Dissipation, Irreversibility and Symplectic Lagrangian Systems on Thermodynamic Space of Dimension 2n+2

R. M. Kiehn

Emeritus, Physics Dept., Univ. Houston
12/06/1996 Updated 09/18/2001
http://www.cartan.pair.com
rkiehn2352@aol.com

Abstract

Recent activity in topological classifications of closed symplectic integrable Hamiltonian systems focuses attention on those properties of a Lagrangian formulation for which the fundamental 2-form is exact. The Lagrangian formulation, based on a Cartan-Hilbert Action which has n degrees of freedom, leads to an unconstrained symplectic system which is dissipative and of dimension 2n+2. Canonical momentum constraints lead to a contact submanifold of dimension 2n+1 with a unique extremal field. If the 2n+2 symplectic system is to exist, it is necessary that the momenta are not defined canonically, $\partial L/\partial v - p \neq 0$, and that there must exist anholonomic differential fluctuations $\Delta v = dv - Adt \neq 0$ in the velocity and/or in position, $\Delta x = dx - Vdt \neq 0$. The implication is that (non-extremal) evolution on the 2n+2 symplectic domain can be dissipative but the process is not described kinematically in terms of a single parameter group. The fluctuations in velocity lead to non-zero temperature gradients and the fluctuations in position lead to non-zero pressure gradients. Both types of fluctuations lead to distinct contributions to a zero point energy. These 2n+2 domains can act as a source of magnetic dynamo action in a plasma, where velocity fluctuations associated with temperature produce a charge acceleration mechanism in regions where $\mathbf{E} \bullet \mathbf{B} \neq 0.$ Anholonomic differential fluctuations in position lead to the dissipative terms in the Navier-Stokes equations. Using the fact that Cartan's Lie derivative of the Action with respect to a vector field $\mathbf{V}$ is a cohomological equivalent to the First Law of Thermodynamics, it is possible to decide if a given process $\mathbf{V}$ is irreversible or not. On the 2n+2 symplectic domain, defined as Thermodynamic Space, two distinct evolutionary processes may be defined in terms of the Adiabatic Vector and the Torsion Current. The first process is a symplectomorphism, and therefore is reversible; the second process is not a symplectomorphism, and is irreversible in a thermodynamic sense.

1. INTRODUCTION

The objective of this article is to construct a non-statisical theory of irreversibility and develop methods to describe processes that decay into stationary or equilibrium states. In 1974 it was suggested that a certain extension to Hamilton's principle [1] could be made such that the evolutionary processes considered would describe dissipative systems. In short, rather than study those extremal vector fields that satisfy the Cartan-Hamilton equation, $i(\mathbf{V})d\mathcal{A} = 0$, it was suggested to consider those processes that satisfy the extended equation $i(\mathbf{V})d\mathcal{A} = \Gamma\mathcal{A} + d\theta$. Both in this current article and in the older article, it is subsumed that a physical system may be described adequately by a 1-form of Action, $\mathcal{A}$, and a physical process may be defined in terms of a dynamical system generated by a vector field, $\mathbf{V}$.

It was not appreciated in 1974 that the topological domain of the extremal conservative systems was a contact manifold of odd dimension, while the topological domain of the suggested dissipative extension was a symplectic manifold of even dimension. In an attempt to understand more recent developments in "Hamiltonian" symplectic topology, especially the successful classifications of topological defects, or invariants, on compact domains [2], it was decided to investigate the modern symplectic developments starting from a Lagrangian (rather than Hamiltonian) point of view. The Lagrangian point of view has its advantages, for the fundamental 2-form is deduced by construction, $\omega = d\mathcal{A}$. The disadvantage is that all symplectic domains so constructed are not compact without boundary. This apparent flaw becomes an advantage when it is appreciated that such non-compact domains are precisely that which is needed to describe closed but not isolated, or open thermodynamic systems.

On a symplectic domain of dimension 2n+2, unique ubiquitous extremal fields of classical Hamiltonian mechanics do not exist. There are no solutions to the extremal equation $i(\mathbf{V})d\mathcal{A} = 0$, on the symplectic domain, but there do exist *non-unique* vector fields $\mathbf{V}$ that satisfy the Helmholtz constraint equation, $d(i(\mathbf{V})d\mathcal{A}) = 0$. In the subset of exact cases, where $i(\mathbf{V})d\mathcal{A} = d\Theta$, these vector fields generate "Hamiltonian" dynamical systems. or processes, (on the 2n+1 submanifold transversal to $d\Theta$), similar to the dynamical systems that are associated with the 2n+1 contact manifolds of classical State Space. The Action integral is a relative (stationary) integral invariant with respect to such Hamiltonian dynamical processes. The function Θ is a Bernoulli-Casimir evolutionary invariant, but these evolutionary invariants (stationary states) are not unique, not independent of gauge conditions, and strongly dependent upon boundary conditions, and are not constants over the domain. The somewhat larger class of vector fields that satisfy the Helmholtz condition $d(i(\mathbf{V})d\mathcal{A}) = 0$ are defined as symplectic vector fields, and as dynamical systems they define symplectic processes. However, the results to be described below imply that all such symplectic processes, exact or not, on symplectic domains of dimension 2n+2, still represent *reversible* thermodynamic processes.

Remarkably, on the 2n+2 symplectic domain there exists a *unique* non Hamiltonian vector field which leaves the Action integral a conformal, not stationary, invariant. This unique vector field, defined as the Torsion Vector, or Topological Torsion Current, does not satisfy the symplectic condition, but instead satisfies the equation, $i(\mathbf{V})d\mathcal{A} = \Gamma\mathcal{A}$ as suggested in the 1974 article. Moreover, it now can be demonstrated that this unique vector field generates dynamical systems that represent irreversible processes in a thermodynamic sense. This unique vector field (to within a factor) is generated by the Top Pfaffian of the Pfaff sequence associated with the Action 1-form, $\mathcal{A}$. The symplectic space of dimension 2n+2 on which the Torsion vector exists is defined as Thermodynamic Space, in order to distinguish it from the classic State Space of dimension 2n+1. The divergence of this Torsion vector field defines a density function on the 2n+2 space. The zero sets of this density function define smooth attractors (inertial maniufolds) of dimension 2n+1 on the 2n+2 dimensional domain. The irreverisible dynamical system generated by the Torsion vector irreversibly decays to these sets of measure zero which form the "stationary" states of a 2n+1 contact manifold. Once in the stationary state, the evolution can take place by a reversible Hamiltonian process.

1.1. Extremal Systems

Cartan's analysis of Hamiltonian extremal systems starts from the concept of an Action 1-form on a 2n+1 dimensional state space:

$$\mathcal{A} = p_k dq^k - H(p,q,t)dt. \tag{1.1}$$

Cartan demonstrates that any vector field, $\mathbf{E}$, that leaves the Action a relative integral invariant is a "Hamiltonian" vector field, and statisfies the extremal equations $i(\mathbf{E})dA = 0$. The components of $\mathbf{E}$ relative to the coordinates $\{p,q,t\}$ are given by the equations, $\{-\partial H/\partial q, \partial H/\partial p, 1\}$. The dynamical system is defined by the equations

$$\frac{dp}{-\partial H/\partial q} = \frac{dq}{\partial H/\partial p} = \frac{dt}{1} \tag{1.2}$$

By direct computation the Pfaff dimension of the Cartan Action 1-form is 2n+1 and the contact manifold volume element is given by the expression

$$\mathcal{A}\hat{}(d\mathcal{A})^n = \{p\partial H/\partial p - H(p,q,t)\}dq^1\hat{}...dq^n\hat{}dp_1\hat{}...dp^n\hat{}dt \tag{1.3}$$

The function $\rho_L(q,p,t) = \{p\partial H/\partial p - H(p,q,t)\}$ defines a Lagrange density on the 2n+1 state space. Its zero set reduces the Pfaff dimension to a 2n dimensional manifold of Phase Space.

If the function L is defined by a Legendre transformation as

$$L(v,q,t) = p_k v^k - H = p\partial H/\partial p - H(p,q,t) \equiv \rho_L(q,p(q,v,t),t) \tag{1.4}$$

then substitution into the Action 1-form leads to the expression,

$$\mathcal{A} = Ldt + p_k(dq^k - \{\partial H(p,q,t)/\partial p_k\}dt) = L(v,q,t)dt + p_k(dq^k - v^k dt). \tag{1.5}$$

The substitutions require that the Hamiltonian is NOT homogeneous of degree 1 in the p.

At first glance it would appear that the Cartan 1-form of Action is equal to the primitive Lagrange function integrand of the Calculus of variations, Ldt, constrained by the anoholonomic constraints, $(dq^k - v^k dt)$, with Lagrange multipliers, p_k. It is precisely this point of view that will be investigated below as the Lagrange formalism. The remarkable result is that the Pfaff dimension of the Lagrange formalism with anholonomic constraints and Lagrange multipliers, p_k, is 2n+2, and not 2n+1.

1.2. Extensions of the Cartan-Hilbert Action 1-form

This article considers those physical systems that can be described by a Lagrange function $L(\mathbf{q}, \mathbf{v}, t)$ and a 1-form of Action given by the expression:

$$\mathcal{A} = L(\mathbf{q}, \mathbf{v}, t)dt + \mathbf{p}\cdot(d\mathbf{q} - \mathbf{v}dt), \tag{1.6}$$

At first glance it appears that the domain of definition is a 3n+1 dimensional variety of independent variables, $\{\mathbf{q}, \mathbf{v}, \mathbf{p}, t\}$. Do not assume that $\mathbf{p}$ is constrained to be a jet; e.g., $\mathbf{p} \neq \partial L/\partial\mathbf{v}$. Instead, consider $\mathbf{p}$ to be a (set of) Lagrange multiplier(s) to be determined later. Note that the Action 1-form has the format used in the Cartan-Hilbert invariant integral [3], except that $\mathbf{p} \neq \partial L/\partial\mathbf{v}$ necessarily. Also, do not assume at this stage that $\mathbf{v}$ is a kinematic velocity function, such that $(d\mathbf{q} - \mathbf{v}dt) \Rightarrow 0$.

For the given Action, construct the Pfaff sequence $\{\mathcal{A}, d\mathcal{A}, \mathcal{A}\hat{}d\mathcal{A}, d\mathcal{A}\hat{}d\mathcal{A}...\}$ in order to determine the Pfaff dimension or class of the 1-form [4]. The top (non-zero) Pfaffian of this sequence is given by the formula,

$$(d\mathcal{A})^{n+1} = (n+1)!\{\Sigma_{k=1}^{n}(\partial L/\partial v^k - p_k) \bullet dv^k\}\hat{}dp_1\hat{}...dp_n\hat{}dq^1\hat{}..dq^n\hat{}dt, \tag{1.7}$$

which indicates that the Pfaff topological dimension is 2n+2 and not the geometrical dimension 3n+1, which might be expected as the 1-form was defined initially on a space of 3n+1 "independent" variables. The implication is that there exists an irreducible number of independent variables equal to 2n+2 which completely characterize the differential topology of the system. It follows that the exact two form $d\mathcal{A}$ satisfies the equations

$$(d\mathcal{A})^{n+1} \neq 0, \; but \quad \mathcal{A}\hat{}(d\mathcal{A})^{n+1} = 0. \tag{1.8}$$

The result is also true for any closed addition γ added to $\mathcal{A}$; e.g., the result is "gauge invariant". Addition of a closed 1-form does not change the Pfaff dimension from even to odd. On the otherhand the result is not renormalizeable, for multiplication of the Action 1-form by a function can change the algebraic Pfaff dimension from even to odd.

On the 2n+2 domain, the components of 2n+1 form $\mathcal{T} = \mathcal{A}\hat{}(d\mathcal{A})^n$ generate what is herein defined as the Torsion Current, a contravariant vector density, $\mathbf{T}^m$, whose line of action is the same as that of the Torsion vector mentioned above. The components of the "Torsion current" are orthogonal (transversal) to the 2n+2 components of the covector, $\mathbf{A}_m$, that make up the coefficients of the Action 1-form. In other words,

$$\mathcal{A}\hat{}\mathcal{T} = \mathcal{A}\hat{}(\mathcal{A}\hat{}(d\mathcal{A})^n) = 0 \Rightarrow i(\mathbf{T})(\mathcal{A}) = \mathbf{T}^m\mathbf{A}_m = 0. \tag{1.9}$$

This topological result does not depend upon geometric ideas such as metric. The Torsion Current will reappear below when it is demonstrated that evolution along the direction of the Torsion current is irreversible in a thermodynamic sense.

The 2n+2 symplectic domain so constructed can not be compact without boundary for it has a volume element which is exact. For the 2n+2 domain to be symplectic, the top Pfaffian (7) can never vanish. The domain is therefore orientable. Examination of the constraint that the symplectic space be of dimension 2n+2 implies that the Lagrange multipliers, $\mathbf{p}$, cannot be used to define momenta in the classical "conjugate or canonical" manner; e.g.,

$$\omega = (\partial L/\partial v^k - p_k) \bullet dv^k \neq 0 \tag{1.10}$$

However , the form ω must be closed; $d\omega = 0$.

$$\begin{aligned} d\omega &= \{\partial(dL)/\partial v^k\}\hat{}dv^k - dp_k\hat{}dv^k \\ &= \{\partial^2 L/\partial q^j\partial v^k\}dq^j\hat{}dv^k + \{\partial^2 L/\partial t\partial v^k\}dt\hat{}dv^k - dp_k\hat{}dv^k\} \\ &= [\{\partial^2 L/\partial q^j\partial v^k\}dq^j + \{\partial^2 L/\partial t\partial v^k\}dt - dp_k]\hat{}dv^k = 0. \end{aligned} \tag{1.11}$$

If, however, the constraints of canonical momenta are subsumed, such that $\partial L/\partial v^k - p_k = 0$, then the 2-form $d\mathcal{A}$ is not symplectic on its maximal dimension 2n+2, but instead the top Pfaffian defines a contact manifold on a state space of topological dimension 2n+1 with the formula

$$\mathcal{A}\hat{}(d\mathcal{A})^n = (n)!\{\Sigma_{k=1}^n(p_k - \partial L/\partial v^k) \bullet dv^k\}\hat{}\{i(p_m)dp_1\hat{}...dp_n\}\hat{}dq^1\hat{}..dq^n\hat{}dt \tag{1.12}$$

$$n!\{L(t,q,v\}dp_1\hat{}...dp_n\hat{}dq^1\hat{}..dq^n\hat{}dt \tag{1.13}$$

The Torsion current reduces to a single component on the contact manifold, when the momenta are defined canonically. It is this 2n+1 dimensional contact manifold that serves as the arena for most of classical mechanics prior to 1965, especially for those theories which were built from the calculus of variations. The 2n+1 dimensional contact manifold, or state space, admits a unique "extremal" evolutionary field, that satisfies "Hamilton's equations" $i(\mathbf{V})d\mathcal{A} = 0$. [5]. The coefficient of the state space volume is to be recognized as the Legendre transform of the physicist's Hamiltonian energy function.

$$L(t, q, v) = p_k v^k - H(t, q, v, p) \tag{1.14}$$

When the constraints of canonical momenta are valid, it follows that $\partial H(t, q, v, p)/\partial v = 0$, This result is interpreted by the statement that the "Hamiltonian" is to be expressed in terms of the variables $\{t, q, p\}$ only. The 2n+1 space maintains its contact structure as long as the "total Hamiltonian energy" is never zero, and the momenta are canonically defined. However, if the Lagrangian is homogeneous of degree 1 in the velocities, $\mathbf{v}$, and if the momenta are canonically defined such that $\partial L/\partial v^k = p_k$, then the top Pfaffian of the sequence, now doubly constrained, defines yet another non-compact symplectic manifold of Pfaff dimension 2n (a distinguished form of Phase Space). These aforementioned constraints are precisely Chern's constraints used to define a Finsler space which admits non-Riemannian geometries (when the Lagrange function contains more than quadratic powers of $\mathbf{v}$) and spaces with torsion.[3] Note that the processes of topological reduction described above are not equivalent to forming an arbitrary section(s) in the form of holonomic constraints.

For the maximal non-canonical symplectic physical system of Pfaff dimension 2n+2, consider evolutionary processes to be representable by vector fields of the form $\gamma \mathbf{W} = \gamma\{\mathbf{V}, \mathbf{A}, \mathbf{F}, 1\}$, relative to the independent variables $\{\mathbf{q}, \mathbf{v}, \mathbf{p}, t\}$. Define the "virtual work" 1-form, W, as $W = i(\mathbf{W})d\mathcal{A}$, a 1-form which must vanish for the extremal case, and be non-zero for the symplectic case. For any n, it may be shown by direct computation that the virtual work 1-form consists of three distinct terms,

$$W = \{\mathbf{p} - \partial L/\partial \mathbf{v}\} \cdot \Delta\mathbf{v} + \{\mathbf{F} - \partial L/\partial \mathbf{q}\} \cdot \Delta\mathbf{q} + \{\mathbf{v} - \mathbf{V}\} \cdot \Delta\mathbf{p} \tag{1.15}$$

where

$$\Delta\mathbf{v} = d\mathbf{v} - \mathbf{A}dt \neq 0, \tag{1.16}$$

$$\Delta\mathbf{q} = d\mathbf{q} - \mathbf{V}dt \neq 0, \tag{1.17}$$

and

$$\Delta\mathbf{p} = (d\mathbf{p} - \mathbf{F}dt). \tag{1.18}$$

The first term involves differential (possibly anholonomic) fluctuations $\Delta \mathbf{v}$ and the second term involves differential fluctuations, $\Delta \mathbf{q}$. The concept of differential fluctuations represents the error in the assumption that the vector field describes an evolution of a singly parametrized group. The third term contains the factor $\Delta \mathbf{p}$ which when zero defines Newton's laws if $\mathbf{p}$ is interpreted as the "momentum". When the evolutionary "velocity" $\mathbf{V}$ is assumed to be equal to the Lagrangian velocity, $\mathbf{v}$, then the third term (with possible differential fluctuations in momentum) can be ignored. Without constraints of zero differential fluctuations, the virtual work one form is zero when the three bracket factors vanish, independent of any differential fluctuations. This special case is the basic assumption of classical mechanics. However, note that the first bracket cannot vanish if the domain is symplectic of dimension 2n+2.

Moreover, in the non-canonical symplectic 2n+2 domain, the work 1-form can never vanish, for there do not exist null eigen vectors of the anti-symmetric matrix of functions that make up the components of the exact non-degenerate 2-form $d\mathcal{A}$. In the contact 2n+1 domain, however, there exists a unique vector field with a null eigen value, such that the virtual work 1-form indeed vanishes: $W = i(\mathbf{W})d\mathcal{A} = 0$. This result serves as the basis of the d'Alembert principle. It is of some interest to consider those points upon which the symplectic 2-form has a null eigen value(s) as topological defects in the symplectic domain of dimension 2n+2. As the eigen values of an anti-symmetric matrix come in pairs, "extremal" vectors representing topological defects of the symplectic domain are not unique, a well known result of the calculus of variations having envelope solutions. Note that the 1-form of virtual work depends on both the system (the Action) and the process (the vector field).

When the symplectic work one form is closed but not zero, such that (locally) $W = i(\mathbf{W})d\mathcal{A} = d\Theta \neq 0$, the process represented by $\mathbf{W}$ is defined to be a symplectic process. Such processes preserve the symplectic structure for

$$\mathcal{L}_{(\mathbf{W})}d\mathcal{A} = 0. \tag{1.19}$$

This requirement for a process to be symplectic is to be recognized as the generalization of the Helmholtz conservation of vorticity law in hydrodynamics. For a symplectic process the functions Θ are never constant and never without a gradient over the symplectic domain. Vector fields that satisfy this closure condition are elements of Lie groups, and in the technical mathematics literature, the functions Θ are often are called "Casimirs" -or somewhat inappropriately, "Hamiltonians". Although not constants over the domain, these "potential" or "energy" functions Θ are evolutionary invariants of a symplectic process. Most engineers and applied scientists have a greater appreciation for these functions when it is pointed out that they are equivalent to the Bernoulli invariants in hydrodynamics. The engineer would call Θ a Bernoulli "constant", a function invariant along a streamline, but of different values for different neighboring streamlines: $\Theta = (P + \rho gh + \rho v^2/2)Vol$.

To prove that the Bernoulli-Casimirs are always evolutionary invariants with respect to symplectic vector fields, construct the Lie derivative of Θ with respect to $\mathbf{W}$.

$$\mathcal{L}_{(\mathbf{W})}\Theta = i(\mathbf{W})d\Theta + d(i(\mathbf{W})\Theta) = i(\mathbf{W})i(\mathbf{W})d\mathcal{A} + d(i(\mathbf{W})\Theta) = 0 + 0. \quad (1.20)$$

Both the first and second terms vanish algebraically. However, for the classic "Hamiltonian" defined above in terms of the Legendre transformation, $H(t, q, v, p) = \{p_k v^k - L(t, q, v\}$, a direct computation indicates that the Hamiltonian need not be an invariant of a symplectic process - even if the Hamiltonian is explicitly time independent. For consider the evolutionary equation,

$$\mathcal{L}_{(\mathbf{W})}H = i(\mathbf{W})dH = \{(\partial H/\partial \mathbf{q})\cdot\mathbf{V} + (\partial H/\partial \mathbf{p})\cdot\mathbf{F} + (\partial H/\partial \mathbf{v})\cdot\mathbf{A} + (\partial H/\partial t)\} \quad (1.21)$$

or equivalently

$$\mathcal{L}_{(\mathbf{W})}H = \{(-\partial L/\partial \mathbf{q} + \mathbf{F})\cdot\mathbf{V} + (\mathbf{p} - \partial L/\partial \mathbf{v}) \cdot \mathbf{A} - (\partial L/\partial t)\}. \quad (1.22)$$

For the domain to remain symplectic and of dimension 2n+2, the first factor of the second term cannot vanish. The first factor of the first term, when set to zero, is equivalent to the classical Lagrange-Euler equations. Hence, even in this case, if the accelerations $\mathbf{A}$ are such that $(\mathbf{p} - \partial L/\partial \mathbf{v}) \cdot \mathbf{A} \neq 0$, then the "Hamiltonian energy" H, is not an evolutionary invariant, even though the Bernoulli-Casimir energies are evolutionary invariants of a symplectic process. A simple example of this situation is where the mechanical (Hamiltonian) energy of a system decays to perhaps some non-zero value at a singular point of the symplectic domain, but the angular momentum stays constant during the process. Numerical simulations of such evolutionary possibilities in fluids have been studied by Carnevale [6].

Even more interesting correspondences can be made to thermodynamics, for then the first term in the expression for W, which depends on differential fluctuations, $\Delta\mathbf{v}$, suggests a relationship to the thermodynamic Helmholtz free energy (functions of the type TS that involve temperature) and the second term , which depends on differential fluctuations, $\Delta\mathbf{q}$, to the thermodynamic Enthalpy (functions of the type PV that involve pressure). The combination of the two types defines the Gibbs free energy (functions of the type $TS - PV$) of closed thermodynamic systems and reversible processes. Hence, these empirical thermodynamic potentials, some 100 years old in concept, are to be recognized as the Bernoulli-Casimirs of the symplectic vector fields on spaces of dimension 2n+2. The thermodynamic potentials are symplectic evolutionary invariants, but the Hamiltonian energy is not. The need for recognizing the differences between mechanical energy and the thermodynamic energies was discussed by Stuke [7], where, without mention of symplectic evolution, he deduces the need for "acceleration " potentials in certain dissipative systems. These acceleration potentials, which can be shown to be the equivalent of Bernoulli-Casimir functions, were used by Stuke to construct the Enthalpy and Gibbs free energy in certain hydrodynamic examples.

The thermodynamic concepts of pressure and temperature are explicitly absent from that classical mechanics which has focused attention on the extremal contact manifolds of dimension 2n+1, and which has ignored the concept of differential fluctuations on symplectic spaces of dimension 2n+2. It is suggested that the occurrence of a pressure gradient, or a temperature gradient should be taken as the signature of a symplectic process.

When the virtual work 1-form is not closed, ($dW \neq 0$ such that the evolutionary processes are not symplectic processes by definition) then the process can become thermodynamically irreversible. These ideas stem from Cartan's definition of an evolutionary process in terms of the equation,

$$\mathcal{L}_{(\mathbf{W})}\mathcal{A} = i(\mathbf{W})d\mathcal{A} + d(i(\mathbf{W})\mathcal{A}) = W + dU = Q, \tag{1.23}$$

and the equation of closure,

$$\mathcal{L}_{(\mathbf{W})}d\mathcal{A} = di(\mathbf{W})d\mathcal{A} = dW = dQ. \tag{1.24}$$

As mentioned above, when dQ is zero, physicists call this last equation the Helmholtz conservation of vorticity equation, but it is essentially the requirement that the vector field $\mathbf{W}$ be a symplectic vector field. Note that Cartan's equation (of topological evolution) in the form, $W + dU = Q$, is precisely the cohomological equivalent of the first law of thermodynamics. This correspondence admits of a useful criteria for connecting dynamical systems and thermodynamics in a non-statistical manner. Note that the dynamical correspondence permits a precise statement to made about the differences between work and heat: the 1-form of heat is not transversal to the evolutionary process, but the 1-form of virtual work is always transversal to the process:

$$i(\mathbf{W})W = i(\mathbf{W})i(\mathbf{W})dA = 0 \;\; but \;\; i(\mathbf{W})Q = i(\mathbf{W})dU \neq 0. \tag{1.25}$$

This idea of transversality is never made clear in most thermodynamic treatments.

The thermodynamic criteria for irreversibility is that the heat 1-form, Q, does not admit an integrating factor [8]. By the Frobenius Theorem, the Pfaff dimension of Q must be greater than 2; e.g. $Q\hat{}dQ \neq 0$. To test for irreversibility of the process $\mathbf{W}$, construct the exterior product $Q\hat{}dQ$ using the above formulas. By the Frobenius theorem, a given process, $\mathbf{W}$, acting on a physical system, $\mathcal{A}$, is irreversible when

$$Q\hat{}dQ = \mathcal{L}_{(\mathbf{W})}\mathcal{A}\hat{}\mathcal{L}_{(\mathbf{W})}d\mathcal{A} \neq 0. \tag{1.26}$$

Before proving the existence of such irreversible processes, note that when the evolutionary vector fields are symplectic (or extremal), such that $dW = dQ = 0$, then such closed processes are reversible in a thermodynamic sense. The Cartan equation for symplectic evolution becomes (for $\mathbf{V} = \mathbf{v}$)

$$\begin{aligned} \mathcal{L}_{(\mathbf{W})}\mathcal{A} &= W + dU = \{\mathbf{p} - \partial L/\partial \mathbf{v}\} \cdot \Delta\mathbf{v} + \{\mathbf{F} - \partial L/\partial \mathbf{x}\} \cdot \Delta\mathbf{q} + dU \quad (1.27) \\ &= d(TS - PV + U) = Q, \quad (1.28) \end{aligned}$$

which defines the heat 1-form Q as the "gradient" of the Gibbs free energy, $G = TS - PV + U$. Compare to Stuke [7]. For symplectic vector fields, the Mechanical Energy of the system need not be an evolutionary invariant, but the "angular momentum" (Casimir) is an evolutionary invariant. By means of a symplectic process the Hamiltonian energy can decay to a singular state where the symplectic condition fails (where the momentum become "canonical") , and then stay in that "equilibrium" state of non-zero energy forever. A special case of symplectic evolution is given by the Adiabatic constraint,

$$\begin{aligned} \mathcal{L}_{(\mathbf{W})}\mathcal{A} &= i(\mathbf{W})d\mathcal{A} + d(i(\mathbf{W})\mathcal{A}) \quad (1.29) \\ &= i(\mathbf{W})d\mathcal{A} + d(\mathbf{p} \bullet (\mathbf{V}(q, v, p, t) - \mathbf{v}) + L(q, v, t)) = 0 \quad (1.30) \end{aligned}$$

For simplicity, first assume that $\mathbf{p} \bullet (\mathbf{V}(q, v, p, t) - \mathbf{v}) = E(q, v, p, t) \Rightarrow 0$, then this special vector $\mathbf{W}$ is uniquely defined in an algebraic manner as the Adiabatic vector, $\mathbf{Z}$. If $[F_{uv}]$ is the matrix of coefficients of the 2-form, $d\mathcal{A}$, then the unique adiabatic process is given by the vector field,

$$\mathbf{Z} = [F_{uv}]^{-1} \circ grad(L). \tag{1.31}$$

The adiabatic process defined by $\mathbf{Z}$ is symplectic and reversible. The function $E(q, v, p, t)$ plays a role similar to the Wieirstrass excess function in the calculus of variations.

To prove the existence of an irreversible process, note that there always exists a 2n+1 form $\mathcal{T}$ =($\mathcal{A}\hat{}(d\mathcal{A})^n$), whose 2n+2 coefficients define the Torsion current, $\mathbf{T}^m$, on the 2n+2 symplectic space. In 4D, the three form $\mathcal{A}\hat{}(d\mathcal{A})$ has been defined as the Topological Torsion 3-form. The Torsion current depends only on the system (the Action) and not upon a process. The divergence of this Torsion current is proportional to the measure of the 2n+2 volume, that defines the symplectic space, and cannot be zero on the symplectic domain. The 2n+2 components of the 2n+1 form $\mathcal{T}$ generate what is called the "subsidiary Pfaffian system" by Forsythe [9].

If the Torsion current, to within a factor, is used as a candidate for an evolutionary process, then the Lie derivative of the Action with respect to the Torsion current satisfies the "conformal" or similarity equation

$$\mathcal{L}_{(\mathbf{T})}\mathcal{A} = \Gamma\mathcal{A} \tag{1.32}$$

The existence of the Torsion vector implies

$$\mathcal{L}_{(\mathbf{T})}d\mathcal{A} = d\Gamma\hat{}\mathcal{A} + \Gamma d\mathcal{A} \tag{1.33}$$

such that the Pfaff dimension of the heat 1-form is greater than 2:

$$Q\hat{}dQ = \mathcal{L}_{(\mathbf{T})}\mathcal{A}\hat{}\mathcal{L}_{(\mathbf{T})}d\mathcal{A} = \Gamma^2\mathcal{A}\hat{}d\mathcal{A} \neq 0. \tag{1.34}$$

Hence the existence of an irreversible process on the symplectic space has been demonstrated, by construction, in the form of the Torsion current. With respect to evolution in the direction of the torsion current, the symplectic volume is contracting or expanding exponentially unless $\Gamma = 0$, and therefore such vector fields cannot represent a symplectic process (which preserves the volume element). The factor, Γ, is a Liapunov function and defines the stability of the process (depending on the sign of Γ). When $\Gamma = 1$, the Torsion vector has been called the "Liouville vector" [10]. Note that herein a constructive process has been displayed for the Torsion current for any given Lagrangian.

Further note that the evolution of the 3-form $\mathcal{A}\hat{}d\mathcal{A}$ is given by the expression,

$$\mathcal{L}_{(\mathbf{T})}\mathcal{A}\hat{}d\mathcal{A} = (\mathcal{L}_{(\mathbf{T})}\mathcal{A})d\mathcal{A} + \mathcal{A}\hat{}\mathcal{L}_{(\mathbf{T})}d\mathcal{A} = 2\Gamma\mathcal{A}\hat{}d\mathcal{A} \tag{1.35}$$

2. An Electromagnetic Example

The best way to exemplify the techniques described above is apply them to an electromagnetic situation. Most everyone has had some experience with electrodynamics on a four dimensional space-time. On the four dimensional space-time of independent variables, (x, y, z, t) the 1-form of Action can be written in the form

$$\mathcal{A} = \Sigma_{k=1}^{3} A_k(x, y, z, t)dx^k - \phi(x, y, z, t)dt. \tag{2.1}$$

A first step is to construct the Pfaff Sequence, $\{\mathcal{A}, d\mathcal{A}, \mathcal{A}\hat{}d\mathcal{A}, d\mathcal{A}\hat{}d\mathcal{A}\}$. Following the usual constructions, the components of the 2-form dA become

$$d\mathcal{A} = \{\partial A_k/\partial x^j - \partial A_j/\partial x^k\}dx^j\hat{}dx^k = F_{jk}dx^j\hat{}dx^k = \mathbf{B}_z dx\hat{}dy...\mathbf{E}_x dx\hat{}dt... \tag{2.2}$$

where in usual notation,

$$\mathbf{E} = -\partial\mathbf{A}/\partial t - grad\phi, \qquad \mathbf{B} = curl\ \mathbf{A}. \tag{2.3}$$

The topological torsion 3-form, $\mathcal{A}\hat{}d\mathcal{A}$, induces the torsion current

$$\mathbf{T} = \{(\mathbf{E}\times\mathbf{A} + \mathbf{B}\phi); \mathbf{A}\bullet\mathbf{B}\} \equiv \{\mathbf{S}, h\}, \tag{2.4}$$

such that

$$\mathcal{A}\hat{}d\mathcal{A} = i(\mathbf{T})dx\hat{}dy\hat{}dz\hat{}dt = \mathbf{S}^x dy\hat{}dz\hat{}dt..... - h dx\hat{}dy\hat{}dz \tag{2.5}$$

The 4-form of topological parity becomes

$$d\mathcal{A}\hat{}d\mathcal{A} = -2(\mathbf{E}\bullet\mathbf{B})dx\hat{}dy\hat{}dz\hat{}dt = (div\,\mathbf{S} + \partial h/\partial t)dx\hat{}dy\hat{}dz\hat{}dt \tag{2.6}$$

It is apparent that a 4D system cannot be symplectic unless $(\mathbf{E}\bullet\mathbf{B})$ never vanishes. Moreover, in the symplectic case the torsion current does not satisfy a conservation law (or the "equation" of continuity")

Note that the Poincare lemma always leads to the first Maxwell pair of (Faraday Induction) equations:

$$dd\mathcal{A} = \{curl\ \mathbf{E} + \partial\mathbf{B}/\partial t\}_x dy\hat{}dz\hat{}dt - .. + .. - div\,\mathbf{B}dx\hat{}dy\hat{}dz\} \Rightarrow 0, \qquad (2.7)$$

or

$$\{curl\ \mathbf{E} + \partial\mathbf{B}/\partial t = 0, \qquad div\,\mathbf{B} = 0\}. \qquad (2.8)$$

The result is actually true for a variety of any dimension $\succeq 4$ and for any set of covariant symbols. The concept of Faraday induction is universal.

Consider an arbitrary process defined by the 4 vector field $\mathbf{W} = \rho\{\mathbf{V}; \mathbf{1}\}$. Then the Work 1-form becomes the Lorentz force law:

$$W = (i(\mathbf{W})d\mathcal{A}) = (\rho\{\mathbf{E} + \mathbf{V} \times \mathbf{B}\}_k dx^k - \{\rho\mathbf{V} \bullet \mathbf{E}\}dt). \qquad (2.9)$$

With these constructions now apply the constraints that produce the Hamiltonian extremal and the Helmholtz symplectic equivalence classes.

2.1. The Hamiltonian Extremal Class

In the extremal Hamiltonian case, dA$\hat{}d$A$= 0 \Rightarrow \mathbf{E} \bullet \mathbf{B} = 0$, and the work 1-form must vanish: $W = i(\mathbf{W})d\mathrm{A} = 0$. Therefore the extremal field constraint requires that

$$\{\mathbf{E} + \mathbf{V} \times \mathbf{B}\}_k dx^k - \{\mathbf{V} \bullet \mathbf{E}\}dt = 0, \qquad (2.10)$$

an equation that is to be satisfied for any value of the "normalization" factor $\rho.$ It is apparent that the extremal constraint forces the Lorentz force to vanish, $\mathbf{E} + \mathbf{V} \times \mathbf{B} \Rightarrow 0$, and the dissipative power to vanish, $\mathbf{V} \bullet \mathbf{E} \Rightarrow 0$. The first condition is the classic constraint for a charge particle moving in crossed magnetic and electric fields.

2.2. The Helmholtz Symplectic Class

The Helmholtz (symplectic) closure requirement, $d(i(\mathbf{W})d\mathrm{A}) \Rightarrow 0$, implies that the Lorentz force need not be zero, but it should have zero curl: $d(\rho\{\mathbf{E} + \mathbf{V} \times \mathbf{B}\}_k dx^k - \{\rho\mathbf{V}\bullet\mathbf{E}\}dt) = 0$. First consider that case where ρ is a constant. Then, the necessary condition to satisfy the closure condition, for arbitrary displacements of the independent variables, is that

$$curl\,\{\mathbf{E} + \mathbf{V} \times \mathbf{B}\} = 0 \tag{2.11}$$

and similarly

$$\partial\{\mathbf{E} + \mathbf{V} \times \mathbf{B}\}/\partial t + grad\{\mathbf{V} \cdot \mathbf{E}\} = 0. \tag{2.12}$$

Substituting the Maxwell result, $curl\,\mathbf{E} = -\partial\mathbf{B}/\partial t$, leads to the Master equation of the Imperfect Plasma:

$$-\partial\mathbf{B}/\partial t + curl\{\mathbf{V} \times \mathbf{B}\} = 0. \tag{2.13}$$

In the symplectic case which is equivalent to a Fomenko system the Lorentz Force cannot vanish, and the symplectic evolutionary process satisfies the equation

$$\rho(\mathbf{E} + \mathbf{V} \times \mathbf{B}) = grad(\Theta). \tag{2.14}$$

The Master equation is modified slightly to account for a non-constant form of the "scaling" function, ρ :

$$-\partial\mathbf{B}/\partial t + curl\{\mathbf{V} \times \mathbf{B}\} = grad\,\ln\rho \times (\mathbf{E} + \mathbf{V} \times \mathbf{B}). \tag{2.15}$$

In elementary physics the scaling function ρ is to recognized as the charge distribution. In elementary mathematics, the scaling function ρ is to be recognized as the integrating factor.

The symplectic *manifold* condition of maximal rank over the 4 dimensional domain requires that the second Poincare invariant is not zero:

$$d\mathcal{A}\hat{}d\mathcal{A} = 2(\mathbf{E} \bullet \mathbf{B})dx\hat{}dy\hat{}dz\hat{}dt \neq 0. \tag{2.16}$$

The symplectic *evolutionary process* condition requires that

$$\rho(\mathbf{E} \bullet \mathbf{B}) = \mathbf{B} \bullet grad\,\Theta \neq 0 \tag{2.17}$$

There must exist a gradient (of pressure or temperature) in the direction of the $\mathbf{B}$ field lines. Similarly, there is a dissipation if the motion is in the direction of the gradient, for then

$$\rho(\mathbf{E} \bullet \mathbf{V}) = \mathbf{V} \bullet grad\,\Theta. \tag{2.18}$$

Note here is no "ohmic" dissipation for evolution $\mathbf{V}$ in the direction orthogonal to $grad\,\Theta$.

Physically, then, in a symplectic system there must exist a component of electric force that accelerates charged particles along the magnetic field lines, and that component of force, as an artifact of the symplectic constraints, is the ultimate source of the magnetic dynamo. A similar situation holds in hydrodynamics where fluid mass can be accelerated along the lines of vorticity. For the extremal, non-symplectic case, the Lorentz force must vanish, and there is no magnetic dynamo action.

From the argument developed above for symplectic systems, the Bernoulli-Casimir energy function Θ is either of the type TS and/or of the type PV. For a solid, assume the former representation dominates. Then the "Lorentz force" must have the form of a spatial gradient of the temperature, $\rho(\mathbf{E}+\mathbf{V}\times\mathbf{B}) = grad(kT)$. For motion that is along the magnetic field lines, the term $\mathbf{V}\times\mathbf{B}\Rightarrow 0$. Then, incorporating the empirical Ohmic relation, $\mathbf{j}=\sigma\mathbf{E}$, it is apparent that the symplectic case leads to a derivation of flux equations in the Thompson format for thermal power.

$$\mathbf{j} = (1/\rho\sigma)grad(kT) \tag{2.19}$$

The suggestion is that the source of magnetic dynamo forces is to be associated with a temperature gradient and the existence of differential velocity fluctuations in a symplectic system. The theory predicts that not only should there exist a Bernoulli-Casimir Pressure gradient, but also there should also exist a Bernoulli-Casimir Temperature gradient that will be exacerbated by domains where $\mathbf{E}\bullet\mathbf{B}\neq 0$.

The Bernoulli-Casimir Pressure gradient is associated with those velocity functions which induce differential fluctuations in position, $\Delta\mathbf{q}$. The (novel and different) Bernoulli-Casimir Temperature gradient is associated with those acceleration functions which induce differential fluctuations in velocity, $\Delta\mathbf{v}$. These ideas are to be compared with the Davies-Unruh concept that uniform acceleration in the vacuum field induces a temperature. [14].

2.3. The Torsion Current and Irreversible Processes (Pfaff dimension 4)

Assume that the Pfaff dimension of the domain of interest is 4, hence the space is symplectic. However, consider evolutionary fields that are not constrained to be symplectic such that $dW\neq 0$. Direct evaluation of the virtual work 1-form,

$W = i(\mathbf{W})d$A yields (the Lorentz force)

$$W = i(\mathbf{W})d\mathcal{A} =(\{\rho\mathbf{E} + \mathbf{J}\times\mathbf{B}\}_k dx^k - \{\mathbf{J}\bullet\mathbf{E}\}dt) \tag{2.20}$$

The obvious first choice for the evolutionary vector field has been based on the classic assumption that $\mathbf{W} =\{\mathbf{J};\rho\} \Rightarrow \rho\{\mathbf{V};1\}$ The expression for virtual work becomes

$$W = \rho(\{\mathbf{E} + \mathbf{V}\times\mathbf{B}\}_k dx^k - \{\mathbf{V}\bullet\mathbf{E}\}dt). \tag{2.21}$$

However, another perhaps not so obvious a candidate for a solution vector field is the expression for the Torsion current. That is, examine the evolution along the four dimensional vector field,

$$\mathbf{T} = \{(\mathbf{E}\times\mathbf{A} + \mathbf{B}\phi); \mathbf{A}\bullet\mathbf{B}\}. \tag{2.22}$$

The expression for virtual work becomes

$$W = i(\mathbf{T})d\mathcal{A} =(\{(\mathbf{A}\bullet\mathbf{B})\mathbf{E} + (\mathbf{E}\times\mathbf{A})\times\mathbf{B}\}_k dx^k - \{\mathbf{E}\bullet\mathbf{B}\phi\}dt) = (\mathbf{E}\bullet\mathbf{B})\mathcal{A}. \tag{2.23}$$

The torsion current is an associated field relative to the 1-form of Action, in the sense that

$$i(\mathbf{T})\mathcal{A} \Rightarrow 0. \tag{2.24}$$

It follows that the Lie derivative of the Action along the direction of the Torsion current is a conformal process in the sense that

$$\mathcal{L}_{(\mathbf{T})}\mathcal{A} = \Gamma\mathcal{A} = (\mathbf{E}\bullet\mathbf{B})\mathcal{A} = Q. \tag{2.25}$$

By direct computation,

$$\mathcal{L}_{(\mathbf{T})}d\mathcal{A} = d\Gamma\hat{}\mathcal{A} + \Gamma d\mathcal{A} = dQ \tag{2.26}$$

from which it follows that

$$Q\hat{}dQ = \Gamma^2\mathcal{A}\hat{}d\mathcal{A}. \tag{2.27}$$

If the topological parity $\Gamma = (\mathbf{E}\bullet\mathbf{B})$ does not vanish, then the Torsion current $\mathbf{T}$ represents an irreversible non-conservative process, as for such cases the heat 1-form, Q, does not admit an integrating factor. When $\Gamma = (\mathbf{E}\bullet\mathbf{B}) \preceq 0$, the process is stable in a Liapunov sense.

The formula $\mathcal{L}_{(\mathbf{w})}$A= ΓA was the fundamental principle used by the present author in 1974 to describe "An Extension of Hamilton's Principle to Include

Dissipative Systems". It was not known at that time the such processes implied the existence of a symplectic structure, nor the fact that the irreversible processes were not symplecto-morphisms.

2.4. The Torsion Current and Reversible Processes (Pfaff dimension 3)

If the non-zero Torsion current has a zero divergence everywhere, then the function $\Gamma = 0$ defines a holonomic constraint of projection on the 4D space, upon which the Pfaff domain generated by the Action 1-form is no longer of dimension 4. The space does not support a symplectic structure of Pfaff dimension 4. Instead the 2-form defines a contact manifold of Pfaff dimension 3. On the 4 dimensional domain of initial independent variables, the contact 2-form is of rank 2, not 4, which implies that on the 4D space there exist TWO extremal fields with null eigenvalues. From the analysis above, the Torsion current is one of these two extremal vector fields. Evolution along the divergence free paths of the torsion current is reversible. It is of particular interest when this hypersurface is minimal.

If the divergence free condition is not global, then evolution along the divergence free paths of the torsion current preserves the contact structure, or singularity in the symplectic domain, for when $\Gamma \Rightarrow 0$,

$$\mathcal{L}_{(\mathbf{T})}(\mathcal{A}\hat{\ }d\mathcal{A}) = (\mathcal{L}_{(\mathbf{T})}(\mathcal{A}))\hat{\ }d\mathcal{A} + \mathcal{A}\hat{\ }\mathcal{L}_{(\mathbf{T})}(d\mathcal{A}) = 0 + 0. \tag{2.28}$$

This divergence free process path is reversible, and stable if $\Gamma < 0$. Such closed process paths form limit cycles.

Any other extremal field of the format $\mathbf{W} = \{\mathbf{J};\rho\}$ must satisfy the equations $\mathbf{J}\bullet\mathbf{E} = 0$,and $\rho\mathbf{E}+\mathbf{J}x\mathbf{B} = 0$. Assuming that $\{\mathbf{J};\rho\} = \rho\{\mathbf{V};1\}$,this second extremal evolutionary process implies that the motion of charges is orthogonal to both the $\mathbf{E}$ and $\mathbf{B}$ fields (the Hall effect).

If it is assumed that the second extremal field is orthogonal to the Torsion vector, then it must be true that

$$\{\mathbf{J};\rho\} = \lambda\{\mathbf{A},\phi\}, \tag{2.29}$$

which is in the format of the London current..

When $\Gamma \neq 0$, any evolutionary field that is to be reversible must be orthogonal to the Torsion vector current. Conversely, in the symplectic case such that $\mathbf{E}\bullet\mathbf{B} \neq 0$, any evolutionary vector field that has a component in the direction of the Torsion vector must be irreversible. Note that the sign of $\Gamma = \mathbf{E}\bullet\mathbf{B}$ determines the "stability" of the process defined by the Torsion vector.

2.5. Submanifolds of the Symplectic Case

The criteria that the Torsion current produce a symplectic manifold implies that the Torsion current does not satisfy an "equation of continuity"; that is, the Torsion current is "not conserved". On the other hand, if the Torsion current satisfies an equation of continuity (has zero divergence), the domain is NOT symplectic. In this case, the exterior derivative of the 3-form vanishes over the 4D domain which implies that there exists a N-2 = 2 form, G, such that $dG = J$. The electromagnetic system is now complete, for both the Fields, $F = d\mathcal{A}$, and the excitations, G, have been defined without a metric. The sources of the electromagnetic field are topological defects in the symplectic structure of space-time. The Poincare lemma $dd\mathcal{A} = dF = 0$ establishes the first Maxwell pair, and the equations $dG = J$ form the second Maxwell pair. No metric constraints, nor connections have been subsumed. This result leads to the concept of quantized flux as the 1-D period integrals of the harmonic components of the Action, $\oint$A, quantized charge as the 2-D period integrals of $\iint_{closed} G$, and quantized spin as 3-D period integrals of the form, $\iiint_{closed}$A$\hat{}G$. [11] Now it is apparent that the existence of such results and the entire Maxwell theory follows from the topological concept that certain physical systems can be described by 1-form of Action which is of Pfaff dimension 3; that is, from the study of non-symplectic systems for which $\mathcal{A}\hat{}F \neq 0$, but for which $d(\mathcal{A}\hat{}dF) = d\mathcal{A}\hat{}d\mathcal{A} = F\hat{}F = 2(\mathbf{E} \bullet \mathbf{B})dx\hat{}dy\hat{}dz\hat{}dt = 0$. It has been demonstrated that ratio of the 3-D period integrals of spin and torsion form a set of rational fractions defining the topological fractional Hall Impedance. [12]

As G is a non-exact 2-form, it could also define a symplectic structure on the 4-D domain, when $dG = 0$. In such a case $G\hat{}G = 2(\mathbf{D} \bullet \mathbf{H})dx\hat{}dy\hat{}dz\hat{}dt \neq 0$. The ratio of the integrals of the two symplectic structures then gives the impedance of free space:

$$Z_0 = \sqrt{\iiiint F\hat{}F \;/\; \iiiint G\hat{}G} = \sqrt{\varepsilon/\mu} = the\ impedance\ of\ freespace. \quad (2.30)$$

The ratio of the period integrals gives the Hall impedance

$$Z_{Hall} = \iiint_{closed} \mathcal{A}\hat{}F \;/\; \iiint_{closed} \mathcal{A}\hat{}G \sim h/e^2 \quad (2.31)$$

such that

$$\frac{2Z_{Hall}}{Z_0} = \alpha = 137.063041 \tag{2.32}$$

It is not uncommon for a variety to support many topologies. For the symplectic topology generated by the exact 2-form, $F = dA$, the topological domain is not compact, while the symplectic topology induced by G can be compact. As the 2-form G is associated with the fields **D** and **H** and sources (charge and currents) it appears that such "quantized" features are to be associated with compact manifolds. However, as the two form F is exact its symplectic topology is non-compact. The associated **E** and **B** fields are empirically related to forces, and therefore to mass. The argument seems to justify the Mach idea that mass is an artifact of a non-compact topology, while the fundamentally different concept of charge is a compact artifact.

Similar examples as applied to hydrodynamic systems will be reported elsewhere. In particular, solutions to the Navier-Stokes equations that satisfy the condition that $curl\mathbf{v} \bullet curl(curl\mathbf{v}) \neq 0$ imply that Action 1-form defines a symplectic domain of Pfaff dimension 4. The condition is therefore a necessary condition for the existence of thermodynamically irreversible turbulent evolution.[13]

2.6. Acknowledgments

The author wishes to express his thanks to Y. Saet who introduced him to the Russian school of symplectic mathematics.

.

3. REFERENCES

[1] R. M. Kiehn, J. Math Phys. 5, 9 (1974). "Extensions of Hamilton's Principle to Include Dissipative System"

[2] A. V. Bolsinov, S. V. Mateveev and A. T. Fomenko, Russian Math Surveys **45**:2 (1990) p59-94

A.V. Bolsinov, A.T.Fomenko, and K. Chang, Proceedings of the Steklov Institute of Mathematics, 1995, Issue 4, p,. 29

[3] Chern, S.S. (1948) Science Reports Nat. Tsing Hua Univ. 5, 95-121.
(1988)

[4] J. A. Schouten and W. Van der Kulk "Pfaffs Problem" , Oxford University Press, (1949)

[5] Cartan, E. (1958) Lecons sur les invariants integrauxs (Hermann, Paris 1958).

[6] G. F. Carnevale and G. K. Vallis, "Isovortical Relaxation to Stable Two-Dimensional Flows" in: H. K. Moffatt and T. S. Tsinober eds, Topological Fluid Mechanics, (Cambridge University Press) 449-458. Also see J. Fluid Mech. (1989) v. **207** pp133-152

[7] B. Stuke, Z. Naturforsch **48a** 883-894 (1993)

[8] P. Morse, "Thermal Physics" (Benjamin, N.Y.) (1964) p 60

I. I. Gol'denblat "Some Problems of the Mechanics of Deformable Media" Norrdhoff-Holland 1962 (p.193)

[9] A.R. Forsythe, "Theory of Differential Equations" Dover (1959) p96.

[10] P. Libermann and C-M Marle, "Symplectic Geometry and Analytical Mechanics" (Riedel) 1978 p 65.

[11] R. M. Kiehn, J. Math Phys. **18**, 614 (1977).

[12] Kiehn R. M.(1991) "Are there Three kinds of Superconductivity" Int. Journ. Mod Phys **5**, 10, p1779

[13] Kiehn, R. M. (1992) "Topological Defects, Coherent Structures and Turbulence in Terms of Cartan's Theory of Differential Topology" in Developments in Theoretical and Applied Mathematics, Proceedings of the SECTAM XVI conference, B. N. Antar, R. Engels, A.A. Prinaris and T. H. Moulden, Editors, The University of Tennessee Space Institute, Tullahoma, TN 37388 USA.

[14] P.W. Milonni, "The Quantum Vacuum" Academic Press (1994) p 60.