

1 Comments on Rotational Magnetism

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1.1 Introductory remarks

First, some more history. As I remember, Barnett did a lot of early work on the ratios of angular momentum to magnetic moments (see Phys Rev 6, 1915, way before Blackett). Blackett was a leader in British Operations Research during WWII. For details on Blackett, see <http://www.nobel.se/laureates/physics-1948-1-bio.html>.

Rotational motion leading to electromagnetism (magnetic moment) is called the Barnett effect. Electro-magnetism inducing rotation is the Einstein-de Haas effect. The magnetic field of the sun and the planets has also been "explained" by the Parker dynamo theory of rotating currents. The fact that the magnetic symmetry axis is not always aligned to the rotation axis often leads to conceptual difficulties with the simple view, but then precession may explain other things.

Why rotation of neutral matter produces magnetism is still a mystery to me. Part of the mystery is that classical EM theory, as taught today, claims that magnetism is produced by currents. However, it is easy to construct vector potentials that produce \mathbf{B} fields, where $\text{curl}\mathbf{B} = \mathbf{J} = 0$. For example, consider

$$\mathbf{A} = y\mathbf{i} - x\mathbf{j}, \quad (1)$$

$$\mathbf{B} = -2\mathbf{k}, \quad (2)$$

$$\text{curl}\mathbf{B} = \mu\mathbf{J} = 0. \quad (3)$$

The BA idea that there are effects due to vector potentials that do not produce \mathbf{B} fields, should, perhaps, be augmented by the fact that there are effects due to vector potentials that produce \mathbf{B} fields, but do not produce currents.

Note that \mathbf{A} above, if interpreted as a velocity field, represents rigid body rotation.

HOWEVER to claim that (as deduced by simple algebraic manipulations between definitions of angular momentum and magnetic moments when combined with the Barnett effect) the formula,

$$\Gamma m^2 = ke^2 = (1/4\pi\epsilon_0)e^2 \quad (4)$$

is a fundamental law, needs much more justification. (MKS units throughout).

Other alternative manipulations are discussed below.

1.2 The fine structure constant

First recall that Gustav Mie was interested in the possibility that the Coulomb energy of the electron was approximately the rest mass energy. The assumption establishes the classical electron radius.

$$ke^2/r_e = m_e c^2 \quad (5)$$

$$\Rightarrow r_e = ke^2/m_e c^2 \approx 2.81...10^{-15} \text{ meters} \quad (6)$$

Then when this number is compared to the Compton wavelength of the electron, the result is exactly the fine structure constant.

$$\lambda_e/r_e = (2\pi\hbar/m_e c)/r_e \Rightarrow \quad (7)$$

$$\alpha^{-1} = (2\pi\hbar/m_e c)(m_e c^2)/ke^2 = 2\pi\hbar c/(ke^2) \approx 137 \quad (8)$$

The Compton wavelength is 137 times bigger than the classical electron radius. The most remarkable thing, however, is that if the fine structure constant is viewed as a sort of "fractal exponent", then the next "scale" is equal to the Bohr orbit (precisely):

$$r_{Bohr} = \{2\pi\hbar c/(ke^2)\}^2 r_e = \{2\pi\hbar c/(ke^2)\} \lambda_e \quad (9)$$

There are many physical objects, including astronomical objects, that have scales that can be put into a "fractal" relationship to powers of the fine structure constant (I think this wild idea was published by Norman Frenkel).

Now consider the same concept for equating the gravitational potential energy to the rest mass energy of an object that will be provisionally labeled m_π . So assume that

$$\Gamma m_\pi M/r_\pi \Rightarrow \Gamma(m_\pi)^2/r_\pi = m_\pi c^2 \quad (10)$$

$$r_\pi = \Gamma(m_\pi)/c^2, \quad (11)$$

and again construct the ratio of the gravitational radius with respect to the Compton wavelength, to define the gravitational fine structure constant:

$$\lambda_\pi/r_\pi = (2\pi\hbar/m_\pi c)/\Gamma(m_\pi)/c^2 = 2\pi\hbar c/(\Gamma m_\pi^2). \quad (12)$$

Now if the **assumption** is made that the Gravitational fine structure constant is the same as the Electromagnetic fine structure constant, then one obtains the rule (Sirag eq. 1)

$$\Gamma m_\pi^2 = ke^2 \quad (13)$$

Why these two "fine structure constants" should be the same is a miracle, and it is not obvious that the ansatz should be true universally.

However, the "derivation" above does not require Kaluza-Klein 5D notions, and does not invoke angular momentum and magnetic moment assumptions, nor assumptions about photons. It depends upon perhaps one of the most important fundamental dimensionless numbers in science, which is known to be a projective invariant over large portions of the astronomical universe.

By working "the numbers" a mass of about 1.8×10^{-8} kg can be computed for when a charged mass has a gravitational radius about equal to the electromagnetic radius. I do not know that any of this has real meaning. The mass is smaller than the Planck mass and therefore has a larger Compton length.

1.3 Impedances and the fine structure constant

Less well known is the fact that the fine structure constant is the ratio of two fundamental "impedances". To quote from the appendix to Kiehn, *Int. J. Mod. Phys B* Vol. 5 p.1779-17790 (1991), where the ideas were mostly due to E. J. Post:

The notion of impedance is an engineering concept, which for a long time has not played a major role in fundamental physics. The idea of a free space impedance goes back to the early days of radio transmission, radar and waveguides. Targets of reduced radar cross section have recently added to its relevance. In this appendix a few remarks will be made about the relationships between the radiation impedance, Z_0 , of free space, the Hall impedance, Z_{Hall} , studied above, and the fine structure constant. In its traditional cgs rendition the fine structure constant,

$$\alpha = 2\pi e^2/hc = 1/137.0360411, \tag{A1}$$

is a dimensionless number determined by the quantum of elementary charge, e , the quantum of action, $h/2\pi$, and the speed of light in matter free space. The fine constant was introduced by Sommerfeld to account for certain relativistic effects in the spectra of hydrogen. In (A1) the elementary charge, e , and the action, $h/2\pi$, are known to be good space-time invariants under all diffeomorphisms. By contrast, the speed of light is only a Lorentz invariant, but not a general space-time invariant. The numerical value of (A1) is a recommended value, which from 1980 onwards has only changed in the last three decimal places. [12] The data include laboratory measurements [13] using Josephson and quantum Hall effects, as well as values deduced from the spectra of far away stellar objects [14] that are subject to very large stellar red shifts. It would appear that α is at least a projective invariant of the universe. Right from the beginning α became surrounded by a lore of mysticism, which has manifested itself in finding independent calculational recipes for the value given in (A1). From the early efforts of Eddington to the more recent attempts by Wyler [15], these calculations have improved to the point of reproducing some six decimal places of the generally accepted measured values. Yet these recipes so far have not exhibited a sufficiently transparent and acceptable rationale to convince the world of physics at large. Their relevance remains, for the time being, in the eye of the beholder. In this appendix no magnitude prediction of α is intended. Instead, the observation is made that when written in terms of the MKS system of units, the fine structure constant becomes,

$$\alpha = 2\pi e^2 / 4\pi\epsilon hc = 1/2(\mu/\epsilon)^{1/2} / (h/e^2) \quad (A2)$$

This formula demonstrates that α is a ratio of two fundamental impedances, the free-space impedance,

$$Z_0 = (\mu/\epsilon)^{1/2} = 376.730313 \text{ } , \quad (A3a)$$

and the Hall impedance,

$$Z_{Hall} = h/e^2 = 25812.81491 \text{ } . \quad (A3b)$$

The relation between the quantum mechanical entities and the free space impedance as given by the equation,

$$\alpha = 1/2(Z_0/Z_{Hall}), \quad (A4)$$

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The radiation impedance is a projective invariant (does not depend upon Lorentz invariance) and the Hall impedance is a topological number. In this sense, the formula A4 has much deeper meaning than anything derived from Lorentz invariance arguments.

The complete article is mostly about 3 types of Superconductivity and how Topological Torsion comes into play in such an arena. It can be downloaded from

<http://www22.pair.com/csdc/pdf/sc3.pdf>