Irreversible Processes, Topological Torsion and Turbulence

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It is a rare thing to attend a conference where on one day a new theoretical statement about thermodynamically irreversible processes, including turbulent flows, is made, and then on the following day experimental evidence is presented to support the abstract theory. At the May 27 meeting of the DTU IUTAM-SIMFLO conference in Copenhagen [1], it was reported that kinematically constrained physical systems which can be defined by a Lagrangian 1-form of Action, A, with non-holonomic differential constraints generate a non-compact symplectic manifold of Pfaff (topological) dimension 2n+2. For each specific physical system, there exists on the symplectic manifold a unique vector field, **T**, defined as the Topological Torsion [2] vector or current. (For mathematical details see the appendix below). According to Cartan's magic formula [3], evolution of the 1-form of Action along the flow lines of the dynamical system generated by the Torsion vector, **T**, creates a 1-form of heat, Q, which does not satisfy the Frobenius integrability condition [4]. Therefore no integrating factor exists and all such processes generated by **T** must be irreversible in a thermodynamic sense.[5] The first occurrence of such irreversible processes is on manifolds of topological dimension greater than 3, giving a non-statistical, thermodynamic, basis for the 4-dimensional foundations of natural processes.

In addition, an example Action 1-form, with constraints that generate the equations describing a barotropic Navier-Stokes fluid, was used to construct a specific realization of the Topological Torsion vector. The example indicated that the Topological Torsion vector for a Navier-Stokes fluid would be represented by lines of vorticity in the form of twisted helices in a space of topological dimension $2n+2 = 4$. The non-zero four dimensional divergence of the Torsion vector generates its signature, $curl\, v \circ curl\; curl\, v \neq 0$, a function [6] whose non-zero values define the domain of support for the 4D symplectic manifold. If the topological dimension of a system defined by a Navier-Stokes fluid is less than 4, then this function must be zero, and the lines of vorticity with a twisted helical signature will not exist. Topological Torsion defects in the Navier-Stokes fluid are represented by the singular sets of this function.

The following day, the Russian scientists, P. A. Kuibin and V. Okulov, [7] presented experimental evidence of, and extensive measurements on, helical twisted lines of vorticity in a swirling fluid. They also presented an interesting analysis of the singular solution sets in an axisymmetric swirling Navier-Stokes fluid, indicating the existence of a coherent structure formed from helical torsional waves of opposite parity, a result also in agreement with the concept of topological torsion.[8] Then on the following day - by private communication to this author - they conveyed the result that their analysis indeed satisfied the signature equation of topological torsion, $curl\, v \circ curl\; curl\, v \neq 0$.

The physical importance of such results is that these experiments lend support to arguments that turbulence and other thermodynamically irreversible processes are inherent artifacts of a space of topological dimension greater than 3. It is remarkable that such hydrodynamic-thermodynamic experiments lead to the irreducible 4-dimensional qualities of nature, yet in a manner entirely different from the concepts generated by the Michelson-Morley experiments.

From a practical point of view for those researchers interested in reducing drag, dissipation and noise, these results should focus attention on the primary problem, that of eliminating or minimizing the source of four dimensional topological torsion defects.

Appendix

Considers those physical systems that can be described by a Lagrange function $L(\mathbf{q}, \mathbf{v}, t)$ and a set of non-holonomic constraints, to yield the integrand of a variational integral, defined as the 1-form of Action:

$$\mathrm{A} = L(\mathbf{q}, \mathbf{v}, t)dt + \mathbf{p} \bullet (d\mathbf{q} - \mathbf{v}dt).$$

At first glance it appears that the domain of definition is a 3n+1 dimensional variety of independent variables, $\{\mathbf{q}, \mathbf{v}, \mathbf{p}, t\}$. Consider $\mathbf{p}$ to be a (set of) Lagrange multiplier(s) to be determined later, and do not assume that $\mathbf{p}$ is constrained to be a jet; e.g., $\mathbf{p} \neq \partial L/\partial \mathbf{v}$.

For the given Action, construct the Pfaff sequence $\{\mathrm{A}, d\mathrm{A}, \mathrm{A}\hat{}d\mathrm{A}, d\mathrm{A}\hat{}d\mathrm{A}...\}$ in order to determine the Pfaff dimension or class of the 1-form. The top (non-zero) Pfaffian of this sequence is given by the formula,

$$(d\mathrm{A})^{n+1} = (n+1)!\{\Sigma_{k=1}^{n}(\partial L/\partial v^k - p_k) \bullet dv^k\}\hat{}dp_1\hat{}...dp_n\hat{}dq^1\hat{}..dq^n\hat{}dt,$$

which indicates that the Pfaff topological dimension is 2n+2 and not the geometrical dimension 3n+1. Similar results are obtained for other forms of non-holonomic constraints: the Top Pfaffian is of dimension 2n+2 and the 2-form $d\mathrm{A}$ generates a non-compact symplectic manifold.

On the 2n+2 domain, the components of 2n+1 form $\mathrm{T} = \mathrm{A}\hat{}(d\mathrm{A})^n$ generate what is herein defined as the Torsion Current, a contravariant vector density, $\mathbf{T}^m$, whose non-zero divergence defines the 2n+2 dimensional domain of support of the non-compact symplectic manifold created by the exact 2-form, $d\mathrm{A}$. That is, $d\mathrm{A}\hat{}(d\mathrm{A})^n = (Div\mathbf{T})\Omega$. The unique (to within a factor) Torsion Current, $\mathbf{T}$, of 2n+2 components, satisfies the equations,

$$\mathrm{L}_{(\mathbf{T})}\mathrm{A} = \Gamma\mathrm{A} \quad and \quad i(\mathbf{T})\mathrm{A} = 0 \quad with\ \Gamma \approx \sigma\, Div\mathbf{T} \neq 0,$$

indicating that evolution of the Action 1-form along the lines of the Torsion current is conformal. Such evolutionary processes on the 2n+2 domain decay exponentially (conformally) to sets of zero divergence, $\Gamma \Rightarrow 0$. These singular sets of measure zero have the properties of an inertial contact manifold of dimension 2n+1. The evolutionary decay in the direction of the Torsion current on the symplectic domain is irreversible in a thermodynamic sense.

To understand what is meant by thermodynamic irreversibility, realize that Cartan's magic formula of topological evolution is equivalent to the first law of thermodynamics.

$$\mathrm{L}_{(\mathbf{V})}\mathrm{A} = i(\mathbf{V})d\mathrm{A} + d(i(\mathbf{V})\mathrm{A}) = W + dU = Q.$$

A is the "Action" 1-form that describes the physical system. $\mathbf{V}$ is the vector field that defines the evolutionary process. W is the 1-form of (virtual) work. Q is the 1-form of heat. From classical thermodynamics, a process is irreversible when the heat 1-form Q does not admit an integrating factor. From the Frobenius theorem, an integrating factor does not exist if $Q\hat{}dQ \neq 0$. Hence a simple test may be made for any process, $\mathbf{V}$, relative to a physical system described by an Action 1-form, A:

$$If\ \ \mathrm{L}_{(\mathbf{v})}\mathrm{A}\hat{}\mathrm{L}_{(\mathbf{v})}d\mathrm{A} \neq 0 \ then\ the\ process\ \mathbf{V}\ \ is\ irreversible.$$

All Symplectic (and Hamiltonian) processes, **S**, are thermodynamically reversible as they satisfy the equation $L_{(\mathbf{S})}dA = dQ = 0.$ However, for processes in the direction of the Torsion vector,

$$L_{(\mathbf{T})}A\hat{}L_{(\mathbf{T})}dA = Q\hat{}dQ = \Gamma^2 A\hat{}dA \neq 0,$$

which implies that such processes are thermodynamically irreversible. The first occurrence of such a result is for an Action whose Pfaff (topological) dimension is 4, indicating the irreducible 4-dimensional qualities of thermodynamically irreversible processes.

For a system with suitable constraints to generate the Navier-Stokes equations for a barotropic fluid on the set {x,y,z,t}, the Topological Torsion vector in engineering format has 4 space time components:

$$\mathbf{T} = \{(\mathbf{v} \circ curl\,\mathbf{v})\mathbf{v} - (\mathbf{v} \circ \mathbf{v}/2)\,curl\,\mathbf{v} - \nu\,curl\,curl\,\mathbf{v};\ \ (\mathbf{v} \circ curl\,\mathbf{v})\},$$

with a 4 divergence given by the expression, $div_4\mathbf{T} = -2\nu\ curl\,\mathbf{v} \circ curl\,curl\,\mathbf{v}$. The compliment of the zero set of this function defines the regions in space-time that form the support of the 4D symplectic manifold and irreversible turbulent processes.

References

[1] R. M. Kiehn, (1997) ”Coherent Structures in Fluids are Deformable Topological Torsion Defects” presented at the IUTAM-SIMFLO Conference at DTU, Denmark, May 25-29, 1997

(Also see http://www.uh.edu./~rkiehn/carfre12.htm)

[2] R. M. Kiehn, (1990) ”Topological Torsion, Pfaff Dimension and Coherent Structures” in: H. K. Moffatt and T. S. Tsinober eds, Topological Fluid Mechanics, (Cambridge University Press), 449-458.

[3] J.E.Marsden and T. S. Riatu ”Introduction to Mechanics and Symmetry” Springer-Verlag, 1994 p.122

[4] H. Flanders, ”Differential Forms” Academic Press, 1965 p.92

[5] P. M. Morse ”Thermal Physics” Benjamin, 1964 p.60

[6] Kiehn, R. M. (1992) ”Topological Defects, Coherent Structures and Turbulence in Terms of Cartan’s Theory of Differential Topology” in Developments in Theoretical and Applied Mathematics, Proceedings of the SECTAM XVI conference, B. N. Antar, R. Engels, A.A. Prinaris and T. H. Moulden, Editors, The University of Tennessee Space Institute, Tullahoma, TN 37388 USA.

[7] S.V. Alekseenko, P.A. Kuibin, V.L. Okulov, S.I. Shtork ”Theoretical and Experimental Investigation of Helical Vortices ” and

P.A. Kuibin, V.L. Okulov ”One Dimensional Solutions for flows with Helical Symmetry” presented at the IUTAM-SIMFLO Conference at DTU, Denmark May 25-29, 1997

[8] R. M. Kiehn, (1991) ”Compact Dissipative Flow Structures with Topological Coherence Embedded in Eulerian Environments”, in: Non-linear Dynamics of Structures, edited by R.Z. Sagdeev, U. Frisch, F. Hussain, S. S. Moiseev and N. S. Erokhin, (World Scientific Press, Singapore) p.139-164.