NEW TRICKS FROM AN OLD DOG

Topological Features of Maxwell's, Equations - without metric, connection or gauge group constraints - lead to new ideas.

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Full Article as a pdf download at

http://www.un.edu/~rkiehn/pdf/ine0899.pdf

(with example solutions to Maxwell's Equations)

Electromagnetism $\cong$ Two Topological Species

Existence of Potentials $\{\mathbf{A}, \phi\}$ implies

the domain of support for

Field Intensities $\mathbf{F}(\mathbf{E}, \mathbf{B})$

$$\mathbf{A} \Rightarrow \mathbf{F} = d\mathbf{A}$$

NOT COMPACT without BOUNDARY

(Open or Compact with boundary.)

Conserved Charge-Current density $\{\mathbf{J}, \rho\}$

implies the existence

Field Excitations $\mathbf{G}(\mathbf{D}, \mathbf{H})$

$$\mathbf{G} \Rightarrow \mathbf{J} = d\mathbf{G}$$

the domain of support for $\mathbf{G}(\mathbf{D}, \mathbf{H})$

can be compact without boundary.

Without gross modifications, classic EM theory over the Reals, Complex functions, or Quaternions leads to the little utilized fields (currents) of

Topological Torsion $:= \mathbf{T}_4$

and Spin$:= \mathbf{S}_4$

In engineering format:

$$\boldsymbol{S}_4 = [\mathbf{A} \times \mathbf{H} + \phi\mathbf{D}, \mathbf{A} \bullet \mathbf{D}]$$

$$\boldsymbol{T}_4 = [\mathbf{E} \times \mathbf{A} + \phi\mathbf{B}, \mathbf{A} \bullet \mathbf{B}]$$

In differential form language:

$$\mathbf{Spin}_{3-form} = \mathbf{A\hat{}G}$$
$$\mathbf{Torsion}_{3-form} = \mathbf{A\hat{}F}$$

Conservation Laws for

Torsion and Spin

Consider

$$4Div\ \mathbf{S_4} \equiv d(A\wedge G) = Poincare1$$

$$4Div\ \mathbf{T_4} \equiv d(A\wedge F) = Poincare2$$

Where in engineering format:

Poincare1:= $(\mathbf{B} \bullet \mathbf{H} - \mathbf{D} \bullet \mathbf{E}) - (\mathbf{A} \bullet \mathbf{J} - \rho\phi)$

Poincare2:= $2\{\mathbf{E} \bullet \mathbf{B}\}$

Over domains where the Poincare invariants vanish, the integrals of the Topological Torsion and Spin integrals form long lived

Coherent Structures

in plasmas

Transverse Waves imply

$$\boldsymbol{A}^\wedge\boldsymbol{F} = \mathbf{0} \supset \boldsymbol{T}_\mathbf{4} = \mathbf{0}$$

(Topological Torsion vanishes)

Waves with longitudinal components require

$$\boldsymbol{A}^\wedge\boldsymbol{F} \neq \mathbf{0} \supset \boldsymbol{T}_\mathbf{4} \neq \mathbf{0}$$

(Topological Torsion NOT zero)

Similarly ..for Magnetic Links and Knots.

The Source of Topological Torsion, **Magnetic Helicity**, **Links and Knots is Poincare2** $\approx -2\{\mathbf{E} \bullet \mathbf{B}\} \neq \mathbf{0}$.

Note: Magnetic Helicity is only the 4th component of a third rank tensor field.

Evolutionary Processes

Define an evolutionary process **V** in terms of a 4-vector field on space time.Evolution of differential forms obey

Cartan's Magic Formula

$$\boldsymbol{L}(\boldsymbol{V})\boldsymbol{A} = \boldsymbol{i}(\boldsymbol{V})\boldsymbol{dA} + \boldsymbol{d}(\boldsymbol{i}(\boldsymbol{V})\boldsymbol{A})$$

$$= \boldsymbol{W} + \boldsymbol{dU} = \boldsymbol{Q}$$

Define a Plasma process as a process that preserves the number of charges.

$$\boldsymbol{L}_{(\boldsymbol{V})}\boldsymbol{G} = \boldsymbol{0} \supset \boldsymbol{J} = \rho\boldsymbol{V}$$

The Ideal Plasma Process.

Define an Ideal Plasma process as a process that preserves the number of charges and is Force Free (the Virtual Work vanishes).

1. $\mathbf{L}_{(\rho\mathbf{V})}\mathbf{G} = \mathbf{0} \supset \mathbf{J} = \rho\mathbf{V}$.
2. $\mathbf{W} = \mathbf{i}(\rho\mathbf{V})\mathbf{dA} = \mathbf{0} \supset \mathbf{E} + \mathbf{V} \times \mathbf{B} = \mathbf{0}$.

Zero Virtual Work implies

$$\mathbf{E} \bullet \mathbf{B} = \mathbf{0}$$

Hence the closed integrals of Torsion current, $\mathbf{T}_4$, are topological invariants of such *Extremal* processes.

Semi-Ideal Plasma Processes.

Define a Semi-Ideal Plasma process as a process that preserves the number of charges, and for which the Virtual Work is exact. (Symplectic, NOT extremal)

1. $\mathbf{L}_{(\rho\mathbf{V})}\mathbf{G} = \mathbf{0} \supset \mathbf{J} = \rho\mathbf{V}$.
2. $\mathbf{W} = \mathbf{i}(\rho\mathbf{V})\mathbf{dA} = \mathbf{d}\Theta \supset \mathbf{E} + \mathbf{V} \times \mathbf{B} = \nabla\Theta$.

For the Virtual Work to be exact

$$\mathbf{E} \bullet \mathbf{B} = \mathbf{B} \bullet \mathbf{grad}\Theta$$

$$\mathbf{E} \bullet \mathbf{V} = \mathbf{V} \bullet \mathbf{grad}\Theta$$

(Compare to Hornig and Schindler)

If $\mathbf{B} \bullet \mathbf{grad}\Theta \neq \mathbf{0}$, then $\mathbf{E} \bullet \mathbf{B} \neq \mathbf{0}$ and it is possible to induce topological change in the Torsion integral, and dissipation.

$\boldsymbol{Grad(\Theta) \bullet B \neq 0}$ induces a dynamo action.

Suppose that the Bernoulli function is

$$\Theta = kT$$

and the temperature gradient has a component in the direction of the **B** field. Such a temperature gradient would be consistent with the production of an Ohmic current flow, and an **E** field parallel to the **B** field.

Conjecture: **Jets** $\supset grad(kT) \bullet \mathbf{B} \neq 0$

Large temperature gradients along the **B** field lines can act as a source of stellar plasma jets in rotating, magnetic neutron stars. (A dynamo effect in a dissipative media).

From Classical thermodyamics: If Heat, **Q**, does not admit an integrating factor the process is irreversible. Then from

$$\textbf{Irreversible} \quad \supset Q \wedge dQ \neq 0,$$

Theorem 1:

All symplectic processes are reversible.

Theorem 2:

Plasma Processes in the direction of the Torsion Current T_4 are Irreversible.

Theorem 3:

Plasma processes in the direction $\mathbf{T}_4$ leave both the $\mathbf{T}_4$ and $\mathbf{S}_4$ ”frozen in”.

$\mathbf{T_4}$ direction field is unique on 4D

The Torsion Current direction field is uniquely defined by the 1-form **A** that defines the Action, for systems of Even Pfaff dimension. This unique Conformal field for even Pfaff dimension is the dual of the Hamiltonian Extremal field in the odd Pfaff dimension case.

1. $\mathbf{L}_{(\mathbf{T}_4)}\mathbf{A} = \Gamma\,\mathbf{A}$
2. $\mathbf{U} = \mathbf{i}(\mathbf{T}_4)\mathbf{A} = \mathbf{0}$
3. For irreversible dissipation
   $\Gamma \cong Div_4(\mathbf{T}_4) = -2\mathbf{E} \bullet \mathbf{B} \neq \mathbf{0}$

For processes in the direction of the Torsion field, the fields decay in a conformal manner, even in the dissipative irreversible regime. The decay can asymptotically approach a long lived ”steady state”.