```
> restart:
```

MapleEM.mws
from maxwell/mws and maxwellplasma.mws
Updated 12/12/97
Updated 11/5/98
Updated 10/24/2002 Correcting sign of T4 and d(A^F)
updated 11/09/2003
-- R. M. Kiehn

**NOTES:**
This Maple program computes Maxwell-Faraday formulas from the postulate of potentials F-dA=0. Given a 1-form of Action on 4D space time, the E and B fields follow.
The Maxwell Ampere equations are computed from the postulate of charge currents, J-dG=0 , and assuming Lorentz vacuum constitutive equations,
D = epsilon E B = mu H.
The procedure has as input the 4 potential, and computes
E,B,D,H,Jamp,Jdisp,Jtot, as well as the Torsion vector = -{ ExA+Bphi, AdotB]
the Spin Vector = A x H + Dphi, AdotD, the First Poincare invariant =
F^G -A^J = (BdotH-DdotE) -(AdotJ-rho.phi), and the second Poincare Invariant = F^F = +2EdotB
see http://www22.pair.com/csdc/pdf/classice.pdf

```
>  with(liesymm):with(linalg):with(plots):with(DEtools):
Warning, the protected name close has been redefined and unprotected

Warning, the protected names norm and trace have been redefined and unprotected

Warning, the name changecoords has been redefined

Warning, the names adjoint and translate have been redefined

> setup(x,y,z,t):defform(x=0,y=0,z=0,t=0,a=const,b=const,c=const,k=const,mu=const,
  m=const,alpha=const,beta=const,m=const,n=const,omega=const,epsilon=const,pi=cons
  t);
```

defform($x = 0, y = 0, z = 0, t = 0, a = const, b = const, c = const, k = const, \mu = const, m = const, \alpha = const, \beta = const,$
$m = const, n = const, \omega = const, \varepsilon = const, \pi = const$)

**The main procedure**

```
> JCM:=proc(Ax,Ay,Az,phi)
     local A,A1,A2,A3,A4,BFC,TFC,EF1,EF2,EF3,JAC,JDC,SFC,ExBC:
     global
A1form,BF,EF,TF,HEL,P1,P2,DF,HF,CD,JA,JD,SPD,SF,JTOT,ExB,NAME,LAGF,AJ,VW,DISS:
     A1:=Ax;A2:=Ay;A3:=Az;A4:=phi: A:=[A1,A2,A3]:
A1form:=A1*d(x)+A2*d(y)+A3*d(z)-A4*d(t):
    EF1:=evalm(-grad(phi,[x,y,z])):
     EF2:=-[diff(A1,t),diff(A2,t),diff(A3,t)];
EF3:=[EF1[1]+EF2[1],EF1[2]+EF2[2],EF1[3]+EF2[3]];

EF:=[factor(simplify(EF3[1])),factor(simplify(EF3[2])),factor(simplify(EF3[3]))];
     BFC:=(curl([A1,A2,A3],[x,y,z])):

BF:=[factor(simplify(BFC[1])),factor(simplify(BFC[2])),factor(simplify(BFC[3]))];
```

```
    HEL:=-factor(simplify(innerprod(A,BF)));

TFC:=-[crossprod(EF,A)[1]+BF[1]*phi,crossprod(EF,A)[2]+BF[2]*phi,crossprod(EF,A)[3
]+BF[3]*phi];

TF:=[factor(simplify(TFC[1])),factor(simplify(TFC[2])),factor(simplify(TFC[3])),fa
ctor(HEL)];
    P2:=factor(simplify(2*innerprod(EF,BF)));

HF:=[factor(simplify(BFC[1]/mu)),factor(simplify(BFC[2]/mu)),factor(simplify(BFC[3
]/mu))];
DF:=[factor(simplify(epsilon*EF3[1])),factor(simplify(epsilon*EF3[2])),factor(simp
lify(epsilon*EF3[3]))];
CD:=factor(diverge([DF[1],DF[2],DF[3]],[x,y,z]));
JAC:=curl([HF[1],HF[2],HF[3]],[x,y,z]);
     JA:=[JAC[1],JAC[2],JAC[3]];
JDC:=-[diff(DF[1],t),diff(DF[2],t),diff(DF[3],t)];
JD:=[factor(JDC[1]),factor(JDC[2]),factor(JDC[3])];JTOT:=[factor(simplify(JA[1]+JD
[1])),factor(simplify(JA[2]+JD[2])),factor(simplify(JA[3]+JD[3]))];DISS:=factor(si
mplify(innerprod(JTOT,EF)));

SPD:=factor(simplify(innerprod(A,DF)));VW:=[factor(simplify(CD*EF[1]+crossprod(JTO
T,BF)[1])),factor(simplify(CD*EF[2]+crossprod(JTOT,BF)[2])),factor(simplify(CD*EF[
3]+crossprod(JTOT,BF)[3]))];

SFC:=[crossprod(A,HF)[1]+DF[1]*phi,crossprod(A,HF)[2]+DF[2]*phi,crossprod(A,HF)[3]
+DF[3]*phi];

SF:=[factor(simplify(SFC[1])),factor(simplify(SFC[2])),factor(simplify(SFC[3])),SP
D];

AJ:=factor(simplify(innerprod(A,JTOT)-CD*phi));LAGF:=factor(simplify(innerprod(BF,
HF)-innerprod(DF,EF)));P1:=factor(simplify(LAGF-AJ));
ExBC:=crossprod(EF,HF);ExB:=[factor(simplify(ExBC[1])),factor(simplify(ExBC[2])),f
actor(simplify(ExBC[3]))];print(NAME);print(`Lorenz constitutive equations, B = mu
H, D = epsilon E`);print(`Action`= A1form);print(`E field`= EF);print(`B field`=
BF);print(`Topological Torsion`= TF);print(`Helicity AdotB`= HEL);print(`Poincare
2 E.B`= P2);print(`D field`= DF);print(`H field`= HF);print(`Poynting vector
ExH`=ExB);print(`Current density`= JTOT);print(`charge density`=
CD);print(`Topological SPIN`=SF);print(`chiralty AdotD`= SPD);print(`LaGrange
field energy density (B.H-D.E)`= LAGF);print(`Interaction energy density
(A.J-rho.phi)`= AJ);print(`Poincare 1 (B.H-D.E)-(A.J-rho.phi)`= P1);print(`Virtual
work `=VW);print(`JdotE power `=DISS);
  end:
>
>
```

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 1a-- Real Linear Polarization A^G<>0, A^F = 0  INBOUND `;
> theta:=(k*z+omega*t);
> Ax:=cos(theta);Ay:=cos(theta);Az:=0;phi:=0;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

************************************************************

*NAME := Example 1a-- Real Linear Polarization A^G<>0, A^F = 0 INBOUND*

$$\theta := k\,z + \omega\,t$$

$$Ax := \cos(k\,z + \omega\,t)$$

$$Ay := \cos(k\,z + \omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

*Example 1a-- Real Linear Polarization A^G<>0, A^F = 0 INBOUND*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$\mathit{Action} = \cos(k\,z + \omega\,t)\,\mathrm{d}(x) + \cos(k\,z + \omega\,t)\,\mathrm{d}(y)$$

$$\mathit{E\ field} = [\sin(k\,z + \omega\,t)\,\omega,\ \sin(k\,z + \omega\,t)\,\omega,\ 0]$$

$$\mathit{B\ field} = [\sin(k\,z + \omega\,t)\,k,\ -\sin(k\,z + \omega\,t)\,k,\ 0]$$

$$\mathit{Topological\ Torsion} = [0, 0, 0, 0]$$

$$\mathit{Helicity\ AdotB} = 0$$

$$\mathit{Poincare\ 2\ E.B} = 0$$

$$\mathit{D\ field} = [\varepsilon\,\sin(k\,z + \omega\,t)\,\omega,\ \varepsilon\,\sin(k\,z + \omega\,t)\,\omega,\ 0]$$

$$\mathit{H\ field} = \left[\frac{\sin(k\,z + \omega\,t)\,k}{\mu},\ -\frac{\sin(k\,z + \omega\,t)\,k}{\mu},\ 0\right]$$

$$\mathit{Poynting\ vector\ ExH} = \left[0,\ 0,\ 2\,\frac{(\cos(k\,z + \omega\,t) - 1)\,(\cos(k\,z + \omega\,t) + 1)\,\omega\,k}{\mu}\right]$$

$$\mathit{Current\ density} = \left[-\frac{\cos(k\,z + \omega\,t)\,(-k^2 + \varepsilon\,\omega^2\,\mu)}{\mu},\ -\frac{\cos(k\,z + \omega\,t)\,(-k^2 + \varepsilon\,\omega^2\,\mu)}{\mu},\ 0\right]$$

$$\mathit{charge\ density} = 0$$

$$\mathit{Topological\ SPIN} = \left[0,\ 0,\ -2\,\frac{\cos(k\,z + \omega\,t)\,\sin(k\,z + \omega\,t)\,k}{\mu},\ 2\,\cos(k\,z + \omega\,t)\,\varepsilon\,\sin(k\,z + \omega\,t)\,\omega\right]$$

$$\mathit{chiralty\ AdotD} = 2\,\cos(k\,z + \omega\,t)\,\varepsilon\,\sin(k\,z + \omega\,t)\,\omega$$

$$\mathit{LaGrange\ field\ energy\ density\ (B.H\text{-}D.E)} = 2\,\frac{(\cos(k\,z + \omega\,t) - 1)\,(\cos(k\,z + \omega\,t) + 1)\,(-k^2 + \varepsilon\,\omega^2\,\mu)}{\mu}$$

$$\mathit{Interaction\ energy\ density\ (A.J\text{-}rho.phi)} = -2\,\frac{\cos(k\,z + \omega\,t)^2\,(-k^2 + \varepsilon\,\omega^2\,\mu)}{\mu}$$

$$\mathit{Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi)} = 2\,\frac{(-k^2 + \varepsilon\,\omega^2\,\mu)\,(-1 + 2\,\cos(k\,z + \omega\,t)^2)}{\mu}$$

$$\mathit{Virtual\ work} = \left[0,\ 0,\ 2\,\frac{\cos(k\,z + \omega\,t)\,(-k^2 + \varepsilon\,\omega^2\,\mu)\,\sin(k\,z + \omega\,t)\,k}{\mu}\right]$$

$$\mathit{JdotE\ power} = -2\,\frac{\cos(k\,z + \omega\,t)\,(-k^2 + \varepsilon\,\omega^2\,\mu)\,\sin(k\,z + \omega\,t)\,\omega}{\mu}$$

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 1b-- Real Linear Polarization A^G<>0, A^F = 0  OUTBOUND `;
> theta:=(k*z-omega*t);
> Ax:=cos(theta);Ay:=cos(theta);Az:=0;phi:=0;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

****************************************************************

*NAME := Example 1b-- Real Linear Polarization A^G<>0, A^F = 0  OUTBOUND*

$$\theta := k\, z - \omega\, t$$

$$Ax := \cos(-k\, z + \omega\, t)$$

$$Ay := \cos(-k\, z + \omega\, t)$$

$$Az := 0$$

$$\phi := 0$$

*Example 1b-- Real Linear Polarization A^G<>0, A^F = 0  OUTBOUND*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \cos(-k\, z + \omega\, t)\, d(x) + \cos(-k\, z + \omega\, t)\, d(y)$$

$$E\ field = [\sin(-k\, z + \omega\, t)\, \omega, \sin(-k\, z + \omega\, t)\, \omega, 0]$$

$$B\ field = [-\sin(-k\, z + \omega\, t)\, k, \sin(-k\, z + \omega\, t)\, k, 0]$$

$$Topological\ Torsion = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = [\varepsilon \sin(-k\, z + \omega\, t)\, \omega, \varepsilon \sin(-k\, z + \omega\, t)\, \omega, 0]$$

$$H\ field = \left[-\frac{\sin(-k\, z + \omega\, t)\, k}{\mu}, \frac{\sin(-k\, z + \omega\, t)\, k}{\mu}, 0\right]$$

$$Poynting\ vector\ ExH = \left[0, 0, -2\, \frac{(\cos(-k\, z + \omega\, t) - 1)\, (\cos(-k\, z + \omega\, t) + 1)\, \omega\, k}{\mu}\right]$$

$$Current\ density = \left[-\frac{\cos(-k\, z + \omega\, t)\, (-k^2 + \varepsilon\, \omega^2\, \mu)}{\mu}, -\frac{\cos(-k\, z + \omega\, t)\, (-k^2 + \varepsilon\, \omega^2\, \mu)}{\mu}, 0\right]$$

$$charge\ density = 0$$

$$Topological\ SPIN = \left[0, 0, 2\, \frac{\cos(-k\, z + \omega\, t) \sin(-k\, z + \omega\, t)\, k}{\mu}, 2 \cos(-k\, z + \omega\, t)\, \varepsilon \sin(-k\, z + \omega\, t)\, \omega\right]$$

$$chiralty\ AdotD = 2 \cos(-k\, z + \omega\, t)\, \varepsilon \sin(-k\, z + \omega\, t)\, \omega$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = 2\, \frac{(\cos(-k\, z + \omega\, t) - 1)\, (\cos(-k\, z + \omega\, t) + 1)\, (-k^2 + \varepsilon\, \omega^2\, \mu)}{\mu}$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = -2\, \frac{\cos(-k\, z + \omega\, t)^2\, (-k^2 + \varepsilon\, \omega^2\, \mu)}{\mu}$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = 2\, \frac{(-k^2 + \varepsilon\, \omega^2\, \mu)\, (-1 + 2 \cos(-k\, z + \omega\, t)^2)}{\mu}$$

$$Virtual\ work\ = \left[0, 0, -2\, \frac{\cos(-k\, z + \omega\, t)\, (-k^2 + \varepsilon\, \omega^2\, \mu) \sin(-k\, z + \omega\, t)\, k}{\mu}\right]$$

$$JdotE\ power\ = -2\, \frac{\cos(-k\, z + \omega\, t)\, (-k^2 + \varepsilon\, \omega^2\, \mu) \sin(-k\, z + \omega\, t)\, \omega}{\mu}$$

```
>
```

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 2a-- Real Circular Polarization A^G=0, A^F <> 0 INBOUND`;
```

```
> theta:=(k*z+omega*t);
> Ax:=cos(theta);Ay:=sin(theta);Az:=0;phi:=0;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*************************************************************

*NAME := Example 2a-- Real Circular Polarization A^G=0, A^F <> 0 INBOUND*

$$\theta := k\,z + \omega\,t$$

$$Ax := \cos(k\,z + \omega\,t)$$

$$Ay := \sin(k\,z + \omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

*Example 2a-- Real Circular Polarization A^G=0, A^F <> 0 INBOUND*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \cos(k\,z + \omega\,t)\,d(x) + \sin(k\,z + \omega\,t)\,d(y)$$

$$E\ field = [\sin(k\,z + \omega\,t)\,\omega, -\cos(k\,z + \omega\,t)\,\omega, 0]$$

$$B\ field = [-\cos(k\,z + \omega\,t)\,k, -\sin(k\,z + \omega\,t)\,k, 0]$$

$$Topological\ Torsion = [0, 0, -\omega, k]$$

$$Helicity\ AdotB = k$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = [\varepsilon\sin(k\,z + \omega\,t)\,\omega, -\varepsilon\cos(k\,z + \omega\,t)\,\omega, 0]$$

$$H\ field = \left[-\frac{\cos(k\,z + \omega\,t)\,k}{\mu}, -\frac{\sin(k\,z + \omega\,t)\,k}{\mu}, 0\right]$$

$$Poynting\ vector\ ExH = \left[0, 0, -\frac{\omega\,k}{\mu}\right]$$

$$Current\ density = \left[-\frac{\cos(k\,z + \omega\,t)\,(-k^2 + \varepsilon\,\omega^2\,\mu)}{\mu}, -\frac{\sin(k\,z + \omega\,t)\,(-k^2 + \varepsilon\,\omega^2\,\mu)}{\mu}, 0\right]$$

$$charge\ density = 0$$

$$Topological\ SPIN = [0, 0, 0, 0]$$

$$chiralty\ AdotD = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{-k^2 + \varepsilon\,\omega^2\,\mu}{\mu}$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = -\frac{-k^2 + \varepsilon\,\omega^2\,\mu}{\mu}$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = 0$$

$$Virtual\ work\ = [0, 0, 0]$$

$$JdotE\ power\ = 0$$

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 2b-- Real Circular Polarization A^G<>0, A^F = 0  OUTBOUND `;
> theta:=(k*z-omega*t);
> Ax:=cos(theta);Ay:=sin(theta);Az:=0;phi:=0;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*************************************************************

$$NAME := Example\ 2b\text{-- } Real\ Circular\ Polarization\ A\text{^}G<>0,\ A\text{^}F = 0\ \ OUTBOUND$$

$$\theta := k\,z - \omega\,t$$

$$Ax := \cos(-k\,z + \omega\,t)$$

$$Ay := -\sin(-k\,z + \omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

*Example 2b-- Real Circular Polarization A^G<>0, A^F = 0  OUTBOUND*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \cos(-k\,z + \omega\,t)\,\mathrm{d}(x) - \sin(-k\,z + \omega\,t)\,\mathrm{d}(y)$$

$$E\ field = [\sin(-k\,z + \omega\,t)\,\omega,\ \cos(-k\,z + \omega\,t)\,\omega,\ 0]$$

$$B\ field = [-\cos(-k\,z + \omega\,t)\,k,\ \sin(-k\,z + \omega\,t)\,k,\ 0]$$

$$Topological\ Torsion = [0,\ 0,\ \omega,\ k]$$

$$Helicity\ AdotB = k$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = [\varepsilon\sin(-k\,z + \omega\,t)\,\omega,\ \varepsilon\cos(-k\,z + \omega\,t)\,\omega,\ 0]$$

$$H\ field = \left[-\frac{\cos(-k\,z + \omega\,t)\,k}{\mu},\ \frac{\sin(-k\,z + \omega\,t)\,k}{\mu},\ 0\right]$$

$$Poynting\ vector\ ExH = \left[0,\ 0,\ \frac{\omega\,k}{\mu}\right]$$

$$Current\ density = \left[-\frac{\cos(-k\,z + \omega\,t)\,(-k^2 + \varepsilon\,\omega^2\,\mu)}{\mu},\ \frac{\sin(-k\,z + \omega\,t)\,(-k^2 + \varepsilon\,\omega^2\,\mu)}{\mu},\ 0\right]$$

$$charge\ density = 0$$

$$Topological\ SPIN = [0,\ 0,\ 0,\ 0]$$

$$chiralty\ AdotD = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{-k^2 + \varepsilon\,\omega^2\,\mu}{\mu}$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = -\frac{-k^2 + \varepsilon\,\omega^2\,\mu}{\mu}$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = 0$$

$$Virtual\ work\ = [0,\ 0,\ 0]$$

$$JdotE\ power\ = 0$$

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 3a-- Complex Linear Polarization A^G<>0, A^F = 0  INBOUND `;
> theta:=(k*z+omega*t);
> Ax:=cos(theta);Ay:=I*cos(theta);Az:=0;phi:=0;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*************************************************************

$$NAME := Example\ 3a\text{-- } Complex\ Linear\ Polarization\ A\text{^}G<>0,\ A\text{^}F = 0\ \ INBOUND$$

$$\theta := k\,z + \omega\,t$$

$$Ax := \cos(k\,z + \omega\,t)$$

$$Ay := I\cos(k\,z+\omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

*Example 3a-- Complex Linear Polarization A^G<>0, A^F = 0 INBOUND*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \cos(k\,z+\omega\,t)\,\mathrm{d}(x) + I\cos(k\,z+\omega\,t)\,\mathrm{d}(y)$$

$$E\ field = [\sin(k\,z+\omega\,t)\,\omega,\ I\sin(k\,z+\omega\,t)\,\omega,\ 0]$$

$$B\ field = [I\sin(k\,z+\omega\,t)\,k,\ -\sin(k\,z+\omega\,t)\,k,\ 0]$$

$$Topological\ Torsion = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = [\varepsilon\sin(k\,z+\omega\,t)\,\omega,\ I\,\varepsilon\sin(k\,z+\omega\,t)\,\omega,\ 0]$$

$$H\ field = \left[\frac{I\sin(k\,z+\omega\,t)\,k}{\mu},\ -\frac{\sin(k\,z+\omega\,t)\,k}{\mu},\ 0\right]$$

$$Poynting\ vector\ ExH = [0, 0, 0]$$

$$Current\ density = \left[-\frac{\cos(k\,z+\omega\,t)\,(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu},\ \frac{-I\cos(k\,z+\omega\,t)\,(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu},\ 0\right]$$

$$charge\ density = 0$$

$$Topological\ SPIN = [0, 0, 0, 0]$$

$$chiralty\ AdotD = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = 0$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = 0$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = 0$$

$$Virtual\ work\ = [0, 0, 0]$$

$$JdotE\ power\ = 0$$

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 3b-- Complex Linear Polarization A^G = 0, A^F = 0   OUTBOUND`;
> theta:=(k*z-omega*t);
> Ax:=cos(theta);Ay:=I*cos(theta);Az:=0;phi:=0;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*******************************************************

*NAME := Example 3b-- Complex Linear Polarization A^G = 0, A^F = 0 OUTBOUND*

$$\theta := k\,z-\omega\,t$$

$$Ax := \cos(-k\,z+\omega\,t)$$

$$Ay := I\cos(-k\,z+\omega\,t)$$

$$Az := 0$$

$$\phi := 0$$

*Example 3b-- Complex Linear Polarization A^G = 0, A^F = 0 OUTBOUND*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \cos(-k\,z+\omega\,t)\,\mathrm{d}(x) + I\cos(-k\,z+\omega\,t)\,\mathrm{d}(y)$$

$$E\ field = [\sin(-k\,z+\omega\,t)\,\omega,\ I\sin(-k\,z+\omega\,t)\,\omega,\ 0]$$

$$B\ field = [-I\sin(-k z+\omega t)\, k,\ \sin(-k z+\omega t)\, k,\ 0]$$

$$Topological\ Torsion = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = [\varepsilon\sin(-k z+\omega t)\,\omega,\ I\,\varepsilon\sin(-k z+\omega t)\,\omega,\ 0]$$

$$H\ field = \left[\frac{-I\sin(-k z+\omega t)\, k}{\mu}, \frac{\sin(-k z+\omega t)\, k}{\mu}, 0\right]$$

$$Poynting\ vector\ ExH = [0, 0, 0]$$

$$Current\ density = \left[-\frac{\cos(-k z+\omega t)\,(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu}, \frac{-I\cos(-k z+\omega t)\,(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu}, 0\right]$$

$$charge\ density = 0$$

$$Topological\ SPIN = [0, 0, 0, 0]$$

$$chiralty\ AdotD = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = 0$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = 0$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = 0$$

$$Virtual\ work\ = [0, 0, 0]$$

$$JdotE\ power\ = 0$$

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 4a-- Complex Circular Polarization A^G<>0, A^F <> 0   INBOUND`;
> theta:=(k*z+omega*t);
> Ax:=cos(theta);Ay:=I*sin(theta);Az:=0;phi:=0;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*********************************************************

*NAME := Example 4a-- Complex Circular Polarization A^G<>0, A^F <> 0   INBOUND*

$$\theta := k z+\omega t$$

$$Ax := \cos(k z+\omega t)$$

$$Ay := I\sin(k z+\omega t)$$

$$Az := 0$$

$$\phi := 0$$

*Example 4a-- Complex Circular Polarization A^G<>0, A^F <> 0   INBOUND*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \cos(k z+\omega t)\,\mathrm{d}(x) + I\sin(k z+\omega t)\,\mathrm{d}(y)$$

$$E\ field = [\sin(k z+\omega t)\,\omega,\ -I\cos(k z+\omega t)\,\omega,\ 0]$$

$$B\ field = [-I\cos(k z+\omega t)\, k,\ -\sin(k z+\omega t)\, k,\ 0]$$

$$Topological\ Torsion = [0, 0, -I\,\omega, I\,k]$$

$$Helicity\ AdotB = I\,k$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = [\varepsilon\sin(k z+\omega t)\,\omega,\ -I\,\varepsilon\cos(k z+\omega t)\,\omega,\ 0]$$

$$H\ field = \left[\frac{-I\cos(k z+\omega t)\, k}{\mu}, -\frac{\sin(k z+\omega t)\, k}{\mu}, 0\right]$$

$$\textit{Poynting vector ExH} = \left[0,\ 0,\ \frac{\omega\, k\, (2 \cos(k\, z + \omega\, t)^2 - 1)}{\mu}\right]$$

$$\textit{Current density} = \left[-\frac{\cos(k\, z + \omega\, t)\, (-k^2 + \varepsilon\, \omega^2\, \mu)}{\mu},\ \frac{-I \sin(k\, z + \omega\, t)\, (-k^2 + \varepsilon\, \omega^2\, \mu)}{\mu},\ 0\right]$$

$$\textit{charge density} = 0$$

$$\textit{Topological SPIN} = \left[0,\ 0,\ -2\, \frac{\cos(k\, z + \omega\, t) \sin(k\, z + \omega\, t)\, k}{\mu},\ 2 \cos(k\, z + \omega\, t)\, \varepsilon \sin(k\, z + \omega\, t)\, \omega\right]$$

$$\textit{chiralty AdotD} = 2 \cos(k\, z + \omega\, t)\, \varepsilon \sin(k\, z + \omega\, t)\, \omega$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \frac{(-k^2 + \varepsilon\, \omega^2\, \mu)\, (2 \cos(k\, z + \omega\, t)^2 - 1)}{\mu}$$

$$\textit{Interaction energy density (A.J-rho.phi)} = -\frac{(-k^2 + \varepsilon\, \omega^2\, \mu)\, (2 \cos(k\, z + \omega\, t)^2 - 1)}{\mu}$$

$$\textit{Poincare 1 (B.H-D.E)-(A.J-rho.phi)} = 2\, \frac{(-k^2 + \varepsilon\, \omega^2\, \mu)\, (2 \cos(k\, z + \omega\, t)^2 - 1)}{\mu}$$

$$\textit{Virtual work } = \left[0,\ 0,\ 2\, \frac{\cos(k\, z + \omega\, t)\, (-k^2 + \varepsilon\, \omega^2\, \mu) \sin(k\, z + \omega\, t)\, k}{\mu}\right]$$

$$\textit{JdotE power } = -2\, \frac{\cos(k\, z + \omega\, t)\, (-k^2 + \varepsilon\, \omega^2\, \mu) \sin(k\, z + \omega\, t)\, \omega}{\mu}$$

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 4b-- Complex Circular Polarization A^G<>0, A^F <> 0   OUTBOUND`;
> theta:=(k*z-omega*t);
> Ax:=cos(theta);Ay:=I*sin(theta);Az:=0;phi:=0;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*************************************************************

*NAME := Example 4b-- Complex Circular Polarization A^G<>0, A^F <> 0   OUTBOUND*

$$\theta := k\, z - \omega\, t$$

$$Ax := \cos(-k\, z + \omega\, t)$$

$$Ay := -I \sin(-k\, z + \omega\, t)$$

$$Az := 0$$

$$\phi := 0$$

*Example 4b-- Complex Circular Polarization A^G<>0, A^F <> 0   OUTBOUND*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$\textit{Action} = \cos(-k\, z + \omega\, t)\, d(x) - I \sin(-k\, z + \omega\, t)\, d(y)$$

$$\textit{E field} = [\sin(-k\, z + \omega\, t)\, \omega,\ I \cos(-k\, z + \omega\, t)\, \omega,\ 0]$$

$$\textit{B field} = [-I \cos(-k\, z + \omega\, t)\, k,\ \sin(-k\, z + \omega\, t)\, k,\ 0]$$

$$\textit{Topological Torsion} = [0,\ 0,\ I\, \omega,\ I\, k]$$

$$\textit{Helicity AdotB} = I\, k$$

$$\textit{Poincare 2 E.B} = 0$$

$$\textit{D field} = [\varepsilon \sin(-k\, z + \omega\, t)\, \omega,\ I\, \varepsilon \cos(-k\, z + \omega\, t)\, \omega,\ 0]$$

$$H\ field = \left[\frac{-I\cos(-k\,z+\omega\,t)\,k}{\mu}, \frac{\sin(-k\,z+\omega\,t)\,k}{\mu}, 0\right]$$

$$Poynting\ vector\ ExH = \left[0, 0, -\frac{\omega\,k\,(2\cos(-k\,z+\omega\,t)^2-1)}{\mu}\right]$$

$$Current\ density = \left[-\frac{\cos(-k\,z+\omega\,t)\,(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu}, \frac{I\sin(-k\,z+\omega\,t)\,(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu}, 0\right]$$

$$charge\ density = 0$$

$$Topological\ SPIN = \left[0, 0, 2\,\frac{\cos(-k\,z+\omega\,t)\sin(-k\,z+\omega\,t)\,k}{\mu}, 2\cos(-k\,z+\omega\,t)\,\varepsilon\sin(-k\,z+\omega\,t)\,\omega\right]$$

$$chiralty\ AdotD = 2\cos(-k\,z+\omega\,t)\,\varepsilon\sin(-k\,z+\omega\,t)\,\omega$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = \frac{(-k^2+\varepsilon\,\omega^2\,\mu)\,(2\cos(-k\,z+\omega\,t)^2-1)}{\mu}$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = -\frac{(-k^2+\varepsilon\,\omega^2\,\mu)\,(2\cos(-k\,z+\omega\,t)^2-1)}{\mu}$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = 2\,\frac{(-k^2+\varepsilon\,\omega^2\,\mu)\,(2\cos(-k\,z+\omega\,t)^2-1)}{\mu}$$

$$Virtual\ work\ = \left[0, 0, -2\,\frac{\cos(-k\,z+\omega\,t)\,(-k^2+\varepsilon\,\omega^2\,\mu)\sin(-k\,z+\omega\,t)\,k}{\mu}\right]$$

$$JdotE\ power\ = -2\,\frac{\cos(-k\,z+\omega\,t)\,(-k^2+\varepsilon\,\omega^2\,\mu)\sin(-k\,z+\omega\,t)\,\omega}{\mu}$$

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 5a-- waveguide TM mode  (group kinematic in, wave in)`;
> theta:=(k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=-vg*f(x,y)*cos(theta);
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*************************************************************

*NAME := Example 5a-- waveguide TM mode  (group kinematic in, wave in)*

$$\theta := k\,z+\omega\,t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := f(x,y)\cos(k\,z+\omega\,t)$$

$$\phi := -vg\,f(x,y)\cos(k\,z+\omega\,t)$$

*Example 5a-- waveguide TM mode  (group kinematic in, wave in)*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = f(x,y)\cos(k\,z+\omega\,t)\,d(z) + vg\,f(x,y)\cos(k\,z+\omega\,t)\,d(t)$$

$$E\ field = \left[vg\left(\frac{\partial}{\partial x}f(x,y)\right)\cos(k\,z+\omega\,t), vg\left(\frac{\partial}{\partial y}f(x,y)\right)\cos(k\,z+\omega\,t), f(x,y)\sin(k\,z+\omega\,t)\,(-vg\,k+\omega)\right]$$

$$B\ field = \left[\left(\frac{\partial}{\partial y}f(x,y)\right)\cos(k\,z+\omega\,t), -\left(\frac{\partial}{\partial x}f(x,y)\right)\cos(k\,z+\omega\,t), 0\right]$$

$$Topological\ Torsion = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = \left[ \varepsilon\, vg \left( \frac{\partial}{\partial x} \mathrm{f}(x, y) \right) \cos(k\, z + \omega\, t),\ \varepsilon\, vg \left( \frac{\partial}{\partial y} \mathrm{f}(x, y) \right) \cos(k\, z + \omega\, t),\ \varepsilon\, \mathrm{f}(x, y) \sin(k\, z + \omega\, t) \left( -vg\, k + \omega \right) \right]$$

$$H\ field = \left[ \frac{\left( \frac{\partial}{\partial y} \mathrm{f}(x, y) \right) \cos(k\, z + \omega\, t)}{\mu},\ -\frac{\left( \frac{\partial}{\partial x} \mathrm{f}(x, y) \right) \cos(k\, z + \omega\, t)}{\mu},\ 0 \right]$$

$$Poynting\ vector\ ExH = \left[ \frac{\mathrm{f}(x, y) \sin(k\, z + \omega\, t) \left( -vg\, k + \omega \right) \left( \frac{\partial}{\partial x} \mathrm{f}(x, y) \right) \cos(k\, z + \omega\, t)}{\mu},\ \frac{\mathrm{f}(x, y) \sin(k\, z + \omega\, t) \left( -vg\, k + \omega \right) \left( \frac{\partial}{\partial y} \mathrm{f}(x, y) \right) \cos(k\, z + \omega\, t)}{\mu},\ -\frac{vg \cos(k\, z + \omega\, t)^2 \left( \left( \frac{\partial}{\partial x} \mathrm{f}(x, y) \right)^2 + \left( \frac{\partial}{\partial y} \mathrm{f}(x, y) \right)^2 \right)}{\mu} \right]$$

$$Current\ density = \left[ \frac{\left( \frac{\partial}{\partial x} \mathrm{f}(x, y) \right) \sin(k\, z + \omega\, t) \left( -k + \varepsilon\, vg\, \omega\, \mu \right)}{\mu},\ \frac{\left( \frac{\partial}{\partial y} \mathrm{f}(x, y) \right) \sin(k\, z + \omega\, t) \left( -k + \varepsilon\, vg\, \omega\, \mu \right)}{\mu},\ -\frac{\cos(k\, z + \omega\, t) \left( \left( \frac{\partial^2}{\partial x^2} \mathrm{f}(x, y) \right) + \left( \frac{\partial^2}{\partial y^2} \mathrm{f}(x, y) \right) - \varepsilon\, \mathrm{f}(x, y)\, \omega\, \mu\, vg\, k + \varepsilon\, \mathrm{f}(x, y)\, \omega^2\, \mu \right)}{\mu} \right]$$

$$charge\ density = \varepsilon \cos(k\, z + \omega\, t) \left( \left( \frac{\partial^2}{\partial x^2} \mathrm{f}(x, y) \right) vg + \left( \frac{\partial^2}{\partial y^2} \mathrm{f}(x, y) \right) vg - \mathrm{f}(x, y)\, k^2\, vg + \mathrm{f}(x, y)\, k\, \omega \right)$$

$$Topological\ SPIN = \left[ -\frac{\mathrm{f}(x, y)\, \%1^2 \left( \frac{\partial}{\partial x} \mathrm{f}(x, y) \right) \left( -1 + \varepsilon\, vg^2\, \mu \right)}{\mu},\ -\frac{\mathrm{f}(x, y)\, \%1^2 \left( \frac{\partial}{\partial y} \mathrm{f}(x, y) \right) \left( -1 + \varepsilon\, vg^2\, \mu \right)}{\mu},\ -\varepsilon\, \mathrm{f}(x, y)^2 \sin(k\, z + \omega\, t) \left( -vg\, k + \omega \right) vg\, \%1,\ \mathrm{f}(x, y)^2\, \%1\, \varepsilon \sin(k\, z + \omega\, t) \left( -vg\, k + \omega \right) \right]$$

$$\%1 := \cos(k\, z + \omega\, t)$$

$$chiralty\ AdotD = \mathrm{f}(x, y)^2 \cos(k\, z + \omega\, t)\, \varepsilon \sin(k\, z + \omega\, t) \left( -vg\, k + \omega \right)$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = \Big( \left( \frac{\partial}{\partial y} \mathrm{f}(x, y) \right)^2 \%1 + \left( \frac{\partial}{\partial x} \mathrm{f}(x, y) \right)^2 \%1 - \varepsilon\, vg^2 \left( \frac{\partial}{\partial x} \mathrm{f}(x, y) \right)^2 \%1\, \mu - \varepsilon\, vg^2 \left( \frac{\partial}{\partial y} \mathrm{f}(x, y) \right)^2 \%1\, \mu - \varepsilon\, \mathrm{f}(x, y)^2\, vg^2\, k^2\, \mu + \varepsilon\, \mathrm{f}(x, y)^2\, vg^2\, k^2\, \mu\, \%1 + 2\, \varepsilon\, \mathrm{f}(x, y)^2\, \omega\, vg\, k\, \mu - 2\, \varepsilon\, \mathrm{f}(x, y)^2\, \omega\, vg\, k\, \mu\, \%1 - \varepsilon\, \mathrm{f}(x, y)^2\, \omega^2\, \mu + \varepsilon\, \mathrm{f}(x, y)^2\, \omega^2\, \mu\, \%1 \Big) / \mu$$

$$\%1 := \cos(k\, z + \omega\, t)^2$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = \mathrm{f}(x, y) \cos(k\, z + \omega\, t)^2 \Big( -\left( \frac{\partial^2}{\partial x^2} \mathrm{f}(x, y) \right) - \left( \frac{\partial^2}{\partial y^2} \mathrm{f}(x, y) \right) + 2\, \varepsilon\, \mathrm{f}(x, y)\, \omega\, \mu\, vg\, k - \varepsilon\, \mathrm{f}(x, y)\, \omega^2\, \mu + \varepsilon\, vg^2\, \mu \left( \frac{\partial^2}{\partial x^2} \mathrm{f}(x, y) \right) + \varepsilon\, vg^2\, \mu \left( \frac{\partial^2}{\partial y^2} \mathrm{f}(x, y) \right) - \varepsilon\, vg^2\, \mu\, \mathrm{f}(x, y)\, k^2 \Big) / \mu$$

$$\mathit{Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi)} = \left(\left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2 \%1 + \left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2 \%1 - \varepsilon\, vg^2 \left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2 \%1\, \mu\right.$$

$$- \varepsilon\, vg^2 \left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2 \%1\, \mu - \varepsilon\, \mathrm{f}(x,y)^2\, vg^2\, k^2\, \mu + 2\, \varepsilon\, \mathrm{f}(x,y)^2\, vg^2\, k^2\, \mu\, \%1 + 2\, \varepsilon\, \mathrm{f}(x,y)^2\, \omega\, vg\, k\, \mu$$

$$- 4\, \varepsilon\, \mathrm{f}(x,y)^2\, \omega\, vg\, k\, \mu\, \%1 - \varepsilon\, \mathrm{f}(x,y)^2\, \omega^2\, \mu + 2\, \varepsilon\, \mathrm{f}(x,y)^2\, \omega^2\, \mu\, \%1 + \mathrm{f}(x,y)\, \%1 \left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right)$$

$$\left. + \mathrm{f}(x,y)\, \%1 \left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right) - \mathrm{f}(x,y)\, \%1\, \varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right) - \mathrm{f}(x,y)\, \%1\, \varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)\right) / \mu$$

$$\%1 := \cos(k\, z + \omega\, t)^2$$

$$\mathit{Virtual\ work} = \left[ -\cos(k\, z + \omega\, t)^2 \left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)\left(-\varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right) - \varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right) + \varepsilon\, vg^2\, \mu\, \mathrm{f}(x,y)\, k^2\right.\right.$$

$$\left. - 2\, \varepsilon\, \mathrm{f}(x,y)\, \omega\, \mu\, vg\, k + \left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right) + \left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right) + \varepsilon\, \mathrm{f}(x,y)\, \omega^2\, \mu\right) / \mu,\ -\cos(k\, z + \omega\, t)^2 \left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)\Bigg($$

$$-\varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right) - \varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right) + \varepsilon\, vg^2\, \mu\, \mathrm{f}(x,y)\, k^2 - 2\, \varepsilon\, \mathrm{f}(x,y)\, \omega\, \mu\, vg\, k + \left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right) + \left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)$$

$$\left. + \varepsilon\, \mathrm{f}(x,y)\, \omega^2\, \mu\right) / \mu,\ \cos(k\, z + \omega\, t)\, \sin(k\, z + \omega\, t) \left(-\varepsilon\, \mathrm{f}(x,y)\, \mu \left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right) vg^2\, k + \varepsilon\, \mathrm{f}(x,y)\, \mu \left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right) vg\, \omega\right.$$

$$- \varepsilon\, \mathrm{f}(x,y)\, \mu \left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right) vg^2\, k + \varepsilon\, \mathrm{f}(x,y)\, \mu \left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right) vg\, \omega + \varepsilon\, \mathrm{f}(x,y)^2\, \mu\, k^3\, vg^2 - 2\, \varepsilon\, \mathrm{f}(x,y)^2\, \mu\, k^2\, vg\, \omega$$

$$\left.\left. + \varepsilon\, \mathrm{f}(x,y)^2\, \mu\, k\, \omega^2 + \left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2 k - \left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2 \varepsilon\, vg\, \omega\, \mu + \left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2 k - \left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2 \varepsilon\, vg\, \omega\, \mu\right) / \mu\right]$$

$$\mathit{JdotE\ power} = -\sin(k\, z + \omega\, t)\, \cos(k\, z + \omega\, t) \left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2 vg\, k - \left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2 vg^2\, \varepsilon\, \omega\, \mu + \left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2 vg\, k\right.$$

$$- \left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2 vg^2\, \varepsilon\, \omega\, \mu - \mathrm{f}(x,y) \left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right) vg\, k + \mathrm{f}(x,y) \left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right) \omega - \mathrm{f}(x,y) \left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right) vg\, k$$

$$\left. + \mathrm{f}(x,y) \left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right) \omega + \varepsilon\, \mathrm{f}(x,y)^2\, \omega\, \mu\, vg^2\, k^2 - 2\, \varepsilon\, \mathrm{f}(x,y)^2\, \omega^2\, \mu\, vg\, k + \varepsilon\, \mathrm{f}(x,y)^2\, \omega^3\, \mu\right) / \mu$$

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 5b-- waveguide TM mode  (group kinematic in, wave out)`;
> theta:=(k*z-omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=-vg*f(x,y)*cos(theta);
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

****************************************************************

$$NAME := \mathit{Example\ 5b\text{-}\text{-}\ waveguide\ TM\ mode\ \ (group\ kinematic\ in,\ wave\ out)}$$

$$\theta := k\, z - \omega\, t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := \mathrm{f}(x,y)\cos(-k\, z + \omega\, t)$$

$$\phi := -vg\, f(x,y)\cos(-k z+\omega t)$$

*Example 5b-- waveguide TM mode (group kinematic in, wave out)*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$\mathit{Action} = f(x,y)\cos(-k z+\omega t)\, d(z) + vg\, f(x,y)\cos(-k z+\omega t)\, d(t)$$

$$\mathit{E\ field} = \left[ vg\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(-k z+\omega t),\ vg\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(-k z+\omega t),\ f(x,y)\sin(-k z+\omega t)(vg\, k+\omega) \right]$$

$$\mathit{B\ field} = \left[ \left(\frac{\partial}{\partial y} f(x,y)\right)\cos(-k z+\omega t),\ -\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(-k z+\omega t),\ 0 \right]$$

$$\mathit{Topological\ Torsion} = [0, 0, 0, 0]$$

$$\mathit{Helicity\ AdotB} = 0$$

$$\mathit{Poincare\ 2\ E.B} = 0$$

$$\mathit{D\ field} = \left[ \varepsilon\, vg\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(-k z+\omega t),\ \varepsilon\, vg\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(-k z+\omega t),\ \varepsilon\, f(x,y)\sin(-k z+\omega t)(vg\, k+\omega) \right]$$

$$\mathit{H\ field} = \left[ \frac{\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(-k z+\omega t)}{\mu},\ -\frac{\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(-k z+\omega t)}{\mu},\ 0 \right]$$

$$\mathit{Poynting\ vector\ ExH} = \left[ \frac{f(x,y)\sin(-k z+\omega t)(vg\, k+\omega)\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(-k z+\omega t)}{\mu},\right.$$
$$\left. \frac{f(x,y)\sin(-k z+\omega t)(vg\, k+\omega)\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(-k z+\omega t)}{\mu},\ -\frac{vg\cos(-k z+\omega t)^2\left(\left(\frac{\partial}{\partial x} f(x,y)\right)^2+\left(\frac{\partial}{\partial y} f(x,y)\right)^2\right)}{\mu} \right]$$

$$\mathit{Current\ density} = \left[ \frac{\left(\frac{\partial}{\partial x} f(x,y)\right)\sin(-k z+\omega t)(k+\varepsilon\, vg\, \omega\, \mu)}{\mu},\ \frac{\left(\frac{\partial}{\partial y} f(x,y)\right)\sin(-k z+\omega t)(k+\varepsilon\, vg\, \omega\, \mu)}{\mu},\right.$$
$$\left. -\frac{\cos(-k z+\omega t)\left(\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)+\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)+\varepsilon\, f(x,y)\,\omega\,\mu\, vg\, k+\varepsilon\, f(x,y)\,\omega^2\mu\right)}{\mu} \right]$$

$$\mathit{charge\ density} = -\varepsilon\cos(-k z+\omega t)\left(-\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)vg-\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)vg+f(x,y)\,k^2\, vg+f(x,y)\,k\,\omega\right)$$

$$\mathit{Topological\ SPIN} = \left[ -\frac{f(x,y)\,\%1^2\left(\frac{\partial}{\partial x} f(x,y)\right)(-1+\varepsilon\, vg^2\mu)}{\mu},\ -\frac{f(x,y)\,\%1^2\left(\frac{\partial}{\partial y} f(x,y)\right)(-1+\varepsilon\, vg^2\mu)}{\mu},\right.$$
$$\left. -\varepsilon\, f(x,y)^2\sin(-k z+\omega t)(vg\, k+\omega)\, vg\, \%1,\ f(x,y)^2\,\%1\,\varepsilon\sin(-k z+\omega t)(vg\, k+\omega) \right]$$

$$\%1 := \cos(-k z+\omega t)$$

$$\mathit{chiralty\ AdotD} = f(x,y)^2\cos(-k z+\omega t)\,\varepsilon\sin(-k z+\omega t)(vg\, k+\omega)$$

$$\mathit{LaGrange\ field\ energy\ density\ (B.H\text{-}D.E)} = \Big( \left(\frac{\partial}{\partial y} f(x,y)\right)^2 \%1 + \left(\frac{\partial}{\partial x} f(x,y)\right)^2 \%1 - \varepsilon\, vg^2 \left(\frac{\partial}{\partial x} f(x,y)\right)^2 \%1\, \mu - \varepsilon\, vg^2 \left(\frac{\partial}{\partial y} f(x,y)\right)^2 \%1\, \mu - \varepsilon\, f(x,y)^2\, vg^2\, k^2\, \mu + \varepsilon\, f(x,y)^2\, vg^2\, k^2\, \mu\, \%1 - 2\, \varepsilon\, f(x,y)^2\, \omega\, vg\, k\, \mu + 2\, \varepsilon\, f(x,y)^2\, \omega\, vg\, k\, \mu\, \%1 - \varepsilon\, f(x,y)^2\, \omega^2\, \mu + \varepsilon\, f(x,y)^2\, \omega^2\, \mu\, \%1 \Big) / \mu$$

$$\%1 := \cos(-k\, z + \omega\, t)^2$$

$$\mathit{Interaction\ energy\ density\ (A.J\text{-}rho.phi)} = -f(x,y)\cos(-k\, z + \omega\, t)^2 \Big( -\varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) - \varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + \varepsilon\, vg^2\, \mu\, f(x,y)\, k^2 + 2\, \varepsilon\, f(x,y)\, \omega\, \mu\, vg\, k + \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) + \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + \varepsilon\, f(x,y)\, \omega^2\, \mu \Big) / \mu$$

$$\mathit{Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi)} = \Big( \left(\frac{\partial}{\partial y} f(x,y)\right)^2 \%1 + \left(\frac{\partial}{\partial x} f(x,y)\right)^2 \%1 - \varepsilon\, vg^2 \left(\frac{\partial}{\partial x} f(x,y)\right)^2 \%1\, \mu - \varepsilon\, vg^2 \left(\frac{\partial}{\partial y} f(x,y)\right)^2 \%1\, \mu - \varepsilon\, f(x,y)^2\, vg^2\, k^2\, \mu + 2\, \varepsilon\, f(x,y)^2\, vg^2\, k^2\, \mu\, \%1 - 2\, \varepsilon\, f(x,y)^2\, \omega\, vg\, k\, \mu + 4\, \varepsilon\, f(x,y)^2\, \omega\, vg\, k\, \mu\, \%1 - \varepsilon\, f(x,y)^2\, \omega^2\, \mu + 2\, \varepsilon\, f(x,y)^2\, \omega^2\, \mu\, \%1 - f(x,y)\, \%1\, \varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) - f(x,y)\, \%1\, \varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + f(x,y)\, \%1 \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) + f(x,y)\, \%1 \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) \Big) / \mu$$

$$\%1 := \cos(-k\, z + \omega\, t)^2$$

$$\mathit{Virtual\ work} = \Big[ -\cos(-k\, z + \omega\, t)^2 \left(\frac{\partial}{\partial x} f(x,y)\right) \Big( -\varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) - \varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + \varepsilon\, vg^2\, \mu\, f(x,y)\, k^2 + 2\, \varepsilon\, f(x,y)\, \omega\, \mu\, vg\, k + \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) + \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + \varepsilon\, f(x,y)\, \omega^2\, \mu \Big) / \mu,\ -\cos(-k\, z + \omega\, t)^2 \left(\frac{\partial}{\partial y} f(x,y)\right) \Big( -\varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) - \varepsilon\, vg^2\, \mu \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + \varepsilon\, vg^2\, \mu\, f(x,y)\, k^2 + 2\, \varepsilon\, f(x,y)\, \omega\, \mu\, vg\, k + \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) + \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + \varepsilon\, f(x,y)\, \omega^2\, \mu \Big) / \mu,\ -\cos(-k\, z + \omega\, t)\sin(-k\, z + \omega\, t) \Big( -\varepsilon\, f(x,y)\, \mu \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) vg^2\, k - \varepsilon\, f(x,y)\, \mu \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) vg\, \omega - \varepsilon\, f(x,y)\, \mu \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) vg^2\, k - \varepsilon\, f(x,y)\, \mu \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) vg\, \omega + \varepsilon\, f(x,y)^2\, \mu\, k^3\, vg^2 + 2\, \varepsilon\, f(x,y)^2\, \mu\, k^2\, vg\, \omega + \varepsilon\, f(x,y)^2\, \mu\, k\, \omega^2 + \left(\frac{\partial}{\partial x} f(x,y)\right)^2 k + \left(\frac{\partial}{\partial x} f(x,y)\right)^2 \varepsilon\, vg\, \omega\, \mu + \left(\frac{\partial}{\partial y} f(x,y)\right)^2 k + \left(\frac{\partial}{\partial y} f(x,y)\right)^2 \varepsilon\, vg\, \omega\, \mu \Big) / \mu \Big]$$

$$\mathit{JdotE\ power} = -\sin(-k\, z + \omega\, t)\cos(-k\, z + \omega\, t) \Big( -\left(\frac{\partial}{\partial x} f(x,y)\right)^2 vg\, k - \left(\frac{\partial}{\partial x} f(x,y)\right)^2 vg^2\, \varepsilon\, \omega\, \mu - \left(\frac{\partial}{\partial y} f(x,y)\right)^2 vg\, k - \left(\frac{\partial}{\partial y} f(x,y)\right)^2 vg^2\, \varepsilon\, \omega\, \mu + f(x,y) \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) vg\, k + f(x,y) \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) \omega + f(x,y) \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) vg\, k$$

$$+ f(x,y)\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)\omega + \varepsilon\, f(x,y)^2\, \omega\, \mu\, vg^2\, k^2 + 2\,\varepsilon\, f(x,y)^2\, \omega^2\, \mu\, vg\, k + \varepsilon\, f(x,y)^2\, \omega^3\, \mu \Bigg) / \mu$$

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 5c-- waveguide TM mode phi (group kinematic out, wave in)`;
> theta:=(k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=+vg*f(x,y)*cos(theta);
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*********************************************************

*NAME* := *Example 5c-- waveguide TM mode phi (group kinematic out, wave in)*

$$\theta := k\, z + \omega\, t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := f(x,y)\cos(k\, z + \omega\, t)$$

$$\phi := vg\, f(x,y)\cos(k\, z + \omega\, t)$$

*Example 5c-- waveguide TM mode phi (group kinematic out, wave in)*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = f(x,y)\cos(k\, z + \omega\, t)\, d(z) - vg\, f(x,y)\cos(k\, z + \omega\, t)\, d(t)$$

$$E\ field = \left[-vg\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(k\, z + \omega\, t), -vg\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(k\, z + \omega\, t), f(x,y)\sin(k\, z + \omega\, t)\,(vg\, k + \omega)\right]$$

$$B\ field = \left[\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(k\, z + \omega\, t), -\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(k\, z + \omega\, t), 0\right]$$

$$Topological\ Torsion = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = \left[-\varepsilon\, vg\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(k\, z + \omega\, t), -\varepsilon\, vg\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(k\, z + \omega\, t), \varepsilon\, f(x,y)\sin(k\, z + \omega\, t)\,(vg\, k + \omega)\right]$$

$$H\ field = \left[\frac{\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(k\, z + \omega\, t)}{\mu}, -\frac{\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(k\, z + \omega\, t)}{\mu}, 0\right]$$

$$Poynting\ vector\ ExH = \left[\frac{f(x,y)\sin(k\, z + \omega\, t)\,(vg\, k + \omega)\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(k\, z + \omega\, t)}{\mu},\right.$$

$$\left.\frac{f(x,y)\sin(k\, z + \omega\, t)\,(vg\, k + \omega)\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(k\, z + \omega\, t)}{\mu}, \frac{vg\cos(k\, z + \omega\, t)^2\left(\left(\frac{\partial}{\partial x} f(x,y)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y)\right)^2\right)}{\mu}\right]$$

$$Current\ density = \left[-\frac{\left(\frac{\partial}{\partial x} f(x,y)\right)\sin(k\, z + \omega\, t)\,(k + \varepsilon\, vg\, \omega\, \mu)}{\mu}, -\frac{\left(\frac{\partial}{\partial y} f(x,y)\right)\sin(k\, z + \omega\, t)\,(k + \varepsilon\, vg\, \omega\, \mu)}{\mu},\right.$$

$$-\frac{\cos(k z+\omega t)\left(\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)+\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)+\varepsilon f(x,y)\,\omega\,\mu\, vg\, k+\varepsilon f(x,y)\,\omega^2\mu\right)}{\mu}\Bigg]$$

$$\mathit{charge\ density}=\varepsilon\cos(k z+\omega t)\left(-\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)vg-\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)vg+f(x,y)\,k^2\,vg+f(x,y)\,k\,\omega\right)$$

$$\mathit{Topological\ SPIN}=\left[-\frac{f(x,y)\,\%1^2\left(\frac{\partial}{\partial x} f(x,y)\right)(-1+\varepsilon\, vg^2\mu)}{\mu},-\frac{f(x,y)\,\%1^2\left(\frac{\partial}{\partial y} f(x,y)\right)(-1+\varepsilon\, vg^2\mu)}{\mu},\right.$$

$$\left.\varepsilon f(x,y)^2\sin(k z+\omega t)\,(vg\,k+\omega)\,vg\,\%1,\ f(x,y)^2\,\%1\,\varepsilon\sin(k z+\omega t)\,(vg\,k+\omega)\right]$$

$$\%1:=\cos(k z+\omega t)$$

$$\mathit{chiralty\ AdotD}=f(x,y)^2\cos(k z+\omega t)\,\varepsilon\sin(k z+\omega t)\,(vg\,k+\omega)$$

$$\mathit{LaGrange\ field\ energy\ density\ (B.H\text{-}D.E)}=-\left(-\%1\left(\frac{\partial}{\partial x} f(x,y)\right)^2-\%1\left(\frac{\partial}{\partial y} f(x,y)\right)^2+\varepsilon\, vg^2\left(\frac{\partial}{\partial x} f(x,y)\right)^2\%1\,\mu\right.$$

$$+\varepsilon\, vg^2\left(\frac{\partial}{\partial y} f(x,y)\right)^2\%1\,\mu+\varepsilon f(x,y)^2\,vg^2\,k^2\mu-\varepsilon f(x,y)^2\,vg^2\,k^2\mu\,\%1+2\,\varepsilon f(x,y)^2\,\omega\, vg\, k\,\mu$$

$$\left.-2\,\varepsilon f(x,y)^2\,\omega\, vg\, k\,\mu\,\%1+\varepsilon f(x,y)^2\,\omega^2\mu-\varepsilon f(x,y)^2\,\omega^2\mu\,\%1\right)/\mu$$

$$\%1:=\cos(k z+\omega t)^2$$

$$\mathit{Interaction\ energy\ density\ (A.J\text{-}rho.phi)}=-f(x,y)\cos(k z+\omega t)^2\left(-\varepsilon\, vg^2\mu\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)-\varepsilon\, vg^2\mu\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)\right.$$

$$\left.+\varepsilon\, vg^2\mu\, f(x,y)\,k^2+2\,\varepsilon f(x,y)\,\omega\,\mu\, vg\, k+\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)+\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)+\varepsilon f(x,y)\,\omega^2\mu\right)/\mu$$

$$\mathit{Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi)}=-\left(-\%1\left(\frac{\partial}{\partial x} f(x,y)\right)^2-\%1\left(\frac{\partial}{\partial y} f(x,y)\right)^2+\varepsilon\, vg^2\left(\frac{\partial}{\partial x} f(x,y)\right)^2\%1\,\mu\right.$$

$$+\varepsilon\, vg^2\left(\frac{\partial}{\partial y} f(x,y)\right)^2\%1\,\mu+\varepsilon f(x,y)^2\,vg^2\,k^2\mu-2\,\varepsilon f(x,y)^2\,vg^2\,k^2\mu\,\%1+2\,\varepsilon f(x,y)^2\,\omega\, vg\, k\,\mu$$

$$-4\,\varepsilon f(x,y)^2\,\omega\, vg\, k\,\mu\,\%1+\varepsilon f(x,y)^2\,\omega^2\mu-2\,\varepsilon f(x,y)^2\,\omega^2\mu\,\%1+f(x,y)\,\%1\,\varepsilon\, vg^2\mu\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)$$

$$\left.+f(x,y)\,\%1\,\varepsilon\, vg^2\mu\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)-f(x,y)\,\%1\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)-f(x,y)\,\%1\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)\right)/\mu$$

$$\%1:=\cos(k z+\omega t)^2$$

$$\mathit{Virtual\ work}=\left[-\cos(k z+\omega t)^2\left(\frac{\partial}{\partial x} f(x,y)\right)\left(-\varepsilon\, vg^2\mu\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)-\varepsilon\, vg^2\mu\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)+\varepsilon\, vg^2\mu\, f(x,y)\,k^2\right.\right.$$

$$\left.+2\,\varepsilon f(x,y)\,\omega\,\mu\, vg\, k+\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)+\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)+\varepsilon f(x,y)\,\omega^2\mu\right)/\mu,\ -\cos(k z+\omega t)^2\left(\frac{\partial}{\partial y} f(x,y)\right)\Bigg($$

$$-\varepsilon\, vg^2\mu\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)-\varepsilon\, vg^2\mu\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)+\varepsilon\, vg^2\mu\, f(x,y)\,k^2+2\,\varepsilon f(x,y)\,\omega\,\mu\, vg\, k+\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)+\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)$$

$$+\varepsilon f(x,y)\,\omega^2\,\mu\Big)/\mu,\ \cos(kz+\omega t)\sin(kz+\omega t)\Big(-\varepsilon f(x,y)\,\mu\left(\frac{\partial^2}{\partial x^2}f(x,y)\right)vg^2 k-\varepsilon f(x,y)\,\mu\left(\frac{\partial^2}{\partial x^2}f(x,y)\right)vg\,\omega$$

$$-\varepsilon f(x,y)\,\mu\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)vg^2 k-\varepsilon f(x,y)\,\mu\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)vg\,\omega+\varepsilon f(x,y)^2\,\mu\,k^3\,vg^2+2\,\varepsilon f(x,y)^2\,\mu\,k^2\,vg\,\omega$$

$$+\varepsilon f(x,y)^2\,\mu\,k\,\omega^2+\left(\frac{\partial}{\partial x}f(x,y)\right)^2 k+\left(\frac{\partial}{\partial x}f(x,y)\right)^2\varepsilon\,vg\,\omega\,\mu+\left(\frac{\partial}{\partial y}f(x,y)\right)^2 k+\left(\frac{\partial}{\partial y}f(x,y)\right)^2\varepsilon\,vg\,\omega\,\mu\Big)/\mu\Big]$$

$$JdotE\ power=\sin(kz+\omega t)\cos(kz+\omega t)\Big(\left(\frac{\partial}{\partial x}f(x,y)\right)^2 vg\,k+\left(\frac{\partial}{\partial x}f(x,y)\right)^2 vg^2\,\varepsilon\,\omega\,\mu+\left(\frac{\partial}{\partial y}f(x,y)\right)^2 vg\,k$$

$$+\left(\frac{\partial}{\partial y}f(x,y)\right)^2 vg^2\,\varepsilon\,\omega\,\mu-f(x,y)\left(\frac{\partial^2}{\partial x^2}f(x,y)\right)vg\,k-f(x,y)\left(\frac{\partial^2}{\partial x^2}f(x,y)\right)\omega-f(x,y)\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)vg\,k$$

$$-f(x,y)\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)\omega-\varepsilon f(x,y)^2\,\omega\,\mu\,vg^2\,k^2-2\,\varepsilon f(x,y)^2\,\omega^2\,\mu\,vg\,k-\varepsilon f(x,y)^2\,\omega^3\,\mu\Big)/\mu$$

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 5d -- waveguide TM mode (group kinematic out, wave out)`;
> theta:=(k*z-omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=+vg*f(x,y)*cos(theta);
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*************************************************************

*NAME := Example 5d -- waveguide TM mode (group kinematic out, wave out)*

$$\theta := kz-\omega t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := f(x,y)\cos(-kz+\omega t)$$

$$\phi := vg\,f(x,y)\cos(-kz+\omega t)$$

*Example 5d -- waveguide TM mode (group kinematic out, wave out)*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action=f(x,y)\cos(-kz+\omega t)\,d(z)-vg\,f(x,y)\cos(-kz+\omega t)\,d(t)$$

$$E\ field=\left[-vg\left(\frac{\partial}{\partial x}f(x,y)\right)\cos(-kz+\omega t),-vg\left(\frac{\partial}{\partial y}f(x,y)\right)\cos(-kz+\omega t),f(x,y)\sin(-kz+\omega t)(-vg\,k+\omega)\right]$$

$$B\ field=\left[\left(\frac{\partial}{\partial y}f(x,y)\right)\cos(-kz+\omega t),-\left(\frac{\partial}{\partial x}f(x,y)\right)\cos(-kz+\omega t),0\right]$$

$$Topological\ Torsion=[0,0,0,0]$$

$$Helicity\ AdotB=0$$

$$Poincare\ 2\ E.B=0$$

$$D\ field=$$

$$\left[-\varepsilon\,vg\left(\frac{\partial}{\partial x}f(x,y)\right)\cos(-kz+\omega t),-\varepsilon\,vg\left(\frac{\partial}{\partial y}f(x,y)\right)\cos(-kz+\omega t),\varepsilon f(x,y)\sin(-kz+\omega t)(-vg\,k+\omega)\right]$$

$$H\ field=\left[\frac{\left(\frac{\partial}{\partial y}f(x,y)\right)\cos(-kz+\omega t)}{\mu},-\frac{\left(\frac{\partial}{\partial x}f(x,y)\right)\cos(-kz+\omega t)}{\mu},0\right]$$

$$\textit{Poynting vector ExH} = \left[ \frac{f(x,y)\sin(-k z+\omega t)(-vg\, k+\omega)\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(-k z+\omega t)}{\mu}, \frac{f(x,y)\sin(-k z+\omega t)(-vg\, k+\omega)\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(-k z+\omega t)}{\mu}, \frac{vg\cos(-k z+\omega t)^2\left(\left(\frac{\partial}{\partial x} f(x,y)\right)^2+\left(\frac{\partial}{\partial y} f(x,y)\right)^2\right)}{\mu}\right]$$

$$\textit{Current density} = \left[ -\frac{\left(\frac{\partial}{\partial x} f(x,y)\right)\sin(-k z+\omega t)(-k+\varepsilon\, vg\, \omega\, \mu)}{\mu}, -\frac{\left(\frac{\partial}{\partial y} f(x,y)\right)\sin(-k z+\omega t)(-k+\varepsilon\, vg\, \omega\, \mu)}{\mu}, -\frac{\cos(-k z+\omega t)\left(\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)+\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)-\varepsilon\, f(x,y)\,\omega\,\mu\, vg\, k+\varepsilon\, f(x,y)\,\omega^2\mu\right)}{\mu}\right]$$

$$\textit{charge density} = -\varepsilon\cos(-k z+\omega t)\left(\left(\frac{\partial^2}{\partial x^2} f(x,y)\right) vg+\left(\frac{\partial^2}{\partial y^2} f(x,y)\right) vg - f(x,y)\, k^2\, vg + f(x,y)\, k\,\omega\right)$$

$$\textit{Topological SPIN} = \left[ -\frac{f(x,y)\,\%1^2\left(\frac{\partial}{\partial x} f(x,y)\right)(-1+\varepsilon\, vg^2\mu)}{\mu}, -\frac{f(x,y)\,\%1^2\left(\frac{\partial}{\partial y} f(x,y)\right)(-1+\varepsilon\, vg^2\mu)}{\mu}, \varepsilon\, f(x,y)^2\sin(-k z+\omega t)(-vg\, k+\omega)\, vg\,\%1, f(x,y)^2\,\%1\,\varepsilon\sin(-k z+\omega t)(-vg\, k+\omega)\right]$$

$$\%1 := \cos(-k z+\omega t)$$

$$\textit{chiralty AdotD} = f(x,y)^2\cos(-k z+\omega t)\,\varepsilon\sin(-k z+\omega t)(-vg\, k+\omega)$$

$$\textit{LaGrange field energy density (B.H-D.E)} = \left(\left(\frac{\partial}{\partial y} f(x,y)\right)^2\%1+\left(\frac{\partial}{\partial x} f(x,y)\right)^2\%1-\varepsilon\, vg^2\left(\frac{\partial}{\partial x} f(x,y)\right)^2\%1\,\mu - \varepsilon\, vg^2\left(\frac{\partial}{\partial y} f(x,y)\right)^2\%1\,\mu+2\,\varepsilon\, f(x,y)^2\,\omega\, vg\, k\,\mu-2\,\varepsilon\, f(x,y)^2\,\omega\, vg\, k\,\mu\,\%1-\varepsilon\, f(x,y)^2\,\omega^2\mu+\varepsilon\, f(x,y)^2\,\omega^2\mu\,\%1 - \varepsilon\, f(x,y)^2\, vg^2\, k^2\,\mu+\varepsilon\, f(x,y)^2\, vg^2\, k^2\,\mu\,\%1\right)/\mu$$

$$\%1 := \cos(-k z+\omega t)^2$$

$$\textit{Interaction energy density (A.J-rho.phi)} = -f(x,y)\cos(-k z+\omega t)^2\left(-\varepsilon\, vg^2\mu\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)-\varepsilon\, vg^2\mu\left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + \varepsilon\, vg^2\mu\, f(x,y)\, k^2-2\,\varepsilon\, f(x,y)\,\omega\,\mu\, vg\, k+\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)+\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)+\varepsilon\, f(x,y)\,\omega^2\mu\right)/\mu$$

$$\textit{Poincare 1 (B.H-D.E)-(A.J-rho.phi)} = \left(\left(\frac{\partial}{\partial y} f(x,y)\right)^2\%1+\left(\frac{\partial}{\partial x} f(x,y)\right)^2\%1-\varepsilon\, vg^2\left(\frac{\partial}{\partial x} f(x,y)\right)^2\%1\,\mu - \varepsilon\, vg^2\left(\frac{\partial}{\partial y} f(x,y)\right)^2\%1\,\mu+2\,\varepsilon\, f(x,y)^2\,\omega\, vg\, k\,\mu-4\,\varepsilon\, f(x,y)^2\,\omega\, vg\, k\,\mu\,\%1-\varepsilon\, f(x,y)^2\,\omega^2\mu+2\,\varepsilon\, f(x,y)^2\,\omega^2\mu\,\%1 - \varepsilon\, f(x,y)^2\, vg^2\, k^2\,\mu+2\,\varepsilon\, f(x,y)^2\, vg^2\, k^2\,\mu\,\%1+f(x,y)\,\%1\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)+f(x,y)\,\%1\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)$$

$$- f(x,y)\,\%1\,\varepsilon\,vg^2\,\mu\left(\frac{\partial^2}{\partial x^2}f(x,y)\right) - f(x,y)\,\%1\,\varepsilon\,vg^2\,\mu\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)\Bigg)\Bigg/\mu$$

$$\%1 := \cos(-k\,z+\omega\,t)^2$$

$$Virtual\ work\ = \Bigg[-\cos(-k\,z+\omega\,t)^2\left(\frac{\partial}{\partial x}f(x,y)\right)\Bigg(-\varepsilon\,vg^2\,\mu\left(\frac{\partial^2}{\partial x^2}f(x,y)\right) - \varepsilon\,vg^2\,\mu\left(\frac{\partial^2}{\partial y^2}f(x,y)\right) + \varepsilon\,vg^2\,\mu\,f(x,y)\,k^2 - 2\,\varepsilon\,f(x,y)\,\omega\,\mu\,vg\,k + \left(\frac{\partial^2}{\partial x^2}f(x,y)\right) + \left(\frac{\partial^2}{\partial y^2}f(x,y)\right) + \varepsilon\,f(x,y)\,\omega^2\,\mu\Bigg)\Bigg/\mu,\ -\cos(-k\,z+\omega\,t)^2\left(\frac{\partial}{\partial y}f(x,y)\right)\Bigg(-\varepsilon\,vg^2\,\mu\left(\frac{\partial^2}{\partial x^2}f(x,y)\right) - \varepsilon\,vg^2\,\mu\left(\frac{\partial^2}{\partial y^2}f(x,y)\right) + \varepsilon\,vg^2\,\mu\,f(x,y)\,k^2 - 2\,\varepsilon\,f(x,y)\,\omega\,\mu\,vg\,k + \left(\frac{\partial^2}{\partial x^2}f(x,y)\right) + \left(\frac{\partial^2}{\partial y^2}f(x,y)\right) + \varepsilon\,f(x,y)\,\omega^2\,\mu\Bigg)\Bigg/\mu,\ \cos(-k\,z+\omega\,t)\sin(-k\,z+\omega\,t)\Bigg(\varepsilon\,f(x,y)\,\mu\left(\frac{\partial^2}{\partial x^2}f(x,y)\right)vg^2\,k - \varepsilon\,f(x,y)\,\mu\left(\frac{\partial^2}{\partial x^2}f(x,y)\right)vg\,\omega + \varepsilon\,f(x,y)\,\mu\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)vg^2\,k - \varepsilon\,f(x,y)\,\mu\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)vg\,\omega - \varepsilon\,f(x,y)^2\,\mu\,k^3\,vg^2 + 2\,\varepsilon\,f(x,y)^2\,\mu\,k^2\,vg\,\omega - \varepsilon\,f(x,y)^2\,\mu\,k\,\omega^2 - \left(\frac{\partial}{\partial x}f(x,y)\right)^2 k + \left(\frac{\partial}{\partial x}f(x,y)\right)^2\varepsilon\,vg\,\omega\,\mu - \left(\frac{\partial}{\partial y}f(x,y)\right)^2 k + \left(\frac{\partial}{\partial y}f(x,y)\right)^2\varepsilon\,vg\,\omega\,\mu\Bigg)\Bigg/\mu\Bigg]$$

$$JdotE\ power\ = \sin(-k\,z+\omega\,t)\cos(-k\,z+\omega\,t)\Bigg(-\left(\frac{\partial}{\partial x}f(x,y)\right)^2 vg\,k + \left(\frac{\partial}{\partial x}f(x,y)\right)^2 vg^2\,\varepsilon\,\omega\,\mu - \left(\frac{\partial}{\partial y}f(x,y)\right)^2 vg\,k + \left(\frac{\partial}{\partial y}f(x,y)\right)^2 vg^2\,\varepsilon\,\omega\,\mu + f(x,y)\left(\frac{\partial^2}{\partial x^2}f(x,y)\right)vg\,k - f(x,y)\left(\frac{\partial^2}{\partial x^2}f(x,y)\right)\omega + f(x,y)\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)vg\,k - f(x,y)\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)\omega - \varepsilon\,f(x,y)^2\,\omega\,\mu\,vg^2\,k^2 + 2\,\varepsilon\,f(x,y)^2\,\omega^2\,\mu\,vg\,k - \varepsilon\,f(x,y)^2\,\omega^3\,\mu\Bigg)\Bigg/\mu$$

\>

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 6a-- Wave guide TTM (kinematic in, wave in)`;
> theta:=(k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=-(omega/k)*f(x,y)*cos(theta);
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*****************************************************************

*NAME := Example 6a-- Wave guide TTM (kinematic in, wave in)*

$$\theta := k\,z+\omega\,t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := f(x,y)\cos(k\,z+\omega\,t)$$

$$\phi := -\frac{\omega\,f(x,y)\cos(k\,z+\omega\,t)}{k}$$

*Example 6a-- Wave guide TTM (kinematic in, wave in)*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = f(x,y)\cos(kz+\omega t)\,d(z) + \frac{\omega\, f(x,y)\cos(kz+\omega t)\,d(t)}{k}$$

$$E\ field = \left[\frac{\omega\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(kz+\omega t)}{k}, \frac{\omega\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(kz+\omega t)}{k}, 0\right]$$

$$B\ field = \left[\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(kz+\omega t), -\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(kz+\omega t), 0\right]$$

$$Topological\ Torsion = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = \left[\frac{\varepsilon\,\omega\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(kz+\omega t)}{k}, \frac{\varepsilon\,\omega\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(kz+\omega t)}{k}, 0\right]$$

$$H\ field = \left[\frac{\left(\frac{\partial}{\partial y} f(x,y)\right)\cos(kz+\omega t)}{\mu}, -\frac{\left(\frac{\partial}{\partial x} f(x,y)\right)\cos(kz+\omega t)}{\mu}, 0\right]$$

$$Poynting\ vector\ ExH = \left[0, 0, -\frac{\omega\cos(kz+\omega t)^2\left(\left(\frac{\partial}{\partial x} f(x,y)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y)\right)^2\right)}{k\,\mu}\right]$$

$$Current\ density = \left[\frac{\left(\frac{\partial}{\partial x} f(x,y)\right)\sin(kz+\omega t)(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu\,k}, \frac{\left(\frac{\partial}{\partial y} f(x,y)\right)\sin(kz+\omega t)(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu\,k},\right.$$
$$\left. -\frac{\cos(kz+\omega t)\left(\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)+\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)\right)}{\mu}\right]$$

$$charge\ density = \frac{\varepsilon\,\omega\cos(kz+\omega t)\left(\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)+\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)\right)}{k}$$

$$Topological\ SPIN =$$
$$\left[-\frac{f(x,y)\cos(kz+\omega t)^2\left(\frac{\partial}{\partial x} f(x,y)\right)(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu\,k^2}, -\frac{f(x,y)\cos(kz+\omega t)^2\left(\frac{\partial}{\partial y} f(x,y)\right)(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu\,k^2}, 0, 0\right]$$

$$chiralty\ AdotD = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{\cos(kz+\omega t)^2\left(\left(\frac{\partial}{\partial x} f(x,y)\right)^2+\left(\frac{\partial}{\partial y} f(x,y)\right)^2\right)(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu\,k^2}$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = \frac{f(x,y)\cos(kz+\omega t)^2\left(\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)+\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)\right)(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu\,k^2}$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) =$$

$$-\frac{\cos(kz+\omega t)^2(-k^2+\varepsilon\omega^2\mu)\left(\left(\frac{\partial}{\partial x}f(x,y)\right)^2+\left(\frac{\partial}{\partial y}f(x,y)\right)^2+f(x,y)\left(\frac{\partial^2}{\partial x^2}f(x,y)\right)+f(x,y)\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)\right)}{\mu k^2}$$

$$\textit{Virtual work} = \left[\frac{\cos(kz+\omega t)^2\left(\left(\frac{\partial^2}{\partial x^2}f(x,y)\right)+\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)\right)\left(\frac{\partial}{\partial x}f(x,y)\right)(-k^2+\varepsilon\omega^2\mu)}{\mu k^2},\right.$$
$$\frac{\cos(kz+\omega t)^2\left(\left(\frac{\partial^2}{\partial x^2}f(x,y)\right)+\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)\right)\left(\frac{\partial}{\partial y}f(x,y)\right)(-k^2+\varepsilon\omega^2\mu)}{\mu k^2},$$
$$\left.-\frac{\sin(kz+\omega t)(-k^2+\varepsilon\omega^2\mu)\cos(kz+\omega t)\left(\left(\frac{\partial}{\partial x}f(x,y)\right)^2+\left(\frac{\partial}{\partial y}f(x,y)\right)^2\right)}{\mu k}\right]$$

$$\textit{JdotE power} = \frac{\sin(kz+\omega t)(-k^2+\varepsilon\omega^2\mu)\,\omega\cos(kz+\omega t)\left(\left(\frac{\partial}{\partial x}f(x,y)\right)^2+\left(\frac{\partial}{\partial y}f(x,y)\right)^2\right)}{\mu k^2}$$

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 6b-- Wave guide TTE (kinematic in, wave out)`;\
> theta:=(k*z-omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=-(omega/k)*f(x,y)*cos(theta);
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*************************************************************

*NAME := Example 6b-- Wave guide TTE (kinematic in, wave out)*

$$\theta := kz-\omega t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := f(x,y)\cos(-kz+\omega t)$$

$$\phi := -\frac{\omega f(x,y)\cos(-kz+\omega t)}{k}$$

*Example 6b-- Wave guide TTE (kinematic in, wave out)*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$\textit{Action} = f(x,y)\cos(-kz+\omega t)\,d(z)+\frac{\omega f(x,y)\cos(-kz+\omega t)\,d(t)}{k}$$

$$\textit{E field} = \left[\frac{\omega\left(\frac{\partial}{\partial x}f(x,y)\right)\cos(-kz+\omega t)}{k}, \frac{\omega\left(\frac{\partial}{\partial y}f(x,y)\right)\cos(-kz+\omega t)}{k}, 2\omega f(x,y)\sin(-kz+\omega t)\right]$$

$$\textit{B field} = \left[\left(\frac{\partial}{\partial y}f(x,y)\right)\cos(-kz+\omega t), -\left(\frac{\partial}{\partial x}f(x,y)\right)\cos(-kz+\omega t), 0\right]$$

$$\textit{Topological Torsion} = [0, 0, 0, 0]$$

$$\textit{Helicity AdotB} = 0$$

$$\textit{Poincare 2 E.B} = 0$$

$$D\ field = \left[ \frac{\varepsilon\,\omega \left(\frac{\partial}{\partial x} f(x,y)\right) \cos(-k\,z+\omega\,t)}{k}, \frac{\varepsilon\,\omega \left(\frac{\partial}{\partial y} f(x,y)\right) \cos(-k\,z+\omega\,t)}{k}, 2\,\varepsilon\,\omega\, f(x,y) \sin(-k\,z+\omega\,t) \right]$$

$$H\ field = \left[ \frac{\left(\frac{\partial}{\partial y} f(x,y)\right) \cos(-k\,z+\omega\,t)}{\mu}, -\frac{\left(\frac{\partial}{\partial x} f(x,y)\right) \cos(-k\,z+\omega\,t)}{\mu}, 0 \right]$$

$$Poynting\ vector\ ExH = \left[ 2\,\frac{\omega\, f(x,y) \sin(-k\,z+\omega\,t) \left(\frac{\partial}{\partial x} f(x,y)\right) \cos(-k\,z+\omega\,t)}{\mu}, \right.$$
$$\left. 2\,\frac{\omega\, f(x,y) \sin(-k\,z+\omega\,t) \left(\frac{\partial}{\partial y} f(x,y)\right) \cos(-k\,z+\omega\,t)}{\mu}, -\frac{\omega \cos(-k\,z+\omega\,t)^2 \left(\left(\frac{\partial}{\partial x} f(x,y)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y)\right)^2\right)}{k\,\mu} \right]$$

$$Current\ density = \left[ \frac{\left(\frac{\partial}{\partial x} f(x,y)\right) \sin(-k\,z+\omega\,t)\,(k^2+\varepsilon\,\omega^2\,\mu)}{k\,\mu}, \frac{\left(\frac{\partial}{\partial y} f(x,y)\right) \sin(-k\,z+\omega\,t)\,(k^2+\varepsilon\,\omega^2\,\mu)}{k\,\mu}, \right.$$
$$\left. -\frac{\cos(-k\,z+\omega\,t)\left(\left(\frac{\partial^2}{\partial x^2} f(x,y)\right) + \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + 2\,\varepsilon\,\omega^2\, f(x,y)\,\mu\right)}{\mu} \right]$$

$$charge\ density = -\frac{\varepsilon\,\omega \cos(-k\,z+\omega\,t)\left(-\left(\frac{\partial^2}{\partial x^2} f(x,y)\right) - \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + 2\, f(x,y)\,k^2\right)}{k}$$

$$Topological\ SPIN = \left[ -\frac{f(x,y)\,\%1^2 \left(\frac{\partial}{\partial x} f(x,y)\right)(-k^2+\varepsilon\,\omega^2\,\mu)}{k^2\,\mu}, -\frac{f(x,y)\,\%1^2 \left(\frac{\partial}{\partial y} f(x,y)\right)(-k^2+\varepsilon\,\omega^2\,\mu)}{k^2\,\mu}, \right.$$
$$\left. -2\,\frac{\varepsilon\,\omega^2\, f(x,y)^2 \sin(-k\,z+\omega\,t)\,\%1}{k}, 2\, f(x,y)^2\,\%1\,\varepsilon\,\omega \sin(-k\,z+\omega\,t) \right]$$

$$\%1 := \cos(-k\,z+\omega\,t)$$

$$chiralty\ AdotD = 2\, f(x,y)^2 \cos(-k\,z+\omega\,t)\,\varepsilon\,\omega \sin(-k\,z+\omega\,t)$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\left(-\%1\,k^2 \left(\frac{\partial}{\partial y} f(x,y)\right)^2 - \%1\,k^2 \left(\frac{\partial}{\partial x} f(x,y)\right)^2 + \varepsilon\,\omega^2\,\mu \left(\frac{\partial}{\partial x} f(x,y)\right)^2 \%1 \right.$$
$$\left. + \varepsilon\,\omega^2\,\mu \left(\frac{\partial}{\partial y} f(x,y)\right)^2 \%1 + 4\, f(x,y)^2\,k^2\,\varepsilon\,\omega^2\,\mu - 4\, f(x,y)^2\,k^2\,\varepsilon\,\omega^2\,\mu\,\%1 \right) \Big/ (k^2\,\mu)$$

$$\%1 := \cos(-k\,z+\omega\,t)^2$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = -f(x,y)\cos(-k\,z+\omega\,t)^2$$
$$\left(-\varepsilon\,\omega^2\,\mu \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) - \varepsilon\,\omega^2\,\mu \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + 4\,\varepsilon\,\omega^2\,\mu\, f(x,y)\,k^2 + k^2 \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) + k^2 \left(\frac{\partial^2}{\partial y^2} f(x,y)\right)\right) \Big/ (k^2\,\mu)$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = -\left(-\%1\,k^2 \left(\frac{\partial}{\partial y} f(x,y)\right)^2 - \%1\,k^2 \left(\frac{\partial}{\partial x} f(x,y)\right)^2 + \varepsilon\,\omega^2\,\mu \left(\frac{\partial}{\partial x} f(x,y)\right)^2 \%1 \right.$$

$$+\varepsilon\,\omega^2\,\mu\left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2\%1+4\,\mathrm{f}(x,y)^2\,k^2\,\varepsilon\,\omega^2\,\mu-8\,\mathrm{f}(x,y)^2\,k^2\,\varepsilon\,\omega^2\,\mu\,\%1+\mathrm{f}(x,y)\,\%1\,\varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right)$$

$$+\mathrm{f}(x,y)\,\%1\,\varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)-\mathrm{f}(x,y)\,\%1\,k^2\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right)-\mathrm{f}(x,y)\,\%1\,k^2\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)\Bigg)\Bigg/(k^2\,\mu)$$

$$\%1:=\cos(-k\,z+\omega\,t)^2$$

$$\mathit{Virtual\ work}\ =\Bigg[-\cos(-k\,z+\omega\,t)^2\left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)$$

$$\left(-\varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right)-\varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)+4\,\varepsilon\,\omega^2\,\mu\,\mathrm{f}(x,y)\,k^2+k^2\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right)+k^2\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)\right)\Bigg/(k^2\,\mu),-$$

$$\cos(-k\,z+\omega\,t)^2\left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)$$

$$\left(-\varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right)-\varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)+4\,\varepsilon\,\omega^2\,\mu\,\mathrm{f}(x,y)\,k^2+k^2\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right)+k^2\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)\right)\Bigg/(k^2\,\mu),-$$

$$\cos(-k\,z+\omega\,t)\sin(-k\,z+\omega\,t)\Bigg(-2\,\varepsilon\,\omega^2\,\mathrm{f}(x,y)\,\mu\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right)-2\,\varepsilon\,\omega^2\,\mathrm{f}(x,y)\,\mu\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)+4\,\mathrm{f}(x,y)^2\,k^2\,\varepsilon\,\omega^2\,\mu$$

$$+\left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2k^2+\left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2\varepsilon\,\omega^2\,\mu+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2k^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2\varepsilon\,\omega^2\,\mu\Bigg)/(k\,\mu)\Bigg]$$

$$\mathit{JdotE\ power}\ =\sin(-k\,z+\omega\,t)\,\omega\cos(-k\,z+\omega\,t)\Bigg(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2k^2+\left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2\varepsilon\,\omega^2\,\mu+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2k^2$$

$$+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2\varepsilon\,\omega^2\,\mu-2\,\mathrm{f}(x,y)\,k^2\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right)-2\,\mathrm{f}(x,y)\,k^2\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)-4\,\mathrm{f}(x,y)^2\,k^2\,\varepsilon\,\omega^2\,\mu\Bigg)\Bigg/(k^2\,\mu)$$

\>

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 6c-- Wave guide TTM (kinematic out, wave in)`;
> theta:=(k*z+omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=+(omega/k)*f(x,y)*cos(theta);
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*************************************************************

*NAME := Example 6c-- Wave guide TTM (kinematic out, wave in)*

$$\theta:=k\,z+\omega\,t$$

$$Ax:=0$$

$$Ay:=0$$

$$Az:=\mathrm{f}(x,y)\cos(k\,z+\omega\,t)$$

$$\phi:=\frac{\omega\,\mathrm{f}(x,y)\cos(k\,z+\omega\,t)}{k}$$

*Example 6c-- Wave guide TTM (kinematic out, wave in)*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$\mathit{Action}=\mathrm{f}(x,y)\cos(k\,z+\omega\,t)\,\mathrm{d}(z)-\frac{\omega\,\mathrm{f}(x,y)\cos(k\,z+\omega\,t)\,\mathrm{d}(t)}{k}$$

$$\mathit{E\ field} = \left[ -\frac{\omega \left(\frac{\partial}{\partial x} f(x,y)\right) \cos(k\,z + \omega\,t)}{k}, -\frac{\omega \left(\frac{\partial}{\partial y} f(x,y)\right) \cos(k\,z + \omega\,t)}{k}, 2\,\omega\, f(x,y) \sin(k\,z + \omega\,t) \right]$$

$$\mathit{B\ field} = \left[ \left(\frac{\partial}{\partial y} f(x,y)\right) \cos(k\,z + \omega\,t), -\left(\frac{\partial}{\partial x} f(x,y)\right) \cos(k\,z + \omega\,t), 0 \right]$$

$$\mathit{Topological\ Torsion} = [0, 0, 0, 0]$$

$$\mathit{Helicity\ AdotB} = 0$$

$$\mathit{Poincare\ 2\ E.B} = 0$$

$$\mathit{D\ field} = \left[ -\frac{\varepsilon\,\omega \left(\frac{\partial}{\partial x} f(x,y)\right) \cos(k\,z + \omega\,t)}{k}, -\frac{\varepsilon\,\omega \left(\frac{\partial}{\partial y} f(x,y)\right) \cos(k\,z + \omega\,t)}{k}, 2\,\varepsilon\,\omega\, f(x,y) \sin(k\,z + \omega\,t) \right]$$

$$\mathit{H\ field} = \left[ \frac{\left(\frac{\partial}{\partial y} f(x,y)\right) \cos(k\,z + \omega\,t)}{\mu}, -\frac{\left(\frac{\partial}{\partial x} f(x,y)\right) \cos(k\,z + \omega\,t)}{\mu}, 0 \right]$$

$$\mathit{Poynting\ vector\ ExH} = \left[ 2\,\frac{\omega\, f(x,y) \sin(k\,z + \omega\,t) \left(\frac{\partial}{\partial x} f(x,y)\right) \cos(k\,z + \omega\,t)}{\mu}, \right.$$

$$\left. 2\,\frac{\omega\, f(x,y) \sin(k\,z + \omega\,t) \left(\frac{\partial}{\partial y} f(x,y)\right) \cos(k\,z + \omega\,t)}{\mu}, \frac{\omega \cos(k\,z + \omega\,t)^2 \left( \left(\frac{\partial}{\partial x} f(x,y)\right)^2 + \left(\frac{\partial}{\partial y} f(x,y)\right)^2 \right)}{k\,\mu} \right]$$

$$\mathit{Current\ density} = \left[ -\frac{\left(\frac{\partial}{\partial x} f(x,y)\right) \sin(k\,z + \omega\,t)\,(k^2 + \varepsilon\,\omega^2\,\mu)}{\mu\,k}, -\frac{\left(\frac{\partial}{\partial y} f(x,y)\right) \sin(k\,z + \omega\,t)\,(k^2 + \varepsilon\,\omega^2\,\mu)}{\mu\,k}, \right.$$

$$\left. -\frac{\cos(k\,z + \omega\,t) \left( \left(\frac{\partial^2}{\partial x^2} f(x,y)\right) + \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + 2\,\varepsilon\,\omega^2\, f(x,y)\,\mu \right)}{\mu} \right]$$

$$\mathit{charge\ density} = \frac{\varepsilon\,\omega \cos(k\,z + \omega\,t) \left( -\left(\frac{\partial^2}{\partial x^2} f(x,y)\right) - \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + 2\, f(x,y)\, k^2 \right)}{k}$$

$$\mathit{Topological\ SPIN} = \left[ -\frac{f(x,y)\,\%1^2 \left(\frac{\partial}{\partial x} f(x,y)\right) (-k^2 + \varepsilon\,\omega^2\,\mu)}{\mu\,k^2}, -\frac{f(x,y)\,\%1^2 \left(\frac{\partial}{\partial y} f(x,y)\right) (-k^2 + \varepsilon\,\omega^2\,\mu)}{\mu\,k^2}, \right.$$

$$\left. 2\,\frac{\varepsilon\,\omega^2\, f(x,y)^2 \sin(k\,z + \omega\,t)\,\%1}{k}, 2\, f(x,y)^2\,\%1\,\varepsilon\,\omega \sin(k\,z + \omega\,t) \right]$$

$$\%1 := \cos(k\,z + \omega\,t)$$

$$\mathit{chiralty\ AdotD} = 2\, f(x,y)^2 \cos(k\,z + \omega\,t)\,\varepsilon\,\omega \sin(k\,z + \omega\,t)$$

$$\mathit{LaGrange\ field\ energy\ density\ (B.H\text{-}D.E)} = \left( \%1\, k^2 \left(\frac{\partial}{\partial y} f(x,y)\right)^2 + \%1\, k^2 \left(\frac{\partial}{\partial x} f(x,y)\right)^2 - \varepsilon\,\omega^2\,\mu \left(\frac{\partial}{\partial x} f(x,y)\right)^2 \%1\right.$$

$$- \varepsilon\,\omega^2\,\mu \left(\frac{\partial}{\partial y} f(x,y)\right)^2 \%1 - 4\,f(x,y)^2\,k^2\,\varepsilon\,\omega^2\,\mu + 4\,\varepsilon\,\omega^2\,f(x,y)^2\,\mu\,k^2\,\%1 \Bigg) \Big/ (\mu\,k^2)$$

$$\%1 := \cos(k\,z + \omega\,t)^2$$

*Interaction energy density (A.J-rho.phi)* $= - f(x,y)\cos(k\,z+\omega\,t)^2$

$$\left(-\varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial x^2} f(x,y)\right) - \varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + 4\,\varepsilon\,\omega^2\,\mu\,f(x,y)\,k^2 + k^2\left(\frac{\partial^2}{\partial x^2} f(x,y)\right) + k^2\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)\right) \Big/ (\mu\,k^2)$$

*Poincare 1 (B.H-D.E)-(A.J-rho.phi)* $= \Bigg(\%1\,k^2\left(\frac{\partial}{\partial y} f(x,y)\right)^2 + \%1\,k^2\left(\frac{\partial}{\partial x} f(x,y)\right)^2 - \varepsilon\,\omega^2\,\mu\left(\frac{\partial}{\partial x} f(x,y)\right)^2 \%1$

$$- \varepsilon\,\omega^2\,\mu\left(\frac{\partial}{\partial y} f(x,y)\right)^2 \%1 - 4\,f(x,y)^2\,k^2\,\varepsilon\,\omega^2\,\mu + 8\,\varepsilon\,\omega^2\,f(x,y)^2\,\mu\,k^2\,\%1 - f(x,y)\,\%1\,\varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial x^2} f(x,y)\right)$$

$$- f(x,y)\,\%1\,\varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + f(x,y)\,\%1\,k^2\left(\frac{\partial^2}{\partial x^2} f(x,y)\right) + f(x,y)\,\%1\,k^2\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)\Bigg) \Big/ (\mu\,k^2)$$

$$\%1 := \cos(k\,z + \omega\,t)^2$$

*Virtual work* $= \Bigg[ -\cos(k\,z+\omega\,t)^2\left(\frac{\partial}{\partial x} f(x,y)\right)$

$$\left(-\varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial x^2} f(x,y)\right) - \varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + 4\,\varepsilon\,\omega^2\,\mu\,f(x,y)\,k^2 + k^2\left(\frac{\partial^2}{\partial x^2} f(x,y)\right) + k^2\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)\right) \Big/ (\mu\,k^2), -$$

$$\cos(k\,z+\omega\,t)^2\left(\frac{\partial}{\partial y} f(x,y)\right)$$

$$\left(-\varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial x^2} f(x,y)\right) - \varepsilon\,\omega^2\,\mu\left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + 4\,\varepsilon\,\omega^2\,\mu\,f(x,y)\,k^2 + k^2\left(\frac{\partial^2}{\partial x^2} f(x,y)\right) + k^2\left(\frac{\partial^2}{\partial y^2} f(x,y)\right)\right) \Big/ (\mu\,k^2),$$

$$\cos(k\,z+\omega\,t)\sin(k\,z+\omega\,t)\Bigg(-2\,\varepsilon\,\omega^2\,f(x,y)\,\mu\left(\frac{\partial^2}{\partial x^2} f(x,y)\right) - 2\,\varepsilon\,\omega^2\,f(x,y)\,\mu\left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + 4\,f(x,y)^2\,k^2\,\varepsilon\,\omega^2\,\mu$$

$$+ \left(\frac{\partial}{\partial x} f(x,y)\right)^2 k^2 + \left(\frac{\partial}{\partial x} f(x,y)\right)^2 \varepsilon\,\omega^2\,\mu + \left(\frac{\partial}{\partial y} f(x,y)\right)^2 k^2 + \left(\frac{\partial}{\partial y} f(x,y)\right)^2 \varepsilon\,\omega^2\,\mu\Bigg) / (\mu\,k)\Bigg]$$

*JdotE power* $= -\sin(k\,z+\omega\,t)\,\omega\cos(k\,z+\omega\,t)\Bigg(-\left(\frac{\partial}{\partial x} f(x,y)\right)^2 k^2 - \left(\frac{\partial}{\partial x} f(x,y)\right)^2 \varepsilon\,\omega^2\,\mu - \left(\frac{\partial}{\partial y} f(x,y)\right)^2 k^2$

$$-\left(\frac{\partial}{\partial y} f(x,y)\right)^2 \varepsilon\,\omega^2\,\mu + 2\,f(x,y)\,k^2\left(\frac{\partial^2}{\partial x^2} f(x,y)\right) + 2\,f(x,y)\,k^2\left(\frac{\partial^2}{\partial y^2} f(x,y)\right) + 4\,f(x,y)^2\,k^2\,\varepsilon\,\omega^2\,\mu\Bigg) \Big/ (\mu\,k^2)$$

**Enter the name of the problem, and the components of the 4 potential.**

```
> NAME:=`Example 6d-- Wave guide TTM (kinematic out, wave out)`;
> theta:=(k*z-omega*t);
> Ax:=0;Ay:=0;Az:=f(x,y)*cos(theta);phi:=+(omega/k)*f(x,y)*cos(theta);
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*****************************************************************

*NAME* := *Example 6d-- Wave guide TTM (kinematic out, wave out)*

$$\theta := k\,z - \omega\,t$$

$$Ax := 0$$

$$Ay := 0$$

$$Az := f(x, y) \cos(-k z + \omega t)$$

$$\phi := \frac{\omega f(x, y) \cos(-k z + \omega t)}{k}$$

*Example 6d-- Wave guide TTM (kinematic out, wave out)*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = f(x, y) \cos(-k z + \omega t)\, d(z) - \frac{\omega f(x, y) \cos(-k z + \omega t)\, d(t)}{k}$$

$$E\ field = \left[ -\frac{\omega \left(\frac{\partial}{\partial x} f(x, y)\right) \cos(-k z + \omega t)}{k}, -\frac{\omega \left(\frac{\partial}{\partial y} f(x, y)\right) \cos(-k z + \omega t)}{k}, 0 \right]$$

$$B\ field = \left[ \left(\frac{\partial}{\partial y} f(x, y)\right) \cos(-k z + \omega t), -\left(\frac{\partial}{\partial x} f(x, y)\right) \cos(-k z + \omega t), 0 \right]$$

$$Topological\ Torsion = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = \left[ -\frac{\varepsilon \omega \left(\frac{\partial}{\partial x} f(x, y)\right) \cos(-k z + \omega t)}{k}, -\frac{\varepsilon \omega \left(\frac{\partial}{\partial y} f(x, y)\right) \cos(-k z + \omega t)}{k}, 0 \right]$$

$$H\ field = \left[ \frac{\left(\frac{\partial}{\partial y} f(x, y)\right) \cos(-k z + \omega t)}{\mu}, -\frac{\left(\frac{\partial}{\partial x} f(x, y)\right) \cos(-k z + \omega t)}{\mu}, 0 \right]$$

$$Poynting\ vector\ ExH = \left[ 0, 0, \frac{\omega \cos(-k z + \omega t)^2 \left( \left(\frac{\partial}{\partial x} f(x, y)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y)\right)^2 \right)}{k \mu} \right]$$

$$Current\ density = \left[ -\frac{\left(\frac{\partial}{\partial x} f(x, y)\right) \sin(-k z + \omega t) (-k^2 + \varepsilon \omega^2 \mu)}{\mu k}, -\frac{\left(\frac{\partial}{\partial y} f(x, y)\right) \sin(-k z + \omega t) (-k^2 + \varepsilon \omega^2 \mu)}{\mu k}, -\frac{\cos(-k z + \omega t) \left( \left(\frac{\partial^2}{\partial x^2} f(x, y)\right) + \left(\frac{\partial^2}{\partial y^2} f(x, y)\right) \right)}{\mu} \right]$$

$$charge\ density = -\frac{\varepsilon \omega \cos(-k z + \omega t) \left( \left(\frac{\partial^2}{\partial x^2} f(x, y)\right) + \left(\frac{\partial^2}{\partial y^2} f(x, y)\right) \right)}{k}$$

$$Topological\ SPIN = \left[ -\frac{f(x, y) \cos(-k z + \omega t)^2 \left(\frac{\partial}{\partial x} f(x, y)\right) (-k^2 + \varepsilon \omega^2 \mu)}{\mu k^2}, -\frac{f(x, y) \cos(-k z + \omega t)^2 \left(\frac{\partial}{\partial y} f(x, y)\right) (-k^2 + \varepsilon \omega^2 \mu)}{\mu k^2}, 0, 0 \right]$$

$$chiralty\ AdotD = 0$$

$$\textit{LaGrange field energy density (B.H-D.E)} = -\frac{\cos(-k\,z+\omega\,t)^2\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2\right)(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu\,k^2}$$

$$\textit{Interaction energy density (A.J-rho.phi)} = \frac{\mathrm{f}(x,y)\cos(-k\,z+\omega\,t)^2\left(\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right)+\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)\right)(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu\,k^2}$$

*Poincare 1 (B.H-D.E)-(A.J-rho.phi) =*

$$-\frac{\cos(-k\,z+\omega\,t)^2(-k^2+\varepsilon\,\omega^2\,\mu)\left(\left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2+\left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2+\mathrm{f}(x,y)\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right)+\mathrm{f}(x,y)\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)\right)}{\mu\,k^2}$$

$$\textit{Virtual work} = \left[\frac{\cos(-k\,z+\omega\,t)^2\left(\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right)+\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)\right)\left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu\,k^2},\right.$$

$$\frac{\cos(-k\,z+\omega\,t)^2\left(\left(\frac{\partial^2}{\partial x^2}\mathrm{f}(x,y)\right)+\left(\frac{\partial^2}{\partial y^2}\mathrm{f}(x,y)\right)\right)\left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)(-k^2+\varepsilon\,\omega^2\,\mu)}{\mu\,k^2},$$

$$\left.\frac{\sin(-k\,z+\omega\,t)(-k^2+\varepsilon\,\omega^2\,\mu)\cos(-k\,z+\omega\,t)\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2\right)}{\mu\,k}\right]$$

$$\textit{JdotE power} = \frac{\sin(-k\,z+\omega\,t)(-k^2+\varepsilon\,\omega^2\,\mu)\,\omega\cos(-k\,z+\omega\,t)\left(\left(\frac{\partial}{\partial x}\mathrm{f}(x,y)\right)^2+\left(\frac{\partial}{\partial y}\mathrm{f}(x,y)\right)^2\right)}{\mu\,k^2}$$

**Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.**

```
> NAME:=`Example 7a = 8a-- Index 1 Irreversible solution EdotB < 0 (kinematic out)
  Type 1`;
> ff:=1;p:=2;n:=4;
> lambda:=(x^p+y^p+z^p-(c*t)^p)^(n/p);
> Ax:=y*ff/lambda;Ay:=-x*ff/lambda;Az:=c*t*ff/lambda;phi:=+z*c*ff/lambda;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

***************************************************************

*NAME := Example 7a = 8a-- Index 1 Irreversible solution EdotB < 0 (kinematic out) Type 1*

$$ff := 1$$

$$p := 2$$

$$n := 4$$

$$\lambda := (x^2+y^2+z^2-c^2\,t^2)^2$$

$$Ax := \frac{y}{(x^2+y^2+z^2-c^2\,t^2)^2}$$

$$Ay := -\frac{x}{(x^2+y^2+z^2-c^2 t^2)^2}$$

$$Az := \frac{c\,t}{(x^2+y^2+z^2-c^2 t^2)^2}$$

$$\phi := \frac{z\,c}{(x^2+y^2+z^2-c^2 t^2)^2}$$

*Example 7a = 8a-- Index 1 Irreversible solution EdotB < 0 (kinematic out) Type 1*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \frac{y\,\mathrm{d}(x)}{\%1^2} - \frac{x\,\mathrm{d}(y)}{\%1^2} + \frac{c\,t\,\mathrm{d}(z)}{\%1^2} - \frac{z\,c\,\mathrm{d}(t)}{\%1^2}$$

$$\%1 := x^2+y^2+z^2-c^2 t^2$$

$$E\ field = \left[4\frac{c\,(z\,x-y\,c\,t)}{(x^2+y^2+z^2-c^2 t^2)^3}, 4\frac{c\,(z\,y+x\,c\,t)}{(x^2+y^2+z^2-c^2 t^2)^3}, -2\frac{c\,(x^2+y^2-z^2+c^2 t^2)}{(x^2+y^2+z^2-c^2 t^2)^3}\right]$$

$$B\ field = \left[-4\frac{y\,c\,t+z\,x}{(x^2+y^2+z^2-c^2 t^2)^3}, 4\frac{-z\,y+x\,c\,t}{(x^2+y^2+z^2-c^2 t^2)^3}, 2\frac{x^2+y^2-z^2+c^2 t^2}{(x^2+y^2+z^2-c^2 t^2)^3}\right]$$

$$Topological\ Torsion = \left[2\frac{c\,x}{\%1^4}, 2\frac{c\,y}{\%1^4}, 2\frac{c\,z}{\%1^4}, 2\frac{c\,t}{\%1^4}\right]$$

$$\%1 := x^2+y^2+z^2-c^2 t^2$$

$$Helicity\ AdotB = 2\frac{c\,t}{(x^2+y^2+z^2-c^2 t^2)^4}$$

$$Poincare\ 2\ E.B = -8\frac{c}{(x^2+y^2+z^2-c^2 t^2)^4}$$

$$D\ field = \left[4\frac{\varepsilon\,c\,(z\,x-y\,c\,t)}{(x^2+y^2+z^2-c^2 t^2)^3}, 4\frac{\varepsilon\,c\,(z\,y+x\,c\,t)}{(x^2+y^2+z^2-c^2 t^2)^3}, -2\frac{\varepsilon\,c\,(x^2+y^2-z^2+c^2 t^2)}{(x^2+y^2+z^2-c^2 t^2)^3}\right]$$

$$H\ field = \left[-4\frac{y\,c\,t+z\,x}{(x^2+y^2+z^2-c^2 t^2)^3\,\mu}, 4\frac{-z\,y+x\,c\,t}{(x^2+y^2+z^2-c^2 t^2)^3\,\mu}, 2\frac{x^2+y^2-z^2+c^2 t^2}{(x^2+y^2+z^2-c^2 t^2)^3\,\mu}\right]$$

$$Poynting\ vector\ ExH = \left[16\frac{c^2\,(x^2+y^2-z^2+c^2 t^2)\,x\,t}{(x^2+y^2+z^2-c^2 t^2)^6\,\mu}, 16\frac{c^2\,(x^2+y^2-z^2+c^2 t^2)\,y\,t}{(x^2+y^2+z^2-c^2 t^2)^6\,\mu}, 32\frac{c^2\,z\,t\,(y^2+x^2)}{(x^2+y^2+z^2-c^2 t^2)^6\,\mu}\right]$$

$$Current\ density = \left[4\frac{(y\,x^2+y^3+y\,z^2+5\,y\,c^2 t^2-6\,c\,t\,z\,x)\,(-1+\varepsilon\,c^2\,\mu)}{(x^2+y^2+z^2-c^2 t^2)^4\,\mu},\right.$$

$$\left. -4\frac{(x^3+x\,y^2+x\,z^2+5\,c^2 t^2\,x+6\,c\,t\,z\,y)\,(-1+\varepsilon\,c^2\,\mu)}{(x^2+y^2+z^2-c^2 t^2)^4\,\mu}, 8\frac{c\,t\,(2\,x^2+2\,y^2-z^2+c^2 t^2)\,(-1+\varepsilon\,c^2\,\mu)}{(x^2+y^2+z^2-c^2 t^2)^4\,\mu}\right]$$

$$charge\ density = 0$$

$$Topological\ SPIN = \left[2\frac{-x^3-x\,y^2+x\,z^2-3\,c^2 t^2\,x+2\,c\,t\,z\,y+2\,\varepsilon\,c^2 z^2\,\mu\,x-2\,\varepsilon\,c^3\,z\,\mu\,y\,t}{\%1^5\,\mu},\right.$$

$$2\frac{-3\,y\,c^2 t^2-2\,c\,t\,z\,x-y\,x^2-y^3+y\,z^2+2\,\varepsilon\,c^2 z^2\,\mu\,y+2\,\varepsilon\,c^3\,z\,\mu\,x\,t}{\%1^5\,\mu},$$

$$-2\frac{z\,(2y^2+2x^2+\varepsilon c^2\mu x^2+\varepsilon c^2\mu y^2-\varepsilon c^2 z^2\mu+\varepsilon c^4\mu t^2)}{\%1^5\,\mu},\ -2\frac{\varepsilon c^2 t\,(3y^2+3x^2-z^2+c^2t^2)}{\%1^5}\Bigg]$$

$\%1 := x^2+y^2+z^2-c^2t^2$

$$\textit{chiralty AdotD} = -2\frac{\varepsilon c^2 t\,(3y^2+3x^2-z^2+c^2t^2)}{(x^2+y^2+z^2-c^2t^2)^5}$$

*LaGrange field energy density (B.H-D.E)* =

$$-4\frac{(6y^2c^2t^2+2z^2x^2+2z^2y^2+6x^2c^2t^2+x^4+2x^2y^2+y^4+z^4-2z^2c^2t^2+c^4t^4)(-1+\varepsilon c^2\mu)}{(x^2+y^2+z^2-c^2t^2)^6\,\mu}$$

*Interaction energy density (A.J-rho.phi)* =

$$4\frac{(-1+\varepsilon c^2\mu)(2x^2y^2+y^4+z^2y^2+9y^2c^2t^2+x^4+z^2x^2+9x^2c^2t^2-2z^2c^2t^2+2c^4t^4)}{(x^2+y^2+z^2-c^2t^2)^6\,\mu}$$

*Poincare 1 (B.H-D.E)-(A.J-rho.phi)* =

$$-4\frac{(-1+\varepsilon c^2\mu)(15y^2c^2t^2+3z^2x^2+3z^2y^2+15x^2c^2t^2+2x^4+4x^2y^2+2y^4+z^4-4z^2c^2t^2+3c^4t^4)}{(x^2+y^2+z^2-c^2t^2)^6\,\mu}$$

$$\textit{Virtual work} = \Big[-8(-1+\varepsilon c^2\mu)(x^5+2x^3y^2+14x^3c^2t^2+xy^4+14xy^2c^2t^2-xz^4-8xz^2c^2t^2+9c^4t^4x$$
$$-2ctzyx^2-2ctzy^3-2ctz^3y+2c^3t^3zy)\big/((x^2+y^2+z^2-c^2t^2)^7\mu),\ -8(-1+\varepsilon c^2\mu)(14c^2t^2x^2y+2ctx^3z$$
$$+14c^2t^2y^3+2cty^2zx-8c^2t^2z^2y+2ctz^3x+9c^4t^4y-2c^3t^3zx+yx^4+2y^3x^2+y^5-yz^4)\big/($$
$$(x^2+y^2+z^2-c^2t^2)^7\mu),\ -16\frac{(-1+\varepsilon c^2\mu)\,z\,(y^2+x^2)(x^2+y^2+11c^2t^2+z^2)}{(x^2+y^2+z^2-c^2t^2)^7\,\mu}\Big]$$

*JdotE power* =

$$-16\frac{(-1+\varepsilon c^2\mu)c^2t\,(6x^2y^2+4z^2y^2+4z^2x^2-2z^2c^2t^2+8x^2c^2t^2+z^4+c^4t^4+3y^4+8y^2c^2t^2+3x^4)}{(x^2+y^2+z^2-c^2t^2)^7\,\mu}$$

**Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.**

```
> NAME:=`Example 7b-- Index 1 Irreversible solution EdotB < 0 t (kinematic in) t
  goes to -t in coefficients, not dt`;
> ff:=1;p:=2;n:=4;
> lambda:=(x^p+y^p+z^p-(c*t)^p)^(n/p);
> Ax:=y*ff/lambda;Ay:=-x*ff/lambda;Az:=-c*t*ff/lambda;phi:=+z*c*ff/lambda;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

************************************************************

*NAME := Example 7b-- Index 1 Irreversible solution EdotB < 0 t (kinematic in) t goes to -t in coefficients, not dt*

$$ff := 1$$

$$p := 2$$

$$n := 4$$

$$\lambda := (x^2+y^2+z^2-c^2t^2)^2$$

$$Ax := \frac{y}{(x^2+y^2+z^2-c^2t^2)^2}$$

$$Ay := -\frac{x}{(x^2+y^2+z^2-c^2 t^2)^2}$$

$$Az := -\frac{c\,t}{(x^2+y^2+z^2-c^2 t^2)^2}$$

$$\phi := \frac{z\,c}{(x^2+y^2+z^2-c^2 t^2)^2}$$

*Example 7b-- Index 1 Irreversible solution EdotB < 0 t (kinematic in) t goes to -t in coefficients, not dt*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \frac{y\,\mathrm{d}(x)}{\%1^2} - \frac{x\,\mathrm{d}(y)}{\%1^2} - \frac{c\,t\,\mathrm{d}(z)}{\%1^2} - \frac{z\,c\,\mathrm{d}(t)}{\%1^2}$$

$$\%1 := x^2+y^2+z^2-c^2 t^2$$

$$E\ field = \left[4\frac{c\,(z\,x-y\,c\,t)}{(x^2+y^2+z^2-c^2 t^2)^3}, 4\frac{c\,(z\,y+x\,c\,t)}{(x^2+y^2+z^2-c^2 t^2)^3}, 4\frac{c\,(z^2+c^2 t^2)}{(x^2+y^2+z^2-c^2 t^2)^3}\right]$$

$$B\ field = \left[-4\frac{z\,x-y\,c\,t}{(x^2+y^2+z^2-c^2 t^2)^3}, -4\frac{z\,y+x\,c\,t}{(x^2+y^2+z^2-c^2 t^2)^3}, 2\frac{x^2+y^2-z^2+c^2 t^2}{(x^2+y^2+z^2-c^2 t^2)^3}\right]$$

$$Topological\ Torsion = \left[0, 0, 2\frac{c\,z}{(x^2+y^2+z^2-c^2 t^2)^4}, -2\frac{c\,t}{(x^2+y^2+z^2-c^2 t^2)^4}\right]$$

$$Helicity\ AdotB = -2\frac{c\,t}{(x^2+y^2+z^2-c^2 t^2)^4}$$

$$Poincare\ 2\ E.B = -16\frac{c\,(z^2+c^2 t^2)}{(x^2+y^2+z^2-c^2 t^2)^5}$$

$$D\ field = \left[4\frac{\varepsilon\,c\,(z\,x-y\,c\,t)}{(x^2+y^2+z^2-c^2 t^2)^3}, 4\frac{\varepsilon\,c\,(z\,y+x\,c\,t)}{(x^2+y^2+z^2-c^2 t^2)^3}, 4\frac{\varepsilon\,c\,(z^2+c^2 t^2)}{(x^2+y^2+z^2-c^2 t^2)^3}\right]$$

$$H\ field = \left[-4\frac{z\,x-y\,c\,t}{(x^2+y^2+z^2-c^2 t^2)^3\,\mu}, -4\frac{z\,y+x\,c\,t}{(x^2+y^2+z^2-c^2 t^2)^3\,\mu}, 2\frac{x^2+y^2-z^2+c^2 t^2}{(x^2+y^2+z^2-c^2 t^2)^3\,\mu}\right]$$

$$Poynting\ vector\ ExH = \left[8\frac{c\,(z\,y+x\,c\,t)\,(x^2+y^2+z^2+3\,c^2 t^2)}{(x^2+y^2+z^2-c^2 t^2)^6\,\mu}, -8\frac{c\,(z\,x-y\,c\,t)\,(x^2+y^2+z^2+3\,c^2 t^2)}{(x^2+y^2+z^2-c^2 t^2)^6\,\mu}, 0\right]$$

$$Current\ density = \Bigg[$$

$$4\frac{-y\,x^2-y^3-y\,z^2-5\,y\,c^2 t^2-6\,c\,t\,z\,x+\varepsilon\,c^2\,\mu\,y\,x^2+\varepsilon\,c^2\,\mu\,y^3+\varepsilon\,c^2 z^2\,\mu\,y+5\,\varepsilon\,c^4\,\mu\,y\,t^2-6\,\varepsilon\,c^3\,z\,\mu\,x\,t}{(x^2+y^2+z^2-c^2 t^2)^4\,\mu},$$

$$-4\frac{-x^3-x\,y^2-x\,z^2-5\,c^2 t^2\,x+6\,c\,t\,z\,y+\varepsilon\,c^2\,\mu\,x^3+\varepsilon\,c^2\,\mu\,x\,y^2+\varepsilon\,c^2 z^2\,\mu\,x+5\,\varepsilon\,c^4\,\mu\,t^2\,x+6\,\varepsilon\,c^3\,z\,\mu\,y\,t}{(x^2+y^2+z^2-c^2 t^2)^4\,\mu},$$

$$-8\frac{c\,t\,(-2\,x^2-2\,y^2+z^2-c^2 t^2+\varepsilon\,c^2\,\mu\,x^2+\varepsilon\,c^2\,\mu\,y^2+4\,\varepsilon\,c^2 z^2\,\mu+2\,\varepsilon\,c^4\,\mu\,t^2)}{(x^2+y^2+z^2-c^2 t^2)^4\,\mu}\Bigg]$$

$$charge\ density = -8\frac{\varepsilon\,c\,z\,(x^2+y^2+z^2+5\,c^2 t^2)}{(x^2+y^2+z^2-c^2 t^2)^4}$$

$$Topological\ SPIN = \left[ 2\,\frac{-x^3 - x\,y^2 + x\,z^2 - 3\,c^2\,t^2\,x - 2\,c\,t\,z\,y + 2\,\varepsilon\,c^2\,z^2\,\mu\,x - 2\,\varepsilon\,c^3\,z\,\mu\,y\,t}{\%1^5\,\mu}, \right.$$

$$2\,\frac{2\,c\,t\,z\,x - 3\,y\,c^2\,t^2 - y\,x^2 - y^3 + y\,z^2 + 2\,\varepsilon\,c^2\,z^2\,\mu\,y + 2\,\varepsilon\,c^3\,z\,\mu\,x\,t}{\%1^5\,\mu}, 4\,\frac{z\,(-y^2 - x^2 + \varepsilon\,c^2\,z^2\,\mu + \varepsilon\,c^4\,\mu\,t^2)}{\%1^5\,\mu},$$

$$\left. -4\,\frac{\varepsilon\,c^2\,t\,(y^2 + x^2 + z^2 + c^2\,t^2)}{\%1^5} \right]$$

$$\%1 := x^2 + y^2 + z^2 - c^2\,t^2$$

$$chiralty\ AdotD = -4\,\frac{\varepsilon\,c^2\,t\,(y^2 + x^2 + z^2 + c^2\,t^2)}{(x^2 + y^2 + z^2 - c^2\,t^2)^5}$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -4\,(-6\,y^2\,c^2\,t^2 - 2\,z^2\,x^2 - 2\,z^2\,y^2 - 6\,x^2\,c^2\,t^2 - x^4 - 2\,x^2\,y^2 - y^4 - z^4 + 2\,z^2\,c^2\,t^2 - c^4\,t^4 + 4\,\varepsilon\,c^2\,\mu\,x^2\,z^2 + 4\,\varepsilon\,c^4\,\mu\,y^2\,t^2 + 4\,\varepsilon\,c^2\,\mu\,y^2\,z^2 + 4\,\varepsilon\,c^4\,\mu\,x^2\,t^2 + 4\,\varepsilon\,c^2\,z^4\,\mu + 8\,\varepsilon\,c^4\,z^2\,\mu\,t^2 + 4\,\varepsilon\,c^6\,\mu\,t^4) \Big/ ((x^2 + y^2 + z^2 - c^2\,t^2)^6\,\mu)$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = 4\,(2\,\varepsilon\,c^2\,\mu\,y^2\,x^2 + \varepsilon\,c^2\,\mu\,y^4 + \varepsilon\,c^2\,\mu\,x^4 + 7\,\varepsilon\,c^4\,\mu\,y^2\,t^2 + 18\,\varepsilon\,c^4\,z^2\,\mu\,t^2 + 3\,\varepsilon\,c^2\,\mu\,y^2\,z^2 + 7\,\varepsilon\,c^4\,\mu\,x^2\,t^2 + 3\,\varepsilon\,c^2\,\mu\,x^2\,z^2 - 2\,x^2\,y^2 - z^2\,y^2 - 2\,c^4\,t^4 + 4\,\varepsilon\,c^6\,\mu\,t^4 - y^4 - x^4 + 2\,z^2\,c^2\,t^2 - 9\,x^2\,c^2\,t^2 - 9\,y^2\,c^2\,t^2 - z^2\,x^2 + 2\,\varepsilon\,c^2\,z^4\,\mu) \Big/ ((x^2 + y^2 + z^2 - c^2\,t^2)^6\,\mu)$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = -4\,(2\,\varepsilon\,c^2\,\mu\,y^2\,x^2 + \varepsilon\,c^2\,\mu\,y^4 + \varepsilon\,c^2\,\mu\,x^4 + 11\,\varepsilon\,c^4\,\mu\,y^2\,t^2 + 26\,\varepsilon\,c^4\,z^2\,\mu\,t^2 + 7\,\varepsilon\,c^2\,\mu\,y^2\,z^2 + 11\,\varepsilon\,c^4\,\mu\,x^2\,t^2 + 7\,\varepsilon\,c^2\,\mu\,x^2\,z^2 - 4\,x^2\,y^2 - 3\,z^2\,y^2 - 3\,c^4\,t^4 + 6\,\varepsilon\,c^2\,z^4\,\mu + 8\,\varepsilon\,c^6\,\mu\,t^4 - 2\,y^4 - 2\,x^4 + 4\,z^2\,c^2\,t^2 - 15\,x^2\,c^2\,t^2 - 15\,y^2\,c^2\,t^2 - 3\,z^2\,x^2 - z^4) \Big/ ((x^2 + y^2 + z^2 - c^2\,t^2)^6\,\mu)$$

$$Virtual\ work\ = [-8\,(\varepsilon\,c^2\,\mu\,x^5 + \varepsilon\,c^2\,\mu\,x\,y^4 + 10\,\varepsilon\,c^4\,\mu\,x\,y^2\,t^2 + 2\,\varepsilon\,c^2\,\mu\,x^3\,y^2 + 10\,\varepsilon\,c^4\,\mu\,x^3\,t^2 + 32\,\varepsilon\,c^4\,z^2\,\mu\,t^2\,x - 6\,\varepsilon\,c^5\,z\,\mu\,t^3\,y + 4\,\varepsilon\,c^2\,z^2\,\mu\,x^3 + 6\,\varepsilon\,c^3\,z\,\mu\,x^2\,y\,t + 4\,\varepsilon\,c^2\,z^2\,\mu\,y^2\,x + 6\,\varepsilon\,c^3\,z\,\mu\,y^3\,t + 3\,\varepsilon\,c^2\,z^4\,\mu\,x + 6\,\varepsilon\,c^3\,z^3\,\mu\,y\,t + 13\,\varepsilon\,c^6\,\mu\,t^4\,x - 2\,c\,t\,z\,y\,x^2 - 9\,c^4\,t^4\,x + x\,z^4 - 2\,x^3\,y^2 - x\,y^4 - 14\,x^3\,c^2\,t^2 - x^5 - 14\,x\,y^2\,c^2\,t^2 + 8\,x\,z^2\,c^2\,t^2 - 2\,c\,t\,z^3\,y - 2\,c\,t\,z\,y^3 + 2\,c^3\,t^3\,z\,y) \Big/ ((x^2 + y^2 + z^2 - c^2\,t^2)^7\,\mu), -8\,(2\,\varepsilon\,c^2\,\mu\,y^3\,x^2 + \varepsilon\,c^2\,\mu\,y^5 + \varepsilon\,c^2\,\mu\,y\,x^4 + 13\,c^6\,t^4\,\varepsilon\,\mu\,y + 10\,c^4\,t^2\,\varepsilon\,\mu\,y^3 + 10\,c^4\,t^2\,\varepsilon\,\mu\,x^2\,y + 3\,\varepsilon\,c^2\,z^4\,\mu\,y - 6\,\varepsilon\,c^3\,z^3\,\mu\,x\,t + 32\,\varepsilon\,c^4\,z^2\,\mu\,t^2\,y - 6\,\varepsilon\,c^3\,z\,\mu\,y^2\,x\,t + 6\,\varepsilon\,c^5\,z\,\mu\,t^3\,x + 4\,\varepsilon\,c^2\,z^2\,\mu\,x^2\,y - 6\,\varepsilon\,c^3\,z\,\mu\,x^3\,t + 4\,\varepsilon\,c^2\,z^2\,\mu\,y^3 - y\,x^4 - 2\,y^3\,x^2 + y\,z^4 + 2\,c\,t\,y^2\,z\,x + 2\,c\,t\,x^3\,z + 8\,c^2\,t^2\,z^2\,y + 2\,c\,t\,z^3\,x - 2\,c^3\,t^3\,z\,x - 14\,c^2\,t^2\,y^3 - 9\,c^4\,t^4\,y - y^5 - 14\,c^2\,t^2\,x^2\,y) \Big/ ((x^2 + y^2 + z^2 - c^2\,t^2)^7\,\mu), -16\,z\,(-z^2\,x^2 + \varepsilon\,c^2\,\mu\,x^4 + 2\,\varepsilon\,c^2\,\mu\,y^2\,x^2 + \varepsilon\,c^2\,\mu\,y^4 - 11\,x^2\,c^2\,t^2 + 12\,\varepsilon\,c^4\,z^2\,\mu\,t^2 + 10\,\varepsilon\,c^6\,\mu\,t^4 - 11\,y^2\,c^2\,t^2 - z^2\,y^2 + \varepsilon\,c^4\,\mu\,x^2\,t^2 + 3\,\varepsilon\,c^2\,\mu\,y^2\,z^2 + \varepsilon\,c^4\,\mu\,y^2\,t^2 + 2\,\varepsilon\,c^2\,z^4\,\mu + 3\,\varepsilon\,c^2\,\mu\,x^2\,z^2 - 2\,x^2\,y^2 - y^4 - x^4) \Big/ ((x^2 + y^2 + z^2 - c^2\,t^2)^7\,\mu)]$$

$$JdotE\ power\ = -16\,c^2\,t\,(\varepsilon\,c^2\,\mu\,x^4 + 7\,\varepsilon\,c^4\,\mu\,y^2\,t^2 + \varepsilon\,c^2\,\mu\,y^4 + 12\,\varepsilon\,c^4\,z^2\,\mu\,t^2 + 8\,\varepsilon\,c^2\,z^4\,\mu + 9\,\varepsilon\,c^2\,\mu\,x^2\,z^2 + 9\,\varepsilon\,c^2\,\mu\,y^2\,z^2 + 2\,\varepsilon\,c^2\,\mu\,y^2\,x^2 + 7\,\varepsilon\,c^4\,\mu\,x^2\,t^2 - 9\,y^2\,c^2\,t^2 - y^4 + 2\,z^4 - 9\,x^2\,c^2\,t^2 - x^4 - 2\,c^4\,t^4 + z^2\,y^2 - 2\,x^2\,y^2 + z^2\,x^2 + 4\,\varepsilon\,c^6\,\mu\,t^4) \Big/ ((x^2 + y^2 + z^2 - c^2\,t^2)^7\,\mu)$$

**Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.**

```
> NAME:=`Example 8b-- Index 1 Irreversible solution EdotB >0 (kinematic out) Type 2`;
> ff:=1;p:=2;n:=4;
> lambda:=(x^p+y^p+z^p-(c*t)^p)^(n/p);
> Ax:=c*t*ff/lambda;Ay:=-z*ff/lambda;Az:=y*ff/lambda;phi:=+x*c*ff/lambda;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

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$$NAME := Example\ 8b\text{-- } Index\ 1\ Irreversible\ solution\ EdotB > 0\ (kinematic\ out)\ Type\ 2$$

$$ff := 1$$

$$p := 2$$

$$n := 4$$

$$\lambda := (x^2+y^2+z^2-c^2 t^2)^2$$

$$Ax := \frac{c\,t}{(x^2+y^2+z^2-c^2 t^2)^2}$$

$$Ay := -\frac{z}{(x^2+y^2+z^2-c^2 t^2)^2}$$

$$Az := \frac{y}{(x^2+y^2+z^2-c^2 t^2)^2}$$

$$\phi := \frac{x\,c}{(x^2+y^2+z^2-c^2 t^2)^2}$$

*Example 8b-- Index 1 Irreversible solution EdotB >0 (kinematic out) Type 2*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \frac{c\,t\,\mathrm{d}(x)}{\%1^2} - \frac{z\,\mathrm{d}(y)}{\%1^2} + \frac{y\,\mathrm{d}(z)}{\%1^2} - \frac{x\,c\,\mathrm{d}(t)}{\%1^2}$$

$$\%1 := x^2+y^2+z^2-c^2 t^2$$

$$E\ field = \left[2\frac{c\,(x^2-y^2-z^2-c^2 t^2)}{(x^2+y^2+z^2-c^2 t^2)^3},\ 4\frac{c\,(x\,y+z\,c\,t)}{(x^2+y^2+z^2-c^2 t^2)^3},\ 4\frac{c\,(z\,x-y\,c\,t)}{(x^2+y^2+z^2-c^2 t^2)^3}\right]$$

$$B\ field = \left[2\frac{x^2-y^2-z^2-c^2 t^2}{(x^2+y^2+z^2-c^2 t^2)^3},\ 4\frac{-z\,c\,t+x\,y}{(x^2+y^2+z^2-c^2 t^2)^3},\ 4\frac{z\,x+y\,c\,t}{(x^2+y^2+z^2-c^2 t^2)^3}\right]$$

$$Topological\ Torsion = \left[-2\frac{c\,x}{\%1^4},\ -2\frac{c\,y}{\%1^4},\ -2\frac{c\,z}{\%1^4},\ -2\frac{c\,t}{\%1^4}\right]$$

$$\%1 := x^2+y^2+z^2-c^2 t^2$$

$$Helicity\ AdotB = -2\frac{c\,t}{(x^2+y^2+z^2-c^2 t^2)^4}$$

$$Poincare\ 2\ E.B = 8\frac{c}{(x^2+y^2+z^2-c^2 t^2)^4}$$

$$D\ field = \left[2\frac{\varepsilon\,c\,(x^2-y^2-z^2-c^2 t^2)}{(x^2+y^2+z^2-c^2 t^2)^3},\ 4\frac{\varepsilon\,c\,(x\,y+z\,c\,t)}{(x^2+y^2+z^2-c^2 t^2)^3},\ 4\frac{\varepsilon\,c\,(z\,x-y\,c\,t)}{(x^2+y^2+z^2-c^2 t^2)^3}\right]$$

$$H\ field = \left[2\frac{x^2-y^2-z^2-c^2 t^2}{(x^2+y^2+z^2-c^2 t^2)^3\,\mu},\ 4\frac{-z\,c\,t+x\,y}{(x^2+y^2+z^2-c^2 t^2)^3\,\mu},\ 4\frac{z\,x+y\,c\,t}{(x^2+y^2+z^2-c^2 t^2)^3\,\mu}\right]$$

$$Poynting\ vector\ ExH = \left[32\frac{c^2\,x\,t\,(z^2+y^2)}{(x^2+y^2+z^2-c^2 t^2)^6\,\mu},\ -16\frac{c^2\,(x^2-y^2-z^2-c^2 t^2)\,y\,t}{(x^2+y^2+z^2-c^2 t^2)^6\,\mu},\ -16\frac{c^2\,(x^2-y^2-z^2-c^2 t^2)\,z\,t}{(x^2+y^2+z^2-c^2 t^2)^6\,\mu}\right]$$

$$Current\ density = \left[-8\frac{c\,t\,(x^2-2\,y^2-2\,z^2-c^2 t^2)\,(-1+\varepsilon\,c^2\,\mu)}{(x^2+y^2+z^2-c^2 t^2)^4\,\mu},\right.$$

$$-4\frac{(z x^2+z y^2+z^3+5 z c^2 t^2+6 c t x y)(-1+\varepsilon c^2 \mu)}{(x^2+y^2+z^2-c^2 t^2)^4 \mu}, 4\frac{(y x^2+y^3+y z^2+5 y c^2 t^2-6 c t z x)(-1+\varepsilon c^2 \mu)}{(x^2+y^2+z^2-c^2 t^2)^4 \mu}\Bigg]$$

$$\textit{charge density}=0$$

$$\textit{Topological SPIN}=\Bigg[2\frac{x(-2 z^2-2 y^2+\varepsilon c^2 x^2 \mu-\varepsilon c^2 \mu y^2-\varepsilon c^2 z^2 \mu-\varepsilon c^4 \mu t^2)}{\%1^5 \mu},$$

$$2\frac{y x^2-y^3-y z^2-3 y c^2 t^2-2 c t z x+2 \varepsilon c^2 x^2 \mu y+2 \varepsilon c^3 z \mu x t}{\%1^5 \mu},$$

$$2\frac{-3 z c^2 t^2+2 c t x y+z x^2-z y^2-z^3+2 \varepsilon c^2 x^2 \mu z-2 \varepsilon c^3 x \mu y t}{\%1^5 \mu}, 2\frac{c^2 \varepsilon t(x^2-3 y^2-3 z^2-c^2 t^2)}{\%1^5}\Bigg]$$

$$\%1 := x^2+y^2+z^2-c^2 t^2$$

$$\textit{chiralty AdotD}=2\frac{c^2 \varepsilon t(x^2-3 y^2-3 z^2-c^2 t^2)}{(x^2+y^2+z^2-c^2 t^2)^5}$$

*LaGrange field energy density (B.H-D.E)* =

$$-4\frac{(x^4+2 x^2 y^2+2 z^2 x^2-2 x^2 c^2 t^2+y^4+2 z^2 y^2+6 y^2 c^2 t^2+z^4+6 z^2 c^2 t^2+c^4 t^4)(-1+\varepsilon c^2 \mu)}{(x^2+y^2+z^2-c^2 t^2)^6 \mu}$$

*Interaction energy density (A.J-rho.phi)* =

$$-4\frac{(-1+\varepsilon c^2 \mu)(2 x^2 c^2 t^2-9 y^2 c^2 t^2-9 z^2 c^2 t^2-2 c^4 t^4-z^2 x^2-2 z^2 y^2-z^4-x^2 y^2-y^4)}{(x^2+y^2+z^2-c^2 t^2)^6 \mu}$$

*Poincare 1 (B.H-D.E)-(A.J-rho.phi)* =

$$-4\frac{(-1+\varepsilon c^2 \mu)(x^4+3 x^2 y^2+3 z^2 x^2-4 x^2 c^2 t^2+2 y^4+4 z^2 y^2+15 y^2 c^2 t^2+2 z^4+15 z^2 c^2 t^2+3 c^4 t^4)}{(x^2+y^2+z^2-c^2 t^2)^6 \mu}$$

$$\textit{Virtual work}=\Bigg[-16\frac{(-1+\varepsilon c^2 \mu) x(z^2+y^2)(y^2+z^2+11 c^2 t^2+x^2)}{(x^2+y^2+z^2-c^2 t^2)^7 \mu}, 8(-1+\varepsilon c^2 \mu)(y x^4+8 c^2 t^2 x^2 y-y^5-2 y^3 z^2$$

$$-14 c^2 t^2 y^3-y z^4-14 c^2 t^2 z^2 y-9 c^4 t^4 y-2 c t x^3 z-2 c t y^2 z x-2 c t z^3 x+2 c^3 t^3 z x) \Big/ ((x^2+y^2+z^2-c^2 t^2)^7 \mu)$$

$$, 8(-1+\varepsilon c^2 \mu)($$

$$8 c^2 t^2 x^2 z+2 c t x^3 y-14 c^2 t^2 y^2 z+2 c t y^3 x-14 c^2 t^2 z^3+2 c t z^2 x y-9 c^4 t^4 z-2 c^3 t^3 x y+z x^4-z y^4-2 z^3 y^2-z^5$$

$$) \Big/ ((x^2+y^2+z^2-c^2 t^2)^7 \mu)\Bigg]$$

*JdotE power* =

$$-16\frac{c^2(-1+\varepsilon c^2 \mu) t(x^4-2 x^2 c^2 t^2+3 y^4+8 y^2 c^2 t^2+3 z^4+8 z^2 c^2 t^2+4 x^2 y^2+4 z^2 x^2+6 z^2 y^2+c^4 t^4)}{(x^2+y^2+z^2-c^2 t^2)^7 \mu}$$

**Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.**

```
> NAME:=`Example 9-- Index 1 Irreversible solution EdotB =0 Type 1 + Type 2`;
> ff:=1;p:=2;n:=4;
> lambda:=(x^p+y^p+z^p-(c*t)^p)^(n/p);
> Ax:=(c*t+y)*ff/lambda;Ay:=(-z-x)*ff/lambda;Az:=(c*t+y)*ff/lambda;phi:=(+x+z)*c*f
  f/lambda;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

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$NAME := Example\ 9\text{-- } Index\ 1\ Irreversible\ solution\ EdotB = 0\ Type\ 1 + Type\ 2$

$$ff := 1$$

$$p := 2$$

$$n := 4$$

$$\lambda := (x^2 + y^2 + z^2 - c^2 t^2)^2$$

$$Ax := \frac{c\,t + y}{(x^2 + y^2 + z^2 - c^2 t^2)^2}$$

$$Ay := \frac{-z - x}{(x^2 + y^2 + z^2 - c^2 t^2)^2}$$

$$Az := \frac{c\,t + y}{(x^2 + y^2 + z^2 - c^2 t^2)^2}$$

$$\phi := \frac{(x + z)\,c}{(x^2 + y^2 + z^2 - c^2 t^2)^2}$$

*Example 9-- Index 1 Irreversible solution EdotB =0 Type 1 + Type 2*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \frac{(c\,t + y)\,\mathrm{d}(x)}{\%1^2} + \frac{(-z - x)\,\mathrm{d}(y)}{\%1^2} + \frac{(c\,t + y)\,\mathrm{d}(z)}{\%1^2} - \frac{(x + z)\,c\,\mathrm{d}(t)}{\%1^2}$$

$$\%1 := x^2 + y^2 + z^2 - c^2 t^2$$

$$E\ field = \left[2\,\frac{c\,(x^2 - y^2 - z^2 - c^2 t^2 + 2\,z\,x - 2\,y\,c\,t)}{(x^2 + y^2 + z^2 - c^2 t^2)^3},\ 4\,\frac{(x + z)\,c\,(c\,t + y)}{(x^2 + y^2 + z^2 - c^2 t^2)^3},\ -2\,\frac{c\,(x^2 + y^2 - z^2 + c^2 t^2 - 2\,z\,x + 2\,y\,c\,t)}{(x^2 + y^2 + z^2 - c^2 t^2)^3}\right]$$

$$B\ field = \left[2\,\frac{x^2 - y^2 - z^2 - c^2 t^2 - 2\,y\,c\,t - 2\,z\,x}{(x^2 + y^2 + z^2 - c^2 t^2)^3},\ 4\,\frac{(c\,t + y)\,(-z + x)}{(x^2 + y^2 + z^2 - c^2 t^2)^3},\ 2\,\frac{x^2 + y^2 - z^2 + c^2 t^2 + 2\,z\,x + 2\,y\,c\,t}{(x^2 + y^2 + z^2 - c^2 t^2)^3}\right]$$

$$Topological\ Torsion = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$D\ field =$

$$\left[2\,\frac{\varepsilon\,c\,(x^2 - y^2 - z^2 - c^2 t^2 + 2\,z\,x - 2\,y\,c\,t)}{(x^2 + y^2 + z^2 - c^2 t^2)^3},\ 4\,\frac{\varepsilon\,(x + z)\,c\,(c\,t + y)}{(x^2 + y^2 + z^2 - c^2 t^2)^3},\ -2\,\frac{\varepsilon\,c\,(x^2 + y^2 - z^2 + c^2 t^2 - 2\,z\,x + 2\,y\,c\,t)}{(x^2 + y^2 + z^2 - c^2 t^2)^3}\right]$$

$$H\ field = \left[2\,\frac{x^2 - y^2 - z^2 - c^2 t^2 - 2\,y\,c\,t - 2\,z\,x}{(x^2 + y^2 + z^2 - c^2 t^2)^3\,\mu},\ 4\,\frac{(c\,t + y)\,(-z + x)}{(x^2 + y^2 + z^2 - c^2 t^2)^3\,\mu},\ 2\,\frac{x^2 + y^2 - z^2 + c^2 t^2 + 2\,z\,x + 2\,y\,c\,t}{(x^2 + y^2 + z^2 - c^2 t^2)^3\,\mu}\right]$$

$Poynting\ vector\ ExH =$

$$\left[16\,\frac{c\,(c\,t + y)\,x\,\%1}{(x^2 + y^2 + z^2 - c^2 t^2)^6\,\mu},\ -8\,\frac{c\,(x^2 + z^2 - y^2 - 2\,y\,c\,t - c^2 t^2)\,\%1}{(x^2 + y^2 + z^2 - c^2 t^2)^6\,\mu},\ 16\,\frac{c\,(c\,t + y)\,z\,\%1}{(x^2 + y^2 + z^2 - c^2 t^2)^6\,\mu}\right]$$

$$\%1 := x^2 + y^2 + z^2 + 2\,y\,c\,t + c^2 t^2$$

$$Current\ density = \left[-4\,\frac{(2\,c\,t\,x^2 - 4\,c\,t\,y^2 - 4\,c\,t\,z^2 - 2\,c^3 t^3 - y\,x^2 - y^3 - y\,z^2 - 5\,y\,c^2 t^2 + 6\,c\,t\,z\,x)\,(-1 + \varepsilon\,c^2\,\mu)}{(x^2 + y^2 + z^2 - c^2 t^2)^4\,\mu},\right.$$

$$-4\,\frac{(x+z)\,(x^2+y^2+z^2+5\,c^2\,t^2+6\,y\,c\,t)\,(-1+\varepsilon\,c^2\,\mu)}{(x^2+y^2+z^2-c^2\,t^2)^4\,\mu},$$

$$4\,\frac{(4\,c\,t\,x^2+4\,c\,t\,y^2-2\,c\,t\,z^2+2\,c^3\,t^3+y\,x^2+y^3+y\,z^2+5\,y\,c^2\,t^2-6\,c\,t\,z\,x)\,(-1+\varepsilon\,c^2\,\mu)}{(x^2+y^2+z^2-c^2\,t^2)^4\,\mu}\Bigg]$$

$$\textit{charge density}=0$$

$$\textit{Topological SPIN}=\Bigg[2\,(-x^3-3\,x\,y^2-x\,z^2+z\,y^2-3\,z\,x^2+z^3+3\,\varepsilon\,c^2\,x^2\,\mu\,z-2\,\varepsilon\,c^3\,x\,\mu\,y\,t-6\,c\,t\,x\,y+\varepsilon\,c^2\,\mu\,x^3$$
$$-\varepsilon\,c^2\,\mu\,z^3-\varepsilon\,c^2\,\mu\,x\,y^2-\varepsilon\,c^2\,\mu\,z\,y^2+\varepsilon\,c^2\,\mu\,x\,z^2-\varepsilon\,c^4\,\mu\,x\,t^2-\varepsilon\,c^4\,\mu\,z\,t^2-2\,\varepsilon\,c^3\,\mu\,z\,y\,t+z\,c^2\,t^2+2\,z\,y\,c\,t-3\,x\,c^2\,t^2)$$
$$\Big/\,(\%1^5\,\mu),\;4\,\frac{(c\,t+y)\,(-y^2-c^2\,t^2-2\,y\,c\,t-2\,z\,x+\varepsilon\,c^2\,x^2\,\mu+2\,\varepsilon\,c^2\,\mu\,z\,x+\varepsilon\,c^2\,z^2\,\mu)}{\%1^5\,\mu},\;-2\,(-x^3-x\,y^2+3\,x\,z^2$$
$$+3\,z\,y^2+z\,x^2+z^3-\varepsilon\,c^2\,x^2\,\mu\,z+2\,\varepsilon\,c^3\,x\,\mu\,y\,t-2\,c\,t\,x\,y+\varepsilon\,c^2\,\mu\,x^3-\varepsilon\,c^2\,\mu\,z^3+\varepsilon\,c^2\,\mu\,x\,y^2+\varepsilon\,c^2\,\mu\,z\,y^2-3\,\varepsilon\,c^2\,\mu\,x\,z^2$$
$$+\varepsilon\,c^4\,\mu\,x\,t^2+\varepsilon\,c^4\,\mu\,z\,t^2+2\,\varepsilon\,c^3\,\mu\,z\,y\,t+3\,z\,c^2\,t^2+6\,z\,y\,c\,t-x\,c^2\,t^2)\,\Big/\,(\%1^5\,\mu),$$
$$-4\,\frac{(c\,t+y)\,\varepsilon\,c\,(x^2+y^2+z^2+2\,y\,c\,t+c^2\,t^2)}{\%1^5}\Bigg]$$

$$\%1:=x^2+y^2+z^2-c^2\,t^2$$

$$\textit{chiralty AdotD}=-4\,\frac{(c\,t+y)\,\varepsilon\,c\,(x^2+y^2+z^2+2\,y\,c\,t+c^2\,t^2)}{(x^2+y^2+z^2-c^2\,t^2)^5}$$

$$\textit{LaGrange field energy density (B.H-D.E)}=-8\,\frac{(x^2+y^2+z^2+2\,y\,c\,t+c^2\,t^2)^2\,(-1+\varepsilon\,c^2\,\mu)}{(x^2+y^2+z^2-c^2\,t^2)^6\,\mu}$$

$$\textit{Interaction energy density (A.J-rho.phi)}=4\,(-1+\varepsilon\,c^2\,\mu)\,(10\,c\,t\,y\,x^2+10\,c\,t\,y\,z^2-2\,c^2\,t^2\,z\,x+3\,x^2\,y^2+3\,z^2\,y^2$$
$$+4\,c^4\,t^4+2\,y^4+x^4+2\,z^3\,x+2\,z\,x^3+7\,z^2\,c^2\,t^2+7\,x^2\,c^2\,t^2+18\,y^2\,c^2\,t^2+2\,z^2\,x^2+2\,z\,x\,y^2+14\,c^3\,t^3\,y+10\,c\,t\,y^3+z^4)$$
$$\Big/\,((x^2+y^2+z^2-c^2\,t^2)^6\,\mu)$$

$$\textit{Poincare 1 (B.H-D.E)-(A.J-rho.phi)}=-4\,(-1+\varepsilon\,c^2\,\mu)\,(18\,c\,t\,y\,x^2+18\,c\,t\,y\,z^2-2\,c^2\,t^2\,z\,x+7\,x^2\,y^2+7\,z^2\,y^2+6\,c^4\,t^4$$
$$+4\,y^4+3\,x^4+2\,z^3\,x+2\,z\,x^3+11\,z^2\,c^2\,t^2+11\,x^2\,c^2\,t^2+30\,y^2\,c^2\,t^2+6\,z^2\,x^2+2\,z\,x\,y^2+22\,c^3\,t^3\,y+18\,c\,t\,y^3+3\,z^4)\,\Big/\,($$
$$(x^2+y^2+z^2-c^2\,t^2)^6\,\mu)$$

$$\textit{Virtual work }=[-8\,(-1+\varepsilon\,c^2\,\mu)\,(2\,x^3\,z^2-2\,c\,t\,x^2\,y\,z+3\,x\,y^4+x\,z^4+2\,x^2\,z^3+x^5+4\,x^3\,y^2-2\,z^3\,y^2-z^5+c^4\,t^4\,z$$
$$+18\,c\,t\,x^3\,y+18\,c\,t\,y^3\,x+18\,c\,t\,z^2\,x\,y+30\,c^3\,t^3\,x\,y-4\,c^2\,t^2\,x^2\,z+3\,z\,x^4-z\,y^4+14\,x\,z^2\,c^2\,t^2+36\,x\,y^2\,c^2\,t^2-2\,z^3\,y\,c\,t$$
$$-2\,z\,y^3\,c\,t+2\,z\,c^3\,t^3\,y+9\,x\,c^4\,t^4+2\,x^2\,y^2\,z+4\,x\,y^2\,z^2+14\,x^3\,c^2\,t^2)\,\Big/\,((x^2+y^2+z^2-c^2\,t^2)^7\,\mu),\;16\,(-1+\varepsilon\,c^2\,\mu)\,($$
$$-y^3\,z^2+3\,c\,t\,z^4-3\,c\,t\,y^2\,z^2+6\,c\,t\,x^2\,z^2-3\,c\,t\,x^2\,y^2-2\,c\,t\,y^2\,z\,x-2\,c\,t\,x^3\,z-3\,c^2\,t^2\,z^2\,y-2\,c\,t\,z^3\,x+2\,c^3\,t^3\,z\,x$$
$$-14\,c^2\,t^2\,y^3-9\,c^4\,t^4\,y-y^5-3\,c^2\,t^2\,x^2\,y+2\,y\,c^2\,t^2\,z\,x-2\,y^3\,z\,x-2\,y\,z^3\,x-2\,y\,x^3\,z+3\,c\,t\,x^4-6\,c\,t\,y^4-c^3\,t^3\,x^2$$
$$-c^3\,t^3\,z^2-16\,c^3\,t^3\,y^2-2\,c^5\,t^5-y^3\,x^2)\,\Big/\,((x^2+y^2+z^2-c^2\,t^2)^7\,\mu),\;8\,(-1+\varepsilon\,c^2\,\mu)\,(-2\,x^3\,z^2-18\,c\,t\,x^2\,y\,z+x\,y^4$$
$$-3\,x\,z^4-2\,x^2\,z^3+x^5+2\,x^3\,y^2-4\,z^3\,y^2-z^5-9\,c^4\,t^4\,z+2\,c\,t\,x^3\,y-36\,c^2\,t^2\,y^2\,z+2\,c\,t\,y^3\,x+2\,c\,t\,z^2\,x\,y-2\,c^3\,t^3\,x\,y$$
$$-14\,c^2\,t^2\,x^2\,z-z\,x^4-3\,z\,y^4-14\,c^2\,t^2\,z^3+4\,x\,z^2\,c^2\,t^2-18\,z^3\,y\,c\,t-18\,z\,y^3\,c\,t-30\,z\,c^3\,t^3\,y-x\,c^4\,t^4-4\,x^2\,y^2\,z$$
$$-2\,x\,y^2\,z^2)\,\Big/\,((x^2+y^2+z^2-c^2\,t^2)^7\,\mu)]$$

$$\textit{JdotE power }=$$
$$-16\,\frac{(-1+\varepsilon\,c^2\,\mu)\,c\,(x^2+y^2+z^2+2\,y\,c\,t+c^2\,t^2)\,(4\,c\,t\,x^2+y\,x^2+5\,y\,c^2\,t^2+4\,c\,t\,z^2+2\,c^3\,t^3+y\,z^2+4\,c\,t\,y^2+y^3)}{(x^2+y^2+z^2-c^2\,t^2)^7\,\mu}$$

**Enter the name of the problem, and the components of the 4 potential, and the Holder norm**

**indices.**

```
> NAME:=`Example 10a -- Plasma Accretion disc -- Hedge Hog solution.`;
> Gamma:=-z*I/(x^2+y^2)*m/(a*x^2+a*y^2+c*z^2)^(1/2);
> Ax:=Gamma*(-y);Ay:=Gamma*x;
> Az:=0; phi:=0;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

************************************************************************

*NAME* := *Example 10a -- Plasma Accretion disc -- Hedge Hog solution.*

$$\Gamma := \frac{-I\,z\,m}{(x^2+y^2)\sqrt{a\,x^2+a\,y^2+z^2\,c}}$$

$$Ax := \frac{I\,z\,m\,y}{(x^2+y^2)\sqrt{a\,x^2+a\,y^2+z^2\,c}}$$

$$Ay := \frac{-I\,z\,m\,x}{(x^2+y^2)\sqrt{a\,x^2+a\,y^2+z^2\,c}}$$

$$Az := 0$$

$$\phi := 0$$

*Example 10a -- Plasma Accretion disc -- Hedge Hog solution.*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \frac{I\,z\,m\,y\,\mathrm{d}(x)}{(x^2+y^2)\sqrt{a\,x^2+a\,y^2+z^2\,c}} - \frac{I\,z\,m\,x\,\mathrm{d}(y)}{(x^2+y^2)\sqrt{a\,x^2+a\,y^2+z^2\,c}}$$

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[\frac{I\,a\,m\,x}{(a\,x^2+a\,y^2+z^2\,c)^{(3/2)}}, \frac{I\,a\,m\,y}{(a\,x^2+a\,y^2+z^2\,c)^{(3/2)}}, \frac{I\,a\,z\,m}{(a\,x^2+a\,y^2+z^2\,c)^{(3/2)}}\right]$$

$$Topological\ Torsion = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = [0, 0, 0]$$

$$H\ field = \left[\frac{I\,a\,m\,x}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^{(3/2)}}, \frac{I\,a\,m\,y}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^{(3/2)}}, \frac{I\,a\,z\,m}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^{(3/2)}}\right]$$

$$Poynting\ vector\ ExH = [0, 0, 0]$$

$$Current\ density = \left[\frac{3\,I\,a\,z\,m\,y\,(-a+c)}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^{(5/2)}}, \frac{-3\,I\,a\,m\,x\,z\,(-a+c)}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^{(5/2)}}, 0\right]$$

$$charge\ density = 0$$

$$Topological\ SPIN = \left[\frac{z^2\,m^2\,x\,a}{(x^2+y^2)\,(a\,x^2+a\,y^2+z^2\,c)^2\,\mu}, \frac{z^2\,m^2\,y\,a}{(x^2+y^2)\,(a\,x^2+a\,y^2+z^2\,c)^2\,\mu}, -\frac{z\,m^2\,a}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^2}, 0\right]$$

$$chiralty\ AdotD = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{m^2\,a^2\,(x^2+y^2+z^2)}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^3}$$

$$\textit{Interaction energy density (A.J-rho.phi)} = -3\,\frac{z^2\,m^2\,a\,(-a+c)}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^3}$$

$$\textit{Poincare 1 (B.H-D.E)-(A.J-rho.phi)} = -\frac{m^2\,a\,(a\,x^2+a\,y^2+4\,a\,z^2-3\,z^2\,c)}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^3}$$

$$\textit{Virtual work} = \left[3\,\frac{a^2\,m^2\,x\,z^2\,(-a+c)}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^4},\ 3\,\frac{a^2\,z^2\,m^2\,y\,(-a+c)}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^4},\ -3\,\frac{a^2\,z\,m^2\,(-a+c)\,(x^2+y^2)}{\mu\,(a\,x^2+a\,y^2+z^2\,c)^4}\right]$$

$$\textit{JdotE power} = 0$$

**Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.**

```
> NAME:=`Example 10b -- Dirac Hedge Hog solution.`;
> r:=(x^2+y^2+1*z^2)^(1/2);Gamma:=factor(I*(m/2)/(r*(z-r)));
> Ax:=Gamma*(-y);Ay:=Gamma*x;
> Az:=0; phi:=0;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

****************************************************************

$$\textit{NAME} := \textit{Example 10b -- Dirac Hedge Hog solution.}$$

$$r := \sqrt{x^2+y^2+z^2}$$

$$\Gamma := \frac{\frac{-1}{2}\,I\,m}{\sqrt{x^2+y^2+z^2}\,(-z+\sqrt{x^2+y^2+z^2})}$$

$$Ax := \frac{\frac{1}{2}\,I\,m\,y}{\sqrt{x^2+y^2+z^2}\,(-z+\sqrt{x^2+y^2+z^2})}$$

$$Ay := \frac{\frac{-1}{2}\,I\,m\,x}{\sqrt{x^2+y^2+z^2}\,(-z+\sqrt{x^2+y^2+z^2})}$$

$$Az := 0$$

$$\phi := 0$$

*Example 10b -- Dirac Hedge Hog solution.*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$\textit{Action} = \frac{\frac{1}{2}\,I\,m\,y\,\mathrm{d}(x)}{\sqrt{\%1}\,(-z+\sqrt{\%1})} - \frac{\frac{1}{2}\,I\,m\,x\,\mathrm{d}(y)}{\sqrt{\%1}\,(-z+\sqrt{\%1})}$$

$$\%1 := x^2+y^2+z^2$$

$$\textit{E field} = [0, 0, 0]$$

$$\textit{B field} = \left[\frac{\frac{1}{2}\,I\,x\,m}{\%1^{(3/2)}},\ \frac{\frac{1}{2}\,I\,y\,m}{\%1^{(3/2)}},\ \frac{\frac{-1}{2}\,I\,(-x^2-y^2-2\,z^2+2\,\sqrt{\%1}\,z)\,z\,m}{(-z+\sqrt{\%1})^2\,\%1^{(3/2)}}\right]$$

$$\%1 := x^2+y^2+z^2$$

$$Topological\ Torsion = [0, 0, 0, 0]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = [0, 0, 0]$$

$$H\ field = \left[\frac{\frac{1}{2} I\, x\, m}{\mu\, \%1^{(3/2)}}, \frac{\frac{1}{2} I\, y\, m}{\mu\, \%1^{(3/2)}}, \frac{\frac{-1}{2} I\, (-x^2 - y^2 - 2\, z^2 + 2\sqrt{\%1}\, z)\, z\, m}{\mu\, (-z + \sqrt{\%1})^2\, \%1^{(3/2)}}\right]$$

$$\%1 := x^2 + y^2 + z^2$$

$$Poynting\ vector\ ExH = [0, 0, 0]$$

$$Current\ density = [0, 0, 0]$$

$$charge\ density = 0$$

*Topological SPIN* =

$$\left[-\frac{1}{4}\frac{m^2\, x\, (-x^2 - y^2 - 2\, z^2 + 2\sqrt{\%1}\, z)\, z}{\%1^2\, (-z + \sqrt{\%1})^3\, \mu}, -\frac{1}{4}\frac{m^2\, y\, (-x^2 - y^2 - 2\, z^2 + 2\sqrt{\%1}\, z)\, z}{\%1^2\, (-z + \sqrt{\%1})^3\, \mu}, -\frac{1}{4}\frac{m^2\, (x^2 + y^2)}{\%1^2\, (-z + \sqrt{\%1})\, \mu}, 0\right]$$

$$\%1 := x^2 + y^2 + z^2$$

$$chiralty\ AdotD = 0$$

*LaGrange field energy density (B.H-D.E)* =

$$\frac{1}{4}\frac{(-x^4 - 8\, x^2\, z^2 - 2\, x^2\, y^2 + 4\, x^2\, z\sqrt{\%1} + 4\, y^2\, z\sqrt{\%1} + 8\sqrt{\%1}\, z^3 - 8\, y^2\, z^2 - 8\, z^4 - y^4)\, m^2}{\mu\, \%1^2\, (-z + \sqrt{\%1})^4}$$

$$\%1 := x^2 + y^2 + z^2$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = 0$$

*Poincare 1 (B.H-D.E)-(A.J-rho.phi)* =

$$\frac{1}{4}\frac{(-x^4 - 8\, x^2\, z^2 - 2\, x^2\, y^2 + 4\, x^2\, z\sqrt{\%1} + 4\, y^2\, z\sqrt{\%1} + 8\sqrt{\%1}\, z^3 - 8\, y^2\, z^2 - 8\, z^4 - y^4)\, m^2}{\mu\, \%1^2\, (-z + \sqrt{\%1})^4}$$

$$\%1 := x^2 + y^2 + z^2$$

$$Virtual\ work\ = [0, 0, 0]$$

$$JdotE\ power\ = 0$$

**Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.**

```
> NAME:=`Example 11-- Coulomb plus Bohm-Aharanov singular vortex string`;
> ff:=b;p:=2;n:=2;
> lambda:=(x^p+y^p)^(n/p);
> Ax:=y*ff/lambda;Ay:=-x*ff/lambda;Az:=0;phi:=m/((4*pi*epsilon)*(x^2+y^2+z^2)^(1/2
  ));
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

******************************************************************

*NAME := Example 11-- Coulomb plus Bohm-Aharanov singular vortex string*

$$ff := b$$

$$p := 2$$

$$n := 2$$

$$\lambda := x^2 + y^2$$

$$Ax := \frac{y\,b}{x^2 + y^2}$$

$$Ay := -\frac{x\,b}{x^2 + y^2}$$

$$Az := 0$$

$$\phi := \frac{1}{4}\,\frac{m}{\pi\,\varepsilon\,\sqrt{x^2 + y^2 + z^2}}$$

*Example 11-- Coulomb plus Bohm-Aharanov singular vortex string*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \frac{y\,b\,\mathrm{d}(x)}{x^2 + y^2} - \frac{x\,b\,\mathrm{d}(y)}{x^2 + y^2} - \frac{1}{4}\,\frac{m\,\mathrm{d}(t)}{\pi\,\varepsilon\,\sqrt{x^2 + y^2 + z^2}}$$

$$E\ field = \left[\frac{1}{4}\,\frac{m\,x}{\pi\,\varepsilon\,(x^2 + y^2 + z^2)^{(3/2)}}, \frac{1}{4}\,\frac{m\,y}{\pi\,\varepsilon\,(x^2 + y^2 + z^2)^{(3/2)}}, \frac{1}{4}\,\frac{m\,z}{\pi\,\varepsilon\,(x^2 + y^2 + z^2)^{(3/2)}}\right]$$

$$B\ field = [0, 0, 0]$$

*Topological Torsion =*

$$\left[-\frac{1}{4}\,\frac{m\,z\,x\,b}{\pi\,\varepsilon\,(x^2 + y^2 + z^2)^{(3/2)}\,(x^2 + y^2)}, -\frac{1}{4}\,\frac{m\,z\,y\,b}{\pi\,\varepsilon\,(x^2 + y^2 + z^2)^{(3/2)}\,(x^2 + y^2)}, \frac{1}{4}\,\frac{m\,b}{(x^2 + y^2 + z^2)^{(3/2)}\,\varepsilon\,\pi}, 0\right]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = \left[\frac{1}{4}\,\frac{m\,x}{\pi\,(x^2 + y^2 + z^2)^{(3/2)}}, \frac{1}{4}\,\frac{m\,y}{\pi\,(x^2 + y^2 + z^2)^{(3/2)}}, \frac{1}{4}\,\frac{m\,z}{\pi\,(x^2 + y^2 + z^2)^{(3/2)}}\right]$$

$$H\ field = [0, 0, 0]$$

$$Poynting\ vector\ ExH = [0, 0, 0]$$

$$Current\ density = [0, 0, 0]$$

$$charge\ density = 0$$

$$Topological\ SPIN = \left[\frac{1}{16}\,\frac{m^2\,x}{\pi^2\,(x^2 + y^2 + z^2)^2\,\varepsilon}, \frac{1}{16}\,\frac{m^2\,y}{\pi^2\,(x^2 + y^2 + z^2)^2\,\varepsilon}, \frac{1}{16}\,\frac{m^2\,z}{\pi^2\,(x^2 + y^2 + z^2)^2\,\varepsilon}, 0\right]$$

$$chiralty\ AdotD = 0$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -\frac{1}{16}\,\frac{m^2}{\pi^2\,(x^2 + y^2 + z^2)^2\,\varepsilon}$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = 0$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = -\frac{1}{16}\,\frac{m^2}{\pi^2\,(x^2 + y^2 + z^2)^2\,\varepsilon}$$

$$Virtual\ work\ = [0, 0, 0]$$

$$JdotE\ power\ = 0$$

>

**Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.**

```
> NAME:=`Example 12 -- Black Hole singular vortex ring `;
>
> phi :=
  e*c/(x^4+2*x^2*y^2+2*z^2*x^2+y^4+2*y^2*z^2+z^4+a^2*z^2)*(x^2+y^2+z^2)^(3/2);Ax
  :=
  e/(x^4+2*x^2*y^2+2*z^2*x^2+y^4+2*y^2*z^2+z^4+a^2*z^2)*(x^2+y^2+z^2)^(1/2)*a*y;Az
  :=0;Ay :=
  -e/(x^4+2*x^2*y^2+2*z^2*x^2+y^4+2*y^2*z^2+z^4+a^2*z^2)*(x^2+y^2+z^2)^(1/2)*a*x;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

************************************************************

*NAME* := *Example 12 -- Black Hole singular vortex ring*

$$\phi := \frac{e\,c\,(x^2+y^2+z^2)^{(3/2)}}{x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2}$$

$$Ax := \frac{e\,\sqrt{x^2+y^2+z^2}\;a\,y}{x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2}$$

$$Az := 0$$

$$Ay := -\frac{e\,\sqrt{x^2+y^2+z^2}\;a\,x}{x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2}$$

*Example 12 -- Black Hole singular vortex ring*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \frac{e\,\sqrt{x^2+y^2+z^2}\;a\,y\,\mathrm{d}(x)}{\%1} - \frac{e\,\sqrt{x^2+y^2+z^2}\;a\,x\,\mathrm{d}(y)}{\%1} - \frac{e\,c\,(x^2+y^2+z^2)^{(3/2)}\,\mathrm{d}(t)}{\%1}$$

$$\%1 := x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2$$

$$E\,field = \left[\frac{e\,c\,\sqrt{x^2+y^2+z^2}\;x\,(x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4-3\,a^2\,z^2)}{\%1^2},\right.$$
$$\frac{e\,c\,\sqrt{x^2+y^2+z^2}\;y\,(x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4-3\,a^2\,z^2)}{\%1^2},$$
$$\left.\frac{e\,c\,\sqrt{x^2+y^2+z^2}\;z\,(x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+2\,a^2\,x^2+2\,a^2\,y^2-a^2\,z^2)}{\%1^2}\right]$$

$$\%1 := x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2$$

$$B\,field = \left[-\frac{e\,a\,x\,z\,(3\,x^4+6\,x^2\,y^2+6\,x^2\,z^2+3\,y^4+6\,y^2\,z^2+3\,z^4+2\,a^2\,x^2+2\,a^2\,y^2+a^2\,z^2)}{\%1^2\,\sqrt{x^2+y^2+z^2}},\right.$$
$$-\frac{e\,a\,y\,z\,(3\,x^4+6\,x^2\,y^2+6\,x^2\,z^2+3\,y^4+6\,y^2\,z^2+3\,z^4+2\,a^2\,x^2+2\,a^2\,y^2+a^2\,z^2)}{\%1^2\,\sqrt{x^2+y^2+z^2}},$$
$$\left.\frac{e\,a\,(3\,x^4\,y^2+x^6+y^6-3\,x^2\,a^2\,z^2-3\,y^2\,a^2\,z^2-2\,a^2\,z^4-2\,z^6-3\,y^2\,z^4+3\,x^2\,y^4-3\,x^2\,z^4)}{\%1^2\,\sqrt{x^2+y^2+z^2}}\right]$$

$$\%1 := x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2$$

$$Topological\ Torsion = \left[2\,\frac{e^2\,c\,(x^2+y^2+z^2)\,z\,a\,x}{\%1^2},\,2\,\frac{e^2\,c\,(x^2+y^2+z^2)\,z\,a\,y}{\%1^2},\,2\,\frac{e^2\,c\,(x^2+y^2+z^2)\,a\,z^2}{\%1^2},\,0\right]$$

$$\%1 := x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = -4\,\frac{e^2\,c\,a\,z\,(x^2+y^2+z^2)\,(z\,a+x^2+y^2+z^2)\,(-z\,a+x^2+y^2+z^2)}{(x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2)^3}$$

$$D\ field = \left[\frac{\varepsilon\,e\,c\,\sqrt{x^2+y^2+z^2}\;x\,(x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4-3\,a^2\,z^2)}{\%1^2},\right.$$
$$\frac{\varepsilon\,e\,c\,\sqrt{x^2+y^2+z^2}\;y\,(x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4-3\,a^2\,z^2)}{\%1^2},$$
$$\left.\frac{\varepsilon\,e\,c\,\sqrt{x^2+y^2+z^2}\;z\,(x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+2\,a^2\,x^2+2\,a^2\,y^2-a^2\,z^2)}{\%1^2}\right]$$
$$\%1 := x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2$$

$$H\ field = \left[-\frac{e\,a\,x\,z\,(3\,x^4+6\,x^2\,y^2+6\,x^2\,z^2+3\,y^4+6\,y^2\,z^2+3\,z^4+2\,a^2\,x^2+2\,a^2\,y^2+a^2\,z^2)}{\%1^2\,\sqrt{x^2+y^2+z^2}\;\mu},\right.$$
$$-\frac{e\,a\,y\,z\,(3\,x^4+6\,x^2\,y^2+6\,x^2\,z^2+3\,y^4+6\,y^2\,z^2+3\,z^4+2\,a^2\,x^2+2\,a^2\,y^2+a^2\,z^2)}{\%1^2\,\sqrt{x^2+y^2+z^2}\;\mu},$$
$$\left.\frac{e\,a\,(3\,x^4\,y^2+x^6+y^6-3\,x^2\,a^2\,z^2-3\,y^2\,a^2\,z^2-2\,a^2\,z^4-2\,z^6-3\,y^2\,z^4+3\,x^2\,y^4-3\,x^2\,z^4)}{\%1^2\,\sqrt{x^2+y^2+z^2}\;\mu}\right]$$
$$\%1 := x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2$$

$$Poynting\ vector\ ExH = [e^2\,c\,y\,a\,(x^2+y^2+z^2)\,(x^8+4\,x^6\,y^2+4\,x^6\,z^2+2\,x^4\,a^2\,z^2+6\,x^4\,z^4+12\,x^4\,y^2\,z^2+6\,x^4\,y^4+4\,x^2\,y^6$$
$$+4\,x^2\,z^6+12\,x^2\,y^2\,z^4+12\,x^2\,y^4\,z^2+4\,x^2\,z^4\,a^2+4\,a^4\,x^2\,z^2+4\,x^2\,y^2\,a^2\,z^2+5\,a^4\,z^4+4\,y^6\,z^2+2\,z^6\,a^2+6\,y^4\,z^4+4\,y^2\,z^6$$
$$+y^8+z^8+4\,a^4\,y^2\,z^2+4\,y^2\,z^4\,a^2+2\,y^4\,a^2\,z^2)\Big/((x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2)^4\,\mu),-e^2\,c\,a\,x$$
$$(x^2+y^2+z^2)\,(x^8+4\,x^6\,y^2+4\,x^6\,z^2+2\,x^4\,a^2\,z^2+6\,x^4\,z^4+12\,x^4\,y^2\,z^2+6\,x^4\,y^4+4\,x^2\,y^6+4\,x^2\,z^6+12\,x^2\,y^2\,z^4$$
$$+12\,x^2\,y^4\,z^2+4\,x^2\,z^4\,a^2+4\,a^4\,x^2\,z^2+4\,x^2\,y^2\,a^2\,z^2+5\,a^4\,z^4+4\,y^6\,z^2+2\,z^6\,a^2+6\,y^4\,z^4+4\,y^2\,z^6+y^8+z^8+4\,a^4\,y^2\,z^2$$
$$+4\,y^2\,z^4\,a^2+2\,y^4\,a^2\,z^2)\Big/((x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2)^4\,\mu),0]$$

$$Current\ density = \left[2\,\frac{\sqrt{x^2+y^2+z^2}\;(x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4-3\,a^2\,z^2)\,y\,a^3\,e}{\mu\,(x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2)^3},\right.$$
$$\left.-2\,\frac{\sqrt{x^2+y^2+z^2}\;(x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4-3\,a^2\,z^2)\,x\,a^3\,e}{\mu\,(x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2)^3},0\right]$$

$$charge\ density = 2\,\frac{\varepsilon\,e\,c\,a^2\,(x^2+y^2+z^2)^{(3/2)}\,(x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4-3\,a^2\,z^2)}{(x^4+2\,x^2\,y^2+2\,x^2\,z^2+y^4+2\,y^2\,z^2+z^4+a^2\,z^2)^3}$$

$$Topological\ SPIN = [e^2\,x\,(\varepsilon\,c^2\,\mu\,z^8+\varepsilon\,c^2\,\mu\,y^8+\varepsilon\,c^2\,\mu\,x^8+2\,z^6\,a^2-a^2\,y^6+2\,a^4\,z^4+3\,x^2\,z^4\,a^2+3\,a^4\,x^2\,z^2-3\,a^2\,x^4\,y^2$$
$$+3\,y^2\,z^4\,a^2+3\,a^4\,y^2\,z^2-6\,\varepsilon\,c^2\,\mu\,x^2\,y^2\,a^2\,z^2+4\,\varepsilon\,c^2\,\mu\,x^6\,y^2+4\,\varepsilon\,c^2\,\mu\,x^2\,z^6+6\,\varepsilon\,c^2\,\mu\,x^4\,y^4+4\,\varepsilon\,c^2\,\mu\,x^2\,y^6+6\,\varepsilon\,c^2\,\mu\,x^4\,z^4$$
$$+4\,\varepsilon\,c^2\,\mu\,y^2\,z^6+6\,\varepsilon\,c^2\,\mu\,y^4\,z^4+4\,\varepsilon\,c^2\,\mu\,y^6\,z^2-3\,\varepsilon\,c^2\,\mu\,z^6\,a^2+4\,\varepsilon\,c^2\,\mu\,x^6\,z^2-a^2\,x^6-3\,a^2\,x^2\,y^4-3\,\varepsilon\,c^2\,\mu\,y^4\,a^2\,z^2$$
$$-3\,\varepsilon\,c^2\,\mu\,x^4\,a^2\,z^2-6\,\varepsilon\,c^2\,\mu\,x^2\,z^4\,a^2+12\,\varepsilon\,c^2\,\mu\,x^2\,y^4\,z^2+12\,\varepsilon\,c^2\,\mu\,x^2\,y^2\,z^4+12\,\varepsilon\,c^2\,\mu\,x^4\,y^2\,z^2-6\,\varepsilon\,c^2\,\mu\,y^2\,z^4\,a^2)\Big/(\mu$$
$$\%1^3),e^2\,y\,(\varepsilon\,c^2\,\mu\,z^8+\varepsilon\,c^2\,\mu\,y^8+\varepsilon\,c^2\,\mu\,x^8+2\,z^6\,a^2-a^2\,y^6+2\,a^4\,z^4+3\,x^2\,z^4\,a^2+3\,a^4\,x^2\,z^2-3\,a^2\,x^4\,y^2+3\,y^2\,z^4\,a^2$$
$$+3\,a^4\,y^2\,z^2-6\,\varepsilon\,c^2\,\mu\,x^2\,y^2\,a^2\,z^2+4\,\varepsilon\,c^2\,\mu\,x^6\,y^2+4\,\varepsilon\,c^2\,\mu\,x^2\,z^6+6\,\varepsilon\,c^2\,\mu\,x^4\,y^4+4\,\varepsilon\,c^2\,\mu\,x^2\,y^6+6\,\varepsilon\,c^2\,\mu\,x^4\,z^4$$
$$+4\,\varepsilon\,c^2\,\mu\,y^2\,z^6+6\,\varepsilon\,c^2\,\mu\,y^4\,z^4+4\,\varepsilon\,c^2\,\mu\,y^6\,z^2-3\,\varepsilon\,c^2\,\mu\,z^6\,a^2+4\,\varepsilon\,c^2\,\mu\,x^6\,z^2-a^2\,x^6-3\,a^2\,x^2\,y^4-3\,\varepsilon\,c^2\,\mu\,y^4\,a^2\,z^2$$
$$-3\,\varepsilon\,c^2\,\mu\,x^4\,a^2\,z^2-6\,\varepsilon\,c^2\,\mu\,x^2\,z^4\,a^2+12\,\varepsilon\,c^2\,\mu\,x^2\,y^4\,z^2+12\,\varepsilon\,c^2\,\mu\,x^2\,y^2\,z^4+12\,\varepsilon\,c^2\,\mu\,x^4\,y^2\,z^2-6\,\varepsilon\,c^2\,\mu\,y^2\,z^4\,a^2)\Big/(\mu$$
$$\%1^3),e^2\,z\,(\varepsilon\,c^2\,\mu\,z^8+\varepsilon\,c^2\,\mu\,y^8+\varepsilon\,c^2\,\mu\,x^8-3\,a^2\,y^6-3\,x^2\,z^4\,a^2-6\,x^4\,a^2\,z^2-6\,y^4\,a^2\,z^2-a^4\,x^2\,z^2-9\,a^2\,x^4\,y^2-3\,y^2\,z^4\,a^2$$
$$+6\,\varepsilon\,c^2\,\mu\,a^2\,x^4\,y^2+6\,\varepsilon\,c^2\,\mu\,a^2\,x^2\,y^4+2\,\varepsilon\,c^2\,\mu\,a^2\,x^6+2\,\varepsilon\,c^2\,\mu\,a^2\,y^6-a^4\,y^2\,z^2-12\,x^2\,y^2\,a^2\,z^2+6\,\varepsilon\,c^2\,\mu\,x^2\,y^2\,a^2\,z^2$$

$+ 4 \varepsilon c^2 \mu x^6 y^2 + 4 \varepsilon c^2 \mu x^2 z^6 + 6 \varepsilon c^2 \mu x^4 y^4 + 4 \varepsilon c^2 \mu x^2 y^6 + 6 \varepsilon c^2 \mu x^4 z^4 + 4 \varepsilon c^2 \mu y^2 z^6 + 6 \varepsilon c^2 \mu y^4 z^4$
$+ 4 \varepsilon c^2 \mu y^6 z^2 - \varepsilon c^2 \mu z^6 a^2 + 4 \varepsilon c^2 \mu x^6 z^2 - 3 a^2 x^6 - 9 a^2 x^2 y^4 + 3 \varepsilon c^2 \mu y^4 a^2 z^2 + 3 \varepsilon c^2 \mu x^4 a^2 z^2 + 12 \varepsilon c^2 \mu x^2 y^4 z^2$
$+ 12 \varepsilon c^2 \mu x^2 y^2 z^4 + 12 \varepsilon c^2 \mu x^4 y^2 z^2 - 2 a^4 x^4 - 2 a^4 y^4 - 4 a^4 y^2 x^2) \big/ (\mu \%1^3), 0]$

$\%1 := x^4 + 2 x^2 y^2 + 2 x^2 z^2 + y^4 + 2 y^2 z^2 + z^4 + a^2 z^2$

*chiralty AdotD* $= 0$

*LaGrange field energy density (B.H-D.E)* $= - e^2 (-4 a^6 z^6 - 9 a^6 y^2 z^4 - 4 a^6 z^2 y^4 - 8 a^4 z^8 - 4 a^2 z^{10} - 17 x^2 z^8 a^2$
$- 22 x^2 z^6 a^4 - 28 x^4 z^6 a^2 - 22 x^6 z^4 a^2 - 20 x^4 z^4 a^4 - 8 x^8 z^2 a^2 - 6 x^6 z^2 a^4 - 17 y^2 z^8 a^2 - 8 y^8 z^2 a^2 - 22 y^2 z^6 a^4$
$- 22 y^6 z^4 a^2 - 28 y^4 z^6 a^2 - 20 y^4 z^4 a^4 - 6 y^6 z^2 a^4 - 32 x^2 z^2 a^2 y^6 - 66 x^2 z^4 y^4 a^2 - 32 x^6 z^2 a^2 y^2 - 56 x^2 z^6 y^2 a^2$
$- 40 x^2 z^4 a^4 y^2 - 66 x^4 z^4 y^2 a^2 - 48 x^4 z^2 a^2 y^4 - 18 x^2 z^2 a^4 y^4 - 18 x^4 z^2 a^4 y^2 - 8 a^6 x^2 y^2 z^2 - 4 a^6 x^4 z^2 - 9 a^6 x^2 z^4$
$- 10 a^2 x^4 y^6 - 10 a^2 x^6 y^4 - 5 a^2 x^2 y^8 - 5 a^2 x^8 y^2 - a^2 y^{10} - a^2 x^{10} - 2 \varepsilon c^2 \mu x^8 z^2 a^2 + 30 \varepsilon c^2 \mu x^2 z^2 y^8$
$- 8 \varepsilon c^2 \mu x^6 z^4 a^2 + 9 \varepsilon c^2 \mu x^4 z^4 a^4 - 12 \varepsilon c^2 \mu x^4 z^6 a^2 + 60 \varepsilon c^2 \mu x^6 z^2 y^4 + \varepsilon c^2 \mu z^{12} + 60 \varepsilon c^2 \mu x^4 z^2 y^6$
$- 8 \varepsilon c^2 \mu x^6 z^2 a^2 y^2 + 60 \varepsilon c^2 \mu x^4 z^6 y^2 + 30 \varepsilon c^2 \mu x^2 z^8 y^2 + 60 \varepsilon c^2 \mu x^2 z^6 y^4 + 60 \varepsilon c^2 \mu x^2 z^4 y^6 + 6 \varepsilon c^2 \mu x^2 z^6 a^4$
$- 8 \varepsilon c^2 \mu x^2 z^8 a^2 + 6 \varepsilon c^2 \mu x^{10} z^2 + 20 \varepsilon c^2 \mu x^6 z^6 + 6 \varepsilon c^2 \mu x^2 z^{10} + 15 \varepsilon c^2 \mu x^8 z^4 + 15 \varepsilon c^2 \mu x^4 z^8 + 15 \varepsilon c^2 \mu x^8 y^4$
$+ 6 \varepsilon c^2 \mu y^2 z^{10} + 20 \varepsilon c^2 \mu x^6 y^6 + 6 \varepsilon c^2 \mu x^{10} y^2 + 15 \varepsilon c^2 \mu x^4 y^8 + 15 \varepsilon c^2 \mu y^4 z^8 + 6 \varepsilon c^2 \mu y^{10} z^2 + 20 \varepsilon c^2 \mu y^6 z^6$
$- 2 \varepsilon c^2 \mu a^2 z^{10} + 15 \varepsilon c^2 \mu y^8 z^4 + 6 \varepsilon c^2 \mu y^{10} x^2 + \varepsilon c^2 \mu a^4 z^8 - 24 \varepsilon c^2 \mu x^4 z^4 y^2 a^2 + 12 \varepsilon c^2 \mu x^4 z^2 a^4 y^2$
$+ 12 \varepsilon c^2 \mu x^2 z^2 a^4 y^4 - 12 \varepsilon c^2 \mu x^4 z^2 a^2 y^4 + 4 \varepsilon c^2 \mu y^6 z^2 a^4 - 24 \varepsilon c^2 \mu x^2 z^6 y^2 a^2 + 18 \varepsilon c^2 \mu x^2 z^4 a^4 y^2$
$+ 90 \varepsilon c^2 \mu x^4 z^4 y^4 + 9 \varepsilon c^2 \mu y^4 z^4 a^4 - 12 \varepsilon c^2 \mu y^4 z^6 a^2 - 8 \varepsilon c^2 \mu x^2 z^2 a^2 y^6 - 8 \varepsilon c^2 \mu y^6 z^4 a^2 - 2 \varepsilon c^2 \mu y^8 z^2 a^2$
$+ 6 \varepsilon c^2 \mu y^2 z^6 a^4 - 8 \varepsilon c^2 \mu y^2 z^8 a^2 + 4 \varepsilon c^2 \mu x^6 z^2 a^4 - 24 \varepsilon c^2 \mu x^2 z^4 y^4 a^2 + 60 \varepsilon c^2 \mu x^6 z^4 y^2 + \varepsilon c^2 \mu x^{12}$
$+ 30 \varepsilon c^2 \mu x^8 z^2 y^2 + \varepsilon c^2 \mu y^{12}) \big/ ((x^4 + 2 x^2 y^2 + 2 x^2 z^2 + y^4 + 2 y^2 z^2 + z^4 + a^2 z^2)^4 \mu)$

*Interaction energy density (A.J-rho.phi)* $= -2 e^2 (x^2 + y^2 + z^2) a^2 (x^4 + 2 x^2 y^2 + 2 x^2 z^2 + y^4 + 2 y^2 z^2 + z^4 - 3 a^2 z^2)$
$(-a^2 x^2 - a^2 y^2 + \varepsilon c^2 \mu x^4 + 2 \varepsilon c^2 \mu x^2 y^2 + 2 \varepsilon c^2 \mu x^2 z^2 + \varepsilon c^2 \mu y^4 + 2 \varepsilon c^2 \mu y^2 z^2 + \varepsilon c^2 \mu z^4) \big/ ($
$(x^4 + 2 x^2 y^2 + 2 x^2 z^2 + y^4 + 2 y^2 z^2 + z^4 + a^2 z^2)^4 \mu)$

*Poincare 1 (B.H-D.E)-(A.J-rho.phi)* $= - e^2 (-4 a^6 z^6 - 15 a^6 y^2 z^4 + 2 a^4 y^8 - 10 a^6 z^2 y^4 + 8 a^4 x^2 y^6 + 2 a^4 x^8$
$- 20 a^2 x^6 \varepsilon c^2 \mu y^4 + 8 a^4 x^6 y^2 - 10 a^2 x^2 y^8 \varepsilon c^2 \mu - 20 a^2 x^4 y^6 \varepsilon c^2 \mu - 2 a^2 x^{10} \varepsilon c^2 \mu - 2 a^2 \varepsilon c^2 \mu y^{10}$
$- 10 a^2 x^8 \varepsilon c^2 \mu y^2 + 12 a^4 x^4 y^4 - 8 a^4 z^8 - 4 a^2 z^{10} - 17 x^2 z^8 a^2 - 20 x^2 z^6 a^4 - 28 x^4 z^6 a^2 - 22 x^6 z^4 a^2 - 14 x^4 z^4 a^4$
$- 8 x^8 z^2 a^2 - 17 y^2 z^8 a^2 - 8 y^8 z^2 a^2 - 20 y^2 z^6 a^4 - 22 y^6 z^4 a^2 - 28 y^4 z^6 a^2 - 14 y^4 z^4 a^4 - 32 x^2 z^2 a^2 y^6 - 66 x^2 z^4 y^4 a^2$
$- 32 x^6 z^2 a^2 y^2 - 56 x^2 z^6 y^2 a^2 - 28 x^2 z^4 a^4 y^2 - 66 x^4 z^4 y^2 a^2 - 48 x^4 z^2 a^2 y^4 - 20 a^6 x^2 y^2 z^2 - 10 a^6 x^4 z^2 - 15 a^6 x^2 z^4$
$- 10 a^2 x^4 y^6 - 10 a^2 x^6 y^4 - 5 a^2 x^2 y^8 - 5 a^2 x^8 y^2 - a^2 y^{10} - a^2 x^{10} - 12 \varepsilon c^2 \mu x^8 z^2 a^2 + 30 \varepsilon c^2 \mu x^2 z^2 y^8$
$- 28 \varepsilon c^2 \mu x^6 z^4 a^2 + 27 \varepsilon c^2 \mu x^4 z^4 a^4 - 32 \varepsilon c^2 \mu x^4 z^6 a^2 + 60 \varepsilon c^2 \mu x^6 z^2 y^4 + \varepsilon c^2 \mu z^{12} + 60 \varepsilon c^2 \mu x^4 z^2 y^6$
$- 48 \varepsilon c^2 \mu x^6 z^2 a^2 y^2 + 60 \varepsilon c^2 \mu x^4 z^6 y^2 + 30 \varepsilon c^2 \mu x^2 z^8 y^2 + 60 \varepsilon c^2 \mu x^2 z^6 y^4 + 60 \varepsilon c^2 \mu x^2 z^4 y^6 + 24 \varepsilon c^2 \mu x^2 z^6 a^4$
$- 18 \varepsilon c^2 \mu x^2 z^8 a^2 + 6 \varepsilon c^2 \mu x^{10} z^2 + 20 \varepsilon c^2 \mu x^6 z^6 + 6 \varepsilon c^2 \mu x^2 z^{10} + 15 \varepsilon c^2 \mu x^8 z^4 + 15 \varepsilon c^2 \mu x^4 z^8 + 15 \varepsilon c^2 \mu x^8 y^4$
$+ 6 \varepsilon c^2 \mu y^2 z^{10} + 20 \varepsilon c^2 \mu x^6 y^6 + 6 \varepsilon c^2 \mu x^{10} y^2 + 15 \varepsilon c^2 \mu x^4 y^8 + 15 \varepsilon c^2 \mu y^4 z^8 + 6 \varepsilon c^2 \mu y^{10} z^2 + 20 \varepsilon c^2 \mu y^6 z^6$
$- 4 \varepsilon c^2 \mu a^2 z^{10} + 15 \varepsilon c^2 \mu y^8 z^4 + 6 \varepsilon c^2 \mu y^{10} x^2 + 7 \varepsilon c^2 \mu a^4 z^8 - 84 \varepsilon c^2 \mu x^4 z^4 y^2 a^2 + 30 \varepsilon c^2 \mu x^4 z^2 a^4 y^2$
$+ 30 \varepsilon c^2 \mu x^2 z^2 a^4 y^4 - 72 \varepsilon c^2 \mu x^4 z^2 a^2 y^4 + 10 \varepsilon c^2 \mu y^6 z^2 a^4 - 64 \varepsilon c^2 \mu x^2 z^6 y^2 a^2 + 54 \varepsilon c^2 \mu x^2 z^4 a^4 y^2$
$+ 90 \varepsilon c^2 \mu x^4 z^4 y^4 + 27 \varepsilon c^2 \mu y^4 z^4 a^4 - 32 \varepsilon c^2 \mu y^4 z^6 a^2 - 48 \varepsilon c^2 \mu x^2 z^2 a^2 y^6 - 28 \varepsilon c^2 \mu y^6 z^4 a^2 - 12 \varepsilon c^2 \mu y^8 z^2 a^2$
$+ 24 \varepsilon c^2 \mu y^2 z^6 a^4 - 18 \varepsilon c^2 \mu y^2 z^8 a^2 + 10 \varepsilon c^2 \mu x^6 z^2 a^4 - 84 \varepsilon c^2 \mu x^2 z^4 y^4 a^2 + 60 \varepsilon c^2 \mu x^6 z^4 y^2 + \varepsilon c^2 \mu x^{12}$
$+ 30 \varepsilon c^2 \mu x^8 z^2 y^2 + \varepsilon c^2 \mu y^{12}) \big/ ((x^4 + 2 x^2 y^2 + 2 x^2 z^2 + y^4 + 2 y^2 z^2 + z^4 + a^2 z^2)^4 \mu)$

*Virtual work* $= [2 e^2 a^2 \%2\, x (\varepsilon c^2 \mu z^8 + \varepsilon c^2 \mu y^8 + \varepsilon c^2 \mu x^8 + 2 z^6 a^2 - a^2 y^6 + 2 a^4 z^4 + 3 x^2 z^4 a^2 + 3 a^4 x^2 z^2$
$- 3 a^2 x^4 y^2 + 3 y^2 z^4 a^2 + 3 a^4 y^2 z^2 - 6 \varepsilon c^2 \mu x^2 y^2 a^2 z^2 + 4 \varepsilon c^2 \mu x^6 y^2 + 4 \varepsilon c^2 \mu x^2 z^6 + 6 \varepsilon c^2 \mu x^4 y^4 + 4 \varepsilon c^2 \mu x^2 y^6$
$+ 6 \varepsilon c^2 \mu x^4 z^4 + 4 \varepsilon c^2 \mu y^2 z^6 + 6 \varepsilon c^2 \mu y^4 z^4 + 4 \varepsilon c^2 \mu y^6 z^2 - 3 \varepsilon c^2 \mu z^6 a^2 + 4 \varepsilon c^2 \mu x^6 z^2 - a^2 x^6 - 3 a^2 x^2 y^4$

$$- 3\,\varepsilon\,c^2\,\mu\,y^4\,a^2\,z^2 - 3\,\varepsilon\,c^2\,\mu\,x^4\,a^2\,z^2 - 6\,\varepsilon\,c^2\,\mu\,x^2\,z^4\,a^2 + 12\,\varepsilon\,c^2\,\mu\,x^2\,y^4\,z^2 + 12\,\varepsilon\,c^2\,\mu\,x^2\,y^2\,z^4 + 12\,\varepsilon\,c^2\,\mu\,x^4\,y^2\,z^2$$
$$- 6\,\varepsilon\,c^2\,\mu\,y^2\,z^4\,a^2) \,\big/\, (\mu\,\%1^5),\ 2\,e^2\,a^2\,\%2\,y\,(\varepsilon\,c^2\,\mu\,z^8 + \varepsilon\,c^2\,\mu\,y^8 + \varepsilon\,c^2\,\mu\,x^8 + 2\,z^6\,a^2 - a^2\,y^6 + 2\,a^4\,z^4 + 3\,x^2\,z^4\,a^2$$
$$+ 3\,a^4\,x^2\,z^2 - 3\,a^2\,x^4\,y^2 + 3\,y^2\,z^4\,a^2 + 3\,a^4\,y^2\,z^2 - 6\,\varepsilon\,c^2\,\mu\,x^2\,y^2\,a^2\,z^2 + 4\,\varepsilon\,c^2\,\mu\,x^6\,y^2 + 4\,\varepsilon\,c^2\,\mu\,x^2\,z^6 + 6\,\varepsilon\,c^2\,\mu\,x^4\,y^4$$
$$+ 4\,\varepsilon\,c^2\,\mu\,x^2\,y^6 + 6\,\varepsilon\,c^2\,\mu\,x^4\,z^4 + 4\,\varepsilon\,c^2\,\mu\,y^2\,z^6 + 6\,\varepsilon\,c^2\,\mu\,y^4\,z^4 + 4\,\varepsilon\,c^2\,\mu\,y^6\,z^2 - 3\,\varepsilon\,c^2\,\mu\,z^6\,a^2 + 4\,\varepsilon\,c^2\,\mu\,x^6\,z^2 - a^2\,x^6$$
$$- 3\,a^2\,x^2\,y^4 - 3\,\varepsilon\,c^2\,\mu\,y^4\,a^2\,z^2 - 3\,\varepsilon\,c^2\,\mu\,x^4\,a^2\,z^2 - 6\,\varepsilon\,c^2\,\mu\,x^2\,z^4\,a^2 + 12\,\varepsilon\,c^2\,\mu\,x^2\,y^4\,z^2 + 12\,\varepsilon\,c^2\,\mu\,x^2\,y^2\,z^4$$
$$+ 12\,\varepsilon\,c^2\,\mu\,x^4\,y^2\,z^2 - 6\,\varepsilon\,c^2\,\mu\,y^2\,z^4\,a^2) \,\big/\, (\mu\,\%1^5),\ 2\,e^2\,a^2\,\%2\,z\,(\varepsilon\,c^2\,\mu\,z^8 + \varepsilon\,c^2\,\mu\,y^8 + \varepsilon\,c^2\,\mu\,x^8 - 3\,a^2\,y^6 - 3\,x^2\,z^4\,a^2$$
$$- 6\,x^4\,a^2\,z^2 - 6\,y^4\,a^2\,z^2 - a^4\,x^2\,z^2 - 9\,a^2\,x^4\,y^2 - 3\,y^2\,z^4\,a^2 + 6\,\varepsilon\,c^2\,\mu\,a^2\,x^4\,y^2 + 6\,\varepsilon\,c^2\,\mu\,a^2\,x^2\,y^4 + 2\,\varepsilon\,c^2\,\mu\,a^2\,x^6$$
$$+ 2\,\varepsilon\,c^2\,\mu\,a^2\,y^6 - a^4\,y^2\,z^2 - 12\,x^2\,y^2\,a^2\,z^2 + 6\,\varepsilon\,c^2\,\mu\,x^2\,y^2\,a^2\,z^2 + 4\,\varepsilon\,c^2\,\mu\,x^6\,y^2 + 4\,\varepsilon\,c^2\,\mu\,x^2\,z^6 + 6\,\varepsilon\,c^2\,\mu\,x^4\,y^4$$
$$+ 4\,\varepsilon\,c^2\,\mu\,x^2\,y^6 + 6\,\varepsilon\,c^2\,\mu\,x^4\,z^4 + 4\,\varepsilon\,c^2\,\mu\,y^2\,z^6 + 6\,\varepsilon\,c^2\,\mu\,y^4\,z^4 + 4\,\varepsilon\,c^2\,\mu\,y^6\,z^2 - \varepsilon\,c^2\,\mu\,z^6\,a^2 + 4\,\varepsilon\,c^2\,\mu\,x^6\,z^2 - 3\,a^2\,x^6$$
$$- 9\,a^2\,x^2\,y^4 + 3\,\varepsilon\,c^2\,\mu\,y^4\,a^2\,z^2 + 3\,\varepsilon\,c^2\,\mu\,x^4\,a^2\,z^2 + 12\,\varepsilon\,c^2\,\mu\,x^2\,y^4\,z^2 + 12\,\varepsilon\,c^2\,\mu\,x^2\,y^2\,z^4 + 12\,\varepsilon\,c^2\,\mu\,x^4\,y^2\,z^2 - 2\,a^4\,x^4$$
$$- 2\,a^4\,y^4 - 4\,a^4\,y^2\,x^2) \,\big/\, (\mu\,\%1^5)]$$

$$\%1 := x^4 + 2\,x^2\,y^2 + 2\,x^2\,z^2 + y^4 + 2\,y^2\,z^2 + z^4 + a^2\,z^2$$

$$\%2 := x^4 + 2\,x^2\,y^2 + 2\,x^2\,z^2 + y^4 + 2\,y^2\,z^2 + z^4 - 3\,a^2\,z^2$$

$$JdotE\ power\ = 0$$

**Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.**

```
> NAME:=`Example A-- Hopf signature index 0. The 1-form is divided by the Holder
  norm p=2, n=4 `;
> ff:=1;p:=2;n:=4;
> lambda:=(x^p+y^p+z^p+(c*t)^p)^(n/p);
> Ax:=y*ff/lambda;Ay:=-x*ff/lambda;Az:=c*t*ff/lambda;phi:=z*c*ff/lambda;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*****************************************************************

*NAME := Example A-- Hopf signature index 0. The 1-form is divided by the Holder norm p=2, n=4*

$$ff := 1$$

$$p := 2$$

$$n := 4$$

$$\lambda := (x^2 + y^2 + z^2 + c^2\,t^2)^2$$

$$Ax := \frac{y}{(x^2 + y^2 + z^2 + c^2\,t^2)^2}$$

$$Ay := -\frac{x}{(x^2 + y^2 + z^2 + c^2\,t^2)^2}$$

$$Az := \frac{c\,t}{(x^2 + y^2 + z^2 + c^2\,t^2)^2}$$

$$\phi := \frac{z\,c}{(x^2 + y^2 + z^2 + c^2\,t^2)^2}$$

*Example A-- Hopf signature index 0. The 1-form is divided by the Holder norm p=2, n=4*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \frac{y\,\mathrm{d}(x)}{\%1^2} - \frac{x\,\mathrm{d}(y)}{\%1^2} + \frac{c\,t\,\mathrm{d}(z)}{\%1^2} - \frac{z\,c\,\mathrm{d}(t)}{\%1^2}$$

$$\%1 := x^2 + y^2 + z^2 + c^2 t^2$$

$$E\ field = \left[ 4 \frac{c\,(x\,z + y\,c\,t)}{(x^2 + y^2 + z^2 + c^2 t^2)^3}, -4 \frac{c\,(-z\,y + x\,c\,t)}{(x^2 + y^2 + z^2 + c^2 t^2)^3}, -2 \frac{c\,(x^2 + y^2 - z^2 - c^2 t^2)}{(x^2 + y^2 + z^2 + c^2 t^2)^3} \right]$$

$$B\ field = \left[ -4 \frac{x\,z + y\,c\,t}{(x^2 + y^2 + z^2 + c^2 t^2)^3}, 4 \frac{-z\,y + x\,c\,t}{(x^2 + y^2 + z^2 + c^2 t^2)^3}, 2 \frac{x^2 + y^2 - z^2 - c^2 t^2}{(x^2 + y^2 + z^2 + c^2 t^2)^3} \right]$$

$$Topological\ Torsion = \left[ 2 \frac{c\,x}{\%1^4}, 2 \frac{c\,y}{\%1^4}, 2 \frac{c\,z}{\%1^4}, 2 \frac{c\,t}{\%1^4} \right]$$

$$\%1 := x^2 + y^2 + z^2 + c^2 t^2$$

$$Helicity\ AdotB = 2 \frac{c\,t}{(x^2 + y^2 + z^2 + c^2 t^2)^4}$$

$$Poincare\ 2\ E.B = -8 \frac{c}{(x^2 + y^2 + z^2 + c^2 t^2)^4}$$

$$D\ field = \left[ 4 \frac{\varepsilon\,c\,(x\,z + y\,c\,t)}{(x^2 + y^2 + z^2 + c^2 t^2)^3}, -4 \frac{\varepsilon\,c\,(-z\,y + x\,c\,t)}{(x^2 + y^2 + z^2 + c^2 t^2)^3}, -2 \frac{\varepsilon\,c\,(x^2 + y^2 - z^2 - c^2 t^2)}{(x^2 + y^2 + z^2 + c^2 t^2)^3} \right]$$

$$H\ field = \left[ -4 \frac{x\,z + y\,c\,t}{(x^2 + y^2 + z^2 + c^2 t^2)^3\,\mu}, 4 \frac{-z\,y + x\,c\,t}{(x^2 + y^2 + z^2 + c^2 t^2)^3\,\mu}, 2 \frac{x^2 + y^2 - z^2 - c^2 t^2}{(x^2 + y^2 + z^2 + c^2 t^2)^3\,\mu} \right]$$

$$Poynting\ vector\ ExH = [0, 0, 0]$$

$$Current\ density = \left[ -4 \frac{(x^2\,y + y^3 + y\,z^2 - 5\,y\,c^2 t^2 - 6\,c\,t\,x\,z)\,(1 + \varepsilon\,c^2\,\mu)}{(x^2 + y^2 + z^2 + c^2 t^2)^4\,\mu}, \right.$$
$$\left. 4 \frac{(x^3 + x\,y^2 + x\,z^2 - 5\,x\,c^2 t^2 + 6\,c\,t\,z\,y)\,(1 + \varepsilon\,c^2\,\mu)}{(x^2 + y^2 + z^2 + c^2 t^2)^4\,\mu}, -8 \frac{c\,t\,(2\,x^2 + 2\,y^2 - z^2 - c^2 t^2)\,(1 + \varepsilon\,c^2\,\mu)}{(x^2 + y^2 + z^2 + c^2 t^2)^4\,\mu} \right]$$

$$charge\ density = 0$$

$$Topological\ SPIN = \left[ 2 \frac{-x^3 - x\,y^2 + x\,z^2 - x\,c^2 t^2 + 2\,c\,t\,z\,y + 2\,\varepsilon\,c^2 z^2\,\mu\,x + 2\,\varepsilon\,c^3\,z\,\mu\,y\,t}{\%1^5\,\mu}, \right.$$
$$-2 \frac{2\,c\,t\,x\,z + y\,c^2 t^2 + x^2\,y + y^3 - y\,z^2 - 2\,\varepsilon\,c^2 z^2\,\mu\,y + 2\,\varepsilon\,c^3\,z\,\mu\,x\,t}{\%1^5\,\mu},$$
$$\left. -2 \frac{z\,(2\,y^2 + 2\,x^2 + \varepsilon\,c^2\,\mu\,x^2 + \varepsilon\,c^2\,\mu\,y^2 - \varepsilon\,c^2 z^2\,\mu - \varepsilon\,c^4\,\mu\,t^2)}{\%1^5\,\mu}, 2 \frac{\varepsilon\,c^2\,t}{\%1^4} \right]$$

$$\%1 := x^2 + y^2 + z^2 + c^2 t^2$$

$$chiralty\ AdotD = 2 \frac{\varepsilon\,c^2\,t}{(x^2 + y^2 + z^2 + c^2 t^2)^4}$$

$$LaGrange\ field\ energy\ density\ (B.H\text{-}D.E) = -4 \frac{-1 + \varepsilon\,c^2\,\mu}{(x^2 + y^2 + z^2 + c^2 t^2)^4\,\mu}$$

$$Interaction\ energy\ density\ (A.J\text{-}rho.phi) = -4 \frac{(x^2 + y^2 - 2\,c^2 t^2)\,(1 + \varepsilon\,c^2\,\mu)}{(x^2 + y^2 + z^2 + c^2 t^2)^5\,\mu}$$

$$Poincare\ 1\ (B.H\text{-}D.E)\text{-}(A.J\text{-}rho.phi) = -4 \frac{-2\,x^2 - 2\,y^2 - z^2 + c^2 t^2 + \varepsilon\,c^2 z^2\,\mu + 3\,\varepsilon\,c^4\,\mu\,t^2}{(x^2 + y^2 + z^2 + c^2 t^2)^5\,\mu}$$

$$Virtual\ work = \left[ 8\,\frac{(x^3 + x\,y^2 + x\,c^2\,t^2 - x\,z^2 - 2\,c\,t\,z\,y)\,(1 + \varepsilon\,c^2\,\mu)}{\mu\,(x^2 + y^2 + z^2 + c^2\,t^2)^6}, \right.$$

$$\left. 8\,\frac{(x^2\,y + 2\,c\,t\,x\,z + y^3 + y\,c^2\,t^2 - y\,z^2)\,(1 + \varepsilon\,c^2\,\mu)}{\mu\,(x^2 + y^2 + z^2 + c^2\,t^2)^6}, 16\,\frac{(x^2 + y^2)\,z\,(1 + \varepsilon\,c^2\,\mu)}{\mu\,(x^2 + y^2 + z^2 + c^2\,t^2)^6} \right]$$

$$JdotE\ power = 16\,\frac{t\,c^2\,(1 + \varepsilon\,c^2\,\mu)}{\mu\,(x^2 + y^2 + z^2 + c^2\,t^2)^5}$$

**Enter the name of the problem, and the components of the 4 potential, and the Holder norm indices.**

```
> NAME:=`Example B-- Hopf signature index 1. The 1-form is divided by the Holder
  norm p=2, n=4 `;
> ff:=1;p:=2;n:=4;
> lambda:=(x^p+y^p+z^p-(c*t)^p)^(n/p);
> Ax:=y*ff/lambda;Ay:=-x*ff/lambda;Az:=c*t*ff/lambda;phi:=z*c*ff/lambda;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

**********************************************************

*NAME := Example B-- Hopf signature index 1. The 1-form is divided by the Holder norm p=2, n=4*

$$ff := 1$$

$$p := 2$$

$$n := 4$$

$$\lambda := (x^2 + y^2 + z^2 - c^2\,t^2)^2$$

$$Ax := \frac{y}{(x^2 + y^2 + z^2 - c^2\,t^2)^2}$$

$$Ay := -\frac{x}{(x^2 + y^2 + z^2 - c^2\,t^2)^2}$$

$$Az := \frac{c\,t}{(x^2 + y^2 + z^2 - c^2\,t^2)^2}$$

$$\phi := \frac{z\,c}{(x^2 + y^2 + z^2 - c^2\,t^2)^2}$$

*Example B-- Hopf signature index 1. The 1-form is divided by the Holder norm p=2, n=4*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \frac{y\,d(x)}{\%1^2} - \frac{x\,d(y)}{\%1^2} + \frac{c\,t\,d(z)}{\%1^2} - \frac{z\,c\,d(t)}{\%1^2}$$

$$\%1 := x^2 + y^2 + z^2 - c^2\,t^2$$

$$E\ field = \left[ 4\,\frac{c\,(x\,z - y\,c\,t)}{(x^2 + y^2 + z^2 - c^2\,t^2)^3}, 4\,\frac{c\,(z\,y + x\,c\,t)}{(x^2 + y^2 + z^2 - c^2\,t^2)^3}, -2\,\frac{c\,(x^2 + y^2 - z^2 + c^2\,t^2)}{(x^2 + y^2 + z^2 - c^2\,t^2)^3} \right]$$

$$B\ field = \left[ -4\,\frac{x\,z + y\,c\,t}{(x^2 + y^2 + z^2 - c^2\,t^2)^3}, 4\,\frac{-z\,y + x\,c\,t}{(x^2 + y^2 + z^2 - c^2\,t^2)^3}, 2\,\frac{x^2 + y^2 - z^2 + c^2\,t^2}{(x^2 + y^2 + z^2 - c^2\,t^2)^3} \right]$$

$$Topological\ Torsion = \left[ 2\,\frac{c\,x}{\%1^4}, 2\,\frac{c\,y}{\%1^4}, 2\,\frac{c\,z}{\%1^4}, 2\,\frac{c\,t}{\%1^4} \right]$$

$$\%1 := x^2 + y^2 + z^2 - c^2 t^2$$

$$\mathit{Helicity\ AdotB} = 2 \frac{c\, t}{(x^2 + y^2 + z^2 - c^2 t^2)^4}$$

$$\mathit{Poincare\ 2\ E.B} = -8 \frac{c}{(x^2 + y^2 + z^2 - c^2 t^2)^4}$$

$$\mathit{D\ field} = \left[ 4 \frac{\varepsilon\, c\, (x\, z - y\, c\, t)}{(x^2 + y^2 + z^2 - c^2 t^2)^3}, 4 \frac{\varepsilon\, c\, (z\, y + x\, c\, t)}{(x^2 + y^2 + z^2 - c^2 t^2)^3}, -2 \frac{\varepsilon\, c\, (x^2 + y^2 - z^2 + c^2 t^2)}{(x^2 + y^2 + z^2 - c^2 t^2)^3} \right]$$

$$\mathit{H\ field} = \left[ -4 \frac{x\, z + y\, c\, t}{(x^2 + y^2 + z^2 - c^2 t^2)^3\, \mu}, 4 \frac{-z\, y + x\, c\, t}{(x^2 + y^2 + z^2 - c^2 t^2)^3\, \mu}, 2 \frac{x^2 + y^2 - z^2 + c^2 t^2}{(x^2 + y^2 + z^2 - c^2 t^2)^3\, \mu} \right]$$

$$\mathit{Poynting\ vector\ ExH} = \left[ 16 \frac{c^2 (x^2 + y^2 - z^2 + c^2 t^2)\, x\, t}{(x^2 + y^2 + z^2 - c^2 t^2)^6\, \mu}, 16 \frac{c^2 (x^2 + y^2 - z^2 + c^2 t^2)\, y\, t}{(x^2 + y^2 + z^2 - c^2 t^2)^6\, \mu}, 32 \frac{c^2\, t\, z\, (x^2 + y^2)}{(x^2 + y^2 + z^2 - c^2 t^2)^6\, \mu} \right]$$

$$\mathit{Current\ density} = \left[ 4 \frac{(x^2 y + y^3 + y\, z^2 + 5\, y\, c^2 t^2 - 6\, c\, t\, x\, z)\, (-1 + \varepsilon\, c^2 \mu)}{(x^2 + y^2 + z^2 - c^2 t^2)^4\, \mu}, \right.$$

$$\left. -4 \frac{(x^3 + x\, y^2 + x\, z^2 + 5\, x\, c^2 t^2 + 6\, c\, t\, z\, y)\, (-1 + \varepsilon\, c^2 \mu)}{(x^2 + y^2 + z^2 - c^2 t^2)^4\, \mu}, 8 \frac{c\, t\, (2\, x^2 + 2\, y^2 - z^2 + c^2 t^2)\, (-1 + \varepsilon\, c^2 \mu)}{(x^2 + y^2 + z^2 - c^2 t^2)^4\, \mu} \right]$$

$$\mathit{charge\ density} = 0$$

$$\mathit{Topological\ SPIN} = \left[ 2 \frac{-x^3 - x\, y^2 + x\, z^2 - 3\, x\, c^2 t^2 + 2\, c\, t\, z\, y + 2\, \varepsilon\, c^2 z^2 \mu\, x - 2\, \varepsilon\, c^3 z\, \mu\, y\, t}{\%1^5\, \mu}, \right.$$

$$2 \frac{-2\, c\, t\, x\, z - 3\, y\, c^2 t^2 - x^2 y - y^3 + y\, z^2 + 2\, \varepsilon\, c^2 z^2 \mu\, y + 2\, \varepsilon\, c^3 z\, \mu\, x\, t}{\%1^5\, \mu},$$

$$\left. -2 \frac{z\, (2\, y^2 + 2\, x^2 + \varepsilon\, c^2 \mu\, x^2 + \varepsilon\, c^2 \mu\, y^2 - \varepsilon\, c^2 z^2 \mu + \varepsilon\, c^4 \mu\, t^2)}{\%1^5\, \mu}, -2 \frac{\varepsilon\, c^2\, t\, (3\, y^2 + 3\, x^2 - z^2 + c^2 t^2)}{\%1^5} \right]$$

$$\%1 := x^2 + y^2 + z^2 - c^2 t^2$$

$$\mathit{chiralty\ AdotD} = -2 \frac{\varepsilon\, c^2\, t\, (3\, y^2 + 3\, x^2 - z^2 + c^2 t^2)}{(x^2 + y^2 + z^2 - c^2 t^2)^5}$$

*LaGrange field energy density (B.H-D.E)* =

$$-4 \frac{(2\, x^2 z^2 + 6\, y^2 c^2 t^2 + 2\, y^2 z^2 + 6\, x^2 c^2 t^2 + x^4 + 2\, x^2 y^2 + y^4 + z^4 - 2\, z^2 c^2 t^2 + c^4 t^4)\, (-1 + \varepsilon\, c^2 \mu)}{(x^2 + y^2 + z^2 - c^2 t^2)^6\, \mu}$$

*Interaction energy density (A.J-rho.phi)* =

$$4 \frac{(-1 + \varepsilon\, c^2 \mu)\, (2\, x^2 y^2 + y^4 + y^2 z^2 + 9\, y^2 c^2 t^2 + x^4 + x^2 z^2 + 9\, x^2 c^2 t^2 - 2\, z^2 c^2 t^2 + 2\, c^4 t^4)}{(x^2 + y^2 + z^2 - c^2 t^2)^6\, \mu}$$

*Poincare 1 (B.H-D.E)-(A.J-rho.phi)* =

$$-4 \frac{(-1 + \varepsilon\, c^2 \mu)\, (3\, x^2 z^2 + 15\, y^2 c^2 t^2 + 3\, y^2 z^2 + 15\, x^2 c^2 t^2 + 2\, x^4 + 4\, x^2 y^2 + 2\, y^4 + z^4 - 4\, z^2 c^2 t^2 + 3\, c^4 t^4)}{(x^2 + y^2 + z^2 - c^2 t^2)^6\, \mu}$$

$$\mathit{Virtual\ work} = \left[ -8\, (-1 + \varepsilon\, c^2 \mu)\, (x^5 + 2\, x^3 y^2 + 14\, x^3 c^2 t^2 + x\, y^4 + 14\, x\, y^2 c^2 t^2 - x\, z^4 - 8\, x\, z^2 c^2 t^2 + 9\, x\, c^4 t^4 \right.$$

$$- 2\, c\, t\, z\, y\, x^2 - 2\, c\, t\, z\, y^3 - 2\, c\, t\, z^3 y + 2\, c^3 t^3 z\, y) \Big/ ((x^2 + y^2 + z^2 - c^2 t^2)^7\, \mu), -8\, (-1 + \varepsilon\, c^2 \mu)\, (2\, t\, c\, x^3 z + 14\, t^2 c^2 x^2 y$$

$$+ 2\, t\, c\, y^2 x\, z + 14\, t^2 c^2 y^3 + 2\, t\, c\, z^3 x - 8\, t^2 c^2 z^2 y - 2\, t^3 c^3 x\, z + 9\, t^4 c^4 y + x^4 y + 2\, x^2 y^3 + y^5 - y\, z^4) \Big/ ($$

$$(x^2+y^2+z^2-c^2t^2)^7\mu), -16\frac{(-1+\varepsilon c^2\mu)z(x^2+y^2)(x^2+y^2+11c^2t^2+z^2)}{(x^2+y^2+z^2-c^2t^2)^7\mu}\Bigg]$$

*JdotE power* =

$$-16\frac{(-1+\varepsilon c^2\mu)c^2t(c^4t^4+3y^4+8y^2c^2t^2+3x^4+8x^2c^2t^2+4y^2z^2+6x^2y^2-2z^2c^2t^2+z^4+4x^2z^2)}{(x^2+y^2+z^2-c^2t^2)^7\mu}$$

```
> NAME:=`Example 12 -- Black Hole 2 singular vortex ring `;
>
> phi := 1; Ax:=a*y/(x^2+y^2+z^2);Ay := -a*x/(x^2+y^2+z^2);Az:=0;
```

**Then call the procedure JCM(Ax,Ay,Az,phi)**

```
> JCM(Ax,Ay,Az,phi):
```

*****************************************************************

```
>
```

*NAME* := *Example 12 -- Black Hole 2 singular vortex ring*

$$\phi := 1$$

$$Ax := \frac{a\,y}{x^2+y^2+z^2}$$

$$Ay := -\frac{a\,x}{x^2+y^2+z^2}$$

$$Az := 0$$

*Example 12 -- Black Hole 2 singular vortex ring*

*Lorenz constitutive equations, B = mu H, D = epsilon E*

$$Action = \frac{a\,y\,\mathrm{d}(x)}{x^2+y^2+z^2} - \frac{a\,x\,\mathrm{d}(y)}{x^2+y^2+z^2} - \mathrm{d}(t)$$

$$E\ field = [0, 0, 0]$$

$$B\ field = \left[-2\frac{a\,x\,z}{(x^2+y^2+z^2)^2}, -2\frac{a\,y\,z}{(x^2+y^2+z^2)^2}, -2\frac{a\,z^2}{(x^2+y^2+z^2)^2}\right]$$

$$Topological\ Torsion = \left[2\frac{a\,x\,z}{(x^2+y^2+z^2)^2}, 2\frac{a\,y\,z}{(x^2+y^2+z^2)^2}, 2\frac{a\,z^2}{(x^2+y^2+z^2)^2}, 0\right]$$

$$Helicity\ AdotB = 0$$

$$Poincare\ 2\ E.B = 0$$

$$D\ field = [0, 0, 0]$$

$$H\ field = \left[-2\frac{a\,x\,z}{(x^2+y^2+z^2)^2\mu}, -2\frac{a\,y\,z}{(x^2+y^2+z^2)^2\mu}, -2\frac{a\,z^2}{(x^2+y^2+z^2)^2\mu}\right]$$

$$Poynting\ vector\ ExH = [0, 0, 0]$$

$$Current\ density = \left[2\frac{a\,y}{(x^2+y^2+z^2)^2\mu}, -2\frac{a\,x}{(x^2+y^2+z^2)^2\mu}, 0\right]$$

$$charge\ density = 0$$

$$Topological\ SPIN = \left[2\frac{a^2\,x\,z^2}{(x^2+y^2+z^2)^3\mu}, 2\frac{a^2\,y\,z^2}{(x^2+y^2+z^2)^3\mu}, -2\frac{a^2\,z\,(x^2+y^2)}{(x^2+y^2+z^2)^3\mu}, 0\right]$$

$$chiralty\ AdotD = 0$$

$$\textit{LaGrange field energy density (B.H-D.E)} = 4\,\frac{a^2 z^2}{(x^2+y^2+z^2)^3\,\mu}$$

$$\textit{Interaction energy density (A.J-rho.phi)} = 2\,\frac{a^2 (x^2+y^2)}{(x^2+y^2+z^2)^3\,\mu}$$

$$\textit{Poincare 1 (B.H-D.E)-(A.J-rho.phi)} = -2\,\frac{a^2 (-2 z^2+x^2+y^2)}{(x^2+y^2+z^2)^3\,\mu}$$

$$\textit{Virtual work} = \left[4\,\frac{a^2 x z^2}{(x^2+y^2+z^2)^4\,\mu}, 4\,\frac{a^2 y z^2}{(x^2+y^2+z^2)^4\,\mu}, -4\,\frac{a^2 z (x^2+y^2)}{(x^2+y^2+z^2)^4\,\mu}\right]$$

$$\textit{JdotE power} = 0$$

>

>

>