```
>
> restart: with (linalg):with(liesymm):with(difforms):
> setup(x,y,z,t):defform(x=0,y=0,z=0,t=0,Vx=0,Vy=0,Vz=0,D1=0,D2=0,D3=0,Ax=0,Ay=0,A
  z=0,C=0,Phi=0,a=const,b=const,c=const,Lx=0,Ly=0,Lz=0,E=0);
Warning, new definition for norm
Warning, new definition for trace
Warning, new definition for close
Warning, new definition for `&^`
Warning, new definition for d
Warning, new definition for mixpar
Warning, new definition for wdegree
```

Using the Lorentz map as a FRAME FIELD

or

Space Time as a Minkowski Fluid

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Cartan's methods will be used to compute torsion and curvature coefficients induced on subspaces of R4 by assuming the Lorentz map, L, to be a Frame Field. The parameters (Vx,Vy,Vz, Lx, Ly, Lz, C,E) will NOT be presumed to be global constants, a priori. For example, it may be true that C = C(x,y,z,t) as it does in real media or in the presence of a gravitational field. The conformal expansion factor E allows for expansion of the manifold and also can be E(x,y,z,t).
If E is a global constant, then all torsion 2-forms of the WF type are zero, as the field "Abnormality" (big Omega) vanishes (See http://www22.pair.com/pdf/projfram.pdf)

Fundamental Assumption:

The Lorentz map,

L: {x,y,z,t} ==> {X,Y,Z,T}

must satisfy the equation

x^2+y^2+z^2-(C*t)^2=0 = => X^2+Y^2+Z^2-(C*T)^2=0

When it is recognized that the Eikonal equation (non-linear first order quadratic form of partial derivatives) is deduced from the Maxwell PDE's as the necessary criteria for the existence of a singular solution set,
(and therefore is the fundamental equation describing the existance of propagating discontinuities), then the key feature of the Lorentz map is that a signal
(a propagating discontinuity - Not an infinite wave train)
is mapped into a signal to all obervers related by the Lorentz map.

The fact that the Eikonal equation is equal to zero, implies that it is possible to scale all quadratic forms by a non zero factor E^2. In other words the map is defined by the equations

x^2+y^2+z^2-(C*t)^2 = E(x,y,z,t)^2{X^2+Y^2+Z^2-(C*T)^2}

where if the RHS is zero, then so is the LHS.

The significance of the Lorentz transformations, L, is that they are the only Linear transformations that preserve the Eikonal quadratic form. -- V. Fock

(AND THEREFOR MAP SIGNALS INTO SIGNALS)

The class of Lorentz maps does not require that C be a domain constant. In fact, when C= C(x,y,z,t), the space-time variety {x,y,z,t} constrained by Lorentz maps can be viewed as a "Minkowski Fluid".

In that which follows, the Lorentz transformations considered will be those that can be constructed from six matrix generators, three rotations (LRx, LRy, LRz) and three translations (LTx, LTy,LTz).. The inverses of these matrices will be designated as (LRxn, LRyn, LRzn) and (LTxn, LTyn,LTzn). The generators may be multiplied by arbitrary non-zero Expansion functions, E, (which merely changes the determinant value of the map) and by 1/E for the inverse generators. The matrix generators consist of parameters(functions of x,y,z,t) and these parameters are not restricted to be domain constants. The only requirement is that the matrix be an element of the Lorentz group that preserves the quadratic form defined as the Eikonal, to within a factor.

Products of these generators will generate new Lorentz transformations. However, the order of application of the generators is significant to the outcome.

LGUN stands for the Minkowski metric

```
> LGUN:=evalm(array([[1,0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,-1]]));r:=[x,y,z,C*t]:
```

In that which follows, six generating matrices will be constructed from maps that take r = {x,y,z,Ct} into R = {XX,YY,ZZ,TT}. The initial rQF (a quadratic form defined as rQF) constructed from <r|1,1,1,-1|r> will be compared to the final RQF defined as <R|1,1,1,-1|R>. The generators will be constructed at first for those that have positive determinant.

```
> LRz:=subs(evalm(E*array([[(1-Lz^2)^(1/2),Lz,0,0],[-Lz,(1-Lz^2)^(1/2),0,0],[0,0,1,0],[0,0,0,1]])));RPRIME:=simplify(innerprod(LRz,r)):XX:=RPRIME[1];YY:=RPRIME[2];ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=det(LRz);R:=innerprod(LRz,r):initial_rQF:=innerprod(r,LGUN,r);LRzn:=inverse(LRz);
```

$$LGUN := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$LRz := \begin{bmatrix} E\sqrt{1-Lz^2} & E\,Lz & 0 & 0 \\ -E\,Lz & E\sqrt{1-Lz^2} & 0 & 0 \\ 0 & 0 & E & 0 \\ 0 & 0 & 0 & E \end{bmatrix}$$

$$XX := E\sqrt{1-Lz^2}\,x + E\,Lz\,y$$

$$YY := -E\,Lz\,x + E\sqrt{1-Lz^2}\,y$$

$$ZZ := E\,z$$

$$TT := E\,C\,t$$

$$DETL := E^4$$

$$initial\_rQF := x^2 + y^2 + z^2 - C^2\,t^2$$

$$LRzn := \begin{bmatrix} \frac{\sqrt{1-Lz^2}}{E} & -\frac{Lz}{E} & 0 & 0 \\ \frac{Lz}{E} & \frac{\sqrt{1-Lz^2}}{E} & 0 & 0 \\ 0 & 0 & \frac{1}{E} & 0 \\ 0 & 0 & 0 & \frac{1}{E} \end{bmatrix}$$

Compute the final quadratic form form the Output vector R

```
> Final_RQF:=(factor(innerprod(R,LGUN,R)));
```

$$Final\_RQF := -E^2\,(-x^2 - y^2 - z^2 + C^2\,t^2)$$

So the map LRz is indeed a Lorentz transformation (LRz = "Lorentz Rotation about z axis").

Similarly for rotations about x and y:

An angular formulation for non-tachyons can be written as:

```
> LRphiz:=subs(evalm(E*array([[cos(phi),sin(phi),0,0],[-sin(phi),cos(phi),0,0],[0,
0,1,0],[0,0,0,1]])));RPRIME:=simplify(innerprod(LRphiz,r)):XX:=RPRIME[1];YY:=RPR
IME[2];ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=det(LRz);R:=innerprod(LRp
hiz,r);initial_rQF:=innerprod(r,LGUN,r);Final_RQF:=factor(simplify(innerprod(R,L
GUN,R)));LRphizn:=inverse(LRphiz);
```

$$LRphiz := \begin{bmatrix} E\cos(\phi) & E\sin(\phi) & 0 & 0 \\ -E\sin(\phi) & E\cos(\phi) & 0 & 0 \\ 0 & 0 & E & 0 \\ 0 & 0 & 0 & E \end{bmatrix}$$

$$XX := E\cos(\phi)\,x + E\sin(\phi)\,y$$

$$YY := -E\sin(\phi)\,x + E\cos(\phi)\,y$$

$$ZZ := E\,z$$

$$TT := E\,C\,t$$

$$DETL := E^4$$

$$R := [E\cos(\phi)\,x + E\sin(\phi)\,y,\ -E\sin(\phi)\,x + E\cos(\phi)\,y,\ E\,z,\ E\,C\,t]$$

$$initial\_rQF := x^2 + y^2 + z^2 - C^2\,t^2$$

$$Final\_RQF := -E^2\,(-x^2 - y^2 - z^2 + C^2\,t^2)$$

$$LRphizn := \begin{bmatrix} \frac{\cos(\phi)}{E\,(\cos(\phi)^2 + \sin(\phi)^2)} & -\frac{\sin(\phi)}{E\,(\cos(\phi)^2 + \sin(\phi)^2)} & 0 & 0 \\ \frac{\sin(\phi)}{E\,(\cos(\phi)^2 + \sin(\phi)^2)} & \frac{\cos(\phi)}{E\,(\cos(\phi)^2 + \sin(\phi)^2)} & 0 & 0 \\ 0 & 0 & \frac{1}{E} & 0 \\ 0 & 0 & 0 & \frac{1}{E} \end{bmatrix}$$

```
> LRx:=subs(evalm(E*array([[1,0,0,0],[0,(1-Lx^2)^(1/2),Lx,0],[0,-Lx,(1-Lx^2)^(1/2)
,0],[0,0,0,1]])));RPRIME:=simplify(innerprod(LRx,r)):XX:=RPRIME[1];YY:=RPRIME[2]
;ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=det(LRz);R:=innerprod(LRx,r);in
itial_rQF:=innerprod(r,LGUN,r);Final_RQF:=innerprod(R,LGUN,R);LRxn:=inverse(LRx)
```

;

>

$$LRx := \begin{bmatrix} E & 0 & 0 & 0 \\ 0 & E\sqrt{1-Lx^2} & E\,Lx & 0 \\ 0 & -E\,Lx & E\sqrt{1-Lx^2} & 0 \\ 0 & 0 & 0 & E \end{bmatrix}$$

$$XX := E\,x$$

$$YY := E\sqrt{1-Lx^2}\,y + E\,Lx\,z$$

$$ZZ := -E\,Lx\,y + E\sqrt{1-Lx^2}\,z$$

$$TT := E\,C\,t$$

$$DETL := E^4$$

$$R := [E\,x,\ E\sqrt{1-Lx^2}\,y + E\,Lx\,z,\ -E\,Lx\,y + E\sqrt{1-Lx^2}\,z,\ E\,C\,t]$$

$$initial\_rQF := x^2 + y^2 + z^2 - C^2\,t^2$$

$$Final\_RQF := E^2\,x^2 + E^2\,y^2 + E^2\,z^2 - E^2\,C^2\,t^2$$

$$LRxn := \begin{bmatrix} \frac{1}{E} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{1-Lx^2}}{E} & -\frac{Lx}{E} & 0 \\ 0 & \frac{Lx}{E} & \frac{\sqrt{1-Lx^2}}{E} & 0 \\ 0 & 0 & 0 & \frac{1}{E} \end{bmatrix}$$

Similary an angular representation is:

>

```
> LRthetax:=subs(evalm(E*array([[1,0,0,0],[0,cos(theta),sin(theta),0],[0,-sin(theta),cos(theta),0],[0,0,0,1]])));RPRIME:=simplify(innerprod(LRthetax,r)):XX:=RPRIME[1];YY:=RPRIME[2];ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=det(LRz);R:=innerprod(LRthetax,r);initial_rQF:=innerprod(r,LGUN,r);Final_RQF:=factor(simplify(innerprod(R,LGUN,R)));LRthetaxn:=inverse(LRthetax);
```

$$LRthetax := \begin{bmatrix} E & 0 & 0 & 0 \\ 0 & E\cos(\theta) & E\sin(\theta) & 0 \\ 0 & -E\sin(\theta) & E\cos(\theta) & 0 \\ 0 & 0 & 0 & E \end{bmatrix}$$

$$XX := E\,x$$

$$YY := E\cos(\theta)\,y + E\sin(\theta)\,z$$

$$ZZ := -E\sin(\theta)\,y + E\cos(\theta)\,z$$

$$TT := E\,C\,t$$

$$DETL := E^4$$

$$R := [E\,x,\ E\cos(\theta)\,y + E\sin(\theta)\,z,\ -E\sin(\theta)\,y + E\cos(\theta)\,z,\ E\,C\,t]$$

$$initial\_rQF := x^2 + y^2 + z^2 - C^2\,t^2$$

$$Final\_RQF := -E^2\,(-x^2 - y^2 - z^2 + C^2\,t^2)$$

$$LRthetaxn := \begin{bmatrix} \frac{1}{E} & 0 & 0 & 0 \\ 0 & \frac{\cos(\theta)}{E\,(\cos(\theta)^2+\sin(\theta)^2)} & -\frac{\sin(\theta)}{E\,(\cos(\theta)^2+\sin(\theta)^2)} & 0 \\ 0 & \frac{\sin(\theta)}{E\,(\cos(\theta)^2+\sin(\theta)^2)} & \frac{\cos(\theta)}{E\,(\cos(\theta)^2+\sin(\theta)^2)} & 0 \\ 0 & 0 & 0 & \frac{1}{E} \end{bmatrix}$$

```
>
> LRy:=subs(evalm(E*array([[(1-Ly^2)^(1/2),0,-Ly,0],[0,1,0,0],[Ly,0,(1-Ly^2)^(1/2)
,0],[0,0,0,1]])));RPRIME:=simplify(innerprod(LRy,r)):XX:=RPRIME[1];YY:=RPRIME[2]
;ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=det(LRz);R:=innerprod(LRy,r);in
itial_rQF:=innerprod(r,LGUN,r);Final_RQF:=innerprod(R,LGUN,R);LRyn:=inverse(LRy)
;
```

$$LRy := \begin{bmatrix} E\sqrt{1-Ly^2} & 0 & -E\,Ly & 0 \\ 0 & E & 0 & 0 \\ E\,Ly & 0 & E\sqrt{1-Ly^2} & 0 \\ 0 & 0 & 0 & E \end{bmatrix}$$

$$XX := E\sqrt{1-Ly^2}\,x - E\,Ly\,z$$

$$YY := E\,y$$

$$ZZ := E\,Ly\,x + E\sqrt{1-Ly^2}\,z$$

$$TT := E\,C\,t$$

$$DETL := E^4$$

$$R := [E\sqrt{1-Ly^2}\,x - E\,Ly\,z,\ E\,y,\ E\,Ly\,x + E\sqrt{1-Ly^2}\,z,\ E\,C\,t]$$

$$initial\_rQF := x^2 + y^2 + z^2 - C^2\,t^2$$

$$Final\_RQF := E^2\,x^2 + E^2\,y^2 + E^2\,z^2 - E^2\,C^2\,t^2$$

$$LRyn := \begin{bmatrix} \frac{\sqrt{1-Ly^2}}{E} & 0 & \frac{Ly}{E} & 0 \\ 0 & \frac{1}{E} & 0 & 0 \\ -\frac{Ly}{E} & 0 & \frac{\sqrt{1-Ly^2}}{E} & 0 \\ 0 & 0 & 0 & \frac{1}{E} \end{bmatrix}$$

The next exercise is to construct the generators for the Lorentz translations.

```
> Delta:=1/(1-(Vz/C)^2)^(1/2):LTz:=subs(evalm(E*array([[1,0,0,0],[0,1,0,0],[0,0,De
lta,Delta*Vz/C],[0,0,Delta*Vz/C,Delta]])));RPRIME:=simplify(innerprod(LTz,r)):XX
:=RPRIME[1];YY:=RPRIME[2];ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=det(LT
z);R:=innerprod(LTz,r):initial_rQF:=innerprod(r,LGUN,r);Final_RQF:=innerprod(R,L
GUN,R);LTzn:=inverse(LTz);
```

$$LTz := \begin{bmatrix} E & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & \frac{E}{\sqrt{1-\frac{Vz^2}{C^2}}} & \frac{E\,Vz}{\sqrt{1-\frac{Vz^2}{C^2}}\,C} \\ 0 & 0 & \frac{E\,Vz}{\sqrt{1-\frac{Vz^2}{C^2}}\,C} & \frac{E}{\sqrt{1-\frac{Vz^2}{C^2}}} \end{bmatrix}$$

$$XX := E\,x$$

$$YY := E\,y$$

$$ZZ := \frac{E\,(z+Vz\,t)}{\sqrt{\frac{C^2-Vz^2}{C^2}}}$$

$$TT := \frac{E\,(Vz\,z+C^2\,t)}{\sqrt{\frac{C^2-Vz^2}{C^2}}\,C}$$

$$DETL := E^4$$

$$initial\_rQF := x^2+y^2+z^2-C^2\,t^2$$

$$Final\_RQF := -(C^2\,t^2-x^2-z^2-y^2)\,E^2$$

$$LTzn := \begin{bmatrix} \frac{1}{E} & 0 & 0 & 0 \\ 0 & \frac{1}{E} & 0 & 0 \\ 0 & 0 & \frac{1}{E\sqrt{\frac{C^2-Vz^2}{C^2}}} & -\frac{Vz}{E\sqrt{\frac{C^2-Vz^2}{C^2}}\,C} \\ 0 & 0 & -\frac{Vz}{E\sqrt{\frac{C^2-Vz^2}{C^2}}\,C} & \frac{1}{E\sqrt{\frac{C^2-Vz^2}{C^2}}} \end{bmatrix}$$

The Lorentz translation can also be given a trigonometric representation as

```
> Delta:=1/cos(Vz/C):LTcosz:=subs(evalm(E*array([[1,0,0,0],[0,1,0,0],[0,0,Delta,Delta*sin(Vz/C)],[0,0,Delta*sin(Vz/C),Delta]])));RPRIME:=simplify(innerprod(LTcosz,r)):XX:=RPRIME[1];YY:=RPRIME[2];ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=det(LTz);R:=innerprod(LTz,r):initial_rQF:=innerprod(r,LGUN,r);Final_RQF:=innerprod(R,LGUN,R);LTcoszn:=inverse(LTcosz);
```

$$LTcosz := \begin{bmatrix} E & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & \frac{E}{\cos\left(\frac{Vz}{C}\right)} & \frac{E\sin\left(\frac{Vz}{C}\right)}{\cos\left(\frac{Vz}{C}\right)} \\ 0 & 0 & \frac{E\sin\left(\frac{Vz}{C}\right)}{\cos\left(\frac{Vz}{C}\right)} & \frac{E}{\cos\left(\frac{Vz}{C}\right)} \end{bmatrix}$$

$$XX := E\,x$$

$$YY := E\,y$$

$$ZZ := \frac{E\left(z + \sin\left(\frac{Vz}{C}\right) C\,t\right)}{\cos\left(\frac{Vz}{C}\right)}$$

$$TT := \frac{E\left(\sin\left(\frac{Vz}{C}\right) z + C\,t\right)}{\cos\left(\frac{Vz}{C}\right)}$$

$$DETL := E^4$$

$$initial\_rQF := x^2 + y^2 + z^2 - C^2\,t^2$$

$$Final\_RQF := -(C^2\,t^2 - x^2 - z^2 - y^2)\,E^2$$

$$LTcoszn := \begin{bmatrix} \frac{1}{E} & 0 & 0 & 0 \\ 0 & \frac{1}{E} & 0 & 0 \\ 0 & 0 & -\frac{\cos\left(\frac{Vz}{C}\right)}{E\left(-1 + \sin\left(\frac{Vz}{C}\right)^2\right)} & \frac{\cos\left(\frac{Vz}{C}\right)\sin\left(\frac{Vz}{C}\right)}{E\left(-1 + \sin\left(\frac{Vz}{C}\right)^2\right)} \\ 0 & 0 & \frac{\cos\left(\frac{Vz}{C}\right)\sin\left(\frac{Vz}{C}\right)}{E\left(-1 + \sin\left(\frac{Vz}{C}\right)^2\right)} & -\frac{\cos\left(\frac{Vz}{C}\right)}{E\left(-1 + \sin\left(\frac{Vz}{C}\right)^2\right)} \end{bmatrix}$$

NOTE If the expansion factor is E = 1/cos(Vz/C) then the coefficients are

```
> subs(E=1/(cos(Vz/C)),evalm(LTcosz));
```

$$\begin{bmatrix} \frac{1}{\cos\left(\frac{Vz}{C}\right)} & 0 & 0 & 0 \\ 0 & \frac{1}{\cos\left(\frac{Vz}{C}\right)} & 0 & 0 \\ 0 & 0 & \frac{1}{\cos\left(\frac{Vz}{C}\right)^2} & \frac{\sin\left(\frac{Vz}{C}\right)}{\cos\left(\frac{Vz}{C}\right)^2} \\ 0 & 0 & \frac{\sin\left(\frac{Vz}{C}\right)}{\cos\left(\frac{Vz}{C}\right)^2} & \frac{1}{\cos\left(\frac{Vz}{C}\right)^2} \end{bmatrix}$$

and related to the generators of

the Raliegh - Taylor the terms: {sin(V/C)/cos^2(V/C)} and the

Kelvin - Helmholtz the terms {1/cos^2(V/C)}

shear or wake discontinuties.!!!!!

Extraordinary that the Lorentz map generates something that can be related to a fluid!!!!.

See http://www22.pair.com/csdc/car/carfre15.htm

(Wakes are related to the eikonal condition again)

```
>
> Delta:=1/(1-(Vx/C)^2)^(1/2):LTx:=subs(E*evalm(array([[Delta,0,0,Delta*Vx/C],[0,1
,0,0],[0,0,1,0],[Delta*Vx/C,0,0,Delta]])));RPRIME:=simplify(innerprod(LTx,r)):XX
:=RPRIME[1];YY:=RPRIME[2];ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=det(LT
z);R:=innerprod(LTx,r):initial_rQF:=innerprod(r,LGUN,r);Final_RQF:=innerprod(R,L
GUN,R);LTxn:=inverse(LTx);
```

$$LTx := E \begin{bmatrix} \frac{1}{\sqrt{1-\frac{Vx^2}{C^2}}} & 0 & 0 & \frac{Vx}{\sqrt{1-\frac{Vx^2}{C^2}}\,C} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{Vx}{\sqrt{1-\frac{Vx^2}{C^2}}\,C} & 0 & 0 & \frac{1}{\sqrt{1-\frac{Vx^2}{C^2}}} \end{bmatrix}$$

$$XX := \frac{E\,(x+Vx\,t)}{\sqrt{\frac{C^2-Vx^2}{C^2}}}$$

$$YY := E\,y$$

$$ZZ := E\,z$$

$$TT := \frac{E\,(Vx\,x+C^2\,t)}{\sqrt{\frac{C^2-Vx^2}{C^2}}\,C}$$

$$DETL := E^4$$

$$initial\_rQF := x^2 + y^2 + z^2 - C^2 t^2$$

$$Final\_RQF := -(C^2 t^2 - x^2 - z^2 - y^2) E^2$$

$$LTxn := \begin{bmatrix} \frac{1}{E\sqrt{\frac{C^2 - Vx^2}{C^2}}} & 0 & 0 & -\frac{Vx}{E\sqrt{\frac{C^2 - Vx^2}{C^2}}\,C} \\ 0 & \frac{1}{E} & 0 & 0 \\ 0 & 0 & \frac{1}{E} & 0 \\ -\frac{Vx}{E\sqrt{\frac{C^2 - Vx^2}{C^2}}\,C} & 0 & 0 & \frac{1}{E\sqrt{\frac{C^2 - Vx^2}{C^2}}} \end{bmatrix}$$

```
> Delta:=1/(1-(Vy/C)^2)^(1/2):LTy:=subs(evalm(E*array([[1,0,0,0],[0,Delta,0,Delta*
Vy/C],[0,0,1,0],[0,Delta*Vy/C,0,Delta]])));RPRIME:=simplify(innerprod(LTy,r)):XX
:=RPRIME[1];YY:=RPRIME[2];ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=det(LT
y);R:=innerprod(LTy,r):initial_rQF:=innerprod(r,LGUN,r);Final_RQF:=innerprod(R,L
GUN,R);LTyn:=inverse(LTy);
```

$$LTy := \begin{bmatrix} E & 0 & 0 & 0 \\ 0 & \frac{E}{\sqrt{1 - \frac{Vy^2}{C^2}}} & 0 & \frac{E\,Vy}{\sqrt{1 - \frac{Vy^2}{C^2}}\,C} \\ 0 & 0 & E & 0 \\ 0 & \frac{E\,Vy}{\sqrt{1 - \frac{Vy^2}{C^2}}\,C} & 0 & \frac{E}{\sqrt{1 - \frac{Vy^2}{C^2}}} \end{bmatrix}$$

$$XX := E\,x$$

$$YY := \frac{E\,(y + Vy\,t)}{\sqrt{\frac{C^2 - Vy^2}{C^2}}}$$

$$ZZ := E\,z$$

$$TT := \frac{E\,(Vy\,y + C^2\,t)}{\sqrt{\frac{C^2 - Vy^2}{C^2}}\,C}$$

$$DETL := E^4$$

$$initial\_rQF := x^2 + y^2 + z^2 - C^2 t^2$$

$$Final\_RQF := -(C^2 t^2 - x^2 - z^2 - y^2) E^2$$

$$LTyn := \begin{bmatrix} \frac{1}{E} & 0 & 0 & 0 \\ 0 & \frac{1}{E\sqrt{\frac{C^2-Vy^2}{C^2}}} & 0 & -\frac{Vy}{E\sqrt{\frac{C^2-Vy^2}{C^2}}\,C} \\ 0 & 0 & \frac{1}{E} & 0 \\ 0 & -\frac{Vy}{E\sqrt{\frac{C^2-Vy^2}{C^2}}\,C} & 0 & \frac{1}{E\sqrt{\frac{C^2-Vy^2}{C^2}}} \end{bmatrix}$$

Note that combinations of products of the six generators produce other Lorentz transformations. The "group" is multiplicative but not additive, and is non-abelian. The inverse of each generator is the matrix where the rotation or translation parameter is the negative value of the original matrix.

Now it is interesting that translation by Vx followed by a translation Vz, then followed by translation of -Vx (the inverse to Vx) and then followed by translation of -Vz (the inverse of Vz), does not close. The product of all four translations is still a Lorentz map, but the map is not the identity: Hence the space of Lorentz generators has curvature or torsion or both.

To show this, compute the "Round Trip" RT product of two translations followed by two inverse translations for a product of 4 generators. The result is still a Lorentz transformation, but it is not the identity. The same is true for a Round Trip of rotations. AS will be shown below, the RT defect of translations can produce curvature and torsion 2-forms. However, the RT defect of rotations does not produce curvature or torsion 2-forms!!!!!

```
> RT:=innerprod(LTzn,LTxn,LTz,LTx);RPRIME:=simplify(innerprod(RT,r)):XX:=simplify(RPRIME[1]);YY:=RPRIME[2];ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=simplify(det(RT));R:=innerprod(RT,r):initial_rQF:=innerprod(r,LGUN,r);Final_RQF:=subs(Lx=1,Ly=1,factor(innerprod(R,LGUN,R)));innerprod(transpose(RT),RT);
```

$RT :=$

$$\left[\frac{C^2\sqrt{\frac{C^2-Vz^2}{C^2}}-Vx^2}{\sqrt{\frac{C^2-Vz^2}{C^2}}\,(C^2-Vx^2)}, 0, -\frac{Vz\,Vx}{\sqrt{\frac{C^2-Vz^2}{C^2}}\,C^2\sqrt{\frac{C^2-Vx^2}{C^2}}}, \frac{C\,Vx\left(-1+\sqrt{\frac{C^2-Vz^2}{C^2}}\right)}{\sqrt{\frac{C^2-Vz^2}{C^2}}\,(C^2-Vx^2)}\right]$$

$$[0, 1, 0, 0]$$

$$\left[\frac{Vz\,Vx\left(C^2-Vz^2+C^2\sqrt{\frac{C^2-Vz^2}{C^2}}\sqrt{\frac{C^2-Vx^2}{C^2}}-C^2\sqrt{\frac{C^2-Vz^2}{C^2}}\right)}{\sqrt{\frac{C^2-Vz^2}{C^2}}\,(C^2-Vx^2)\,(C^2-Vz^2)}, 0, \frac{C^2\sqrt{\frac{C^2-Vx^2}{C^2}}-Vz^2}{(C^2-Vz^2)\sqrt{\frac{C^2-Vx^2}{C^2}}},\right.$$
$$\left.\frac{Vz\left(Vx^2C^2-Vx^2Vz^2+C^4\sqrt{\frac{C^2-Vz^2}{C^2}}\sqrt{\frac{C^2-Vx^2}{C^2}}-C^4\sqrt{\frac{C^2-Vz^2}{C^2}}\right)}{\sqrt{\frac{C^2-Vz^2}{C^2}}\,C\,(C^2-Vx^2)\,(C^2-Vz^2)}\right]$$

$$\left[-\frac{C\,Vx\left(C^2-Vz^2+\sqrt{\frac{C^2-Vz^2}{C^2}}\,Vz^2\sqrt{\frac{C^2-Vx^2}{C^2}}-C^2\sqrt{\frac{C^2-Vz^2}{C^2}}\right)}{\sqrt{\frac{C^2-Vz^2}{C^2}}\,(C^2-Vx^2)\,(C^2-Vz^2)},\ 0\ ,-\frac{C\,Vz\left(\sqrt{\frac{C^2-Vx^2}{C^2}}-1\right)}{(C^2-Vz^2)\sqrt{\frac{C^2-Vx^2}{C^2}}},\right.$$
$$\left.-\frac{Vx^2\,C^2-Vx^2\,Vz^2+C^2\sqrt{\frac{C^2-Vz^2}{C^2}}\,Vz^2\sqrt{\frac{C^2-Vx^2}{C^2}}-C^4\sqrt{\frac{C^2-Vz^2}{C^2}}}{\sqrt{\frac{C^2-Vz^2}{C^2}}\,(C^2-Vx^2)\,(C^2-Vz^2)}\right]$$

$$XX := \left(x\,C^4\sqrt{\frac{C^2-Vx^2}{C^2}}\sqrt{\frac{C^2-Vz^2}{C^2}}-x\,C^2\sqrt{\frac{C^2-Vx^2}{C^2}}\,Vx^2-Vz\,Vx\,z\,C^2+Vz\,Vx^3\,z\right.$$
$$\left.+C^4\,Vx\,t\sqrt{\frac{C^2-Vx^2}{C^2}}\sqrt{\frac{C^2-Vz^2}{C^2}}-C^4\,Vx\,t\sqrt{\frac{C^2-Vx^2}{C^2}}\right)\Big/\left(\sqrt{\frac{C^2-Vz^2}{C^2}}\,(C^2-Vx^2)\,C^2\sqrt{\frac{C^2-Vx^2}{C^2}}\right)$$

$$YY := y$$

$$ZZ := -\left(-Vz\,Vx\,x\sqrt{\frac{C^2-Vx^2}{C^2}}\,C^2+Vz^3\,Vx\,x\sqrt{\frac{C^2-Vx^2}{C^2}}-Vz\,Vx\,x\,C^2\sqrt{\frac{C^2-Vz^2}{C^2}}+Vz\,Vx^3\,x\sqrt{\frac{C^2-Vz^2}{C^2}}\right.$$
$$+Vz\,Vx\,x\sqrt{\frac{C^2-Vx^2}{C^2}}\,C^2\sqrt{\frac{C^2-Vz^2}{C^2}}-z\sqrt{\frac{C^2-Vz^2}{C^2}}\,C^4\sqrt{\frac{C^2-Vx^2}{C^2}}+z\sqrt{\frac{C^2-Vz^2}{C^2}}\,C^2\sqrt{\frac{C^2-Vx^2}{C^2}}\,Vx^2$$
$$+z\sqrt{\frac{C^2-Vz^2}{C^2}}\,C^2\,Vz^2-z\sqrt{\frac{C^2-Vz^2}{C^2}}\,Vx^2\,Vz^2-Vz\,t\sqrt{\frac{C^2-Vx^2}{C^2}}\,Vx^2\,C^2+Vz^3\,t\sqrt{\frac{C^2-Vx^2}{C^2}}\,Vx^2$$
$$\left.-Vz\,t\,C^4\sqrt{\frac{C^2-Vz^2}{C^2}}+Vz\,t\,C^2\sqrt{\frac{C^2-Vz^2}{C^2}}\,Vx^2+Vz\,t\sqrt{\frac{C^2-Vx^2}{C^2}}\,C^4\sqrt{\frac{C^2-Vz^2}{C^2}}\right)\Big/\Bigg($$
$$\left.\sqrt{\frac{C^2-Vz^2}{C^2}}\,(C^2-Vx^2)\,(C^2-Vz^2)\sqrt{\frac{C^2-Vx^2}{C^2}}\right)$$

$$TT := \left(-Vx\,x\sqrt{\frac{C^2-Vx^2}{C^2}}\,C^4+Vx\,x\sqrt{\frac{C^2-Vx^2}{C^2}}\,Vz^2\,C^2-Vx\,x\sqrt{\frac{C^2-Vz^2}{C^2}}\,Vz^2\,C^2+Vx^3\,x\sqrt{\frac{C^2-Vz^2}{C^2}}\,Vz^2\right.$$
$$+Vx\,x\sqrt{\frac{C^2-Vx^2}{C^2}}\,C^4\sqrt{\frac{C^2-Vz^2}{C^2}}-Vz\,z\sqrt{\frac{C^2-Vz^2}{C^2}}\,C^4\sqrt{\frac{C^2-Vx^2}{C^2}}+Vz\,z\sqrt{\frac{C^2-Vz^2}{C^2}}\sqrt{\frac{C^2-Vx^2}{C^2}}\,Vx^2\,C^2$$
$$+Vz\,z\sqrt{\frac{C^2-Vz^2}{C^2}}\,C^4-Vz\,z\sqrt{\frac{C^2-Vz^2}{C^2}}\,Vx^2\,C^2-t\sqrt{\frac{C^2-Vx^2}{C^2}}\,Vx^2\,C^4+t\sqrt{\frac{C^2-Vx^2}{C^2}}\,Vx^2\,Vz^2\,C^2$$
$$\left.-t\,C^4\sqrt{\frac{C^2-Vz^2}{C^2}}\,Vz^2+t\,C^2\sqrt{\frac{C^2-Vz^2}{C^2}}\,Vz^2\,Vx^2+t\sqrt{\frac{C^2-Vx^2}{C^2}}\,C^6\sqrt{\frac{C^2-Vz^2}{C^2}}\right)\Big/\Bigg($$
$$\left.C\sqrt{\frac{C^2-Vz^2}{C^2}}\,(C^2-Vx^2)\,(C^2-Vz^2)\sqrt{\frac{C^2-Vx^2}{C^2}}\right)$$

$$DETL := 1$$
$$initial\_rQF := x^2+y^2+z^2-C^2\,t^2$$
$$Final\_RQF := x^2+y^2+z^2-C^2\,t^2$$

$$\Bigg[\Bigg(C^8-4\sqrt{\frac{C^2-Vz^2}{C^2}}\,Vx^2\,C^6-2\,C^6\,Vz^2+2\,Vx^2\,C^6+Vx^4\,C^4+Vz^4\,C^4+4\sqrt{\frac{C^2-Vx^2}{C^2}}\sqrt{\frac{C^2-Vz^2}{C^2}}\,Vx^2\,C^4\,Vz^2$$

$$- 4\, Vx^2\, Vz^2\, C^4 \sqrt{\frac{C^2-Vx^2}{C^2}} + 2\, Vx^2\, Vz^2\, C^4 - 2\, Vx^4\, C^2\, Vz^2 - Vx^4\, Vz^4 \Bigg) \Bigg/ \left(\left(C^2-Vx^2\right)^2 \left(C^2-Vz^2\right)^2\right), 0, -2\, Vz\, Vx \Bigg(C^4$$

$$- C^2\, Vz^2 + \sqrt{\frac{C^2-Vz^2}{C^2}}\, Vx^2\, Vz^2 - C^4 \sqrt{\frac{C^2-Vx^2}{C^2}} + C^2 \sqrt{\frac{C^2-Vx^2}{C^2}}\, Vz^2 - C^4 \sqrt{\frac{C^2-Vz^2}{C^2}}$$

$$+ C^2 \sqrt{\frac{C^2-Vz^2}{C^2}}\, Vz^2 \sqrt{\frac{C^2-Vx^2}{C^2}} + C^4 \sqrt{\frac{C^2-Vz^2}{C^2}} \sqrt{\frac{C^2-Vx^2}{C^2}} - C^2\, Vz^2 \sqrt{\frac{C^2-Vz^2}{C^2}} \Bigg) \Bigg/ \Bigg($$

$$\sqrt{\frac{C^2-Vz^2}{C^2}} \left(C^2-Vx^2\right) \left(C^2-Vz^2\right)^2 \sqrt{\frac{C^2-Vx^2}{C^2}} \Bigg), 2 \Bigg(\Bigg(\Bigg(-C^6 \sqrt{\frac{C^2-Vz^2}{C^2}} + C^6 + Vz^2\, C^4 \sqrt{\frac{C^2-Vz^2}{C^2}} \sqrt{\frac{C^2-Vx^2}{C^2}}$$

$$+ C^4\, Vx^2 - 2\, C^4 \sqrt{\frac{C^2-Vx^2}{C^2}}\, Vz^2 - \sqrt{\frac{C^2-Vz^2}{C^2}}\, Vx^2\, C^4 + C^2\, Vz^4 - C^2\, Vx^2\, Vz^2$$

$$+ \sqrt{\frac{C^2-Vx^2}{C^2}} \sqrt{\frac{C^2-Vz^2}{C^2}}\, C^2\, Vx^2\, Vz^2 - Vx^2\, Vz^4 \Bigg) Vx\, C \Bigg) \Bigg/ \left(\left(C^2-Vx^2\right)^2 \left(C^2-Vz^2\right)^2\right) \Bigg]$$

$[0, 1, 0, 0]$

$$\Bigg[-2\, Vz\, Vx \Bigg(C^4 - C^2\, Vz^2 + \sqrt{\frac{C^2-Vz^2}{C^2}}\, Vx^2\, Vz^2 - C^4 \sqrt{\frac{C^2-Vx^2}{C^2}} + C^2 \sqrt{\frac{C^2-Vx^2}{C^2}}\, Vz^2 - C^4 \sqrt{\frac{C^2-Vz^2}{C^2}}$$

$$+ C^2 \sqrt{\frac{C^2-Vz^2}{C^2}}\, Vz^2 \sqrt{\frac{C^2-Vx^2}{C^2}} + C^4 \sqrt{\frac{C^2-Vz^2}{C^2}} \sqrt{\frac{C^2-Vx^2}{C^2}} - C^2\, Vz^2 \sqrt{\frac{C^2-Vz^2}{C^2}} \Bigg) \Bigg/ \Bigg($$

$$\sqrt{\frac{C^2-Vz^2}{C^2}} \left(C^2-Vx^2\right) \left(C^2-Vz^2\right)^2 \sqrt{\frac{C^2-Vx^2}{C^2}} \Bigg), 0,$$

$$- \frac{Vx^2\, Vz^4 - C^6 + C^4\, Vx^2 + 4\, C^4 \sqrt{\frac{C^2-Vx^2}{C^2}}\, Vz^2 - C^2\, Vz^4 - 2\, Vz^2\, C^4}{\left(C^2-Vz^2\right)^2 \left(C^2-Vx^2\right)}, -2\, Vz\, C \Bigg(\sqrt{\frac{C^2-Vz^2}{C^2}}\, Vx^2\, Vz^2 + Vx^2\, C^2$$

$$- Vx^2\, Vz^2 - \sqrt{\frac{C^2-Vx^2}{C^2}}\, C^2\, Vx^2 + \sqrt{\frac{C^2-Vx^2}{C^2}}\, Vx^2\, Vz^2 - C^4 \sqrt{\frac{C^2-Vz^2}{C^2}} + C^4 \sqrt{\frac{C^2-Vz^2}{C^2}} \sqrt{\frac{C^2-Vx^2}{C^2}}$$

$$+ C^2 \sqrt{\frac{C^2-Vz^2}{C^2}}\, Vz^2 \sqrt{\frac{C^2-Vx^2}{C^2}} - C^2\, Vz^2 \sqrt{\frac{C^2-Vz^2}{C^2}} \Bigg) \Bigg/ \Bigg($$

$$\left(C^2-Vz^2\right)^2 \sqrt{\frac{C^2-Vx^2}{C^2}} \sqrt{\frac{C^2-Vz^2}{C^2}} \left(C^2-Vx^2\right) \Bigg) \Bigg]$$

$$\Bigg[2 \Bigg(\Bigg(\Bigg(-C^6 \sqrt{\frac{C^2-Vz^2}{C^2}} + C^6 + Vz^2\, C^4 \sqrt{\frac{C^2-Vz^2}{C^2}} \sqrt{\frac{C^2-Vx^2}{C^2}} + C^4\, Vx^2 - 2\, C^4 \sqrt{\frac{C^2-Vx^2}{C^2}}\, Vz^2$$

$$- \sqrt{\frac{C^2-Vz^2}{C^2}}\, Vx^2\, C^4 + C^2\, Vz^4 - C^2\, Vx^2\, Vz^2 + \sqrt{\frac{C^2-Vx^2}{C^2}} \sqrt{\frac{C^2-Vz^2}{C^2}}\, C^2\, Vx^2\, Vz^2 - Vx^2\, Vz^4 \Bigg) Vx\, C \Bigg) \Bigg/ \Big($$

$$\left(C^2-Vx^2\right)^2 \left(C^2-Vz^2\right)^2 \Big), 0, -2\, Vz\, C \Bigg(\sqrt{\frac{C^2-Vz^2}{C^2}}\, Vx^2\, Vz^2 + Vx^2\, C^2 - Vx^2\, Vz^2 - \sqrt{\frac{C^2-Vx^2}{C^2}}\, C^2\, Vx^2$$

$$+ \sqrt{\frac{C^2-Vx^2}{C^2}}\, Vx^2\, Vz^2 - C^4 \sqrt{\frac{C^2-Vz^2}{C^2}} + C^4 \sqrt{\frac{C^2-Vz^2}{C^2}} \sqrt{\frac{C^2-Vx^2}{C^2}} + C^2 \sqrt{\frac{C^2-Vz^2}{C^2}}\, Vz^2 \sqrt{\frac{C^2-Vx^2}{C^2}}$$

$$- C^2\, Vz^2 \sqrt{\frac{C^2 - Vz^2}{C^2}}\Bigg) \Bigg/ \Bigg((C^2 - Vz^2)^2 \sqrt{\frac{C^2 - Vx^2}{C^2}} \sqrt{\frac{C^2 - Vz^2}{C^2}}\, (C^2 - Vx^2) \Bigg), \Bigg(C^8 + 2\, Vx^2\, C^6$$

$$- 4\, C^6 \sqrt{\frac{C^2 - Vx^2}{C^2}}\, Vz^2 - 4 \sqrt{\frac{C^2 - Vz^2}{C^2}}\, Vx^2\, C^6 + 2\, C^6\, Vz^2 + Vx^4\, C^4 + Vz^4\, C^4$$

$$+ 4 \sqrt{\frac{C^2 - Vx^2}{C^2}} \sqrt{\frac{C^2 - Vz^2}{C^2}}\, Vx^2\, C^4\, Vz^2 - 4\, Vx^2\, Vz^2\, C^4 - Vx^4\, Vz^4 \Bigg) \Bigg/ \left((C^2 - Vx^2)^2\, (C^2 - Vz^2)^2 \right) \Bigg]$$

The same is true for a Round Trip RT product of two rotations followed by two inverse rotations for a product of 4 generators. The result is still a Lorentz transformation, but it is not the identity.

```
> RT:=evalm(innerprod(LRphizn,LRthetaxn,LRphiz,LRthetax));RPRIME:=simplify(innerprod(RT,r)):XX:=simplify(RPRIME[1]);YY:=RPRIME[2];ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=simplify(det(RT));R:=innerprod(RT,r):initial_rQF:=innerprod(r,LGUN,r);Final_RQF:=simplify(factor(innerprod(R,LGUN,R)));GUN:=innerprod(transpose(RT),RT);
```

$RT :=$

$$\Bigg[\frac{\cos(\phi)^2 \cos(\theta)^2 + \cos(\phi)^2 \sin(\theta)^2 + \cos(\theta) \sin(\phi)^2}{(\cos(\phi)^2 + \sin(\phi)^2)\, (\cos(\theta)^2 + \sin(\theta)^2)},$$

$$\frac{\sin(\phi)\, (\cos(\phi) \cos(\theta)^3 + \cos(\phi) \cos(\theta) \sin(\theta)^2 - \cos(\phi) \cos(\theta)^2 - \sin(\theta)^2)}{(\cos(\phi)^2 + \sin(\phi)^2)\, (\cos(\theta)^2 + \sin(\theta)^2)},$$

$$\frac{\sin(\phi) \sin(\theta)\, (\cos(\phi) \cos(\theta)^2 + \cos(\phi) \sin(\theta)^2 - \cos(\phi) \cos(\theta) + \cos(\theta))}{(\cos(\phi)^2 + \sin(\phi)^2)\, (\cos(\theta)^2 + \sin(\theta)^2)}, 0 \Bigg]$$

$$\Bigg[\frac{\cos(\phi) \sin(\phi)\, (\cos(\theta)^2 + \sin(\theta)^2 - \cos(\theta))}{(\cos(\phi)^2 + \sin(\phi)^2)\, (\cos(\theta)^2 + \sin(\theta)^2)},$$

$$\frac{\sin(\phi)^2 \cos(\theta)^3 + \cos(\theta) \sin(\phi)^2 \sin(\theta)^2 + \cos(\phi)^2 \cos(\theta)^2 + \cos(\phi) \sin(\theta)^2}{(\cos(\phi)^2 + \sin(\phi)^2)\, (\cos(\theta)^2 + \sin(\theta)^2)},$$

$$\frac{\sin(\theta)\, (\sin(\phi)^2 \cos(\theta)^2 + \sin(\phi)^2 \sin(\theta)^2 + \cos(\phi)^2 \cos(\theta) - \cos(\phi) \cos(\theta))}{(\cos(\phi)^2 + \sin(\phi)^2)\, (\cos(\theta)^2 + \sin(\theta)^2)}, 0 \Bigg]$$

$$\Bigg[-\frac{\sin(\theta) \sin(\phi)}{\cos(\theta)^2 + \sin(\theta)^2}, \frac{\cos(\theta) \sin(\theta)\, (\cos(\phi) - 1)}{\cos(\theta)^2 + \sin(\theta)^2}, \frac{\cos(\phi) \sin(\theta)^2 + \cos(\theta)^2}{\cos(\theta)^2 + \sin(\theta)^2}, 0 \Bigg]$$

$$[0, 0, 0, 1]$$

$$XX := x \cos(\phi)^2 + x \cos(\theta) - x \cos(\theta) \cos(\phi)^2 + \sin(\phi)\, y \cos(\phi) \cos(\theta) - \sin(\phi)\, y \cos(\phi) \cos(\theta)^2 - \sin(\phi)\, y$$
$$+ \sin(\phi)\, y \cos(\theta)^2 + \sin(\phi)\, z \cos(\phi) \sin(\theta) - \sin(\phi) \sin(\theta)\, z \cos(\phi) \cos(\theta) + \sin(\phi) \sin(\theta)\, z \cos(\theta)$$

$$YY := \cos(\phi) \sin(\phi)\, x - \cos(\phi) \sin(\phi)\, x \cos(\theta) + y \cos(\theta) - y \cos(\theta) \cos(\phi)^2 + y \cos(\phi)^2 \cos(\theta)^2 + y \cos(\phi)$$
$$- \cos(\theta)^2\, y \cos(\phi) + \sin(\theta)\, z - z \cos(\phi)^2 \sin(\theta) + \sin(\theta)\, z \cos(\phi)^2 \cos(\theta) - \sin(\theta)\, z \cos(\phi) \cos(\theta)$$

$$ZZ := -\sin(\theta) \sin(\phi)\, x + \cos(\theta) \sin(\theta)\, y \cos(\phi) - \cos(\theta) \sin(\theta)\, y + z \cos(\phi) - z \cos(\phi) \cos(\theta)^2 + z \cos(\theta)^2$$

$$TT := C\, t$$

$$DETL := 1$$

$$initial\_rQF := x^2 + y^2 + z^2 - C^2\, t^2$$

$$Final\_RQF := x^2 + y^2 + z^2 - C^2\, t^2$$

$GUN :=$

$$[(\sin(\phi)^2 \cos(\theta)^2 + 2 \cos(\theta)^2 \sin(\theta)^2 \cos(\phi)^2 + \cos(\phi)^2 \sin(\phi)^2 \sin(\theta)^2 + \cos(\phi)^2 \sin(\theta)^4 + \cos(\phi)^2 \cos(\theta)^4$$
$$+ \sin(\phi)^4 \sin(\theta)^2) \Big/ \left((\cos(\phi)^2 + \sin(\phi)^2)\, (\cos(\theta)^2 + \sin(\theta)^2)^2 \right), - ((\cos(\phi)^3 \sin(\theta)^2 - \cos(\phi)^2 \sin(\theta)^2$$
$$- 2 \cos(\phi) \sin(\theta)^2 \cos(\theta)^2 - \cos(\phi) \cos(\theta)^4 + \cos(\phi) \cos(\theta)^2 + \cos(\phi) \sin(\phi)^2 \sin(\theta)^2 - \cos(\phi) \sin(\theta)^4$$

$- \sin(\phi)^2 \sin(\theta)^2 + \sin(\theta)^2) \cos(\theta) \sin(\phi)) / ((\cos(\phi)^2 + \sin(\phi)^2)(\cos(\theta)^2 + \sin(\theta)^2)^2), -((\cos(\phi)^3 \sin(\theta)^2$
$+ \cos(\phi)^2 \cos(\theta)^2 - 2\cos(\phi)\sin(\theta)^2\cos(\theta)^2 - \cos(\phi)\cos(\theta)^4 + \cos(\phi)\cos(\theta)^2 + \cos(\phi)\sin(\phi)^2\sin(\theta)^2$
$- \cos(\phi)\sin(\theta)^4 + \sin(\phi)^2\cos(\theta)^2 - \cos(\theta)^2)\sin(\phi)\sin(\theta)) / ((\cos(\phi)^2 + \sin(\phi)^2)(\cos(\theta)^2 + \sin(\theta)^2)^2), 0]$

$[-((\cos(\phi)^3\sin(\theta)^2 - \cos(\phi)^2\sin(\theta)^2 - 2\cos(\phi)\sin(\theta)^2\cos(\theta)^2 - \cos(\phi)\cos(\theta)^4 + \cos(\phi)\cos(\theta)^2$
$+ \cos(\phi)\sin(\phi)^2\sin(\theta)^2 - \cos(\phi)\sin(\theta)^4 - \sin(\phi)^2\sin(\theta)^2 + \sin(\theta)^2)\cos(\theta)\sin(\phi)) / ($
$(\cos(\phi)^2 + \sin(\phi)^2)(\cos(\theta)^2 + \sin(\theta)^2)^2), (\sin(\theta)^4 - 2\cos(\phi)\cos(\theta)^2\sin(\phi)^2\sin(\theta)^2 + \sin(\phi)^2\cos(\theta)^6$
$+ 2\sin(\phi)^2\sin(\theta)^2\cos(\theta)^4 + \cos(\phi)^2\cos(\theta)^4 + \cos(\phi)^4\cos(\theta)^2\sin(\theta)^2 + 2\cos(\phi)\sin(\theta)^2\cos(\theta)^2$
$+ \cos(\theta)^2\sin(\theta)^2\cos(\phi)^2 + \sin(\phi)^2\sin(\theta)^4\cos(\theta)^2 - 2\cos(\phi)^3\cos(\theta)^2\sin(\theta)^2 + \sin(\phi)^2\cos(\theta)^2\sin(\theta)^2$
$+ \cos(\phi)^2\sin(\phi)^2\cos(\theta)^2\sin(\theta)^2) / ((\cos(\phi)^2 + \sin(\phi)^2)(\cos(\theta)^2 + \sin(\theta)^2)^2), ((\sin(\theta)^2\cos(\phi)^4$
$- \cos(\phi)^3\sin(\theta)^2 + \cos(\phi)^3\cos(\theta)^2 + \cos(\phi)^2\sin(\phi)^2\sin(\theta)^2 - \cos(\phi)\sin(\phi)^2\sin(\theta)^2 + \cos(\phi)\sin(\theta)^2$
$+ \cos(\phi)\sin(\phi)^2\cos(\theta)^2 - \cos(\phi)\cos(\theta)^2 - \sin(\theta)^2 + \sin(\phi)^2\cos(\theta)^4 + 2\sin(\phi)^2\cos(\theta)^2\sin(\theta)^2 - \sin(\phi)^2\cos(\theta)^2$
$+ \sin(\phi)^2\sin(\theta)^4)\sin(\theta)\cos(\theta)) / ((\cos(\phi)^2 + \sin(\phi)^2)(\cos(\theta)^2 + \sin(\theta)^2)^2), 0]$

$[-((\cos(\phi)^3\sin(\theta)^2 + \cos(\phi)^2\cos(\theta)^2 - 2\cos(\phi)\sin(\theta)^2\cos(\theta)^2 - \cos(\phi)\cos(\theta)^4 + \cos(\phi)\cos(\theta)^2$
$+ \cos(\phi)\sin(\phi)^2\sin(\theta)^2 - \cos(\phi)\sin(\theta)^4 + \sin(\phi)^2\cos(\theta)^2 - \cos(\theta)^2)\sin(\phi)\sin(\theta)) / ($
$(\cos(\phi)^2 + \sin(\phi)^2)(\cos(\theta)^2 + \sin(\theta)^2)^2), ((\sin(\theta)^2\cos(\phi)^4 - \cos(\phi)^3\sin(\theta)^2 + \cos(\phi)^3\cos(\theta)^2$
$+ \cos(\phi)^2\sin(\phi)^2\sin(\theta)^2 - \cos(\phi)\sin(\phi)^2\sin(\theta)^2 + \cos(\phi)\sin(\theta)^2 + \cos(\phi)\sin(\phi)^2\cos(\theta)^2 - \cos(\phi)\cos(\theta)^2$
$- \sin(\theta)^2 + \sin(\phi)^2\cos(\theta)^4 + 2\sin(\phi)^2\cos(\theta)^2\sin(\theta)^2 - \sin(\phi)^2\cos(\theta)^2 + \sin(\phi)^2\sin(\theta)^4)\sin(\theta)\cos(\theta)) / ($
$(\cos(\phi)^2 + \sin(\phi)^2)(\cos(\theta)^2 + \sin(\theta)^2)^2), (\cos(\phi)^2\cos(\theta)^4 + 2\cos(\phi)^3\cos(\theta)^2\sin(\theta)^2 + \cos(\phi)^4\sin(\theta)^4$
$+ 2\sin(\phi)^2\sin(\theta)^4\cos(\theta)^2 + \sin(\phi)^2\sin(\theta)^2\cos(\theta)^4 + \cos(\theta)^2\sin(\theta)^2 + \sin(\phi)^2\cos(\theta)^4 + \cos(\phi)^2\sin(\phi)^2\sin(\theta)^4$
$+ 2\cos(\phi)\cos(\theta)^2\sin(\phi)^2\sin(\theta)^2 + \sin(\phi)^2\sin(\theta)^6 - 2\cos(\phi)\sin(\theta)^2\cos(\theta)^2 + \cos(\theta)^2\sin(\theta)^2\cos(\phi)^2) / ($
$(\cos(\phi)^2 + \sin(\phi)^2)(\cos(\theta)^2 + \sin(\theta)^2)^2), 0]$

$[0, 0, 0, 1]$

```
> RT1:=evalm(innerprod(LRzn,LRxn,LRz,LRx));RT:=innerprod(transpose(
RT1),RT1);RPRIME:=simplify(innerprod(RT,r)):XX:=simplify(RPRIME[1]);YY:=RPRIME[2
];ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=simplify(det(RT));R:=innerprod
(RT,r):initial_rQF:=innerprod(r,LGUN,r);Final_RQF:=subs(Lx=1,Ly=1,factor(innerpr
od(R,LGUN,R)));innerprod(transpose(RT),RT);innerprod(transpose(LRz),LRz);innerpr
od(transpose(LTz),LTz);
```

$RT1 :=$

$[1 - Lz^2 + Lz^2\sqrt{1 - Lx^2}, Lz\sqrt{1 - Lx^2}\sqrt{1 - Lz^2} - \sqrt{1 - Lz^2}\,Lz + \sqrt{1 - Lz^2}\,Lz\,Lx^2 - Lz\,Lx^2,$
$Lz\,Lx\sqrt{1 - Lz^2} - Lz\,Lx\sqrt{1 - Lz^2}\sqrt{1 - Lx^2} + Lz\,Lx\sqrt{1 - Lx^2}, 0]$

$[-\sqrt{1 - Lz^2}\,Lz(-1 + \sqrt{1 - Lx^2}), Lz^2\sqrt{1 - Lx^2} + 1 - Lx^2 - Lz^2 + Lx^2 Lz^2 + Lx^2\sqrt{1 - Lz^2},$
$Lz^2 Lx + Lx\sqrt{1 - Lx^2} - Lx\,Lz^2\sqrt{1 - Lx^2} - Lx\sqrt{1 - Lx^2}\sqrt{1 - Lz^2}, 0]$

$[-Lz\,Lx, Lx\sqrt{1 - Lx^2}\sqrt{1 - Lz^2} - Lx\sqrt{1 - Lx^2}, 1 + Lx^2\sqrt{1 - Lz^2} - Lx^2, 0]$

$[0, 0, 0, 1]$

$$RT := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$XX := x$$

$$YY := y$$

$$ZZ := z$$

$$TT := C\,t$$

$$DETL := 1$$

$$initial\_rQF := x^2 + y^2 + z^2 - C^2\,t^2$$

$$Final\_RQF := x^2 + y^2 + z^2 - C^2\,t^2$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} E^2 & 0 & 0 & 0 \\ 0 & E^2 & 0 & 0 \\ 0 & 0 & E^2 & 0 \\ 0 & 0 & 0 & E^2 \end{bmatrix}$$

$$\begin{bmatrix} E^2 & 0 & 0 & 0 \\ 0 & E^2 & 0 & 0 \\ 0 & 0 & \frac{E^2\,(C^2 + Vz^2)}{C^2 - Vz^2} & 2\,\frac{E^2\,C\,Vz}{C^2 - Vz^2} \\ 0 & 0 & 2\,\frac{E^2\,C\,Vz}{C^2 - Vz^2} & \frac{E^2\,(C^2 + Vz^2)}{C^2 - Vz^2} \end{bmatrix}$$

On the other hand a translation along an axis, followed by a rotation about the same axis, followed by the inverse translation, followed by the inverse rotation about the same axis is the Identity. The process closes.

```
> RT:=innerprod(LRzn,LTzn,LRz,LTz);RPRIME:=simplify(innerprod(RT,r)):XX:=simplify(
  RPRIME[1]);YY:=RPRIME[2];ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=simplif
  y(det(RT));R:=innerprod(RT,r):initial_rQF:=innerprod(r,LGUN,r);Final_RQF:=subs(L
  x=1,Ly=1,factor(innerprod(R,LGUN,R)));
```

$$RT := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$XX := x$$

$$YY := y$$

$$ZZ := z$$

$$TT := C\,t$$

$$DETL := 1$$

$$initial\_rQF := x^2 + y^2 + z^2 - C^2\,t^2$$

$$Final\_RQF := x^2 + y^2 + z^2 - C^2\,t^2$$

However this is not the case if the translation and rotation are not about the same axis. THen the RT matrix is not the identity and the process of translate - rotate - inverse translate - inverse rotate is not the identity map. The RT product is still a Lorentz map even though it does not close.

```
> RT:=innerprod(LRxn,LTzn,LRx,LTz);RPRIME:=simplify(innerprod(RT,r)):XX:=simplify(
  RPRIME[1]);YY:=RPRIME[2];ZZ:=(RPRIME[3]);TT:=(simplify(RPRIME[4]));DETL:=simplif
  y(det(RT));R:=innerprod(RT,r):initial_rQF:=innerprod(r,LGUN,r);Final_RQF:=subs(L
  x=1,Ly=1,factor(innerprod(R,LGUN,R)));
```

$RT :=$

$$[1, 0, 0, 0]$$

$$\left[0, -\frac{-\sqrt{-\frac{-C^2+Vz^2}{C^2}} + Lx^2\sqrt{-\frac{-C^2+Vz^2}{C^2}} - Lx^2}{\sqrt{-\frac{-C^2+Vz^2}{C^2}}}, -\frac{Lx\left(\sqrt{1-Lx^2}\, C^2\sqrt{-\frac{-C^2+Vz^2}{C^2}} - \sqrt{1-Lx^2}\, C^2 + Vz^2\right)}{-C^2+Vz^2},\right.$$
$$\left. -\frac{Lx\, C\, Vz\left(\sqrt{1-Lx^2}\sqrt{-\frac{-C^2+Vz^2}{C^2}} - \sqrt{1-Lx^2} + 1\right)}{-C^2+Vz^2}\right]$$

$$\left[0, \frac{\sqrt{1-Lx^2}\, Lx\left(\sqrt{-\frac{-C^2+Vz^2}{C^2}} - 1\right)}{\sqrt{-\frac{-C^2+Vz^2}{C^2}}}, -\frac{C^2\, Lx^2\sqrt{-\frac{-C^2+Vz^2}{C^2}} + C^2 - C^2\, Lx^2 - \sqrt{1-Lx^2}\, Vz^2}{-C^2+Vz^2},\right.$$
$$\left. -\frac{C\, Vz\left(Lx^2\sqrt{-\frac{-C^2+Vz^2}{C^2}} + 1 - Lx^2 - \sqrt{1-Lx^2}\right)}{-C^2+Vz^2}\right]$$

$$\left[0, \frac{Lx\, Vz}{\sqrt{-\frac{-C^2+Vz^2}{C^2}}\, C}, \frac{C\, Vz\,(-1+\sqrt{1-Lx^2}\,)}{-C^2+Vz^2}, \frac{\sqrt{1-Lx^2}\, Vz^2 - C^2}{-C^2+Vz^2}\right]$$

$$XX := x$$

$$YY := -\left(y\, C^2\sqrt{-\frac{-C^2+Vz^2}{C^2}} - y\sqrt{-\frac{-C^2+Vz^2}{C^2}}\, Vz^2 - y\, C^2\, Lx^2\sqrt{-\frac{-C^2+Vz^2}{C^2}} + y\, Lx^2\sqrt{-\frac{-C^2+Vz^2}{C^2}}\, Vz^2\right.$$
$$+ y\, C^2\, Lx^2 - y\, Lx^2\, Vz^2 + Lx\, z\sqrt{1-Lx^2}\, C^2 - z\sqrt{1-Lx^2}\, Lx\, Vz^2 - Lx\, z\sqrt{-\frac{-C^2+Vz^2}{C^2}}\sqrt{1-Lx^2}\, C^2$$
$$+ Lx\, z\sqrt{-\frac{-C^2+Vz^2}{C^2}}\, Vz^2 + Lx\, C^2\, Vz\, t\sqrt{1-Lx^2} - Lx\, Vz^3\, t\sqrt{1-Lx^2} - Lx\, C^2\, Vz\, t\sqrt{-\frac{-C^2+Vz^2}{C^2}}\sqrt{1-Lx^2}$$
$$\left. + Lx\, C^2\, Vz\, t\sqrt{-\frac{-C^2+Vz^2}{C^2}}\right) \Bigg/ \left(\sqrt{-\frac{-C^2+Vz^2}{C^2}}\,(-C^2+Vz^2)\right)$$

$$ZZ := \left(-y\sqrt{1-Lx^2}\, Lx\, C^2\sqrt{-\frac{-C^2+Vz^2}{C^2}} + y\sqrt{1-Lx^2}\, Lx\sqrt{-\frac{-C^2+Vz^2}{C^2}}\, Vz^2 + y\sqrt{1-Lx^2}\, Lx\, C^2\right.$$
$$- y\sqrt{1-Lx^2}\, Lx\, Vz^2 - z\, C^2\, Lx^2 + z\, Lx^2\, Vz^2 - z\sqrt{-\frac{-C^2+Vz^2}{C^2}}\, C^2 + z\sqrt{-\frac{-C^2+Vz^2}{C^2}}\, C^2\, Lx^2$$
$$+ z\sqrt{-\frac{-C^2+Vz^2}{C^2}}\sqrt{1-Lx^2}\, Vz^2 - C^2\, Vz\, t\, Lx^2 + Vz^3\, t\, Lx^2 - C^2\, Vz\, t\sqrt{-\frac{-C^2+Vz^2}{C^2}} + C^2\, Vz\, t\sqrt{-\frac{-C^2+Vz^2}{C^2}}\, Lx^2$$
$$\left. + C^2\, Vz\, t\sqrt{-\frac{-C^2+Vz^2}{C^2}}\sqrt{1-Lx^2}\right) \Bigg/ \left(\sqrt{-\frac{-C^2+Vz^2}{C^2}}\,(-C^2+Vz^2)\right)$$

$$TT := \left(-Lx\, Vz\, y\, C^2 + Lx\, Vz^3\, y - C^2\, Vz\, z\sqrt{-\frac{-C^2+Vz^2}{C^2}} + C^2\, Vz\, z\sqrt{-\frac{-C^2+Vz^2}{C^2}}\sqrt{1-Lx^2}\right.$$

$$+C^2\, t \sqrt{-\frac{-C^2+Vz^2}{C^2}}\sqrt{1-Lx^2}\; Vz^2 - C^4\, t \sqrt{-\frac{-C^2+Vz^2}{C^2}}\Bigg) \Bigg/ \left(\sqrt{-\frac{-C^2+Vz^2}{C^2}}\; C\,(-C^2+Vz^2)\right)$$

$$DETL := 1$$

$$initial\_rQF := x^2+y^2+z^2-C^2\, t^2$$

$$Final\_RQF := x^2+y^2+z^2-C^2\, t^2$$

>

>

>

Now for any product combination of the Lorentz map generators it is possible to compute the Structural equations of Cartan based on the Frame field constructed from the given (or presumed) Lorentz map. The Cartan connection linearly connects the given Frame Field to its neighbors.

The algebra can be quite messy, but Maple with do the work.

>

Example 1:

THE TORSION and CURVATURE of a LORENTZ translation

along the z axis

```
> FFINV:=LTz:
```

```
> Z:=innerprod(FFINV,[d(x),d(y),d(z),d(C*t)]):sigma1:=Z[1];sigma2:=Z[2];sigma3:=Z[3];omega:=simplify((Z[4]));
```

The Vierbein 1-forms.

$$\sigma 1 := E\, \mathrm{d}(x)$$

$$\sigma 2 := E\, \mathrm{d}(y)$$

$$\sigma 3 := \frac{E\,(\mathrm{d}(z)\, C + Vz\, t\, \mathrm{d}(C) + Vz\, C\, \mathrm{d}(t))}{C\sqrt{-\frac{-C^2+Vz^2}{C^2}}}$$

$$\omega := \frac{E\,(Vz\, \mathrm{d}(z) + C\, t\, \mathrm{d}(C) + C^2\, \mathrm{d}(t))}{C\sqrt{-\frac{-C^2+Vz^2}{C^2}}}$$

```
> #Vol4:=wcollect(simplify(sigma1&^sigma2&^sigma3&^Z[4]));rho:=subs(getcoeff(Vol4));
```

The density (determinant)

The determinant cannot go to zero for the projective domain. The zero sets of the density function determine a hypersurface. IF the hypersurface is harmonic then it can be a boundary.

There is an induced metric on R4

```
> FF:=inverse(FFINV):Gun:=subs(innerprod(transpose(FF),FF));
```

$$Gun := \begin{bmatrix} \frac{1}{E^2} & 0 & 0 & 0 \\ 0 & \frac{1}{E^2} & 0 & 0 \\ 0 & 0 & -\frac{C^2+Vz^2}{E^2(-C^2+Vz^2)} & 2\frac{C\,Vz}{E^2(-C^2+Vz^2)} \\ 0 & 0 & 2\frac{C\,Vz}{E^2(-C^2+Vz^2)} & -\frac{C^2+Vz^2}{E^2(-C^2+Vz^2)} \end{bmatrix}$$

From the Frame Field use the standard methods to compute the

Cartan Matrix of connection 1-forms.

See http://www.uh.edu/~rkiehn/pdf/projfram.pdf

for details of the Cartan method for an arbitrary Repere Mobile.

```
> dFF:=array([[d(FF[1,1]),d(FF[1,2]),d(FF[1,3]),d(FF[1,4])],[d(FF[2,1]),d(FF[2,2])
,d(FF[2,3]),d(FF[2,4])],[d(FF[3,1]),d(FF[3,2]),d(FF[3,3]),d(FF[3,4])],[d(FF[4,1]
),d(FF[4,2]),d(FF[4,3]),d(FF[4,4])]]):
> cartan:=(evalm(FFINV&*dFF)):
```

The Interior (space-space) Connection 1 forms

```
> Gamma11:=factor(wcollect(cartan[1,1]));Gamma21:=factor(wcollect(cartan[2,1]));Ga
mma31:=factor(wcollect(cartan[3,1]));
```

$$\Gamma 11 := -\frac{\mathrm{d}(E)}{E}$$

$$\Gamma 21 := 0$$

$$\Gamma 31 := 0$$

```
> Gamma12:=factor(wcollect(cartan[1,2]));Gamma22:=factor(wcollect(cartan[2,2]));Ga
mma32:=factor(wcollect(cartan[3,2]));
```

$$\Gamma 12 := 0$$

$$\Gamma 22 := -\frac{\mathrm{d}(E)}{E}$$

$$\Gamma 32 := 0$$

```
> Gamma13:=wcollect(factor(wcollect(cartan[1,3])));Gamma23:=factor(wcollect(cartan
[2,3]));Gamma33:=simplify(wcollect(cartan[3,3]));
```

$$\Gamma 13 := 0$$

$$\Gamma 23 := 0$$

$$\Gamma 33 := -\frac{\mathrm{d}(E)}{E}$$

```
>
```

The "space-time" connection 1-forms are:

```
> hh1:=simplify(wcollect(cartan[4,1]));hh2:=factor(wcollect(cartan[4,2]));hh3:=wco
llect(factor(wcollect(cartan[4,3])));
>
```

$$hh1 := 0$$

$$hh2 := 0$$

$$hh3 := \frac{C\,\mathrm{d}(Vz)}{(-C+Vz)\,(C+Vz)} - \frac{Vz\,\mathrm{d}(C)}{(-C+Vz)\,(C+Vz)}$$

The "time-space connection" 1-forms are

```
> gg1:=factor(wcollect(factor(wcollect(cartan[1,4]))));gg2:=factor(wcollect(cartan
[2,4]));gg3:=wcollect(factor(wcollect(cartan[3,4])));
```

$$gg1 := 0$$

$$gg2 := 0$$

$$gg3 := \frac{C\,\mathrm{d}(Vz)}{(-C+Vz)\,(C+Vz)} - \frac{Vz\,\mathrm{d}(C)}{(-C+Vz)\,(C+Vz)}$$

The abnormality (time-time) connection 1-form

Note that the abnormality (Big Omega vanishes if expansion is a global constant. (no red shift?)

```
> Omega:=wcollect(subs(simplify(wcollect(cartan[4,4]))));
```

$$\Omega := -\frac{\mathrm{d}(E)}{E}$$

```
> L:=factor(wcollect(hh1&^sigma1+hh2&^sigma2+hh3&^sigma3));
```

$$L := E\,\left(C\,(\mathrm{d}(Vz)\ \&\!\wedge\ \mathrm{d}(z)) + Vz\,t\,(\mathrm{d}(Vz)\ \&\!\wedge\ \mathrm{d}(C)) + Vz\,C\,(\mathrm{d}(Vz)\ \&\!\wedge\ \mathrm{d}(t)) - Vz\,(\mathrm{d}(C)\ \&\!\wedge\ \mathrm{d}(z)) - Vz^2\,(\mathrm{d}(C)\ \&\!\wedge\ \mathrm{d}(t))\right) \Big/ \left(\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vz)\,(C+Vz)\right)$$

```
> S:=(wcollect(factor(hh1&^gg1+hh2&^gg2+hh3&^gg3)));
```

$$S := 0$$

There are in general two sets of torsion two forms.

1. Particle AFFINE (**PA**) torsion 2-forms which depend upon the product of little omega (the timelike part of the Vierbein) and the (time-space) connection components, little gamma.

2. WaveAFFINE (**WA**) torsion 2-forms which depend upon Big Omega (the time-time connection component or abnormality) and again the (time-space) connection components, little gamma.

If the time-space connection 1-forms vanish, small gamma, neither form of torsion exists.

See http://www.uh.edu/~rkiehn/pdf/projfram.pdf

PA TORSION 2-forms

```
> Sigma1:=wcollect(simplify((wcollect(factor(omega&^gg1)))));
  Sigma2:=factor(simplify(wcollect(factor(omega&^gg2))));Sigma3:=wcollect(factor(s
  implify((omega&^gg3))));
```

$$\Sigma 1 := 0$$

$$\Sigma 2 := 0$$

$$\Sigma 3 := \frac{E\,Vz\,(\mathrm{d}(z)\ \&\!\wedge\ \mathrm{d}(Vz))}{\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vz)\,(C+Vz)} - \frac{E\,Vz\,C\,(\mathrm{d}(t)\ \&\!\wedge\ \mathrm{d}(C))}{\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vz)\,(C+Vz)} + \frac{E\,C^2\,(\mathrm{d}(t)\ \&\!\wedge\ \mathrm{d}(Vz))}{\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vz)\,(C+Vz)} + \frac{E\,C\,t\,(\mathrm{d}(C)\ \&\!\wedge\ \mathrm{d}(Vz))}{\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vz)\,(C+Vz)} - \frac{E\,Vz^2\,(\mathrm{d}(z)\ \&\!\wedge\ \mathrm{d}(C))}{C\,\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vz)\,(C+Vz)}$$

WA TORSION 2-forms

```
> Phi1:=simplify((wcollect((factor(Omega&^gg1)))));Phi2:=wcollect(factor(Omega&^gg
  2));Phi3:=wcollect((factor(Omega&^gg3)));
```

$$\Phi1 := 0$$

$$\Phi2 := 0$$

$$\Phi3 := -\frac{C\,(\mathrm{d}(E)\ \&\hat{}\ \mathrm{d}(Vz))}{E\,(-C+Vz)\,(C+Vz)} + \frac{Vz\,(\mathrm{d}(E)\ \&\hat{}\ \mathrm{d}(C))}{E\,(-C+Vz)\,(C+Vz)}$$

Next compute the matrix of curvature 2-forms on the x,y,z subspace

Curvature 2-forms

```
> Theta:=array([[gg1&^hh1,gg1&^hh2,gg1&^hh3],[gg2&^hh1,gg2&^hh2,gg2&^hh3],[(gg3&^h
  h1),(gg3&^hh2),(gg3&^hh3)]]);
>
```

$$\Theta := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The curvature 2-forms on the interior space all vanish.

DISCUSSION

This example is rather remarkable in that it points out that **THIS** Lorentz transformation does not introduce curvature on the 3D subspace (translational motions) even if accelerations and variable speeds C are admitted.

The WA Torsion coefficients vanish if the Expansion is constant d(E)=0

The PA torsion coefficients are dependent on the Expansion as a factor.

The Torsion coefficients vanish if both C and V are uniform and time independent, d(C)=0, d(V)=0.

The PA torsion is not zero (even though d(C)=0) if the differential of the Velocity field does not vanish. (accelerations)

```
>
```

Example 2:

THE TORSION and CURVATURE of a LORENTZ rotation

about the z axis

```
> FFINV:=LRz:
```

```
> Z:=innerprod(FFINV,[d(x),d(y),d(z),d(C*t)]):sigma1:=Z[1];sigma2:=Z[2];sigma3:=Z[
  3];omega:=simplify((Z[4]));
```

The Vierbein 1-forms.

$$\sigma1 := E\sqrt{1-Lz^2}\ \mathrm{d}(x) + E\,Lz\,\mathrm{d}(y)$$

$$\sigma2 := -E\,Lz\,\mathrm{d}(x) + E\sqrt{1-Lz^2}\ \mathrm{d}(y)$$

$$\sigma3 := E\,\mathrm{d}(z)$$

$$\omega := E\,(t\,\mathrm{d}(C) + C\,\mathrm{d}(t))$$

```
> #Vol4:=wcollect(simplify(sigma1&^sigma2&^sigma3&^Z[4]));rho:=subs(getcoeff(Vol4)
  );
```

The density (determinant)

The determinant cannot go to zero for the projective domain. The zero sets of the density function determine a hypersurface. IF the hypersurface is harmonic then it can be a boundary.

There is an induced metric on R4

```
> FF:=inverse(FFINV):Gun:=subs(innerprod(transpose(FF),FF));
```

$$Gun := \begin{bmatrix} \frac{1}{E^2} & 0 & 0 & 0 \\ 0 & \frac{1}{E^2} & 0 & 0 \\ 0 & 0 & \frac{1}{E^2} & 0 \\ 0 & 0 & 0 & \frac{1}{E^2} \end{bmatrix}$$

From the Frame Field use the standard methods to compute the

Cartan Matrix of connection 1-forms.

See http://www.uh.edu/~rkiehn/pdf/projfram.pdf

for details of the Cartan method for an arbitrary Repere Mobile.

```
> dFF:=array([[d(FF[1,1]),d(FF[1,2]),d(FF[1,3]),d(FF[1,4])],[d(FF[2,1]),d(FF[2,2]),d(FF[2,3]),d(FF[2,4])],[d(FF[3,1]),d(FF[3,2]),d(FF[3,3]),d(FF[3,4])],[d(FF[4,1]),d(FF[4,2]),d(FF[4,3]),d(FF[4,4])]]):
> cartan:=(evalm(FFINV&*dFF)):
```

The Interior (space-space) Connection 1 forms

```
> Gamma11:=factor(wcollect(cartan[1,1]));Gamma21:=factor(wcollect(cartan[2,1]));Gamma31:=factor(wcollect(cartan[3,1]));
```

$$\Gamma 11 := -\frac{\mathrm{d}(E)}{E}$$

$$\Gamma 21 := \frac{\mathrm{d}(Lz)}{\sqrt{-(Lz-1)\,(Lz+1)}}$$

$$\Gamma 31 := 0$$

```
> Gamma12:=factor(wcollect(cartan[1,2]));Gamma22:=factor(wcollect(cartan[2,2]));Gamma32:=factor(wcollect(cartan[3,2]));
```

$$\Gamma 12 := -\frac{\mathrm{d}(Lz)}{\sqrt{-(Lz-1)\,(Lz+1)}}$$

$$\Gamma 22 := -\frac{\mathrm{d}(E)}{E}$$

$$\Gamma 32 := 0$$

```
> Gamma13:=wcollect(factor(wcollect(cartan[1,3])));Gamma23:=factor(wcollect(cartan[2,3]));Gamma33:=simplify(wcollect(cartan[3,3]));
```

$$\Gamma 13 := 0$$

$$\Gamma 23 := 0$$

$$\Gamma 33 := -\frac{\mathrm{d}(E)}{E}$$

```
>
```

The "space-time" connection 1-forms are:

```
> hh1:=simplify(wcollect(cartan[4,1]));hh2:=factor(wcollect(cartan[4,2]));hh3:=wcollect(factor(wcollect(cartan[4,3])));
>
```

$$hh1 := 0$$
$$hh2 := 0$$
$$hh3 := 0$$

The "time-space connection" 1-forms are

```
> gg1:=factor(wcollect(factor(wcollect(cartan[1,4]))));gg2:=factor(wcollect(cartan
  [2,4]));gg3:=wcollect(factor(wcollect(cartan[3,4])));
```

$$gg1 := 0$$
$$gg2 := 0$$
$$gg3 := 0$$

The abnormality (time-time) connection 1-form

Note that the abnormality (Big Omega vanishes if expansion is a global constant. (no red shift?)

```
> Omega:=wcollect(subs(simplify(wcollect(cartan[4,4]))));
```

$$\Omega := -\frac{\mathrm{d}(E)}{E}$$

```
> L:=factor(wcollect(hh1&^sigma1+hh2&^sigma2+hh3&^sigma3));
```

$$L := 0$$

```
> S:=(wcollect(factor(hh1&^gg1+hh2&^gg2+hh3&^gg3)));
```

$$S := 0$$

There are in general two sets of torsion two forms.

1. Particle AFFINE (**PA**) torsion 2-forms which depend upon the product of little omega (the timelike part of the Vierbein) and the (time-space) connection components, little gamma.

2. WaveAFFINE (**WA**) torsion 2-forms which depend upon Big Omega (the time-time connection component or abnormality) and again the (time-space) connection components, little gamma.

If the time-space connection 1-forms vanish, small gamma, neither form of torsion exists.

See http://www.uh.edu/~rkiehn/pdf/projfram.pdf

PA TORSION 2-forms

```
> Sigma1:=wcollect(simplify((wcollect(factor(omega&^gg1)))));
  Sigma2:=factor(simplify(wcollect(factor(omega&^gg2))));Sigma3:=wcollect(factor(s
  implify((omega&^gg3))));
```

$$\Sigma 1 := 0$$
$$\Sigma 2 := 0$$
$$\Sigma 3 := 0$$

WA TORSION 2-forms

```
> Phi1:=simplify((wcollect((factor(Omega&^gg1)))));Phi2:=wcollect(factor(Omega&^gg
  2));Phi3:=wcollect((factor(Omega&^gg3)));
```

$$\Phi 1 := 0$$
$$\Phi 2 := 0$$
$$\Phi 3 := 0$$

Next compute the matrix of curvature 2-forms on the x,y,z subspace

Curvature 2-forms

```
> Theta:=array([[gg1&^hh1,gg1&^hh2,gg1&^hh3],[gg2&^hh1,gg2&^hh2,gg2&^hh3],[(gg3&^hh1),(gg3&^hh2),(gg3&^hh3)]]);
>
```

$$\Theta := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The curvature 2-forms on the interior space all vanish.

DISCUSSION

Single axis Lorentz Rotations do not generate torsion or curvature on the 3D subspace !
(and as is shown below, multiple Lorentz rotations do not generate torsion or curvature on 3D.

```
>
```

Example 3:

THE TORSION and CURVATURE of a LORENTZ translation combined with a Lorentz rotation

both relative to the z axis

```
> FFINV:=evalm(innerprod(LRz,LTz)):
>
```

```
> Z:=innerprod(FFINV,[d(x),d(y),d(z),d(C*t)]):sigma1:=Z[1];sigma2:=Z[2];sigma3:=Z[3];omega:=simplify((Z[4]));
```

The Vierbein 1-forms.

$$\sigma1 := E^2 \sqrt{1 - Lz^2}\, \mathrm{d}(x) + E^2\, Lz\, \mathrm{d}(y)$$

$$\sigma2 := -E^2\, Lz\, \mathrm{d}(x) + E^2 \sqrt{1 - Lz^2}\, \mathrm{d}(y)$$

$$\sigma3 := \frac{E^2\, (\mathrm{d}(z)\, C + Vz\, t\, \mathrm{d}(C) + Vz\, C\, \mathrm{d}(t))}{\sqrt{-\frac{-C^2 + Vz^2}{C^2}}\, C}$$

$$\omega := \frac{E^2\, (Vz\, \mathrm{d}(z) + C\, t\, \mathrm{d}(C) + C^2\, \mathrm{d}(t))}{\sqrt{-\frac{-C^2 + Vz^2}{C^2}}\, C}$$

```
> #Vol4:=wcollect(simplify(sigma1&^sigma2&^sigma3&^Z[4]));rho:=subs(getcoeff(Vol4));
```

The density (determinant)

The determinant cannot go to zero for the projective domain. The zero sets of the density function determine a hypersurface. IF the hypersurface is harmonic then it can be a boundary.

There is an induced metric on R4

```
> FF:=inverse(FFINV):Gun:=subs(innerprod(transpose(FF),FF));
```

$$Gun := \begin{bmatrix} \frac{1}{E^4} & 0 & 0 & 0 \\ 0 & \frac{1}{E^4} & 0 & 0 \\ 0 & 0 & -\frac{C^2+Vz^2}{E^4(-C^2+Vz^2)} & 2\frac{C\,Vz}{E^4(-C^2+Vz^2)} \\ 0 & 0 & 2\frac{C\,Vz}{E^4(-C^2+Vz^2)} & -\frac{C^2+Vz^2}{E^4(-C^2+Vz^2)} \end{bmatrix}$$

From the Frame Field use the standard methods to compute the
Cartan Matrix of connection 1-forms.
See http://www.uh.edu/~rkiehn/pdf/projfram.pdf
for details of the Cartan method for an arbitrary Repere Mobile.

```
> dFF:=array([[d(FF[1,1]),d(FF[1,2]),d(FF[1,3]),d(FF[1,4])],[d(FF[2,1]),d(FF[2,2]),d(FF[2,3]),d(FF[2,4])],[d(FF[3,1]),d(FF[3,2]),d(FF[3,3]),d(FF[3,4])],[d(FF[4,1]),d(FF[4,2]),d(FF[4,3]),d(FF[4,4])]]):
> cartan:=(evalm(FFINV&*dFF)):
```

The Interior (space-space) Connection 1 forms

```
> Gamma11:=factor(wcollect(cartan[1,1]));Gamma21:=factor(wcollect(cartan[2,1]));Gamma31:=factor(wcollect(cartan[3,1]));
```

$$\Gamma 11 := -2\frac{\mathrm{d}(E)}{E}$$

$$\Gamma 21 := \frac{\mathrm{d}(Lz)}{\sqrt{-(Lz-1)(Lz+1)}}$$

$$\Gamma 31 := 0$$

```
> Gamma12:=factor(wcollect(cartan[1,2]));Gamma22:=factor(wcollect(cartan[2,2]));Gamma32:=factor(wcollect(cartan[3,2]));
```

$$\Gamma 12 := -\frac{\mathrm{d}(Lz)}{\sqrt{-(Lz-1)(Lz+1)}}$$

$$\Gamma 22 := -2\frac{\mathrm{d}(E)}{E}$$

$$\Gamma 32 := 0$$

```
> Gamma13:=wcollect(factor(wcollect(cartan[1,3])));Gamma23:=factor(wcollect(cartan[2,3]));Gamma33:=simplify(wcollect(cartan[3,3]));
```

$$\Gamma 13 := 0$$

$$\Gamma 23 := 0$$

$$\Gamma 33 := -2\frac{\mathrm{d}(E)}{E}$$

```
>
```

The "space-time" connection 1-forms are:

```
> hh1:=simplify(wcollect(cartan[4,1]));hh2:=factor(wcollect(cartan[4,2]));hh3:=wcollect(factor(wcollect(cartan[4,3])));
>
```

$$hh1 := 0$$

$$hh2 := 0$$

$$hh3 := \frac{C\,\mathrm{d}(Vz)}{(-C+Vz)(C+Vz)} - \frac{Vz\,\mathrm{d}(C)}{(-C+Vz)(C+Vz)}$$

The "time-space connection" 1-forms are

```
> gg1:=factor(wcollect(factor(wcollect(cartan[1,4]))));gg2:=factor(wcollect(cartan[2,4]));gg3:=wcollect(factor(wcollect(cartan[3,4])));
```

$$gg1 := 0$$

$$gg2 := 0$$

$$gg3 := \frac{C\,\mathrm{d}(Vz)}{(-C+Vz)\,(C+Vz)} - \frac{Vz\,\mathrm{d}(C)}{(-C+Vz)\,(C+Vz)}$$

The abnormality (time-time) connection 1-form

Note that the abnormality (Big Omega vanishes if expansion is a global constant. (no red shift?)

```
> Omega:=wcollect(subs(simplify(wcollect(cartan[4,4]))));
```

$$\Omega := -2\,\frac{\mathrm{d}(E)}{E}$$

```
> L:=factor(wcollect(hh1&^sigma1+hh2&^sigma2+hh3&^sigma3));
```

$$L := E^2 \left(C\,(\mathrm{d}(Vz)\,\&\!\wedge\,\mathrm{d}(z)) + Vz\,t\,(\mathrm{d}(Vz)\,\&\!\wedge\,\mathrm{d}(C)) + Vz\,C\,(\mathrm{d}(Vz)\,\&\!\wedge\,\mathrm{d}(t)) - Vz\,(\mathrm{d}(C)\,\&\!\wedge\,\mathrm{d}(z)) - Vz^2\,(\mathrm{d}(C)\,\&\!\wedge\,\mathrm{d}(t))\right) \Big/ \left(\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vz)\,(C+Vz)\right)$$

```
> S:=(wcollect(factor(hh1&^gg1+hh2&^gg2+hh3&^gg3)));
```

$$S := 0$$

There are in general two sets of torsion two forms.

1. Particle AFFINE (**PA**) torsion 2-forms which depend upon the product of little omega (the timelike part of the Vierbein) and the (time-space) connection components, little gamma.

2. WaveAFFINE (**WA**) torsion 2-forms which depend upon Big Omega (the time-time connection component or abnormality) and again the (time-space) connection components, little gamma.

If the time-space connection 1-forms vanish, small gamma, neither form of torsion exists.

See http://www.uh.edu/~rkiehn/pdf/projfram.pdf

PA TORSION 2-forms

```
> Sigma1:=wcollect(simplify((wcollect(factor(omega&^gg1)))));
Sigma2:=factor(simplify(wcollect(factor(omega&^gg2))));Sigma3:=wcollect(factor(simplify((omega&^gg3))));
```

$$\Sigma 1 := 0$$

$$\Sigma 2 := 0$$

$$\Sigma 3 := \frac{E^2\,Vz\,(\mathrm{d}(z)\,\&\!\wedge\,\mathrm{d}(Vz))}{\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vz)\,(C+Vz)} - \frac{E^2\,Vz\,C\,(\mathrm{d}(t)\,\&\!\wedge\,\mathrm{d}(C))}{\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vz)\,(C+Vz)} + \frac{E^2\,C^2\,(\mathrm{d}(t)\,\&\!\wedge\,\mathrm{d}(Vz))}{\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vz)\,(C+Vz)} + \frac{E^2\,C\,t\,(\mathrm{d}(C)\,\&\!\wedge\,\mathrm{d}(Vz))}{\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vz)\,(C+Vz)}$$

$$-\frac{E^2 Vz^2 (\mathrm{d}(z) \,\&\wedge\, \mathrm{d}(C))}{\sqrt{-\frac{(-C+Vz)(C+Vz)}{C^2}}\, C(-C+Vz)(C+Vz)}$$

WA TORSION 2-forms

```
> Phi1:=simplify((wcollect((factor(Omega&^gg1)))));Phi2:=wcollect(factor(Omega&^gg2));Phi3:=wcollect((factor(Omega&^gg3)));
```

$$\Phi 1 := 0$$

$$\Phi 2 := 0$$

$$\Phi 3 := -2\,\frac{C\,(\mathrm{d}(E) \,\&\wedge\, \mathrm{d}(Vz))}{E\,(-C+Vz)(C+Vz)} + 2\,\frac{Vz\,(\mathrm{d}(E) \,\&\wedge\, \mathrm{d}(C))}{E\,(-C+Vz)(C+Vz)}$$

Next compute the matrix of curvature 2-forms on the x,y,z subspace

Curvature 2-forms

```
> Theta:=array([[gg1&^hh1,gg1&^hh2,gg1&^hh3],[gg2&^hh1,gg2&^hh2,gg2&^hh3],[(gg3&^hh1),(gg3&^hh2),(gg3&^hh3)]]);
>
```

$$\Theta := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

```
>
>
```

The curvature 2-forms on the interior space all vanish.

DISCUSSION

This example is rather remarkable in that it points out that **THIS** Lorentz transformation does not introduce curvature on the 3D subspace (Helical like motion) even if accelerations and variable speeds C are admitted.

The WA Torsion coefficients vanish if the Expansion is constant d(E)=0

The PA torsion coefficients are dependent on the Expansion as a factor.

The Torsion coefficients vanish if both C and V are uniform and time independent, d(C)=0, d(V)=0.

The PA torsion is not zero (even though d(C)=0) if the differential of the Velocity field does not vanish.

The Lorentz rotation parameters do not enter into the formulas !!!

Example 4:

THE TORSION and CURVATURE of a LORENTZ translation along the z axis and a Lorentz translation along the x axis.

```
> FFINV:=evalm(innerprod(LTx,LTz));
```

$$FFINV := \begin{bmatrix} \frac{E^2}{\sqrt{-\frac{-C^2+Vx^2}{C^2}}} & 0 & \frac{E^2\, Vx\, Vz}{\sqrt{-\frac{-C^2+Vx^2}{C^2}}\, C^2 \sqrt{-\frac{-C^2+Vz^2}{C^2}}} & \frac{E^2\, Vx}{\sqrt{-\frac{-C^2+Vx^2}{C^2}}\, C \sqrt{-\frac{-C^2+Vz^2}{C^2}}} \\ 0 & E^2 & 0 & 0 \\ 0 & 0 & \frac{E^2}{\sqrt{-\frac{-C^2+Vz^2}{C^2}}} & \frac{E^2\, Vz}{\sqrt{-\frac{-C^2+Vz^2}{C^2}}\, C} \\ \frac{E^2\, Vx}{\sqrt{-\frac{-C^2+Vx^2}{C^2}}\, C} & 0 & \frac{E^2\, Vz}{\sqrt{-\frac{-C^2+Vx^2}{C^2}} \sqrt{-\frac{-C^2+Vz^2}{C^2}}\, C} & \frac{E^2}{\sqrt{-\frac{-C^2+Vx^2}{C^2}} \sqrt{-\frac{-C^2+Vz^2}{C^2}}} \end{bmatrix}$$

```
>
> Z:=innerprod(FFINV,[d(x),d(y),d(z),d(C*t)]):sigma1:=Z[1];sigma2:=Z[2];sigma3:=Z[
  3];omega:=simplify((Z[4]));
```

The Vierbein 1-forms.

$$\sigma 1 := \frac{E^2 \left(\mathrm{d}(x)\, C^2 \sqrt{-\frac{-C^2+Vz^2}{C^2}} + Vx\, Vz\, \mathrm{d}(z) + Vx\, C\, t\, \mathrm{d}(C) + Vx\, C^2\, \mathrm{d}(t) \right)}{\sqrt{-\frac{-C^2+Vx^2}{C^2}}\, C^2 \sqrt{-\frac{-C^2+Vz^2}{C^2}}}$$

$$\sigma 2 := E^2\, \mathrm{d}(y)$$

$$\sigma 3 := \frac{E^2\, (\mathrm{d}(z)\, C + Vz\, t\, \mathrm{d}(C) + Vz\, C\, \mathrm{d}(t))}{\sqrt{-\frac{-C^2+Vz^2}{C^2}}\, C}$$

$$\omega := \frac{E^2 \left(Vx\, \mathrm{d}(x) \sqrt{-\frac{-C^2+Vz^2}{C^2}} + Vz\, \mathrm{d}(z) + C\, t\, \mathrm{d}(C) + C^2\, \mathrm{d}(t) \right)}{\sqrt{-\frac{-C^2+Vx^2}{C^2}} \sqrt{-\frac{-C^2+Vz^2}{C^2}}\, C}$$

```
> #Vol4:=wcollect(simplify(sigma1&^sigma2&^sigma3&^Z[4]));rho:=subs(getcoeff(Vol4)
  );
```

The density (determinant)

The determinant cannot go to zero for the projective domain. The zero sets of the density function determine a hypersurface. IF the hypersurface is harmonic then it can be a boundary.

There is an induced metric on R4

```
> FF:=inverse(FFINV):Gun:=subs(innerprod(transpose(FF),FF));
```

$$Gun := \begin{bmatrix} \frac{C^4 - C^2 Vz^2 + Vx^2 Vz^2 + C^2 Vx^2}{E^4 (-C^2+Vx^2)(-C^2+Vz^2)}, & 0, & -2 \frac{Vx\, Vz}{E^4 \sqrt{-\frac{-C^2+Vx^2}{C^2}}\,(-C^2+Vz^2)}, & -2 \frac{C^3\, Vx}{E^4 (-C^2+Vx^2)(-C^2+Vz^2)} \\ 0, & \frac{1}{E^4}, & 0, & 0 \\ -2 \frac{Vx\, Vz}{E^4 \sqrt{-\frac{-C^2+Vx^2}{C^2}}\,(-C^2+Vz^2)}, & 0, & -\frac{C^2+Vz^2}{E^4(-C^2+Vz^2)}, & 2 \frac{C\, Vz}{E^4(-C^2+Vz^2)\sqrt{-\frac{-C^2+Vx^2}{C^2}}} \\ -2 \frac{C^3\, Vx}{E^4(-C^2+Vx^2)(-C^2+Vz^2)}, & 0, & 2 \frac{C\, Vz}{E^4(-C^2+Vz^2)\sqrt{-\frac{-C^2+Vx^2}{C^2}}}, & -\frac{-C^2 Vx^2 + Vx^2 Vz^2 - C^2 Vz^2 - C^4}{E^4(-C^2+Vx^2)(-C^2+Vz^2)} \end{bmatrix}$$

From the Frame Field use the standard methods to compute the

Cartan Matrix of connection 1-forms.

See http://www.uh.edu/~rkiehn/pdf/projfram.pdf

for details of the Cartan method for an arbitrary Repere Mobile.

```
> dFF:=array([[d(FF[1,1]),d(FF[1,2]),d(FF[1,3]),d(FF[1,4])],[d(FF[2,1]),d(FF[2,2]),d(FF[2,3]),d(FF[2,4])],[d(FF[3,1]),d(FF[3,2]),d(FF[3,3]),d(FF[3,4])],[d(FF[4,1]),d(FF[4,2]),d(FF[4,3]),d(FF[4,4])]]):
> cartan:=(evalm(FFINV&*dFF)):
```

The Interior (space-space) Connection 1 forms

```
> Gamma11:=factor(wcollect(cartan[1,1]));Gamma21:=factor(wcollect(cartan[2,1]));Gamma31:=factor(wcollect(cartan[3,1]));
```

$$\Gamma 11 := -2\,\frac{d(E)}{E}$$

$$\Gamma 21 := 0$$

$$\Gamma 31 := -\frac{(-C+Vx)\,(Vx+C)\,(Vz\, d(C) - d(Vz)\, C)\, Vx}{\left(-\frac{(-C+Vx)\,(Vx+C)}{C^2}\right)^{(3/2)} C^3\,(-C+Vz)\,(C+Vz)}$$

```
> Gamma12:=factor(wcollect(cartan[1,2]));Gamma22:=factor(wcollect(cartan[2,2]));Gamma32:=factor(wcollect(cartan[3,2]));
```

$$\Gamma 12 := 0$$

$$\Gamma 22 := -2\,\frac{d(E)}{E}$$

$$\Gamma 32 := 0$$

```
> Gamma13:=wcollect(factor(wcollect(cartan[1,3])));Gamma23:=factor(wcollect(cartan[2,3]));Gamma33:=simplify(wcollect(cartan[3,3]));
```

$$\Gamma 13 := \frac{Vx\, d(Vz)}{\sqrt{-\frac{(-C+Vx)\,(Vx+C)}{C^2}}\,(-C+Vz)\,(C+Vz)} - \frac{Vz\, d(C)\, Vx}{\sqrt{-\frac{(-C+Vx)\,(Vx+C)}{C^2}}\, C\,(-C+Vz)\,(C+Vz)}$$

$$\Gamma 23 := 0$$

$$\Gamma 33 := -2\,\frac{d(E)}{E}$$

```
>
```

The "space-time" connection 1-forms are:

```
> hh1:=simplify(wcollect(cartan[4,1]));hh2:=factor(wcollect(cartan[4,2]));hh3:=wco
```

```
llect(factor(wcollect(cartan[4,3])));
>
```

$$hh1 := -\frac{-C\,d(Vx) + Vx\,d(C)}{-C^2 + Vx^2}$$

$$hh2 := 0$$

$$hh3 := \frac{C\,d(Vz)}{(-C+Vz)\,(C+Vz)\sqrt{-\frac{(-C+Vx)\,(Vx+C)}{C^2}}} - \frac{Vz\,d(C)}{(-C+Vz)\,(C+Vz)\sqrt{-\frac{(-C+Vx)\,(Vx+C)}{C^2}}}$$

The "time-space connection" 1-forms are

```
> gg1:=factor(wcollect(factor(wcollect(cartan[1,4]))));gg2:=factor(wcollect(cartan[2,4]));gg3:=wcollect(factor(wcollect(cartan[3,4])));
```

$$gg1 := -\frac{-C\,d(Vx) + Vx\,d(C)}{(-C+Vx)\,(Vx+C)}$$

$$gg2 := 0$$

$$gg3 := -\frac{(-C+Vx)\,(Vx+C)\,d(Vz)}{C\left(-\frac{(-C+Vx)\,(Vx+C)}{C^2}\right)^{(3/2)}(-C+Vz)\,(C+Vz)} + \frac{(-C+Vx)\,(Vx+C)\,Vz\,d(C)}{\left(-\frac{(-C+Vx)\,(Vx+C)}{C^2}\right)^{(3/2)}C^2\,(-C+Vz)\,(C+Vz)}$$

The abnormality (time-time) connection 1-form

Note that the abnormality (Big Omega vanishes if expansion is a global constant. (no red shift?)

```
> Omega:=wcollect(subs(simplify(wcollect(cartan[4,4]))));
```

$$\Omega := -2\,\frac{d(E)}{E}$$

```
> L:=factor(wcollect(hh1&^sigma1+hh2&^sigma2+hh3&^sigma3));
```

$$\begin{aligned} L := -E^2\sqrt{-\frac{(-C+Vx)\,(Vx+C)}{C^2}}\Big(&-Vx\,t\,(d(Vx)\,\&\wedge\,d(C))\,C^4 + Vx\,t\,(d(Vx)\,\&\wedge\,d(C))\,C^2\,Vz^2 \\ &- Vx\,Vz\,(d(Vx)\,\&\wedge\,d(z))\,C^3 + Vx\,Vz^3\,(d(Vx)\,\&\wedge\,d(z))\,C - C^5\,Vx\,(d(Vx)\,\&\wedge\,d(t)) + C^3\,Vx\,(d(Vx)\,\&\wedge\,d(t))\,Vz^2 \\ &+ Vx\,(d(C)\,\&\wedge\,d(x))\,C^4\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}} - Vx\,(d(C)\,\&\wedge\,d(x))\,C^2\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,Vz^2 \\ &- Vz^3\,(d(C)\,\&\wedge\,d(z))\,Vx^2 + Vz\,(d(C)\,\&\wedge\,d(z))\,C^4 + (d(C)\,\&\wedge\,d(t))\,C^4\,Vx^2 - 2\,(d(C)\,\&\wedge\,d(t))\,C^2\,Vz^2\,Vx^2 \\ &+ (d(C)\,\&\wedge\,d(t))\,C^4\,Vz^2 - C^5\,(d(Vx)\,\&\wedge\,d(x))\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}} \\ &+ C^3\,(d(Vx)\,\&\wedge\,d(x))\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,Vz^2 - C^5\,Vz\,(d(Vz)\,\&\wedge\,d(t)) + C^3\,Vz\,(d(Vz)\,\&\wedge\,d(t))\,Vx^2 \\ &- C^5\,(d(Vz)\,\&\wedge\,d(z)) + C^3\,(d(Vz)\,\&\wedge\,d(z))\,Vx^2 - Vz\,t\,(d(Vz)\,\&\wedge\,d(C))\,C^4 + Vz\,t\,(d(Vz)\,\&\wedge\,d(C))\,C^2\,Vx^2\Big) \\ &\Big/\left((-C+Vx)^2\,(Vx+C)^2\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vz)\,(C+Vz)\right) \end{aligned}$$

```
> S:=(wcollect(factor(hh1&^gg1+hh2&^gg2+hh3&^gg3)));
```

$$S := 0$$

There are in general two sets of torsion two forms.

1. Particle AFFINE (**PA**) torsion 2-forms which depend upon the product of little omega (the

timelike part of the Vierbein) and the (time-space) connection components, little gamma.

2. WaveAFFINE (**WA**) torsion 2-forms which depend upon Big Omega (the time-time connection component or abnormality) and again the (time-space) connection components, little gamma.

If the time-space connection 1-forms vanish, small gamma, neither form of torsion exists.

See http://www.uh.edu/~rkiehn/pdf/projfram.pdf

PA TORSION 2-forms

```
> Sigma1:=wcollect(simplify((wcollect(factor(omega&^gg1)))));
  Sigma2:=factor(simplify(wcollect(factor(omega&^gg2))));Sigma3:=wcollect(factor(s
  implify((omega&^gg3))));
```

$$\Sigma 1 := -\frac{E^2\, C\, Vx\,(\mathrm{d}(t)\ \&\hat{}\ \mathrm{d}(C))}{\sqrt{-\frac{(-C+Vx)(Vx+C)}{C^2}}\sqrt{-\frac{(-C+Vz)(C+Vz)}{C^2}}\,(Vx+C)(-C+Vx)} - \frac{E^2\, Vx\, Vz\,(\mathrm{d}(z)\ \&\hat{}\ \mathrm{d}(C))}{\sqrt{-\frac{(-C+Vx)(Vx+C)}{C^2}}\sqrt{-\frac{(-C+Vz)(C+Vz)}{C^2}}\,C\,(Vx+C)(-C+Vx)}$$
$$+\frac{E^2\, Vx\,(\mathrm{d}(x)\ \&\hat{}\ \mathrm{d}(Vx))}{\sqrt{-\frac{(-C+Vx)(Vx+C)}{C^2}}\,(Vx+C)(-C+Vx)} - \frac{E^2\, Vx^2\,(\mathrm{d}(x)\ \&\hat{}\ \mathrm{d}(C))}{\sqrt{-\frac{(-C+Vx)(Vx+C)}{C^2}}\,C\,(Vx+C)(-C+Vx)}$$
$$+\frac{E^2\, C^2\,(\mathrm{d}(t)\ \&\hat{}\ \mathrm{d}(Vx))}{\sqrt{-\frac{(-C+Vx)(Vx+C)}{C^2}}\sqrt{-\frac{(-C+Vz)(C+Vz)}{C^2}}\,(Vx+C)(-C+Vx)} + \frac{E^2\, Vz\,(\mathrm{d}(z)\ \&\hat{}\ \mathrm{d}(Vx))}{\sqrt{-\frac{(-C+Vx)(Vx+C)}{C^2}}\sqrt{-\frac{(-C+Vz)(C+Vz)}{C^2}}\,(Vx+C)(-C+Vx)}$$
$$+\frac{E^2\, C\, t\,(\mathrm{d}(C)\ \&\hat{}\ \mathrm{d}(Vx))}{\sqrt{-\frac{(-C+Vx)(Vx+C)}{C^2}}\sqrt{-\frac{(-C+Vz)(C+Vz)}{C^2}}\,(Vx+C)(-C+Vx)}$$

$$\Sigma 2 := 0$$

$$\Sigma 3 := -\frac{Vz\, C^2\, E^2\,(\mathrm{d}(z)\ \&\hat{}\ \mathrm{d}(Vz))}{(C+Vz)(-C+Vz)\sqrt{-\frac{(-C+Vz)(C+Vz)}{C^2}}\,(-C+Vx)(Vx+C)} - \frac{Vx\, C^2\, E^2\,(\mathrm{d}(x)\ \&\hat{}\ \mathrm{d}(Vz))}{(C+Vz)(-C+Vz)(-C+Vx)(Vx+C)}$$
$$+\frac{Vz\, C^3\, E^2\,(\mathrm{d}(t)\ \&\hat{}\ \mathrm{d}(C))}{(C+Vz)(-C+Vz)\sqrt{-\frac{(-C+Vz)(C+Vz)}{C^2}}\,(-C+Vx)(Vx+C)} - \frac{C^4\, E^2\,(\mathrm{d}(t)\ \&\hat{}\ \mathrm{d}(Vz))}{(C+Vz)(-C+Vz)\sqrt{-\frac{(-C+Vz)(C+Vz)}{C^2}}\,(-C+Vx)(Vx+C)}$$

$$-\frac{C^3\, t\, E^2\, (\mathrm{d}(C)\ \&\wedge\ \mathrm{d}(Vz))}{(C+Vz)\,(-C+Vz)\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vx)\,(Vx+C)}$$
$$+\frac{C\, Vz^2\, E^2\, (\mathrm{d}(z)\ \&\wedge\ \mathrm{d}(C))}{(C+Vz)\,(-C+Vz)\sqrt{-\frac{(-C+Vz)\,(C+Vz)}{C^2}}\,(-C+Vx)\,(Vx+C)}+\frac{C\, Vx\, Vz\, (\mathrm{d}(x)\ \&\wedge\ \mathrm{d}(C))\, E^2}{(C+Vz)\,(-C+Vz)\,(-C+Vx)\,(Vx+C)}$$

WA TORSION 2-forms

```
> Phi1:=simplify((wcollect((factor(Omega&^gg1)))));Phi2:=wcollect(factor(Omega&^gg
  2));Phi3:=wcollect((factor(Omega&^gg3)));
```

$$\Phi 1 := 2\,\frac{-C\,(\mathrm{d}(E)\ \&\wedge\ \mathrm{d}(Vx))+Vx\,(\mathrm{d}(E)\ \&\wedge\ \mathrm{d}(C))}{E\,(Vx+C)\,(-C+Vx)}$$

$$\Phi 2 := 0$$

$\Phi 3 :=$

$$2\,\frac{(-C+Vx)\,(Vx+C)\,(\mathrm{d}(E)\ \&\wedge\ \mathrm{d}(Vz))}{E\, C\left(-\frac{(-C+Vx)\,(Vx+C)}{C^2}\right)^{(3/2)}(-C+Vz)\,(C+Vz)}-2\,\frac{(-C+Vx)\,(Vx+C)\, Vz\,(\mathrm{d}(E)\ \&\wedge\ \mathrm{d}(C))}{E\left(-\frac{(-C+Vx)\,(Vx+C)}{C^2}\right)^{(3/2)}C^2\,(-C+Vz)\,(C+Vz)}$$

Next compute the matrix of curvature 2-forms on the x,y,z subspace

Curvature 2-forms

```
> Theta:=array([[gg1&^hh1,gg1&^hh2,gg1&^hh3],[gg2&^hh1,gg2&^hh2,gg2&^hh3],[(gg3&^h
  h1),(gg3&^hh2),(gg3&^hh3)]]);
>
```

$\Theta :=$

$$\Big[0\,,0\,,\frac{C^2\,(\mathrm{d}(Vx)\ \&\wedge\ \mathrm{d}(Vz))}{(Vx+C)\,(-C+Vx)\,(-C+Vz)\,(C+Vz)\sqrt{-\frac{(-C+Vx)\,(Vx+C)}{C^2}}}$$
$$-\frac{C\, Vz\,(\mathrm{d}(Vx)\ \&\wedge\ \mathrm{d}(C))}{(Vx+C)\,(-C+Vx)\,(-C+Vz)\,(C+Vz)\sqrt{-\frac{(-C+Vx)\,(Vx+C)}{C^2}}}$$
$$-\frac{Vx\, C\,(\mathrm{d}(C)\ \&\wedge\ \mathrm{d}(Vz))}{(Vx+C)\,(-C+Vx)\,(-C+Vz)\,(C+Vz)\sqrt{-\frac{(-C+Vx)\,(Vx+C)}{C^2}}}\Big]$$

$$[0\,,0\,,0]$$

$$\Big[-\frac{(-C+Vx)\,(Vx+C)\,(\mathrm{d}(Vz)\ \&\wedge\ \mathrm{d}(Vx))}{(-C^2+Vx^2)\left(-\frac{(-C+Vx)\,(Vx+C)}{C^2}\right)^{(3/2)}(-C+Vz)\,(C+Vz)}$$
$$+\frac{(-C+Vx)\,(Vx+C)\, Vx\,(\mathrm{d}(Vz)\ \&\wedge\ \mathrm{d}(C))}{(-C^2+Vx^2)\, C\left(-\frac{(-C+Vx)\,(Vx+C)}{C^2}\right)^{(3/2)}(-C+Vz)\,(C+Vz)}$$
$$+\frac{(-C+Vx)\,(Vx+C)\, Vz\,(\mathrm{d}(C)\ \&\wedge\ \mathrm{d}(Vx))}{(-C^2+Vx^2)\left(-\frac{(-C+Vx)\,(Vx+C)}{C^2}\right)^{(3/2)}C\,(-C+Vz)\,(C+Vz)}\,,0\,,0\Big]$$

DISCUSSION

This example is rather remarkable in that it points out that **THIS** Lorentz transformation **DOES** introduce curvature on the 3D subspace (when accelerations and variable speeds C are admitted).

The curvature coefficients are not dependent upon the Expansion, but the WA torsion coefficients require that the d(E) is not zero over the domain.

```
>
```

Example 5:

THE TORSION and CURVATURE of a LORENTZ rotation along the z axis and a Lorentz rotation along the x axis.

```
> FFINV:=evalm(innerprod(LRx,LRzn,LRx,LRz)):
>
```

```
> Z:=innerprod(FFINV,[d(x),d(y),d(z),d(C*t)]):sigma1:=Z[1];sigma2:=Z[2];sigma3:=Z[3];omega:=simplify((Z[4]));
```

The Vierbein 1-forms.

$$\sigma 1 := d(x)\,E^2 - d(x)\,E^2\,Lz^2 + d(x)\,Lz^2\,E^2\sqrt{1-Lx^2} + d(y)\,E^2\sqrt{1-Lz^2}\,Lz - d(y)\,Lz\,E^2\sqrt{1-Lx^2}\sqrt{1-Lz^2} - Lz\,E^2\,Lx\,d(z)$$

$$\sigma 2 := d(x)\,Lz\,E^2\sqrt{1-Lx^2}\sqrt{1-Lz^2} - d(x)\,E^2\sqrt{1-Lz^2}\,Lz + d(x)\,E^2\,Lz\sqrt{1-Lz^2}\,Lx^2 + d(x)\,E^2\,Lz\,Lx^2 + d(y)\,Lz^2\,E^2\sqrt{1-Lx^2} + E^2\,d(y) - d(y)\,E^2\,Lz^2 - d(y)\,E^2\,Lx^2 + d(y)\,E^2\,Lx^2\,Lz^2 - d(y)\,E^2\sqrt{1-Lz^2}\,Lx^2 + Lx\,E^2\sqrt{1-Lx^2}\,d(z)\sqrt{1-Lz^2} + Lx\,E^2\sqrt{1-Lx^2}\,d(z)$$

$$\sigma 3 := -d(x)\,Lz\,E^2\,Lx\sqrt{1-Lz^2} + d(x)\,E^2\,Lz\sqrt{1-Lz^2}\sqrt{1-Lx^2}\,Lx + d(x)\,E^2\,Lz\,Lx\sqrt{1-Lx^2} - d(y)\,Lz^2\,E^2\,Lx - d(y)\,E^2\sqrt{1-Lx^2}\,Lx + d(y)\,E^2\sqrt{1-Lx^2}\,Lx\,Lz^2 - d(y)\,E^2\sqrt{1-Lz^2}\,Lx\sqrt{1-Lx^2} - E^2\,d(z)\sqrt{1-Lz^2}\,Lx^2 + E^2\,d(z) - E^2\,d(z)\,Lx^2$$

$$\omega := E^2\,(t\,d(C) + C\,d(t))$$

```
> #Vol4:=wcollect(simplify(sigma1&^sigma2&^sigma3&^Z[4]));rho:=subs(getcoeff(Vol4));
```

The density (determinant)

The determinant cannot go to zero for the projective domain. The zero sets of the density function determine a hypersurface. IF the hypersurface is harmonic then it can be a boundary.

There is an induced metric on R4

```
> FF:=inverse(FFINV):Gun:=subs(innerprod(transpose(FF),FF));
```

$$Gun := \begin{bmatrix} \frac{1}{E^4} & 0 & 0 & 0 \\ 0 & \frac{1}{E^4} & 0 & 0 \\ 0 & 0 & \frac{1}{E^4} & 0 \\ 0 & 0 & 0 & \frac{1}{E^4} \end{bmatrix}$$

From the Frame Field use the standard methods to compute the

Cartan Matrix of connection 1-forms.

See http://www.uh.edu/~rkiehn/pdf/projfram.pdf

for details of the Cartan method for an arbitrary Repere Mobile.

```
> dFF:=array([[d(FF[1,1]),d(FF[1,2]),d(FF[1,3]),d(FF[1,4])],[d(FF[2,1]),d(FF[2,2]),d(FF[2,3]),d(FF[2,4])],[d(FF[3,1]),d(FF[3,2]),d(FF[3,3]),d(FF[3,4])],[d(FF[4,1]),d(FF[4,2]),d(FF[4,3]),d(FF[4,4])]]):
> cartan:=(evalm(FFINV&*dFF)):
```

The Interior (space-space) Connection 1 forms

```
> Gamma11:=factor(wcollect(cartan[1,1]));Gamma21:=factor(wcollect(cartan[2,1]));Gamma31:=factor(wcollect(cartan[3,1]));
```

$$\Gamma 11 := 0$$

$$\Gamma 21 := (-\mathrm{d}(Lz)\sqrt{-(Lx-1)(Lx+1)}\,Lz^2 + \mathrm{d}(Lz)(-(Lx-1)(Lx+1))^{(3/2)}\,Lz^2 + Lz\sqrt{-(Lz-1)(Lz+1)}\,Lx\,\mathrm{d}(Lx) + Lz^3\,Lx^3\,\mathrm{d}(Lx)\sqrt{-(Lz-1)(Lz+1)} + Lz\,Lx^3\,\mathrm{d}(Lx)(-(Lz-1)(Lz+1))^{(3/2)} + \mathrm{d}(Lz)\sqrt{-(Lx-1)(Lx+1)}\sqrt{-(Lz-1)(Lz+1)}\,Lx^2 + \mathrm{d}(Lz)\sqrt{-(Lx-1)(Lx+1)}\,Lz^2\,Lx^2 - \mathrm{d}(Lz) + \mathrm{d}(Lz)\sqrt{-(Lx-1)(Lx+1)} - \mathrm{d}(Lz)\sqrt{-(Lx-1)(Lx+1)}\,Lx^2 + Lx^2\,\mathrm{d}(Lz) - Lz\,Lx^3\,\mathrm{d}(Lx)\sqrt{-(Lz-1)(Lz+1)}) \Big/ (\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)})$$

$$\Gamma 31 := -(Lx\,\mathrm{d}(Lz) + Lz\sqrt{-(Lz-1)(Lz+1)}\,\mathrm{d}(Lx) - Lx\sqrt{-(Lx-1)(Lx+1)}\,\mathrm{d}(Lz) + Lz^3\,\mathrm{d}(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lx^2 + Lz\,\mathrm{d}(Lx)(-(Lz-1)(Lz+1))^{(3/2)}\,Lx^2 - Lz\,\mathrm{d}(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lx^2 + Lx\sqrt{-(Lx-1)(Lx+1)}\sqrt{-(Lz-1)(Lz+1)}\,\mathrm{d}(Lz)) \Big/ \sqrt{-(Lz-1)(Lz+1)}$$

```
> Gamma12:=factor(wcollect(cartan[1,2]));Gamma22:=factor(wcollect(cartan[2,2]));Gamma32:=factor(wcollect(cartan[3,2]));
```

$$\Gamma 12 := (-2\,Lz\,Lx\,\mathrm{d}(Lx)\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)} + 2\,Lz\,Lx\,\mathrm{d}(Lx)(-(Lz-1)(Lz+1))^{(3/2)}\sqrt{-(Lx-1)(Lx+1)} + 2\,Lz^3\,Lx\,\mathrm{d}(Lx)\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)} + Lz\sqrt{-(Lz-1)(Lz+1)}\,Lx\,\mathrm{d}(Lx) + \mathrm{d}(Lz)\sqrt{-(Lx-1)(Lx+1)}\,Lx^2 + \mathrm{d}(Lz)\sqrt{-(Lx-1)(Lx+1)}\,Lz^2 - \mathrm{d}(Lz)\sqrt{-(Lx-1)(Lx+1)} - 2\,Lz^3\,Lx\,\mathrm{d}(Lx)\sqrt{-(Lz-1)(Lz+1)} - 2\,Lz\,Lx\,\mathrm{d}(Lx)(-(Lz-1)(Lz+1))^{(3/2)} - \mathrm{d}(Lz)(-(Lx-1)(Lx+1))^{(3/2)}\,Lz^2 - 2\,Lz\,Lx^3\,\mathrm{d}(Lx)\sqrt{-(Lz-1)(Lz+1)} - \mathrm{d}(Lz)\sqrt{-(Lx-1)(Lx+1)}\sqrt{-(Lz-1)(Lz+1)}\,Lx^2 - \mathrm{d}(Lz)\sqrt{-(Lx-1)(Lx+1)}\,Lz^2\,Lx^2 + \mathrm{d}(Lz) - Lx^2\,\mathrm{d}(Lz) + 2\,Lz^3\,Lx^3\,\mathrm{d}(Lx)\sqrt{-(Lz-1)(Lz+1)} + 2\,Lz\,Lx^3\,\mathrm{d}(Lx)(-(Lz-1)(Lz+1))^{(3/2)}) \Big/ (\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)})$$

$$\Gamma 22 := 0$$

$$\Gamma 32 := (-3\,\mathrm{d}(Lx)(-(Lz-1)(Lz+1))^{(3/2)}\,Lz^2\,Lx^2 - Lz\,Lx\,\mathrm{d}(Lz)\sqrt{-(Lx-1)(Lx+1)}\sqrt{-(Lz-1)(Lz+1)} + 3\,\mathrm{d}(Lx)\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)}\,Lz^4\,Lx^2 + Lz\,Lx^3\,\mathrm{d}(Lz)\sqrt{-(Lx-1)(Lx+1)}\sqrt{-(Lz-1)(Lz+1)}$$

$$+Lz\,Lx\,d(Lz)\,(-(Lx-1)(Lx+1))^{(3/2)}\sqrt{-(Lz-1)(Lz+1)}$$
$$+3\,d(Lx)\,(-(Lz-1)(Lz+1))^{(3/2)}Lz^2Lx^2\sqrt{-(Lx-1)(Lx+1)}-3\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lz^4Lx^2$$
$$-Lz\,Lx\,d(Lz)\,(-(Lx-1)(Lx+1))^{(3/2)}-3\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)}\,Lz^2Lx^2$$
$$-d(Lx)\,(-(Lz-1)(Lz+1))^{(3/2)}Lx^2+d(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lx^2-Lz\,Lx^3\,d(Lz)\sqrt{-(Lx-1)(Lx+1)}$$
$$+2\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lz^2Lx^2+2\,d(Lx)\,(-(Lz-1)(Lz+1))^{(3/2)}Lz^2Lx^4$$
$$+2\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lz^4Lx^4-2\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lz^2Lx^4-Lz^2\,d(Lx)$$
$$-d(Lx)\sqrt{-(Lz-1)(Lz+1)}+d(Lx))\Big/\left(\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)}\right)$$

```
> Gamma13:=wcollect(factor(wcollect(cartan[1,3])));Gamma23:=factor(wcollect(cartan
  [2,3]));Gamma33:=simplify(wcollect(cartan[3,3]));
```

$$\Gamma13 := ((-2\,Lz^3\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)}\,Lx^2$$
$$-2\,Lz\,(-(Lz-1)(Lz+1))^{(3/2)}Lx^2\sqrt{-(Lx-1)(Lx+1)}+2\,Lz\sqrt{-(Lz-1)(Lz+1)}\,Lx^2\sqrt{-(Lx-1)(Lx+1)}$$
$$+Lz^3\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)}+Lz\,(-(Lz-1)(Lz+1))^{(3/2)}\sqrt{-(Lx-1)(Lx+1)}$$
$$-Lz^3\sqrt{-(Lz-1)(Lz+1)}-Lz\,(-(Lz-1)(Lz+1))^{(3/2)}+Lz\sqrt{-(Lz-1)(Lz+1)}$$
$$+2\,Lz^3\sqrt{-(Lz-1)(Lz+1)}\,Lx^2+2\,Lz\,(-(Lz-1)(Lz+1))^{(3/2)}Lx^2-2\,Lz\sqrt{-(Lz-1)(Lz+1)}\,Lx^2)\,d(Lx))\Big/($$
$$\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)})+((-Lx+Lx\,(-(Lx-1)(Lx+1))^{(3/2)}Lz^2$$
$$+Lx^3\sqrt{-(Lx-1)(Lx+1)}\,Lz^2+\sqrt{-(Lz-1)(Lz+1)}\,Lx-Lx^3\sqrt{-(Lz-1)(Lz+1)}+\sqrt{-(Lx-1)(Lx+1)}\,Lx$$
$$-Lx\sqrt{-(Lx-1)(Lx+1)}\,Lz^2+Lx^3)\,d(Lz))\Big/\left(\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)}\right)$$

$$\Gamma23 := (-2\,d(Lx)\,(-(Lz-1)(Lz+1))^{(3/2)}Lz^2\sqrt{-(Lx-1)(Lx+1)}$$
$$+2\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)}\,Lz^2-2\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lz^4\sqrt{-(Lx-1)(Lx+1)}$$
$$-2\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lz^2+2\,d(Lx)\,(-(Lz-1)(Lz+1))^{(3/2)}Lz^2+2\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lz^4$$
$$+2\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lz^4Lx^4-2\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lz^2Lx^4$$
$$-3\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)}\,Lz^2Lx^2$$
$$+3\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)}\,Lz^4Lx^2$$
$$-Lz\,Lx^3\,d(Lz)\sqrt{-(Lx-1)(Lx+1)}\sqrt{-(Lz-1)(Lz+1)}$$
$$-Lz\,Lx\,d(Lz)\,(-(Lx-1)(Lx+1))^{(3/2)}\sqrt{-(Lz-1)(Lz+1)}$$
$$+3\,d(Lx)\,(-(Lz-1)(Lz+1))^{(3/2)}Lz^2Lx^2\sqrt{-(Lx-1)(Lx+1)}+Lz\,Lx\,d(Lz)\,(-(Lx-1)(Lx+1))^{(3/2)}$$
$$+5\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lz^2Lx^2+2\,d(Lx)\,(-(Lz-1)(Lz+1))^{(3/2)}Lz^2Lx^4$$
$$+Lz\,Lx^3\,d(Lz)\sqrt{-(Lx-1)(Lx+1)}+Lz\,Lx\,d(Lz)\sqrt{-(Lx-1)(Lx+1)}\sqrt{-(Lz-1)(Lz+1)}$$
$$-4\,d(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lz^4Lx^2+d(Lx)\sqrt{-(Lz-1)(Lz+1)}+d(Lx)\,(-(Lz-1)(Lz+1))^{(3/2)}Lx^2$$
$$-d(Lx)\sqrt{-(Lz-1)(Lz+1)}\,Lx^2-4\,d(Lx)\,(-(Lz-1)(Lz+1))^{(3/2)}Lz^2Lx^2-d(Lx)+Lz^2\,d(Lx))\Big/($$
$$\sqrt{-(Lz-1)(Lz+1)}\sqrt{-(Lx-1)(Lx+1)})$$

$$\Gamma33 := 0$$

```
>
```

The "space-time" connection 1-forms are:

```
> hh1:=simplify(wcollect(cartan[4,1]));hh2:=factor(wcollect(cartan[4,2]));hh3:=wco
  llect(factor(wcollect(cartan[4,3])));
>
```

$$hh1 := 0$$

$$hh2 := 0$$

$$hh3 := 0$$

The "time-space connection" 1-forms are

```
> gg1:=factor(wcollect(factor(wcollect(cartan[1,4]))));gg2:=factor(wcollect(cartan[2,4]));gg3:=wcollect(factor(wcollect(cartan[3,4])));
```

$$gg1 := 0$$

$$gg2 := 0$$

$$gg3 := 0$$

The abnormality (time-time) connection 1-form

Note that the abnormality (Big Omega vanishes if expansion is a global constant. (no red shift?)

```
> Omega:=wcollect(subs(simplify(wcollect(cartan[4,4]))));
```

$$\Omega := 0$$

```
> L:=factor(wcollect(hh1&^sigma1+hh2&^sigma2+hh3&^sigma3));
```

$$L := 0$$

```
> S:=(wcollect(factor(hh1&^gg1+hh2&^gg2+hh3&^gg3)));
```

$$S := 0$$

There are in general two sets of torsion two forms.

1. Particle AFFINE (**PA**) torsion 2-forms which depend upon the product of little omega (the timelike part of the Vierbein) and the (time-space) connection components, little gamma.

2. WaveAFFINE (**WA**) torsion 2-forms which depend upon Big Omega (the time-time connection component or abnormality) and again the (time-space) connection components, little gamma.

If the time-space connection 1-forms vanish, small gamma, neither form of torsion exists.

See http://www.uh.edu/~rkiehn/pdf/projfram.pdf

PA TORSION 2-forms

```
> Sigma1:=wcollect(simplify((wcollect(factor(omega&^gg1)))));
Sigma2:=factor(simplify(wcollect(factor(omega&^gg2))));Sigma3:=wcollect(factor(simplify((omega&^gg3))));
```

$$\Sigma 1 := 0$$

$$\Sigma 2 := 0$$

$$\Sigma 3 := 0$$

WA TORSION 2-forms

```
> Phi1:=simplify((wcollect((factor(Omega&^gg1)))));Phi2:=wcollect(factor(Omega&^gg2));Phi3:=wcollect((factor(Omega&^gg3)));
```

$$\Phi 1 := 0$$

$$\Phi 2 := 0$$

$$\Phi 3 := 0$$

Next compute the matrix of curvature 2-forms on the x,y,z subspace

Curvature 2-forms

```
> Theta:=array([[gg1&^hh1,gg1&^hh2,gg1&^hh3],[gg2&^hh1,gg2&^hh2,gg2&^hh3],[(gg3&^hh1),(gg3&^hh2),(gg3&^hh3)]]);
>
```

$$\Theta := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The curvature 2-forms on the interior space all vanish.

DISCUSSION

Lorentz rotations do not seem to generate torsion or curvature on the 3D supspace, which must be due to the fact (somehow) that the transpose of a Lorentz rotation is proportional to its inverse. This fact is not true for Lorentz translations.

It is remarkable that the Lorentz Round Trip Rotations do not seem to effect curvature or torsion, yet the represent a defect in that the Identity is not the result of the round trip. I would have suspected that this would induce a curvature or torsion defect of some kind. But that does not seem to be the case. **Strange.**

>