A REMARK ON THE SYMMETRY BREAKING OF SPACE TIME

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Abstract. The algebraic differences between the space-time signature (-,-,-,+) and the space-time signature (+,+,+,-) suggest that there may be a physical effect associated with such a symmetry breaking.

Introduction

In this journal, almost twenty years ago [1,2], an argument was presented to show how the properties of the four forces in physics could be deduced from the features of the four distinct Pfaffian equivalence classes of differential geometry that can be constructed on a space of four dimensions. The four equivalence classes were determined from the metric solutions, $g_{\mu\nu}$, to the Einstein field equations, by constructing a 1-form of action, A, in terms of the space time, $g_{4\mu}$, components of the metric field: $A = g_{4\mu}dx^{\mu}$. The methods of Pfaff reduction can be used to generate four equivalence classes in terms of the Pfaff dimension, or class, of this 1-form. Summarizing the previous results, the equivalence class of Pfaff dimension 1 class will support long range gravitation (mass) and is parity preserving. The second equivalence class of Pfaff dimension 2 will support both gravity (mass) and electromagnetism (charge) and is to be associated with long range parity preserving forces. The third equivalence class of Pfaff dimension 3 will support both mass and charge, but the forces - although parity preserving-- are of short range. The last equivalence class of Pfaff dimension 4 involved short range interactions that can violate time reversal and symmetry breaking . Examples were given in terms of known solutions to the field equations.

Although the previous methods were motivated by ideas of differential geometry, it is now known that the concepts used to generate the four equivalence classes associated with the four forces are not of a geometrical nature, but instead the equivalence classes have their foundations in topology. Indeed, the older analysis concluded that two of the equivalence classes are to be associated with forces that are *long range*, in the sense of having distance limits going to infinity, while the other two equivalence classes are to be associated with forces that are of *short range*. However, the concept of distance is more of a geometrical idea, not a topological idea.

At the present time of writing this article it is perceived that the true nature of the equivalence classes is based on the topological issue of *connectedness*, and does not reflect the geometrical idea of *distance* necessarily. Following the work of Baldwin, two of the equivalence classes belong to a connected topology (Pfaff dimension 1 and 2), and the other two equivalence classes belong to a disconnected topology (Pfaff dimension 3 and 4). Hence the topological features of the strong and the weak forces do not involve short range, but instead reflect the concepts of accessibility. That is, the topology of the "long range" forces is connected, while the topology of the "short range" forces is disconnected. The topological idea of connectedness is to be exchanged for the geometrical idea of "long range". There is a difference between the concepts of whether or not the point b is not reachable by a continuous process and not reachable in a finite time.

These ideas are most readily understood in terms of the Cartan topology built on a Pfaffian system, and its differential closure. Such an exercise is presented in Appendix A, and is a result of P. Baldwin for a single Pfaffian A [3]. Another method emphasizing the topological features is to realize that the existence of a global 1-form of Action, A, on a space of N+1 dimensions induces a line bundle on the variety N. Intrinsic geometric concepts and certain topological properties can be evaluated in two ways. The first method uses techniques of fiber bundle theory [4], but the second method generates all of the interesting features more simply from the Jacobian matrix of the vector field adjoint to the global 1-form, A. The two even dimensional equivalence classes mentioned in the older article, and discussed in the appendix below, are elements of the Chern characteristic classes for the line bundle. These sets have global properties, and therefore carry topological significance. These concepts of Pfaff equivalence classes have application not only to the microcosm of atoms and elementary particles, as well as the cosmological arena of galaxies, but also to the mundane physics of hydrodynamics. Such methods have been used recently to obtain a better understanding of the production of wake patterns, and the creation and decay of turbulence in fluids.

Signature Symmetry Breaking

However, over the years a new feature of the analysis has appeared, and it is to this new feature that this letter is directed. Note that in the 1975 reference, the signature of the quadratic form was taken to be { +,+,+,- }. The question now arises: Is there a symmetry to be broken if one considers the often used but opposite signature { -,-,-,+ }. The idea is that the wave equation

$$+ \partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2 + \partial^2\psi/\partial z^2 = + (1/c^2)\partial^2\psi/\partial t^2 ,$$

has a set of characteristics which satisfy the partial differential system:

$$+ (\partial\psi/\partial x)^2 + (\partial\psi/\partial y)^2 + (\partial\psi/\partial z)^2 = +(1/c^2)(\partial\psi/\partial t)^2.$$

Hence, there are two ways to write this constraint as an algebraic variety (a null set) :

$$+ (\partial\psi/\partial x)^2 + (\partial\psi/\partial y)^2 + (\partial\psi/\partial z)^2 - (1/c^2)(\partial\psi/\partial t)^2 = 0, \text{ or}$$

$$- (\partial\psi/\partial x)^2 - (\partial\psi/\partial y)^2 - (\partial\psi/\partial z)^2 + (1/c^2)(\partial\psi/\partial t)^2 = 0.$$

Each quadratic form is the complete mirror symmetry (the negative) of the other, but it turns out that the signatures are intrinsically different from a topological point of view in the neighborhood of the null variety.

The analytic question that remains is: Does this symmetry of space time signatures have distinguishable consequences? The physical question is: Are there experiments that can be done to distinguish the symmetry breaking between {-,-,-,+} and {+,+,+,-}?

The analytic answer, based on the idea that the Clifford Algebras of such systems are not isomorphic to one another [5], is yes! The mathematical argument is similar to that used to distinguish the two species of angular momentum algebras in quantum mechanics, an argument which is based on the different signatures of the raising or lowering operators (commutator or anti-commutator brackets) for Bosons vs. Fermions. The fact that the differences in angular momentum signature are physically observable implies that the differences in space-time signatures may also be measurable.

Consider the Clifford Algebra with signature {+,+,+,-}. As discussed in reference [5], this algebra is isomorphic to the algebra of 4x4 matrices with real numbers as matrix elements. This matrix algebra is the usual representation used for waves in 4 dimensions. Next consider the Clifford Algebra with signature {-,-,-,+}. This algebra is isomorphic to the algebra of 2x2 matrices with quaternions as matrix elements. The non-abelian quality of the quaternions makes this algebra have extraordinary differences from the algebra of 4x4 matrices over the real numbers.

This positive analytic result which breaks the symmetry between the two space-time signatures implies there must be a physical difference between the two types of space-time, one with signature {+,+,+,-}, and the other with signature {-,-,-,+}. These differences imply that there exist two species of waves. What are they? A possible answer was first given by Schultz [6] who found exact quaternionic solutions to Maxwell's equations that indicated that the speed of propagation in the inbound and outbound directions would be different for such waves. This result was in agreement with the ring laser experiments of Sanders [7]. These sets of experiments indicated that the electromagnetic four fold degeneracy of the Lorentz equivalence class could be broken such that all four waves of left - right polarizations and of to - fro propagation directions would propagate at four distinct speeds. A further more general analysis on the macroscopic parity and time reversal symmetry breaking effects in electromagnetic systems was presented in reference [8]. The question of whether or not these waves, or the effects of {+,+,+,-} vs. {-,-,-,+} signatures, produce any quantum or hydromechanical effects is open.

APPENDIX A : THE CARTAN TOPOLOGY

Starting in 1899, Cartan [8,9,10,11] developed his theory of exterior differential systems built on the Grassmann algebraic concept of exterior multiplication, and the novel calculus concept of exterior differentiation. These operations are applied to sets called exterior p-forms, which are often described as the objects that form an integrand under the integral sign. The Cartan concepts may still seem unconventional to the engineer, and only during the past few years have they slowly crept into the mainstream of physics. There are several texts at an introductory level that the uninitiated will find useful [4,12,13,14,15,16,17]. A reading of Cartan's many works in the original French will yield a wealth of ideas that have yet to be exploited in the physical sciences. It is not the purpose of this article to provide such a tutorial of Cartan's methods, but suffice it to say the "raison d'être" for these, perhaps unfamiliar but simple and useful, methods is that they permit topological properties of physical systems and processes to be sifted out from the chaff of geometric ideas that, at present, seem to dominate the engineering and physical sciences.

Cartan built his theory around an exterior differential system, Σ, which consists of a collection of 0-forms, 1-forms, 2-forms, etc. [18]. He defined the closure of this collection as the union of the original collection with those forms which are obtained by forming the exterior derivatives of every p-form in the initial collection. In general, the collection of exterior derivatives will be denoted by $d\Sigma$, and the closure of Σ by the symbol, Σ^c, where

$$\Sigma^c = \Sigma \cup d\Sigma. \tag{a1}$$

Cartan's interest in this closure was that founded on the idea that he was able to prove that the system of 1-forms adjoint to the closure were completely integrable. The result allowed him to devise schemes for prolonging a non-integrable system until it became integrable.

For notational simplicity in this article the systems of p-forms will be assumed to consist of the single 1-form, A. Then the exterior derivative of A is the 2-form F = dA, and the closure of A is the union of A and F: $A^c = A \cup F$. The other logical operation is the concept of intersection, so that from the exterior product it is possible to construct the set A^F defined collectively as H: H = A^F. The exterior derivative of H produces the set defined as K = dH, and the closure of H is the union of H and K: $H^c = H \cup K$.

This ladder process of constructing exterior derivatives, and exterior products, may be continued until a null set is produced, or the largest p-form so constructed is equal to the dimension of the space under consideration. The collection of sets so generated is defined as a Pfaff sequence. The largest rank of the sequence determines the Pfaff dimension of the domain (or class of the form, [19], which is a topological invariant.

The idea for evolutionary systems is that the 1-form A (in general the exterior differential system, Σ) generates on space-time, {x,y,z,t}, four equivalence classes of points that act as domains of support for the elements of the Pfaff sequence, { A, F, H, K}. The union of all such points will be denoted by $X = A \cup F \cup H \cup K$. The fundamental equivalence classes are given specific names:

TOPOLOGICAL ACTION	$A = A_\mu dx^\mu$
TOPOLOGICAL VORTICITY	$F = dA = F_{\mu\nu} dx^\mu dx^\nu$
TOPOLOGICAL TORSION	$H = A\wedge dA = H_{\mu\nu\rho} dx^\mu dx^\nu dx^\rho$
TOPOLOGICAL PARITY	$K = dA\wedge dA = K_{\mu\nu\rho\sigma} dx^\mu dx^\nu dx^\rho dx^\sigma$.

The Cartan topology is constructed from a basis of open sets, which are defined as follows: First consider the domain of support of A. Define this "point set" by the symbol A. A is the first open set of the Cartan topology. Next construct the exterior derivative, F

= dA, and determine its domain of support. Next, form the closure of A by constructing the union of these two domains of support, $A \cup F = A^c$. $A \cup F$ forms the second open set of the Cartan topology.

Next construct the intersection H = A^F, and determine its domain of support. Define this "point set" by the symbol H. H forms the third open set of the Cartan topology. Now follow the procedure established in the preceding paragraph. Construct the closure of H as the union of the domains of support of H and K = dH. The construction forms the fourth open set of the Cartan topology. In 4 dimensions, the process stops, but for N > 4, the process may be continued.

Now consider the basis collection of open sets that consists of the subsets,

$$B = \{ A, A^c, H, H^c \} = \{ A, A\cup F, H, H\cup K\}$$

The collection of all possible unions of these base elements, and the null set, {0} generate the Cartan topology of open sets***:***

$$T_{open} = \{ X, 0, A, H, A^c, H^c, A\cup H, A\cup H^c, A^c\cup H\}.$$

These nine subsets form the open point sets of the Cartan topology constructed from the domains of support of the Pfaff sequence {A,F,H,K}.

Subsets σ	**Limit Pts** $d\sigma$	**Interior**	**Boundary** $\partial\sigma$	**Closure** $\sigma\cup d\sigma$
0	0	0	0	0
A	F	A	F	A∪F
F	0	0	F	F
H	K	H	K	H∪K
K	0	0	K	K
A∪F	F	A∪F	0	A∪F
A∪H	F,K	A∪H	F∪K	X
A∪K	F	A	F∪K	A∪F∪K
F∪H	K	H	F∪K	F∪H∪K
F∪K	0	0	F∪K	F∪K
H∪K	K	H∪K	0	H∪K
A∪F∪H	F,K	A∪F∪H	K	X
F∪H∪K	K	H∪K	F	F∪H∪K
A∪H∪K	F,K	A∪H∪K	F	X
A∪F∪K	F	A∪F	K	A∪F∪K
X	F,K	X	0	X

Table 1 Cartan's Topological Structure

The closed sets of the Cartan topology are the compliments of the open sets :

$$T_{closed} = \{ 0, X, F\cup H^c , A^c\cup K, H^c, A^c, F , K \}.$$

It is apparent that the Cartan topology as given in Table 1 is composed of the union of two subsets which are both open and closed ($X = A^c\cup H^c$), a result that implies that the Cartan topology is not connected, unless the Topological Torsion, H, and hence its closure, vanishes. This extraordinary result has a number of physical consequences.

It is possible to compute the limit points for every subset relative to the Cartan topology. The classical definition of a topological limit point is that a point p is a limit point of the subset Y relative to the topology T if and only if for every open set which contains p there exists another point of Y other than p [20]. The results of this definition are presented in Table I which is due to P. Baldwin [3]. Note that the Cartan exterior derivative is a *limit point operator* relative to the Cartan topology. In this sense, the Field Intensities of electromagnetism, **E** and **B**, generated as elements of F = dA, are the limit sets of the potentials, A.

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