

Some remarks about magnetism.

Observational evidence indicates that there exist field intensities, denoted by \mathbf{B} , that can be associated with currents flowing in conductors, and also with permanent magnets which apparently have no flowing currents. How does one know that such \mathbf{B} fields exist? One method is to move a charged particle in the \mathbf{B} field and observe that there are forces on the charge particle that do not change its kinetic energy, but do change its direction of travel. Another method is to insert a small current loop in the \mathbf{B} field and observe that loop oscillates with a frequency that can be correlated with the strength of the \mathbf{B} field.

Such observations indicate that there are forces on a charged particle moving with a velocity, \mathbf{v} , in a \mathbf{B} field, such that the magnetic force is perpendicular to both the velocity and the \mathbf{B} field, and has a value given by the Lorentz formula

$$\mathbf{F} = q\{\mathbf{E} + \mathbf{v} \times \mathbf{B}\}.$$

Under the assumption that currents are (free) charges in motion, the Lorentz formula gives an expression for the force on a current of length dl in a \mathbf{B} field as

$$\mathbf{F} = \mathbf{I}dl \times \mathbf{B}.$$

All of this becomes more complicated if the magnetic field \mathbf{B} is not uniform

For regions with or without \mathbf{E} field, the Lorentz force law produces a mathematical expression which is in apparent quantitative agreement with observation. In particular, if there is a charged ion moving at constant velocity and at right angles to a uniform magnetic field, \mathbf{B} , then there is a force on the positive ion to cause it to execute clock-wise circular motion (clock wise on looking down on the arrow that represents the \mathbf{B} field). The kinetic energy of the ion is not changed, as the applied magnetic force is perpendicular to the velocity. (This is the principle of the mass spectrometer). A negative ion would execute counter clockwise motion. If the motion of the ion has a constant speed component parallel to the magnetic field, that motion continues unchanged, and the ion executes a helical screw-like motion in the uniform magnetic field, with the helical axis in the direction of the \mathbf{B} field. The positive ion has a trajectory of a left hand screw, and the negative ion has a trajectory of a right and screw relative to the direction of the \mathbf{B} field. The field is presumed to be constant and non-rotating, but induces a circular motion on charge particles that are injected in it.

Topological arguments indicate that the \mathbf{B} field is generated by the "curl" (similar to vorticity) of a vector potential \mathbf{A} . From my point of view the \mathbf{B} field is the limit point set of the vector potentials.

The functional relationship between \mathbf{B} and \mathbf{A} is not unique.

There are many vector potentials \mathbf{A} that yield the same \mathbf{B} field.

How is the \mathbf{B} field produced? There are two methods that have been observed. The first method is that there exists a correlation between a current flowing in a wire and the forces on a moving charged particle near the wire. That is, there is a \mathbf{B} field generated somehow by flowing currents. So one reproducible source of magnetic \mathbf{B} field is somehow related to current flows. If the currents occur in media, the \mathbf{B} field produced can depend upon the magnetic properties of the media as well as the currents.

Actually the concept of currents can be related to another electromagnetic field, \mathbf{H} , of magnetic "excitation" (using the words of A. Sommerfeld). According to Maxwell, the field \mathbf{H} can produce currents, if it has a curl ("vorticity" of $\mathbf{H} =$ the current density, \mathbf{J}). The relationship between \mathbf{H} and \mathbf{J} is *independent* of the media. Note: the functional relationship between \mathbf{H} and \mathbf{J} is not unique. From a topological perspective, the current densities \mathbf{J} are the limit points of the magnetic excitations \mathbf{H} .

Often it is assumed that the \mathbf{B} and the \mathbf{H} field are linearly related. This certainly seems to be the experimental evidence for the vacuum. It is most certainly not true for magnetic media.

The bottom line is that there are differences between \mathbf{B} and \mathbf{H} that do not admit even a linear relationship. In fact, the well known hysteresis phenomena is a non-linear relationship between the quantities of \mathbf{B} and \mathbf{H} .

However, it is observed that there are certain materials that produce magnetic \mathbf{B} fields without an obvious source of current flow. The classic example is the bar magnet. By measuring forces on moving charged particles, and mapping out the \mathbf{B} field lines, it appears as if the magnetic field of a bar magnet - at distances which are much greater than the linear dimension of the bar magnet - is approximately the same as the \mathbf{B} field produced by a circular loop of current. The shape of the field lines is called a magnetic dipole.

The magnetic field of the earth can be *approximated* by a bar magnet embedded in the earth but inclined at an angle with respect to the rotation axis of the earth. It also can be approximated by a current loop presumed to be located in the outer molten layer of the earth's core. All of this is a bit speculative, but that is the state of affairs as known today. The North magnetic pole, from which the \mathbf{B} field lines emanate, is actually near the south rotation pole of the earth; The South pole of the magnet, into which the \mathbf{B} field lines seem to enter, is near the north rotation pole of the earth. The angular offset is of the order of 11 degrees. There is evidence that the magnetic poles of the earth flip every million years or so. No explanation is available.

The dipole field is symmetric about the the magnetic axis, the central axis perpendicular to the plane of the equivalent current loop. The \mathbf{B} field lines are not twisted in this classic sophomore exposition of a magnetic dipole. If the \mathbf{B} field lines are integrable in the sense of Frobenius, then there is no twist to the dipole lines. If however, there is a component of the \mathbf{B} in the direction of the curl of the \mathbf{B} field, then there exists a twisted form of the dipole field. It is curious, but the analogue in fluid dynamics would be when the flow vorticity is proportional to the curl of the vorticity (not velocity) of the fluid. For a Navier-Stokes fluid, I have a topological argument that says that such processes are thermodynamically irreversible.

I do not know if there are any field line measurements for the earth's field that indicate that the dipole field lines are twisted. Actually, the earth's magnetic field is highly distorted by the flow of charged particles from the sun (the so called solar wind). However, it bothers me to think of a continuous flow of charged particles of one sign coming from the sun, for then the sun would become charged, and it would seem that emission of charge particles would stop. It must be that there is a return current somewhere.

Now consider two bar magnets. It is apparent from observation that like magnetic poles repel and unlike magnetic poles attract. If things are symmetrical the "push" and the "pull" are equal. If a North magnetic pole is placed above a South magnetic pole with a piece of paper inbetween, the \mathbf{B} field lines emanate from the North pole and enter the South magnetic pole. If the area of the poles is large and are of the same in size, the field in between the poles is more or less uniform. That is the field lines are straight and of uniform density. Fringing occurs at the edges.

The magnetic field is neither paramagnetic nor diamagnetic. Matter put into a magnetic field *responds* in either a paramagnetic or a diamagnetic manner. (depending on whether the matter aligns

or anti-aligns with the external magnetic field.)

It is a topological fact that the divergence of the **B** field lines is always zero. This means that the **B** field lines either form closed loops, or stop and start only on topological boundary points. In general, such statements cannot be said about **H**. The lines of **H** can be discontinuous.

Now comes a dilemma for me, for if I rotate a fluid like water - which supposedly contains both positive and negative ions - and do this in the presence of a magnetic field which is symmetrically collinear with the rotation axis of the field, I ask what happens? I think the system will act like a homopolar generator and produce a charge separation (Zeta potential?) in the fluid. The fluid will become polarized and should conduct current. As far as I know such experiments have never been done.

More later.