ARE THERE THREE KINDS OF SUPERCONDUCTIVITY?

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Abstract

A Maxwell system of differential forms induces a course topology on a space time variety. This topology can be refined by certain extremal constraints of domain closure that lead to topologically distinct, reactive impedances, Z_1, Z_2, Z_3. These non-dissipative impedances are rational fraction ratios of topological invariants. The closure conditions induce physical constraints that permit Z_1 to be identified with the ordinary type I and II superconductive domains, and Z_3 with the fractional quantum Hall effect. Only Z_2 admits the time-reversal and parity symmetry breaking associated with Anyon theories.

1. Introduction

The idea that superconductivity may have its origins in a topological theory was given further credence by VonKlitzing's [1] remarks regarding the quantum Hall effect. The fact that the quantization is independent of sample size and shape, and to a certain extent independent of impurity content, makes it appear that the QHE is a topological phenomena. Earlier recognition of the Bohm-Aharanov phenomena and flux quantization in type II superconductors demonstrated that superconductivity was to be associated with cohomology theory [2]. E. J. Post even predicted the rational fraction quantum Hall effect from cohomological arguments [3] some two years before its experimental verification. Thouless [4] developed a two dimensional QM theory which ultimately led to the idea that half band filling fractions correspond to a topological invariant. More recently, application of superstring ideas has led to the Anyon theory [5] of superconductivity, where topological concepts are offered to explain high TC effects. Bellissard [6] and Xia [7] have offered theories that state the Hall impedance is proportional to the Chern-Simons invariant.

In this article, the point of departure is based on Cartan's theory of differential systems. The topological structure induced on the variety, $\{x, y, z, t\}$, by the complete Maxwell system of exterior differential forms, can be refined by certain closure constraints. Each of the four possible closure constraints leads to a distinct cohomology structure on the variety. Remarkably, three of these ratios of the cohomology structures have the physical dimensions of an impedance (ohms). As these ratios of topological invariants are rational fractions, they ultimately have a periodic presentation, and therefore it is subsumed that they are representative of dissipation free phenomena. The closure refinements necessary to insure the dissipation free, rational fraction impedances indicate or predict certain qualities of the Maxwell fields on the topological domains. In one case, the closure conditions imply that the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$, are excluded from the domain, but the potentials, $\mathbf{A}$ and ϕ are not. This result is remindful of Meisner expulsion of the bulk magnetic field from type I and type II superconductors. In two cases the Euler characteristic of the domain must vanish, implying that time-reversal and parity symmetry are conserved. One of these cases admits the presence of large $\mathbf{B}$ fields, and appears to be related to the rational fraction quantum Hall effect. The third case admits a non-zero Euler characteristic ($\mathbf{E} \circ \mathbf{B} \neq 0$), and therefore permits the parity and time-reversal symmetry breaking alluded to be one of the features of Anyon superconductivity.

2. Exterior Differential Systems

The idea that an exterior differential system, Σ can induce a topology on a variety, $\{x, y, z, t\}$, has its roots in Cartan's theory of exterior differential forms. In its most elementary physical realization, the differential system, Σ, is generated by the single 1-form of Lagrange-Cartan action,

$$A = \mathbf{A}.d\mathbf{r} - \phi dt. \tag{2.1}$$

From this system, Σ, it is possible to construct the closure of the system by adding to A its exterior derivative, $dA = F$. The closure is represented by the sets Σ and dΣ, or A , $dA = F$. The original system may be further augmented by adding to it all of the possible intersections that can be formed from Σ and dΣ. The closure plus the intersections form the complete Pfaff sequence, or topological structure, which for 4-dimensions is given by the system:

$$\{A,\ dA = F,\ H = A\hat{}F,\ K = F\hat{}F\} \tag{2.2}$$

The 4-dimensional form, $F\hat{}F$, is equivalent to the "Top Pfaffian" of Chern [8], who proved that its integral yields the Euler characteristic, $\mathcal{X}$, (perhaps the most important topological invariant) of the domain. This demonstration of how topological properties can be assessed from a differential system on the cotangent space has been accomplished WITHOUT the geometric constraint of a metric, or the geometric specialization to a particular group structure or fiber bundle.

If the differential system is composed from more than one differential form, the same basic procedure is to be followed. First construct the closure by adding to the original system the exterior derivatives of the original set, then form all possible set intersections to build the complete Pfaff sequence, which acts as a topological base. The additional features of the added forms refine the induced topology, which can be further specialized by placing constraints on the domain. The constraints often take the form of extremal conditions where some of the elements of the complete Pfaff sequence, usually the limit sets (the exterior derivatives) of the system, are set equal to zero. This is the procedure to be used for the complete Maxwell system.

3. The Maxwell System

To the fundamental 1-form of action, A, is adjoined an N-2 form density, G, with an exterior derivative, $dG = J$ (which in a sense is the limit set of G and generates coefficients which are the sources). The statement $F = dA$ defines the field intensities $(\mathbf{E}, \mathbf{B})$ in terms of the derivatives of the potentials, and the statement , $dG = J$ defines the charge current densities in terms of the field excitations, $(\mathbf{D}, \mathbf{H})$. The exterior differential system, $\{A, G\}$ along with its closure, $\{dA = F$, $dG = J\}$ forms a Maxwell system of equations, for by the Poincare lemma $ddA =$

$dF = 0$ and $ddG = dJ = 0$, become the first Maxwell Faraday pair of equations, and the conservation of charge current, respectively. Again it is important to remark that these results are independent from a metric, a connection, or any constitutive constraint on the space time variety. These metric free ideas were first championed by Von Dantzig [9].

Explicitly, the 1-form of Action on the four dimensional space-time of independent variables, (x, y, z, t) is given by the expression,

$$A{=}\Sigma_{k=1}^{3} A_k(x, y, z, t)dx^k - \phi(x, y, z, t)dt. \tag{3.1}$$

with physical dimensions (for the Maxwell field) of angular momentum per unit source, or charge, $[h/e]$. The induced 2-form of electromagnetic field intensities, $\mathbf{E}$ and $\mathbf{B}$, as coefficients, has the same physical dimensions,

$$\begin{aligned} F &= \mathbf{B}_z dx\hat{\ }dy + \mathbf{B}_x dy\hat{\ }dz + \mathbf{B}_y dz\hat{\ }dx + \\ &\quad \mathbf{E}_x dx\hat{\ }dt + \mathbf{E}_y dy\hat{\ }dt + \mathbf{E}_z dz\hat{\ }dt \end{aligned} \tag{3.2}$$

(where $\mathbf{B} = curl\ \mathbf{A}$ and $\mathbf{E} = -\partial\mathbf{A}/\partial t - grad\phi$)

The N-2 form density, G, consists of components of excitation, $\mathbf{D}$ and $\mathbf{H}$, and is of the form

$$\begin{aligned} G &= -\mathbf{D}_z dx\hat{\ }dy + \mathbf{D}_x dy\hat{\ }dz + \mathbf{D}_y dz\hat{\ }dx + \\ &\quad \mathbf{H}_x dx\hat{\ }dt + \mathbf{H}_y dy\hat{\ }dt + \mathbf{H}_z dz\hat{\ }dt \end{aligned} \tag{3.3}$$

with physical dimensions of charge, [e]. The induced 3-form density, $\mathcal{J}$ becomes

$$J{=}\mathbf{J}^x dy\hat{\ }dz\hat{\ }dt - \mathbf{J}^y dz\hat{\ }dx\hat{\ }dt + \mathbf{J}^z dx\hat{\ }dy\hat{\ }dt - \rho dx\hat{\ }dy\hat{\ }dz \tag{3.4}$$

with the same physical dimensions of [e] (where $\mathbf{J} = curl\mathbf{H} + \partial D/\partial t$, and $div\mathbf{D} = \rho$).

Additional forms, $\{A\hat{\ }F, F\hat{\ }F, A\hat{\ }G, F\hat{\ }G, A\hat{\ }J, G\hat{\ }G\}$, constructed from non-null set intersections (based on the exterior product) may be added to the Maxwell closure,$\{A,\ dA = F,\ G, dG = J\}$, to form the complete Pfaff sequence. The complete set of forms determines a topological base on the space time variety, with the Maxwell closure acting as a subbase. The induced topology has been constructed *without* a metric or a constitutive tensor. Further constraints of a geometrical nature can be imposed on the system by specifying a metric (for example, a Lorentz metric), or a constitutive linkage ($G^{\mu\nu} = \chi^{\mu\nu\alpha\beta}F_{\alpha\beta}$) between G and F, or constraint of isochronism (dt = 0). These constraints are not examined in this article.

4. Period Integrals of the Maxwell System as Topological Invariants

The complete system admits of four natural extremal constraints of closure in the sense that the following combination of elements of the topological base are exact p-forms:

$$
\begin{array}{lll}
dA = F & p = 2 & \text{preimage} = A \\
dG = J & p = N-1 & \text{preimage} = G \\
d(A\hat{}F) = F\hat{}F & p = N & \text{preimage} = A\hat{}F \\
d(A\hat{}G) = F\hat{}G - A\hat{}J & p = N & \text{preimage} = A\hat{}G
\end{array}
\tag{4.1}
$$

The integrals of exact forms over compact oriented manifolds ordinarily are considered to be zero, but such an assumption assumes that the pre-image of the form does not go to zero on the domain of interest. The singular (null) points of a global 1-form are known to determine the Euler index of the variety on which the 1-form is defined. For example there must be two such singular points for a vector field on a sphere. Similarly, the null points of *all* elements of the Pfaff sequence determine topological information. The integral of $F\hat{}F$ (an exact 4-form - which in local coordinates has the expression $2\mathbf{E} \circ \mathbf{B}(dx\hat{}dy\hat{}dz\hat{}dt)$ over a domain will lead to the Euler characteristic or Chern index of the topology induced on space time by the complete system of forms on that domain. The integral of $F\hat{}G - A\hat{}J$ (an exact N-form - which in local coordinates becomes $\{(\mathbf{B} \circ \mathbf{H} - \mathbf{D} \circ \mathbf{E}) - (\mathbf{A} \circ \mathbf{J} - \rho\phi)\}(dx\hat{}dy\hat{}dz\hat{}dt)$ will be defined as the Lagrange index [10] . These integrals are known to be topological invariants. If the RHS of each of the exact forms vanishes, then the pre-images are closed in an exterior derivative sense. Hence, integrals of these closed forms over closed (but not bounding) integration chains are period integrals, and their values (by deRham's theorems) are integers times some smallest value of scaling. The number of independent such integrals (the number of non-condensable closed chains) determines the p-1 dimensional cyclic cohomology of space time variety. The result is essentially the number of p-1 dimensional "holes" or obstructions on the space time variety. These numbers are topological invariants of homeomorphisms. The reason that they are integers is related to the physical impossibility of having "half a hole".

If the closed integration chains (which are composed of cycles) are boundaries then the values of period integrals are zero. If singularities occur, they must occur in canceling pairs. There is a dominant cycle which would be the boundary if all interior cycles collapsed. This cycle defines the edge of a domain with a heterogeneous interior. The value of this period integral is the sum of all the other period integrals. The dominant cycle is the usual integration chain over a physical object. It is the integration chain assumed in most of that which follows.

Forcing the RHS of each of the exact forms to zero acts as a set of extremal constraints or refinements on the variety. The constraints may be interpreted physically to be the limit of zero fluctuations in the pre-images, and in this sense would correspond to the limit of absolute zero in temperature. It follows that there are four cases of such period integrals to consider:

$$\begin{array}{lcc} 1.\ F=0 & \begin{array}{c}\oint A = int * c1 \\ of\ physical\ \dim ension\ h/e\end{array} & = the\ flux\ quantum \\ 2.\ J=0 & \begin{array}{c}\iint_{closed} G = int * c2 \\ of\ physical\ \dim ension\ e\end{array} & = the\ ch\arg e\, quantum \\ 3.\ F\hat{}F=0 & \begin{array}{c}\iiint_{closed} A\hat{}F = int * c3 \\ of\ physical\ \dim ension\ (h/e)^2\end{array} & = the\ torsion\ quantum \\ 4.\ F\hat{}G - A\hat{}J = 0 & \begin{array}{c}\iiint_{closed} A\hat{}G = int * c4 \\ of\ physical\ \dim ension\ h\end{array} & = the\ spin\ quantum \end{array} \tag{4.2}$$

Each of these period integrals, subject to the appropriate constraint, is a topological invariant of the domain.

5. Transverse Reactive Impedances as Ratios of Period Integrals

Note that ratios of the topological invariant period integrals may be used to construct 3 distinct transverse impedances of physical dimensions, $[h/e^2]$, and each of these ratios is a rational fraction of some smallest scaling:

$$\begin{array}{ll} Z_1 = \oint A / \iint_{closed} G & = m/n\ \ (c1/c2) \\ Z_2 = \iint_{closed} A\hat{}G / (\iint_{closed} G)^2 & = r/s\ \ (c4/(c2\cdot c2)) \\ Z_3 = \iiint_{closed} A\hat{}F / \iint_{closed} A\hat{}G & = p/q\ \ (c3/c4) \end{array} \tag{5.1}$$

The coefficients c1,c2,c3 and c4 are the unknown smallest scaling values (which are legislated quantities and are to be ignored in a topological analysis) , and m,n,r,s,p,q which are integers. When the appropriate physical conditions of null fluctuations are true such that any of these topological impedances are rational fractions, they then are expected to represent reversible reactive phenomena, and are dissipation free. In that which follows, the features induced by the null fluctuation topological constraints are put into correspondence with the physical features of superconducting domains.

5.1. THE TOPOLOGICAL FRACTIONAL HALL IMPEDANCE Z_3

The experimental observations of the Quantum Hall Effect indicate that the "quantization" conditions are independent from the size and shape of the sample, and are more or less independent from sample impurities. [1] Such results imply that the phenomena is of topological rather than geometrical origin. Indeed it appears that the features of constraint that generate the rational fraction topological impedance, Z_3 , indicate that it represents the Fractional Quantum Hall impedance. Of the three possible topological impedances, the least constrained situation is for Z_3. The constraints of closure correspond to a domain where the finite variables can be written as,

finite: $\{\mathbf{A}, \phi, \mathbf{E}, \mathbf{B}, \mathbf{D}, \mathbf{H}, \mathbf{J}, \rho\}$ with

null sets $\{\mathbf{E} \circ \mathbf{B} = 0 \quad and \quad [(\mathbf{B}.\mathbf{H} - \mathbf{D}.\mathbf{E}) - (\mathbf{A}.\mathbf{J} - \rho\phi)] = 0.\}$

The 3 dimensional integral in the numerator of Z3 may be written in engineering format as

$$\iiint_{closed} A\hat{}F = \iiint_{closed} \{\mathbf{E} \times \mathbf{A} + \mathbf{B}\phi\}_x dy\hat{}dz\hat{}dt - .. + .. - \mathbf{A} \circ \mathbf{B} dx\hat{}dy\hat{}dz \tag{5.2}$$

and the three dimensional integral in the denominator may be written as

$$\iiint_{closed} A\hat{}G = \iiint_{closed} \{\mathbf{A} \times \mathbf{H} + \mathbf{D}\phi\}_x dy\hat{}dz\hat{}dt - .. + .. - \mathbf{A} \circ \mathbf{D} dx\hat{}dy\hat{}dz \tag{5.3}$$

Topological domains that satisfy the conditions to produce the rational fraction transverse impedance, Z_3 , can sustain large $\mathbf{B}$ fields and fluctuation currents of the normal variety. Such conditions lead to the statement that Z_3 represents the fractional quantum Hall impedance, which occurs experimentally in the presence of large $\mathbf{B}$ fields. Subject to the stated constraints, both the Euler and the

Lagrange indices are zero. Setting the denominator to 1 leads to the earlier cited results that the Hall impedance is related to the integral of $A\hat{}F$. [3,4,5]

The admissibility of charge currents implies that such systems can be used to emulate traveling wave amplifiers, whereby RF energy can be created at the expense of DC currents, or charge density waves.

The topological argument that Z_3 is the transverse Quantum Hall impedance implies that the fundamental idea is independent from some set of Schroedinger wave functions (and a probability normalization condition which acts a yet another topological constraint) which may be used to model the topological concept. The arguments presented above reverse the usual theoretical attack, which starts from the postulate that all physics has a quantum mechanical foundation. Here the notion is that a topological foundation can lead to the quantum theory. In fact, it can be shown that the Heisenberg uncertainty principle can be associated with the curvature 2-forms, F, of a space that admits fluctuations in its dynamics and kinematics $(dx^i - V^i dt \neq 0)$.

It is to be noted that only for space-time topologies of Euler characteristic zero is it possible to construct an evolutionary vector field without singularities (the world lines never intersect). Furthermore, note that the transverse Hall effect as represented by Z_3 , is, topologically, an irreducibly 3-dimensional concept (although the three dimensions may be composed of two space variables and time).

5.2. ORDINARY SUPERCONDUCTORS $\mathbf{Z}_1$

The most constrained configuration for the Maxwell system under consideration corresponds to a domain where the finite variables are,

finite: $\{\mathbf{A}, \phi, \mathbf{D}, \mathbf{H}\}$ with

null sets $\{\mathbf{E} = 0,\ \mathbf{B} = 0\,,\ \mathbf{J} = 0,\ \rho = 0.\}$

which of course implies that both the Euler and the Lagrange indices are zero. The one dimensional integral in the numerator of Z1 represents quantized flux in the sense of Bohm-Aharanov,

$$\oint A = \oint(\mathbf{A} \circ d\mathbf{r} - \phi dt) \tag{5.4}$$

and the 2 dimensional integral in the denominator represents quantized charge,

$$\iint_{closed} G = \iint_{closed}(-\mathbf{D}_z dx\hat{}dy.. - .. - .. + .. + .. + H_z dz\hat{}dt) \tag{5.5}$$

time independent case. This constraint configuration, in which the **B** field is expelled from the domain, is in correspondence with the fundamental ideas of ordinary superconductivity, and predicts that there exists a rational fraction transverse impedance associated with such phenomena that has yet to be measured. (Note that the additional constraint, dt = 0, makes the choice of ϕ and **E** ambiguous.)

5.3. ARE HIGH TC SUPERCONDUCTORS RELATED TO $\mathbf{Z}_2$?

The next least constrained system corresponds to the domain where the finite variables are:

finite: $\{\mathbf{A}, \phi, \mathbf{E}, \mathbf{B}, \mathbf{D}, \mathbf{H}, \mathbf{E} \circ \mathbf{B}\}$ with

null sets $\{\mathbf{J} = 0, \rho = 0, (\mathbf{B}.\mathbf{H} - \mathbf{D}.\mathbf{E}) = 0, .\}$

This topological configuration corresponds to a zero value for the Lagrange index, but the Euler index may take on non-zero values ($\mathbf{E} \circ \mathbf{B} \neq 0$). If the Euler index is identically zero, the constraint mimics the usual conditions for the propagation of plain waves in "free space", where the domain is charge current free, and the magnetic and the electric energy densities are balanced. However, when the Euler index is not zero, this constraint produces the only reactive impedance built from rational fractions that will admit the longitudinal **E** and **B** fields necessary to generate helical modes. The lack of free charge and currents forbids the interactions to used to produce traveling wave amplifiers, as is possible for the impedances of type Z_3 . When $\mathbf{E} \circ \mathbf{B}$ is not zero, parity and time reversal symmetries can be broken at a macroscopic level, and waves may propagate with different phase velocities in different directions [11]. (Such concepts have led to the conjecture that this impedance may be associated with High Tc superconductivity.) Anyon theories that lead to broken symmetries of parity and time reversal must be associated with domains for which the Euler characteristic is not necessarily zero.

5.4. A FEW CONTACTS WITH EXPERIMENT

A certain number of pertinent statements can be made from the crystallographic experiments made on experimental high TC superconducting materials. Certain compounds indicate that their charge distributions cannot be mapped on to a euclidean periodic lattice in 3-dimensions. The recent thesis of Xiaobo Kan at the University of Houston indicates that, for the Bismuth 2212 type material, a

4-dimensional euclidean structure is required to yield a periodic tiling for crystalline structure as seen and measured in 3-D; that is, the 2212 material does not exhibit periodic tiling of 3-dimensional space. The x-ray data indicate satellite diffraction peaks occur along one of the reciprocal lattice vectors, such that 4-dimensions are required for euclidean periodicity. Such materials could support an electromagnetic vector and scalar potential describing the charge distribution in the material such that $\mathbf{E} \circ \mathbf{B}$ is not zero in such a domain. A greater than 3 dimensional periodic tiling is necessary for materials that can support $\mathbf{E} \circ \mathbf{B} \neq 0$. However, just because the tiling is 4-dimensional, it is not possible to conclude that $\mathbf{E} \circ \mathbf{B} \neq 0$. Four dimensions is a necessary but not sufficient condition for $\mathbf{E} \circ \mathbf{B} \neq 0$. However, if lowest dimension for periodic euclidean tiling of the lattice is in 5 dimensions or higher, then the only form of superconductivity is of type Z_2 , for $\mathbf{E} \circ \mathbf{B} \neq 0$ is then a necessary requirement for such systems. On the other hand, if the crystallographic structure of a high TC material is describable by a 3-dimensional periodic tiling, then the vector and scalar potential required to describe such a material requires only 3-parameters, and the associated distribution of charge cannot support a domain where $\mathbf{E} \circ \mathbf{B} \neq 0$. The conclusion is that periodic tiling in 3-dimensions implies that time reversal symmetry is preserved, and periodic tiling in 4 or more dimensions is required if time reversal symmetry is not preserved. As some high TC materials are periodic in 3-dimensions time reversal symmetry breaking is not required in high TC superconductors. The question of parity symmetry breaking is still open. In the language of the presentation given above, $A\hat{}dA$ need not be 0 in a 3-dimensional tiling, but the tiling can have dislocation and disclination defects. In order for $dA\hat{}dA \neq 0$, the space must be irreducibly 4-dimensional, or more. $dA\hat{}dA$ must be zero for domains where Z_1 and Z_3 are not zero. Only Z_2 domains can support $dA\hat{}dA \neq 0$. For domains where $A\hat{}dA \neq 0$, a chiral symmetry can be broken, and this can happen for materials that support Z_2 or Z_3 superconductivity. The concept of defects complicates the issue, for the defects are representatives of the periodic tiling imperfections.

6. APPENDIX A :

6.1. TOPOLOGICAL FEATURES OF THE FINE STRUCTURE CONSTANT (written in conjunction, and by correspondence, with E.J. POST)

The notion of impedance is an engineering concept, which for a long time has not played a major role in fundamental physics. The idea of a free space impedance goes back to the early days of radio transmission, radar and waveguides. Targets of reduced radar cross section have recently added to its relevance. In this appendix a few remarks will be made about the relationships between the radiation impedance, Z_0 , of free space, the Hall impedance, Z_{Hall} , studied above, and the fine structure constant. In its traditional cgs rendition the fine structure constant,

$$\alpha = 2\pi e^2/hc = 1/137.0360411, \tag{6.1}$$

is a dimensionless number determined by the quantum of elementary charge, e, the quantum of action, $h/2\pi$, and the speed of light in matter free space. The fine constant was introduced by Sommerfeld to account for certain relativistic effects in the spectra of hydrogen. In (A1) the elementary charge, e, and the action, $h/2\pi$, are known to be good space-time invariants under all diffeomorphisms. By contrast, the speed of light is only a Lorentz invariant, but not a general space-time invariant. The numerical value of (A1) is a recommended value, which from 1980 onwards has only changed in the last three decimal places. [12] The data include laboratory measurements [13] using Josephson and quantum Hall effects, as well as values deduced from the spectra of far away stellar objects [14] that are subject to very large stellar red shifts. It would appear that α is at least a projective invariant of the universe. Right from the beginning α became surrounded by a lore of mysticism, which has manifested itself in finding independent calculational recipes for the value given in (A1). From the early efforts of Eddington to the more recent attempts by Wyler [15], these calculations have improved to the point of reproducing some six decimal places of the generally accepted measured values. Yet these recipes so far have not exhibited a sufficiently transparent and acceptable rationale to convince the world of physics at large. Their relevance remains, for the time being, in the eye of the beholder. In this appendix no magnitude prediction of α is intended. Instead, the observation is made that when written in terms of the MKS system of units, the fine structure constant becomes,

$$\alpha = 2\pi e^2/4\pi\varepsilon\, hc = 1/2(\mu/\varepsilon)^{1/2}/(h/e^2) \tag{6.2}$$

This formula demonstrates that α is a ratio of two fundamental impedances, the free-space impedance,

$$Z_0 = (\mu/\varepsilon)^{1/2} = 376.730313\Omega, \tag{6.3}$$

and the Hall impedance,

$$Z_{Hall} = h/e^2 = 25812.81491\Omega. \tag{6.4}$$

The relation between the quantum mechanical entities and the free space impedance as given by the equation,

$$\alpha = 1/2(Z_0/Z_{Hall}), \tag{6.5}$$

elevates the importance of the free space impedance, Z_0 . Furthermore, recent measurements of the quantized "ballistic" impedance of electrons in mesoscopic channels focuses attention on the relationship between the Hall impedance for electron waves and the free space impedance for photon waves [16]. The option expressed by (A4) has been around for some time [17], yet the question remains: Why should (A4) be preferred over (A1)? A dimensionless number can always be rewritten as the ratio of any two physical quantities of equal dimension and proper numerical values. One important argument that can be brought to bear on the situation is the general invariance of (A4) contrasts with the Lorentz invariance of (A1). As shown in reference [17], the free space impedance, Z_0 , (unlike the speed of light c) is a general invariant of the constitutive tensor $\chi^{\mu\nu\alpha\beta}$. The constitutive tensor is a map between the pair two form, F, of electric intensities (**E** and **B**), and the impair N-2 form, G, of electric excitations, (**D** and **H**). The impedance Z_0 is an algebraic invariant of the constitutive tensor [17]. Moreover, the impedance, Z_0 is a conformal invariant in 4-dimensions. These results suggest that a deeper and perhaps topological result is behind equation (A4). From dimensional arguments, the idea that the Giorgi choice of *four* fundamental dimensional units (length, time,mass, and charge) over the classical cgs system of *three* fundamental units (length, time, mass) induces such a relation as (A4) also suggests that a topological concept is involved. General invariance is necessary but not sufficient to make Z_0 a topological constant of nature. In support of this constancy note that a varying Z_0 in space-time would cause a distributed reflection of radiation throughout space. Such a phenomenon would indeed be incompatible with what astronomers see [18]. The α values reported by Bahcall and Schmidt [14] and derived from the observed spectra of quasi-stellar objects strongly support an α value constant through the known universe. The experiments of the quantum Hall effect [1] support the idea that the Hall impedance is a topological entity. The associated recognition that h/2π and e are topological properties, when combined with the constancy of α and equation (A4) suggest that Z_0 is also a topological property of the universe, and its measured projective invariant properties suggest that it should be determined somehow by a topological cross ratio. The topological base defined by a Maxwell system (and used in the main portion of this article, above, to define certain rational fraction topological impedances) focuses attention on another integral ratio, which, although not rational, in simple examples is exactly the square of the free space impedance,

$$\begin{aligned} (Z_0)^2 &= \iiiint F\hat{}F / \iiiint G\hat{}G \\ &= \iiiint \mathbf{E} \circ \mathbf{B} / \iiiint \mathbf{D} \circ \mathbf{H} \\ &= \mu/\varepsilon \end{aligned} \tag{6.6}$$

The integrals above are open integrals and do not necessarily form a rational ratio, as do the integrals that make up the cyclic impedances discussed above. While Z_0 seems to be relevant anywhere in space-time, the other universal impedance, Z_{Hall} , relates to the presence of matter and properties of matter. Unlike the homogeneous nature of matter-free space, the Hall impedance relates to the inhomogeneous structures that exist in space-time. An integral representation of such physical structural elements would have to account for the structure, yet at the same time reflect the universality of Z_{Hall} . Such inhomogeneity properties are to be associated with the cyclic cohomology of the Maxwell differential system, and was described above. The question remaining is how to relate the cyclic impedances, Z_1 , Z_2 , $Z_{3,}$ and the free space impedance, Z_0, which is based on a non-cyclic ratio. By use of Stokes theorem, and assuming that the integrations on the boundary do not vanish, eq. (A5) leads algebraically to the equations,

$$\begin{aligned} (Z_0)^2 &= \iiiint F\hat{}F / \iiiint G\hat{}G \\ &= Z_3 Z_1 / Z_2 \{ \iiiint F\hat{}G - A\hat{}J / \iiiint G\hat{}G \} \\ &= Z_3 Z_1 / Z_2 \{ \iiint_{closed} A\hat{}G / \iiiint G\hat{}G \} \end{aligned} \tag{6.7}$$

$$= \quad Z_3 Z_1 \{ (\iint_{closed} G)^2 / \iiiint G\hat{\ }G \}$$

These formulas (assuming that $Z_{Hall} = Z_1 = Z_3$) suggest the possibility that the fine structure constant is related to the charge distribution ratio of a "point" electron to a "distributed" electron:

$$\alpha^2 = 4 \{ (\iint_{closed} G)^2 / \iiiint G\hat{\ }G \} \tag{6.8}$$

The numeric value is, of course, the ratio of the "size" of the classical electron radius to the Bohr orbit.

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