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A Topological Perspective of COSMOLOGY

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http://www.cartan.pair.com
http://www22.pair.com/csdc/pdf/cosmos.pdf

Topological Cosmology in 3 Parts

1 **Topological Thermodynamics** Space - Time as a turbulent non-equilibrium dilute gas near its critical point, of Pfaff topological dimension 4.

2 **Topological Evolution** to produce stars and galaxies as topological defects of Pfaff dimension 3.

3 **Topological Defects** Falaco solitons are examples of topological defects that resemble Cosmic strings in a swimming pool.

Topological Cosmology

Lev Landau 1958

- **A low density van der Waals gas near its critical point exhibits large fluctuations in density.**
- **The density fluctuations are correlated and indicate a force of attractive interaction equivalent to the law of Newtonian gravity.**

Topological Cosmology

Conjectures RMK 1965

- The Universe is a dilute gas or plasma near its critical point.
- The large fluctuations in density form topological defects of Pfaff dimension 3, called stars and galaxies.
- The density defects attract as $1/r^2$.

Topological Cosmology

Conjectures from Topological Thermodynamics RMK 2003

- **The Universe is a dilute TURBULENT gas or plasma, NOT in EQUILIBRIUM due to expansion, of Pfaff Topological Dimension 4, near its critical point.**
- **The large fluctuations in density are the equivalent to the creation, by irreversible topological evolution, of defects of Pfaff Topological Dimension 3. The condensation defects, are called stars and galaxies.**
- **The density defects attract as $1/r^2$.**

Topological Cosmology

From Topological Thermodynamics RMK 2003

- The thermodynamics of a non-equilibrium, turbulent, dissipative system of Pfaff Topological Dimension 4, explains

  - the granularity of the night sky,
  - the $1/r^2$ gravitation attraction,
  - the expansion of the Universe,

creating stars and galaxies by formation of topological defects as "stationary" states of Pfaff topological dimension 3, far from equilibrium (which requires Pfaff dimension 2).

Topological Cosmology

Based on Topological Thermodynamics RMK 2003

- **A Neat Theory, but WHY? Must show:**
- **That universal topological features of any 4 dimensional space-time system can be homeomorphic to a van der Waals gas.**
- **That irreversible topological evolution can create topological defects (stars) that are long lived stationary states far from Equilibrium.**

Part 1: Topological Thermodynamics

The objective is to show that

Universal topological features of a 4 dimensional space-time system exhibit homeomorphic properties equivalent to a

van der Waals gas.

Topological Thermodynamics

- **Topological Structure of Physical Systems** is encoded in an Action differential 1-form **A**.

- **Physical Processes** can be defined in terms of contravariant vector direction fields, **V**.

- **Continuous Topological Evolution** is encoded by Cartan's magic formula :

$$L_{(V)}A = i(V)dA + di(V)A$$

($L_{(V)}A$ is the Lie differential with respect to the direction field V acting on the 1-form A)

Topological Thermodynamics

- **Open Thermodynamic System:**
  Pfaff Topological Dimension 4
- **Closed Thermodynamic System**
  Pfaff Topological Dimension 3
- **Isolated-Equilibrium Thermodynamic System Pfaff Topological Dimension 2**
  (Pfaff Topological Dimension > 2 implies non-equilibrium

Topological Thermodynamics

- Consider **ANY** vector field V with a Jacobian matrix of rank (topological dimension) 4.
- The Cayley-Hamilton theorem creates a characteristic polynomial of 4th degree describing a non-equilibrium open system. The result is:

- **A Universal Thermodynamic Phase Function**

$$\Theta = \Psi^4 - M\,\Psi^3 + G\,\Psi^2 - A\,\Psi + K \Rightarrow 0$$

Topological Thermodynamics

- The Universal Topological Phase Function generated by a non-singular Jacobian matrix is holomorphic in the complex variable Ψ.
- THEN, from a theorem of Sophus Lie, The Universal Phase Function creates:

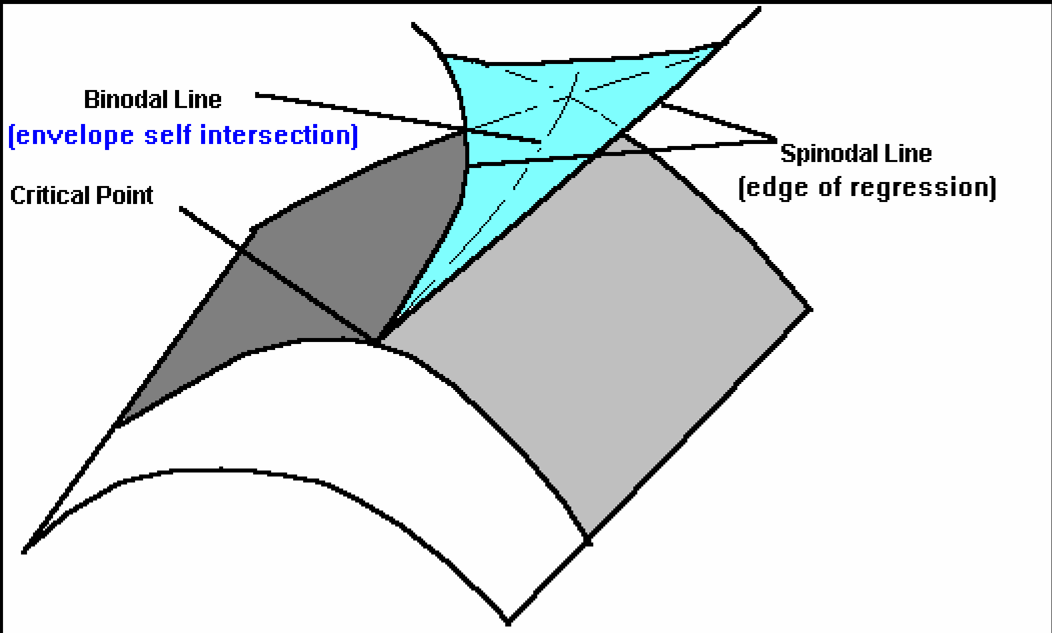
Conjugate Minimal Surfaces in 4D.

Topological Thermodynamics

- **The UNIVERSAL TOPOLOGICAL PHASE function supports an envelope, which when constrained to a minimal surface (M = 0), generates the Swallow-Tail bifurcation set. The result is that the UNIVERSAL minimal surface ENVELOPE is homeomorphic to the:**

Gibbs Surface of a van der Waals gas

Topological Thermodynamics



A Universal Thermodynamic Swallowtail

The Gibbs surface of a Van der Waals gas

Topological Thermodynamics

- The determinant of the Jacobian matrix is given by the similarity coefficient K.

$$K = \Psi^4 - M\Psi^3 + G\Psi^2 - A\Psi$$

- Every determinant can be related to the divergence of a current. Hence

$$\mathrm{div}J + \partial\rho/\partial t = \Psi(\Psi^3 - M\Psi^2 + G\Psi - A)$$

A Universal Landau Ginsburg format

Topological Thermodynamics

- If K = 0, the Jacobian matrix is singular.
  - The Pfaff topological dimension ⇒ 3.

$$K = \Psi(\Psi^3 - M\Psi^2 + G\Psi - A) \Rightarrow 0$$

- The singular UNIVERSAL Phase function becomes homeomorphic to the cubic equation of state for a

Universal van der Waals gas!

Conclusions: Part 1

- **If the Universe is of Topological Dimension 4, it will exhibit the features of a**

non-equilibrium, turbulent

van der Waals Gas

undergoing irreversible topological evolution to produce stars and galaxies.

Conclusions: Part 1

- **The thermodynamics of a non-equilibrium, turbulent, dissipative system of Pfaff Topological Dimension 4, explains,**
  - **the granularity of the night sky,**
  - **the $1/r^2$ gravitational attraction,**
  - **the expansion of the Universe,**

creating stars and galaxies by formation of topological defects as "stationary" states of Pfaff topological dimension 3, far from equilibrium (which requires Pfaff dimension 2).

COSMOLOGICAL notes

- **It is important to realize that cosmology from a topological perspective does not depend upon metric, connection, scales, equilibrium statistics, gauge symmetries, or quantum hypotheses !!!**

- **WARNING**

- **The failure of a symmetric METRIC approach**

A symmetric METRIC matrix has no complex eigenvalues.   Such matrices can not represent a non-equilibrium thermodynamic system.

Part 2: Topological Evolution

The objective is to show that

Continuous irreversible topological evolution can create topological defects (stars) from a dissipative turbulent background. These topological defects are long lived

Stationary States far from Equilibrium.

Irreversible Decay from 4D to Topological Defects in 3D

Continuous Topological Evolution can describe the irreversible evolution on an

“Open” symplectic domain of Pfaff dimension **4**, with evolutionary orbits being irreversibly attracted to a

“Closed” contact domain of Pfaff dimension **3**, with topological defects (stationary states), and a possible ultimate decay to the

“Isolated-Equilibrium” domain of Pfaff dimension **2** or less (integrable Caratheodory surface).

Basic Ideas of Topology Evolution

- Properties that are independent from size and shape are topological properties. Such objects are deformation invariants.
- Topological evolution can take place continuously or discontinuously.
- Cutting is a discontinuous process. Pasting is a continuous process.

Simple Topological Examples

- A condensed fluid is a thermodynamic phase with a connected components.
- A vapor is a thermodynamic phase with disconnected components.
- A change of phase implies a change of topology. Condensation is continuous. Vaporization is discontinuous.

Pfaff Topological Dimension

A physical system represented by a 1-form of Action, A, has a minimum number of functions required for its topological definition. This number,

PTD = Pfaff Topological Dimension,

is equal or less than the geometrical dimension N of the domain of support. The PTD is also equal to the number of non-zero terms in the

Pfaff Sequence = {A,dA,A^dA,dA^dA…}.

Subspaces of lesser PTD form coherent topological structures, defects, or thermodynamic phases.

Pfaff Topological Dimension

A physical system for which the 1-forms of Action, A, of Work, W, and Heat, Q, are all of PTD = 4 defines a dissipative, non-equilibrium turbulent system.

The Jacobian matrix of A has a characteristic polynomial with 4 non-zero roots. As shown in Part 1, the universal phase functions is homeomorphic to a

van der Waals gas.

Connectivity and the Arrow of time

Regions where PTD ≤ 2 generate a connected topology;

PTD ≥ 3 generate a disconnected topology.

Continuous Processes can represent the evolution from a disconnected topology (≥ 3) to a connected topology (≤ 2) .

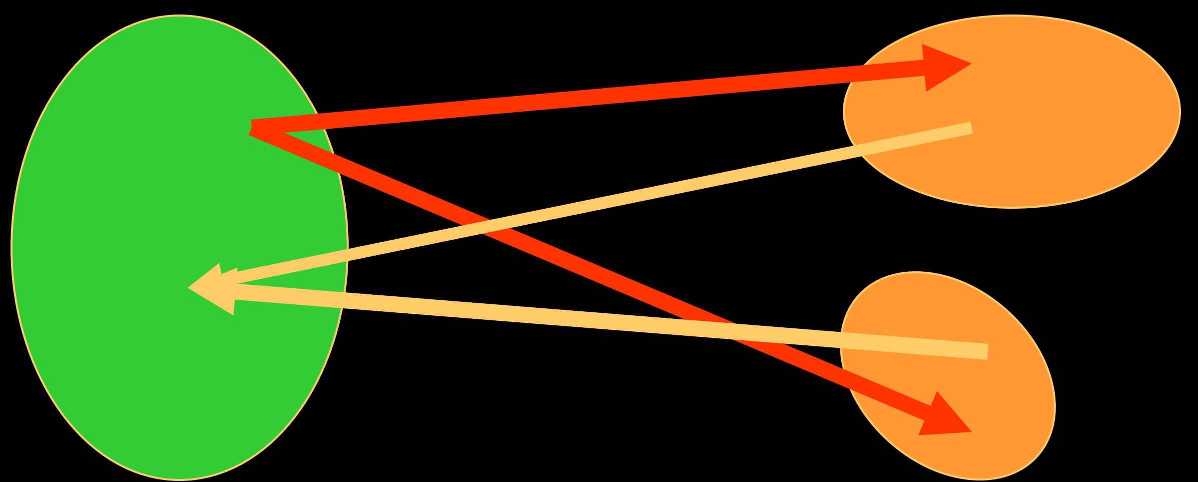
Continuous Processes can NOT represent the evolution from a connected topology (≤ 2) to a disconnected topology (≥ 3) .

(An Arrow of Time.)

You can describe the decay of turbulence continuously, but NOT the creation of turbulence.

Arrow of Time and Turbulence

Creation of Turbulence is a Discontinuous Process



Decay of Turbulence is a Continuous Process

Streamline Flow

Connected Topology $D_{pfaff} \leq \mathbf{2}$

Turbulent Flow

Disconnected Topology $D_{pfaff} > \mathbf{3}$

Emphasis on

Continuous Topological Evolution

- Continuous Evolution can change Topology.
- An arrow of time and thermodynamic irreversibility require **Topological Change.**

- **Exterior Differential Forms,** unlike tensors, are functionally well behaved - with respect to those C1 maps which are neither diffeomorphisms nor homeomorphisms.

- Hence Cartan's methods can be used to describe **Continuous Topological Evolution.**

Objectives of CTE

(Continuous Topological Evolution)

- Establish the long sought for connection between **Irreversible Thermodynamics and Dynamical Systems -- without Statistics!**
- Demonstrate the connection between **Thermodynamic Irreversibility and Topological (Pfaff) Dimension 4.**

Theorems of CTE

- **Topological evolution** is a necessary condition for both time asymmetry and thermodynamic irreversibility
- **A unique extremal direction field** which represents a conservative reversible Hamiltonian process always exists on subspaces of topological dimension **2n+1**.
- **A unique torsion direction field** which represents a thermodynamically irreversible process always exists on subspaces of even topological dimension **2n+2**.

Cartan's Magic Formula

Define the exterior differential forms:

Work = W= i(V)dA, Energy = U= i(V)A, Heat = Q.

Then Cartan's Magic Formula of CTE,

$$L_{(V)}A \;=\; i(V)dA \;+\; di(V)A \;=\; Q,$$

becomes the First Law of Thermodynamics,

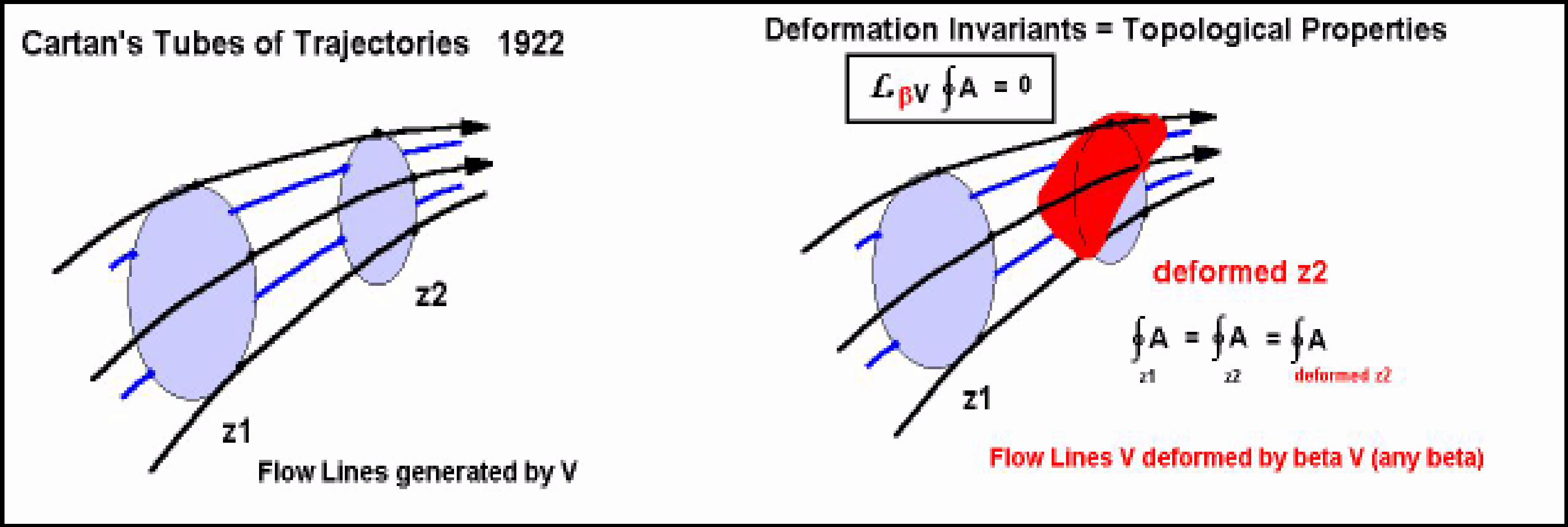
$$L_{(V)}A = W + dU = Q,$$

connecting Dynamical Systems and Thermodynamics.

($L_{(V)}A$ is the Lie differential with respect to the direction field V acting on the 1-form A)

Cartan's Magic Formula

Cartan used an integral version of his Lie derivative formula - in terms of hydrodynamic flow along a tube of trajectories - to prove that all Hamiltonian processes preserve the closed integrals of Action, and conversely.



Thermodynamic Processes page 1

- **Classical Thermodynamics:**

  A Process acting on a Physical System that creates a 1-form of Heat Q, is an irreversible process unless Q admits an integrating factor.

- **Frobenius Theorem:**

  An integrating factor exists iff $Q \wedge dQ = 0$.
  (Pfaff dimension of $Q < 3$.)

Thermodynamic Processes page 2

For a Process V acting on a Physical System represented by a 1-form of Action, A:

A Dynamical Test for a Reversible Process is

$$L_{(V)}A \wedge L_{(V)}\, dA = Q \wedge dQ = 0.$$

A Dynamical Test for an Irreversible Process is

$$L_{(V)}A \wedge L_{(V)}\, dA = Q \wedge dQ \neq 0.$$

($Q \wedge dQ = 0$ implies a Pfaff dimension of <3; $Q \wedge dQ \neq 0$ implies a Pfaff dimension of 3 or more.)

Thermodynamic Processes page 3

All classic Hamiltonian, Symplectic, Bernoulli and Stokes Processes, satisfy the Helmholtz-Poincare constraint (“conservation of vorticity”).

$$L_{(V)}dA = dQ = 0.$$

and are therefore

Thermodynamically Reversible.

as

$$L_{(V)}A \wedge L_{(V)}\, dA = Q \wedge dQ = 0.$$

Thermodynamic Processes

When $dQ = 0$, then $dW = 0$, and the first law becomes a precise statement of deRham cohomology:

The difference between the closed but not exact 1-form of heat, Q, and the closed but not exact 1-form of work, W, is an exact differential of the internal energy, dU.

$$Q - W = dU$$

Thermodynamics is a Topological Theory !

Thermodynamic Processes page 5

R. Bott "..deRham cohomology ignores **Torsion**."

deRham cohomology is Thermodynamically **REVERSIBLE**.

TORSION is the source of Thermodynamic IRREVERSIBILITY

Riemannian metric Geometry is Torsion Free, and CANNOT describe IRREVERSIBILITY.

Contact Manifolds, n = 2k+1 .

On subspaces of Pfaff dimension $n = 2k + 1 \leq m$, called contact manifolds, the Principle of Least Action implies that the evolution obeys the Helmholtz-Poincare constraint ("conservation of vorticity")

$$L_{(V)}dA = dQ = 0.$$

and all such evolutionary processes are therefore

Thermodynamically Reversible.

A unique direction field, **V**, completely determined by the topological features of the Action, A, of odd Pfaff dimension, such that $W = i(V)dA=0$ is called the **Extremal Field,** and if $U = i(V)A = 1$, a **Reeb** field.

Symplectic Manifolds n = 2k+2.

On subspaces of Pfaff dimension $n = 2k + 2 \leq m$, called symplectic manifolds, extremal fields do not exist. However, a unique direction field T can be defined in terms of the topological features of the physical system, A:

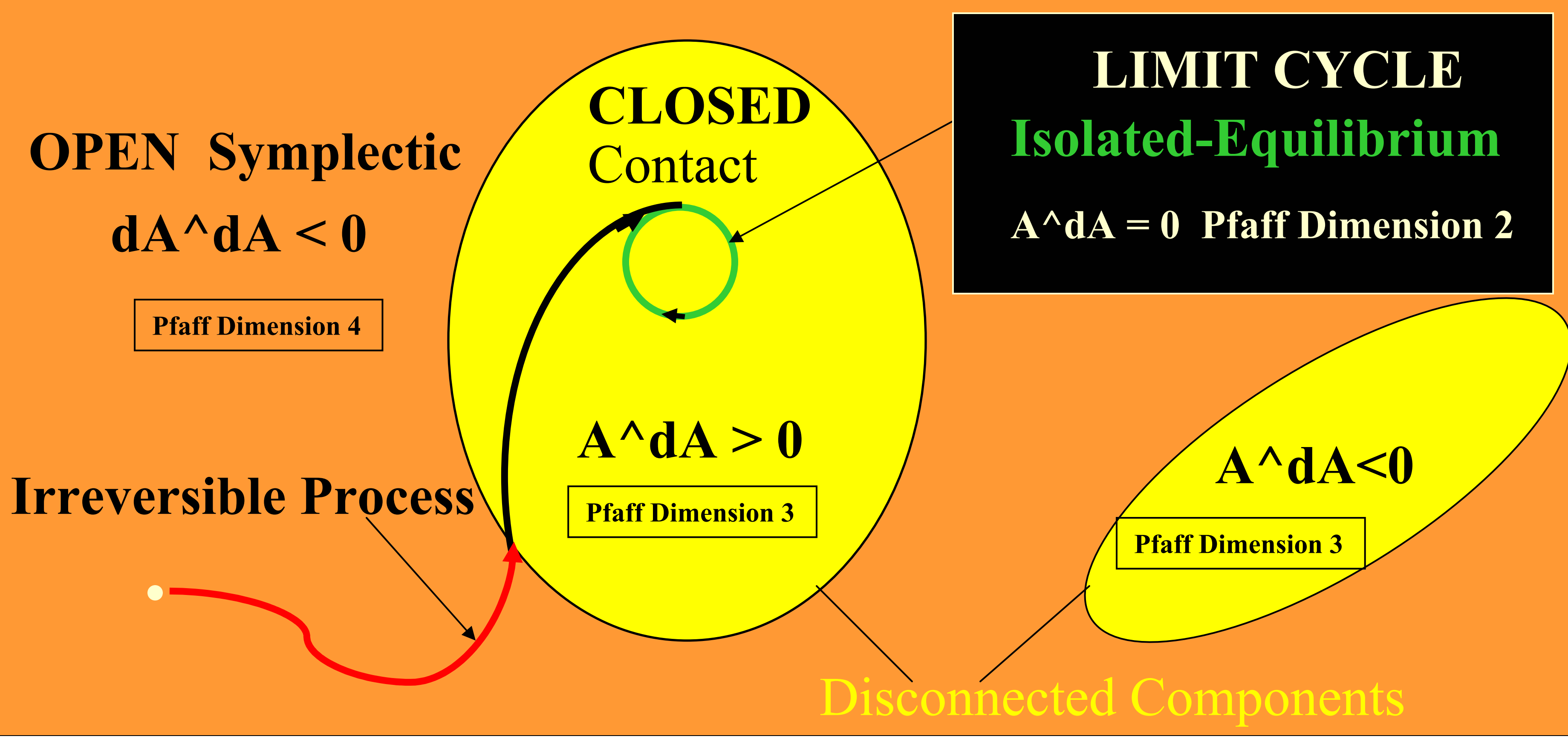
$$i(T)\{dx\wedge dy\wedge dz\wedge dt\}=A\wedge dA.$$

Processes in the direction of the Torsion Vector, T, are Thermodynamically Irreversible, as

$$L_{(T)} A\wedge L_{(T)} dA = Q\wedge dQ = \Gamma^2 A\wedge dA \neq 0.$$

Irreversible Decay on a Symplectic Manifold (PTD=4) to a Contact Manifold (PTD=3) of disconnected components, then possibly to an Isolated-Equilibrium (PTD = 2) State.

Turbulent Non-Equilibrium Pfaff dimension 4



Lagrangian Example

Evolution starts on the 2k+2 symplectic manifold with orbits being attracted to 2k+1 domains where the momenta become canonical: $p - \partial L/\partial v \Rightarrow 0$.

- Topological evolution can either continue to reduce the Pfaff topological dimension, or
- the process on the Contact 2k+1 manifold can become "extremal", and the topological change stops.

The resulting contact manifold becomes a "stationary" non-dissipating Hamiltonian state,

"Far from Equilibrium".

The Sliding - Rolling Ball page 1

Consider a bowling ball with initial translational and rotational energy, thrown to the floor of the bowling alley.

Initially the ball skids or slips on a 2k + 2 symplectic manifold irreversibly reducing its energy and angular momentum via “friction” forces.

From arbitrary initial conditions, the evolution is attracted to a 2k + 1 contact manifold, where the ball rolls without slipping, and the anholonomic constraint vanishes.

$$dx - \lambda \, d\Theta = 0$$

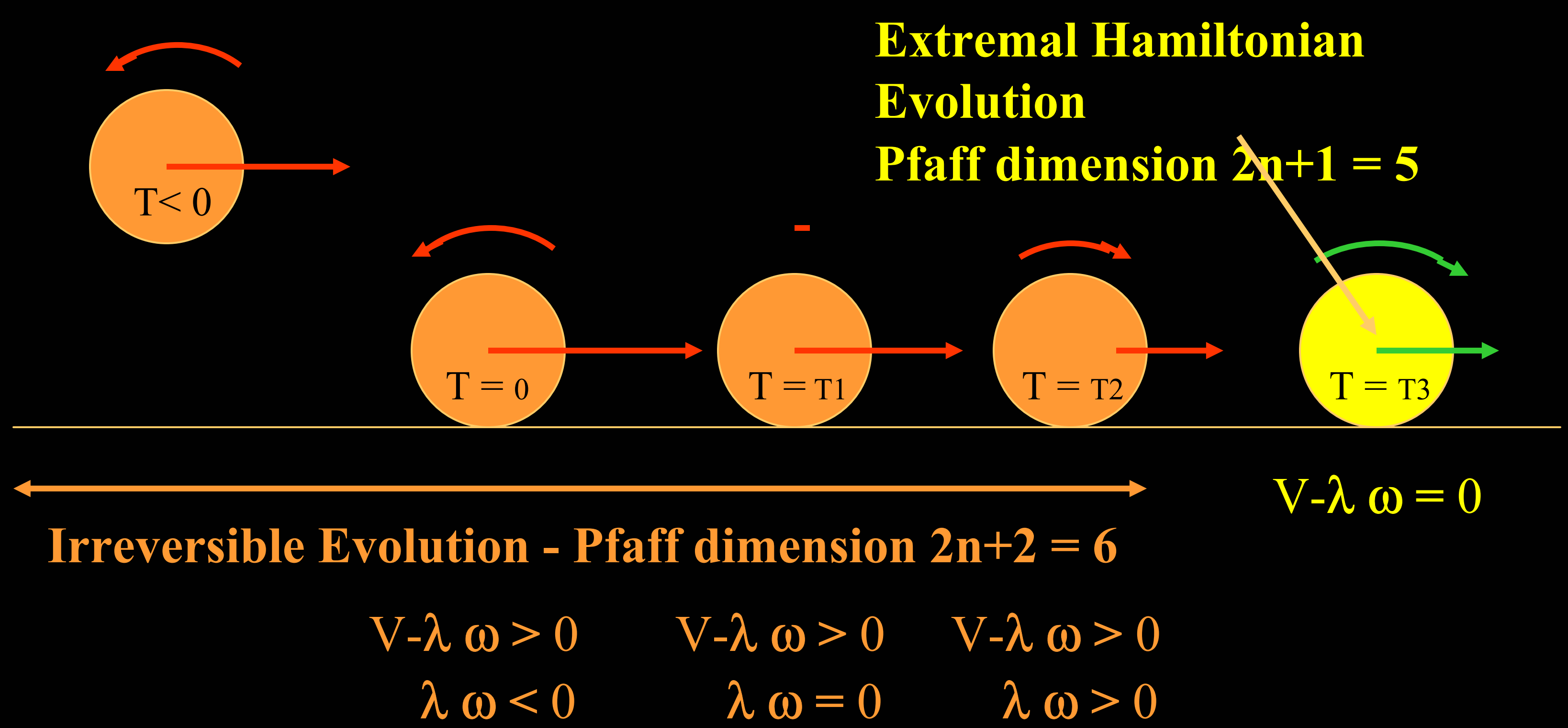
The Sliding - Rolling Ball page 2

The subsequent motion, neglecting air resistance, continues in a Hamiltonian manner without change of Kinetic Energy or Angular Momentum.

The 1-form of Action can be written as:

$$A = L(t,x,\Theta,v,\omega)dt + \ldots + s \bullet (dx - \lambda\, d\Theta)$$

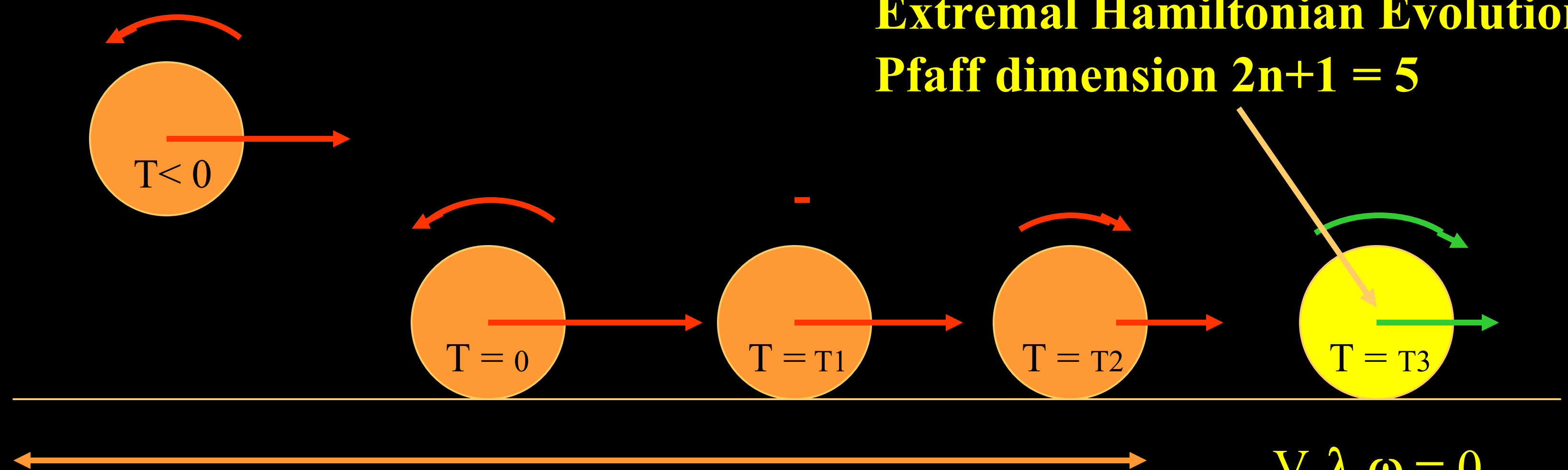
The Sliding - Rolling Ball page 3



Note how friction changes Angular Momentum

The Sliding - Rolling Ball page 3

Extremal Hamiltonian Evolution
Pfaff dimension 2n+1 = 5



$V\text{-}\lambda\,\omega = 0$

Irreversible Evolution - Pfaff dimension 2n+2 = 6

$V\text{-}\lambda\,\omega > 0$	$V\text{-}\lambda\,\omega > 0$	$V\text{-}\lambda\,\omega > 0$
$\lambda\,\omega < 0$	$\lambda\,\omega = 0$	$\lambda\,\omega > 0$

Note how friction changes Angular Momentum

Summary

- Without Topological Evolution, there is no Arrow of Time and no Thermodynamic Irreversibility.
- Physical Systems of Pfaff dimension 4 generate a unique continuous evolutionary process which is thermodynamically irreversible.
- Cartan's Magic formula combines continuous topological evolution and thermodynamics

Part 3 Falaco Solitons

RMK 1986

Are

- **Falaco Solitons**

(created in a Swimming Pool)

related to

Cosmological Strings and Topological Defects

Falaco Solitons 1986

Topological Defects in a swimming pool

History of Falaco Solitons

- 1986 visit to Rio de Janeiro and the mountain side house of my MIT roommate, Jose Haraldo H. Falcao.

- On the mountain side above the beach

History of Falaco Solitons

- 1986 visit to Rio de Janeiro at the house of my MIT roommate, Jose Haraldo H. Falcao.

- To the swimming pool

Topological Defects in a swimming pool

Optical Properties of Falaco Solitons

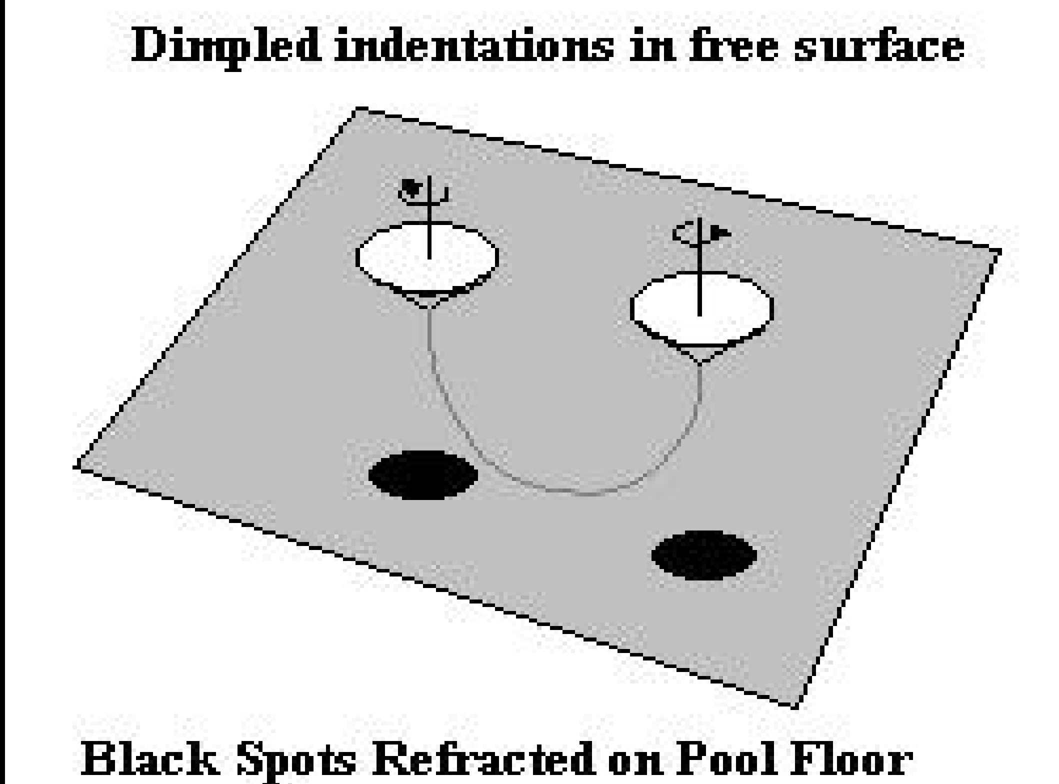


Figure 2. The Falaco Soliton

Optical Properties of Falaco Solitons

- Snell Refraction from a rotational vertex.

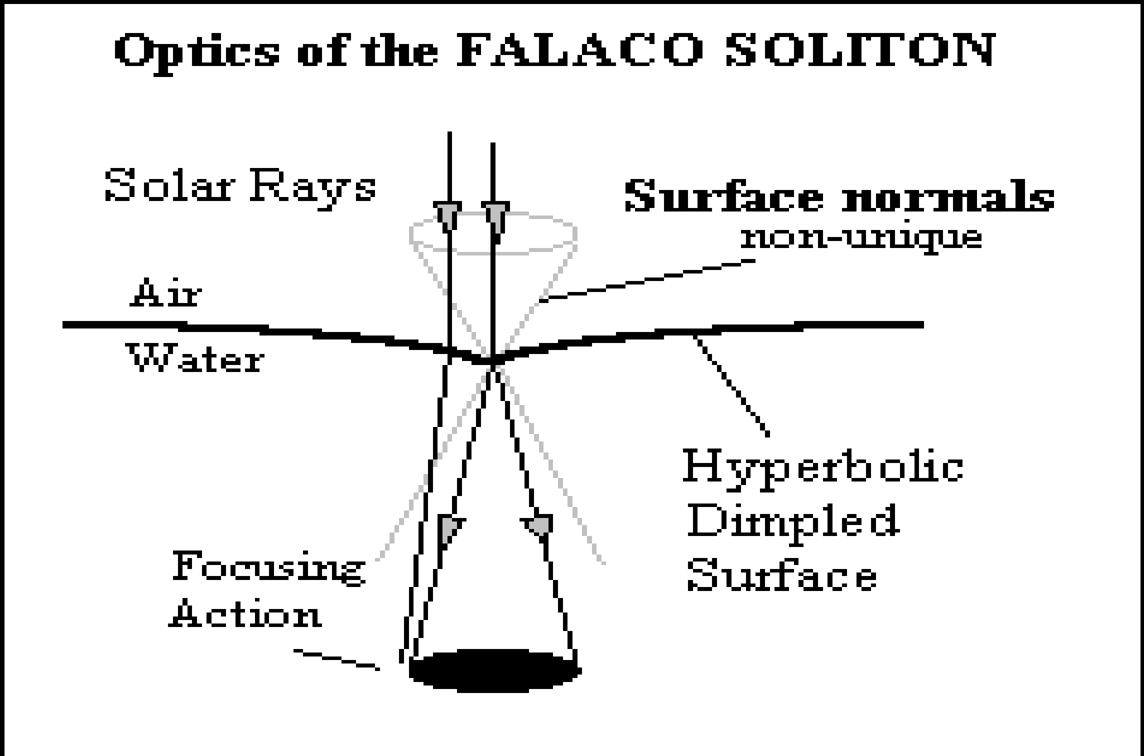
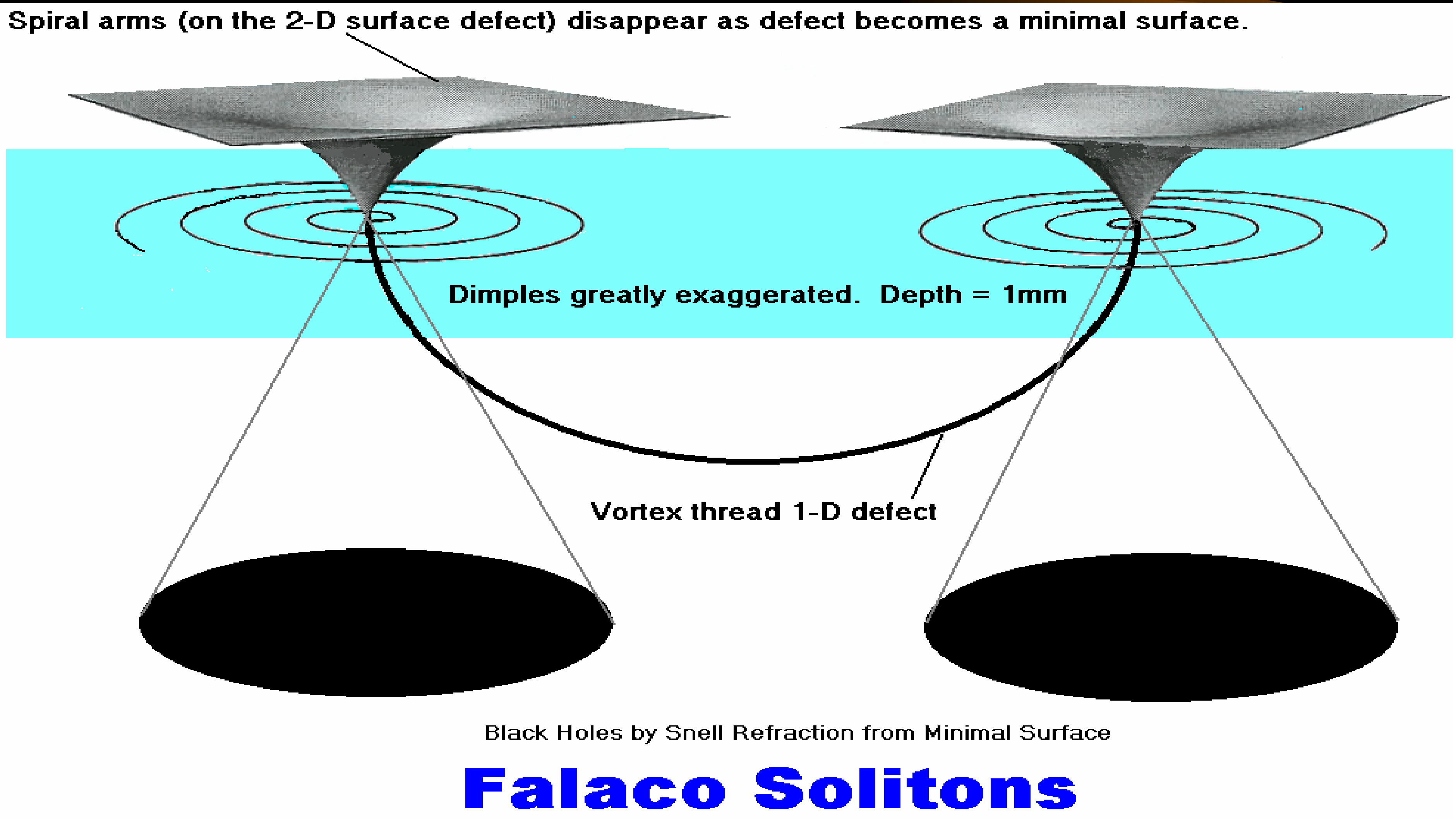


Figure 3. The Snell Black Disk

CGL theory applied to Falaco Solitons



Adapted from O. Tornkvist and E. Schroeder, PRL, 78, 10 1997 p.1980

Topology of Falaco Solitons

String (vortex) defect related to GPL theory.

Spiral arms obvious during formation phase indicate end caps are related to CGL theory.

Stability and confinement of 2D topological defect end caps produced by 1D defect string.

Topology of Falaco Solitons

- The topological defects and CGL analysis do not depend upon a Low Temperature !!!

- The Falaco Soliton is a long lived state far from equilibrium (Pfaff Dim=3), produced in a dissipative media by irreversible processes, (Pfaff Dim=4).

Motivation from Falaco Solitons

- **Falaco Solitons** are universal topological defects that can be created easily at a macroscopic level - without low temperature.
- **Falaco Solitons** mimic microscopic quarks with a confinement problem.
- **Falaco Solitons** mimic cosmological strings on a space-time manifold.

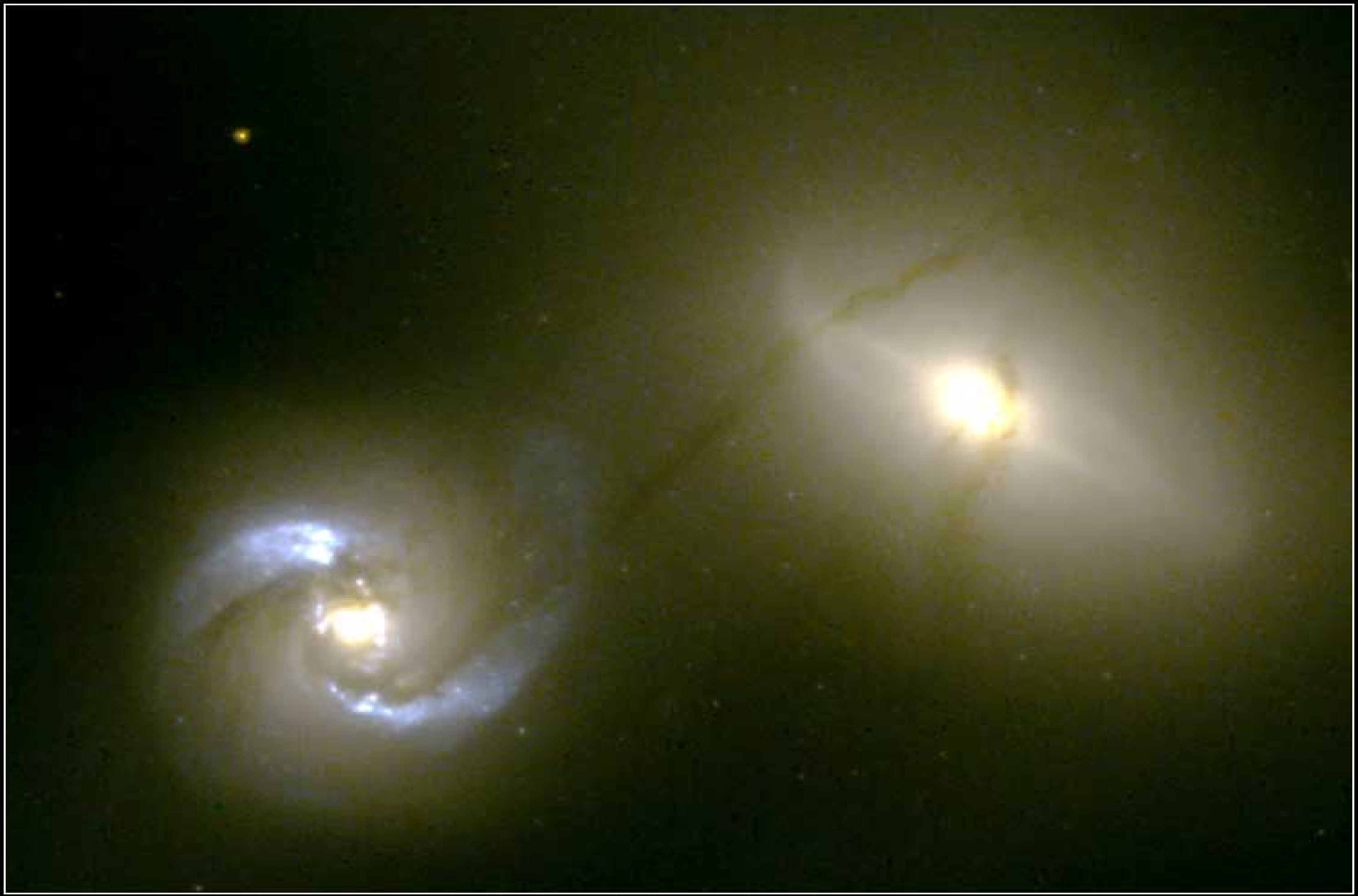
Topological Cosmology

Conjectures 1987 - 2003

- **Are the thin spiral arm galaxies formed on discontinuity layers, as are the endcaps of the Falaco solitons?**

- **Are the spiral arm galaxies M31 and the Milky Way confined by a singular, globally stabilizing thread connecting the dimpled vertex points of the galactic cores - as is observed in the Falaco Solitons?**

Cosmic Strings from Hubble ?



Interacting Galaxies NGC 1409 and NGC 1410 **HST** • WFPC2

NASA and W. Keel (University of Alabama) • STScI-PRC01-02