

# Hi

Thanks for the Corum thesis. I will study it in more detail. However there are some immediate comments.

1. In the Sagnac effect, the Corum treatment does not admit the 4 fold symmetry breaking that is observed experimentally, and is captured theoretically by the method of exterior differential systems. There exist 4 distinct sets of (measured) beat frequencies dependent upon polarization and direction of propagation.
2. The object of anholonomicity is not the same as the affine connection torsion. However, if one is zero, the other is zero. (See below)
3. Orthogonal frames do not require metric, **orthonormal** frames (Corum preference) require a **metric**. EM theory (Maxwell PDE's) is independent from metric and connection. (VanDantzig, Cartan, Schouten, Cottler, Post, ... (and Kiehn))
4. Affine transformations are transitive, Projective transformations are not. Projective transformations cover Affine transformations (Cayley).
5. Many of the differences in approach are illuminated by the discrete transformations. For example note the differences in Parity and Time reflexive operations between the current (Henley Sakurai) dogma, and the (Post-Schouten) method based on tensor analysis and equivalent to a consistent use (Kiehn) of exterior differential forms (Foundations of Physics 1977).

**Table I Parity and Time inversion behavior of field coefficients.**

Transformation ( <i>P</i> or <i>T</i> )	<i>A</i>	$\phi$	<i>E</i>	<i>B</i>	$\tilde{D}$	$\tilde{H}$	<i>J</i>	$\rho$	$\Phi$	<i>q</i>	<i>m</i>	$S = A^G$	<i>p</i>	<i>V</i>	$T = A^F$	<i>L</i>	
Sakurai-Henley	<i>P</i> :	-	+	-	+	-	+	-	+	+	-	-	-	+	-	+	-
Post (differential forms)	<i>P</i> :	-	+	-	+	-	+	-	+	+	-	-	-	+	-	+	-
Sakurai-Henley	<i>T</i> :	-	+	+	-	+	-	-	+	-	+	+	-	-	-	+	+
Post (differential forms)	<i>T</i> :	+	-	-	+	-	+	+	-	+	-	+	-	-	-	+	+

The Post method demonstrates that charge is a P and T pseudo scalar, and gives enantiomorphic significance to plus and minus charge pairs. This is also the result of Truesdell and Toupin in the Handbuch der Physik.

The Sakurai method (to be consistent) indicates that charge is a P pseudo-scalar but not a T pseudo Scalar. HOWEVER, it is a classic (error) that the prejudicial dogma in use in particle theory often states that charge is a scalar, not a pseudo scalar.!

The ratio of charge to mass is a P scalar in both schemes but not a T scalar in the Post scheme.

6. The Lie derivative (acting on differential forms) IS NOT EQUIVALENT to the covariant derivative (See R. Hermann and Ehresman). The Lie derivative does not depend upon metric or connection. I believe SLEBODZINSKI was the first to coin the word Lie derivative.

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## CLOSURE 2-forms (a component of Cartan Torsion 2-forms)

**The curl format (apparently is favored by Kleinert.)**

Consider the vector array of 1-forms:  $|\sigma^k\rangle = [F_a^k(q^b)] \circ |dq^a\rangle$  and the vector of "closure" 2-forms:

$$|d\sigma^k\rangle = [dF_a^k(q^b)] \wedge |dq^a\rangle \Rightarrow |\{\partial F_a^k/\partial q^b - \partial F_b^k/\partial q^a\} dq^b \wedge dq^a\rangle$$

### The Object of Anholonomicity format ( apparently is favored by Corum.)

Recall the Cartan-Darboux idea that if the 1-forms  $\sigma^k$  are complete, such that the product,  $\sigma^1 \wedge \sigma^2 \wedge \dots \wedge \sigma^N \neq 0$ , then every 2-form  $d\sigma^k$  can be expanded as:  $d\sigma^k = \Omega_{mn}^k \sigma^m \wedge \sigma^n$ . The vector of "closure" 2-forms becomes:

$$|d\sigma^k\rangle = |\Omega_{mn}^k \sigma^m \wedge \sigma^n\rangle = |\Omega_{mn}^k \{(F_b^m F_a^n - F_a^m F_b^n)\} dq^b \wedge dq^a\rangle.$$

The three index symbols  $\Omega_{mn}^k$  form the components of the Object of Anholonomicity.

### The Affine Torsion format ( apparently is favored by Cartan.)

If the basis frame is complete, then there exists a right Cartan matrix of connection 1-forms  $[dF_a^k(q^b)] = [F_c^k(q^b)] \circ [C_{ab}^c dq^b]$  such that the vector of "closure" 2-forms becomes:

$$|d\sigma^k\rangle = [F_c^k(q^b)] \circ |C_{[ab]}^c dq^b \wedge dq^a\rangle$$

The three index symbols  $C_{[ab]}^c$  are the coefficients of the Affine torsion object.

### Summary

Hence there are three equivalent formulations for the vector of closure 2-forms:

$$|d\sigma^k\rangle = |\Omega_{mn}^k \sigma^m \wedge \sigma^n\rangle = |\{\partial F_a^k/\partial q^b - \partial F_b^k/\partial q^a\} dq^b \wedge dq^a\rangle = [F_c^k(q^b)] \circ |C_{[ab]}^c dq^b \wedge dq^a\rangle$$

$$\Omega_{mn}^k \neq C_{[ab]}^c$$

The Affine torsion 3 index symbols are not the same as the Object of Anholonomicity, but all three formulations express the **same** vector of closure 2-forms  $|d\sigma^k\rangle$ . Due to linear independence, a zero of any of the three representations implies a zero for the other representations.

It follows that a zero closure condition (all elements of  $|d\sigma^k\rangle$  are of Pfaff dimension 1) implies zero "curl", zero Affine Torsion and Zero Object of anholonomicity.

However, Zero closure does not imply Exactness.