Dear Dr Hehl

In going over your ps file ”birkkiehn ” I am glad to see that you have emphasized that the gauge transformations include the non-trivial closed but not exact parts, γ

$A \Rightarrow A' = A + df + \gamma.$

It is a common error to exclude this idea.

I am also pleased to see that you are including the ideas of exterior differential systems. The best reference to these ideas is ”Exterior Differntial Systems ”R. L. Bryant, S.S. Chern, R. B. Gardner, H.L. Goldschmidt, P. A. Griffiths.

Springer Verlag (about 1998?)

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However, now I must say that the topological features of A due to γ are measureable.

Your statement that the potentials are not measurable is not correct.

I give examples in the next few paragraphs.

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Even in regions where $dA = 0,$ it is the closed integrals of $\oint \gamma$ that are measureable and give substance to the B-A effect. In a topological sense, thiese numbers are measureable as ”holes ” in a surface. The measureable topological numbers (period integrals) are the basis of flux quantization. It is not the magnetic field through a surface that is being measured, it is the circulation integral around a topological defect (the hole) that yields the measurable value.

In hydrodynamics, 2- dimensional potential flow around an obstacle does not produce drag or lift. Hoever, the addition of non-zero Kelvin circulation to the potential flow produces lift, but not drag. The Joukowski method investigates flows about a wing (the hole) where the flow has zero vorticity! It is a topological effect that causes the circulation that leads to Lift on a 2D wing.

The velocity field is given by $V = df + \gamma$ with $dV = 0.$ It is the non-zero circulation $\oint \gamma$ that gives the Joukowski lift. That is certainly a measurable quantity. A remarkable fact is that the lift coefficient of a given airfoil depends upon the interaction between the potential flow df and the circulation induced by $\gamma.$ A round cylinder produces lift if it is rotating such as to produce circulation. An airfoil shape induces the circulation because of a sharp trailing edge. However, it is remarkable that a given shape will produce a certain lift independent dependent on the angle of attack, but independent of its direction of motion. That is, you could fly an airplane with a wing such that the sharp edge is the leading edge, and the blunt end is the trailing edge and the lift performance would be the same. (NAsa has wind tunnel tests that demonstrate this non-intuitive idea.)

The moral is that the potentials do have measurable content through the period integrals $\oint \gamma.$

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I believe that measurements relate to topological features, such as period integrals and limit points. The field intensities of $F = dA$ are measureable for they are limit points of the topology induced by the exterior derivative, which is a Kuratowski closure operator.

Similarly the charge-current densities are limit points of the field excitations, $J = dG,$ and are therefor measurable. However, I do not know of any direct measurement methods of either D or H that do not involve some form of closed cycle. Charge is the period integral of G and exists in domains where $J = 0.$ The concept of charge density is a measurement of the limit sets again. If the domains are simply connected, then the closed integrals are over boundaries and there must be interior currents and charge densities to satisfy Stokes theorem. However, the integral over the closed but not exact parts of G vanish if the integration chain is a boundary.

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Now you can measure (locally) the forces on test charges on matter and currents in wires, but these involve the field intensities F and not the excitations, G. The field intensities are limit sets, hence measurable; the field excitations are not. I believe this to be the message of Arnold Sommerfeld.

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I find your 1+3 decomposition to be confusing. It essentially subsumes that the domain of interest is foliated (simply connected?). It is not clear that this is always possible. In effect it appears to me that if true, then the assumption is latent that the 1-form of potentials, A, is of Pfaff dimension 2 at most. It follows that the 3-form of topological torsion, $A\hat{}F \Rightarrow 0$. The Frobenius conditions of unique integrability are then satisfied.

This limitation is, of course, the standard treatment that appears in most text books, for there exists a syndrome amongst most scientists to assume that given initial data the only useful theories are those that produce unique solutions. When $A\hat{}F \neq 0$ the integrability conditions are not satisfied, and the solutions, if they exist, are NOT unique. (check my article on envelopes). That is just the point of why and when the 3-forms $A\hat{}G$ and $A\hat{}F$ are important.

I have talked with Chern about the 3-form $A\hat{}F$, and its relationship to the Chern-Simons form. I am not sure that I approve of calling the electro-dynamic 3-form $A\hat{}F$ a Chern-Simons form (although there is certainly a connection to the Chern - Simon Lagrangian 3D field theories) The Chern-Simons concepts (with group valued 1-forms A) were published in 1978 as applied to differential geometry of the connection coefficients.. The applications to field theory appeared about 1985. I used the idea that $A\hat{}F = 0$ described Pfaff dimension 2 hydrodynamics (streamline flows and Stokes flows) and the idea that the turbulent state implied that $A\hat{}F \neq 0$ about 1976 (in an old NASA report) The form $A\hat{}dA$ appears in several of my articles in the 1976-1977 era. I knew nothing of Chern-Simons theory at the time.

My objection to the use of the Chern-Simons name for the 3-form $A\hat{}F$ has nothing to do with credits; the fact is that the Chern-Simons name has a tendency to obscure the relationship of the 3-form $A\hat{}F \neq 0$ to the failure of the Frobenius integrability theorem, and to the very important idea that the topology induced by the Pfaff sequence is a disconnected topology when $A\hat{}F \neq 0$, and a connected topology when $A\hat{}F = 0$. This result led to the concept that the decay of turbulence could be studied with continuous methods, but the creation of turbulence requires a discontinuous process.

Moreover, it is important to realize that the direction field generated by the 3-form $A\hat{}F$ in 4 dimensional space time is the direction field for any process that will induce thermodynamic irreversibility, if $d(A\hat{}F) \neq 0$. In EM theory this means that $\mathbf{E} \bullet \mathbf{B} \neq 0$ is a necessary condition for thermodynamic irreversibility.

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I have translated the Original article on the $A\hat{}H$ 3-form (1969) for you into pdf format. Recall the new notation is $A\hat{}G$.

see

http://www22.pair.com/csdc/pdf/inttrans.pdf

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Regards

RMK