Propagating Topological Singularities in the lightcone: the Photon

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ABSTRACT

At its foundations, Maxwell's theory of Electrodynamics, like thermodynamics, is a topological theory independent from geometric constraints of metric, scales, or gauge symmetries. One of the most interesting features of Electromagnetism is its relationship to the transport of momentum and energy by means of photons. This article utilizes a topological perspective to discuss the classical features and quantum concepts associated with the photon, including Topological Spin, Topological Torsion, Helicity and Chirality. A comparison is made between the experimental realizations of Falaco Solitons as propagating topological defect singularities in a fluid discontinuity, and the concept of a photon as propagating topological defect singularities in the light cone discontinuity.

Keywords: Photon, Propagating topological singularities, Topological Torsion and Spin, Polarization and Helicity,

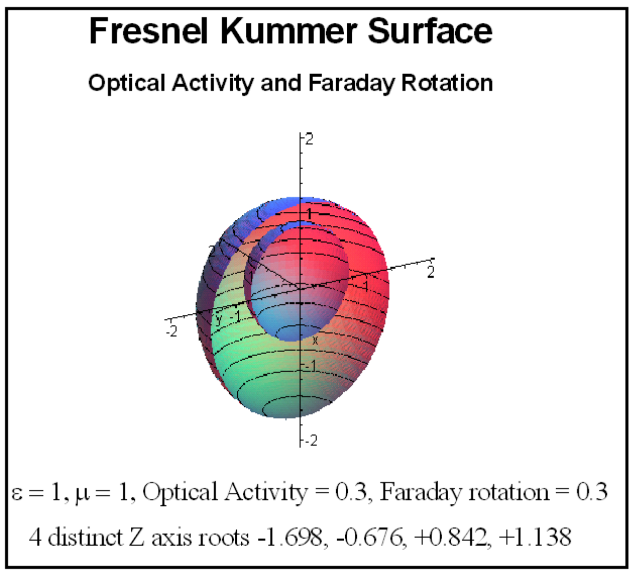
1. A TOPOLOGICAL PERSPECTIVE

At its foundations, Maxwell's Electrodynamics is a topological theory independent from the geometric constraints of metric scales or gauge symmetries. The fundamental partial differential equations were shown to be metric free by Van Dantzig[17] in the 1930's. It is now appreciated that the Partial Differential Equations of Maxwell's Electromagnetism can be deduced from topological arguments, independent from a choice of linear connection, as well as metric.[21] In the first half of the 20th century the dogma and successes of quantum mechanics combined with those successes of relativity, led to the idea that electromagnetic radiation was composed of corpuscular "quanta" or "photons" carrying integer spin (angular momentum independent from a choice of origin), $L = n\hbar$, and a ratio of energy $E = n\hbar\omega$, and momentum, $p = n\hbar/\lambda$ that is a constant equal to C^2. The object of this presentation is to demonstrate that a topological approach to Electromagnetism not only generates the Maxwell system of Partial Differential Equations, but also serves to demonstrate the "quantum" features of electromagnetic radiation, without ad hoc additions and assumptions. The fundamental idea is to relate the concept of the "photon" to those singular solutions of the Maxwell PDE's which represent point sets upon which the solutions are not uniquely defined. A class of such solutions can be represented by (complex) direction fields which are isotropic. Such objects were defined as "Spinors" by E. Cartan.[2] As isotropic direction fields, Spinors in a 4D space have a quadratic form with zero value, even though its components are not zero. A comparison of formal properties of spinors and vectors leads to the "strange" idea that the spinor has "zero geometric" length, and yet a pair of conjugate spinors can generate a non - zero bivector, representing a "finite" geometric area.

In 1932 Fock[4] demonstrated that the singular solutions to the Maxwell PDE's satisfied the null non-linear Eikonal partial differential equation (see eq. 9) and the zero sets of these solutions represented hypersurfaces of discontinuities of field intensities, **E** and **B**. Moreover these surfaces of field discontinuity propagated through space time with the phase velocity, C. Fock thereby gave formal realization of what constituted an electromagnetic signal (a definition well beyond the early Einstein conjecture that "needle" radiation would follow geodesic paths). The zero sets of the Eikonal singular solutions represent *propagating* discontinuities (a topological defect formed by limit sets) in the field strengths. The energy content and the field intensities of the EM field in front of a propagating discontinuity can be zero (it is dark), while the energy content and the field intensities of the EM field behind the propagating discontinuity are not zero (it is not dark). A "signal" is the abrupt recognition of the propagating discontinuity, from dark to light.

The finite propagation speeds of the singular solution discontinuities are 4 fold degenerate in spaces constrained by the special class of geometric symmetries known as the Lorentz constitutive equations. In general, there can be four speeds of discontinuity propagation, one for each direction and polarization pair, and all four speeds need not be the same. In the degenerate (Lorentz) case, all 4 speeds are the same and are equal to the same constant, C. Equivalence classes of "inertial" frames of reference can be defined such that each observer in the equivalence class would agree that an electromagnetic signal was a propagating singularity. Each inertial observer "sees" the signal as a propagating discontinuity. Fock demonstrated that the only *linear* transformations that preserved the signal discontinuity feature were the Lorentz transformations. It is this invariance feature of the field discontinuity that gives physical stature to the equivalence class of reference frames constructed with Lorentz transformations.

For those symmetry situations where the singular solutions are not 4 fold degenerate, then the propagation speeds of field discontinuities can be different for different polarizations and for different directions of propagation; i.e., the speed of light need not be the same in the outbound and inbound directions, say, in a rotating expanding plasma. Such results have been demonstrated in experiments with dual polarized ring lasers, and have been demonstrated theoretically by exact singular solutions (Fresnel Kummer surfaces) to Maxwell's equations.[11] As an example consider the case that is isotropic in a birefingence sense, but admits both Faraday rotation and Optical Activity. The Fresnel Kummer wave surface is given in the Figure below. The radius vector to a point on the surface is a measure of the reciprocal wave phase velocity. Each surface component represents a different polarization sense. It is obvious that there is no center of symmetry, and that there are 4 distinct phase velocities depending on polarization and direction.



It is important to realize that for a physical variety that can be encoded in terms of exterior differential forms, Cartan's methods of exterior differential systems effectively employ global, topological (rather than geometrical) constraints on the variety. The geometric methods (diffeomorphic equivalence) of tensor analysis restrict the analysis to those where topological evolution is impossible. The method of exterior differential forms goes beyond tensor analysis, and admits topological change to be considered. The topological theory of classical electromagnetism[21] can be constructed in terms of two exterior differential systems, which have a correspondence with thermodynamics in that the first ("pair" or even in French) exterior differential system deals with thermodynamic intensities (**E** and **B**) and potentials (**A**, ϕ), and the second ("impair" or odd in French) exterior differential system deals with thermodynamic quantities (or differential densities, **D** and **H**) and charge current densities (**J**, ρ). The pair exterior differential 2-form system can be written as $F - dA = 0$, and the impair exterior differential 3-form system can be written as, $J - dG = 0$. In each case, the exterior differential system acts as a topological constraint on the variety of exterior differential forms constructed in terms of the independent base variables, say $\{x, y, z, t\}$ and their differentials. These two fundamental topological constraints lead algebraically to two other independent topological concepts (exterior differential systems) of topological torsion, $A\hat{}F$, and topological spin, $A\hat{}G$. Both 3-forms vanish in isolated-equilibrium thermodynamic systems, and are explicitly dependent upon the concept of potentials, $\{\mathbf{A}, \phi\}$.

The additional exterior differential systems are deduced as $\{F\hat{}F\} - d(A\hat{}F) = 0$, and $\{F\hat{}G - A\hat{}J\} - d(A\hat{}G) = 0$. The exterior derivative of the 3-forms, $A\hat{}F$,and $A\hat{}G$, creates the two familiar Poincare deformation "invariants", $\{F\hat{}F\}$ and $\{F\hat{}G - A\hat{}J\}$, as additional topological limit sets of an electromagnetic system, valid in the excited vacuum and the plasma state. Non-zero values of the Poincare invariants are the source of topological change and irreversible phenomena in non-equilibrium thermodynamics. When and where the Poincare "invariants" are equal to zero, the closed integrals of the 3-forms, $A\hat{}F$ and $A\hat{}G$, exhibit topological invariant properties similar to the "quantized" chiral and spin properties of a photon. The "quantization" result is a topological result (independent from any microscopic or macroscopic constraint of scale) related to the integers (by deRham theory). The "quantized" result is related to the obvious topological fact that the number of holes in a surface is always an integer; 1.439 holes does not make sense. In the opinion of the author, the new 3-forms, $A\hat{}F$ and $A\hat{}G$, and their dynamics (which vanish in equilibrium electrodynamic systems) will lead to many new practical applications that will utilize non equilibrium thermodynamic properties of electromagnetic systems.

2. DOWN WITH DOGMA

This presentation may startle the reader (or listener) with what might appear to be a bit of heresy relative to the classical teachings of Electromagnetism, which currently are presented dogmatically in terms of a geometrical perspective. The ultimate topic of discussion herein is the Photon. The perspective of this presentation is based upon Topology, not Geometry.

The first somewhat heretical claim is:

1. Maxwell's theory of Electromagnetism is a Topological Theory,
not a Geometric theory, and can be deduced from logical principles.

Although admittedly "discovered" through a historical series of geometrically dominated or constrained experiments, and then summarized and augmented with an inspired guess by J. C. Maxwell, it should be recognized that the PDE's of electrodynamic theory can be deduced (in a Platonic manner) from mathematical logic, without the use of geometric constraints of metric, size and shape, or even experiment. For example, the sequence of logical steps which produce, universally, the Maxwell Faraday Partial Differential Equations starts with:

1. An ordered set {1,2,3,4...}, followed by an ordered set of independent variables, with neighborhoods,
$$\{\xi^1, \xi^2, \xi^3, \xi^4...; d\xi^1, d\xi^2, d\xi^3, d\xi^4...\}, \tag{1}$$
upon which an ordered set of C2 functions $\{A_k(\xi^1, \xi^2, \xi^3, \xi^4...)\}$ is used to construct a C2 differentiable 1-form of Action, A. For electromagnetism, the coefficients of the 1-form play the role of the classic vector and scalar potentials:
$$A\{\xi^1, \xi^2, \xi^3, \xi^4...; d\xi^1, d\xi^2, d\xi^3, d\xi^4...\} = A_k(\xi^j)d\xi^k. \tag{2}$$
2. An abstract topological (neighborhood) constraint is imposed in terms of an (pair) exterior differential system,
$$\text{Constraint of thermodynamic Intensities } F - dA = 0. \tag{3}$$
The 2-form $F = dA$ is required to be exact, which leads to the classic electromagnetic flux conservation law. The topological constraint implies that the domain of support of the 2-form F (in engineering language, the **E** and the **B** field intensities) cannot be compact without a boundary. In effect, it topologically denies the existence of magnetic monopoles. Relative to even dimensional spaces, a 2-form of maximal rank generates a Symplectic manifold as the domain of support of the field intensities.

3. Exterior differentiation of the topological constraint, and use of the Poincare theorem on C2 differentiable functions, creates an ordered set of Partial Differential Equations from the coefficients of the equations (which are differential 3-forms): $dF = ddA = 0$.The first four equations of this ordered set of of PDE's have the format of the Maxwell Faraday Partial Differential Equations, which, by relabeling the partial derivatives[21] of the abstract coefficients, $A_k(\xi^j)$, are equivalent to the expressions,

$$\text{Maxwell Faraday} \tag{4}$$

$$div\ \mathbf{B} = 0,\ curl\ \mathbf{E} + \partial\mathbf{B}/\partial t = 0. \tag{5}$$

There are no additional terms, and no other field functions, no matter how many independent variables (≥ 4) are used in the construction of the abstract 1-form of Action. If more than 4 independent variables (geometric dimensions) are used, the new "coordinates" add new PDE's that couple "new" field variables to the **E** and **B** field variables of the first four (Maxwell) equations, but do not alter the format of the first four PDE's - the Maxwell Faraday equations - in any way! The Maxwell Faraday equations are valid in a universal sense, nested in the totality of the ordered set of variables. No metric ideas were used in this logical non-experimental "deduction" of the Maxwell Faraday PDE's. The concept of Faraday induction is universal for all thermodynamic systems that can be encoded by a 1 form of Action, A. It may be startling, but true, that Hydrodynamics and Mechanics, as well as Electromagnetism, when encoded in terms of a 1-form of Action, are governed by the Faraday induction law. If experimental results do not obey these "laws", then something is wrong with the human logic, or something has been forgotten in the experiment.

From a thermodynamic point of view, the 2-form F, is related to thermodynamic intensities (objects which are homogeneous of degree 0 - like temperature and pressure). However, the complete Maxwell system utilizes not only an exact (and pair) 2-form, F , but also recognizes that there exists another thermodynamic set of conjugate variables, related to quantities or excitations (objects which are homogeneous of degree 1 - like Entropy and Volume). In short, topological electromagnetism presumes that there exists a impair 2-form density G, which need not be closed, nor exact. The non-exact (and impair) 2-form $G\{\xi^1, \xi^2, \xi^3, \xi^4 ...; d\xi^1, d\xi^2, d\xi^3, d\xi^4 ...\}$ can have domains of support which are compact (closed) without boundary, while the exact 2-form F cannot. Exterior differentiation of G produces a 3-form of charge - current density, J, equivalent to a second topological constraint:

$$\text{Constraint of thermodynamic quantities (densities) } J - dG = 0. \tag{6}$$

The 3-form J, like F, is exact. This topological constraint leads to the Maxwell Ampere PDE's, and as J is exact, leads to the conservation of charge. With appropriate relabeling, the Maxwell Ampere equations are:

$$\text{Maxwell Ampere} \tag{7}$$

$$div\ \mathbf{D} = \rho,\quad curl\ \mathbf{H} = \partial\mathbf{D}/\partial t + \mathbf{J}. \tag{8}$$

The guess of a $\partial\mathbf{D}/\partial t$ term (as introduced by Maxwell) is automatic in the topological system.

Note that differential form densities, such as G and J, can be integrated without metric. The two systems of PDE's generated by exterior differentiation of the topological constraints are diffeomorphically invariant, meaning they are functionally well defined for all diffeomorphically equivalent coordinate systems, be they Galilean, Lorentz, Spherical, or anything else if the mapping functions are homeomorphically equivalent and differentiable. However, the differential form constraints are not constrained to diffeomorphic (tensor) equivalences. The topological differential form statements, and therefor Maxwell PDE's, are well defined (via the pullback substitutions) with respect to submersions from higher dimensional spaces (think fiber bundles) which are not invertible, but are differentiable. The maps from $\{\xi^1, \xi^2, \xi^3, \xi^4 ...; d\xi^1,\ d\xi^2,\ d\xi^3,\ d\xi^4 ...\}$ to A and G are essentially projections from the "tangent" fiber bundle. The only difference is that it must be remembered that the differentials $\{d\xi^1,\ d\xi^2,\ d\xi^3,\ d\xi^4 ...\}$ need not necessarily be *global* generators of a 1-parameter group.

The bottom line is that Lorentz (diffeomorphic) invariance of the PDE's is trivial, as they are tensor equations. So what makes the Lorentz equations so dogmatically important? The answer resides with the fact that the *singular solutions* of the PDE's, *not the equations*, have a linear (hence local) equivalence class generated by only the Lorentz transformations.

The second somewhat heretical claim is:

2. An electromagnetic signal is a propagating discontinuity
(a propagating topological defect), not a sinusoidal wave train!

Actually this idea was developed in detail by V. Fock[4] about 1932, where he demonstrated (following Hadamard's ideas[5] of characteristics) that the hyperbolic PDE's of Maxwell admitted singular solutions upon which the field intensities were not *uniquely* defined. These singular point sets can admit zero field intensities on one side and finite non zero field intensities on the other side of the singular solution submanifold. The singular point sets are not stationary and represent propagating discontinuities, with a speed $C = \pm 1/\sqrt{\epsilon\mu}$ in simple cases. The equivalence class of reference systems which are linearly related and preserve the singular solutions have a common fact: the singularities propagate at a finite constant and invariant speed, C. The singular set was defined by Fock in terms of a solution, ϕ, to the null Eikonal equation, which is a non-linear first order quadratic PDE equal to zero:

$$\{\pm(\partial\phi/\partial x)^2 \pm (\partial\phi/\partial y)^2 \pm (\partial\phi/\partial z)^2 \mp (\partial\phi/c\partial t)^2\} = 0. \tag{9}$$

The zero set of the singular solution set defines an implicit hypersurface in space time. (Note that Majorana, Weyl and Dirac Spinors are related to the differences in signs and their solution representations.)

More importantly, Fock demonstrated that the only *linear* transformation of coordinates that preserved the propagating field discontinuity was the Lorentz transformation. That is, if two observers were related by a Lorentz transformation, then if the first observer claimed to see a propagating discontinuity (signal), then so would the (Lorentz related) second observer claim to see a propagating discontinuity (signal) and each would say the propagation speed was the same constant C. The importance of the Lorentz transformation is that it defines an equivalence class of ("inertial") systems (for observers that use electromagnetic means of measurement) that preserve the idea of a propagating discontinuity (signal). The Maxwell PDE's are well defined with respect to **all** diffeomorphic observers, but the singular Eikonal singular solutions at constant speed C are well defined only with respect to the linear Lorentz equivalent observers. However, it is now known, but not widely utilized in engineering practice, that the extended (conformal or Poincare) Lorentz transformations also preserve the concept of signal discontinuity, and so do the non-linear fractional projective transformations.

It should be noted that Fock also demonstrated that there was a non-linear transformation that also preserved the concept of a propagating discontinuity. It is the fractional projective (Moebius) transformation. The speed of discontinuity propagation is not a constant, and can range from zero to infinity. Hence for Moebius related observers, the speed of a signal is not the constant value C of the Lorentz equivalent class. Such situations are also related to the conformal group. This mathematical result of Fock has yet to be exploited in practical electromagnetism, but the possibility of detecting signals that travel faster than C is exciting. The conjecture is on the expanding spherical sections of the light cone (which expand with the speed C) are topological defects which can move *on the spherical sections* with speeds greater than C.

The correspondence of the Fock - Eikonal idea to the Einstein - Null Geodesic idea is that both are based upon quadratic forms of a Minkowski signature. However, the Fock concept makes an explicit correspondence to Maxwell's electromagnetic theory, while the Einstein concept does not.

$$\text{Null geodesic} \quad : \quad (\text{đ}s)^2 = (dx)^2 + (dy)^2 + (dz)^2 - (dt)^2 \Rightarrow 0, \tag{10}$$

$$\text{Eikonal} \quad : \quad (\partial\phi/\partial x)^2 + (\partial\phi/\partial y)^2 + (\partial\phi/\partial z)^2 - (\partial\phi/c\partial t)^2 \Rightarrow 0. \tag{11}$$

Note that the square of the line element is not the square of an exact differential form; $đs$ can have path dependent values (an important fact in the resolution of the twin paradox).

The Eikonal solutions are not necessarily solutions to the wave equation. However, if an eikonal solution is also a solution to the wave equation, then any function of the Eikonal solution is also a solution to the wave equation. The classic example is given by the (linear) phase function, $\phi = kz \pm \omega t$, which satisfies both the eikonal equation and the wave equation, if the constants satisfy the constraint, $\omega/k = \pm\, c$.

In 1914, in a small monograph entitled Electrical and Optical Wave Motion, H. Bateman, introduced a number of interesting solutions to Maxwell's equations that emulate propagating singular strings (not plane waves). Bateman is perhaps more famous for his work on the equations that describe the decay chains of radioactive species. However, as pointed out by Whittaker,[23] it was Bateman who determined in 1910 that the Maxwell equations were invariant with respect to the conformal group, a much wider group than the Lorentz transformations. Bateman in 1910 also recognized the relationship of his work to the tensor calculus of Ricci and Levi-Civita, several years before the Einstein development of general relativity. Bateman[1] discusses various forms of transformations which lead to forming one wave function from another, including the Moebius transformation. He even describes methods for constructing a wave function on a space of dimension N+1 by transforming a solution to the diffusion equation on a space of dimension N. Bateman mentions that Stokes and Wiechert thought of X-rays as "pulses traveling through the aether, the energy being confined within a thin shell" (of discontinuities). However, there are solutions to the eikonal equation that are not solutions to the wave equation. This difference distinguishes a "signal" from a "wave".

The third somewhat heretical claim is:

3. The concept of Spinor singular solutions to Maxwell's equations
is a topological idea that does not depend upon microphysical scales.

The impact of quantum mechanics, starting with Planck's concept of the "quantized" oscillator energy enabling the thermodynamic deduction of the blackbody radiation distribution law, the Einstein model for explaining the photoelectric effect, the Bohr atom description of the emission of light carrying off integer units of angular momentum and rational amounts of energy, the Compton analysis of the relativisitic particle-like distribution peaks in the scattering of electromagnetic radiation by electrons, the deBroglie conjecture that energy and momentum were related to a "wave" analysis involving Planck's constant, frequency and reciprocal wavelength, and Dirac's description of the relativistic hydrogen atom, all have led to the idea that the "bundle" of energy and momentum now known as the Photon has a deep relationship to microphysics, and appears to be associated with what Cartan called spinors. Yet Spinors are not dependent upon scales, and should have physical importance in the macroscopic as well as the microscopic domain. The philosophical problem is that these bundles of energy and momentum, these photons, can have extent and coherent interactions that are many orders of magnitude greater than the dimensions of the excited atoms and molecules, from which they supposedly originate. A fundamental question is how do the quantal properties of the Photon emerge from a topological perspective of electromagnetism? The bottom line is the result that propagating discontinuities as topological defects are independent from scale.

First consider the concept of Spinors. Without the use of micro scales, the idea of Spinor solutions to Maxwell's Electrodynamics comes from the topological perspective that the 2-form of field intensities, $F = dA$, can be represented by an anti-symmetric matrix $[\mathbb{F}]$ of functions representing the 6 components of the electric and magnetic field, $\mathbf{E}$ and $\mathbf{B}$. Then, depending on the rank of the matrix $[\mathbb{F}]$ (in say 4D) the eigenvectors either have zero eigenvalues, or complex eigenvalues. In every case, if $\mathbf{e}$ is an eigenvector with eigenvalue γ, such that

$$[\mathbb{F}] \circ |\mathbf{e}\rangle = \gamma\, |\mathbf{e}\rangle\,, \tag{12}$$

$$\langle \mathbf{e}| \circ [\mathbb{F}] \circ |\mathbf{e}\rangle = \gamma\, \langle \mathbf{e}| \circ |\mathbf{e}\rangle\,. \tag{13}$$

Due to antisymmetry of $[\mathbb{F}]$, it follows that

$$\langle \mathbf{e}| \circ [\mathbb{F}] \circ |\mathbf{e}\rangle = \gamma \langle \mathbf{e}| \circ |\mathbf{e}\rangle \Rightarrow 0. \tag{14}$$

For division algebras there are two choices: either $\gamma = 0$, or $\langle \mathbf{e}| \circ |\mathbf{e}\rangle = 0$. The implication is that for non zero eigenvalues γ, the quadratic form $\langle \mathbf{e}|\circ|\mathbf{e}\rangle$ must vanish. (This concept can be extended to a diagonal unit matrix of any signature.) For $\gamma \neq 0$, the eigenvector of the antisymmetric matrix is a complex vector, which has been defined as an "isotropic vector" in the theory of differential geometry. The isotropic eigenvector direction fields are similar to vectors, but have complex components and generate non-zero complex eigenvalues of the matrix $[\mathbb{F}]$. Such isotropic vectors define Spinors.[2] Spinors have "metric" properties in the sense of a quadratic form (which has value zero), but not the *unique* affine properties (see p3, in Cartan's book[2]) of tensors. Spinors are polarization, or chirally, sensitive.

If the rank of the N x N matrix $[\mathbb{F}]$ is less than N, then the matrix can have real vectors with eigen value zero. These vectors of eigen value zero will be defined as "null" vectors, and are not the same as Spinors. Spinors represent topological defects in complex domains. Why? - because the zero value of the quadratic form, $\langle \mathbf{e}| \circ |\mathbf{e}\rangle = 0$, defines an implicit hypersurface in the complex domain. The topological dimension of the complex domain is reduced by 2, on the spinor hypersurface as a subspace of the complex domain.

As an example, consider the 1-form of Action, $A(x, y, z, s, dx, dy, dz, ds)$ and its associated Pfaff sequence given by the expressions

$$\begin{aligned}
A &= -ydx + xdy - sdz + zds, & (15)\\
F &= dA = 2dx\hat{}dy + 2dz\hat{}ds \Rightarrow B_z dx\hat{}dy + E_z dz\hat{}dt, & (16)\\
A\hat{}F &= 2\{xdy\hat{}dz\hat{}ds - ydx\hat{}dz\hat{}ds + zdx\hat{}dy\hat{}ds - sdx\hat{}dy\hat{}dz, & (17)\\
F\hat{}F &= 8dx\hat{}dy\hat{}dz\hat{}ds \Rightarrow 2\,(\mathbf{E}\cdot\mathbf{B})\,dx\hat{}dy\hat{}dz\hat{}ds. & (18)
\end{aligned}$$

If the 1-form is interpreted as a representation of a Maxwell system, then there exists both an **E** and a **B** field, both equal to the constant value 2 (times a factor to adjust the physical dimensions), and both parallel to one another. That is, $\mathbf{E} \circ \mathbf{B} \neq \mathbf{0}$. The 3-form of topological torsion, $A\hat{}F$, is not zero, and has components proportional to a 4 dimensional position vector, $\mathbf{R}^T = [x, y, z, t]$ representing a 4D expansion.

Note that the 4x4 antisymmetric matrix representation of the 2-form F is of the form

$$[\mathbb{F}] = \begin{bmatrix} 0 & 2 & 0 & 0 \\ -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & B_z & 0 & 0 \\ -B_z & 0 & 0 & 0 \\ 0 & 0 & 0 & E_z \\ 0 & 0 & -E_z & 0 \end{bmatrix}. \tag{19}$$

The matrix has 4 eigenvalues and 4 eigenvectors given by the formulas,

$$\begin{aligned}
\text{Isotropic Eigenvector 1} &= [0, 0, -i, 1] \text{ with eigenvalue} = iE_z, & (20)\\
\text{Isotropic Eigenvector 2} &= [0, 0, i, 1] \text{ with eigenvalue} = -iE_z, & (21)\\
\text{Isotropic Eigenvector 3} &= [1, i, 0, 0] \text{ with eigenvalue} = iB_z, & (22)\\
\text{Isotropic Eigenvector 4} &= [1, -i, 0, 0] \text{ with eigenvalue} = -iB_z. & (23)
\end{aligned}$$

Each eigenvector is null isotropic such that the sum of squares (with euclidean signature) of the coefficients is zero. This example is a simple case generated by the 1-form of Action (potentials), A, whose coefficients form the adjoint field to the three exact differentials generated by the Hopf map (a submersion from 4D to 3D).[21] All eigenvectors of a 2-form F with non-zero determinant have "zero" (euclidean) length. The idea can be extended to Minkowski or anti-Minkowski signatures. The fundamental result is that Spinors are the natural format of propagating singularities generated from the Eikonal equation. It is the objective of this article to exploit the connection between propagating singularities, spinors and the photon.

It should be noted, in the example above, that, as $F\hat{}F \neq 0$, the Pfaff topological dimension of the 1-form, A, and the rank of the 2-form $F = dA$ is 4. It is also true that the topological torsion 3-form is not zero, $A\hat{}dA \neq 0$. It follows that evolution in the direction of $A\hat{}F$ is thermodynamically irreversible, and the topology induced by the 1-form of Action is not an evolutionary invariant. Electrodynamic expansion of the 4D universe is thermodynamically irreversible. Note that the associated $\mathbf{E}$ and $\mathbf{B}$ fields have parallel components; the value $\mathbf{E} \cdot \mathbf{B} \neq 0$ is a measure of the dissipation.[18] This situation implies that the determinant of the 4D matrix, $[\mathbb{F}]$, is non-zero, and there are no eigenvectors with zero eigenvalues.

The existence of Spinors is a topological property related to the Pfaff topological dimension[18] of the 1-form, A, that generates the 2-form of EM field intensities, F. If the Pfaff topological dimension is 4, all four eigenvectors of the matrix $[\mathbb{F}]$ are spinors. If the Pfaff topological dimension of A is 3 or 2, then two of the eigenvectors of the matrix $[\mathbb{F}]$ are spinors with $\gamma \neq 0$, and the other eigenvectors have $\gamma = 0$. If the Pfaff topological dimension of A is 1, then the matrix $[\mathbb{F}]$ is null (as $F = dA \Rightarrow 0$), and there are no spinor eigenvalues. Of particular interest are those cases where the Pfaff topological dimension of A is 3 or 4, for then in each case the 3-form of "topological torsion exists; $A\hat{}F \neq 0$. Explicit formulas in engineering format are given below. Recall that for any 4x4 matrix, the characteristic polynomial (in terms of eigenvalues) has either 4 real solutions, two pairs of complex conjugate solutions, or 1-pair of complex conjugate solutions along with 2 real solutions.

A search of the more recent mathematics literature[6] indicates that Spinors can be related to harmonic forms, and also to conjugate pairs of minimal surfaces. Yet little application of this correspondence has been made in the engineering physical sciences. For purposes herein, the conclusion reached is that Spinors are normal consequences of antisymmetric matrices, and, as topological artifacts, are not restricted to physical microscopic or quantum constraints of scale. According to the topological thermodynamic arguments, they should appear at all scales of Pfaff topological dimension 2 or greater. The fundamental result is that Spinors are the natural format of propagating singularities generated from the Eikonal equation.

Note that the 2-form F can be considered to be a "vector" in a 6 dimensional space. If the components of this 6-vector are such that $\mathbf{E} \cdot \mathbf{B} = 0$, then the determinant of matrix $[\mathbb{F}]$ is zero. This induced topological constraint has been has called the Klein quadric; the projection hypersurface reduces the 6 dimensional space to a 5 dimensional projective subspace. This result forms an interesting connection between modern Kaluza-Klein theory and the embedding of a curved 4D manifold into a Ricci flat space of 5D.[22] The important point is that when $\mathbf{E} \cdot \mathbf{B} = 0$ is zero, the Pfaff topological dimension is at most 3, and there exist on a 4D space two eigenvectors of the matrix $[\mathbb{F}]$ which have zero eigenvalues, and two eigenvectors which are spinors. These results will be used below to formulate a model of a photon deduced from a topological perspective of Electromagnetic theory.

The fourth somewhat heretical claim is:

4. The concept of Photon Quantization is a topological idea
that does not depend upon microphysical scales.

From the topological formulation given above, in terms of exterior differential forms, $\{ A, F, G, J \}$ the question arises as to how discrete (quantum) features of the photon enter into the topological theory. From thermodynamic arguments, if the Maxwell equations are uniquely integrable, then the maximum topological dimension of the 1-form of Action is 2. That is, there exist two functions on the geometrical domain of 4D which generate all of the differential topology associated with the field intensities. Such is the domain of an isolated, or equilibrium, thermodynamic system. Exterior differential 3-forms, such as $A\hat{}F$, do not exist on domains of an isolated topology of Pfaff dimension 2 (or less); the topological structure consists of a single connected component. On non-equilibrium domains, the topological dimension of the 1-form of Action potentials, A, can be 3 or 4. Such domains support exterior differential 3-forms and 4-forms on multiple components of the topological structure. The question is how do you formulate the possible multiple component topological structures of non-equilibrium electrodynamic system? The answer is in terms of closed, but not exact, exterior differential 3-forms which are homogeneous of degree zero.

By inspection, from the set of exterior forms $\{ A,\ F,\ G,\ J\}$, it is possible to construct two important 3-forms that are related to 4 component "currents" on a 4D domain of $\{x, y, z, t\}$. The 3-forms are written in terms of engineering variables, representing the coefficients of the 3-forms, in the following equations. The objects are zero in isolated-equilibrium systems. They are (topological) artifacts of non-equilibrium electromagnetic systems:

$$\begin{aligned}
\text{Topological Torsion} &= A\hat{}F = i(\mathbf{T}_4)dx\hat{}dy\hat{}dz\hat{}dt, \quad \text{units h/e} && (24)\\
\mathbf{T}_4 &= [\mathbf{E}\times\mathbf{A}+\mathbf{B}\phi,\ \mathbf{A}\circ\mathbf{B}] && (25)\\
\text{Topological Spin} &= A\hat{}G = i(\mathbf{S}_4)dx\hat{}dy\hat{}dz\hat{}dt,\ \text{units h} && (26)\\
\mathbf{S}_4 &= [\mathbf{A}\times\mathbf{H}+\mathbf{D}\phi,\ \mathbf{A}\circ\mathbf{D}], && (27)
\end{aligned}$$

These topological objects are universally defined for non equilibrium electromagnetic systems, yet their dynamics and properties have been little utilized. These 3-forms can have non-zero exterior differentials (which are exact exterior differential 4-forms) related to the historical Poincare invariants of the electromagnetic field:

$$\begin{aligned}
\text{Poincare II}\ \ d(A\hat{}F) &= F\hat{}F = 2(\mathbf{E}\circ\mathbf{B})dx\hat{}dy\hat{}dz\hat{}dt && (28)\\
\text{Poincare I}\ \ d(A\hat{}G) &= F\hat{}G - A\hat{}J = \{\mathbf{B}\circ\mathbf{H}-\mathbf{D}\circ\mathbf{E}\} - \{\mathbf{A}\circ\mathbf{J}-\rho\phi\}dx\hat{}dy\hat{}dz\hat{}dt. && (29)
\end{aligned}$$

When and where the exterior derivatives of each 3-form vanish, then by deRham's topological theorems, the closed cyclic integrals of each 3-form will have values that have rational integer ratios; i.e., the closed cyclic integrals are integers times some universal constant. The cyclic integrals therefor are "quantized" relative to the physical constant, h/e, for topological torsion, and to the physical constant, h, for topological spin. These concepts have not made any use of geometric ideas of size and shape, yet yield "quantum" numbers. The do not depend upon geometric scales, nor any explicit use of quantum theory.

$$\begin{aligned}
\underset{closed}{\iiint}(A\hat{}F) &= m\ h/e \quad \text{if Poincare II} = 0 && (30)\\
\underset{closed}{\iiint}(A\hat{}G) &= n\ h \qquad \text{if Poincare I} = 0 && (31)
\end{aligned}$$

Further note that the ratios of these two topological quantum numbers yields the Hall impedance, $Z_{Hall} = h/e^2$ (to within a rational fraction).[10]

In terms of topological thermodynamics, the manifolds upon which $d(A\hat{}F) = F\hat{}F = 0$ are non-equilibrium domains of Pfaff topological dimension 3. These submanifolds of space time can emerge (as if by a droplet condensation process) from dissipative thermodynamic systems of Pfaff topological dimension 4 ($d(A\hat{}F) = F\hat{}F \neq 0$). The Cartan topology of Pfaff dimension 3 (or more) is a topology of disconnected multiple components. Each component has a connected topological coherence.

The fifth somewhat heretical claim is:

5. Long lived propagating topological coherent states can occur in non equilibrium electrodynamic systems, and the photon is an example of such a soliton.

The non-equilibrium electrodynamic system consists of disconnected components where the Pfaff topological dimension is greater than 2. For a 4D space time set of independent variables, the possibilities are that the domain of interest is of Pfaff dimension 3 or Pfaff dimension 4. Pfaff dimension 3 domains can emerge from Pfaff topological dimension 4 domains by means of continuous thermodynamic irreversible processes in the direction of the topological torsion vector, generated by the components of the 3-form $A\hat{}F$ with $F\hat{}F \neq 0$. What is remarkable is that, on thermodynamic domains of Pfaff topological dimension 3 ($F\hat{}F = 0,\ A\hat{}F \neq 0$) a continuous evolutionary process in the direction of the topological torsion vector becomes equivalent to the unique extremal Hamiltonian field that generates "stationary, or excited, states". Such submanifold domains then can evolve as Soliton structures ("stationary" excited states) maintaining a topological coherence and a long life time. The submanifold structures of Pfaff topological dimension 3 do not depend upon geometric scales, yet they are precisely the domains required such that the 3-forms of topological torsion and topological spin have zero divergence. They are sets that have the properties required for the "quantized" topological properties of spin quanta and flux quanta.

Such unique Hamiltonian fields exist for all odd Pfaff topological dimensional systems greater than 2. Such manifolds belong to the class of Contact manifolds. All even Pfaff topological manifolds belong to the class of Symplectic manifolds, and do NOT admit such extremal Hamiltonian processes. In fact, it appears that the class of thermodynamically irreversible processes is an artifact of Pfaff topological dimension 4. The important idea is that non equilibrium electromagnetic systems involve the 3-forms of topological torsion, $A\hat{}F$, and topological Spin, $A\hat{}G$, whose closed homogeneous forms furnish the quantum numbers associated with photons[15].[12]

The subspaces of Pfaff topological dimension 3 may be viewed as topological defects or discontinuities in domains of Pfaff topological dimension 4. For example the generation of the light cone as propagating discontinuities in the 2-form F is directly related to the Klein quadratic implicit surface function, $F\hat{}F = 0$, which constrains the 4D domain of Pfaff topological dimension 4 to subspaces of Pfaff topological dimension 3. The eigenvectors of the matrix $[\mathbb{F}]$ are of two types: they are either eigenvectors of zero eigenvalue, or complex isotropic direction fields commonly called Spinors. Recall that the isotropic spinor eigenvectors are generators of 2D surfaces of zero mean curvature, within the 3D domains. It is these defect structures that are to be associated with the photon. On spaces with a euclidean metric, these 2D defect structures of zero mean curvature are called minimal surfaces, but on spaces with a Minkowski metric these defect structures of zero mean curvature are called maximal surfaces. Both defect surface types are amenable to isotropic spinor formulations.

3. MAXIMAL SURFACES

Maximal surfaces are 2D surfaces of zero mean curvatures that are generated by immersive maps from a two dimension space into a 3 dimensional space with a Lorentz metric.[3] The maximal surface is defined in terms of a space like immersion with positive Gauss curvature and with zero mean curvature. Such maximal surfaces are to be compared to minimal surfaces in a space with a Euclidean metric, but note that minimal surfaces in Euclidean space have negative Gauss curvature. Maximal surfaces can admit isolated, or "conical", singularities, where Minimal surfaces do not. Maximal surfaces can mimic catenoidal and helical surfaces of Euclidean theory, but may exhibit singular subsets of points. It is remarkable (and discussed in the next section) that such maximal surfaces can appear in fluids as propagating long lived topological defects. These observable propagating defects have been named Falaco Solitons,[19] and give credence to the model of a photon presented below.

Consider a 3D space with a Minkowski - Lorentz metric of the form

$$(ds)^2 = (dx)^2 + (dy)^2 - (dz)^2. \tag{32}$$

The immersion

$$R(u,v) = [(\sinh(v)\cos(u), (p\sinh(a)\sin(u), h\ v] \tag{33}$$

generates a space-like maximal surface of zero mean curvature in a space with a Minkowski metric. The coefficient p is related to the handedness of the rotation about the z axis, and h is related to the helicity along the z axis. The surface is of zero mean curvature, but the metric vanishes at the conical singular point: the Gauss curvature becomes infinite. The surface is similar to the hyperbolic minimal surface (Catenoid) in Euclidean geometry, but here, unlike the Euclidean catenoid, the Minkowski catenoid has a singular point. The surface is sensitive to the sign of the helicity ($h = \pm 1$), but is not sensitive to the handedness of polarization, p.

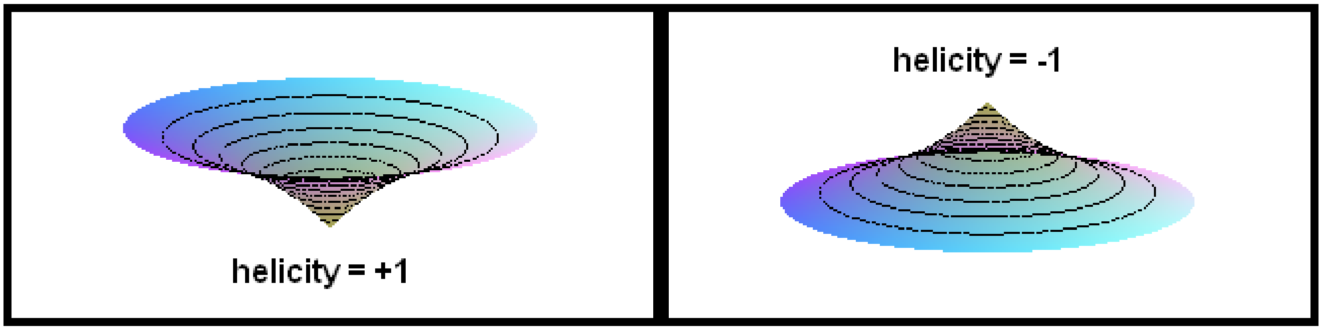


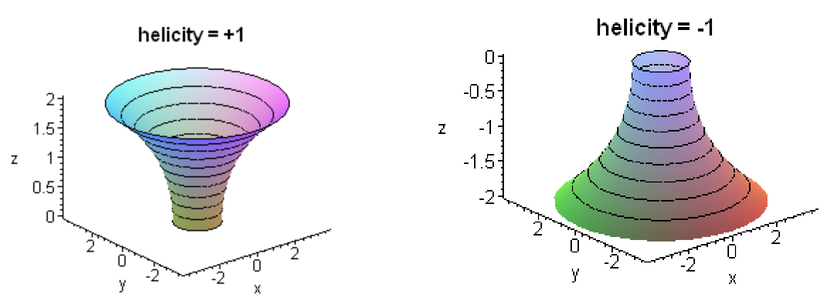
Fig. 1a and 1b. Maximal hyperbolic surfaces of zero mean curvature, but with different helicity in a 3D Minkowski space.

Other examples of zero mean curvature surfaces in both Euclidean and Minkowski spaces can be found at http://www22.pair.com/csdc/download/maxlor.pdf

The hyperbolic immersion

$$R(u, v) = [\cosh(v)\cos(u), p\cosh(v)\sin(u), hv] \tag{34}$$

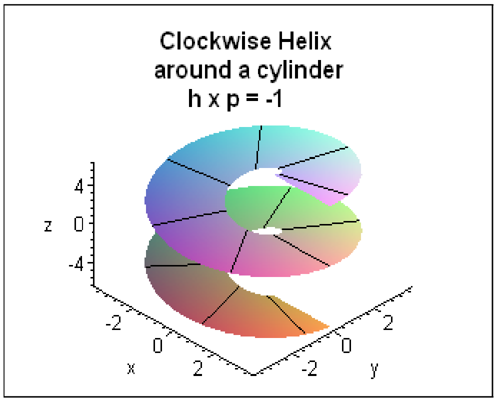
generates a minimal surface of zero mean curvature in a space with a Euclidean metric. The surface mimics a Wheeler wormhole, and the soap film between two rings separated by a diameter. The zero mean curvature surface is also sensitive to the sign of the helicity ($h = \pm 1$), but is not sensitive to the handedness of polarization, p.

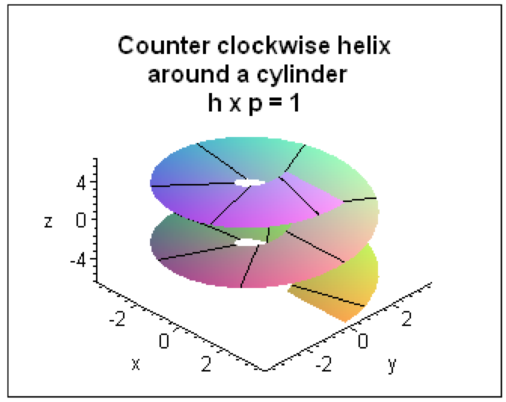


On the otherhand, the conjugate surface generated by the immersion

$$R(u, v) = [(\cosh(v)\cos(u), (p\cosh(v)\sin(u), h\ u] \tag{35}$$

generates a surface of zero mean curvature in *both* a Euclidean space or in Minkowski space. The surfaces are ruled helices rapped around a "hole" of radius unity. The Helical surface is sensitive to the sigh of the product of the polarization p and the helicity h.





The Gauss curvature of the immersion is negative and bounded in Euclidean Space. The Gauss curvature of the immersion is positive and singular for $v = 0$ in the Minkowski space.

The zero mean curvature surfaces, with a singular point, can be formed experimentally in a fluid. The experimental evidence is presented below. The idea that 3-dimensional space may or may not be Euclidean challenges a dogmatic precept of modern physics, where it is rarely perceived that physical 3D space can be anything but Euclidean. However, as discussed in the following section, the occurence of long lived rotational structures in the free surface of a water, which have been described as Falaco Solitons, exhibit the features of maximal surfaces in a Lorentz - Minkowski space. The Falaco Solitons are topological defect structures easily replicated in an experimental sense. Optical measurements indicate that the surface defect structures have a zero mean curvature. In addition, the surface defect structures have an apparent conical singularity which is an artifact of the signature of a maximal space-like surface in Minkowski space.

3.1. Falaco Solitons

During March of 1986, while visiting an old MIT roomate (class of 1950) in Rio de Janeiro, Brazil, the present author became aware of a significant topological event involving solitons that can be replicated experimentally by almost everyone with access to a swimming pool. The Falaco Solitons are topologically universal phenomena created experimentally by a macroscopic rotational dynamics in a continuous media with a discontinuity surface, such as that found in a swimming pool. The topologically coherent structure of Falaco Solitons appears to replicate certain features found at all physical scales, from spiral arm galaxies and cosmic strings to microscopic hadrons. The easy to replicate experiment indicates the creation of "stationary" thermodynamic states (or solitons) far from equilibrium. Such states are locally unstable in a Euclidean sense, but are experimentally globally stabilized.[19]

Study the photo which was taken by David Radabaugh, in the late afternoon, Houston, TX 1986.

Fig 2. Three Falaco Soliton pairs

The extraordinary photo is an image of 3 pairs of what are now called Falaco Solitons, a few minutes after their creation. The Falaco Soliton consists of a pair of globally stabilized rotational indentations in the free water-air surface of the swimming pool. The pair of contra-rotating dimples are connected by an (unseen in the photograph) interconnecting thread from the vertex of one dimple to the vertex of the other dimple of the rotational pair. These Solitons are apparently long-lived non-equilibrium states of matter far from thermodynamic equilibirum. They will persist for many minutes in a still pool of water. Their stability is presumed to be globally established by a connecting thread (under tension) that connects the two dimpled singularities. If the singular thread is abruptly severed (experimentally), the endcaps disappear in a rapid non-diffusive manner.

The black discs are formed on the bottom of the pool by Snell refraction of a rotationally induced dimpled surface of zero mean curvature. Careful examination of the contrast in the photo will indicate the region of the dimpled surface as deformed artifacts to the left of each black spot at a distance about equal to the separation distance of the top right pair and elevated above the horizon by about 25 degrees. The photo was taken in late afternoon. The fact that the projections are circular and not ellipses indicates that the dimpled surface is a surface of zero mean curvature. (Photo by David Radabaugh, Schlumberger, Houston, 1986.) A better photo, also taken by D. Radabaugh in 2004 in a swimming pool in Mazan, France, demonstrates the dimpled surface and the Snell refraction.

Fig 3. Surface Indentations of a Falaco Soilton

The photo is in effect a single frame of a digital movie that may be downloaded from http://www22.pair.com/csdc/download/blackspots.avi.

The fluid motion is a local (non-rigid body) rotation motion about the interconnecting thread. In the photos note that the actual indentations of the free surface are of a few millimeters at most. The lighting and contrast optics enables the dimpled surface structures to be seen (although highly distorted) above and to the left of the black spots on the bottom of the pool. The experimental details of creating these objects are described below. From a mathematical point of view, the Falaco Soliton is a connected pair of two dimensional topological defects connected by a one dimensional topological defect or thread. The Falaco soliton is easily observed in terms of the black spots associated with the surface indentations. The black circular discs on the bottom of the pool are created by Snell refraction of sunlight on the dimpled surfaces of zero mean curvature. Also the vestiges of mushroom spirals in the surface structures around each pair can be seen. Such surface spiral arms can be visually enhanced by spreading chalk dust on the free surface of the pool.

The surface defects of the Falaco Soliton are observed dramatically due the formation of circular black discs on the bottom of the swimming pool. The very dark black discs are emphasized in contrast by a bright ring or halo of focused light surrounding the black disc. All of these visual effects can be explained by means of the unique optics of Snell refraction from a surface of zero mean curvature. (This explanation was reached on the day, and about 30 minutes after, the present author became aware of the Falaco effect, while standing under a brilliant Brazilian sun and in the white marble swimming pool of his friend in Rio de Janeiro. An anecdotal history of the discovery is described in.[19]) During the initial few seconds of decay to the metastable soliton state, each large black disk is decorated with spiral arm caustics, remindful of spiral arm galaxies. The spiral arm caustics contract around the large black disk during the stabilization process, and ultimately disappear when the steady soliton state is achieved. It should be noted that if chalk dust is sprinkled on the surface of the pool during the formative stages of the Falaco soliton, then the topological signature of the familiar Mushroom Spiral pattern is exposed. The dimpled surface created appears to be (almost) a surface of zero mean curvature. This conclusion is justified by the fact that the Snell projection to the floor of the pool is almost conformal, preserving the circular appearance of the black disc, independent from the angle of solar incidence. (Notice that the black spots on the bottom of the pool in the photo are circular and not distorted ellipses, even though the solar elevation is less than 30 degrees.) The conformal projection property is a property of normal projection from surfaces of zero mean curvature.[16]

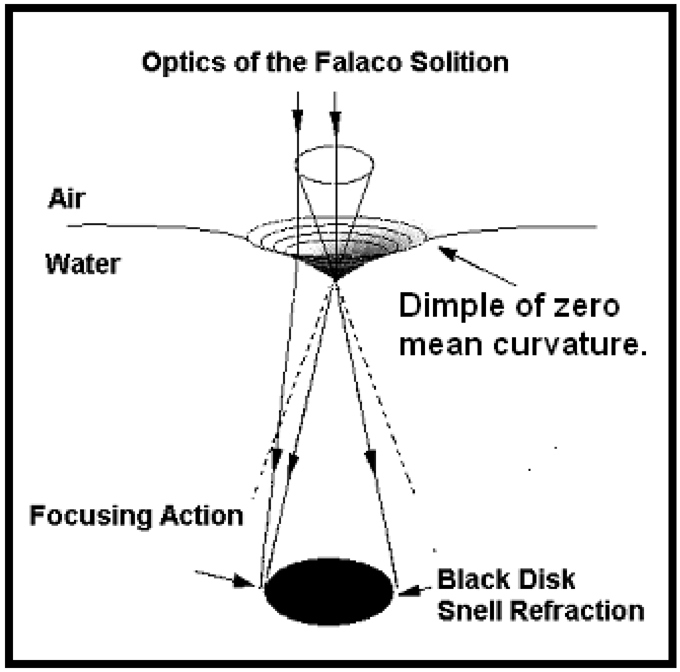


Fig 4. Optics of the Falaco Soliton

The Figure presented above was originally constructed in 1987-1989, before the concept of a Maximal Surface of zero mean curvature in Minkowski 3 Space was appreciated. However, note the similarity of the visually observed dimples presented phenomenologically in 1987 and the computed Minkowski maximal surface constructed in Figure 1.

A feature of the Falaco Soliton[19] that is not immediately obvious is that it consists of a pair of two dimensional topological defects, in a surface of fluid discontinuity, which are *connected* by means of a topological singular thread.

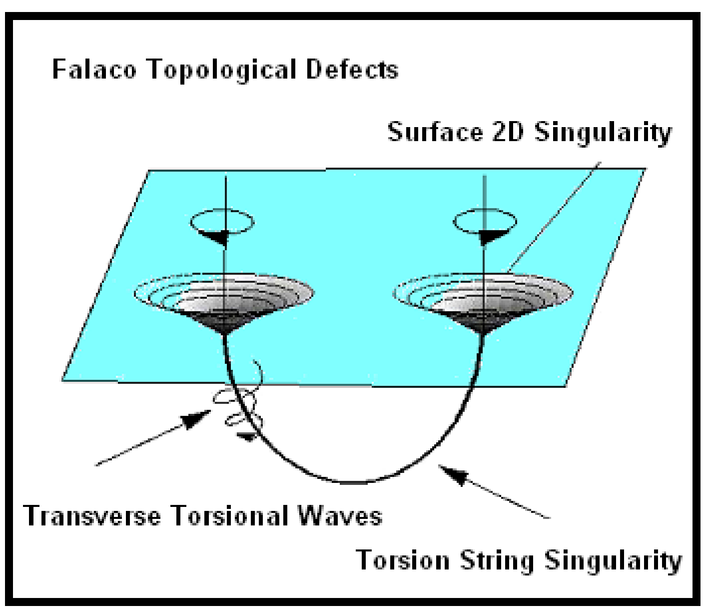


Fig 5. Falaco Topological Defects

Dye injection near an axis of rotation during the formative stages of the Falaco Soliton indicates that there is a unseen thread, or 1-dimensional string singularity, in the form of a circular arc that connects the two 2-dimensional surface singularities or dimples. Transverse Torsional waves of dye streaks can be observed to propagate, back and forth, from one dimple vertex to the other dimple vertex, guided by the "string" singularity. The effect is remindful of the whistler propagation of electrons along the guiding center of the earth's magnetic field lines. It is conjectured that the tension in the singular connecting thread provides the force that maintains the global stability of the pair of locally unstable, dimpled surface structures. The equilibrium mode for the free surface implies that the surface should be flat, of zero Gauss curvature and zero mean curvature, without dimpled distortions. If the thread is severed, the endcap singularities disappear almost immediately, and not diffusively.

However, as a soliton, the topological system retains its coherence for remarkably long time - more than 15 minutes in a still pool. The long lifetime of the Falaco Soliton is due to this *global stabilization* of the connecting string singularity, even though the surface of negative Gauss curvature is locally unstable (in euclidean space). The long life of the soliton state in the presence of a viscous media indicates that the flow vector field describing the dynamics is probably harmonic. This result is in agreement with the assumption that the fluid can be represented by a Navier-Stokes equation with a dissipation that is represented by the product of a shear viscosity and the vector Laplacian of the velocity field. If the velocity field is harmonic, the vector Laplacian vanishes, and the shear dissipation goes to zero no matter what is the magnitude of the shear viscosity term. Hence a palatable argument is offered for the existence of the Soliton's long lifetime. More over it is known that surfaces of zero mean curvature (in both Euclidean of Minkowski space) are generated by harmonic vector fields, hence the surface endcaps of zero mean curvature give further credence to the idea of a harmonic velocity field.

The bottom line is that it is possible to produce, hydrodynamically, in a viscous fluid with a surface of discontinuity, a long lived coherent structure that consists of a set of macroscopic topological defects. The Falaco Solitons are representative of non-equilibrium long lived structures, or "stationary states", far from equilibrium. These observation were first reported at the 1987 Dynamics Days conference in Austin, Texas, and subsequently in many other places, mostly in the hydrodynamic literature, as well as at several APS meetings.

These, long-lived topologically coherent objects, dubbed the Falaco Solitons, have several features equivalent to those reported for models of the sub-microscopic hadron. String theorists take note, for the structure consists of a pair of topological 2-dimensional locally unstable rotational defects in a surface of discontinuity, globally connected and stabilized globally in the fluid by a 1 dimensional topological defect or string with tension. (In Euclidean space, the surface defects are of negative Gauss curvature, and are, therefor, locally unstable.) As mentioned above, the experimental equilibrium state is a flat surface of zero Gauss curvature and zero mean curvature. However, it is conjectured that the local instability is overcome globally by a string whose tension globally stabilizes the locally unstable endcaps.

The reader must remember that the Falaco Soliton is a topological object that can and will appear at all scales, from the microscopic to the macroscopic, from the sub-submicroscopic world of strings connection branes to the cosmological level of spiral arm galaxies connected by threads. At the microscopic level, the method offers a view of forming spin pairs that is different from Cooper pairs and could offer insight into Superconductivity. At the level of Cosmology, the concept of Falaco Solitons could lead to explanations of the formation of flat spiral arm companion galaxies. At the submicroscopic level, the Falaco Solitons mimic quarks on a string and vortex structures in Boson condensates.

3.2. The Analysis

The original analysis was conducted without considering the possibility of 3D space having Minkowski signature properties. Several exact solutions to the Navier-Stokes equations, *in a rotating frame of reference but with a Euclidean metric assumption*, had been used to demonstrate bifurcations to catenoidal defect structures that in appearance are close to Falaco Solitons. The Navier-Stokes solutions were locally catenoidal with "open throat". However, the Navier-Stokes solutions (based upon euclidean vector methods) found so far, were locally catenoidal with an "open throat". They do not replicate the formation of the conical singularity that is observed experimentally. These results suggest that the catenoidal open throat structures are minimal surface realizations of topological defects of zero mean curvature in Euclidean 3 space, but the closed throat, string connected, Falaco Solitons are maximal surface realizations of topological defects of zero mean curvature in Minkowski 3 space. The difference leads to the definition of Euclidean Falaco Solitons (equivalent to Wheeler's wormholes) and Minkowski Falaco Solitons (with the appearance of string connected Hadrons, or of strings connecting "branes"). The fundamental idea that is missing in the classic Navier-Stokes analysis is that the dynamics could have Spinor (not vector) solutions, which are ignored classically.

In fact, in the original attempts to analyze the Falaco Soliton experiments, it was thought that the Falaco Solitons might be macroscopic realizations of the Wheeler wormhole (open vortex throat). This structure was presented early on by Wheeler (1955), but was considered to be unattainable in a practical sense. To quote Zeldovich p. 126[24]

> "The throat or "wormhole" (in a Kruskal metric) as Wheeler calls it, connects regions of the same physical space which are extremely remote from each other. (Zeldovich then gives a sketch that topologically is equivalent to the Falaco Soliton in a Euclidean metric). Such a topology implies the existence of 'truly geometrodynamic objects' which are unknown to physics. Wheeler suggests that such objects have a bearing on the nature of elementary particles and anti particles and the relationships between them. However, this idea has not yet borne fruit; and there are no macroscopic "geometrodynamic objects" in nature that we know of. Thus we shall not consider such a possibility."

This quotation dates back to the period 1967-1971. Now the experimental evidence justifies (again) Wheeler's intuition. However, the concept of a wormhole as a catenoidal minimal surface of zero mean curvature in a Euclidean space (with an open throat) is transformed into a Falaco Soliton as a catenoidal maximal surface of zero mean curvature in a Minkowski 3 space. The catenoidal surface of zero mean curvature, and negative Gauss curvature, in a 3D Euclidean space is a Wheeler Wormhole, while the catenoidal surface of zero mean curvature, and positive Gauss curvature, and its conical singular point in a 3D Minkowski space is a part of the fluid Falaco Soliton.

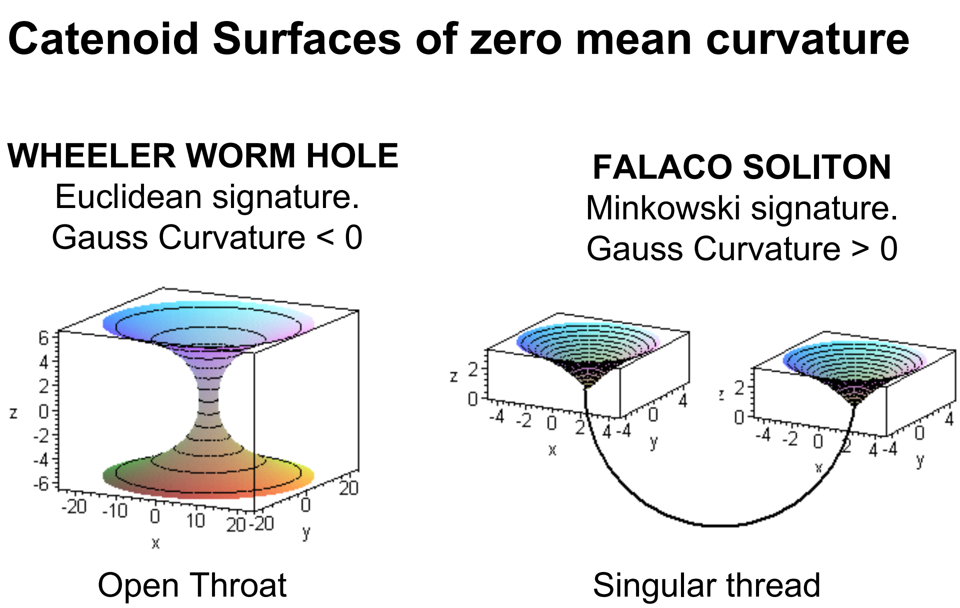
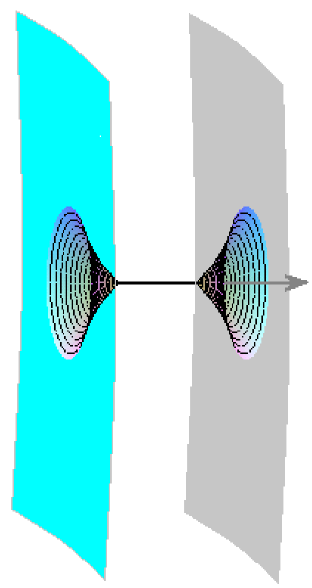


Fig 6. Surfaces of Zero mean Curvature

If the Maximal surfaces appear as deformations in disconnected hypersurfaces of discontinuity, the topological structure has the appearance of "strings connecting branes", a concept touted by the string theorists. The new feature is that the "brane" surface of discontinuity is deformed by the Maximal surface dimple. This structure motivates the next section in which the idea is used to model the photon.

Falaco Solitons



2D Maximal surface deformations in a 3D discontinuity surface, connected with a 1D string

The key experimental features of the Falaco Soliton are that the topological structure deforms a local discontinuity surface in a catenoidal-cone-like manner of zero mean curvature, and that the deformation appears to be stabilized by a connecting (elastic?) string or thread between a pair of deformation structures. In the next section, it is assumed that a thermodynamic (electromagentic) system can be encoded by a 1-form of Action potentials, A, which leads by exterior differentiation to a 2-form of field intensities, $F = dA$. The null eigenvectors of the antisymmetric matrix representation of F will form 3D expanding spherical surfaces of propagating field discontinuities (related to the spatial portions of the Minkowski lightcone where $F\hat{}F = 0$). In addition, the isotropic Spinor eigenvectors of F will form surfaces of zero mean curvature as defect structures on the spherical spatial portions of the lightcone. The result is a Falaco Soliton pair (with $A\hat{}F \neq 0$) between the two bounding cycles of a spherical shell. The claim is that this concept serves as a model for the Photon.

4. A TOPOLOGICAL MODEL FOR A PHOTON.

The idea is to combine the topological features of the Minkowski signature, the possibilities of coherent states of "stationary" topology (solitons) for non-equilibrium, but thermodynamically closed, systems of Pfaff topological dimension 3 (with $A\hat{}F \neq 0$), and the fact that for such systems the electromagnetic 2-form, $F = dA$, has one pair of eigenvectors of eigenvalue zero, and one pair of complex conjugate isotropic null eigenvector arrays with imaginary eigenvalues. The eigenvectors with zero eigenvalues form Minkowski lightcones with $F\hat{}F = 0$. Consider two causal expanding spheres (two light cones) representing the "on" and "off" propagating discontinuity defects (as expanding concentric spheres in 3D). The concentric spherical surfaces of field discontinuity bound an interior region of finite electromagnetic field intensities, **E** and **B**. The conjugate pair of Spinor eigenvectors define 2D surfaces of zero mean curvature as conical topological deformation defects on the light cones.

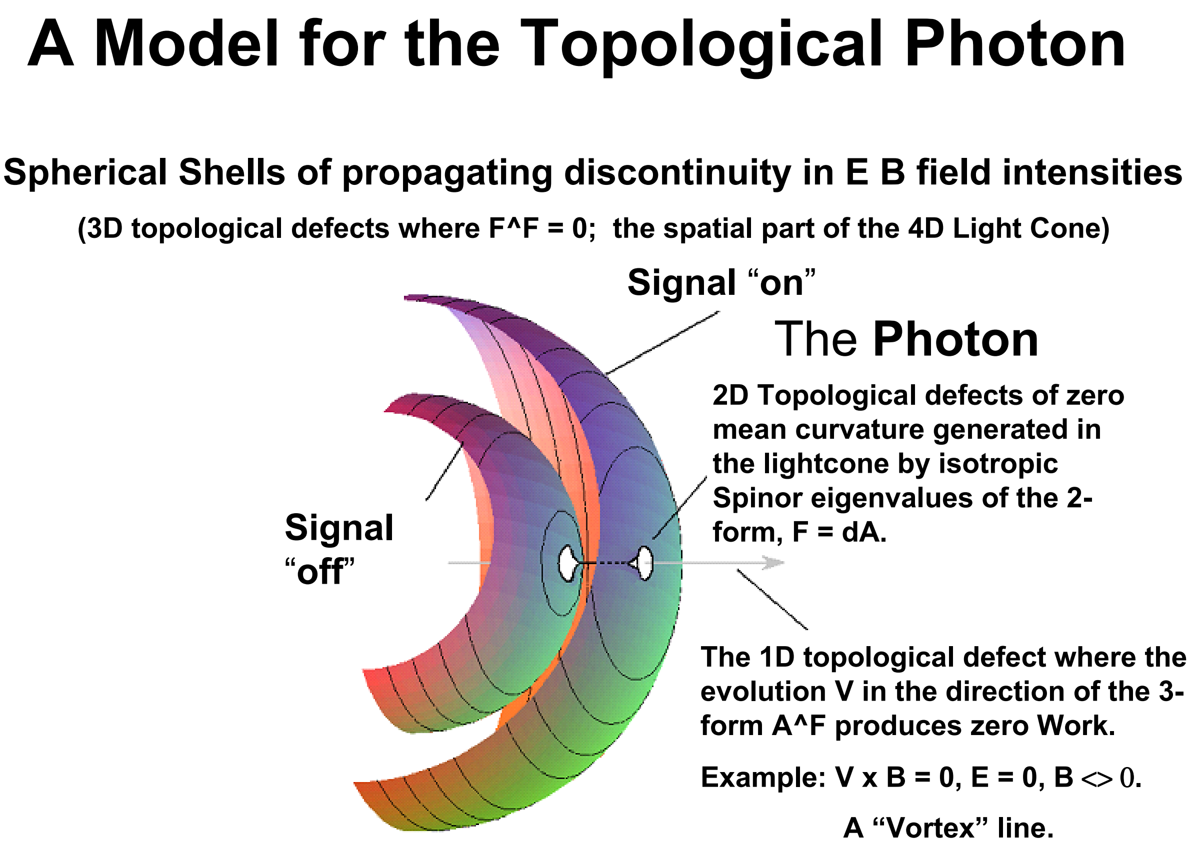


Fig. 7. The Photon as a Falaco Soliton between lightcone shells, which bound a closed non equilibrium thermodynamic state, $\mathbf{A}\hat{}\mathbf{F} \neq 0$.

The conical defects on each light cone are connected by a 1D "string", or "vortex tube", of zero radius, determined by the condition that evolution, V, in the direction of the components of the 3-form, $A\hat{}F$, of topological torsion, are extremal. That is, the thermodynamic work vanishes: $W = i(V)dA = i(V)F \Rightarrow 0$.

As an example, consider the 1-form of Action given by,

$$A = (m_e/e)\{\omega(xdy - ydx) - c^2dt\}, \tag{36}$$

where the constancts $(m_{electron}/e = h/(ec\lambda_{Compton})), \omega$ and c have been chosen on grounds of dimensional analysis. The Pfaff sequence demonstrates the the Pfaff topological dimension relative to the 1-form A is 3:

$$\begin{aligned} F &= dA = (m_e/e)2\omega\ dx\hat{}dy = B_z dx\hat{}dy, & (37)\\ A\hat{}F &= (m_e c^2/e)B_z dx\hat{}dy\hat{}dt, & (38)\\ F\hat{}F &= 0. & (39) \end{aligned}$$

There is no **E** field, but there is a **B** field component along the z axis. Hence the example has the properties

$$\mathbf{E} = 0,\ \mathbf{B} \neq 0,\ \mathbf{A} \neq 0,\quad \mathbf{E} \circ \mathbf{B} = 0,\quad \mathbf{A} \circ \mathbf{B} = 0 \tag{40}$$

Note that the scalar and vector potentials are given by the expressions

$$\phi = (m_e c^2/e), \tag{41}$$
$$\mathbf{A} = (m_e/e)\omega[-y, x, 0]. \tag{42}$$

The vector potential is tangent to a circle about the origin in the z = 0 plane. The direction field generated by the toplogical torsion vector is

$$\mathbf{T}_4 = [\mathbf{E}\times\mathbf{A}+\mathbf{B}\phi,\ \mathbf{A}\circ\mathbf{B}] \tag{43}$$
$$= [0, 0, B_z(m_e c^2/e), 0]. \tag{44}$$

For evolutionary processes $\mathbf{V}_4$ in the direction of $\mathbf{T}_4$, it follows that the Work 1-form vanishes.

$$W = i(\mathbf{V}_4)F = -\{\mathbf{E}+\mathbf{V}\times\mathbf{B}\}\circ d\mathbf{r} + (\mathbf{E}\circ\mathbf{V})dt, \tag{45}$$
$$W = i(\mathbf{T}_4)F = -\{\mathbf{0}+\mathbf{B}\phi\times\mathbf{B}\}\circ d\mathbf{r} + (0)dt \Rightarrow 0. \tag{46}$$

The evolutionary velocity field $\mathbf{V}$ in the direction of $\mathbf{T}_4$ is proportional to the $\mathbf{B}$ field.

This result gains credence from the observations of similar topological defects in fluid systems, called Falaco Solitons.[19] Thermodynamic systems of Pfaff topological dimension 3 (based on the 1-form, A) are non equilibrium, thermodynamically closed systems that can exchange energy (radiation) but not mass with their environments. When the Photon is "created" the Pfaff topological dimension is presumed to be 4, with evolution along a space time direction field given by the Topological Torsion vector, $\mathbf{T}_4$. The processes is thermodynamically irreversible, and $(\mathbf{E}\circ\mathbf{B}) \neq 0$. The process evolves continuously to domains of Pfaff topological dimension 3, forming the "condensations" - or Photons - of topological coherence as stationary, but excited, states of a Hamiltonian process. It is conjectured that the conical topological defects are not constrained by a limiting speed C, but can move (transversely on the light cone) with speeds given by the projective Moebius transformations, which can vary from zero to infinity.

5. CAN PHOTONS DETECT VACUUM CHIRALITY.

From the disciplines of Astronomy, General Relativity, and Quantum Mechanics comes an increased interest in possible chiral phenomena that could be associated with the cosmological vacuum state. Note that the cosmological vacuum state need not be a euclidean "void" and yet can be "flat" in the sense that metric and torsion components can cancel. The classic literature of electromagnetism does not seem to address the possibility of such a chiral effect. The conventional Lorentz Vacuum state for classical electromagnetism is defined in terms of solutions to the Maxwell Faraday equations for the intensities, $\mathbf{E}$ and $\mathbf{B}$, and the Maxwell Ampere equations for the excitations, $\mathbf{D}$ and $\mathbf{H}$, which produce no charge densities or current densities, and satisfy the constitutive equations of constraint, $\mathbf{D} = \boldsymbol{\varepsilon}_0\mathbf{E}$ and $\mathbf{H} = \mathbf{B}/\mu_0$. Such solutions for the field intensities satisfy not only both Maxwell equations, but also the vector wave equation with a propagation speed of $c = 1/\sqrt{\varepsilon_0\mu_0}$. The permittivity, ε_0, and the permeability, μ_0, of the Lorentz Vacuum domain are presumed to be isotropic and homogeneous constants.

It is remarkable that a chiral constitutive relation of the form $\mathbf{D} = \boldsymbol{\varepsilon}_0\mathbf{E} + [\gamma]\circ\mathbf{B}$ and $\mathbf{H} = \mathbf{B}/\mu_0 - [\gamma^{\ddagger}]\circ\mathbf{E}$ will also satisfy both Maxwell equations, without generating real charge densities and real current densities. The assumption of a simple complex scalar form for chiral constitutive matrix, $[\gamma] = (g + i\gamma)$, leads to two general cases. In one case, the only detectable difference between the Chiral vacuum and the Lorentz vacuum is to be found in the value for radiation impedance, Z, a value which depends on the chiral coefficients g and γ, as well as the ratio $\sqrt{\mu_0/\boldsymbol{\varepsilon}_0}$, through the determinant of the constitutive matrix. In the other case, the propagation phase velocities of left handed and right handed helical waves can be slightly different leading to a reactive impedance contribution to the classic radiation impedance of the Lorentz vacuum.

The Lorentz vacuum will be defined as the case where $\gamma = 0, \gamma^{\dagger} = 0$, and the Chiral vacuum will be defined as the case when $\gamma \neq 0, \gamma^{\dagger} \neq 0$. Substitution of the Lorentz Vacuum constraints, $\mathbf{D} = \varepsilon_0 \mathbf{E} \,.\, \mathbf{H} = \mathbf{B}/\mu_0$ into the Maxwell-Ampere equation yields

$$grad\, div\mathbf{E} - curl\, curl\, \mathbf{E} - \varepsilon\mu\partial^2\mathbf{E}/\partial t^2 \tag{47}$$

In other words a necessary condition for the Lorentz vacuum is that the fields satisfy the Vector Wave Equation (with $div\mathbf{E} = 0$).

Following Bateman, form the inner 3D product of the Maxwell Faraday equation with $\mathbf{H} = \mathbf{B}/\mu$, and the inner product of the source free Maxwell Ampere equation with $\mathbf{E}$. Use the constitutive definitions for the Lorentz vacuum where $\mathbf{H} = \mathbf{B}/\mu$ and $\mathbf{D} = \varepsilon\mathbf{E}$. Subtract the second resultant from the first, (assuming $\gamma = 0$), to produce the famous Poynting equation,

$$\begin{aligned} div(\mathbf{E} \times \mathbf{H}) + \mathbf{H} \circ \partial\mathbf{B}/\partial t + \mathbf{E} \circ \partial\mathbf{D}/\partial t \quad &\Rightarrow & (48)\\ div(\mathbf{E} \times \mathbf{H}) + \partial(1/2\mathbf{B}^2/\mu + 1/2\varepsilon\mathbf{E}^2)/\partial t \quad &= \quad 0. & (49)\end{aligned}$$

The result is an equation of continuity in terms of the field variables. By comparison to a "fluid", this "equation of continuity" yields a field energy density, ρ_e, and an energy current density, $\rho_e\mathbf{v}$, given by the expressions:

$$\rho_e c^2\mathbf{v} = (\mathbf{E} \times \mathbf{H}) = (\mathbf{D} \times \mathbf{B})c^2 \quad and \quad \rho_e c^2 = (1/2\mathbf{B}^2/\mu + 1/2\varepsilon\mathbf{E}^2). \tag{50}$$

It is important to note that the "energy flux, $(\mathbf{E} \times \mathbf{H})$", and the "momentum flux", $(\mathbf{D} \times \mathbf{B})$, are in the same direction and propagate with the same speed.

It should be remembered that these equations can be complex. The energy current density and the energy density can be formed from complex numbers. Bateman finds the extraordinary result, equivalent to the expression,

$$\begin{aligned} \rho_e^2(1/\mu\varepsilon - \mathbf{v} \circ \mathbf{v}) \quad &= \quad \rho_e^2(c^2 - \mathbf{v} \circ \mathbf{v}) & (51)\\ &\equiv \quad (1/c^2)\{[(1/2)(\mathbf{D} \circ \mathbf{E}) - (1/2)(\mathbf{B} \circ \mathbf{H})]^2 + (\mathbf{E} \circ \mathbf{B}/Z_{freespace})^2\}. & (52)\end{aligned}$$

under the assumption that $\varepsilon\mu c^2 = 1$. The factor (μ/ε) is the square of the radiation impedance of free space, $Z_{freespace} = \sqrt{\mu/\varepsilon}$. It is apparent that the first term on the right is the first Poincare (conformal) invariant equivalent to the Lagrange energy density of the field (the difference between the deformation and the kinetic energy densities). The second term is the second Poincare invariant of the field, and is to be associated with topological parity and thermodynamic irreversibility.[13] Bateman remarks that "the rate at which energy flows through the field is less that the velocity of light", unless the two Poincare invariants on the RHS vanish. The importance of the null Poincare invariants becomes obvious, as they furnish the requirement that the field energy propagates with the speed of light. It is important to remember that these equations can involve complex vector fields.

In general, for the Lorentz vacuum, the Hamiltonian energy density of the field is defined as

$$Ham = (1/2)(\mathbf{D} \circ \mathbf{E}) + (1/2)(\mathbf{B} \circ \mathbf{H}) = 1/2\mathbf{B}^2/\mu + 1/2\varepsilon\mathbf{E}^2 \tag{53}$$

while the field Lagrangian is defined classically as

$$Lag = (1/2)(\mathbf{D} \circ \mathbf{E}) - (1/2)(\mathbf{B} \circ \mathbf{H}) = 1/2\varepsilon\mathbf{E}^2 - 1/2\mathbf{B}^2/\mu. \tag{54}$$

The development above describes classic results valid for a Lorentz Vacuum, but now the question arises as to how these results change for a Chiral Vacuum. A Chiral Vacuum will be defined as a vacuum for which the constitutive matrices represented by $[\gamma]$ are not zero, but for which there are no real charge densities or current densities. The objective of this article is to assume that $[\gamma]$ is a complex domain constant, not zero, and then to determine what are the consequences of such an assumption. Such an assumption, which if applicable to the vacuum, would imply that the Chiral vacuum, and therefor the universe itself, may not have a center of symmetry. The Chiral adjective is appropriate, for a pure imaginary $[\gamma]$ replicates certain features of media which are optically active. The classic example of an optically active media is a solution of right handed helical molecules, such as sugar, in water. The phenomena has practical use in the wine industry and has been used to permit the grower to determine the sugar content of his grapes. (This is the basis of the words $\circ$*brix* often found on French wine labels).

Once a constitutive matrix is assumed it is possible to compute the characteristics of the combined Maxwell Faraday and Maxwell Ampere partial differential system. These surfaces, independent from any gauge assumptions, define point sets upon which the solutions to the partial differential system are not unique. The characteristic point sets, in general, form non-stationary Kummer-Fresnel quartic surfaces, of which the constitutive equations of the Chiral Vacuum generate a special case.[21] The theory for such surfaces has been worked out in detail, and the references below contain links to Maple programs that will generate such surfaces for arbitrary constitutive equations. There is an added importance to the recognition that the characteristic surfaces are Kummer surfaces, for then a connection between classical electromagnetism and Clifford algebras can be made, with the possibility that classical solutions to Maxwell's equations can involve spinors. Examples of such quaternionic solutions that indicate that the phase velocity of propagation in the inbound and outbound directions are not the same have been published.[11]

Along these lines, it is of interest to note that, in 1914, Bateman[1] realized that a complex 3-dimensional vector, $\mathbf{M} = \mathbf{B} \pm i\sqrt{\varepsilon\mu}\mathbf{E}$ could be used to express both the Maxwell Faraday and the Maxwell Ampere equations for the Lorentz vacuum as one combined set of complex vector equations. Bateman determined that it is possible to find a conjugate pair of solutions $\mathbf{M}$ and $\mathbf{M}'$ that satisfy the complex equation

$$\mathbf{M} \circ \mathbf{M}' = 0. \tag{55}$$

Each solution satisfies the equation

$$\mathbf{M} \circ \mathbf{M} = (\mathbf{B}^2 - \varepsilon\mu\mathbf{E}^2) \pm 2i(\sqrt{\varepsilon\mu}\ \mathbf{E} \circ \mathbf{B}) = \mathbf{I}_1 \pm 2i\,\mathbf{I}_2, \tag{56}$$

where $\mathbf{I}_1$ and $\mathbf{I}_2$ are the Poincare conformal invariants of the field, $\mathbf{M}$.

If the complex solution vector satisfies the complex equation of constraint,

$$\mathbf{M} \circ \mathbf{M} = (\mathbf{B}^2 - \varepsilon\mu\mathbf{E}^2) + 2i(\sqrt{\varepsilon\mu}\ \mathbf{E} \circ \mathbf{B}) = 0, \tag{57}$$

then such a vector not only satisfies both the Maxwell Faraday and the Maxwell Ampere (source free) equations for a Lorentz vacuum, but also propagates the field energy with the speed of light. Such solutions were defined by Bateman as self conjugate solutions. (Translate to self dual solutions in modern day language.) The self dual equation of constraint also leads to the Clifford algebras, and therefor indicates that the Bateman solutions can have spinor representations, as well as complex number representations.

The Bateman self conjugate condition requires that the (complex) magnetic energy density be the same as the (complex) electric energy density, and the (complex) Electric field be orthogonal to the (complex) Magnetic field, $\mathbf{E} \circ \mathbf{B} = 0$. Both of these Poincare conformal invariants must be zero to satisfy the Bateman self duality condition. It is the self dual solutions, these self conjugate solutions, that satisfy the Eikonal expression, and therefore, as Bateman points out, can represent propagating electromagnetic discontinuities.[7] The Poincare invariants are additive, such that it is conceivable to construct a self-conjugate solution from two or more non-self conjugate solutions, each of which has different Poincare invariants, but which are equal to zero under addition.

Bateman apparently did not notice that the complex constraint equation of self duality on $\mathbf{M}$ is precisely the conditions that the complex position vector generated by $\mathbf{M}$ defines a minimal surface.[8] Moreover, Bateman did not notice that most of his results are to be obtained also for a Chiral vacuum.

5.1. Details of a chiral vacuum

Use the (complex) Chiral Vacuum constitutive equations in the format of Post,[9]

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + [\gamma] \circ \mathbf{B} \qquad \mathbf{H} = -[\gamma^\dagger] \circ \mathbf{E} + \mathbf{B}/\mu_0 \ , \tag{58}$$

along with the Maxwell Faraday equations and the Maxwell Ampere equations, and replicate the steps of the preceding section. For simplicity, assume that the matrix

$$[\gamma] = (g + \sqrt{-1}\gamma)\sqrt{\mu/\varepsilon}\,[1], \quad [\gamma^\dagger] = (\alpha \cdot g - \sqrt{-1}\beta \cdot \gamma)\sqrt{\mu/\varepsilon}\,[1] \tag{59}$$

where $\alpha, \beta = \pm 1$. Note that if $\alpha = +1, \beta = +1$, then $[\gamma^\dagger]$ is the Hermitean conjugate of $[\gamma]$. If $\alpha = 1, \beta = -1$, then the imaginary part of $[\gamma]$ is anti-Hermitean. The Fresnel-Kummer wave surface equation for the characteristic of the Maxwell equations may be written as the polynomial,

$$\{R^4 + 1 - [2 - g^2(1-\alpha)^2 + \gamma^2(1+\beta)^2]R^2\} - i2\{g\gamma(1-\alpha)(1+\beta)\} = 0, \tag{60}$$

where $R^2 = {n_x}^2 + {n_y}^2 + {n_z}^2 = \mathbf{n} \circ \mathbf{n}$ represents the norm of the projectivized wave vector (index of refraction vector), $\mathbf{n} = \mathbf{k}/\omega$. Solutions of the characteristic polynomial yield the phase velocities of propagation in terms of the magnitude of the reciprocal index of refraction vector, $\mathbf{n}$. The phase velocity solutions are isotropic and homogeneous constants, determined by the root of the characteristic polynomial. The phase velocity is complex unless $\alpha = +1$, or the numeric factors are zero, e.g., $g = 0$ or $\gamma = 0$. For this reason, the case of $\alpha = -1$ is ignored in this article.

If the Hermitean conjugate constraints are used, $\alpha = 1,$ and $\beta = 1,$ then the phase velocity is determined from the formula for the (homogeneous, isotropic) index of refraction, $n = \pm\gamma \pm \sqrt{\gamma^2 + 1}$. For finite γ any g, there is a time-like dispersion of two helical waves. These chiral waves have phase velocities a bit greater and a bit less that the velocity of light $c = \sqrt{1/\varepsilon\mu}$, as determined by the chiral factor γ, and these phase velocities are independent of the chiral factor g.

If the constraints $\alpha = 1$,and $\beta = -1$ are used, then the phase velocities are those of the Lorentz Vacuum, ($n = 1$), for any value of chiral factors, g and/or γ. The fundamental result is that the Chiral Vacuum and the Lorentz Vacuum are almost indistinguishable.

For the case $\alpha = 1$, and $\beta = 1$, the determinant of the constitutive matrix is real and equal to

$$\det[Constitutive] = -(\varepsilon/\mu + g^2 + \gamma^2)^3, \tag{61}$$

a value which is proportional to the reciprocal of the free space impedance cubed. For $\gamma = 0$, the only difference between the Chiral Vacuum and the Lorentz Vacuum would be in the value of the free space impedance, $Z = \sqrt{1/(\varepsilon/u + g^2)}$. If $\gamma \neq 0$, then there could exist a slight dispersion (in time) between left handed and right handed polarization states.

For the case $\alpha = 1$, and $\beta = -1$, the determinant of the constitutive tensor is more complicated. The determinant has complex values (implying dissipation) unless either $\gamma = 0$, or $g = 0$. In each non-dissipative case,

$$Z = \sqrt{1/(\varepsilon/u + g^2)} \ \ for\ \gamma = 0, \quad Z = \sqrt{1/(\varepsilon/u - \gamma^2)} \ \ for\ g = 0, \quad n = 1. \tag{62}$$

Reality constraints imply that all cases of interest to this article are such that $\alpha = 1$. Substitution of the constitutive equations into the Maxwell Ampere equation yields

$$\mathbf{J} = curl\,\mathbf{H} - \partial\mathbf{D}/\partial t = \{curl\mathbf{B} - \varepsilon\mu\partial\mathbf{E}/\partial t\}/\mu \tag{63}$$

$$+g(-curl\,E - \partial B/\partial t) + \sqrt{-1}\gamma(\beta \cdot curl\,E - \partial B/\partial t) \tag{64}$$

$$\rho = div\mathbf{D} = \varepsilon div\mathbf{E} + (g + \sqrt{-1}\gamma)(div\,\mathbf{B}) \tag{65}$$

The point of this exercise is to note that in virtue of the Maxwell Faraday equation, the Chiral Vacuum constitutive relations produce no real charge currents or charge densities if $\beta = -1$, independent of the choice of chiral coefficients. The field intensities satisfy the vector wave equation with phase velocities that are those of the Lorentz Vacuum.

If $\beta = +1$ then only an imaginary current density is created for non-zero γ. It is then possible to compute the reactive power, $\mathbf{J} \circ \mathbf{E}$, and therefor a reactive impedance that depends upon γ. It is tempting to identify the chiral coefficient with the reciprocal Hall impedance, $\gamma = e^2/h)$. The field intensities then satisfy a wave equation with a phase velocity that depends upon γ.

In no case do the Chiral Vacuum constitutive equations yield a free charge density, if $div\mathbf{E} = 0$ and $div\mathbf{B} = 0$. This result is valid if the field intensities are derived from a set of potentials. A second point is that the chiral factors of the type, g, do not have any effect on the Lorentz Vacuum except to modify the Radiation Impedance, Z.

Similar substitutions of the Chiral constitutive equations lead to the Poynting equation in the form:

$$\begin{aligned} div(\mathbf{E} \times \mathbf{H}) + \mathbf{H} \circ \partial \mathbf{B}/\partial t + \mathbf{E} \circ \partial \mathbf{D}/\partial t &= \\ div(\mathbf{E} \times \mathbf{H}) + \partial(1/2\mathbf{B}^2/\mu + 1/2\varepsilon \mathbf{E}^2)/\partial t &= \{(\alpha - 1)g - \sqrt{-1}(\beta + 1)\gamma\}\mathbf{E} \circ \partial \mathbf{B}/\partial t. \end{aligned} \tag{66}$$

If the RHS of the equation above vanishes, then the Poynting theorem of equation 48 is retrieved without change in form. For the choice $\alpha = +1$, $\beta = -1$, again there are no differences between the Chiral Vacuum and the Lorentz Vacuum, for any value of the chiral factors. For the choice $\alpha = +1$, $\beta = +1$, the equation implies a chiral (imaginary or reactive) component to the Poynting equation, related to the time-like dispersion of the left handed and right handed helical waves. This term vanishes for $\gamma = 0$, and is independent from g.

The next step is to evaluate the expressions for the total field Hamiltonian energy density and the Lagrange density of the Chiral Vacuum. The expression for the Hamiltonian energy density becomes

$$Ham = 1/2\mathbf{B}^2/\mu + 1/2\varepsilon \mathbf{E}^2 + \{(\alpha - 1)g + \sqrt{-1}(\beta + 1)\gamma\}\mathbf{E} \circ \mathbf{B}/2$$

while the field Lagrangian is becomes:

$$Lag = 1/2\varepsilon \mathbf{E}^2 - 1/2\mathbf{B}^2/\mu + \{(\alpha + 1)g + \sqrt{-1}(1 - \beta)\gamma\}\mathbf{E} \circ \mathbf{B}/2$$

These results indicate that there are slight modifications to the energy density formulas, modifications that are dependent upon the second Poincare invariant. However, for systems where the field intensities are deducible from a 1-form of potentials, and the 1-form is of Pfaff dimension 3 or less, then $\mathbf{E} \circ \mathbf{B}$ vanishes, and all computations of Hamiltonian or Lagrangian energy densities are identical for the Lorentz Vacuum, or for the Chiral vacuum. It is only for cases where the 1-form of potentials is of Pfaff dimension 4, such that $\mathbf{E} \circ \mathbf{B} \neq 0$, that the Chiral factors can make a difference in the expressions for Hamiltonian or Lagrangian energy density.

Again study the case $\alpha = 1$. Then the choice $\beta = -1$, implies that the Hamiltonian energy density is the same as the Lorentz Vacuum, but the Lagrangian depends upon the chiral factors. The choice $\beta = +1$, implies that the Lagrangian depends upon the chiral factor g and the Hamiltonian depends upon the chiral factor γ. All chiral effects on the energy densities disappear if $F\hat{}F = -2(\mathbf{E} \circ \mathbf{B})dx\hat{}dy\hat{}dz\hat{}dt = 0$.

> **These are a rather startling results for they demonstrate that the Lorentz vacuum and the Chiral vacuum can be formally indistinguishable, except for the impedance of free space (which is related to the determinant of the constitutive tensor and therefor to the chiral coefficients).**

6. SUMMARY

From a topological and thermodynamic perspective of the electromagnetic field, there appears to be a common thread between Eikonal solutions, Spinors, propagating topological discontinuities or defects, minimal surfaces, and "topological quantization". All of these properties suggest that the common topological thread is that which is usually perceived as the Photon. A topological perspective of Electromagnetism not only includes features attributed to the Photon, but also points out that non equilibrium thermodynamic concepts can be formulated to produce interesting experiments and practical devices. For example, the fact the irreversible dissipation occurs when the field intensities have a collinear component ($\mathbf{E} \circ \mathbf{B} \neq 0$) could be used to influence condensation. Stable long lived states in a plasma should be designed about the (Klein quadric) constraint that ($\mathbf{E} \circ \mathbf{B} = 0$) which yields "stationary" non equilibrium dynamical systems, or excited states described by Hamiltonian processes. Each of these ideas involve the concepts of topological torsion and topological spin, and hence the quantal properties of the photon.

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