2$

```
> Gamma12:=(wcollect(cartan[1,2]));
```

$$\Gamma 12 := \left(-\frac{(\sinh(v)\, a \cosh(v) \sin(u) + b \sinh(v)^2 \cos(u) + \cos(u)\, b)\, (-a \cosh(v) \sin(u) - b \sinh(v) \cos(u))}{\cosh(v)\, \%1} \right.$$

$$+\frac{(\sinh(v)\,a\cosh(v)\cos(u)-b\sinh(v)^2\sin(u)-\sin(u)\,b)\,(a\cosh(v)\cos(u)-b\sinh(v)\sin(u))}{\cosh(v)\,\%1}\Bigg)\mathrm{d}(u)+\Bigg($$

$$-\frac{(\sinh(v)\,a\cosh(v)\sin(u)+b\sinh(v)^2\cos(u)+\cos(u)\,b)\,(a\sinh(v)\cos(u)-b\cosh(v)\sin(u))}{\cosh(v)\,\%1}$$

$$+\frac{(\sinh(v)\,a\cosh(v)\cos(u)-b\sinh(v)^2\sin(u)-\sin(u)\,b)\,(a\sinh(v)\sin(u)+b\cosh(v)\cos(u))}{\cosh(v)\,\%1}\Bigg)\mathrm{d}(v)$$

$$\%1:=a^2\sinh(v)^2\sin(u)^2+b^2\cos(u)^2\sinh(v)^2+b^2\cos(u)^2+a^2\sinh(v)^2\cos(u)^2+b^2\sin(u)^2\sinh(v)^2+b^2\sin(u)^2+a^2\sin(u)^2+a^2\cos(u)^2$$

```
> Gamma21:=(wcollect(cartan[2,1]));
```

$$\Gamma21:=\Bigg(\frac{(a\sinh(v)^2\cos(u)-\sinh(v)\,b\cosh(v)\sin(u)+\cos(u)\,a)\,(-a\sinh(v)\cos(u)+b\cosh(v)\sin(u))}{\cosh(v)\,\%1}$$

$$+\frac{(a\sinh(v)^2\sin(u)+\sinh(v)\,b\cosh(v)\cos(u)+\sin(u)\,a)\,(-a\sinh(v)\sin(u)-b\cosh(v)\cos(u))}{\cosh(v)\,\%1}\Bigg)\mathrm{d}(u)+\Bigg($$

$$\frac{(a\sinh(v)^2\cos(u)-\sinh(v)\,b\cosh(v)\sin(u)+\cos(u)\,a)\,(-a\cosh(v)\sin(u)-b\sinh(v)\cos(u))}{\cosh(v)\,\%1}$$

$$+\frac{(a\sinh(v)^2\sin(u)+\sinh(v)\,b\cosh(v)\cos(u)+\sin(u)\,a)\,(a\cosh(v)\cos(u)-b\sinh(v)\sin(u))}{\cosh(v)\,\%1}\Bigg)\mathrm{d}(v)$$

$$\%1:=a^2\sinh(v)^2\sin(u)^2+b^2\cos(u)^2\sinh(v)^2+b^2\cos(u)^2+a^2\sinh(v)^2\cos(u)^2+b^2\sin(u)^2\sinh(v)^2+b^2\sin(u)^2+a^2\sin(u)^2+a^2\cos(u)^2$$

```
> Gamma22:=(wcollect(cartan[2,2]));
```

$$\Gamma22:=\Bigg(\frac{(a\sinh(v)^2\cos(u)-\sinh(v)\,b\cosh(v)\sin(u)+\cos(u)\,a)\,(-a\cosh(v)\sin(u)-b\sinh(v)\cos(u))}{\cosh(v)\,\%1}$$

$$+\frac{(a\sinh(v)^2\sin(u)+\sinh(v)\,b\cosh(v)\cos(u)+\sin(u)\,a)\,(a\cosh(v)\cos(u)-b\sinh(v)\sin(u))}{\cosh(v)\,\%1}\Bigg)\mathrm{d}(u)+\Bigg($$

$$\frac{(a\sinh(v)^2\cos(u)-\sinh(v)\,b\cosh(v)\sin(u)+\cos(u)\,a)\,(a\sinh(v)\cos(u)-b\cosh(v)\sin(u))}{\cosh(v)\,\%1}$$

$$+\frac{(a\sinh(v)^2\sin(u)+\sinh(v)\,b\cosh(v)\cos(u)+\sin(u)\,a)\,(a\sinh(v)\sin(u)+b\cosh(v)\cos(u))}{\cosh(v)\,\%1}\Bigg)\mathrm{d}(v)$$

$$\%1:=a^2\sinh(v)^2\sin(u)^2+b^2\cos(u)^2\sinh(v)^2+b^2\cos(u)^2+a^2\sinh(v)^2\cos(u)^2+b^2\sin(u)^2\sinh(v)^2+b^2\sin(u)^2+a^2\sin(u)^2+a^2\cos(u)^2$$

The second fundamental form or shape matrix comes from the third row of the Cartan matrix

```
> h1:=wcollect(cartan[3,1]);
```

$$h1:=\left(-\frac{\sqrt{\%2}\,\sin(u)\,(-a\sinh(v)\cos(u)+b\cosh(v)\sin(u))}{\cosh(v)\,\%1}+\frac{\sqrt{\%2}\,\cos(u)\,(-a\sinh(v)\sin(u)-b\cosh(v)\cos(u))}{\cosh(v)\,\%1}\right)$$

$$\mathrm{d}(u)+$$

$$\left(-\frac{\sqrt{\%2}\,\sin(u)\,(-a\cosh(v)\sin(u)-b\sinh(v)\cos(u))}{\cosh(v)\,\%1}+\frac{\sqrt{\%2}\,\cos(u)\,(a\cosh(v)\cos(u)-b\sinh(v)\sin(u))}{\cosh(v)\,\%1}\right)$$

$$\mathrm{d}(v)$$

$$\%1:=a^2\sinh(v)^2\sin(u)^2+b^2\cos(u)^2\sinh(v)^2+b^2\cos(u)^2+a^2\sinh(v)^2\cos(u)^2+b^2\sin(u)^2\sinh(v)^2+b^2\sin(u)^2+a^2\sin(u)^2+a^2\cos(u)^2$$

$$\%2:=b^4\cosh(v)^2\sin(u)^2+2\,b^2\cosh(v)^2\sin(u)^2\,a^2+a^4\cosh(v)^2\sin(u)^2+a^4\cosh(v)^2\cos(u)^2$$

$+ 2 a^2 \cosh(v)^2 \cos(u)^2 b^2 + b^4 \cosh(v)^2 \cos(u)^2 + a^4 \sinh(v)^2 \sin(u)^4 \cosh(v)^2$
$+ 4 a^2 \sinh(v)^2 \sin(u)^2 \cosh(v)^2 b^2 \cos(u)^2 + 2 a^4 \sinh(v)^2 \sin(u)^2 \cosh(v)^2 \cos(u)^2$
$+ 2 a^2 \sinh(v)^2 \sin(u)^4 \cosh(v)^2 b^2 + b^4 \cosh(v)^2 \cos(u)^4 \sinh(v)^2 + 2 b^2 \cosh(v)^2 \cos(u)^4 \sinh(v)^2 a^2$
$+ 2 b^4 \cosh(v)^2 \cos(u)^2 \sinh(v)^2 \sin(u)^2 + a^4 \sinh(v)^2 \cos(u)^4 \cosh(v)^2 + b^4 \cosh(v)^2 \sin(u)^4 \sinh(v)^2$

```
> gamma1:=wcollect(cartan[1,3]);
```

$$\gamma 1 := \left(\frac{(\sinh(v)\, a \cosh(v) \sin(u) + b \sinh(v)^2 \cos(u) + \cos(u)\, b) \cos(u)\, (b^2 + a^2)}{\%3 \sqrt{\%2}} - \frac{(\sinh(v)\, a \cosh(v) \cos(u) - b \sinh(v)^2 \sin(u) - \sin(u)\, b) \sin(u)\, (b^2 + a^2)}{\%3 \sqrt{\%2}} \right) \mathrm{d}(u) + \left(- (\sinh(v)\, a \cosh(v) \sin(u) + b \sinh(v)^2 \cos(u) + \cos(u)\, b) \left(- \frac{\sinh(v) \sin(u)\, (b^2 + a^2)}{\sqrt{\%2}} + \frac{1}{2} \frac{\cosh(v) \sin(u)\, (b^2 + a^2)\, \%1}{\%2^{3/2}} \right) / (\cosh(v)\, \%3) + (\sinh(v)\, a \cosh(v) \cos(u) - b \sinh(v)^2 \sin(u) - \sin(u)\, b) \left(\frac{\sinh(v) \cos(u)\, (b^2 + a^2)}{\sqrt{\%2}} - \frac{1}{2} \frac{\cosh(v) \cos(u)\, (b^2 + a^2)\, \%1}{\%2^{3/2}} \right) / (\cosh(v)\, \%3) + \frac{a \left(\frac{-a^2 \cosh(v)^2 - a^2 \sinh(v)^2 - b^2 \sinh(v)^2 - b^2 \cosh(v)^2}{\sqrt{\%2}} - \frac{1}{2} \frac{(-a^2 \sinh(v) \cosh(v) - b^2 \cosh(v) \sinh(v))\, \%1}{\%2^{3/2}} \right)}{a^2 \sinh(v)^2 + b^2 \sinh(v)^2 + b^2 + a^2} \right) \mathrm{d}(v)$$

$\%1 := 2 b^4 \cosh(v) \sin(u)^2 \sinh(v) + 4 b^2 \cosh(v) \sin(u)^2 a^2 \sinh(v) + 2 a^4 \cosh(v) \sin(u)^2 \sinh(v)$
$+ 2 a^4 \cosh(v) \cos(u)^2 \sinh(v) + 4 a^2 \cosh(v) \cos(u)^2 b^2 \sinh(v) + 2 b^4 \cosh(v) \cos(u)^2 \sinh(v)$
$+ 2 a^4 \sinh(v) \sin(u)^4 \cosh(v)^3 + 2 a^4 \sinh(v)^3 \sin(u)^4 \cosh(v) + 8 a^2 \sinh(v) \sin(u)^2 \cosh(v)^3 b^2 \cos(u)^2$
$+ 8 a^2 \sinh(v)^3 \sin(u)^2 \cosh(v) b^2 \cos(u)^2 + 4 a^4 \sinh(v) \sin(u)^2 \cosh(v)^3 \cos(u)^2$
$+ 4 a^4 \sinh(v)^3 \sin(u)^2 \cosh(v) \cos(u)^2 + 4 a^2 \sinh(v) \sin(u)^4 \cosh(v)^3 b^2 + 4 a^2 \sinh(v)^3 \sin(u)^4 \cosh(v) b^2$
$+ 2 b^4 \cosh(v) \cos(u)^4 \sinh(v)^3 + 2 b^4 \cosh(v)^3 \cos(u)^4 \sinh(v) + 4 b^2 \cosh(v) \cos(u)^4 \sinh(v)^3 a^2$
$+ 4 b^2 \cosh(v)^3 \cos(u)^4 \sinh(v) a^2 + 4 b^4 \cosh(v) \cos(u)^2 \sinh(v)^3 \sin(u)^2 + 4 b^4 \cosh(v)^3 \cos(u)^2 \sinh(v) \sin(u)^2$
$+ 2 a^4 \sinh(v) \cos(u)^4 \cosh(v)^3 + 2 a^4 \sinh(v)^3 \cos(u)^4 \cosh(v) + 2 b^4 \cosh(v) \sin(u)^4 \sinh(v)^3$
$+ 2 b^4 \cosh(v)^3 \sin(u)^4 \sinh(v)$

$\%2 := b^4 \cosh(v)^2 \sin(u)^2 + 2 b^2 \cosh(v)^2 \sin(u)^2 a^2 + a^4 \cosh(v)^2 \sin(u)^2 + a^4 \cosh(v)^2 \cos(u)^2$
$+ 2 a^2 \cosh(v)^2 \cos(u)^2 b^2 + b^4 \cosh(v)^2 \cos(u)^2 + a^4 \sinh(v)^2 \sin(u)^4 \cosh(v)^2$
$+ 4 a^2 \sinh(v)^2 \sin(u)^2 \cosh(v)^2 b^2 \cos(u)^2 + 2 a^4 \sinh(v)^2 \sin(u)^2 \cosh(v)^2 \cos(u)^2$
$+ 2 a^2 \sinh(v)^2 \sin(u)^4 \cosh(v)^2 b^2 + b^4 \cosh(v)^2 \cos(u)^4 \sinh(v)^2 + 2 b^2 \cosh(v)^2 \cos(u)^4 \sinh(v)^2 a^2$
$+ 2 b^4 \cosh(v)^2 \cos(u)^2 \sinh(v)^2 \sin(u)^2 + a^4 \sinh(v)^2 \cos(u)^4 \cosh(v)^2 + b^4 \cosh(v)^2 \sin(u)^4 \sinh(v)^2$

$\%3 := a^2 \sinh(v)^2 \sin(u)^2 + b^2 \cos(u)^2 \sinh(v)^2 + b^2 \cos(u)^2 + a^2 \sinh(v)^2 \cos(u)^2 + b^2 \sin(u)^2 \sinh(v)^2 + b^2 \sin(u)^2$
$+ a^2 \sin(u)^2 + a^2 \cos(u)^2$

```
> h2:=(wcollect(cartan[3,2]));
```

$h2 :=$

$$\left(-\frac{\sqrt{\%2}\,\sin(u)\,(-a\cosh(v)\sin(u)-b\sinh(v)\cos(u))}{\cosh(v)\,\%1}+\frac{\sqrt{\%2}\,\cos(u)\,(a\cosh(v)\cos(u)-b\sinh(v)\sin(u))}{\cosh(v)\,\%1}\right)$$

$$d(u)+$$

$$\left(-\frac{\sqrt{\%2}\,\sin(u)\,(a\sinh(v)\cos(u)-b\cosh(v)\sin(u))}{\cosh(v)\,\%1}+\frac{\sqrt{\%2}\,\cos(u)\,(a\sinh(v)\sin(u)+b\cosh(v)\cos(u))}{\cosh(v)\,\%1}\right)$$

$$d(v)$$

$$\%1 := a^2\sinh(v)^2\sin(u)^2+b^2\cos(u)^2\sinh(v)^2+b^2\cos(u)^2+a^2\sinh(v)^2\cos(u)^2+b^2\sin(u)^2\sinh(v)^2+b^2\sin(u)^2 + a^2\sin(u)^2+a^2\cos(u)^2$$

$$\%2 := b^4\cosh(v)^2\sin(u)^2+2\,b^2\cosh(v)^2\sin(u)^2a^2+a^4\cosh(v)^2\sin(u)^2+a^4\cosh(v)^2\cos(u)^2 + 2\,a^2\cosh(v)^2\cos(u)^2b^2+b^4\cosh(v)^2\cos(u)^2+a^4\sinh(v)^2\sin(u)^4\cosh(v)^2 + 4\,a^2\sinh(v)^2\sin(u)^2\cosh(v)^2b^2\cos(u)^2+2\,a^4\sinh(v)^2\sin(u)^2\cosh(v)^2\cos(u)^2 + 2\,a^2\sinh(v)^2\sin(u)^4\cosh(v)^2b^2+b^4\cosh(v)^2\cos(u)^4\sinh(v)^2+2\,b^2\cosh(v)^2\cos(u)^4\sinh(v)^2a^2 + 2\,b^4\cosh(v)^2\cos(u)^2\sinh(v)^2\sin(u)^2+a^4\sinh(v)^2\cos(u)^4\cosh(v)^2+b^4\cosh(v)^2\sin(u)^4\sinh(v)^2$$

```
> gamma2:=(wcollect(cartan[2,3]));
```

$$\gamma 2 := \left(-\frac{(a\sinh(v)^2\cos(u)-\sinh(v)\,b\cosh(v)\sin(u)+\cos(u)\,a)\cos(u)\,(b^2+a^2)}{\%3\sqrt{\%2}} - \frac{(a\sinh(v)^2\sin(u)+\sinh(v)\,b\cosh(v)\cos(u)+\sin(u)\,a)\sin(u)\,(b^2+a^2)}{\%3\sqrt{\%2}}\right)d(u)+\Bigg($$

$$(a\sinh(v)^2\cos(u)-\sinh(v)\,b\cosh(v)\sin(u)+\cos(u)\,a)\left(-\frac{\sinh(v)\sin(u)\,(b^2+a^2)}{\sqrt{\%2}}+\frac{1}{2}\frac{\cosh(v)\sin(u)\,(b^2+a^2)\,\%1}{\%2^{3/2}}\right)/(\cosh(v)\,\%3)+$$

$$(a\sinh(v)^2\sin(u)+\sinh(v)\,b\cosh(v)\cos(u)+\sin(u)\,a)\left(\frac{\sinh(v)\cos(u)\,(b^2+a^2)}{\sqrt{\%2}}-\frac{1}{2}\frac{\cosh(v)\cos(u)\,(b^2+a^2)\,\%1}{\%2^{3/2}}\right)/(\cosh(v)\,\%3)$$

$$+\frac{b\left(\frac{-a^2\cosh(v)^2-a^2\sinh(v)^2-b^2\sinh(v)^2-b^2\cosh(v)^2}{\sqrt{\%2}}-\frac{1}{2}\frac{(-a^2\sinh(v)\cosh(v)-b^2\cosh(v)\sinh(v))\,\%1}{\%2^{3/2}}\right)}{a^2\sinh(v)^2+b^2\sinh(v)^2+b^2+a^2}$$

$$\Bigg)d(v)$$

$$\%1 := 2\,b^4\cosh(v)\sin(u)^2\sinh(v)+4\,b^2\cosh(v)\sin(u)^2a^2\sinh(v)+2\,a^4\cosh(v)\sin(u)^2\sinh(v) + 2\,a^4\cosh(v)\cos(u)^2\sinh(v)+4\,a^2\cosh(v)\cos(u)^2b^2\sinh(v)+2\,b^4\cosh(v)\cos(u)^2\sinh(v) + 2\,a^4\sinh(v)\sin(u)^4\cosh(v)^3+2\,a^4\sinh(v)^3\sin(u)^4\cosh(v)+8\,a^2\sinh(v)\sin(u)^2\cosh(v)^3b^2\cos(u)^2 + 8\,a^2\sinh(v)^3\sin(u)^2\cosh(v)\,b^2\cos(u)^2+4\,a^4\sinh(v)\sin(u)^2\cosh(v)^3\cos(u)^2 + 4\,a^4\sinh(v)^3\sin(u)^2\cosh(v)\cos(u)^2+4\,a^2\sinh(v)\sin(u)^4\cosh(v)^3b^2+4\,a^2\sinh(v)^3\sin(u)^4\cosh(v)\,b^2 + 2\,b^4\cosh(v)\cos(u)^4\sinh(v)^3+2\,b^4\cosh(v)^3\cos(u)^4\sinh(v)+4\,b^2\cosh(v)\cos(u)^4\sinh(v)^3a^2 + 4\,b^2\cosh(v)^3\cos(u)^4\sinh(v)\,a^2+4\,b^4\cosh(v)\cos(u)^2\sinh(v)^3\sin(u)^2+4\,b^4\cosh(v)^3\cos(u)^2\sinh(v)\sin(u)^2 + 2\,a^4\sinh(v)\cos(u)^4\cosh(v)^3+2\,a^4\sinh(v)^3\cos(u)^4\cosh(v)+2\,b^4\cosh(v)\sin(u)^4\sinh(v)^3 + 2\,b^4\cosh(v)^3\sin(u)^4\sinh(v)$$

$$\%2 := b^4\cosh(v)^2\sin(u)^2+2\,b^2\cosh(v)^2\sin(u)^2a^2+a^4\cosh(v)^2\sin(u)^2+a^4\cosh(v)^2\cos(u)^2$$

$$+ 2\, a^2 \cosh(v)^2 \cos(u)^2\, b^2 + b^4 \cosh(v)^2 \cos(u)^2 + a^4 \sinh(v)^2 \sin(u)^4 \cosh(v)^2$$
$$+ 4\, a^2 \sinh(v)^2 \sin(u)^2 \cosh(v)^2\, b^2 \cos(u)^2 + 2\, a^4 \sinh(v)^2 \sin(u)^2 \cosh(v)^2 \cos(u)^2$$
$$+ 2\, a^2 \sinh(v)^2 \sin(u)^4 \cosh(v)^2\, b^2 + b^4 \cosh(v)^2 \cos(u)^4 \sinh(v)^2 + 2\, b^2 \cosh(v)^2 \cos(u)^4 \sinh(v)^2\, a^2$$
$$+ 2\, b^4 \cosh(v)^2 \cos(u)^2 \sinh(v)^2 \sin(u)^2 + a^4 \sinh(v)^2 \cos(u)^4 \cosh(v)^2 + b^4 \cosh(v)^2 \sin(u)^4 \sinh(v)^2$$
$$\%3 := a^2 \sinh(v)^2 \sin(u)^2 + b^2 \cos(u)^2 \sinh(v)^2 + b^2 \cos(u)^2 + a^2 \sinh(v)^2 \cos(u)^2 + b^2 \sin(u)^2 \sinh(v)^2 + b^2 \sin(u)^2 + a^2 \sin(u)^2 + a^2 \cos(u)^2$$

The abnormality for the parametric surface will show up as a non-zero entry in the [3,3] slot of the Cartan Matrix. Always an exact differental for parametric and Monge surfaces. Therefore implicit Monge surfaces will admit disclination defects (Torsion of the second kind due to rotations)

```
> Omega:=(wcollect(factor(simpform(cartan[3,3]))));
```

$$\Omega := -\frac{(\cos(u)^2 + \sin(u)^2 - 1)\, d(v) \cosh(v) \sinh(v)}{(\sinh(v)^2 \cos(u)^2 + 1 + \sinh(v)^2 \sin(u)^2)\,(\sinh(v)^2 + 1)}$$

Omega vanishes for a given normalization.

```
> wcollect(factor(simpform(d(Omega))));
```

$$0$$

```
> FROBOMEGA:=simpform(Omega&^d(Omega));
```

$$FROBOMEGA := 0$$

The coefficients of the shape matrix determined from the Cartan matrix.

```
> factor(simpform(Omega&^gamma1));
```

$$(\cos(u)^2 + \sin(u)^2 - 1) \cosh(v) \sinh(v)\, b\, ((d(u))\ \&\wedge\ (d(v))) \Big/ \Big(\sqrt{\cosh(v)^2 (\sin(u)^2 + \cos(u)^2)(\sinh(v)^2 \cos(u)^2 + 1 + \sinh(v)^2 \sin(u)^2)(b^2 + a^2)^2}\ (\sinh(v)^2 \cos(u)^2 + 1 + \sinh(v)^2 \sin(u)^2)(\sinh(v)^2 + 1)\Big)$$

```
> simplify(Omega&^gamma1);
```

$$0$$

The components of the disclination 2-form are given above. Note that they are proportional to the Square Root of the Gauss Curvature (for scaling = 1) and form the "Stream" vector relative to the gradient of the Monge function g -- a symplectic rotation

```
> shape11:=-factor(gamma1&^d(v)/d(u)&^d(v));
```

$$shape11 := -\frac{b}{\sqrt{\cosh(v)^2 (\sin(u)^2 + \cos(u)^2)(\sinh(v)^2 \cos(u)^2 + 1 + \sinh(v)^2 \sin(u)^2)(b^2 + a^2)^2}}$$

```
> shape12:=-factor(gamma1&^d(u)/d(v)&^d(u));
```

$$shape12 := \frac{a \cosh(v)^2}{\sqrt{\cosh(v)^2 (\sin(u)^2 + \cos(u)^2)(\sinh(v)^2 \cos(u)^2 + 1 + \sinh(v)^2 \sin(u)^2)(b^2 + a^2)^2}\ (\sinh(v)^2 + 1)}$$

```
> shape21:=-factor(gamma2&^d(v)/d(u)&^d(v));
```

$$shape21 := \frac{a}{\sqrt{\cosh(v)^2 (\sin(u)^2 + \cos(u)^2)(\sinh(v)^2 \cos(u)^2 + 1 + \sinh(v)^2 \sin(u)^2)(b^2 + a^2)^2}}$$

```
> shape22:=-factor(gamma2&^d(u)/d(v)&^d(u));
```

$$shape22 := \frac{b \cosh(v)^2}{\sqrt{\cosh(v)^2 (\sin(u)^2 + \cos(u)^2)(\sinh(v)^2 \cos(u)^2 + 1 + \sinh(v)^2 \sin(u)^2)(b^2 + a^2)^2}\ (\sinh(v)^2 + 1)}$$

```
>
> SHAPE:=array([[shape11,shape12],[shape21,shape22]]):
```

```
> HH:=simplify(trace(SHAPE)/2):
> print(`Mean Curvature is `,HH);
```

$$\textit{Mean Curvature is}\ ,\ 0$$

```
> KK:=simplify(det(SHAPE)):
> print(`Gauss Curvature is `,KK);
```

$$\textit{Gauss Curvature is}\ ,\ -\frac{1}{\cosh(v)^4\,(b^2+a^2)}$$

```
>
```

Note that the scaling of the normal or adjoint vector is a common factor of the formulas for the mean curvature and the Gauss curvature. Note the appearance of the Hessian of the Monge function.

The induce metric appears below

```
> GUN:=innerprod(transpose(FF),FF);
```

$GUN :=$

$$\left[a^2\sinh(v)^2\sin(u)^2+b^2\cos(u)^2\cosh(v)^2+a^2\sinh(v)^2\cos(u)^2+b^2\cosh(v)^2\sin(u)^2+a^2\ ,\right.$$
$$a\sinh(v)^2\sin(u)^2\,b-b\cosh(v)^2\cos(u)^2\,a+a\sinh(v)^2\cos(u)^2\,b-b\cosh(v)^2\sin(u)^2\,a+a\,b\ ,$$
$$\left.\frac{\cosh(v)\,(b^2+a^2)\,a\sinh(v)\,\%2}{\sqrt{\%1}}\right]$$

$$\left[a\sinh(v)^2\sin(u)^2\,b-b\cosh(v)^2\cos(u)^2\,a+a\sinh(v)^2\cos(u)^2\,b-b\cosh(v)^2\sin(u)^2\,a+a\,b\ ,\right.$$
$$a^2\cos(u)^2\cosh(v)^2+b^2\sin(u)^2\sinh(v)^2+a^2\cosh(v)^2\sin(u)^2+b^2\cos(u)^2\sinh(v)^2+b^2\ ,$$
$$\left.\frac{\cosh(v)\,(b^2+a^2)\,b\sinh(v)\,\%2}{\sqrt{\%1}}\right]$$

$$\left[\frac{\cosh(v)\,(b^2+a^2)\,a\sinh(v)\,\%2}{\sqrt{\%1}}\ ,\ \frac{\cosh(v)\,(b^2+a^2)\,b\sinh(v)\,\%2}{\sqrt{\%1}}\ ,\right.$$
$$\left.\frac{\sin(u)^2+\cos(u)^2+\sinh(v)^2}{\sin(u)^2+\cos(u)^2+\sinh(v)^2\sin(u)^4+2\sinh(v)^2\sin(u)^2\cos(u)^2+\sinh(v)^2\cos(u)^4}\right]$$

$$\%1 := b^4\cosh(v)^2\sin(u)^2+2\,b^2\cosh(v)^2\sin(u)^2\,a^2+a^4\cosh(v)^2\sin(u)^2+a^4\cosh(v)^2\cos(u)^2$$
$$+\,2\,a^2\cosh(v)^2\cos(u)^2\,b^2+b^4\cosh(v)^2\cos(u)^2+a^4\sinh(v)^2\sin(u)^4\cosh(v)^2$$
$$+\,4\,a^2\sinh(v)^2\sin(u)^2\cosh(v)^2\,b^2\cos(u)^2+2\,a^4\sinh(v)^2\sin(u)^2\cosh(v)^2\cos(u)^2$$
$$+\,2\,a^2\sinh(v)^2\sin(u)^4\cosh(v)^2\,b^2+b^4\cosh(v)^2\cos(u)^4\sinh(v)^2+2\,b^2\cosh(v)^2\cos(u)^4\sinh(v)^2\,a^2$$
$$+\,2\,b^4\cosh(v)^2\cos(u)^2\sinh(v)^2\sin(u)^2+a^4\sinh(v)^2\cos(u)^4\cosh(v)^2+b^4\cosh(v)^2\sin(u)^4\sinh(v)^2$$
$$\%2 := \cos(u)^2+\sin(u)^2-1$$

```
>
>
>
```