A Topological Theory of the Physical Vacuum

R. M. Kiehn
Emeritus Professor of Physics
University of Houston
http://www.cartan.pair.com

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Abstract

DRAFT 2

Shipov's concept of a "Physical Vacuum" is extended to include sets that do not require the global features of "Absolute Parallelism". The theory is developed in terms of vectors and matrices of exterior differential forms, which permit the topological coherent structures of fields and particles that make up a "Physical Vacuum", as well as their topological fluctuations, to be readily evaluated in terms of a more or less "universal" theory. The sole requirement is that the "Physical Vacuum" be defined as a vector space of infinitesimal linear neighborhoods whose points admit a vector Basis Frame, as a matrix of C2 functions with non-zero determinant, to be defined over the domain of the "Physical Vacuum". The topological universality of a Basis Frame over infinitesimal neighborhoods can be specialized by particular choices of a subgroup structure. Such specializations appear to include the field structures of all four forces, from gravity fields to Yang Mills fields.

1 Preface

In 1993-1998, Gennady Shipov presented his pioneering concept of a "Physical Vacuum" as a space of "Absolute Parallelism".

The point of departure in this article consists of two parts:

I Shipov's vision that a "Physical Vacuum" is a space of Absolute Parallelism is extended to include a larger set of admissible systems. The larger set is based on the sole requirement that *infinitesimal* neighborhoods of a "Physical Vacuum" are linearly connected as a vector space. The additional (*global*) constraint of "Absolute Parallelism" is not utilized (necessarily). The sole requirement implies that the points $\{y^a\}$ of the "Physical Vacuum" support a matrix of C2 functions, with a non-zero determinant. This matrix of functions is defined as a Basis Frame, $[\mathbb{B}(y^a)]$, for the "Physical Vacuum", and represents the vector space properties of infinitesimal neighborhoods.

II It is recognized that topological coherent structures (fields and particles, along with fluctuations) in a "Physical Vacuum" can be put into correspondence with the concepts of topological thermodynamics. The mathematics will utilize vector and matrix arrays of exterior differential forms (which are automatically covariant with respect to diffeomorphisms). The topological properties of such mathematical structures also can be used to describe topological evolution and change.

2 The fundamentals of the theory in 4D

1. Assume the existence of a matrix array of 0-forms (functions), $[\mathbb{B}]$, with non-zero determinant, and therefore with an inverse matrix $[\mathbb{B}]^{-1}$.

2. The Basis Frame $[\mathbb{B}(y)]$ can be used to map vector arrays of exact differentials $|dy^a\rangle$ into nearby vector arrays of 1-forms $|\sigma^a\rangle$

$$[\mathbb{B}(y)] \circ |dy^a\rangle \;\Rightarrow\; |\sigma^a\rangle\,. \tag{1}$$

3. From the identity $[\mathbb{B}] \circ [\mathbb{B}]^{-1} = [\mathbb{I}]$, use exterior differentiation to derive the (right) Cartan Connection $[\mathbb{C}]$ as a matrix of 1-forms.

$$\begin{aligned}
\text{Right} &: \text{Cartan Connection } [\mathbb{C}] && (2)\\
d\,[\mathbb{B}] &= [\mathbb{B}] \circ [\mathbb{C}]\,, && (3)\\
[\mathbb{C}] &= -d\,[\mathbb{B}]^{-1} \circ [\mathbb{B}] && (4)\\
&= +\,[\mathbb{B}]^{-1} \circ d\,[\mathbb{B}]\,. && (5)
\end{aligned}$$

4. It is also possible to define a left Cartan Connection, $[\mathbf{\Delta}]$,

$$\begin{aligned} \text{Left} &: \quad \text{Cartan Connection } [\mathbf{\Delta}] & (6)\\ d\,[\mathbb{B}] &= [\mathbf{\Delta}] \circ [\mathbb{B}]\,, & (7)\\ [\mathbf{\Delta}] &= -\,[\mathbb{B}] \circ d\,[\mathbb{B}]^{-1} & (8)\\ &= +d\,[\mathbb{B}] \circ [\mathbb{B}]^{-1}\,. & (9) \end{aligned}$$

5. The Right and Left Cartan connections are not (usually) identical. They are equivalent in terms of the similarity transformation:

$$[\mathbb{C}] = [\mathbb{B}]^{-1} \circ [\mathbf{\Delta}] \circ [\mathbb{B}]\,, \tag{10}$$

Also note that inverse matrix also enjoys differential closure properties.

$$\begin{aligned} d\,[\mathbb{B}]^{-1} &= [\mathbb{B}]^{-1} \circ [-\mathbf{\Delta}]\,, & (11)\\ &= [-\mathbb{C}] \circ [\mathbb{B}]^{-1}\,. & (12) \end{aligned}$$

6. If the 1-forms $|\sigma^a\rangle$ were written to include a constant factor of physical dimensions, $\hbar/e$, the resulting 1-forms are formally equivalent to a set of electromagnetic 1-forms of Action (the vector potentials), $|A^a\rangle$ for each index a. For purposes of more rapid comprehension - based on the assumption of familiarity with electromagnetic theory - the Basis Frame infinitesimal mapping formula is rewritten as:

$$[\mathbb{B}(y)] \circ |dy^a\rangle \;\Rightarrow\; |A^a\rangle\,. \tag{13}$$

7. Exterior differentiation of the infinitesimal mapping equation generates a vector of (exact) 2-forms $|F^a\rangle$ which is formally equivalent (for each element, a) to the (pair, or even) 2-form of $\mathbf{E}, \mathbf{B}$ field intensities of electromagnetic theory.

$$\begin{aligned} \text{Vectors of Torsion 2-forms} &: & (14)\\ [\mathbb{B}(y)] \circ [\mathbb{C}]\,\hat{}\,|dy^a\rangle &\Rightarrow |dA^a\rangle = |F^a\rangle\,, & (15)\\ [\mathbb{B}(y)] \circ |G\rangle &= |F\rangle\,. & (16)\\ \text{Field Intensity 2-forms} &= |F^a\rangle & (17)\\ \text{Field Excitation 2-forms} &= \left|G^k\right\rangle. & (18) \end{aligned}$$

8. The vector of 2-forms $\left|G^k\right\rangle = \left[\mathbb{C}^k_a\right]\,\hat{}\,|dy^a\rangle$ defines precisely the vector of Excitation Torsion 2-forms, relative to the connection $[\mathbb{C}]$. It is unfortunate that, historically,

this anti-symmetry property of the coefficients of the Connection has been described as the vector of "affine"[1] Torsion 2-forms. However, the anti-symmetry concept is described by the same formula, $\left|G^k\right\rangle = \left[\mathbb{C}^k_a\right] \hat{} \left|dy^a\right\rangle$, even though the Basis Frame $[\mathbb{B}(y)]$ is not an element of the affine group (of 13 parameters in 4D), or one of its subgroups. For example, the Torsion formula holds equally well for the 15 parameter projective group, which is not affine.

$$\text{(Affine) Torsion 2-forms } \left|G^k\right\rangle = [\mathbb{C}] \hat{} \left|dy^a\right\rangle, \tag{19}$$
$$\text{Vector of Field Excitation 2-forms} = \left|G^k\right\rangle \tag{20}$$
$$= [\mathbb{B}]^{-1} \circ \left|F^a\right\rangle \tag{21}$$

The vector of 2-forms $\left|G^k\right\rangle$ is formally equivalent (for each element k) to the (impair, or odd) 2-form (density) of $\mathbf{D}, \mathbf{H}$ field excitations in electromagnetic theory. The matrix $[\mathbb{B}]^{-1}$ plays the role of the Constitutive map between $\mathbf{E}, \mathbf{B}$ and $\mathbf{D}, \mathbf{H}$. Note that none of this development depends upon the specification of a metric.

9. It is possible to use the left Cartan Connection to define another vector of torsion 2-forms. Exterior differentiation of the infinitesimal mapping equation produces the equation yields

$$[\boldsymbol{\Delta}] \hat{} ([\mathbb{B}(y)] \circ \left|dy^a\right\rangle) \Rightarrow [\boldsymbol{\Delta}] \hat{} \left|\sigma^a\right\rangle = \left|d\sigma^a\right\rangle. \tag{22}$$

The algebraic combination defined below as $\left|\Sigma^a\right\rangle$, is the Cartan vector of Torsion 2-forms. It is not the same as the vector of excitation Torision 2-forms defined by $\left|G^k\right\rangle$.

$$\text{Cartan} : \text{vector of Torsion 2-forms } \left|\Sigma^a\right\rangle \tag{23}$$
$$\left|\Sigma^a\right\rangle = \left|d\sigma^a\right\rangle - [\boldsymbol{\Delta}^a_b] \hat{} \left|\sigma^b\right\rangle \neq \left|G^k\right\rangle. \tag{24}$$

The Cartan vector of Torsion 2-forms, $\left|\Sigma^a\right\rangle$, is always Zero for "Physical Vacuums", while the excitation Torsion 2-forms, $\left|G^k\right\rangle$ is not.

10. The exterior derivative of the vector of field intensity 2-forms $\left|F^a\right\rangle$ vanishes, resulting in a sets of partial differential equations of the Maxwell Faraday type:

$$\left|dF^a\right\rangle \Rightarrow 0 \tag{25}$$
$$= \text{Maxwell Faraday PDE,s.} \tag{26}$$

[1] I use the word "affine" in very small print to reinforce that this idea of Torsion is not constrained to be that associated with Affine Matrices. Hopefully, in the future, people will say the "the excitation Torsion coefficients relative to the conformal group, or relative to the affine group with a fixed point, etc.

11. The exterior derivative of the vector of 2-form excitations $\left|G^k\right\rangle$ produces a vector of 3-forms (formally) equivalent (element by element k) to the conserved electromagnetic charge-current density.

$$\begin{aligned} \left|dG^k\right\rangle &= \left|J^k\right\rangle, & (27)\\ d\left|J^k\right\rangle &= 0. & (28)\end{aligned}$$

12. Construct the (pair - even) Topological Torsion scalar of 3-forms defined as

$$\text{Topological Torsion:} \qquad H = \langle A^a|\,\hat{}\,|F^a\rangle \qquad (29)$$
$$= A^1\hat{}F^1 + A^2\hat{}F^2 + A^3\hat{}F^3 + A^4\hat{}F^4. \qquad (30)$$

13. Construct the (impair-odd) Topological Spin (pseudo) scalar of 3-forms defined as

$$\text{Topological Spin:} \qquad S = \langle A^m|\,\hat{}\,|G^m\rangle, \qquad (31)$$
$$= A^1\hat{}G^1 + A^2\hat{}G^2 + A^3\hat{}G^3 + A^4\hat{}G^4. \qquad (32)$$

14. Exterior differentiation of these two 3-forms produces the Poincare invariants

$$\begin{aligned} dH &= \langle F^a|\,\hat{}\,|F^a\rangle = \sum_a 2(\mathbf{E}\cdot\mathbf{B})^a \quad = \text{Poincare II} & (33)\\ dS &= \langle F^a|\,\hat{}\,|G^a\rangle - \langle A^a|\,\hat{}\,|J^a\rangle \qquad = \text{Poincare I.} & (34)\\ &= \sum_a [\{(\mathbf{B}\cdot\mathbf{H})^a - (\mathbf{D}\cdot\mathbf{E})^a\} - \{(\mathbf{A}\cdot\mathbf{J})^a - (\rho\phi)^a\}]. & (35)\end{aligned}$$

The notation in terms of $\mathbf{E}, \mathbf{B}, \mathbf{D}, \mathbf{H}$ is symbolic, and refer only to the formal equivalence of the formulas to electromagnetic theory. If the Poincare invariants vanish, the closed integrals of the closed 3-forms could become topologically quantized through the concept of deRham period integrals. There are many ways that this result could happen, due to the fact that there are four (Yang Mills) components to the excitation 2-forms, $\left|G^k\right\rangle$. Note that if there exists only 1-component of $\left|G^k\right\rangle$ with $\left|dG^k\right\rangle = 0$, say G^4, then, formally, the integral over a closed 2D integration chain of G^4 defines the Quantized Charge, e, of classical electromagnetic theory.

15. Use the quadratic congruence to define the symmetric (metric) qualities of $[\mathbb{B}]$

$$[g] = [\mathbb{B}]^T \circ [\eta] \circ [\mathbb{B}]\,. \qquad (36)$$

The matrix $[\eta]$ is a (diagonal) Sylvestor signature matrix whose elements are ± 1.

16. Compute the Christoffel connection, and its matrix of 1-forms, $[\Gamma]$, from the quadratic metric matrix $[g]$ using the Levi-Cevita-Christoffel formulas.

17. Decompose the Cartan Connection matrix of 1-forms as follows:

$$[\mathbb{C}] = [\Gamma] + [\mathbb{T}] \tag{37}$$

18. Construct the matrix of Cartan Curvature 2-forms, $[\Phi]$, derived from the second exterior differentiation of the Basis Frame, $dd[\mathbb{B}]$.

$$\begin{aligned} dd[\mathbb{B}] &= d[\mathbb{B}] \circ [\mathbb{C}] + [\mathbb{B}] \circ d[\mathbb{C}], & (38)\\ &= [\mathbb{B}] \circ \{d[\mathbb{C}] + [\mathbb{C}] \hat{} [\mathbb{C}]\}, & (39)\\ &= [\mathbb{B}] \circ [\Phi] \Rightarrow 0, & (40)\\ [\Phi] &= \{d[\mathbb{C}] + [\mathbb{C}] \hat{} [\mathbb{C}]\} \Rightarrow 0 & (41) \end{aligned}$$

19. Note that exterior differentiation of the Cartan structure matrix of curvature 2-forms is equivalent to the Bianchi identity:

$$\begin{aligned} [d\Phi] + [d\mathbb{C}] \hat{} [\mathbb{C}] - [\mathbb{C}] \hat{} [d\mathbb{C}] &= & (42)\\ [d\Phi] + [\Phi] \hat{} [\mathbb{C}] - [\mathbb{C}] \hat{} [\Phi] &\Rightarrow 0 & (43) \end{aligned}$$

This concept of a Bianchi identity is valid for all forms of the Cartan structure equations. The Bianchi statements are essentially definitions of cohomology, in that the difference between two non-exact p-forms is equal to a perfect differential.

20. Substitute $[\Gamma] + [\mathbb{T}]$ for $[\mathbb{C}]$ in the definition of the matrix of curvature 2-forms, and recall that for the Vacuum the matrix of curvature 2-forms is zero.

$$\begin{aligned} [\Phi_{\mathbb{C}}] &= \{d[\mathbb{C}] + [\mathbb{C}] \hat{} [\mathbb{C}]\} \Rightarrow 0, & (44)\\ &= \{d([\Gamma] + [\mathbb{T}]) + ([\Gamma] + [\mathbb{T}])\hat{}([\Gamma] + [\mathbb{T}])\} & (45)\\ &= \{d[\Gamma] + [\Gamma]\hat{}[\Gamma]\} + \{[\mathbb{T}] \hat{} [\Gamma] + [\Gamma] \hat{} [\mathbb{T}]\} + \{d[\mathbb{T}] + [\mathbb{T}]\hat{}[\mathbb{T}]\}, & (46)\\ &= \{[\Phi_\Gamma]\} + \{interaction\_2 - forms\} + \{[\Phi_{\mathbb{T}}]\} & (47) \end{aligned}$$

21. Separate the matrices of 2-forms into metric curvature 2-forms, $\{d[\Gamma] + [\Gamma] \hat{} [\Gamma]\} = [Field\ metric\ 2 - forms]$,

   and the remainder, defined as $[-(Inertial\quad 2 - forms)]$. The decomposition leads to the strong equivalence equation,

$$\begin{aligned} \text{Principle of} &: \text{Strong Equivalence} & (48)\\ [\text{Field curvature 2-forms}] &= [\text{Inertial curvature 2-forms}]. & (49) \end{aligned}$$

At this point, there has been no indication that the problem being investigated has anything to do with the Gravitational Field. The issue is how the quadratic symmetries of $[\mathbb{B}]$ are established, by its group structure. Different choices for the group structure of the Basis Frame will strongly influence the application to any particular physical system.

3 Example 1. The Schwarzschild Metric embedded in a Basis Frame. $[\mathbb{B}]$ as a 10 parameter subgroup of an affine group.

3.1 The metric

In this example, the isotropic form of the Schwarzschild metric will be incorporated into a Cartan Connection $[\mathbb{C}]$. The technique is easily evaluated for diagonal metrics. However, the symmetry properties of the Cartan Connection are not limited to metrics of the "gravitational" type. Once the Schwarzschild metric is embedded in to the Basis Frame, then the universal methods described above will be applied, and evaluated.

The isotropic Schwarzschild metric is a diagonal metric of the form,

$$(\delta s)^2 = -(1+m/2r)^4\{(dx)^2+(dy)^2+(dz)^2\}+\frac{(1-2m/r)^2}{(1+2m/r)^2}(dt)^2 \quad (50)$$

$$= -(\alpha)^2\{(dx)^2+(dy)^2+(dz)^2\}+(\beta)^2(dt)^2 \quad (51)$$

$$\text{with } r = \sqrt{(x)^2+(y)^2+(z)^2}, \quad (52)$$

As Eddington points out, the isotropic form is palatable with the idea that the speed of light is equivalent in any direction. That is not true for the non-isotropic Schwarzschild metric, where transverse and longitudinal null geodesics do not have the same speed.

For the isotropic Schwarzschild example, the metric $[g_{jk}]$ can be constructed from the triple matrix product:

$$[g_{jk}] = [\widetilde{f}] \circ [\eta] \circ [f]\,, \tag{53}$$

$$\text{where } f = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \beta \end{bmatrix}, \tag{54}$$

$$\text{and} \quad \alpha = (1 + m/2r)^2\,, \quad \beta = \frac{(1 - 2m/r)}{(1 + 2m/r)}\,, \tag{55}$$

$$\text{and} \quad \eta = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{56}$$

3.2 The Diffeomorphic Jacobian Basis Frame

At first, consider the diffeomorphic map ϕ^k from spherical to Cartesian coordinates:

$$\{y^a\} = \{r, \theta, \varphi, \tau\} \Rightarrow \{x^k\} = \{x, y, z, t\} \tag{57}$$

$$\phi^k : [r\sin(\theta)\cos(\varphi), r\sin(\theta)\sin(\varphi), rcos(\theta), \tau] \Rightarrow [x, y, z, t] \tag{58}$$

$$\{dy^a\} = \{dr, d\theta, d\phi.d\tau\}. \tag{59}$$

The Jacobian of the diffeomorphic map ϕ^k can be utilized as an integrable Basis Frame matrix $[\mathbb{B}]$ which is an element of the 10 parameter F-Affine group (The Affine subgroup with a fixed point):

$$[\mathbb{B}] = \begin{bmatrix} \sin(\theta)\cos(\varphi) & r\cos(\theta)\cos(\varphi) & -r\sin(\theta)\sin(\varphi) & 0 \\ \sin(\theta)\sin(\varphi) & r\cos(\theta)\sin(\varphi) & r\sin(\theta)\cos(\varphi) & 0 \\ \cos(\theta) & -r\sin(\theta) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{60}$$

Theorem 1 *The effects of a diagonal metric $[g_{jk}]$ can be absorbed into a re-definition of the Frame matrix:*

$$[\widehat{\mathbb{B}}] = [f] \circ [\mathbb{B}]. \tag{61}$$

The integrable Jacobian Basis Frame matrix given above will be perturbed by multiplication on the left by the diagonal matrix, $[f]$. The perturbed Basis Frame becomes

$$[\widehat{\mathbb{B}}] = [f]\circ[\mathbb{B}] \text{ the Schwarzschild Cartan Basis Frame.} \tag{62}$$

$$= \begin{bmatrix} \alpha\sin(\theta)\cos(\varphi) & \alpha r\cos(\theta)\cos(\varphi) & -\alpha r\sin(\theta)\sin(\varphi) & 0 \\ \alpha\sin(\theta)\sin(\varphi) & \alpha r\cos(\theta)\sin(\varphi) & \alpha r\sin(\theta)\cos(\varphi) & 0 \\ \alpha\cos(\theta) & -\alpha r\sin(\theta) & 0 & 0 \\ 0 & 0 & 0 & \beta \end{bmatrix}. \tag{63}$$

Use of the congruent pullback formula based on the perturbed Basis Frame, $[\widehat{\mathbb{B}}]$, yields,

$$[g_{mn}] = [\widehat{\mathbb{B}}_{transpose}]\circ\eta\circ[\widehat{\mathbb{B}}], \tag{64}$$

$$= \begin{bmatrix} -\alpha^2 & 0 & 0 & 0 \\ 0 & -\alpha^2 r^2 & 0 & 0 \\ 0 & 0 & -\alpha^2 r^2\sin^2(\theta) & 0 \\ 0 & 0 & 0 & +\beta^2 \end{bmatrix}, \tag{65}$$

which agrees with formula given above for the isotropic Schwarzschild metric in spherical coordinates. It actually includes a more general idea, for the coefficients, α,and β, can be dependent upon both r and τ.

3.3 The Schwarzschild-Cartan connection.

The Schwarzschild-Cartan (right) Connection $[\widehat{\mathbb{C}}]$, as a matrix of 1-forms relative to the Basis Frame $[\widehat{\mathbb{B}}]$, becomes

$$[\widehat{\mathbb{C}}] = [\widehat{\mathbb{B}^{-1}}]\circ d[\widehat{\mathbb{B}}], \tag{66}$$

$$[\widehat{\mathbb{C}}] = \begin{bmatrix} -2mdr/r\gamma & -rd\theta & \sin^2(\theta)rd\phi & 0 \\ d\theta/r & \delta dr/\gamma & -\cos(\theta)\sin(\theta)d\phi & 0 \\ d\phi/r & \cot(\theta)d\phi & \cot(\theta)d\theta+\delta dr/\gamma & 0 \\ 0 & 0 & 0 & 4mdr/(\gamma\delta) \end{bmatrix}. \tag{67}$$

$$\gamma = (2r+m), \quad \delta = (2r-m) \tag{68}$$

$$\begin{bmatrix} -2\frac{d(r)\,M}{r\,(2\,r+M)} & -r\,d(\theta) & d(\phi)\,(-1+\cos(\theta)^2)\,r & 0 \\ \frac{d(\theta)}{r} & \frac{d(r)\,(2\,r-M)}{r\,(2\,r+M)} & -d(\phi)\,\cos(\theta)\,\sin(\theta) & 0 \\ \frac{d(\phi)}{r} & \frac{d(\phi)\,\cos(\theta)}{\sin(\theta)} & \frac{d(r)\,(2\,r-M)}{r\,(2\,r+M)}+\frac{\cos(\theta)\,d(\theta)}{\sin(\theta)} & 0 \\ 0 & 0 & 0 & 4\frac{M\,d(r)}{4\,r^2-M^2} \end{bmatrix}$$

Perturbed Cartan Connection

Surprisingly, for the perturbed Basis Frame $[\widehat{\mathbb{B}}]$ representing the field of a massive object, the vector of excitation torsion 2-forms, based on the affine subgroup with a fixed point, are not zero, and can be evaluated as:

$$\left|\widehat{\Sigma}_{torsion\_2-forms}\right\rangle \Rightarrow |\widehat{G}\rangle = [\widehat{\mathbb{C}}]\hat{}\, |dy^a\rangle \tag{69}$$

$$\text{Torsion 2-forms} \quad : \quad \text{of the Affine subgroup with a fixed point} \tag{70}$$

$$|\widehat{G}\rangle = \left| \begin{array}{c} 0 \\ (2m/r\gamma)(d\theta\hat{}dr) \\ (2m/r\gamma)(d\phi\hat{}dr) \\ (4m/\gamma\delta)(dr\hat{}d\tau) \end{array} \right\rangle \text{"Excitations"} \tag{71}$$

Similarly the perturbed 2-forms of field intensities can be evaluated:

$$|\widehat{F}\rangle = d([\widehat{\mathbb{B}}]\circ|dy^a\rangle) = \left|dA^k\right\rangle \tag{72}$$

$$: \quad \text{Intensity 2-forms of the Affine subgroup with a fixed point} \tag{73}$$

$$|\widehat{F}\rangle = \left| \begin{array}{c} +(m\gamma/2r^2)\{(\sin(\phi)\cos(\theta)d\theta\hat{}dr) - (\sin(\theta)\cos(\phi)d\phi\hat{}dr) \\ +(m\gamma/2r^2)\{(\sin(\phi)\cos(\theta)d\theta\hat{}dr) + (\sin(\theta)\cos(\phi)d\phi\hat{}dr) \\ +(m\gamma/2r^2)(\sin(\theta)d\theta\hat{}dr)+ \\ (4m/\gamma^2)(dr\hat{}d\tau) \end{array} \right\rangle. \text{"Intensities"} \tag{74}$$

The constitutive map relating the field intensities and the field excitations

$$|\widehat{G}\rangle = [\widehat{\mathbb{B}}]^{-1}\circ|\widehat{F}\rangle \tag{75}$$

is determined by the inverse of the perturbed Basis Frame, $[\widehat{\mathbb{B}}]^{-1}$:

$$\begin{bmatrix} 4\frac{r^2\sin(\Theta)\cos(\phi)}{(2r+M)^2} & 4\frac{r^2\sin(\Theta)\sin(\phi)}{(2r+M)^2} & 4\frac{r^2\cos(\Theta)}{(2r+M)^2} & 0 \\ 4\frac{\cos(\Theta)\,r\cos(\phi)}{(2r+M)^2} & 4\frac{\cos(\Theta)\,r\sin(\phi)}{(2r+M)^2} & -4\frac{r\sin(\Theta)}{(2r+M)^2} & 0 \\ -4\frac{r\sin(\phi)}{\sin(\Theta)(2r+M)^2} & 4\frac{r\cos(\phi)}{\sin(\Theta)(2r+M)^2} & 0 & 0 \\ 0 & 0 & 0 & \frac{2r+M}{2r-M} \end{bmatrix}$$

Schwarzschild Constitutive Map

The Topological Torsion for the Schwarzschild example vanishes: $H = \langle A| \hat{} \, |F\rangle \Rightarrow 0$. The implication is that the system is of Pfaff dimension 2 (and therefor is an equilibrium thermodynamic system). The exterior derivative of the vector of excitations is zero, hence there are no current 3-forms (the existence of currents implies non-equilibrium):

$$\text{Charge Current 3-form} \quad |J\rangle = d\,|G\rangle = 0. \tag{76}$$

Both Poincare invariants vanish, but the Topological Spin 3-form is NOT zero.

$$\begin{aligned} \text{Topological Spin 3-form} \qquad S &= \langle A| \hat{} \, |G\rangle \qquad dA = 0. && (77) \\ &= (-m\gamma\sin(\theta)/2r^2)\{\cos(\phi)+1)(dr\hat{}d\theta\hat{}d\phi)\}. && (78) \end{aligned}$$

The matrices of Connection 1-forms are presented below for each (perturbed) connection, $[\Gamma]$, $[\mathbb{C}]$, $[\mathbb{T}]$.

$$[\,\Gamma\,] = \begin{bmatrix} -2\dfrac{d(r)\,M}{r\,(2r+M)} & -\dfrac{(2r-M)\,r\,d(\theta)}{2r+M} & \dfrac{(2r-M)\,(-1+\cos(\theta)^2)\,r\,d(\phi)}{2r+M} & 64\,\dfrac{r^4\,(2r-M)\,M\,d(\tau)}{(2r+M)^7} \\ \dfrac{(2r-M)\,d(\theta)}{r\,(2r+M)} & \dfrac{d(r)\,(2r-M)}{r\,(2r+M)} & -d(\phi)\cos(\theta)\sin(\theta) & 0 \\ \dfrac{(2r-M)\,d(\phi)}{r\,(2r+M)} & -\dfrac{\cos(\theta)\sin(\theta)\,d(\phi)}{-1+\cos(\theta)^2} & \dfrac{d(r)\,(2r-M)}{r\,(2r+M)} - \dfrac{\cos(\theta)\sin(\theta)\,d(\theta)}{-1+\cos(\theta)^2} & 0 \\ 4\,\dfrac{M\,d(\tau)}{(2r-M)\,(2r+M)} & 0 & 0 & 4\,\dfrac{M\,d(r)}{(2r-M)\,(2r+M)} \end{bmatrix}$$

$$[C] = \begin{bmatrix} -2\dfrac{d(r)\,M}{r\,(2r+M)} & -r\,d(\theta) & d(\phi)\,r\,(-1+\cos(\theta)^2) & 0 \\ \dfrac{d(\theta)}{r} & \dfrac{d(r)\,(2r-M)}{r\,(2r+M)} & -d(\phi)\cos(\theta)\sin(\theta) & 0 \\ \dfrac{d(\phi)}{r} & \dfrac{d(\phi)\cos(\theta)}{\sin(\theta)} & \dfrac{d(r)\,(2r-M)}{r\,(2r+M)} + \dfrac{\cos(\theta)\,d(\theta)}{\sin(\theta)} & 0 \\ 0 & 0 & 0 & 4\,\dfrac{M\,d(r)}{4r^2-M^2} \end{bmatrix}$$

$$[T] = \begin{bmatrix} 0 & -2\dfrac{r\,d(\theta)\,M}{2r+M} & 2\,\dfrac{d(\phi)\,r\,(-1+\cos(\theta)^2)\,M}{2r+M} & -64\,\dfrac{r^4\,(2r-M)\,M\,d(\tau)}{(2r+M)^7} \\ 2\,\dfrac{d(\theta)\,M}{r\,(2r+M)} & 0 & 0 & 0 \\ 2\,\dfrac{d(\phi)\,M}{r\,(2r+M)} & 0 & 0 & 0 \\ -4\,\dfrac{M\,d(\tau)}{4r^2-M^2} & 0 & 0 & 0 \end{bmatrix}$$

Schwarzschild Perturbed Connections

The matrix of (metric) curvature 2-forms, $[\Phi_\Gamma]$, based on the formula

$$[\Phi_\Gamma] = d[\Gamma] + [\Gamma]\,\hat{}\,[\Gamma], \tag{79}$$

is computed to be:

$$[\Phi_\Gamma] = \begin{bmatrix} 0 & -4\,\dfrac{r\,M\,(d(r)\wedge d(\theta))}{(2r+M)^2} & 4\,\dfrac{r\,M\,(-1+\cos(\theta)^2)\,(d(r)\wedge d(\phi))}{(2r+M)^2} & -128\,\dfrac{r^3\,M\,(-4rM+M^2+4r^2)\,(d(r)\wedge d(\tau))}{(2r+M)^8} \\ 4\,\dfrac{M\,(d(r)\wedge d(\theta))}{r\,(2r+M)^2} & 0 & -8\,\dfrac{r\,M\,(-1+\cos(\theta)^2)\,(d(\theta)\wedge d(\phi))}{(2r+M)^2} & 64\,\dfrac{(2r-M)^2\,r^3\,M\,(d(\theta)\wedge d(\tau))}{(2r+M)^8} \\ 4\,\dfrac{M\,(d(r)\wedge d(\phi))}{r\,(2r+M)^2} & -8\,\dfrac{(d(\theta)\wedge d(\phi))\,r\,M}{(2r+M)^2} & 0 & 64\,\dfrac{(2r-M)^2\,r^3\,M\,(d(\phi)\wedge d(\tau))}{(2r+M)^8} \\ -8\,\dfrac{M\,(d(r)\wedge d(\tau))}{r\,(2r+M)^2} & -4\,\dfrac{r\,M\,(d(\tau)\wedge d(\theta))}{(2r+M)^2} & 4\,\dfrac{r\,M\,(-1+\cos(\theta)^2)\,(d(\tau)\wedge d(\phi))}{(2r+M)^2} & 0 \end{bmatrix}$$

Curvature 2-forms for the Schwarzschild perturbed Christoffel Connection

By the Strong equivalence Principle,

$$: \quad \{d[\Gamma]+[\Gamma]\hat{}[\Gamma]\} \quad +\{[\mathbb{T}]\hat{}[\Gamma]+[\Gamma]\hat{}[\mathbb{T}]\} \quad +\{d[\mathbb{T}]+[\mathbb{T}]\hat{}[\mathbb{T}]\} \tag{80}$$

$$= \quad \{[\Phi_\Gamma]\} \quad +\{[Interaction\ 2-forms]\}+ \quad \{[\Phi_T]\}\} \Rightarrow 0. \tag{81}$$

which can be checked using Maple algebra.

http://www22.pair.com/csdc/pdf/mapleep1.pdf

3.4 In Summary

The idea that has been exploited is that the arbitrary Basis Frame (a linear form), without metric, can be perturbed algebraically to produce a new Basis Frame that absorbs the properties of a quadratic metric system. For the Schwarzschild example, another remarkable feature is that the 1-forms $\left|\sigma^k\right\rangle$ constructed according to the formula

$$[\widehat{\mathbb{B}}]\circ|dy^a\rangle \Rightarrow \left|\sigma^k\right\rangle = \left|A^k\right\rangle, \tag{82}$$

are all integrable (as the Topological Torsion term is Zero). The symbol $|dy^a\rangle$ stands for the set $[dr, d\theta, d\varphi, d\tau]$ (transposed into a column vector), and $[\widehat{\mathbb{B}}]$ is the "perturbed" Schwarzschild metric. The integrability condition means that there exist integrating factors $\lambda^{(k)}$ for each σ^k such that a new Basis Frame can be constructed from $[\widehat{\mathbb{B}}]$ algebraically. Relative to this new Basis Frame, the vector of torsion 2-forms is zero, $\left|d\sigma^k\right\rangle = \left|dA^k\right\rangle = \left|F^k\right\rangle = 0$! The Coriolis acceleration which is related to the 2-form of torsion 2-forms $\left|F^k\right\rangle$ can be eliminated algebraically.! Encroyable! Formidable!

Of course this reduction is impossible if any of the 1-forms, σ^k, is of Pfaff dimension 3 or more. The Basis Frame then admits Topological Torsion, which is irreducible.

4 Remarks

This set ideas enumerated in Section 2 startles me. There is only ONE fundamental assumption, and the rest of the 18 equations are derived following the rules of the Cartan Calculus. It seems to appear that these 18 properties are universal!

Frankly, I can use some help and suggestions on how to interpret and apply this "universal" formalism. I am not sure that it is absolutely useful, but it has been my experience and intuition that universal, derived, results (not just an Ansatz, or guessing another term to be used in a Lagrangian) generally work out into something useful.

THINGS I WOULD LIKE TO TRY:

5 Example 2. $[\mathbb{B}]$ as a 13 parameter intransitive group (Electromagnetism)

To be worked out using Maple

6 Example 3. $[\mathbb{B}]$ as a 13 parameter transitive group (Particles)

To be worked out using Maple.

7 Example 4. $[\mathbb{B}]$ as an Immersion with two parameters (strings).

To be worked out using Maple.

8 Example 5. $[\mathbb{B}]$ as an Projection in terms of the Hopf map.

To be worked out using Maple.

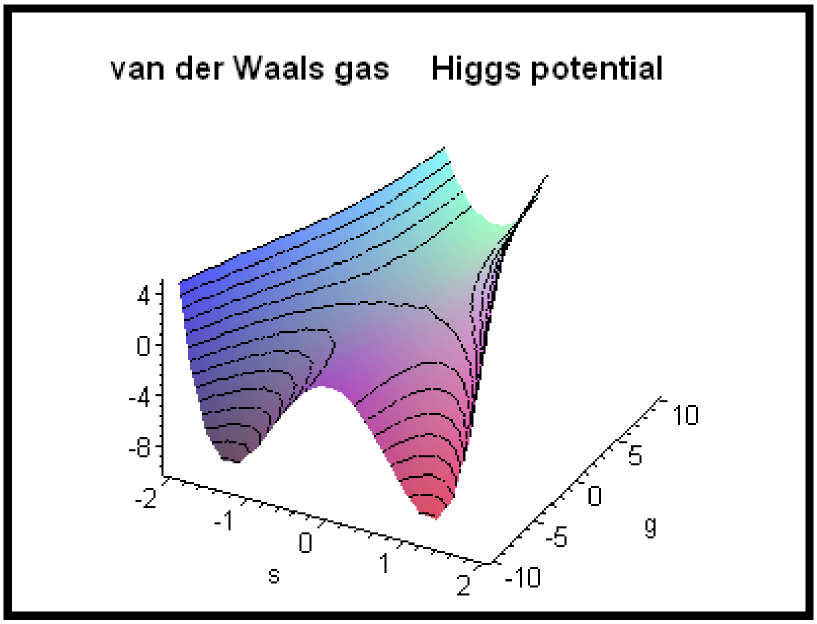
Note that the matter Field parts are related to quadratic forms, while other fields are related to the linear and anti symmetric properties of the connection decomposition (see equation 37).

9 Thermodynamic Phase Functions from $[\mathbb{B}]$

As pointed out in my first volume[2] "Non Equilibrium Thermodynamics and Irreversible Processes", that given any matrix, useful information (related to Mean and Gaussian Curvatures of the domain described by the matrix) can be obtained from the Cayley-Hamilton

[2] See http://www.lulu.com/kiehn

Characteristic Polynomial. This polynomial exists for any square matrix. In particular, for 4 x 4 matrices, if the matrix has no null eigenvalues, (the domain is then Symplectic), and the characteristic polynomial is of 4th degree in the polynomial variable. By treating the characteristic polynomial as an expression in the 4D space of the similarity invaraints of the 4 x 4 matrix, with the polynomial variable as a family parameter, the envelope of the reduced quartic polynomial yields a thermodynamic phase function that establishes an extraordinary correspondence between the binodal, spinodal lines of a van der Waals gas. These curves act as bifurcation limit sets on the quartic surface. I call this envelope the Higgs Phase function. The conjecture is that Higgs features (related to mass and inertial) have a basis in topological thermodynamics.



It is remarkable to me that all of this starts from the sole assumption of a Basis Frame $[\mathbb{B}]$. The Higgs idea and the Yang Mills theory (and weak force) seem to be related to an irreducible Pfaff topological dimension 4 (Evolutionary irreversible processes are possible and do not preserve parity).

I conjecture that if the Phase function has 1 null eigen value, then the space is related to a non equilibrium configuration of Pfaff dimension 3, for which parity is always preserved (the strong Force). The electromagnetic domain only requires Pfaff dimension 2, and the gravity domain is embedded in Pfaff topological dimension 1. This follows from an old I argument (based on differential geometry) that I presented in 1975.

Kiehn, R. M. (1975), "Submersive equivalence classes for metric fields", Lett. al Nuovo Cimento 14, p. 308.

The idea is that spaces of Pfaff Topological dimension 2 or less (hence thermodyamically in equilibrium) are topologically *connected*, and interactions can range over the entire domain (long range forces - gravity and electromagnetism). However, spaces of Pfaff Dimension 3 or more are topologically *disconnected* domains of multiple components, and are not in thermodynamic equilibrium. Hence interactions and forces are short range (strong and weak forces, that may or may not preserve parity)