```
> restart;
> with(liesymm):with(linalg):setup(x,y,z,t,s,Ct);deforms(a=const,b=const,c=const,d=const,p=const,n=const,k=const,omega=const,e=const);
Warning, new definition for close
Warning, new definition for norm
Warning, new definition for trace
```

$$[x, y, z, t, s, Ct]$$

$$\mathrm{deforms}(a = const, b = const, c = const, d = const, p = const, n = const, k = const, \omega = const, e = const)$$

HOLDER NORMS 4D examples including the Hopf Map

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Updated 11/26/2000-12/02/2000-12/22/2000

***

An Implicit 4D surface linear in Z:

The similarity invariants for the Jacobian matrix of the surface normal scaled by the Holder norm with a=b=c=e=1,n=1,p=2

(In 3D, the result yields the classic partial differential equations for the mean and Gauss curvature.)

```
> Phi:=z-f(x,y,t);
```

$$\Phi := z - \mathrm{f}(x, y, t)$$

```
> A1:=diff(Phi,x);A2:=diff(Phi,y);A3:=diff(Phi,z);A4:=+diff(Phi,t);
```

$$A1 := -\left(\frac{\partial}{\partial x}\mathrm{f}(x, y, t)\right)$$

$$A2 := -\left(\frac{\partial}{\partial y}\mathrm{f}(x, y, t)\right)$$

$$A3 := 1$$

$$A4 := -\left(\frac{\partial}{\partial t}\mathrm{f}(x, y, t)\right)$$

```
> A:=[A1,A2,A3];phi:=-A4;
```

$$A := \left[-\left(\frac{\partial}{\partial x}\mathrm{f}(x, y, t)\right), -\left(\frac{\partial}{\partial y}\mathrm{f}(x, y, t)\right), 1\right]$$

$$\phi := \frac{\partial}{\partial t}\mathrm{f}(x, y, t)$$

```
> B:=curl(A,[x,y,z]);EP:=evalm(-grad(phi,[x,y,z]));EV:=-[diff(A[1],t),diff(A[2],t),diff(A[3],t)];E:=evalm(EV+EP);Parity:=innerprod(E,B);Torsion_current:=evalm(crossprod(E,A)+evalm(B*phi));Helicity:=innerprod(A,B);
>
```

$$B := [0, 0, 0]$$

$$EP := \left[-\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x, y, t)\right), -\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x, y, t)\right), 0\right]$$

$$EV := \left[\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x, y, t), \frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x, y, t), 0\right]$$

$$E := [0, 0, 0]$$

$$Parity := 0$$

$$Torsion\_current := [0, 0, 0]$$

$$Helicity := 0$$

The gradient normal produces zero E and B fields.

```
> lambda:=(A[1]^2+A[2]^2+A[3]^2+e*phi^2)^(1/2);
```

>

$$\lambda := \sqrt{\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2 + \left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2 + 1 + e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2}$$

```
> NA:=evalm([A1,A2,A3,phi]/lambda);
```

$$NA := \left[-\frac{\frac{\partial}{\partial x}\mathrm{f}(x,y,t)}{\sqrt{\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2 + \left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2 + 1 + e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2}}, -\frac{\frac{\partial}{\partial y}\mathrm{f}(x,y,t)}{\sqrt{\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2 + \left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2 + 1 + e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2}}, \frac{1}{\sqrt{\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2 + \left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2 + 1 + e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2}}, \frac{\frac{\partial}{\partial t}\mathrm{f}(x,y,t)}{\sqrt{\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2 + \left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2 + 1 + e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2}} \right]$$

```
> BB:=curl([NA[1],NA[2],NA[3]],[x,y,z]);EEP:=evalm(-grad(NA[4],[x,y,z]));EEV:=-[diff(NA[1],t),diff(NA[2],t),diff(NA[3],t)];E:=evalm(EV+EP);Parity:=innerprod(E,B);Torsion_current:=evalm(crossprod(E,A)+evalm(B*phi));Helicity:=innerprod(A,B);
```

$$BB := \left[-\frac{1}{2}\frac{2\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right) + 2\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right) + 2e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)}{\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2 + \left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2 + 1 + e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^{(3/2)}}, \frac{1}{2}\frac{2\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right) + 2\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right) + 2e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)}{\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2 + \left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2 + 1 + e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^{(3/2)}}, \frac{1}{2}\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(2\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right) + 2\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right) + 2e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)\right) \Big/ \left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2 + \left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2 + 1 + e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^{(3/2)} - \frac{1}{2}\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(2\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right) + 2\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right) + 2e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)\right) \Big/$$

$$\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^{(3/2)}\Bigg]$$

$$EEP := \Bigg[\frac{1}{2}\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)$$

$$\left(2\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)+2\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)+2\,e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)\right) \Big/$$

$$\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^{(3/2)}$$

$$-\frac{\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)}{\sqrt{\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2}},\ \frac{1}{2}\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)$$

$$\left(2\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)+2\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)+2\,e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)\right) \Big/$$

$$\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^{(3/2)}$$

$$-\frac{\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)}{\sqrt{\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2}},\ 0\Bigg]$$

$$EEV := \Bigg[-\frac{1}{2}\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)$$

$$\left(2\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)+2\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)+2\,e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)\right) \Big/$$

$$\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^{(3/2)}$$

$$+\frac{\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)}{\sqrt{\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2}},\ -\frac{1}{2}\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)$$

$$\left(2\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)+2\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)+2\,e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)\right) \Big/$$

$$\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^{(3/2)}$$

$$+\frac{\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)}{\sqrt{\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2}},$$

$$\left.\frac{1}{2}\,\frac{2\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)+2\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)+2\,e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)}{\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^{(3/2)}}\right]$$

$$E := [0,0,0]$$

$$Parity := 0$$

$$Torsion\_current := [0,0,0]$$

$$Helicity := 0$$

```
>
>
>
> JAC:=jacobian(NA,[x,y,z,t]):
```

The printing of the 4x4 JAcobian matrix has been suppressed.

```
>
> MEAN_CURVATURE:=factor(trace(JAC)/2);
```

$$MEAN\_CURVATURE := \frac{1}{2}\Bigg(2\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)$$
$$+\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)-\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2-\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)$$
$$-\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)$$
$$-\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2-\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)-\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2$$
$$-\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)-\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)$$
$$+\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)\Bigg)\Bigg/$$
$$\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^{(3/2)}$$

Note the classic formula for the mean curvature of an implicit surface in 3D xyt space is obtained.

```
> S2:=factor(trace(innerprod(JAC,JAC))):
  Gauss:=factor(-(1/2)*(-trace(JAC)*trace(JAC)+S2));
```

$$Gauss := \Bigg(\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2-\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)^2e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2$$
$$-\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)$$
$$-\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)$$
$$+\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)$$
$$+\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)$$
$$+\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)$$
$$+\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)$$

$$-\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)$$
$$-\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)$$
$$-2\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)+\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)^2$$
$$-\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)^2+2\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)$$
$$+\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)-\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)+\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)^2\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2$$
$$+\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)^2\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2-\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2$$
$$\left.-\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)-\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2\right)\Bigg/$$
$$\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^2$$

```
> ADJAC:=adjoint(JAC):
> ADJOINT_CURVATURE:=factor(trace(ADJAC));
```

$$ADJOINT\_CURVATURE := \left(-\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)^2-\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)^2\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)\right.$$
$$+\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)-\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)^2\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)$$
$$\left.+2\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)\right)\Bigg/$$
$$\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^{(5/2)}$$

```
>
```

Note that the classic formula for the ADJOINT Gauss curvature of an implicit surface in 3D xyt space is obtained

```
> CurrentJ:=innerprod(ADJAC,NA);Interaction:=innerprod(CurrentJ,NA);DivJ:=factor(diverge(CurrentJ,[x,y,z,t]));
```

$$CurrentJ := \left[0,\,0,\,\left(-\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)^2-\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)^2\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)\right.\right.$$
$$+\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)-\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)^2\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)$$
$$\left.+2\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)\right)\Bigg/$$
$$\left.\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^2,\,0\right]$$

$$Interaction := \left(-\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)^2-\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)^2\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)\right.$$
$$+\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)-\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)^2\left(\frac{\partial^2}{\partial t^2}\mathrm{f}(x,y,t)\right)$$
$$\left.+2\left(\frac{\partial^2}{\partial x\,\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial t}\mathrm{f}(x,y,t)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{f}(x,y,t)\right)\right)\Bigg/$$

$$\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,t)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,t)\right)^2+1+e\left(\frac{\partial}{\partial t}\mathrm{f}(x,y,t)\right)^2\right)^{(5/2)}$$

$$DivJ := 0$$

It is now apparent that the interaction between the potentials and the conserved current is equal to the Adjoint curvature.

```
>
> Adjointcurv:=simplify(factor(trace(ADJAC))):Mean:=factor(trace(JAC)):Net:=factor(Adjointcurv-Interaction):Gnet:=factor(subs(a=1,b=1,c=1,p=2,n=1,Net));
```

$$Gnet := 0$$

```
> Vorticity:=curl([A[1],A[2],A[3]],[x,y,z]);Helicity:=innerprod([A[1],A[2],A[3]],Vorticity);Diss:=innerprod([CurrentJ[1],CurrentJ[2],CurrentJ[3]],E);
```

$$Vorticity := [0, 0, 0]$$

$$Helicity := 0$$

$$Diss := 0$$

Note that the formulas are valid for any signature e

```
>
>
```

An Implicit 4D surface linear in t:

The similarity invariants for the Jacobian matrix of the surface normal scaled by the Holder norm with a=b=c=1,n=1,p=2
fields the classic partial differential equations for the mean and Gauss curvature.

```
> Phi:=-t+f(x,y,z);
```

$$\Phi := -t+\mathrm{f}(x,y,z)$$

```
> A1:=diff(Phi,x);A2:=diff(Phi,y);A3:=diff(Phi,z);A4:=diff(Phi,t);
```

$$A1 := \frac{\partial}{\partial x}\mathrm{f}(x,y,z)$$

$$A2 := \frac{\partial}{\partial y}\mathrm{f}(x,y,z)$$

$$A3 := \frac{\partial}{\partial z}\mathrm{f}(x,y,z)$$

$$A4 := -1$$

```
> A:=[A1,A2,A3];phi:=-A4;
```

$$A := \left[\frac{\partial}{\partial x}\mathrm{f}(x,y,z), \frac{\partial}{\partial y}\mathrm{f}(x,y,z), \frac{\partial}{\partial z}\mathrm{f}(x,y,z)\right]$$

$$\phi := 1$$

```
> B:=curl(A,[x,y,z]);EP:=evalm(-grad(phi,[x,y,z]));EV:=-[diff(A[1],t),diff(A[2],t),diff(A[3],t)];E:=evalm(EV+EP);Parity:=innerprod(E,B);Torsion_current:=evalm(crossprod(E,A)+evalm(B*phi));Helicity:=innerprod(A,B);
>
```

$$B := [0, 0, 0]$$

$$EP := [0, 0, 0]$$

$$EV := [0, 0, 0]$$

$$E := [0, 0, 0]$$

$$Parity := 0$$

$$Torsion\_current := [0, 0, 0]$$

$$Helicity := 0$$

Zero E and B fields as expected for a gradient norm.

```
> lambda:=(A[1]^2+A[2]^2+A[3]^2+A4^2)^(1/2);
>
>
```

$$\lambda := \sqrt{\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1}$$

```
> NA:=evalm([A1,A2,A3,phi]/lambda);
```

$$NA := \left[\frac{\frac{\partial}{\partial x} f(x, y, z)}{\sqrt{\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1}},\right.$$
$$\frac{\frac{\partial}{\partial y} f(x, y, z)}{\sqrt{\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1}},$$
$$\frac{\frac{\partial}{\partial z} f(x, y, z)}{\sqrt{\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1}},$$
$$\left.\frac{1}{\sqrt{\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1}}\right]$$

```
> BB:=curl([NA[1],NA[2],NA[3]],[x,y,z]);EEP:=evalm(-grad(NA[4],[x,y,z]));EEV:=-[diff(NA[1],t),diff(NA[2],t),diff(NA[3],t)];EE:=evalm(EEV+EEP);Parity:=innerprod(EE,BB);Torsion_current:=evalm(crossprod(EE,[NA[1],NA[2],NA[3]])+evalm(BB*NA[4]));Helicity:=innerprod([NA[1],NA[2],NA[3]],BB);
```

$$BB := \left[-\frac{1}{2}\left(\frac{\partial}{\partial z} f(x, y, z)\right)\right.$$
$$\left(2\left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y^2} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x, y, z)\right)\right)$$
$$\Big/ \left(\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1\right)^{(3/2)} + \frac{1}{2}\left(\frac{\partial}{\partial y} f(x, y, z)\right)$$
$$\left(2\left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z^2} f(x, y, z)\right)\right) \Big/$$
$$\left(\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1\right)^{(3/2)}, -\frac{1}{2}\left(\frac{\partial}{\partial x} f(x, y, z)\right)$$
$$\left(2\left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z^2} f(x, y, z)\right)\right) \Big/$$
$$\left(\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1\right)^{(3/2)} + \frac{1}{2}\left(\frac{\partial}{\partial z} f(x, y, z)\right)$$
$$\left(2\left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial x^2} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x, y, z)\right)\right)$$
$$\Big/ \left(\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1\right)^{(3/2)}, -\frac{1}{2}\left(\frac{\partial}{\partial y} f(x, y, z)\right)$$
$$\left(2\left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial x^2} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x, y, z)\right)\right)$$

$$\Big/ \left(\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1\right)^{(3/2)} + \frac{1}{2}\left(\frac{\partial}{\partial x} f(x, y, z)\right)$$

$$\left(2\left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y^2} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x, y, z)\right)\right)$$

$$\Big/ \left(\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1\right)^{(3/2)} \Bigg]$$

$$EEP := \Bigg[$$

$$\frac{1}{2}\,\frac{2\left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial x^2} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x, y, z)\right)}{\left(\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1\right)^{(3/2)}},$$

$$\frac{1}{2}\,\frac{2\left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y^2} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x, y, z)\right)}{\left(\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1\right)^{(3/2)}},$$

$$\frac{1}{2}\,\frac{2\left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z^2} f(x, y, z)\right)}{\left(\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1\right)^{(3/2)}}\Bigg]$$

$$EEV := [0, 0, 0]$$

$$EE := \Bigg[$$

$$\frac{1}{2}\,\frac{2\left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial x^2} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x, y, z)\right)}{\left(\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1\right)^{(3/2)}},$$

$$\frac{1}{2}\,\frac{2\left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y^2} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x, y, z)\right)}{\left(\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1\right)^{(3/2)}},$$

$$\frac{1}{2}\,\frac{2\left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z^2} f(x, y, z)\right)}{\left(\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1\right)^{(3/2)}}\Bigg]$$

$$Parity := 0$$

$$Torsion\_current := \Bigg[\frac{1}{2}\Bigg($$

$$\left(2\left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y^2} f(x, y, z)\right) + 2\left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x, y, z)\right)\right)$$

$$\left(\frac{\partial}{\partial z} f(x, y, z)\right)) \Big/ (\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1)^2 - \frac{1}{2}($$

$$(2 \left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial x} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial y} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z^2} f(x, y, z)\right))$$

$$\left(\frac{\partial}{\partial y} f(x, y, z)\right)) \Big/ (\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1)^2 + (-\frac{1}{2} \left(\frac{\partial}{\partial z} f(x, y, z)\right)$$

$$(2 \left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\, \partial x} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y^2} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial y} f(x, y, z)\right))$$

$$\Big/ ((\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1)^{(3/2)} + \frac{1}{2} \left(\frac{\partial}{\partial y} f(x, y, z)\right)$$

$$(2 \left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial x} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial y} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z^2} f(x, y, z)\right)) \Big/$$

$$((\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1)^{(3/2)}) \Big/$$

$$\sqrt{\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1}, \frac{1}{2}($$

$$(2 \left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial x} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial y} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z^2} f(x, y, z)\right))$$

$$\left(\frac{\partial}{\partial x} f(x, y, z)\right)) \Big/ (\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1)^2 - \frac{1}{2}($$

$$(2 \left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial x^2} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\, \partial x} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial x} f(x, y, z)\right))$$

$$\left(\frac{\partial}{\partial z} f(x, y, z)\right)) \Big/ (\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1)^2 + (-\frac{1}{2} \left(\frac{\partial}{\partial x} f(x, y, z)\right)$$

$$(2 \left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial x} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial y} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z^2} f(x, y, z)\right)) \Big/$$

$$((\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1)^{(3/2)} + \frac{1}{2} \left(\frac{\partial}{\partial z} f(x, y, z)\right)$$

$$(2 \left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial x^2} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\, \partial x} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial x} f(x, y, z)\right))$$

$$\Big/ ((\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1)^{(3/2)}) \Big/$$

$$\sqrt{\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1}, \frac{1}{2}($$

$$(2 \left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial x^2} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\, \partial x} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial x} f(x, y, z)\right))$$

$$\left(\frac{\partial}{\partial y} f(x, y, z)\right)) \Big/ (\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1)^2 - \frac{1}{2}($$

$$(2 \left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\, \partial x} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y^2} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial y} f(x, y, z)\right))$$

$$\left(\frac{\partial}{\partial x} f(x, y, z)\right)) \Big/ (\left(\frac{\partial}{\partial x} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y, z)\right)^2 + \left(\frac{\partial}{\partial z} f(x, y, z)\right)^2 + 1)^2 + (-\frac{1}{2} \left(\frac{\partial}{\partial y} f(x, y, z)\right)$$

$$(2 \left(\frac{\partial}{\partial x} f(x, y, z)\right)\left(\frac{\partial^2}{\partial x^2} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial y} f(x, y, z)\right)\left(\frac{\partial^2}{\partial y\, \partial x} f(x, y, z)\right) + 2 \left(\frac{\partial}{\partial z} f(x, y, z)\right)\left(\frac{\partial^2}{\partial z\, \partial x} f(x, y, z)\right))$$

$$\Big/ \left(\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + 1\right)^{(3/2)} + \frac{1}{2}\left(\frac{\partial}{\partial x} f(x,y,z)\right)$$

$$\left(2\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x,y,z)\right) + 2\left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right) + 2\left(\frac{\partial}{\partial z} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x,y,z)\right)\right)$$

$$\Big/ \left(\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + 1\right)^{(3/2)}\Bigg) \Big/$$

$$\sqrt{\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + 1}\;\Bigg]$$

$$Helicity := 0$$

```
>
>
> NA:=evalm([A1,A2,A3,phi]/lambda);
```

$$NA := \left[\frac{\frac{\partial}{\partial x} f(x,y,z)}{\sqrt{\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + 1}},\right.$$

$$\frac{\frac{\partial}{\partial y} f(x,y,z)}{\sqrt{\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + 1}},$$

$$\frac{\frac{\partial}{\partial z} f(x,y,z)}{\sqrt{\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + 1}},$$

$$\left.\frac{1}{\sqrt{\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + 1}}\right]$$

```
> JAC:=jacobian(NA,[x,y,z,t]):
> MEAN_CURVATURE:=factor(trace(JAC)/2);
```

$$MEAN\_CURVATURE := \frac{1}{2}\Bigg(-2\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x,y,z)\right)$$

$$-2\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x,y,z)\right) + \left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)^2$$

$$+\left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + \left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right) - 2\left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x,y,z)\right)$$

$$+\left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + \left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)$$

$$+\left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right)\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right)\Bigg) \Big/$$

$$\left(\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + 1\right)^{(3/2)}$$

Note the classic formula for the mean curvature of a 3D implicit surface in xyz is obtained.

```
> S2:=factor(trace(innerprod(JAC,JAC))):
  Gauss:=factor(-(1/2)*(-trace(JAC)*trace(JAC)+S2));
```

$$Gauss := \left(\left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 - \left(\frac{\partial^2}{\partial y\,\partial x} f(x,y,z)\right)^2\left(\frac{\partial}{\partial z} f(x,y,z)\right)^2\right.$$

$$- 2\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x,y,z)\right)\left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)$$
$$+ 2\left(\frac{\partial^2}{\partial y\,\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x,y,z)\right)$$
$$- 2\left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)$$
$$+ 2\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x,y,z)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x,y,z)\right) - \left(\frac{\partial^2}{\partial z\,\partial x} f(x,y,z)\right)^2$$
$$+ \left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right) + 2\left(\frac{\partial^2}{\partial z\,\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x,y,z)\right)$$
$$- \left(\frac{\partial^2}{\partial z\,\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right)\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right)$$
$$- \left(\frac{\partial^2}{\partial z\,\partial x} f(x,y,z)\right)^2\left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)^2$$
$$- 2\left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x,y,z)\right) - \left(\frac{\partial^2}{\partial y\,\partial x} f(x,y,z)\right)^2$$
$$+ \left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right) - \left(\frac{\partial^2}{\partial z\,\partial y} f(x,y,z)\right)^2\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2\Bigg) \Bigg/$$
$$\left(\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + 1\right)^2$$

```
> ADJAC:=adjoint(JAC):
> ADJOINT_CURVATURE:=factor(trace(ADJAC));
```

$$ADJOINT\_CURVATURE := -\left(\left(\frac{\partial^2}{\partial y\,\partial x} f(x,y,z)\right)^2\left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right) + \left(\frac{\partial^2}{\partial z\,\partial x} f(x,y,z)\right)^2\left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)\right.$$
$$+ \left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x,y,z)\right)^2 - \left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right)$$
$$\left. - 2\left(\frac{\partial^2}{\partial y\,\partial x} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x,y,z)\right)\right) \Bigg/$$
$$\left(\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + 1\right)^{(5/2)}$$

```
>
```

Note that the classic formula for the ADJOINT Gauss curvature of a 3D implicit surface in xyz is obtained.

```
> CurrentJ:=innerprod(ADJAC,NA);Interaction:=innerprod(CurrentJ,NA);DivJ:=facto
  r(diverge(CurrentJ,[x,y,z,t]));
```

$$CurrentJ := \left[0, 0, 0, -\left(\left(\frac{\partial^2}{\partial y\,\partial x} f(x,y,z)\right)^2\left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right) + \left(\frac{\partial^2}{\partial z\,\partial x} f(x,y,z)\right)^2\left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)\right.\right.$$
$$+ \left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x,y,z)\right)^2 - \left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right)$$
$$\left. - 2\left(\frac{\partial^2}{\partial y\,\partial x} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x,y,z)\right)\right) \Bigg/$$
$$\left.\left(\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + 1\right)^2\right]$$

$$Interaction := -\left(\left(\frac{\partial^2}{\partial y\,\partial x} f(x,y,z)\right)^2\left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right) + \left(\frac{\partial^2}{\partial z\,\partial x} f(x,y,z)\right)^2\left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)\right.$$

$$+\left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x,y,z)\right)^2 - \left(\frac{\partial^2}{\partial x^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial y^2} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z^2} f(x,y,z)\right)$$

$$-2\left(\frac{\partial^2}{\partial y\,\partial x} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y} f(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial x} f(x,y,z)\right)\Bigg) \Bigg/$$

$$\left(\left(\frac{\partial}{\partial x} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y,z)\right)^2 + \left(\frac{\partial}{\partial z} f(x,y,z)\right)^2 + 1\right)^{(5/2)}$$

$$DivJ := 0$$

It is now apparent that the interaction between the potentials and the conserved current is equal to the Adjoint curvature of the simple surface.

```
>
> Adjointcurv:=simplify(factor(trace(ADJAC))):Mean:=factor(trace(JAC)):Net:=factor(Adjointcurv-Interaction):Gnet:=factor(subs(a=1,b=1,c=1,p=2,n=1,Net));
```

$$Gnet := 0$$

```
> Vorticity:=curl([A[1],A[2],A[3]],[x,y,z]);Helicity:=innerprod([A[1],A[2],A[3]],Vorticity);Diss:=innerprod([CurrentJ[1],CurrentJ[2],CurrentJ[3]],E);
```

$$Vorticity := [0, 0, 0]$$

$$Helicity := 0$$

$$Diss := 0$$

```
>
>
>
```

Coulomb potential only:

The Coulomb potential may used as the sole component of the 1-form of Action; As the normal field has only one component, the Jacobian matrix constructed from the renormalized 1-form is the Zero matrix. All curvatures are zeo. The associated hypersurface is flat in a euclidean sense and without shape.

```
>
> A1:=0;A2:=0;A3:=0;A4:=-k/(x^2+y^2+z^2)^(1/2);
>
```

$$A1 := 0$$

$$A2 := 0$$

$$A3 := 0$$

$$A4 := -\frac{k}{\sqrt{x^2+y^2+z^2}}$$

```
> A:=[A1,A2,A3];phi:=A4;
```

$$A := [0, 0, 0]$$

$$\phi := -\frac{k}{\sqrt{x^2+y^2+z^2}}$$

```
>
> B:=curl(A,[x,y,z]);EP:=evalm(-grad(phi,[x,y,z]));EV:=-[diff(A[1],t),diff(A[2],t),diff(A[3],t)];E:=evalm(EV+EP);Parity:=innerprod(E,B);Torsion_current:=evalm(crossprod(E,A)+evalm(B*phi));Helicity:=innerprod(A,B);
>
```

$$B := [0, 0, 0]$$

$$EP := \left[-\frac{k\,x}{(x^2+y^2+z^2)^{(3/2)}}, -\frac{k\,y}{(x^2+y^2+z^2)^{(3/2)}}, -\frac{k\,z}{(x^2+y^2+z^2)^{(3/2)}}\right]$$

$$EV := [0, 0, 0]$$

$$E := \left[-\frac{k\,x}{(x^2+y^2+z^2)^{(3/2)}}, -\frac{k\,y}{(x^2+y^2+z^2)^{(3/2)}}, -\frac{k\,z}{(x^2+y^2+z^2)^{(3/2)}}\right]$$

$$Parity := 0$$

$$Torsion\_current := [0, 0, 0]$$

$$Helicity := 0$$

```
> lambda:=(A[1]^2+A[2]^2+A[3]^2+e*A4^2)^(n/2);
>
```

$$\lambda := \left(\frac{e\,k^2}{x^2+y^2+z^2}\right)^{(1/2\,n)}$$

```
> NA:=evalm([A1,A2,A3,phi]/lambda);
```

$$NA := \left[0, 0, 0, -\frac{k}{\left(\frac{e\,k^2}{x^2+y^2+z^2}\right)^{(1/2\,n)}\sqrt{x^2+y^2+z^2}}\right]$$

```
> JAC:=jacobian(NA,[x,y,z,t]);
```

$JAC :=$

$$[0, 0, 0, 0]$$
$$[0, 0, 0, 0]$$
$$[0, 0, 0, 0]$$
$$\left[-\frac{k\,n\,x}{\left(\frac{e\,k^2}{x^2+y^2+z^2}\right)^{(1/2\,n)}(x^2+y^2+z^2)^{(3/2)}} + \frac{k\,x}{\left(\frac{e\,k^2}{x^2+y^2+z^2}\right)^{(1/2\,n)}(x^2+y^2+z^2)^{(3/2)}},\right.$$
$$-\frac{k\,n\,y}{\left(\frac{e\,k^2}{x^2+y^2+z^2}\right)^{(1/2\,n)}(x^2+y^2+z^2)^{(3/2)}} + \frac{k\,y}{\left(\frac{e\,k^2}{x^2+y^2+z^2}\right)^{(1/2\,n)}(x^2+y^2+z^2)^{(3/2)}},$$
$$\left.-\frac{k\,n\,z}{\left(\frac{e\,k^2}{x^2+y^2+z^2}\right)^{(1/2\,n)}(x^2+y^2+z^2)^{(3/2)}} + \frac{k\,z}{\left(\frac{e\,k^2}{x^2+y^2+z^2}\right)^{(1/2\,n)}(x^2+y^2+z^2)^{(3/2)}}, 0\right]$$

The Jacobian matrix has only 1 row not zero.

```
> MEAN_CURVATURE:=factor(trace(JAC)/2);
```

$$MEAN\_CURVATURE := 0$$

```
> S2:=factor(trace(innerprod(JAC,JAC))):
  Gauss:=factor(-(1/2)*(-trace(JAC)*trace(JAC)+S2));
```

$$Gauss := 0$$

```
> ADJAC:=adjoint(JAC);
```

$$ADJAC := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The adjoint matrix in this case is identically ZERO.

```
> ADJOINT_CURVATURE:=factor(trace(ADJAC));
```

$$ADJOINT\_CURVATURE := 0$$

```
>
> CurrentJ:=innerprod(ADJAC,NA);Interaction:=innerprod(CurrentJ,NA);DivJ:=facto
  r(diverge(CurrentJ,[x,y,z,t]));
```

$$CurrentJ := [0, 0, 0, 0]$$

$$Interaction := 0$$

$$DivJ := 0$$

It is apparent that the Coulomb potential yields zero Adjoint currents and charge densities.

```
>
> Adjointcurv:=simplify(factor(trace(ADJAC))):Mean:=factor(trace(JAC)):Net:=factor(Adjointcurv-Interaction):Gnet:=factor(subs(a=1,b=1,c=1,p=2,n=1,Net));
```

$$Gnet := 0$$

```
> Vorticity:=curl([A[1],A[2],A[3]],[x,y,z]);Helicity:=innerprod([A[1],A[2],A[3]],Vorticity);Diss:=innerprod([CurrentJ[1],CurrentJ[2],CurrentJ[3]],E);
```

$$Vorticity := [0, 0, 0]$$

$$Helicity := 0$$

$$Diss := 0$$

NOTE that the Jacobian matrix is not zero, but all of its similarity invariants are zero.
The current density J is zero, implying that G is closed.

```
>
>
>
```

Scalar potential only:

The 1-form of Action has only 1 component, a scalar potential, but now the potential is an arbitrary function of space and time.

```
> A1:=0;A2:=0;A3:=0;A4:=-U(x,y,z,t);
>
```

$$A1 := 0$$

$$A2 := 0$$

$$A3 := 0$$

$$A4 := -\mathrm{U}(x, y, z, t)$$

```
> A:=[A1,A2,A3];phi:=A4;
```

$$A := [0, 0, 0]$$

$$\phi := -\mathrm{U}(x, y, z, t)$$

```
>
> B:=curl(A,[x,y,z]);EP:=evalm(-grad(phi,[x,y,z]));EV:=-[diff(A[1],t),diff(A[2],t),diff(A[3],t)];E:=evalm(EV+EP);Parity:=innerprod(E,B);Torsion_current:=evalm(crossprod(E,A)+evalm(B*phi));Helicity:=innerprod(A,B);
>
```

$$B := [0, 0, 0]$$

$$EP := \left[\frac{\partial}{\partial x}\mathrm{U}(x, y, z, t), \frac{\partial}{\partial y}\mathrm{U}(x, y, z, t), \frac{\partial}{\partial z}\mathrm{U}(x, y, z, t)\right]$$

$$EV := [0, 0, 0]$$

$$E := \left[\frac{\partial}{\partial x}\mathrm{U}(x, y, z, t), \frac{\partial}{\partial y}\mathrm{U}(x, y, z, t), \frac{\partial}{\partial z}\mathrm{U}(x, y, z, t)\right]$$

$$Parity := 0$$

$$Torsion\_current := [0, 0, 0]$$

$$Helicity := 0$$

The 1-form with only a scalar potential generates an E field, but no B field.

```
> lambda:=(A[1]^2+A[2]^2+A[3]^2+A4^2)^(1/2);
>
```

$$\lambda := \sqrt{\mathrm{U}(x, y, z, t)^2}$$

```
> NA:=evalm([A1,A2,A3,phi]/lambda);
```

$$NA := \left[0, 0, 0, -\frac{\mathrm{U}(x, y, z, t)}{\sqrt{\mathrm{U}(x, y, z, t)^2}}\right]$$

```
> JAC:=jacobian(NA,[x,y,z,t]);
```

$JAC :=$

$[0, 0, 0, 0]$

[0, 0, 0, 0]

[0, 0, 0, 0]

$$\left[\frac{U(x,y,z,t)^2\left(\frac{\partial}{\partial x}U(x,y,z,t)\right)}{\left(U(x,y,z,t)^2\right)^{(3/2)}}-\frac{\frac{\partial}{\partial x}U(x,y,z,t)}{\sqrt{U(x,y,z,t)^2}},\frac{U(x,y,z,t)^2\left(\frac{\partial}{\partial y}U(x,y,z,t)\right)}{\left(U(x,y,z,t)^2\right)^{(3/2)}}-\frac{\frac{\partial}{\partial y}U(x,y,z,t)}{\sqrt{U(x,y,z,t)^2}},\frac{U(x,y,z,t)^2\left(\frac{\partial}{\partial z}U(x,y,z,t)\right)}{\left(U(x,y,z,t)^2\right)^{(3/2)}}-\frac{\frac{\partial}{\partial z}U(x,y,z,t)}{\sqrt{U(x,y,z,t)^2}},\frac{U(x,y,z,t)^2\left(\frac{\partial}{\partial t}U(x,y,z,t)\right)}{\left(U(x,y,z,t)^2\right)^{(3/2)}}-\frac{\frac{\partial}{\partial t}U(x,y,z,t)}{\sqrt{U(x,y,z,t)^2}}\right]$$

The Jacobian matrix has entries in only one row.

```
> MEAN_CURVATURE:=factor(trace(JAC)/2);
```

$$MEAN\_CURVATURE := 0$$

Note the classic formula for the mean curvature of a Monge surface is obtained.

```
> S2:=factor(trace(innerprod(JAC,JAC))):
  Gauss:=factor(-(1/2)*(-trace(JAC)*trace(JAC)+S2));
```

$$Gauss := 0$$

```
> ADJAC:=adjoint(JAC);
```

$$ADJAC := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> ADJOINT_CURVATURE:=factor(trace(ADJAC));
```

$$ADJOINT\_CURVATURE := 0$$

```
>
> CurrentJ:=innerprod(ADJAC,NA);Interaction:=innerprod(CurrentJ,NA);DivJ:=facto
  r(diverge(CurrentJ,[x,y,z,t]));
```

$$CurrentJ := [0, 0, 0, 0]$$

$$Interaction := 0$$

$$DivJ := 0$$

A normal field that consists of one time like component generates zero charge-current densities.

All curvature invariants are zero.

```
>
> Adjointcurv:=simplify(factor(trace(ADJAC))):Mean:=factor(trace(JAC)):Net:=fac
  tor(Adjointcurv-Interaction):Gnet:=factor(subs(a=1,b=1,c=1,p=2,n=1,Net));
```

$$Gnet := 0$$

```
> Vorticity:=curl([A[1],A[2],A[3]],[x,y,z]);Helicity:=innerprod([A[1],A[2],A[3]
  ],Vorticity);Diss:=innerprod([CurrentJ[1],CurrentJ[2],CurrentJ[3]],E);
```

$$Vorticity := [0, 0, 0]$$

$$Helicity := 0$$

$$Diss := 0$$

NOTE that the Jacobian matrix is not zero, but all of its similarity invaraints are zero.

J has no non-closed components

```
>
>
```

Vector potential only (time independent):

The 1-form of potentials is presumed to have a vector potential which can be time dependent, but no scalar potential

```
>
> A1:=u(x,y,z,t);A2:=v(x,y,z,t);A3:=w(x,y,z,t);A4:=0;
> #A1:=x^3*y-z^2;A2:=z*x;A3:=y*x;A4:=0;
```

$$A1 := u(x, y, z, t)$$
$$A2 := v(x, y, z, t)$$
$$A3 := w(x, y, z, t)$$
$$A4 := 0$$

```
> A:=[A1,A2,A3];phi:=A4;
```

$$A := [u(x, y, z, t), v(x, y, z, t), w(x, y, z, t)]$$
$$\phi := 0$$

```
>
> B:=curl(A,[x,y,z]);EP:=evalm(-grad(phi,[x,y,z]));EV:=-[diff(A[1],t),diff(A[2],t),diff(A[3],t)];E:=evalm(EV+EP);Parity:=factor(innerprod(E,B));Torsion_current:=evalm(crossprod(E,A)+evalm(B*phi));Helicity:=innerprod(A,B);
>
```

$$B := \left[\left(\frac{\partial}{\partial y} w(x, y, z, t)\right) - \left(\frac{\partial}{\partial z} v(x, y, z, t)\right), \left(\frac{\partial}{\partial z} u(x, y, z, t)\right) - \left(\frac{\partial}{\partial x} w(x, y, z, t)\right), \left(\frac{\partial}{\partial x} v(x, y, z, t)\right) - \left(\frac{\partial}{\partial y} u(x, y, z, t)\right)\right]$$

$$EP := [0, 0, 0]$$

$$EV := \left[-\left(\frac{\partial}{\partial t} u(x, y, z, t)\right), -\left(\frac{\partial}{\partial t} v(x, y, z, t)\right), -\left(\frac{\partial}{\partial t} w(x, y, z, t)\right)\right]$$

$$E := \left[-\left(\frac{\partial}{\partial t} u(x, y, z, t)\right), -\left(\frac{\partial}{\partial t} v(x, y, z, t)\right), -\left(\frac{\partial}{\partial t} w(x, y, z, t)\right)\right]$$

$$Parity := -\left(\frac{\partial}{\partial t} u(x, y, z, t)\right)\left(\frac{\partial}{\partial y} w(x, y, z, t)\right) + \left(\frac{\partial}{\partial t} u(x, y, z, t)\right)\left(\frac{\partial}{\partial z} v(x, y, z, t)\right) - \left(\frac{\partial}{\partial t} v(x, y, z, t)\right)\left(\frac{\partial}{\partial z} u(x, y, z, t)\right) + \left(\frac{\partial}{\partial t} v(x, y, z, t)\right)\left(\frac{\partial}{\partial x} w(x, y, z, t)\right) - \left(\frac{\partial}{\partial t} w(x, y, z, t)\right)\left(\frac{\partial}{\partial x} v(x, y, z, t)\right) + \left(\frac{\partial}{\partial t} w(x, y, z, t)\right)\left(\frac{\partial}{\partial y} u(x, y, z, t)\right)$$

$$Torsion\_current := \left[-\left(\frac{\partial}{\partial t} v(x, y, z, t)\right) w(x, y, z, t) + \left(\frac{\partial}{\partial t} w(x, y, z, t)\right) v(x, y, z, t), -\left(\frac{\partial}{\partial t} w(x, y, z, t)\right) u(x, y, z, t) + \left(\frac{\partial}{\partial t} u(x, y, z, t)\right) w(x, y, z, t), -\left(\frac{\partial}{\partial t} u(x, y, z, t)\right) v(x, y, z, t) + \left(\frac{\partial}{\partial t} v(x, y, z, t)\right) u(x, y, z, t)\right]$$

$$Helicity := u(x, y, z, t)\left(\frac{\partial}{\partial y} w(x, y, z, t)\right) - u(x, y, z, t)\left(\frac{\partial}{\partial z} v(x, y, z, t)\right) + v(x, y, z, t)\left(\frac{\partial}{\partial z} u(x, y, z, t)\right) - v(x, y, z, t)\left(\frac{\partial}{\partial x} w(x, y, z, t)\right) + w(x, y, z, t)\left(\frac{\partial}{\partial x} v(x, y, z, t)\right) - w(x, y, z, t)\left(\frac{\partial}{\partial y} u(x, y, z, t)\right)$$

The 1-form generates an E field, and a B field.

```
> lambda:=(A[1]^2+A[2]^2+A[3]^2+e*A4^2)^(1/2);
>
```

$$\lambda := \sqrt{u(x, y, z, t)^2 + v(x, y, z, t)^2 + w(x, y, z, t)^2}$$

```
> NA:=evalm([A1,A2,A3,phi]/lambda);
```

$$NA := \left[\frac{u(x, y, z, t)}{\sqrt{u(x, y, z, t)^2 + v(x, y, z, t)^2 + w(x, y, z, t)^2}}, \frac{v(x, y, z, t)}{\sqrt{u(x, y, z, t)^2 + v(x, y, z, t)^2 + w(x, y, z, t)^2}}, \frac{w(x, y, z, t)}{\sqrt{u(x, y, z, t)^2 + v(x, y, z, t)^2 + w(x, y, z, t)^2}}, 0\right]$$

```
> JAC:=jacobian(NA,[x,y,z,t]):
> MEAN_CURVATURE:=factor(trace(JAC)/2);
```

$$MEAN\_CURVATURE := -\frac{1}{2}\Big(u(x, y, z, t)\, v(x, y, z, t) \left(\frac{\partial}{\partial x} v(x, y, z, t)\right)$$
$$+ u(x, y, z, t)\, w(x, y, z, t) \left(\frac{\partial}{\partial x} w(x, y, z, t)\right) - \left(\frac{\partial}{\partial x} u(x, y, z, t)\right) v(x, y, z, t)^2 - \left(\frac{\partial}{\partial x} u(x, y, z, t)\right) w(x, y, z, t)^2$$
$$+ v(x, y, z, t)\, u(x, y, z, t) \left(\frac{\partial}{\partial y} u(x, y, z, t)\right) + v(x, y, z, t)\, w(x, y, z, t) \left(\frac{\partial}{\partial y} w(x, y, z, t)\right)$$
$$- \left(\frac{\partial}{\partial y} v(x, y, z, t)\right) u(x, y, z, t)^2 - \left(\frac{\partial}{\partial y} v(x, y, z, t)\right) w(x, y, z, t)^2 + w(x, y, z, t)\, u(x, y, z, t) \left(\frac{\partial}{\partial z} u(x, y, z, t)\right)$$
$$+ w(x, y, z, t)\, v(x, y, z, t) \left(\frac{\partial}{\partial z} v(x, y, z, t)\right) - \left(\frac{\partial}{\partial z} w(x, y, z, t)\right) u(x, y, z, t)^2 - \left(\frac{\partial}{\partial z} w(x, y, z, t)\right) v(x, y, z, t)^2 \Big) \Big/$$
$$(u(x, y, z, t)^2 + v(x, y, z, t)^2 + w(x, y, z, t)^2)^{(3/2)}$$

The mean curvature is not zero.

```
> S2:=factor(trace(innerprod(JAC,JAC))):
  GAUSS_CURVATURE:=factor(-(1/2)*(-trace(JAC)*trace(JAC)+S2));
```

$$GAUSS\_CURVATURE := -\Big(w(x, y, z, t)^2 \left(\frac{\partial}{\partial y} u(x, y, z, t)\right)\left(\frac{\partial}{\partial x} v(x, y, z, t)\right)$$
$$- w(x, y, z, t)^2 \left(\frac{\partial}{\partial x} u(x, y, z, t)\right)\left(\frac{\partial}{\partial y} v(x, y, z, t)\right) + w(x, y, z, t) \left(\frac{\partial}{\partial y} v(x, y, z, t)\right) u(x, y, z, t) \left(\frac{\partial}{\partial z} u(x, y, z, t)\right)$$
$$+ w(x, y, z, t) \left(\frac{\partial}{\partial x} u(x, y, z, t)\right) v(x, y, z, t) \left(\frac{\partial}{\partial y} w(x, y, z, t)\right)$$
$$- w(x, y, z, t) \left(\frac{\partial}{\partial y} u(x, y, z, t)\right) v(x, y, z, t) \left(\frac{\partial}{\partial x} w(x, y, z, t)\right)$$
$$+ w(x, y, z, t) \left(\frac{\partial}{\partial x} u(x, y, z, t)\right) v(x, y, z, t) \left(\frac{\partial}{\partial z} v(x, y, z, t)\right)$$
$$- w(x, y, z, t) \left(\frac{\partial}{\partial z} u(x, y, z, t)\right) v(x, y, z, t) \left(\frac{\partial}{\partial x} v(x, y, z, t)\right)$$
$$- w(x, y, z, t) \left(\frac{\partial}{\partial z} v(x, y, z, t)\right) u(x, y, z, t) \left(\frac{\partial}{\partial y} u(x, y, z, t)\right)$$
$$- w(x, y, z, t)\, u(x, y, z, t) \left(\frac{\partial}{\partial y} w(x, y, z, t)\right)\left(\frac{\partial}{\partial x} v(x, y, z, t)\right)$$
$$+ w(x, y, z, t)\, u(x, y, z, t) \left(\frac{\partial}{\partial y} v(x, y, z, t)\right)\left(\frac{\partial}{\partial x} w(x, y, z, t)\right)$$
$$+ v(x, y, z, t) \left(\frac{\partial}{\partial z} w(x, y, z, t)\right) u(x, y, z, t) \left(\frac{\partial}{\partial y} u(x, y, z, t)\right)$$
$$+ u(x, y, z, t) \left(\frac{\partial}{\partial z} w(x, y, z, t)\right) v(x, y, z, t) \left(\frac{\partial}{\partial x} v(x, y, z, t)\right)$$
$$- u(x, y, z, t)\, v(x, y, z, t) \left(\frac{\partial}{\partial z} v(x, y, z, t)\right)\left(\frac{\partial}{\partial x} w(x, y, z, t)\right)$$
$$- v(x, y, z, t)\, u(x, y, z, t) \left(\frac{\partial}{\partial z} u(x, y, z, t)\right)\left(\frac{\partial}{\partial y} w(x, y, z, t)\right) + \left(\frac{\partial}{\partial z} v(x, y, z, t)\right)\left(\frac{\partial}{\partial y} w(x, y, z, t)\right) u(x, y, z, t)^2$$
$$- \left(\frac{\partial}{\partial x} u(x, y, z, t)\right)\left(\frac{\partial}{\partial z} w(x, y, z, t)\right) v(x, y, z, t)^2 - \left(\frac{\partial}{\partial y} v(x, y, z, t)\right)\left(\frac{\partial}{\partial z} w(x, y, z, t)\right) u(x, y, z, t)^2$$
$$+ \left(\frac{\partial}{\partial z} u(x, y, z, t)\right)\left(\frac{\partial}{\partial x} w(x, y, z, t)\right) v(x, y, z, t)^2 \Big) \Big/ (u(x, y, z, t)^2 + v(x, y, z, t)^2 + w(x, y, z, t)^2)^2$$

```
>
> ADJAC:=adjoint(JAC);
```

$$ADJAC := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The adjoint matrix is the Zero matrix.

```
> ADJOINT_CURVATURE:=factor(trace(ADJAC));
```

$$ADJOINT\_CURVATURE := 0$$

```
>
> CurrentJ:=innerprod(ADJAC,NA);Interaction:=innerprod(CurrentJ,NA);DivJ:=factor(diverge(CurrentJ,[x,y,z,t]));Diss:=innerprod([CurrentJ[1],CurrentJ[2],CurrentJ[3]],E);
```

$$CurrentJ := [0, 0, 0, 0]$$

$$Interaction := 0$$

$$DivJ := 0$$

$$Diss := 0$$

A normal field that consists of vector potential alone does not generate non- zero charge-current densities.

All curvature invariants are NOT zero as in the scalar potential only case.

```
>
> Adjointcurv:=simplify(factor(trace(ADJAC))):Mean:=factor(trace(JAC)):Net:=factor(Adjointcurv-Interaction):Gnet:=factor(subs(a=1,b=1,c=1,p=2,n=1,Net));
```

$$Gnet := 0$$

```
> Vorticity:=curl([A[1],A[2],A[3]],[x,y,z]);Helicity:=innerprod([A[1],A[2],A[3]],Vorticity);
```

$$Vorticity := \left[\left(\frac{\partial}{\partial y} \mathrm{w}(x, y, z, t)\right) - \left(\frac{\partial}{\partial z} \mathrm{v}(x, y, z, t)\right), \left(\frac{\partial}{\partial z} \mathrm{u}(x, y, z, t)\right) - \left(\frac{\partial}{\partial x} \mathrm{w}(x, y, z, t)\right), \left(\frac{\partial}{\partial x} \mathrm{v}(x, y, z, t)\right) - \left(\frac{\partial}{\partial y} \mathrm{u}(x, y, z, t)\right)\right]$$

$$Helicity := \mathrm{u}(x, y, z, t)\left(\frac{\partial}{\partial y} \mathrm{w}(x, y, z, t)\right) - \mathrm{u}(x, y, z, t)\left(\frac{\partial}{\partial z} \mathrm{v}(x, y, z, t)\right) + \mathrm{v}(x, y, z, t)\left(\frac{\partial}{\partial z} \mathrm{u}(x, y, z, t)\right) - \mathrm{v}(x, y, z, t)\left(\frac{\partial}{\partial x} \mathrm{w}(x, y, z, t)\right) + \mathrm{w}(x, y, z, t)\left(\frac{\partial}{\partial x} \mathrm{v}(x, y, z, t)\right) - \mathrm{w}(x, y, z, t)\left(\frac{\partial}{\partial y} \mathrm{u}(x, y, z, t)\right)$$

```
> factor(innerprod(E,B));
```

$$-\left(\frac{\partial}{\partial t} \mathrm{u}(x, y, z, t)\right)\left(\frac{\partial}{\partial y} \mathrm{w}(x, y, z, t)\right) + \left(\frac{\partial}{\partial t} \mathrm{u}(x, y, z, t)\right)\left(\frac{\partial}{\partial z} \mathrm{v}(x, y, z, t)\right) - \left(\frac{\partial}{\partial t} \mathrm{v}(x, y, z, t)\right)\left(\frac{\partial}{\partial z} \mathrm{u}(x, y, z, t)\right) + \left(\frac{\partial}{\partial t} \mathrm{v}(x, y, z, t)\right)\left(\frac{\partial}{\partial x} \mathrm{w}(x, y, z, t)\right) - \left(\frac{\partial}{\partial t} \mathrm{w}(x, y, z, t)\right)\left(\frac{\partial}{\partial x} \mathrm{v}(x, y, z, t)\right) + \left(\frac{\partial}{\partial t} \mathrm{w}(x, y, z, t)\right)\left(\frac{\partial}{\partial y} \mathrm{u}(x, y, z, t)\right)$$

A stationary surface (no t dependence)

The similarity invariants for the Jacobian matrix of the surface normal scaled by the Holder norm with a=b=c=1,n=1,p=2

fields the classic partial differential equations for the mean and Gauss curvature.

```
>
> A1:=u(x,y,z);A2:=v(x,y,z);A3:=w(x,y,z);A4:=-f(x,y,z);
```

$$A1 := \mathrm{u}(x, y, z)$$

$$A2 := \mathrm{v}(x, y, z)$$

$$A3 := \mathrm{w}(x, y, z)$$

$$A4 := -\mathrm{f}(x, y, z)$$

```
> A:=[A1,A2,A3];phi:=A4;
```

$$A := [\mathrm{u}(x, y, z), \mathrm{v}(x, y, z), \mathrm{w}(x, y, z)]$$

$$\phi := -f(x, y, z)$$

```
>
> B:=curl(A,[x,y,z]);EP:=evalm(-grad(phi,[x,y,z]));EV:=-[diff(A[1],t),diff(A[2],t),diff(A[3],t)];E:=evalm(EV+EP);Parity:=innerprod(E,B);Torsion_current:=evalm(crossprod(E,A)+evalm(B*phi));Helicity:=innerprod(A,B);
>
```

$$B := \left[\left(\frac{\partial}{\partial y} w(x,y,z)\right) - \left(\frac{\partial}{\partial z} v(x,y,z)\right), \left(\frac{\partial}{\partial z} u(x,y,z)\right) - \left(\frac{\partial}{\partial x} w(x,y,z)\right), \left(\frac{\partial}{\partial x} v(x,y,z)\right) - \left(\frac{\partial}{\partial y} u(x,y,z)\right)\right]$$

$$EP := \left[\frac{\partial}{\partial x} f(x,y,z), \frac{\partial}{\partial y} f(x,y,z), \frac{\partial}{\partial z} f(x,y,z)\right]$$

$$EV := [0, 0, 0]$$

$$E := \left[\frac{\partial}{\partial x} f(x,y,z), \frac{\partial}{\partial y} f(x,y,z), \frac{\partial}{\partial z} f(x,y,z)\right]$$

$$Parity := \left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right) - \left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right) + \left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial}{\partial z} u(x,y,z)\right)$$
$$- \left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial}{\partial x} w(x,y,z)\right) + \left(\frac{\partial}{\partial z} f(x,y,z)\right)\left(\frac{\partial}{\partial x} v(x,y,z)\right) - \left(\frac{\partial}{\partial z} f(x,y,z)\right)\left(\frac{\partial}{\partial y} u(x,y,z)\right)$$

$$Torsion\_current := \Bigg[$$
$$\left(\frac{\partial}{\partial y} f(x,y,z)\right) w(x,y,z) - \left(\frac{\partial}{\partial z} f(x,y,z)\right) v(x,y,z) - f(x,y,z)\left(\left(\frac{\partial}{\partial y} w(x,y,z)\right) - \left(\frac{\partial}{\partial z} v(x,y,z)\right)\right),$$
$$\left(\frac{\partial}{\partial z} f(x,y,z)\right) u(x,y,z) - \left(\frac{\partial}{\partial x} f(x,y,z)\right) w(x,y,z) - f(x,y,z)\left(\left(\frac{\partial}{\partial z} u(x,y,z)\right) - \left(\frac{\partial}{\partial x} w(x,y,z)\right)\right),$$
$$\left(\frac{\partial}{\partial x} f(x,y,z)\right) v(x,y,z) - \left(\frac{\partial}{\partial y} f(x,y,z)\right) u(x,y,z) - f(x,y,z)\left(\left(\frac{\partial}{\partial x} v(x,y,z)\right) - \left(\frac{\partial}{\partial y} u(x,y,z)\right)\right)\Bigg]$$

$$Helicity := u(x,y,z)\left(\frac{\partial}{\partial y} w(x,y,z)\right) - u(x,y,z)\left(\frac{\partial}{\partial z} v(x,y,z)\right) + v(x,y,z)\left(\frac{\partial}{\partial z} u(x,y,z)\right)$$
$$- v(x,y,z)\left(\frac{\partial}{\partial x} w(x,y,z)\right) + w(x,y,z)\left(\frac{\partial}{\partial x} v(x,y,z)\right) - w(x,y,z)\left(\frac{\partial}{\partial y} u(x,y,z)\right)$$

The potentials produce a non-zero, but static E and B field

```
> lambda:=(A[1]^2+A[2]^2+A[3]^2+e*A4^2)^(1/2);
>
```

$$\lambda := \sqrt{u(x,y,z)^2 + v(x,y,z)^2 + w(x,y,z)^2 + e\, f(x,y,z)^2}$$

```
> NA:=evalm([A1,A2,A3,phi]/lambda);
```

$$NA := \left[\frac{u(x,y,z)}{\sqrt{u(x,y,z)^2 + v(x,y,z)^2 + w(x,y,z)^2 + e\, f(x,y,z)^2}}, \frac{v(x,y,z)}{\sqrt{u(x,y,z)^2 + v(x,y,z)^2 + w(x,y,z)^2 + e\, f(x,y,z)^2}},\right.$$
$$\left.\frac{w(x,y,z)}{\sqrt{u(x,y,z)^2 + v(x,y,z)^2 + w(x,y,z)^2 + e\, f(x,y,z)^2}}, -\frac{f(x,y,z)}{\sqrt{u(x,y,z)^2 + v(x,y,z)^2 + w(x,y,z)^2 + e\, f(x,y,z)^2}}\right]$$

```
> JAC:=jacobian(NA,[x,y,z,t]):
```

The Jacobian matrix has not been printed, but now there are entries in more than 1 row.

```
> MEAN_CURVATURE:=factor(trace(JAC)/2);
```

$$MEAN\_CURVATURE := -\frac{1}{2}\Bigg(u(x,y,z)\, v(x,y,z)\left(\frac{\partial}{\partial x} v(x,y,z)\right) + u(x,y,z)\, w(x,y,z)\left(\frac{\partial}{\partial x} w(x,y,z)\right)$$
$$+ u(x,y,z)\, e\, f(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right) - \left(\frac{\partial}{\partial x} u(x,y,z)\right) v(x,y,z)^2 - \left(\frac{\partial}{\partial x} u(x,y,z)\right) w(x,y,z)^2$$
$$- \left(\frac{\partial}{\partial x} u(x,y,z)\right) e\, f(x,y,z)^2 + v(x,y,z)\, u(x,y,z)\left(\frac{\partial}{\partial y} u(x,y,z)\right) + v(x,y,z)\, w(x,y,z)\left(\frac{\partial}{\partial y} w(x,y,z)\right)$$

$$+ v(x,y,z)\, e\, f(x,y,z) \left(\frac{\partial}{\partial y} f(x,y,z)\right) - \left(\frac{\partial}{\partial y} v(x,y,z)\right) u(x,y,z)^2 - \left(\frac{\partial}{\partial y} v(x,y,z)\right) w(x,y,z)^2$$

$$- \left(\frac{\partial}{\partial y} v(x,y,z)\right) e\, f(x,y,z)^2 + w(x,y,z)\, u(x,y,z) \left(\frac{\partial}{\partial z} u(x,y,z)\right) + w(x,y,z)\, v(x,y,z) \left(\frac{\partial}{\partial z} v(x,y,z)\right)$$

$$+ w(x,y,z)\, e\, f(x,y,z) \left(\frac{\partial}{\partial z} f(x,y,z)\right) - \left(\frac{\partial}{\partial z} w(x,y,z)\right) u(x,y,z)^2 - \left(\frac{\partial}{\partial z} w(x,y,z)\right) v(x,y,z)^2$$

$$- \left(\frac{\partial}{\partial z} w(x,y,z)\right) e\, f(x,y,z)^2 \Bigg) \Big/ \left(u(x,y,z)^2 + v(x,y,z)^2 + w(x,y,z)^2 + e\, f(x,y,z)^2\right)^{(3/2)}$$

Note the classic formula for the mean curvature of an implicit surface in 3D xyt space is obtained.

```
> S2:=factor(trace(innerprod(JAC,JAC))):
  Gauss:=factor(-(1/2)*(-trace(JAC)*trace(JAC)+S2));
```

$$Gauss := -\Bigg(\left(\frac{\partial}{\partial y} v(x,y,z)\right) w(x,y,z)\, u(x,y,z) \left(\frac{\partial}{\partial z} u(x,y,z)\right)$$

$$+ \left(\frac{\partial}{\partial x} u(x,y,z)\right) w(x,y,z)\, v(x,y,z) \left(\frac{\partial}{\partial z} v(x,y,z)\right) + \left(\frac{\partial}{\partial z} u(x,y,z)\right)\left(\frac{\partial}{\partial x} w(x,y,z)\right) v(x,y,z)^2$$

$$+ u(x,y,z)\, w(x,y,z) \left(\frac{\partial}{\partial x} w(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right) + \left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial x} v(x,y,z)\right) w(x,y,z)^2$$

$$- \left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right) v(x,y,z)^2 + \left(\frac{\partial}{\partial z} v(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right) u(x,y,z)^2$$

$$- \left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right) u(x,y,z)^2 - \left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right) w(x,y,z)^2$$

$$+ f(x,y,z)\, e\, v(x,y,z) \left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right) + f(x,y,z)\, e\, u(x,y,z) \left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)$$

$$+ f(x,y,z)\, e \left(\frac{\partial}{\partial y} v(x,y,z)\right) w(x,y,z) \left(\frac{\partial}{\partial z} f(x,y,z)\right) + f(x,y,z)\, e\, u(x,y,z) \left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)$$

$$- f(x,y,z)\, e\, u(x,y,z) \left(\frac{\partial}{\partial z} f(x,y,z)\right)\left(\frac{\partial}{\partial x} w(x,y,z)\right) - f(x,y,z)\, e\, u(x,y,z) \left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial}{\partial x} v(x,y,z)\right)$$

$$- f(x,y,z)\, e \left(\frac{\partial}{\partial y} u(x,y,z)\right) v(x,y,z) \left(\frac{\partial}{\partial x} f(x,y,z)\right) + f(x,y,z)\, e \left(\frac{\partial}{\partial x} u(x,y,z)\right) v(x,y,z) \left(\frac{\partial}{\partial y} f(x,y,z)\right)$$

$$- f(x,y,z)\, e\, v(x,y,z) \left(\frac{\partial}{\partial z} f(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right) - f(x,y,z)^2\, e \left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)$$

$$- f(x,y,z)^2\, e \left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right) + f(x,y,z)^2\, e \left(\frac{\partial}{\partial z} v(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)$$

$$- f(x,y,z)^2\, e \left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right) - \left(\frac{\partial}{\partial z} v(x,y,z)\right) w(x,y,z)\, u(x,y,z) \left(\frac{\partial}{\partial y} u(x,y,z)\right)$$

$$+ u(x,y,z)\, v(x,y,z) \left(\frac{\partial}{\partial x} v(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right) - u(x,y,z)\, v(x,y,z) \left(\frac{\partial}{\partial z} v(x,y,z)\right)\left(\frac{\partial}{\partial x} w(x,y,z)\right)$$

$$+ f(x,y,z)\, e \left(\frac{\partial}{\partial x} u(x,y,z)\right) w(x,y,z) \left(\frac{\partial}{\partial z} f(x,y,z)\right) - u(x,y,z)\, w(x,y,z) \left(\frac{\partial}{\partial y} w(x,y,z)\right)\left(\frac{\partial}{\partial x} v(x,y,z)\right)$$

$$- \left(\frac{\partial}{\partial z} u(x,y,z)\right) w(x,y,z)\, v(x,y,z) \left(\frac{\partial}{\partial x} v(x,y,z)\right) + \left(\frac{\partial}{\partial x} u(x,y,z)\right) v(x,y,z)\, w(x,y,z) \left(\frac{\partial}{\partial y} w(x,y,z)\right)$$

$$+ v(x,y,z)\, u(x,y,z) \left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right) - \left(\frac{\partial}{\partial y} u(x,y,z)\right) v(x,y,z)\, w(x,y,z) \left(\frac{\partial}{\partial x} w(x,y,z)\right)$$

$$- v(x,y,z)\, u(x,y,z) \left(\frac{\partial}{\partial z} u(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right) - f(x,y,z)\, e \left(\frac{\partial}{\partial z} u(x,y,z)\right) w(x,y,z) \left(\frac{\partial}{\partial x} f(x,y,z)\right)$$

$$- f(x,y,z)\, e \left(\frac{\partial}{\partial z} v(x,y,z)\right) w(x,y,z) \left(\frac{\partial}{\partial y} f(x,y,z)\right) + f(x,y,z)^2\, e \left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial x} v(x,y,z)\right)$$

$$+ f(x,y,z)^2 e \left(\frac{\partial}{\partial z} u(x,y,z)\right)\left(\frac{\partial}{\partial x} w(x,y,z)\right)\Big) \Big/ \left(u(x,y,z)^2 + v(x,y,z)^2 + w(x,y,z)^2 + e\, f(x,y,z)^2\right)^2$$

```
> ADJAC:=adjoint(JAC):
```

The Adjoint is no longer the Zero matrix, and has a non-zero trace implying the existence of Adjoint curvature.

```
> ADJOINT_CURVATURE:=factor(trace(ADJAC));
```

$$ADJOINT\_CURVATURE := - f(x,y,z)\, e \Bigg(f(x,y,z) \left(\frac{\partial}{\partial x} w(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial z} u(x,y,z)\right)$$
$$+ f(x,y,z)\left(\frac{\partial}{\partial x} v(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)\left(\frac{\partial}{\partial y} u(x,y,z)\right)$$
$$- f(x,y,z)\left(\frac{\partial}{\partial x} v(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)\left(\frac{\partial}{\partial z} u(x,y,z)\right)$$
$$- f(x,y,z)\left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)$$
$$- f(x,y,z)\left(\frac{\partial}{\partial x} w(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right)\left(\frac{\partial}{\partial y} u(x,y,z)\right)$$
$$+ f(x,y,z)\left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)$$
$$- \left(\frac{\partial}{\partial x} w(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right) u(x,y,z)$$
$$- \left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right) w(x,y,z)\left(\frac{\partial}{\partial y} f(x,y,z)\right)$$
$$+ \left(\frac{\partial}{\partial x} w(x,y,z)\right) v(x,y,z)\left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)$$
$$+ \left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right) w(x,y,z)\left(\frac{\partial}{\partial z} f(x,y,z)\right)$$
$$+ \left(\frac{\partial}{\partial x} v(x,y,z)\right) w(x,y,z)\left(\frac{\partial}{\partial z} u(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)$$
$$+ v(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial z} u(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)$$
$$+ \left(\frac{\partial}{\partial x} w(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right) u(x,y,z)$$
$$- u(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)$$
$$- v(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)$$
$$- \left(\frac{\partial}{\partial x} v(x,y,z)\right) w(x,y,z)\left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)$$
$$+ u(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)$$
$$+ w(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right)$$
$$- \left(\frac{\partial}{\partial x} v(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right) u(x,y,z)$$
$$- \left(\frac{\partial}{\partial x} w(x,y,z)\right) v(x,y,z)\left(\frac{\partial}{\partial z} u(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)$$

$$- w(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial z} u(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)$$
$$+\left(\frac{\partial}{\partial x} u(x,y,z)\right) v(x,y,z)\left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)$$
$$-\left(\frac{\partial}{\partial x} u(x,y,z)\right) v(x,y,z)\left(\frac{\partial}{\partial z} f(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)$$
$$+\left(\frac{\partial}{\partial x} v(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right) u(x,y,z)\Bigg) \Big/$$
$$\left(u(x,y,z)^2 + v(x,y,z)^2 + w(x,y,z)^2 + e\, f(x,y,z)^2\right)^{(5/2)}$$

These terms all cancel algebraically.

```
> CurrentJ:=innerprod(ADJAC,NA);Interaction:=innerprod(CurrentJ,NA);DivJ:=facto
  r(diverge(CurrentJ,[x,y,z,t]));rho:=CurrentJ[4];
```

$$CurrentJ := \Bigg[0, 0, 0, \Bigg(f(x,y,z)\left(\frac{\partial}{\partial x} w(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial z} u(x,y,z)\right)$$
$$+ f(x,y,z)\left(\frac{\partial}{\partial x} v(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)\left(\frac{\partial}{\partial y} u(x,y,z)\right)$$
$$- f(x,y,z)\left(\frac{\partial}{\partial x} v(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)\left(\frac{\partial}{\partial z} u(x,y,z)\right)$$
$$- f(x,y,z)\left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)$$
$$- f(x,y,z)\left(\frac{\partial}{\partial x} w(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right)\left(\frac{\partial}{\partial y} u(x,y,z)\right)$$
$$+ f(x,y,z)\left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)$$
$$-\left(\frac{\partial}{\partial x} w(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right) u(x,y,z)$$
$$-\left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right) w(x,y,z)\left(\frac{\partial}{\partial y} f(x,y,z)\right)$$
$$+\left(\frac{\partial}{\partial x} w(x,y,z)\right) v(x,y,z)\left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)$$
$$+\left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right) w(x,y,z)\left(\frac{\partial}{\partial z} f(x,y,z)\right)$$
$$+\left(\frac{\partial}{\partial x} v(x,y,z)\right) w(x,y,z)\left(\frac{\partial}{\partial z} u(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)$$
$$+ v(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial z} u(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)$$
$$+\left(\frac{\partial}{\partial x} w(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right) u(x,y,z)$$
$$- u(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)$$
$$- v(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)$$
$$-\left(\frac{\partial}{\partial x} v(x,y,z)\right) w(x,y,z)\left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)$$

$$+ \mathrm{u}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{w}(x,y,z)\right)$$

$$+ \mathrm{w}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{v}(x,y,z)\right)$$

$$-\left(\frac{\partial}{\partial x}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,z)\right)\mathrm{u}(x,y,z)$$

$$-\left(\frac{\partial}{\partial x}\mathrm{w}(x,y,z)\right)\mathrm{v}(x,y,z)\left(\frac{\partial}{\partial z}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,z)\right)$$

$$- \mathrm{w}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{v}(x,y,z)\right)$$

$$+\left(\frac{\partial}{\partial x}\mathrm{u}(x,y,z)\right)\mathrm{v}(x,y,z)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{w}(x,y,z)\right)$$

$$-\left(\frac{\partial}{\partial x}\mathrm{u}(x,y,z)\right)\mathrm{v}(x,y,z)\left(\frac{\partial}{\partial z}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{w}(x,y,z)\right)$$

$$+\left(\frac{\partial}{\partial x}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{f}(x,y,z)\right)\mathrm{u}(x,y,z)\Bigg)\Bigg/$$

$$\left(\mathrm{u}(x,y,z)^2+\mathrm{v}(x,y,z)^2+\mathrm{w}(x,y,z)^2+e\,\mathrm{f}(x,y,z)^2\right)^2\Bigg]$$

$$Interaction := -\Bigg(\Bigg(\mathrm{f}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{u}(x,y,z)\right)$$

$$+ \mathrm{f}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{u}(x,y,z)\right)$$

$$- \mathrm{f}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{u}(x,y,z)\right)$$

$$- \mathrm{f}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{w}(x,y,z)\right)$$

$$- \mathrm{f}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{u}(x,y,z)\right)$$

$$+ \mathrm{f}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{w}(x,y,z)\right)$$

$$-\left(\frac{\partial}{\partial x}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{f}(x,y,z)\right)\mathrm{u}(x,y,z)$$

$$-\left(\frac{\partial}{\partial x}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{v}(x,y,z)\right)\mathrm{w}(x,y,z)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,z)\right)$$

$$+\left(\frac{\partial}{\partial x}\mathrm{w}(x,y,z)\right)\mathrm{v}(x,y,z)\left(\frac{\partial}{\partial y}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{f}(x,y,z)\right)$$

$$+\left(\frac{\partial}{\partial x}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{v}(x,y,z)\right)\mathrm{w}(x,y,z)\left(\frac{\partial}{\partial z}\mathrm{f}(x,y,z)\right)$$

$$+\left(\frac{\partial}{\partial x}\mathrm{v}(x,y,z)\right)\mathrm{w}(x,y,z)\left(\frac{\partial}{\partial z}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,z)\right)$$

$$+ \mathrm{v}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{w}(x,y,z)\right)$$

$$+\left(\frac{\partial}{\partial x}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{v}(x,y,z)\right)\mathrm{u}(x,y,z)$$

$$- \mathrm{u}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{w}(x,y,z)\right)$$

$$- v(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)$$
$$- \left(\frac{\partial}{\partial x} v(x,y,z)\right) w(x,y,z)\left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)$$
$$+ u(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)$$
$$+ w(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right)$$
$$- \left(\frac{\partial}{\partial x} v(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right) u(x,y,z)$$
$$- \left(\frac{\partial}{\partial x} w(x,y,z)\right) v(x,y,z)\left(\frac{\partial}{\partial z} u(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)$$
$$- w(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial z} u(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)$$
$$+ \left(\frac{\partial}{\partial x} u(x,y,z)\right) v(x,y,z)\left(\frac{\partial}{\partial y} f(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)$$
$$- \left(\frac{\partial}{\partial x} u(x,y,z)\right) v(x,y,z)\left(\frac{\partial}{\partial z} f(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)$$
$$+ \left(\frac{\partial}{\partial x} v(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right) u(x,y,z)\Bigg) f(x,y,z)\Bigg) \Big/$$
$$\left(u(x,y,z)^2 + v(x,y,z)^2 + w(x,y,z)^2 + e\, f(x,y,z)^2\right)^{(5/2)}$$

$$DivJ := 0$$

$$\rho := \Bigg(f(x,y,z)\left(\frac{\partial}{\partial x} w(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial z} u(x,y,z)\right)$$
$$+ f(x,y,z)\left(\frac{\partial}{\partial x} v(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)\left(\frac{\partial}{\partial y} u(x,y,z)\right)$$
$$- f(x,y,z)\left(\frac{\partial}{\partial x} v(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)\left(\frac{\partial}{\partial z} u(x,y,z)\right)$$
$$- f(x,y,z)\left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)$$
$$- f(x,y,z)\left(\frac{\partial}{\partial x} w(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right)\left(\frac{\partial}{\partial y} u(x,y,z)\right)$$
$$+ f(x,y,z)\left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)$$
$$- \left(\frac{\partial}{\partial x} w(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right) u(x,y,z)$$
$$- \left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial z} v(x,y,z)\right) w(x,y,z)\left(\frac{\partial}{\partial y} f(x,y,z)\right)$$
$$+ \left(\frac{\partial}{\partial x} w(x,y,z)\right) v(x,y,z)\left(\frac{\partial}{\partial y} u(x,y,z)\right)\left(\frac{\partial}{\partial z} f(x,y,z)\right)$$
$$+ \left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right) w(x,y,z)\left(\frac{\partial}{\partial z} f(x,y,z)\right)$$
$$+ \left(\frac{\partial}{\partial x} v(x,y,z)\right) w(x,y,z)\left(\frac{\partial}{\partial z} u(x,y,z)\right)\left(\frac{\partial}{\partial y} f(x,y,z)\right)$$
$$+ v(x,y,z)\left(\frac{\partial}{\partial x} f(x,y,z)\right)\left(\frac{\partial}{\partial z} u(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)$$

$$+\left(\frac{\partial}{\partial x}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{v}(x,y,z)\right)\mathrm{u}(x,y,z)$$
$$-\mathrm{u}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{w}(x,y,z)\right)$$
$$-\mathrm{v}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{w}(x,y,z)\right)$$
$$-\left(\frac{\partial}{\partial x}\mathrm{v}(x,y,z)\right)\mathrm{w}(x,y,z)\left(\frac{\partial}{\partial y}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{f}(x,y,z)\right)$$
$$+\mathrm{u}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{w}(x,y,z)\right)$$
$$+\mathrm{w}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{v}(x,y,z)\right)$$
$$-\left(\frac{\partial}{\partial x}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,z)\right)\mathrm{u}(x,y,z)$$
$$-\left(\frac{\partial}{\partial x}\mathrm{w}(x,y,z)\right)\mathrm{v}(x,y,z)\left(\frac{\partial}{\partial z}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,z)\right)$$
$$-\mathrm{w}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{v}(x,y,z)\right)$$
$$+\left(\frac{\partial}{\partial x}\mathrm{u}(x,y,z)\right)\mathrm{v}(x,y,z)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{w}(x,y,z)\right)$$
$$-\left(\frac{\partial}{\partial x}\mathrm{u}(x,y,z)\right)\mathrm{v}(x,y,z)\left(\frac{\partial}{\partial z}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{w}(x,y,z)\right)$$
$$+\left(\frac{\partial}{\partial x}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{f}(x,y,z)\right)\mathrm{u}(x,y,z)\Bigg)\Big/$$
$$\left(\mathrm{u}(x,y,z)^2+\mathrm{v}(x,y,z)^2+\mathrm{w}(x,y,z)^2+e\,\mathrm{f}(x,y,z)^2\right)^2$$

It is now apparent that the interaction between the potentials and the conserved current is equal to the adjoint curvature of the implicit surface. The system of stationary potentials produces a non-zero charge density, but not a non-zero current density.

```
>
> Adjointcurv:=simplify(factor(trace(ADJAC))):Mean:=factor(trace(JAC)):Net:=factor(Adjointcurv-Interaction):Gnet:=factor(subs(a=1,b=1,c=1,p=2,n=1,Net));
```

$$Gnet := -\Bigg(\Bigg(\mathrm{f}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{u}(x,y,z)\right)$$
$$+\mathrm{f}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{u}(x,y,z)\right)$$
$$-\mathrm{f}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{u}(x,y,z)\right)$$
$$-\mathrm{f}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{w}(x,y,z)\right)$$
$$-\mathrm{f}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{u}(x,y,z)\right)$$
$$+\mathrm{f}(x,y,z)\left(\frac{\partial}{\partial x}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{w}(x,y,z)\right)$$
$$-\left(\frac{\partial}{\partial x}\mathrm{w}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{v}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{f}(x,y,z)\right)\mathrm{u}(x,y,z)$$
$$-\left(\frac{\partial}{\partial x}\mathrm{u}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{v}(x,y,z)\right)\mathrm{w}(x,y,z)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,z)\right)$$

$$+\left(\frac{\partial}{\partial x}w(x,y,z)\right)v(x,y,z)\left(\frac{\partial}{\partial y}u(x,y,z)\right)\left(\frac{\partial}{\partial z}f(x,y,z)\right)$$
$$+\left(\frac{\partial}{\partial x}u(x,y,z)\right)\left(\frac{\partial}{\partial y}v(x,y,z)\right)w(x,y,z)\left(\frac{\partial}{\partial z}f(x,y,z)\right)$$
$$+\left(\frac{\partial}{\partial x}v(x,y,z)\right)w(x,y,z)\left(\frac{\partial}{\partial z}u(x,y,z)\right)\left(\frac{\partial}{\partial y}f(x,y,z)\right)$$
$$+v(x,y,z)\left(\frac{\partial}{\partial x}f(x,y,z)\right)\left(\frac{\partial}{\partial z}u(x,y,z)\right)\left(\frac{\partial}{\partial y}w(x,y,z)\right)$$
$$+\left(\frac{\partial}{\partial x}w(x,y,z)\right)\left(\frac{\partial}{\partial y}f(x,y,z)\right)\left(\frac{\partial}{\partial z}v(x,y,z)\right)u(x,y,z)$$
$$-u(x,y,z)\left(\frac{\partial}{\partial x}f(x,y,z)\right)\left(\frac{\partial}{\partial z}v(x,y,z)\right)\left(\frac{\partial}{\partial y}w(x,y,z)\right)$$
$$-v(x,y,z)\left(\frac{\partial}{\partial x}f(x,y,z)\right)\left(\frac{\partial}{\partial y}u(x,y,z)\right)\left(\frac{\partial}{\partial z}w(x,y,z)\right)$$
$$-\left(\frac{\partial}{\partial x}v(x,y,z)\right)w(x,y,z)\left(\frac{\partial}{\partial y}u(x,y,z)\right)\left(\frac{\partial}{\partial z}f(x,y,z)\right)$$
$$+u(x,y,z)\left(\frac{\partial}{\partial x}f(x,y,z)\right)\left(\frac{\partial}{\partial y}v(x,y,z)\right)\left(\frac{\partial}{\partial z}w(x,y,z)\right)$$
$$+w(x,y,z)\left(\frac{\partial}{\partial x}f(x,y,z)\right)\left(\frac{\partial}{\partial y}u(x,y,z)\right)\left(\frac{\partial}{\partial z}v(x,y,z)\right)$$
$$-\left(\frac{\partial}{\partial x}v(x,y,z)\right)\left(\frac{\partial}{\partial z}w(x,y,z)\right)\left(\frac{\partial}{\partial y}f(x,y,z)\right)u(x,y,z)$$
$$-\left(\frac{\partial}{\partial x}w(x,y,z)\right)v(x,y,z)\left(\frac{\partial}{\partial z}u(x,y,z)\right)\left(\frac{\partial}{\partial y}f(x,y,z)\right)$$
$$-w(x,y,z)\left(\frac{\partial}{\partial x}f(x,y,z)\right)\left(\frac{\partial}{\partial z}u(x,y,z)\right)\left(\frac{\partial}{\partial y}v(x,y,z)\right)$$
$$+\left(\frac{\partial}{\partial x}u(x,y,z)\right)v(x,y,z)\left(\frac{\partial}{\partial y}f(x,y,z)\right)\left(\frac{\partial}{\partial z}w(x,y,z)\right)$$
$$-\left(\frac{\partial}{\partial x}u(x,y,z)\right)v(x,y,z)\left(\frac{\partial}{\partial z}f(x,y,z)\right)\left(\frac{\partial}{\partial y}w(x,y,z)\right)$$
$$+\left(\frac{\partial}{\partial x}v(x,y,z)\right)\left(\frac{\partial}{\partial y}w(x,y,z)\right)\left(\frac{\partial}{\partial z}f(x,y,z)\right)u(x,y,z)\Bigg)f(x,y,z)\,(e-1)\Bigg)\Bigg/$$
$$\left(u(x,y,z)^2+v(x,y,z)^2+w(x,y,z)^2+e\,f(x,y,z)^2\right)^{(5/2)}$$

The terms cancel algebraically

```
>
> Vorticity:=curl([A[1],A[2],A[3]],[x,y,z]);Helicity:=innerprod([A[1],A[2],A[3]],Vorticity);Diss:=innerprod([CurrentJ[1],CurrentJ[2],CurrentJ[3]],E);
```

$$Vorticity := \left[\left(\frac{\partial}{\partial y}w(x,y,z)\right)-\left(\frac{\partial}{\partial z}v(x,y,z)\right),\left(\frac{\partial}{\partial z}u(x,y,z)\right)-\left(\frac{\partial}{\partial x}w(x,y,z)\right),\left(\frac{\partial}{\partial x}v(x,y,z)\right)-\left(\frac{\partial}{\partial y}u(x,y,z)\right)\right]$$

$$Helicity := u(x,y,z)\left(\frac{\partial}{\partial y}w(x,y,z)\right)-u(x,y,z)\left(\frac{\partial}{\partial z}v(x,y,z)\right)+v(x,y,z)\left(\frac{\partial}{\partial z}u(x,y,z)\right)$$
$$-v(x,y,z)\left(\frac{\partial}{\partial x}w(x,y,z)\right)+w(x,y,z)\left(\frac{\partial}{\partial x}v(x,y,z)\right)-w(x,y,z)\left(\frac{\partial}{\partial y}u(x,y,z)\right)$$

$$Diss := 0$$

THere is no induced current density BUT there is an induced charge density proportional to the Adjoint curvature

```
> factor(innerprod(E,B));
```

$$\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{w}(x,y,z)\right)-\left(\frac{\partial}{\partial x}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{v}(x,y,z)\right)+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial z}\mathrm{u}(x,y,z)\right)$$
$$-\left(\frac{\partial}{\partial y}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial x}\mathrm{w}(x,y,z)\right)+\left(\frac{\partial}{\partial z}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial x}\mathrm{v}(x,y,z)\right)-\left(\frac{\partial}{\partial z}\mathrm{f}(x,y,z)\right)\left(\frac{\partial}{\partial y}\mathrm{u}(x,y,z)\right)$$

```
>
```

A Whittaker surface (time harmonic case)

The 1-form of Action consists of a product of a function alpha and a gradient of a second function beta. The 1-form is integrable in the sense of Frobenius. Hence the helicity is zero, (Topological Torsion is zero). The three curvature invariants depend only on the function beta. If beta is not a function of time, then the Adjoint curvature vanishes, and there is no curvature induced charge current density.

```
>
> A1:=alpha(x,y,z,t)*diff(beta(x,y,z),x);A2:=alpha(x,y,z,t)*diff(beta(x,y,z),y);A3:=alpha(x,y,z,t)*diff(beta(x,y,z),z);A4:=alpha(x,y,z,t)*diff(beta(x,y,z),t);
```

$$A1 := \alpha(x,y,z,t)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)$$
$$A2 := \alpha(x,y,z,t)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)$$
$$A3 := \alpha(x,y,z,t)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)$$
$$A4 := 0$$

```
> A:=[A1,A2,A3];phi:=-A4;
```

$$A := \left[\alpha(x,y,z,t)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right), \alpha(x,y,z,t)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right), \alpha(x,y,z,t)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)\right]$$
$$\phi := 0$$

```
>
> B:=curl(A,[x,y,z]);EP:=evalm(-grad(phi,[x,y,z]));EV:=-[diff(A[1],t),diff(A[2],t),diff(A[3],t)];E:=evalm(EV+EP);Parity:=innerprod(E,B);Torsion_current:=evalm(crossprod(E,A)+evalm(B*phi));Helicity:=innerprod(A,B);
```

$$B := \left[\left(\frac{\partial}{\partial y}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)-\left(\frac{\partial}{\partial z}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right),\right.$$
$$\left(\frac{\partial}{\partial z}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)-\left(\frac{\partial}{\partial x}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right),$$
$$\left.\left(\frac{\partial}{\partial x}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)-\left(\frac{\partial}{\partial y}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)\right]$$
$$EP := [0,0,0]$$
$$EV := \left[-\left(\frac{\partial}{\partial t}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right), -\left(\frac{\partial}{\partial t}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right), -\left(\frac{\partial}{\partial t}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)\right]$$
$$E := \left[-\left(\frac{\partial}{\partial t}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right), -\left(\frac{\partial}{\partial t}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right), -\left(\frac{\partial}{\partial t}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)\right]$$
$$Parity := 0$$
$$Torsion\_current := [0,0,0]$$
$$Helicity := 0$$

The potentials produce a non-zero E and B field

```
> lambda:=(A[1]^2+A[2]^2+A[3]^2+A4^2)^(1/2);
>
```

$$\lambda := \sqrt{\alpha(x,y,z,t)^2 \left(\frac{\partial}{\partial x}\beta(x,y,z)\right)^2 + \alpha(x,y,z,t)^2 \left(\frac{\partial}{\partial y}\beta(x,y,z)\right)^2 + \alpha(x,y,z,t)^2 \left(\frac{\partial}{\partial z}\beta(x,y,z)\right)^2}$$

```
> NA:=evalm([A1,A2,A3,phi]/lambda);
```

$$NA := \left[\frac{\alpha(x,y,z,t)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)}{\sqrt{\alpha(x,y,z,t)^2 \left(\frac{\partial}{\partial x}\beta(x,y,z)\right)^2 + \alpha(x,y,z,t)^2 \left(\frac{\partial}{\partial y}\beta(x,y,z)\right)^2 + \alpha(x,y,z,t)^2 \left(\frac{\partial}{\partial z}\beta(x,y,z)\right)^2}}, \frac{\alpha(x,y,z,t)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)}{\sqrt{\alpha(x,y,z,t)^2 \left(\frac{\partial}{\partial x}\beta(x,y,z)\right)^2 + \alpha(x,y,z,t)^2 \left(\frac{\partial}{\partial y}\beta(x,y,z)\right)^2 + \alpha(x,y,z,t)^2 \left(\frac{\partial}{\partial z}\beta(x,y,z)\right)^2}}, \frac{\alpha(x,y,z,t)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)}{\sqrt{\alpha(x,y,z,t)^2 \left(\frac{\partial}{\partial x}\beta(x,y,z)\right)^2 + \alpha(x,y,z,t)^2 \left(\frac{\partial}{\partial y}\beta(x,y,z)\right)^2 + \alpha(x,y,z,t)^2 \left(\frac{\partial}{\partial z}\beta(x,y,z)\right)^2}}, 0 \right]$$

```
> JAC:=jacobian(NA,[x,y,z,t]):
```

The Jacobian matrix has not been printed, but now there are entries in more than 1 row.

```
> MEAN_CURVATURE:=factor(trace(JAC)/2);
```

$$MEAN\_CURVATURE := \frac{1}{2}\alpha(x,y,z,t)^3 \left(-2\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y}\beta(x,y,z)\right) - 2\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial x}\beta(x,y,z)\right) - 2\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\beta(x,y,z)\right) + \left(\frac{\partial^2}{\partial y^2}\beta(x,y,z)\right)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)^2 + \left(\frac{\partial^2}{\partial y^2}\beta(x,y,z)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)^2 + \left(\frac{\partial^2}{\partial z^2}\beta(x,y,z)\right)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)^2 + \left(\frac{\partial^2}{\partial z^2}\beta(x,y,z)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)^2 + \left(\frac{\partial^2}{\partial x^2}\beta(x,y,z)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)^2 + \left(\frac{\partial^2}{\partial x^2}\beta(x,y,z)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)^2\right) \Big/ \left(\alpha(x,y,z,t)^2\left(\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)^2 + \left(\frac{\partial}{\partial y}\beta(x,y,z)\right)^2 + \left(\frac{\partial}{\partial z}\beta(x,y,z)\right)^2\right)\right)^{(3/2)}$$

Note the classic formula for the mean curvature of an implicit surface in 3D xyt space is obtained.

```
> S2:=factor(trace(innerprod(JAC,JAC))):
  Gauss:=factor(-(1/2)*(-trace(JAC)*trace(JAC)+S2));
```

$$Gauss := -\left(\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)^2\left(\frac{\partial^2}{\partial z\,\partial y}\beta(x,y,z)\right)^2 - \left(\frac{\partial}{\partial x}\beta(x,y,z)\right)^2\left(\frac{\partial^2}{\partial y^2}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial z^2}\beta(x,y,z)\right) + 2\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial x}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial y^2}\beta(x,y,z)\right) + 2\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial z^2}\beta(x,y,z)\right) - 2\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial x}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y}\beta(x,y,z)\right) - 2\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial y\,\partial x}\beta(x,y,z)\right) - \left(\frac{\partial^2}{\partial x^2}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial y^2}\beta(x,y,z)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)^2 + 2\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial y}\beta(x,y,z)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial x^2}\beta(x,y,z)\right) + \left(\frac{\partial^2}{\partial z\,\partial x}\beta(x,y,z)\right)^2\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)^2$$

$$+\left(\frac{\partial^2}{\partial y\,\partial x}\beta(x,y,z)\right)^2\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)^2-2\left(\frac{\partial^2}{\partial y\,\partial x}\beta(x,y,z)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial z\,\partial x}\beta(x,y,z)\right)$$

$$-\left(\frac{\partial^2}{\partial x^2}\beta(x,y,z)\right)\left(\frac{\partial^2}{\partial z^2}\beta(x,y,z)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)^2\Bigg)\Bigg/$$

$$\left(\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)^2+\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)^2+\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)^2\right)^2$$

```
> ADJAC:=adjoint(JAC):
```

```
> ADJOINT_CURVATURE:=factor(trace(ADJAC));
```

$$ADJOINT\_CURVATURE := 0$$

```
>
```

```
> CurrentJ:=innerprod(ADJAC,NA);Interaction:=innerprod(CurrentJ,NA);DivJ:=facto
r(diverge(CurrentJ,[x,y,z,t]));rho:=CurrentJ[4];
```

$$CurrentJ := [0, 0, 0, 0]$$

$$Interaction := 0$$

$$DivJ := 0$$

$$\rho := 0$$

It is now apparent that the interaction between the potentials and the conserved current is equal to the adjoint curvature of the implicit surface. Both are zero for the time harmonic case where beta is independent from time.

```
>
> Adjointcurv:=simplify(factor(trace(ADJAC))):Mean:=factor(trace(JAC)):Net:=fac
tor(Adjointcurv-Interaction):Gnet:=factor(subs(a=1,b=1,c=1,p=2,n=1,Net));
```

$$Gnet := 0$$

```
> Vorticity:=curl([A[1],A[2],A[3]],[x,y,z]);Helicity:=factor(innerprod([A[1],A[
2],A[3]],Vorticity));Parity:=factor(innerprod(E,B));Diss:=factor(innerprod([C
urrentJ[1],CurrentJ[2],CurrentJ[3]],E));
```

$$Vorticity := \left[\left(\frac{\partial}{\partial y}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right)-\left(\frac{\partial}{\partial z}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right),\right.$$

$$\left(\frac{\partial}{\partial z}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)-\left(\frac{\partial}{\partial x}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial z}\beta(x,y,z)\right),$$

$$\left.\left(\frac{\partial}{\partial x}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial y}\beta(x,y,z)\right)-\left(\frac{\partial}{\partial y}\alpha(x,y,z,t)\right)\left(\frac{\partial}{\partial x}\beta(x,y,z)\right)\right]$$

$$Helicity := 0$$

$$Parity := 0$$

$$Diss := 0$$

```
>
```

A Whittaker surface (not time harmonic)

The 1-form of Action consists of a product of a function alpha and a gradient of a second function beta. The 1-form is integrable in the sense of Frobenius. Hence the helicity is zero, (Topological Torsion is zero). The three curvature invariants depend only on the function beta. ***If beta is not a function of time, then the Adjoint curvature vanishes, and there is no curvature induced charge current density. see the time harmonic example above.***

```
> #A1:=alpha(x,y,z,t)*diff(beta(x,y,z,t),x);A2:=alpha(x,y,z,t)*diff(beta(x,y,z,
t),y);A3:=alpha(x,y,z,t)*diff(beta(x,y,z,t),z);A4:=alpha(x,y,z,t)*diff(beta(x
,y,z,t),t);
```

```
> A1:=x*y/z*t^2*diff(x^3*y+3*z^2,x);A2:=x*y/z*t^2*diff(x^3*y+3*z^2,y);A3:=x*y/z*t^2*diff(x^3*y+3*z^2,z);A4:=x*y/z*t*diff(x^3*y+3*z^2,t);
```

$$A1 := 3\,\frac{x^3\,y^2\,t^2}{z}$$

$$A2 := \frac{x^4\,y\,t^2}{z}$$

$$A3 := 6\,x\,y\,t^2$$

$$A4 := 0$$

```
> A:=[A1,A2,A3];phi:=-A4;
```

$$A := \left[3\,\frac{x^3\,y^2\,t^2}{z},\ \frac{x^4\,y\,t^2}{z},\ 6\,x\,y\,t^2\right]$$

$$\phi := 0$$

```
>
> B:=curl(A,[x,y,z]);EP:=evalm(-grad(phi,[x,y,z]));EV:=-[diff(A[1],t),diff(A[2],t),diff(A[3],t)];E:=evalm(EV+EP);Parity:=innerprod(E,B);Torsion_current:=evalm(crossprod(E,A)+evalm(B*phi));Helicity:=innerprod(A,B);
>
```

$$B := \left[6\,x\,t^2 + \frac{x^4\,y\,t^2}{z^2},\ -3\,\frac{x^3\,y^2\,t^2}{z^2} - 6\,y\,t^2,\ -2\,\frac{x^3\,y\,t^2}{z}\right]$$

$$EP := [0, 0, 0]$$

$$EV := \left[-6\,\frac{x^3\,y^2\,t}{z},\ -2\,\frac{x^4\,y\,t}{z},\ -12\,x\,y\,t\right]$$

$$E := \left[-6\,\frac{x^3\,y^2\,t}{z},\ -2\,\frac{x^4\,y\,t}{z},\ -12\,x\,y\,t\right]$$

$$Parity := 0$$

$$Torsion\_current := [0, 0, 0]$$

$$Helicity := 0$$

```
>
```

The potentials produce a non-zero, but static E and B field

```
> lambda:=(A[1]^2+A[2]^2+A[3]^2+A4^2)^(1/2);
>
```

$$\lambda := \sqrt{9\,\frac{x^6\,y^4\,t^4}{z^2} + \frac{x^8\,y^2\,t^4}{z^2} + 36\,x^2\,y^2\,t^4}$$

```
> NA:=evalm([A1,A2,A3,phi]/lambda);
```

$$NA := \left[3\,\frac{x^3\,y^2\,t^2}{\sqrt{9\,\frac{x^6\,y^4\,t^4}{z^2} + \frac{x^8\,y^2\,t^4}{z^2} + 36\,x^2\,y^2\,t^4}\ z},\ \frac{x^4\,y\,t^2}{\sqrt{9\,\frac{x^6\,y^4\,t^4}{z^2} + \frac{x^8\,y^2\,t^4}{z^2} + 36\,x^2\,y^2\,t^4}\ z},\ 6\,\frac{x\,y\,t^2}{\sqrt{9\,\frac{x^6\,y^4\,t^4}{z^2} + \frac{x^8\,y^2\,t^4}{z^2} + 36\,x^2\,y^2\,t^4}},\ 0\right]$$

```
> JAC:=jacobian(NA,[x,y,z,t]):
```

The Jacobian matrix has not been printed, but now there are entries in more than 1 row.

```
> MEAN_CURVATURE:=factor(trace(JAC));
```

$$MEAN\_CURVATURE := -6\,\frac{x^4\,t^6\,y^3\,(2\,x^6\,y - 36\,y\,z^2 - 9\,x^3\,y^2 - x^5)}{\left(\frac{x^2\,y^2\,t^4\,(9\,x^4\,y^2 + x^6 + 36\,z^2)}{z^2}\right)^{(3/2)}\,z^3}$$

```
> S2:=factor(trace(innerprod(JAC,JAC))):
Gauss:=factor(-(1/2)*(-trace(JAC)*trace(JAC)+S2));
```

$$Gauss := -36\,\frac{(2\,y\,x^3+9\,z^2)\,x^4}{(9\,x^4\,y^2+x^6+36\,z^2)^2}$$

```
> ADJAC:=adjoint(JAC):
```

The Adjoint is no longer the Zero matrix, and has a non-zero trace implying the existence of Adjoint curvature.

```
> ADJOINT_CURVATURE:=factor(trace(ADJAC));
```

$$ADJOINT\_CURVATURE := 0$$

```
>
```

None of the similarity invariants depend upon the first factor alpha.

```
> CurrentJ:=innerprod(ADJAC,NA);Interaction:=innerprod(CurrentJ,NA);DivJ:=factor(diverge(CurrentJ,[x,y,z,t]));rho:=CurrentJ[4];J1:=factor(CurrentJ[1]);
```

$$CurrentJ := [0,0,0,0]$$

$$Interaction := 0$$

$$DivJ := 0$$

$$\rho := 0$$

$$J1 := 0$$

It is now apparent that the interaction between the potentials and the conserved current is equal to the adjoint curvature of the implicit surface. The Bateman system produces a non-zero charge density, and a non-zero current density.

```
>
> Adjointcurv:=simplify(factor(trace(ADJAC))):Mean:=factor(trace(JAC)):Net:=factor(Adjointcurv-Interaction):Gnet:=factor(subs(a=1,b=1,c=1,p=2,n=1,Net));
```

$$Gnet := 0$$

```
> Vorticity:=curl(A,[x,y,z]);Helicity:=factor(innerprod(A,Vorticity));Parity:=innerprod(E,B);Diss:=factor(innerprod([CurrentJ[1],CurrentJ[2],CurrentJ[3]],E));
```

$$Vorticity := \left[6\,x\,t^2+\frac{x^4\,y\,t^2}{z^2},\,-3\,\frac{x^3\,y^2\,t^2}{z^2}-6\,y\,t^2,\,-2\,\frac{x^3\,y\,t^2}{z}\right]$$

$$Helicity := 0$$

$$Parity := 0$$

$$Diss := 0$$

```
> GMN:=innerprod(transpose(JAC),JAC):DETGMN:=det(GMN);
>
```

$$DETGMN := 0$$

The induced metric is a long expression, and printing has been suppresed. It is a singular metric.

```
>
```

A Hopf surface

The 1-form is modeled after the Hopf map 1-form. In the example, constant coefficients are used to show that topological torsion and topological pairity need not be zero. By permuting variables and signs it is to be observed that there are 2 distinct pairs of triples of Hopf implicit surfaces. Three with positive parity and three with negative parity. (orientation) For more details, see http://www22.pair.com/csdc/pdf/vig2000.pdf

```
> A1:=a*y;A2:=-a*x;A3:=b*t;A4:=-b*z;
```

$$A1 := a\,y$$

$$A2 := -a\,x$$

$$A3 := b\,t$$

$$A4 := -b\,z$$

```
>
> A:=evalm([A1,A2,A3]);phi:=-A4;
```

$$A := [a\,y, -a\,x, b\,t]$$

$$\phi := b\,z$$

```
> B:=curl([A[1],A[2],A[3]],[x,y,z]);EP:=evalm(-grad(phi,[x,y,z]));EV:=-[diff(A[1],t),diff(A[2],t),diff(A[3],t)];E:=evalm(EV+EP);Parity:=innerprod(E,B);Torsion_current:=evalm(crossprod(E,A)+evalm(B*phi));Helicity:=innerprod(A,B);
>
```

$$B := [0, 0, -2\,a]$$

$$EP := [0, 0, -b]$$

$$EV := [0, 0, -b]$$

$$E := [0, 0, -2\,b]$$

$$Parity := 4\,b\,a$$

$$Torsion\_current := [-2\,b\,a\,x, -2\,b\,a\,y, -2\,b\,z\,a]$$

$$Helicity := -2\,b\,t\,a$$

The potentials produce a non-zero, but static E and B field. It ie noteworthy that the two fields are PARALLEL or ANTI PARALLEL depending on the signs of A3 and A4, or A1 and A2. That is A1= -y, A2 = +x has negative parity compared to the example under study.

```
> lambda:=factor(subs(a^2=1,b^2=1,A[1]^2+A[2]^2+A[3]^2+phi^2))^(1/2);
>
```

$$\lambda := \sqrt{y^2+x^2+t^2+z^2}$$

```
> NA:=evalm([A1,A2,A3,-phi]/lambda);
```

$$NA := \left[\frac{a\,y}{\sqrt{y^2+x^2+t^2+z^2}}, -\frac{a\,x}{\sqrt{y^2+x^2+t^2+z^2}}, \frac{b\,t}{\sqrt{y^2+x^2+t^2+z^2}}, -\frac{b\,z}{\sqrt{y^2+x^2+t^2+z^2}}\right]$$

```
>
> JAC:=jacobian(NA,[x,y,z,t]):det(JAC);`curvatures`:=(solve(factor(minpoly(JAC,alpha))=0,alpha));
```

$$0$$

$$curvatures := 0, 0, \frac{\sqrt{-a^2\,t^2-a^2\,z^2-b^2\,x^2-b^2\,y^2}}{y^2+x^2+t^2+z^2}, -\frac{\sqrt{-a^2\,t^2-a^2\,z^2-b^2\,x^2-b^2\,y^2}}{y^2+x^2+t^2+z^2}$$

The minimum polynomial indicates that two eigen values (curvatures) are zero and two are pure imaginary conjugates.
The sum of the two imaginary conjugates gives zero (a Minimal Surface) but the Gauss curvature is positive as the product of the two imaginary curvatures.

```
> MEAN_CURVATURE:=factor(subs(trace(JAC)));
```

$$MEAN\_CURVATURE := 0$$

```
> S2:=factor(trace(innerprod(JAC,JAC))):
Gauss:=factor(subs((-(1/2)*(-trace(JAC)*trace(JAC)+S2))));
```

$$Gauss := \frac{a^2\,t^2+a^2\,z^2+b^2\,y^2+b^2\,x^2}{(y^2+x^2+t^2+z^2)^2}$$

```
> ADJAC:=adjoint(JAC):
> ADJOINT_CURVATURE:=factor(subs(trace(ADJAC)));
```

$$ADJOINT\_CURVATURE := 0$$

```
>
> CurrentJ:=((innerprod(ADJAC,NA)));Interaction:=innerprod(CurrentJ,NA);DivJ:=factor(diverge(CurrentJ,[x,y,z,t]));rho:=CurrentJ[4];parity:=innerprod(E,B);Diss:=innerprod([CurrentJ[1],CurrentJ[2],CurrentJ[3]],E);
```

$$CurrentJ := \left[\frac{a^2\, x\, b^2}{(y^2+x^2+t^2+z^2)^2}, \frac{a^2\, b^2\, y}{(y^2+x^2+t^2+z^2)^2}, \frac{a^2\, b^2\, z}{(y^2+x^2+t^2+z^2)^2}, \frac{a^2\, b^2\, t}{(y^2+x^2+t^2+z^2)^2}\right]$$

$$Interaction := 0$$

$$DivJ := 0$$

$$\rho := \frac{a^2\, b^2\, t}{(y^2+x^2+t^2+z^2)^2}$$

$$parity := 4\, b\, a$$

$$Diss := -2\,\frac{a^2\, b^3\, z}{(y^2+x^2+t^2+z^2)^2}$$

```
> )`AdotJ`:=CurrentJ[1]*A[1]+CurrentJ[2]*A[2]+CurrentJ[3]*A[3];rho_phi:=rho*phi
  ;
```

$$AdotJ := \frac{a^2\, b^3\, z\, t}{(y^2+x^2+t^2+z^2)^2}$$

$$\sim rho\_phi := \frac{a^2\, b^3\, z\, t}{(y^2+x^2+t^2+z^2)^2}$$

What is remarkable about the Hopf map is that the Jacobian matrix has two zero curvatures and two imaginary equal and opposite curvatures. Yet the Gauss sectional curvaure is real and positive. The surface is an "imaginary 2 surface" in 4D. IT can have both a right handed and a left handed realization. The right and left handed concepts relate to the parity (whether E and B are parallel or anti-parallel) and to orientation of the 4 form, F^F.

```
>
> Adjointcurv:=simplify(factor(trace(ADJAC))):Mean:=factor(trace(JAC)):Net:=fac
  tor(Adjointcurv-Interaction):Gnet:=factor(subs(c=1,n=1,Net));
```

$$Gnet := 0$$

```
> Vorticity:=curl([A[1],A[2],A[3]],[x,y,z]);Helicity:=factor(innerprod([A[1],A[
  2],A[3]],Vorticity));Torsion_current:=evalm(crossprod(E,A)+evalm(B*phi));Pari
  ty:=innerprod(E,B);
```

$$Vorticity := [0, 0, -2\, a]$$

$$Helicity := -2\, b\, t\, a$$

$$Torsion\_current := [-2\, b\, a\, x, -2\, b\, a\, y, -2\, b\, z\, a]$$

$$Parity := 4\, b\, a$$

When a=b the charge density is positive but time dependent, parity is zero and the torsion current is not equal to the adjoint current. The E field is zero, but the magnetic field is not zero.
The interaction energy is zero, the odd curvatures are zero, and the gauss curvature is that of a 4D euclidean sphere.

If a = -b non of the curvatures are zero, the parity is not zero, and the charge density is negative, and the torsion current is proportional to the charge-current density.

```
> GMN:=subs(a^2=1,b^2=1,innerprod(transpose(JAC),JAC));DETGMN:=det(GMN);
```

$GMN :=$

$$\left[\frac{t^4+t^2x^2+2\,t^2z^2+2\,t^2y^2+x^2z^2+y^4+z^4+y^2x^2+2\,y^2z^2}{(y^2+x^2+t^2+z^2)^3}, -\frac{y\,x}{(y^2+x^2+t^2+z^2)^2}, -\frac{x\,z}{(y^2+x^2+t^2+z^2)^2}, -\frac{t\,x}{(y^2+x^2+t^2+z^2)^2}\right]$$

$$\left[-\frac{y\,x}{(y^2+x^2+t^2+z^2)^2}, \frac{t^4+t^2y^2+2\,t^2z^2+2\,t^2x^2+x^4+y^2z^2+2\,x^2z^2+z^4+y^2x^2}{(y^2+x^2+t^2+z^2)^3}, -\frac{y\,z}{(y^2+x^2+t^2+z^2)^2},\right.$$

$$-\frac{t\,y}{(y^2+x^2+t^2+z^2)^2}\Bigg]$$

$$\Bigg[-\frac{x\,z}{(y^2+x^2+t^2+z^2)^2},\ -\frac{y\,z}{(y^2+x^2+t^2+z^2)^2},\ \frac{t^4+t^2z^2+2\,t^2y^2+2\,t^2x^2+y^2z^2+y^4+x^4+2\,y^2x^2+x^2z^2}{(y^2+x^2+t^2+z^2)^3},$$
$$\frac{z\,t\,(-t^2-y^2-z^2-x^2)}{(y^2+x^2+t^2+z^2)^3}\Bigg]$$

$$\Bigg[-\frac{t\,x}{(y^2+x^2+t^2+z^2)^2},\ -\frac{t\,y}{(y^2+x^2+t^2+z^2)^2},\ \frac{z\,t\,(-t^2-y^2-z^2-x^2)}{(y^2+x^2+t^2+z^2)^3},$$
$$\frac{t^2x^2+t^2y^2+t^2z^2+z^4+x^4+2\,y^2x^2+y^4+2\,y^2z^2+2\,x^2z^2}{(y^2+x^2+t^2+z^2)^3}\Bigg]$$

$$DETGMN := 0$$

The induced metric is singular and is rank 2, not 3.

\>

\>

A Hopf surface2

The 1-form is modeled after the Hopf map 1-form. In the example, constant coefficients are used to show that topological torsion and topological pairity need not be zero. By permuting variables and signs it is to be observed that there are 2 distinct pairs of triples of Hopf implicit surfaces. Three with positive parity and three with negative parity. (orientation) For more details, see http://www22.pair.com/csdc/pdf/vig2000.pdf

\>

```
> A1:=2*t*y-2*x*z;A2:=2*y*z+2*t*x;A3:=x^2+y^2-t^2-z^2;A4:=0*(x^2+y^2+z^2+t^2);
```

$$A1 := 2\,t\,y-2\,x\,z$$
$$A2 := 2\,y\,z+2\,t\,x$$
$$A3 := x^2+y^2-t^2-z^2$$
$$A4 := 0$$

\>

```
> A:=evalm([A1,A2,A3]);phi:=-A4;
```

$$A := [2\,t\,y-2\,x\,z,\ 2\,y\,z+2\,t\,x,\ x^2+y^2-t^2-z^2]$$
$$\phi := 0$$

```
> B:=curl([A[1],A[2],A[3]],[x,y,z]);EP:=evalm(-grad(phi,[x,y,z]));EV:=-[diff(A[1],t),diff(A[2],t),diff(A[3],t)];E:=evalm(EV+EP);Parity:=innerprod(E,B);Torsion_current:=evalm(crossprod(E,A)+evalm(B*phi));Helicity:=innerprod(A,B);
>
```

$$B := [0,\ -4\,x,\ 0]$$
$$EP := [0,\ 0,\ 0]$$
$$EV := [-2\,y,\ -2\,x,\ 2\,t]$$
$$E := [-2\,y,\ -2\,x,\ 2\,t]$$
$$Parity := 8\,x^2$$
$$Torsion\_current := [-2\,x\,(x^2+y^2-t^2-z^2)-2\,t\,(2\,y\,z+2\,t\,x),\ 2\,t\,(2\,t\,y-2\,x\,z)+2\,y\,(x^2+y^2-t^2-z^2),$$
$$-2\,y\,(2\,y\,z+2\,t\,x)+2\,x\,(2\,t\,y-2\,x\,z)]$$
$$Helicity := -8\,(y\,z+t\,x)\,x$$

The potentials produce a non-zero, but static E and B field. It ie noteworthy that the two fields are PARALLEL or ANTI PARALLEL depending on the signs of A3 and A4, or A1 and A2. That is A1= -y, A2 = +x has negative parity compared to the example under study.

```
> lambda:=factor(subs(b=1,a=1,A[1]^2+A[2]^2+A[3]^2+phi^2))^(1/2);
>
```

$$\lambda := \sqrt{(y^2+x^2+t^2+z^2)^2}$$

```
> NA:=evalm([A1,A2,A3,-phi]/lambda);
```

$$NA := \left[\frac{2\,t\,y-2\,x\,z}{\sqrt{(y^2+x^2+t^2+z^2)^2}}, \frac{2\,y\,z+2\,t\,x}{\sqrt{(y^2+x^2+t^2+z^2)^2}}, \frac{x^2+y^2-t^2-z^2}{\sqrt{(y^2+x^2+t^2+z^2)^2}}, 0\right]$$

```
>
> JAC:=jacobian(NA,[x,y,z,t]):det(JAC);MM:=minpoly(JAC,beta);CC:=coeff(MM,beta)
;BB:=(coeff(MM,beta^2));R1:=-subs(t=0,lambda^2*BB)+(factor(subs(t=0,(lambda^4
*(simplify(BB*BB)-4*CC)))))^(1/2);
>
>
```

$$0$$

$$MM := -4\frac{(t^4+2\,t^2\,z^2-3\,t^2\,x^2+t^2\,y^2-8\,y\,x\,z\,t-3\,y^2\,z^2+z^4+x^2\,z^2)\,\beta}{(y^2+x^2+t^2+z^2)^3}+8\frac{(y^2+x^2+t^2+z^2)\,y\,(y\,z+t\,x)\,\beta^2}{((y^2+x^2+t^2+z^2)^2)^{(3/2)}}+\beta^3$$

$$CC := -4\frac{t^4+2\,t^2\,z^2-3\,t^2\,x^2+t^2\,y^2-8\,y\,x\,z\,t-3\,y^2\,z^2+z^4+x^2\,z^2}{(y^2+x^2+t^2+z^2)^3}$$

$$BB := 8\frac{(y^2+x^2+t^2+z^2)\,y\,(y\,z+t\,x)}{((y^2+x^2+t^2+z^2)^2)^{(3/2)}}$$

$$R1 := -8\frac{(x^2+y^2+z^2)^3\,y^2\,z}{((x^2+y^2+z^2)^2)^{(3/2)}}+\sqrt{16}\,\sqrt{z^2\,(z^2+x^2-y^2)^2}$$

The minimum polynomial indicates that two eigen values (curvatures) are zero and two are pure imaginary conjugates.
The sum of the two imaginary conjugates gives zero (a Minimal Surface) but the Gauss curvature is positive as the product of the two imaginary curvatures.

```
> MEAN_CURVATURE:=factor(subs(t=t,trace(JAC)));
```

$$MEAN\_CURVATURE := -8\frac{(y^2+x^2+t^2+z^2)\,y\,(y\,z+t\,x)}{((y^2+x^2+t^2+z^2)^2)^{(3/2)}}$$

```
> S2:=factor(trace(innerprod(JAC,JAC))):
Gauss:=factor(subs(t=t,(-(1/2)*(-trace(JAC)*trace(JAC)+S2))));
```

$$Gauss := -4\frac{t^4+2\,t^2\,z^2-3\,t^2\,x^2+t^2\,y^2-8\,y\,x\,z\,t-3\,y^2\,z^2+z^4+x^2\,z^2}{(y^2+x^2+t^2+z^2)^3}$$

```
> ADJAC:=adjoint(JAC):
> ADJOINT_CURVATURE:=factor(subs(trace(ADJAC)));
```

$$ADJOINT\_CURVATURE := 0$$

```
>
> CurrentJ:=innerprod(ADJAC,NA);Interaction:=innerprod(CurrentJ,NA);DivJ:=facto
r(diverge(CurrentJ,[x,y,z,t]));rho:=CurrentJ[4];innerprod(E,B);Diss:=innerpro
d([CurrentJ[1],CurrentJ[2],CurrentJ[3]],E);
```

$$CurrentJ := [0, 0, 0, 0]$$

$$Interaction := 0$$

$$DivJ := 0$$

$$\rho := 0$$

$$8\,x^2$$

$$Diss := 0$$

>

What is remarkable about the Hopf map is that the Jacobian matrix has two zero curvatures and two imaginary equal and opposite curvatures. Yet the Gauss sectional curvature is real and positive. The surface is an "imaginary 2 surface" in 4D. IT can have both a right handed and a left handed realization. The right and left handed concepts relate to the parity (whether E and B are parallel or anti-parallel) and to orientation of the 4 form, F^F.

>

```
> Adjointcurv:=simplify(factor(trace(ADJAC))):Mean:=factor(trace(JAC)):Net:=factor(Adjointcurv-Interaction):Gnet:=factor(subs(c=1,n=1,Net));
```

$$Gnet := 0$$

```
> Vorticity:=curl([A[1],A[2],A[3]],[x,y,z]);Helicity:=factor(innerprod([A[1],A[2],A[3]],Vorticity));Torsion_current:=evalm(crossprod(E,A)+evalm(B*phi));Parity:=innerprod(E,B);
```

$$Vorticity := [0, -4\,x, 0]$$

$$Helicity := -8\,(y\,z + t\,x)\,x$$

$$Torsion\_current := [-2\,x\,(x^2+y^2-t^2-z^2) - 2\,t\,(2\,y\,z+2\,t\,x),\ 2\,t\,(2\,t\,y-2\,x\,z) + 2\,y\,(x^2+y^2-t^2-z^2),\ -2\,y\,(2\,y\,z+2\,t\,x) + 2\,x\,(2\,t\,y-2\,x\,z)]$$

$$Parity := 8\,x^2$$

When a=b the charge density is positive but time dependent, parity is zero and the torsion current is not equal to the adjoint current. The E field is zero, but the magnetic field is not zero.
The interaction energy is zero, the odd curvatures are zero, and the gauss curvature is that of a 4D euclidean sphere.

If a = -b non of the curvatures are zero, the parity is not zero, and the charge density is negative, and the torsion current is proportional to the charge-current density.

```
> GMN:=innerprod(transpose(JAC),JAC);DETGMN:=det(GMN);
```

$$GMN := \begin{bmatrix} 4\dfrac{t^2+z^2}{(y^2+x^2+t^2+z^2)^2} & 0 & 4\dfrac{t\,y-x\,z}{(y^2+x^2+t^2+z^2)^2} & -4\dfrac{y\,z+t\,x}{(y^2+x^2+t^2+z^2)^2} \\ 0 & 4\dfrac{t^2+z^2}{(y^2+x^2+t^2+z^2)^2} & -4\dfrac{y\,z+t\,x}{(y^2+x^2+t^2+z^2)^2} & -4\dfrac{t\,y-x\,z}{(y^2+x^2+t^2+z^2)^2} \\ 4\dfrac{t\,y-x\,z}{(y^2+x^2+t^2+z^2)^2} & -4\dfrac{y\,z+t\,x}{(y^2+x^2+t^2+z^2)^2} & 4\dfrac{y^2+x^2}{(y^2+x^2+t^2+z^2)^2} & 0 \\ -4\dfrac{y\,z+t\,x}{(y^2+x^2+t^2+z^2)^2} & -4\dfrac{t\,y-x\,z}{(y^2+x^2+t^2+z^2)^2} & 0 & 4\dfrac{y^2+x^2}{(y^2+x^2+t^2+z^2)^2} \end{bmatrix}$$

$$DETGMN := 0$$

The induced metric is singular and is rank 2, not 3.

>

>

>

>